**🔍 Key Components of TD-VAE**

TD-VAE learns:

• **Belief states**: deterministic codes summarizing past observations (like filtering posteriors in POMDPs).

• **Latent state transitions**: stochastic transitions between hidden states.

• **Jumpy transitions**: directly modeling transitions from time t1 to t2 > t1 without traversing all intermediate steps.

• **Observation model**: reconstructs observations from latent states.

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**📐 Model Structure**

Given a pair of timepoints t1 and t2:

1. Compute belief states bt1, bt2 from observations up to t1, t2.

2. Sample a future latent state zt2 from bt2.

3. Use a smoothing network to infer what the earlier latent state zt1 might have been, given zt2.

4. Predict zt2 from zt1 via a jumpy transition model.

5. Ground the state in observation: reconstruct xt2 from zt2.

The training loss is a **belief-state-based ELBO** designed to:

• Encourage accurate prediction and reconstruction,

• Keep belief states coherent across time,

• Allow smooth gradient updates across temporally distant steps (via temporal difference).