Национальный исследовательский университет ИТМО
Факультет программной инженерии и компьютерной техники
Дисциплина «Дискретная математика»

Курсовая работа
Часть 1
Вариант 116

Студент
Казарин Андрей Максимович
P3108

Поверил
Поляков Владимир Иванович

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Функция $f(x_1, x_2, x_3, x_4, x_5)$ принимает значение 1 при $-2 \leq x_4x_5 - x_1x_2x_3 < 1$ и неопределенное значение при $x_4x_5 - x_1x_2x_3 = -5$

## Таблица истинности

| № | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_4x_5$ | $x_1x_2x_3$ | $x_4x_5$ | $x_1x_2x_3$ | f |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 2 | 0 | 2 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 1 | 3 | 0 | 3 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 5 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 0 | 1 | 1 | 0 | 2 | 1 | 2 | 1 | 0 |
| 7 | 0 | 0 | 1 | 1 | 1 | 3 | 1 | 3 | 1 | 0 |
| 8 | 0 | 1 | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 1 |
| 9 | 0 | 1 | 0 | 0 | 1 | 1 | 2 | 1 | 2 | 1 |
| 10 | 0 | 1 | 0 | 1 | 0 | 2 | 2 | 2 | 2 | 1 |
| 11 | 0 | 1 | 0 | 1 | 1 | 3 | 2 | 3 | 2 | 0 |
| 12 | 0 | 1 | 1 | 0 | 0 | 0 | 3 | 0 | 3 | 0 |
| 13 | 0 | 1 | 1 | 0 | 1 | 1 | 3 | 1 | 3 | 1 |
| 14 | 0 | 1 | 1 | 1 | 0 | 2 | 3 | 2 | 3 | 1 |
| 15 | 0 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 1 |
| 16 | 1 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 4 | 0 |
| 17 | 1 | 0 | 0 | 0 | 1 | 1 | 4 | 1 | 4 | 0 |
| 18 | 1 | 0 | 0 | 1 | 0 | 2 | 4 | 2 | 4 | 1 |
| 19 | 1 | 0 | 0 | 1 | 1 | 3 | 4 | 3 | 4 | 1 |
| 20 | 1 | 0 | 1 | 0 | 0 | 0 | 5 | 0 | 5 | d |
| 21 | 1 | 0 | 1 | 0 | 1 | 1 | 5 | 1 | 5 | 0 |
| 22 | 1 | 0 | 1 | 1 | 0 | 2 | 5 | 2 | 5 | 0 |
| 23 | 1 | 0 | 1 | 1 | 1 | 3 | 5 | 3 | 5 | 1 |
| 24 | 1 | 1 | 0 | 0 | 0 | 0 | 6 | 0 | 6 | 0 |
| 25 | 1 | 1 | 0 | 0 | 1 | 1 | 6 | 1 | 6 | d |
| 26 | 1 | 1 | 0 | 1 | 0 | 2 | 6 | 2 | 6 | 0 |
| 27 | 1 | 1 | 0 | 1 | 1 | 3 | 6 | 3 | 6 | 0 |
| 28 | 1 | 1 | 1 | 0 | 0 | 0 | 7 | 0 | 7 | 0 |
| 29 | 1 | 1 | 1 | 0 | 1 | 1 | 7 | 1 | 7 | 0 |
| 30 | 1 | 1 | 1 | 1 | 0 | 2 | 7 | 2 | 7 | d |
| 31 | 1 | 1 | 1 | 1 | 1 | 3 | 7 | 3 | 7 | 0 |

## Аналитический вид

### Каноническая ДНФ:

$$f = \overline{x_1}\,\overline{x_2}\,\overline{x_3}\,\overline{x_4}\,\overline{x_5} \vee \overline{x_1}\,\overline{x_2}\,x_3\,\overline{x_4}\,\overline{x_5} \vee \overline{x_1}\,\overline{x_2}\,x_3\,\overline{x_4}\,x_5 \vee \overline{x_1}\,x_2\,\overline{x_3}\,\overline{x_4}\,\overline{x_5} \vee \overline{x_1}\,x_2\,\overline{x_3}\,\overline{x_4}\,x_5 \vee \overline{x_1}\,x_2\,\overline{x_3}\,x_4\,\overline{x_5} \vee$$
$$\vee \overline{x_1}\,x_2\,x_3\,\overline{x_4}\,x_5 \vee \overline{x_1}\,x_2\,x_3\,x_4\,\overline{x_5} \vee \overline{x_1}\,x_2\,x_3\,x_4\,x_5 \vee x_1\,\overline{x_2}\,\overline{x_3}\,x_4\,\overline{x_5} \vee x_1\,\overline{x_2}\,\overline{x_3}\,x_4\,x_5 \vee x_1\,\overline{x_2}\,x_3\,x_4\,x_5$$

### Каноническая КНФ:

$$f = (x_1 \vee x_2 \vee x_3 \vee x_4 \vee \overline{x_5})\,(x_1 \vee x_2 \vee x_3 \vee \overline{x_4} \vee x_5)\,(x_1 \vee x_2 \vee x_3 \vee \overline{x_4} \vee \overline{x_5})\,(x_1 \vee x_2 \vee \overline{x_3} \vee \overline{x_4} \vee x_5)$$
$$(x_1 \vee x_2 \vee \overline{x_3} \vee \overline{x_4} \vee \overline{x_5})\,(x_1 \vee \overline{x_2} \vee x_3 \vee \overline{x_4} \vee \overline{x_5})\,(x_1 \vee \overline{x_2} \vee \overline{x_3} \vee x_4 \vee x_5)\,(\overline{x_1} \vee x_2 \vee x_3 \vee x_4 \vee x_5)$$
$$(\overline{x_1} \vee x_2 \vee x_3 \vee x_4 \vee \overline{x_5})\,(\overline{x_1} \vee x_2 \vee \overline{x_3} \vee x_4 \vee \overline{x_5})\,(\overline{x_1} \vee x_2 \vee \overline{x_3} \vee \overline{x_4} \vee x_5)\,(\overline{x_1} \vee \overline{x_2} \vee x_3 \vee x_4 \vee x_5)$$
$$(\overline{x_1} \vee \overline{x_2} \vee x_3 \vee \overline{x_4} \vee x_5)\,(\overline{x_1} \vee \overline{x_2} \vee x_3 \vee \overline{x_4} \vee \overline{x_5})\,(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3} \vee x_4 \vee x_5)\,(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3} \vee x_4 \vee \overline{x_5})$$
$$(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3} \vee \overline{x_4} \vee \overline{x_5})$$

# Минимизация булевой функции методом Квайна–Мак-Класки

## Кубы различной размерности и простые импликанты

| $K^0(f)$ | | |
|---|---|---|
| $m_0$ | 00000 | ✓ |
| $m_4$ | 00100 | ✓ |
| $m_8$ | 01000 | ✓ |
| $m_5$ | 00101 | ✓ |
| $m_9$ | 01001 | ✓ |
| $m_{10}$ | 01010 | ✓ |
| $m_{18}$ | 10010 | ✓ |
| $m_{20}$ | 10100 | ✓ |
| $m_{13}$ | 01101 | ✓ |
| $m_{14}$ | 01110 | ✓ |
| $m_{19}$ | 10011 | ✓ |
| $m_{25}$ | 11001 | ✓ |
| $m_{15}$ | 01111 | ✓ |
| $m_{23}$ | 10111 | ✓ |
| $m_{30}$ | 11110 | ✓ |

| $K^1(f)$ | |
|---|---|
| $m_0$-$m_4$ | 00X00 |
| $m_0$-$m_8$ | 0X000 |
| $m_4$-$m_5$ | 0010X |
| $m_8$-$m_9$ | 0100X |
| $m_8$-$m_{10}$ | 010X0 |
| $m_4$-$m_{20}$ | X0100 |
| $m_9$-$m_{13}$ | 01X01 |
| $m_{10}$-$m_{14}$ | 01X10 |
| $m_5$-$m_{13}$ | 0X101 |
| $m_{18}$-$m_{19}$ | 1001X |
| $m_9$-$m_{25}$ | X1001 |
| $m_{14}$-$m_{15}$ | 0111X |
| $m_{13}$-$m_{15}$ | 011X1 |
| $m_{19}$-$m_{23}$ | 10X11 |
| $m_{14}$-$m_{30}$ | X1110 |

| $Z(f)$ |
|---|
| 00X00 |
| 0X000 |
| 0010X |
| 0100X |
| 010X0 |
| X0100 |
| 01X01 |
| 01X10 |
| 0X101 |
| 1001X |
| X1001 |
| 0111X |
| 011X1 |
| 10X11 |
| X1110 |

### Таблица импликант

Вычеркнем строки, соответствующие существенным импликантам, а также столбцы, соответствующие вершинам, покрываемым существенными импликантами. Затем вычеркнем импликанты, не покрывающие ни одной вершины.

| Простые импликанты | | 0-кубы | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| | | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| | | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| | | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| | | 0 | 4 | 5 | 8 | 9 | 10 | 13 | 14 | 15 | ~~18~~ | ~~19~~ | ~~23~~ |
| A | 00X00 | X | X | | | | | | | | | | |
| B | 0X000 | X | | | X | | | | | | | | |
| C | 0010X | | X | X | | | | | | | | | |
| D | 0100X | | | | X | X | | | | | | | |
| E | 010X0 | | | | X | | X | | | | | | |
| F | X0100 | | X | | | | | | | | | | |
| G | 01X01 | | | | | X | | X | | | | | |
| H | 01X10 | | | | | | X | | X | | | | |
| I | 0X101 | | | X | | | | X | | | | | |
| | ~~1001X~~ | | | | | | | | | | ~~X~~ | ~~X~~ | |
| J | X1001 | | | | | X | | | | | | | |
| K | 0111X | | | | | | | | X | X | | | |
| L | 011X1 | | | | | | | X | | X | | | |
| | ~~10X11~~ | | | | | | | | | | | ~~X~~ | ~~X~~ |
| M | X1110 | | | | | | | | X | | | | |

Ядро покрытия:

$$T = \begin{Bmatrix} 1001X \\ 10X11 \end{Bmatrix}$$

Получим следующую упрощенную импликантную таблицу:

<table>
<tr><td colspan="2" rowspan="3">Простые импликанты</td><td colspan="9">0-кубы</td></tr>
<tr><td>0<br>0<br>0<br>0<br>0</td><td>0<br>0<br>1<br>0<br>0</td><td>0<br>0<br>1<br>0<br>1</td><td>0<br>1<br>0<br>0<br>0</td><td>0<br>1<br>0<br>0<br>1</td><td>0<br>1<br>0<br>1<br>0</td><td>0<br>1<br>1<br>0<br>1</td><td>0<br>1<br>1<br>1<br>0</td><td>0<br>1<br>1<br>1<br>1</td></tr>
<tr><td>0</td><td>4</td><td>5</td><td>8</td><td>9</td><td>10</td><td>13</td><td>14</td><td>15</td></tr>
<tr><td>A</td><td>00X00</td><td>X</td><td>X</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>0X000</td><td>X</td><td></td><td></td><td>X</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>C</td><td>0010X</td><td></td><td>X</td><td>X</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>D</td><td>0100X</td><td></td><td></td><td></td><td>X</td><td>X</td><td></td><td></td><td></td><td></td></tr>
<tr><td>E</td><td>010X0</td><td></td><td></td><td></td><td>X</td><td></td><td>X</td><td></td><td></td><td></td></tr>
<tr><td>F</td><td>X0100</td><td></td><td>X</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>G</td><td>01X01</td><td></td><td></td><td></td><td></td><td>X</td><td></td><td>X</td><td></td><td></td></tr>
<tr><td>H</td><td>01X10</td><td></td><td></td><td></td><td></td><td></td><td>X</td><td></td><td>X</td><td></td></tr>
<tr><td>I</td><td>0X101</td><td></td><td></td><td>X</td><td></td><td></td><td></td><td>X</td><td></td><td></td></tr>
<tr><td>J</td><td>X1001</td><td></td><td></td><td></td><td></td><td>X</td><td></td><td></td><td></td><td></td></tr>
<tr><td>K</td><td>0111X</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>X</td><td>X</td></tr>
<tr><td>L</td><td>011X1</td><td></td><td></td><td></td><td></td><td></td><td></td><td>X</td><td></td><td>X</td></tr>
<tr><td>M</td><td>X1110</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>X</td><td></td></tr>
</table>

## Метод Петрика

Запишем булево выражение, определяющее условие покрытия всех вершин:

$$Y = (A \vee B)\,(A \vee C \vee F)\,(C \vee I)\,(B \vee D \vee E)\,(D \vee G \vee J)\,(E \vee H)\,(G \vee I \vee L)\,(H \vee K \vee M)\,(K \vee L)$$

Приведем выражение в ДНФ:

$$Y = ACDHL \vee ACEGK \vee ADEIK \vee ADHIK \vee ADHIL \vee AEGIK \vee AEIJK \vee BCDHL \vee BCEGK \vee BCGHK \vee BCGHL \vee BCHJL \vee ABGHIK \vee ABGHIL \vee ABHIJK \vee ABHIJL \vee ACDEKL \vee ACDELM \vee ACDGHK \vee ACEGHL \vee ACEGLM \vee ACEHJL \vee ACEJKL \vee ACEJLM \vee ADEILM \vee AEGHIL \vee AEGILM \vee AEHIJL \vee AEIJLM \vee BCDEIK \vee BCDEKL \vee BCDELM \vee BCDHIK \vee BCEGLM \vee BCEIJK \vee BCEJKL \vee BCEJLM \vee BCHIJK \vee BDEFIK \vee BDFHIK \vee BDFHIL \vee BEFGIK \vee BEFIJK \vee BFGHIK \vee BFGHIL \vee BFHIJK \vee BFHIJL \vee BDEFILM \vee BEFGILM \vee BEFIJLM$$

Возможны следующие покрытия:

$$C_1 = \left\{\begin{matrix} T \\ A \\ C \\ D \\ H \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 0100X \\ 01X10 \\ 011X1 \end{matrix}\right\} \qquad C_2 = \left\{\begin{matrix} T \\ A \\ C \\ E \\ G \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 010X0 \\ 01X01 \\ 0111X \end{matrix}\right\} \qquad C_3 = \left\{\begin{matrix} T \\ A \\ D \\ E \\ I \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0100X \\ 010X0 \\ 0X101 \\ 0111X \end{matrix}\right\}$$

$$S_1^a = 28 \quad S_1^b = 35 \qquad S_2^a = 28 \quad S_2^b = 35 \qquad S_3^a = 28 \quad S_3^b = 35$$

$$C_4 = \left\{\begin{matrix} T \\ A \\ D \\ H \\ I \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0100X \\ 01X10 \\ 0X101 \\ 0111X \end{matrix}\right\} \qquad C_5 = \left\{\begin{matrix} T \\ A \\ D \\ H \\ I \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0100X \\ 01X10 \\ 0X101 \\ 011X1 \end{matrix}\right\} \qquad C_6 = \left\{\begin{matrix} T \\ A \\ E \\ G \\ I \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 010X0 \\ 01X01 \\ 0X101 \\ 0111X \end{matrix}\right\}$$

$$S_4^a = 28 \quad S_4^b = 35 \qquad S_5^a = 28 \quad S_5^b = 35 \qquad S_6^a = 28 \quad S_6^b = 35$$

$$C_7 = \left\{\begin{matrix} T \\ A \\ E \\ I \\ J \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 010X0 \\ 0X101 \\ X1001 \\ 0111X \end{matrix}\right\}$$

$$S_7^a = 28$$
$$S_7^b = 35$$

$$C_8 = \left\{\begin{matrix} T \\ B \\ C \\ D \\ H \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 0100X \\ 01X10 \\ 011X1 \end{matrix}\right\}$$

$$S_8^a = 28$$
$$S_8^b = 35$$

$$C_9 = \left\{\begin{matrix} T \\ B \\ C \\ E \\ G \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 010X0 \\ 01X01 \\ 0111X \end{matrix}\right\}$$

$$S_9^a = 28$$
$$S_9^b = 35$$

$$C_{10} = \left\{\begin{matrix} T \\ B \\ C \\ G \\ H \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 01X01 \\ 01X10 \\ 0111X \end{matrix}\right\}$$

$$S_{10}^a = 28$$
$$S_{10}^b = 35$$

$$C_{11} = \left\{\begin{matrix} T \\ B \\ C \\ G \\ H \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 01X01 \\ 01X10 \\ 011X1 \end{matrix}\right\}$$

$$S_{11}^a = 28$$
$$S_{11}^b = 35$$

$$C_{12} = \left\{\begin{matrix} T \\ B \\ C \\ H \\ J \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 01X10 \\ X1001 \\ 011X1 \end{matrix}\right\}$$

$$S_{12}^a = 28$$
$$S_{12}^b = 35$$

$$C_{13} = \left\{\begin{matrix} T \\ A \\ B \\ G \\ H \\ I \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0X000 \\ 01X01 \\ 01X10 \\ 0X101 \\ 0111X \end{matrix}\right\}$$

$$S_{13}^a = 32$$
$$S_{13}^b = 40$$

$$C_{14} = \left\{\begin{matrix} T \\ A \\ B \\ G \\ H \\ I \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0X000 \\ 01X01 \\ 01X10 \\ 0X101 \\ 011X1 \end{matrix}\right\}$$

$$S_{14}^a = 32$$
$$S_{14}^b = 40$$

$$C_{15} = \left\{\begin{matrix} T \\ A \\ B \\ H \\ I \\ J \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0X000 \\ 01X10 \\ 0X101 \\ X1001 \\ 0111X \end{matrix}\right\}$$

$$S_{15}^a = 32$$
$$S_{15}^b = 40$$

$$C_{16} = \left\{\begin{matrix} T \\ A \\ B \\ H \\ I \\ J \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0X000 \\ 01X10 \\ 0X101 \\ X1001 \\ 011X1 \end{matrix}\right\}$$

$$S_{16}^a = 32$$
$$S_{16}^b = 40$$

$$C_{17} = \left\{\begin{matrix} T \\ A \\ C \\ D \\ E \\ K \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 0100X \\ 010X0 \\ 0111X \\ 011X1 \end{matrix}\right\}$$

$$S_{17}^a = 32$$
$$S_{17}^b = 40$$

$$C_{18} = \left\{\begin{matrix} T \\ A \\ C \\ D \\ E \\ L \\ M \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 0100X \\ 010X0 \\ 011X1 \\ X1110 \end{matrix}\right\}$$

$$S_{18}^a = 32$$
$$S_{18}^b = 40$$

$$C_{19} = \left\{\begin{matrix} T \\ A \\ C \\ D \\ G \\ H \\ K \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 0100X \\ 01X01 \\ 01X10 \\ 0111X \end{matrix}\right\}$$

$$S_{19}^a = 32$$
$$S_{19}^b = 40$$

$$C_{20} = \left\{\begin{matrix} T \\ A \\ C \\ E \\ G \\ H \\ L \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 010X0 \\ 01X01 \\ 01X10 \\ 011X1 \end{matrix}\right\}$$

$$S_{20}^a = 32$$
$$S_{20}^b = 40$$

$$C_{21} = \left\{\begin{matrix} T \\ A \\ C \\ E \\ G \\ L \\ M \end{matrix}\right\} = \left\{\begin{matrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 010X0 \\ 01X01 \\ 011X1 \\ X1110 \end{matrix}\right\}$$

$$S_{21}^a = 32$$
$$S_{21}^b = 40$$

$$C_{22} = \begin{Bmatrix} T \\ A \\ C \\ E \\ H \\ J \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 010X0 \\ 01X10 \\ X1001 \\ 011X1 \end{Bmatrix} \qquad C_{23} = \begin{Bmatrix} T \\ A \\ C \\ E \\ J \\ K \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 010X0 \\ X1001 \\ 0111X \\ 011X1 \end{Bmatrix} \qquad C_{24} = \begin{Bmatrix} T \\ A \\ C \\ E \\ J \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 010X0 \\ X1001 \\ 011X1 \\ X1110 \end{Bmatrix}$$

$S^a_{22} = 32$
$S^b_{22} = 40$

$S^a_{23} = 32$
$S^b_{23} = 40$

$S^a_{24} = 32$
$S^b_{24} = 40$

$$C_{25} = \begin{Bmatrix} T \\ A \\ D \\ E \\ I \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 0100X \\ 010X0 \\ 0X101 \\ 011X1 \\ X1110 \end{Bmatrix} \qquad C_{26} = \begin{Bmatrix} T \\ A \\ E \\ G \\ H \\ I \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 010X0 \\ 01X01 \\ 01X10 \\ 0X101 \\ 011X1 \end{Bmatrix} \qquad C_{27} = \begin{Bmatrix} T \\ A \\ E \\ G \\ I \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 010X0 \\ 01X01 \\ 0X101 \\ 011X1 \\ X1110 \end{Bmatrix}$$

$S^a_{25} = 32$
$S^b_{25} = 40$

$S^a_{26} = 32$
$S^b_{26} = 40$

$S^a_{27} = 32$
$S^b_{27} = 40$

$$C_{28} = \begin{Bmatrix} T \\ A \\ E \\ H \\ I \\ J \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 010X0 \\ 01X10 \\ 0X101 \\ X1001 \\ 011X1 \end{Bmatrix} \qquad C_{29} = \begin{Bmatrix} T \\ A \\ E \\ I \\ J \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 010X0 \\ 0X101 \\ X1001 \\ 011X1 \\ X1110 \end{Bmatrix} \qquad C_{30} = \begin{Bmatrix} T \\ B \\ C \\ D \\ E \\ I \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 0100X \\ 010X0 \\ 0X101 \\ 0111X \end{Bmatrix}$$

$S^a_{28} = 32$
$S^b_{28} = 40$

$S^a_{29} = 32$
$S^b_{29} = 40$

$S^a_{30} = 32$
$S^b_{30} = 40$

$$C_{31} = \begin{Bmatrix} T \\ B \\ C \\ D \\ E \\ K \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 0100X \\ 010X0 \\ 0111X \\ 011X1 \end{Bmatrix} \qquad C_{32} = \begin{Bmatrix} T \\ B \\ C \\ D \\ E \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 0100X \\ 010X0 \\ 011X1 \\ X1110 \end{Bmatrix} \qquad C_{33} = \begin{Bmatrix} T \\ B \\ C \\ D \\ H \\ I \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 0100X \\ 01X10 \\ 0X101 \\ 0111X \end{Bmatrix}$$

$S^a_{31} = 32$
$S^b_{31} = 40$

$S^a_{32} = 32$
$S^b_{32} = 40$

$S^a_{33} = 32$
$S^b_{33} = 40$

$$C_{34} = \begin{Bmatrix} T \\ B \\ C \\ E \\ G \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 010X0 \\ 01X01 \\ 011X1 \\ X1110 \end{Bmatrix} \qquad C_{35} = \begin{Bmatrix} T \\ B \\ C \\ E \\ I \\ J \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 010X0 \\ 0X101 \\ X1001 \\ 0111X \end{Bmatrix} \qquad C_{36} = \begin{Bmatrix} T \\ B \\ C \\ E \\ J \\ K \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 010X0 \\ X1001 \\ 0111X \\ 011X1 \end{Bmatrix}$$

$S^a_{34} = 32$
$S^b_{34} = 40$

$S^a_{35} = 32$
$S^b_{35} = 40$

$S^a_{36} = 32$
$S^b_{36} = 40$

$$C_{37} = \begin{Bmatrix} T \\ B \\ C \\ E \\ J \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 010X0 \\ X1001 \\ 011X1 \\ X1110 \end{Bmatrix}$$

$S^a_{37} = 32$
$S^b_{37} = 40$

$$C_{38} = \begin{Bmatrix} T \\ B \\ C \\ H \\ I \\ J \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0010X \\ 01X10 \\ 0X101 \\ X1001 \\ 0111X \end{Bmatrix}$$

$S^a_{38} = 32$
$S^b_{38} = 40$

$$C_{39} = \begin{Bmatrix} T \\ B \\ D \\ E \\ F \\ I \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0100X \\ 010X0 \\ X0100 \\ 0X101 \\ 0111X \end{Bmatrix}$$

$S^a_{39} = 32$
$S^b_{39} = 40$

$$C_{40} = \begin{Bmatrix} T \\ B \\ D \\ F \\ H \\ I \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0100X \\ X0100 \\ 01X10 \\ 0X101 \\ 0111X \end{Bmatrix}$$

$S^a_{40} = 32$
$S^b_{40} = 40$

$$C_{41} = \begin{Bmatrix} T \\ B \\ D \\ F \\ H \\ I \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0100X \\ X0100 \\ 01X10 \\ 0X101 \\ 011X1 \end{Bmatrix}$$

$S^a_{41} = 32$
$S^b_{41} = 40$

$$C_{42} = \begin{Bmatrix} T \\ B \\ E \\ F \\ G \\ I \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 010X0 \\ X0100 \\ 01X01 \\ 0X101 \\ 0111X \end{Bmatrix}$$

$S^a_{42} = 32$
$S^b_{42} = 40$

$$C_{43} = \begin{Bmatrix} T \\ B \\ E \\ F \\ I \\ J \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 010X0 \\ X0100 \\ 0X101 \\ X1001 \\ 0111X \end{Bmatrix}$$

$S^a_{43} = 32$
$S^b_{43} = 40$

$$C_{44} = \begin{Bmatrix} T \\ B \\ F \\ G \\ H \\ I \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ X0100 \\ 01X01 \\ 01X10 \\ 0X101 \\ 0111X \end{Bmatrix}$$

$S^a_{44} = 32$
$S^b_{44} = 40$

$$C_{45} = \begin{Bmatrix} T \\ B \\ F \\ G \\ H \\ I \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ X0100 \\ 01X01 \\ 01X10 \\ 0X101 \\ 011X1 \end{Bmatrix}$$

$S^a_{45} = 32$
$S^b_{45} = 40$

$$C_{46} = \begin{Bmatrix} T \\ B \\ F \\ H \\ I \\ J \\ K \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ X0100 \\ 01X10 \\ 0X101 \\ X1001 \\ 0111X \end{Bmatrix}$$

$S^a_{46} = 32$
$S^b_{46} = 40$

$$C_{47} = \begin{Bmatrix} T \\ B \\ F \\ H \\ I \\ J \\ L \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ X0100 \\ 01X10 \\ 0X101 \\ X1001 \\ 011X1 \end{Bmatrix}$$

$S^a_{47} = 32$
$S^b_{47} = 40$

$$C_{48} = \begin{Bmatrix} T \\ B \\ D \\ E \\ F \\ I \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 0100X \\ 010X0 \\ X0100 \\ 0X101 \\ 011X1 \\ X1110 \end{Bmatrix}$$

$S^a_{48} = 36$
$S^b_{48} = 45$

$$C_{49} = \begin{Bmatrix} T \\ B \\ E \\ F \\ G \\ I \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 010X0 \\ X0100 \\ 01X01 \\ 0X101 \\ 011X1 \\ X1110 \end{Bmatrix}$$

$S^a_{49} = 36$
$S^b_{49} = 45$

$$C_{50} = \begin{Bmatrix} T \\ B \\ E \\ F \\ I \\ J \\ L \\ M \end{Bmatrix} = \begin{Bmatrix} 1001X \\ 10X11 \\ 0X000 \\ 010X0 \\ X0100 \\ 0X101 \\ X1001 \\ 011X1 \\ X1110 \end{Bmatrix}$$

$S^a_{50} = 36$
$S^b_{50} = 45$

Рассмотрим следующее минимальное покрытие:

$$C_{\min} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 0100X \\ 01X10 \\ 011X1 \end{Bmatrix}$$

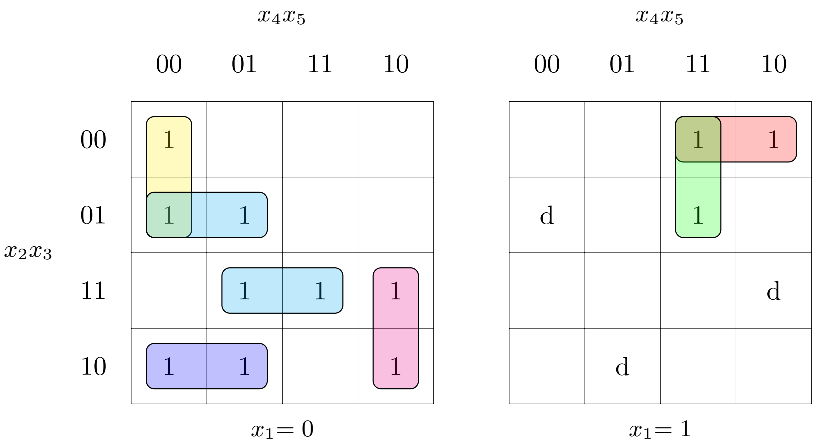
$$S^a = 28$$
$$S^b = 35$$

Этому покрытию соответствует следующая МДНФ:

$$f = x_1\,\overline{x_2}\,\overline{x_3}\,x_4 \vee x_1\,\overline{x_2}\,x_4\,x_5 \vee \overline{x_1}\,\overline{x_2}\,\overline{x_4}\,\overline{x_5} \vee \overline{x_1}\,\overline{x_2}\,x_3\,\overline{x_4} \vee \overline{x_1}\,x_2\,\overline{x_3}\,\overline{x_4} \vee \overline{x_1}\,x_2\,x_4\,\overline{x_5} \vee \overline{x_1}\,x_2\,x_3\,x_5$$

## Минимизация булевой функции на картах Карно

### Определение МДНФ

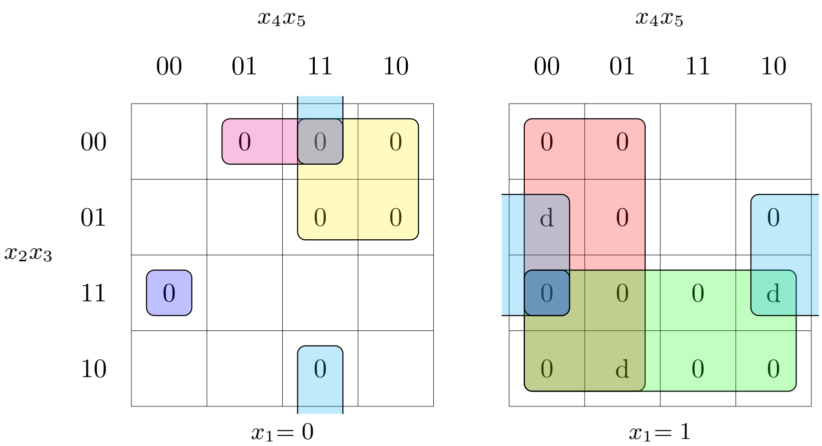



$$C_{\min} = \begin{Bmatrix} 1001X \\ 10X11 \\ 00X00 \\ 0010X \\ 0100X \\ 01X10 \\ 011X1 \end{Bmatrix}$$

$$S^a = 28$$
$$S^b = 35$$

$$f = x_1\,\overline{x_2}\,\overline{x_3}\,x_4 \vee x_1\,\overline{x_2}\,x_4\,x_5 \vee \overline{x_1}\,\overline{x_2}\,\overline{x_4}\,\overline{x_5} \vee \overline{x_1}\,\overline{x_2}\,x_3\,\overline{x_4} \vee \overline{x_1}\,x_2\,\overline{x_3}\,\overline{x_4} \vee \overline{x_1}\,x_2\,x_4\,\overline{x_5} \vee \overline{x_1}\,x_2\,x_3\,x_5$$

### Определение МКНФ




$$C_{\min} = \begin{Bmatrix} 1XX0X \\ 11XXX \\ 00X1X \\ 1X1X0 \\ X1100 \\ 000X1 \\ 0X011 \end{Bmatrix}$$

$$S^a = 22$$
$$S^b = 29$$

$$f = (\overline{x_1} \vee x_4)\,(\overline{x_1} \vee \overline{x_2})\,(x_1 \vee x_2 \vee \overline{x_4})\,(\overline{x_1} \vee \overline{x_3} \vee x_5)\,(\overline{x_2} \vee \overline{x_3} \vee x_4 \vee x_5)\,(x_1 \vee x_2 \vee x_3 \vee \overline{x_5})\,(x_1 \vee x_3 \vee \overline{x_4} \vee \overline{x_5})$$

# Преобразование минимальных форм булевой функции

## Факторизация и декомпозиция МДНФ

$$f = x_1 \overline{x_2}\,\overline{x_3} x_4 \vee x_1 \overline{x_2}\,\overline{x_4} x_5 \vee x_1 \overline{x_2}\,\overline{x_4}\,\overline{x_5} \vee x_1 \overline{x_2}\,\overline{x_3} x_4 \vee x_1 \overline{x_2} x_3 \overline{x_4} \vee x_1 \overline{x_2}\,\overline{x_4} x_5 \vee \overline{x_1} x_2 \overline{x_3} x_5 \; S_Q = 35 \; \tau = 2$$

Декомпозиция невозможна

$$f = \overline{x_1}\, x_2\, (\overline{x_3}\,\overline{x_4} \vee x_4 \overline{x_5} \vee x_3 x_5) \vee x_1 \overline{x_2}\, x_4\, (\overline{x_3} \vee x_5) \vee \overline{x_1}\,\overline{x_2}\,\overline{x_4}\, (x_3 \vee \overline{x_5}) \qquad S_Q = 27 \quad \tau = 4$$

## Факторизация и декомпозиция МКНФ

$$\begin{aligned} f = {} & (\overline{x_1} \vee x_4)\,(\overline{x_1} \vee \overline{x_2})\,(x_1 \vee x_2 \vee \overline{x_4})\,(\overline{x_1} \vee \overline{x_3} \vee x_5)\,(\overline{x_2} \vee \overline{x_3} \vee x_4 \vee x_5) \\ & (x_1 \vee x_2 \vee x_3 \vee \overline{x_5})\,(x_1 \vee x_3 \vee \overline{x_4} \vee \overline{x_5}) \end{aligned} \qquad S_Q = 29 \quad \tau = 2$$

$$f = (\overline{x_1} \vee \overline{x_2}\, x_4)\,(\overline{x_3} \vee x_5 \vee \overline{x_1}\,(\overline{x_2} \vee x_4))\,(x_1 \vee x_2 \vee \overline{x_4})\,(x_1 \vee x_3 \vee \overline{x_5} \vee x_2 \overline{x_4}) \qquad S_Q = 24 \quad \tau = 4$$

$$\varphi = \overline{x_1}\,(\overline{x_2} \vee x_4)$$

$$\overline{\varphi} = x_1 \vee x_2 \overline{x_4}$$

$$f = (\overline{x_1} \vee \overline{x_2}\, x_4)\,(\overline{x_3} \vee x_5 \vee \varphi)\,(x_1 \vee x_2 \vee \overline{x_4})\,(\overline{\varphi} \vee x_3 \vee \overline{x_5}) \qquad S_Q = 22 \quad \tau = 5$$

# Синтез комбинационных схем

Будем анализировать схемы на следующих наборах аргументов:

$$f([x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 1]) = 0$$
$$f([x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 1, x_5 = 0]) = 0$$
$$f([x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0]) = 1$$
$$f([x_1 = 0, x_2 = 0, x_3 = 1, x_4 = 0, x_5 = 0]) = 1$$

## Булев базис

Схема по упрощенной МДНФ:

$$f = \overline{x_1}\, x_2\, (\overline{x_3}\,\overline{x_4} \vee x_4 \overline{x_5} \vee x_3 x_5) \vee x_1 \overline{x_2}\, x_4\, (\overline{x_3} \vee x_5) \vee \overline{x_1}\,\overline{x_2}\,\overline{x_4}\, (x_3 \vee \overline{x_5}) \quad (S_Q = 27, \tau = 4)$$

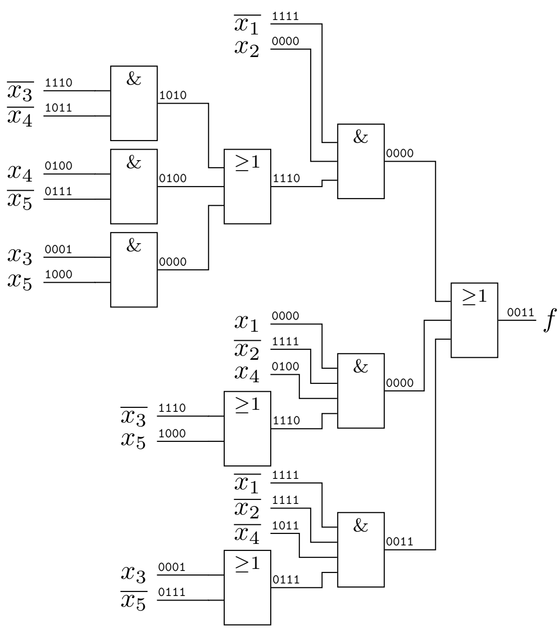
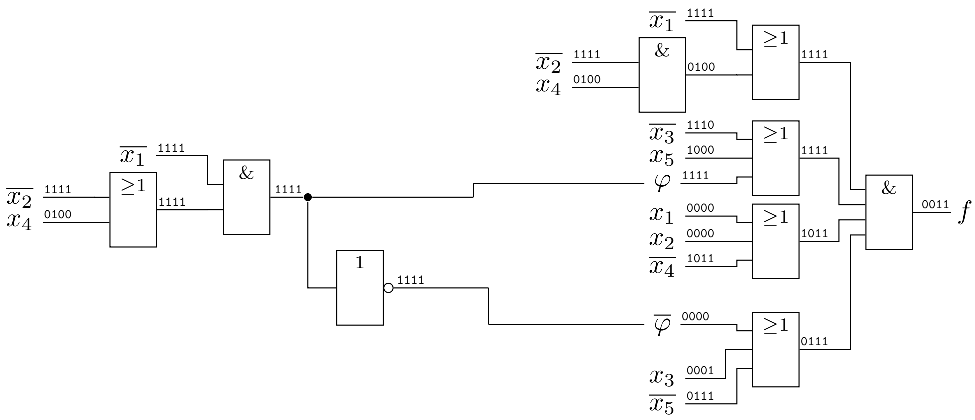


Схема по упрощенной МКНФ:

$$f = (\overline{x_1} \vee \overline{x_2}\, x_4)\ (\overline{x_3} \vee x_5 \vee \varphi)\ (x_1 \vee x_2 \vee \overline{x_4})\ (\overline{\varphi} \vee x_3 \vee \overline{x_5}) \quad (S_Q = 22, \tau = 5)$$

$$\varphi = \overline{x_1}\ (\overline{x_2} \vee x_4)$$



## Сокращенный булев базис (И, НЕ)

Схема по упрощенной МДНФ в базисе И, НЕ:

$$f = \overline{\overline{x_1\, \overline{x_2}\, x_4\, \overline{x_3\, \overline{x_5}}}\ \overline{\overline{x_1}\, \overline{x_2}\, \overline{x_4}\, \overline{\overline{x_3}\, x_5}}\ \overline{\overline{x_1}\, x_2\, \overline{x_3}\, \overline{x_4}}\ \overline{\overline{x_1}\, x_2\, x_4\, \overline{x_5}}\ \overline{\overline{x_1}\, x_2\, x_3\, x_5}} \quad (S_Q = 37, \tau = 6)$$

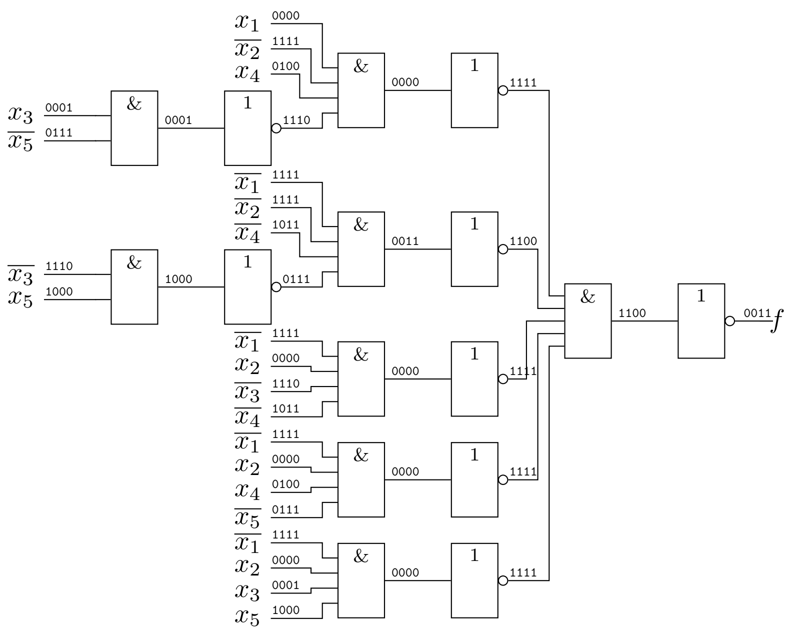
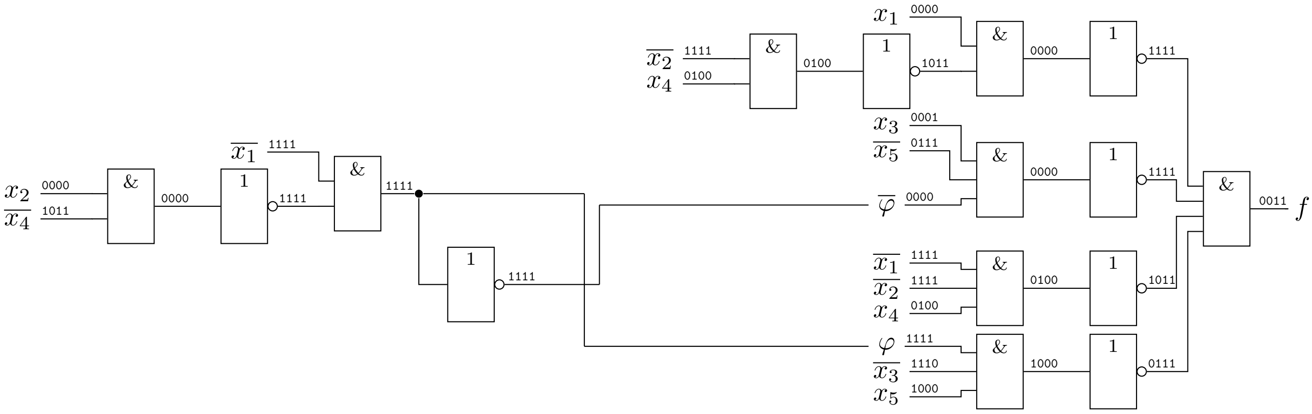


Схема по упрощенной МКНФ в базисе И, НЕ:

$$f = \overline{x_1 \overline{\overline{x_2}\, x_4}}\; \overline{x_3\, \overline{x_5}\, \overline{\varphi}}\; \overline{\overline{x_1}\, \overline{x_2}\, x_4}\; \overline{\varphi\, \overline{x_3}\, x_5} \quad (S_Q = 28, \tau = 7)$$

$$\varphi = \overline{x_1}\, \overline{x_2\, \overline{x_4}}$$



## Универсальный базис (И-НЕ, 2 входа)

Схема по упрощенной МДНФ в базисе И-НЕ с ограничением на число входов:

$$f = \overline{ \overline{ \overline{x_2}\, \overline{ \overline{ x_1 \overline{\overline{x_4 \overline{x_3 \overline{x_5}}}} }\; \overline{ \overline{x_1}\, \overline{\overline{\overline{x_4}\, \overline{\overline{x_3}\, x_5}}} } } }\; \overline{ \overline{x_1}\, \overline{\overline{ x_2 \overline{ \overline{\overline{x_3}\,\overline{x_4}}\; \overline{\overline{\overline{x_4 \overline{x_5}}\; \overline{x_3 x_5}}} } }} } } \quad (S_Q = 40, \tau = 8)$$

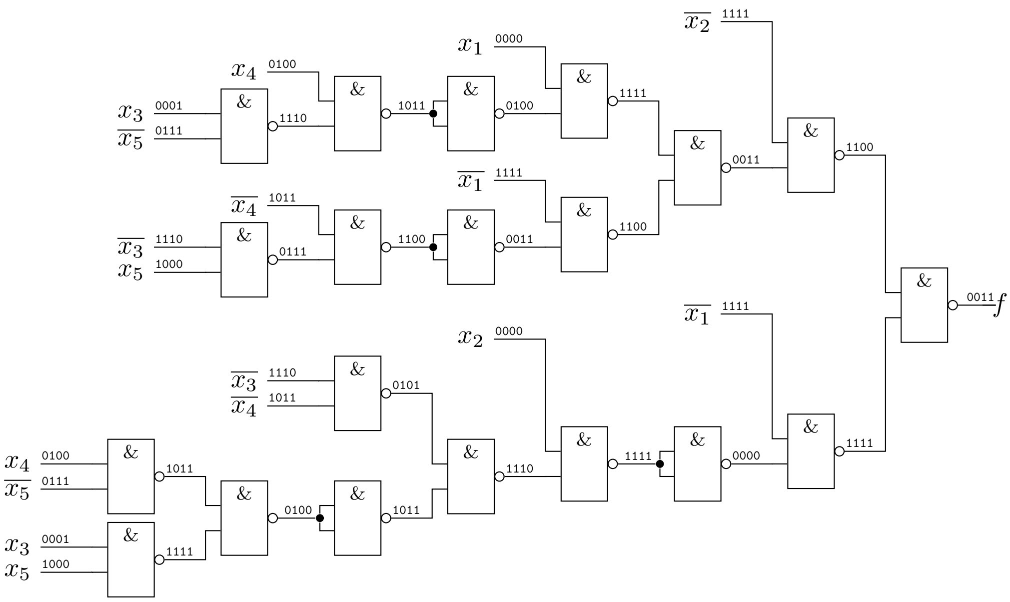


Схема по упрощенной МКНФ в базисе И-НЕ с ограничением на число входов:

$$f = \overline{\overline{\overline{\overline{x_1}\,\overline{\overline{\overline{x_2}\,x_4}\,\overline{\overline{x_3}\,\overline{\overline{x_5\,\overline{x_2\,\overline{x_4}}}}}}}\;\overline{\overline{\overline{x_1\,\overline{\overline{x_2}\,x_4}}\,\overline{x_3\,\overline{\overline{\overline{x_5}\,\overline{\overline{x_1}\,\overline{x_2\,\overline{x_4}}}}}}}}}} \quad (S_Q = 36, \tau = 9)$$