IAML: Decision Trees

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Semester 1

Predict if John will play tennis

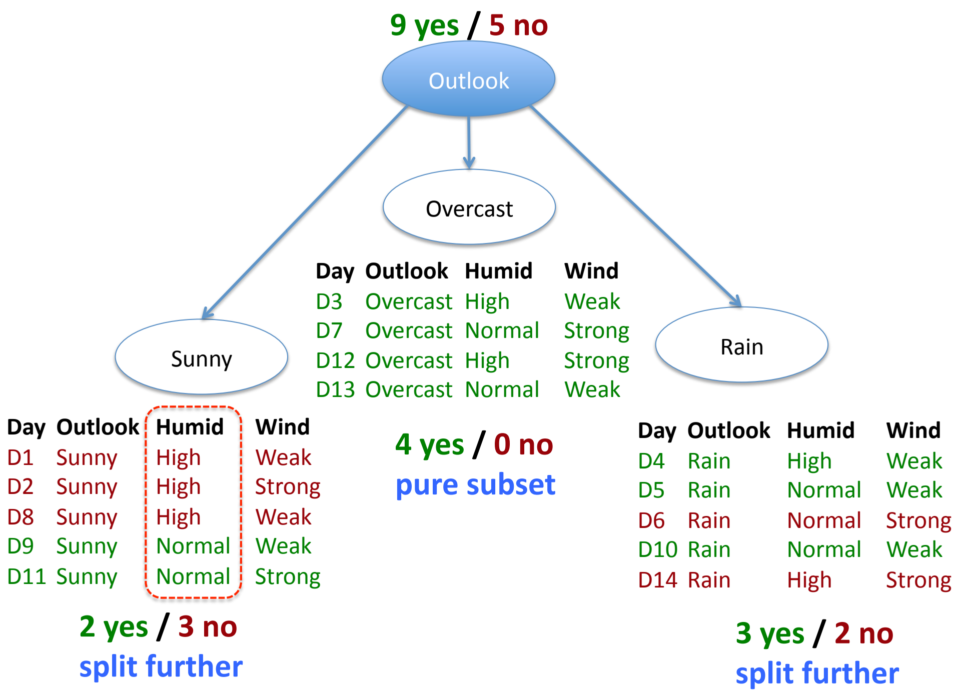
- Hard to guess
- Divide & conquer:
  - split into subsets
  - are they pure? (all yes or all no)
  - if yes: stop
  - if not: repeat
- See which subset new data falls into

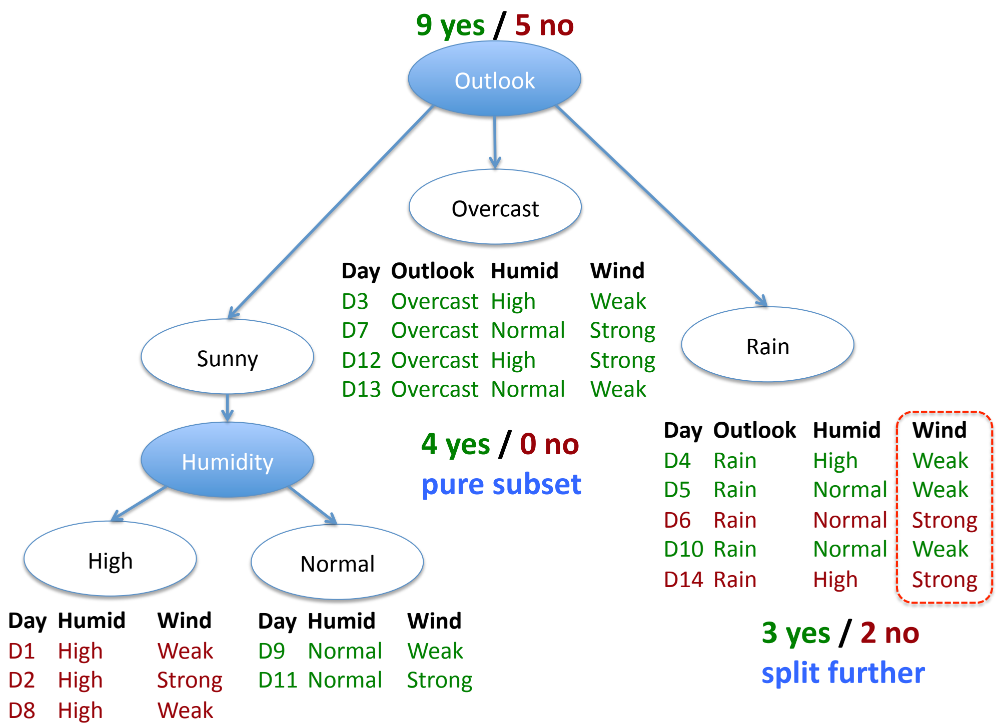
Training examples: 9 yes / 5 no

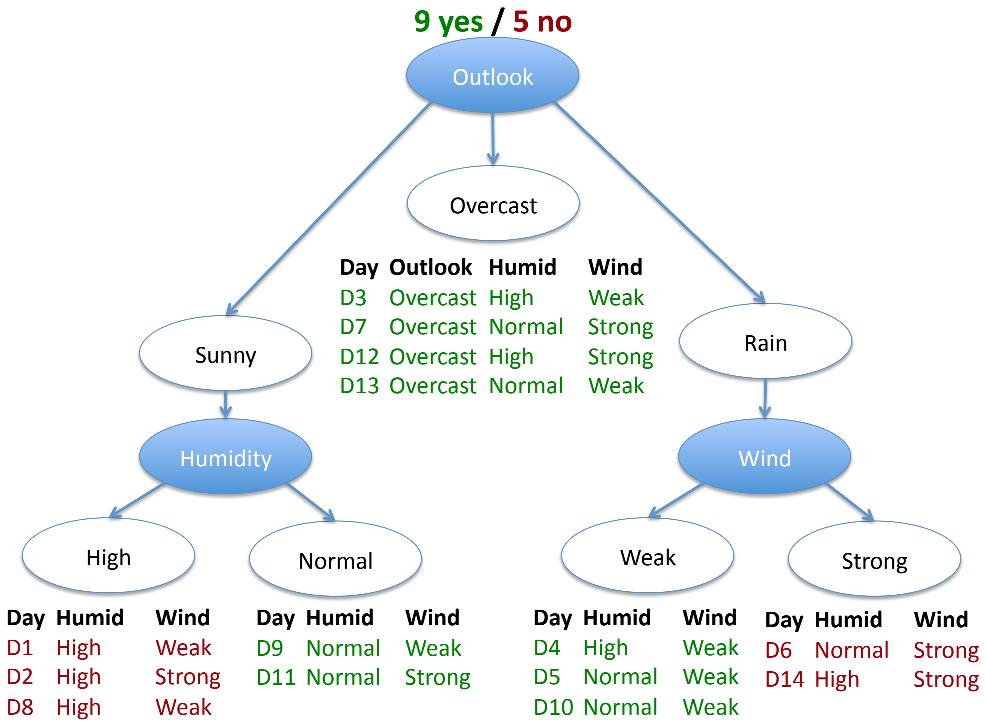
Day	Outlook	Humidity	Wind	Play
D1	Sunny	High	Weak	No
D2	Sunny	High	Strong	No
D3	Overcast	High	Weak	Yes
D4	Rain	High	Weak	Yes
D5	Rain	Normal	Weak	Yes
D6	Rain	Normal	Strong	No
D7	Overcast	Normal	Strong	Yes
D8	Sunny	High	Weak	No
D9	Sunny	Normal	Weak	Yes
D10	Rain	Normal	Weak	Yes
D11	Sunny	Normal	Strong	Yes
D12	Overcast	High	Strong	Yes
D13	Overcast	Normal	Weak	Yes
D14	Rain	High	Strong	No

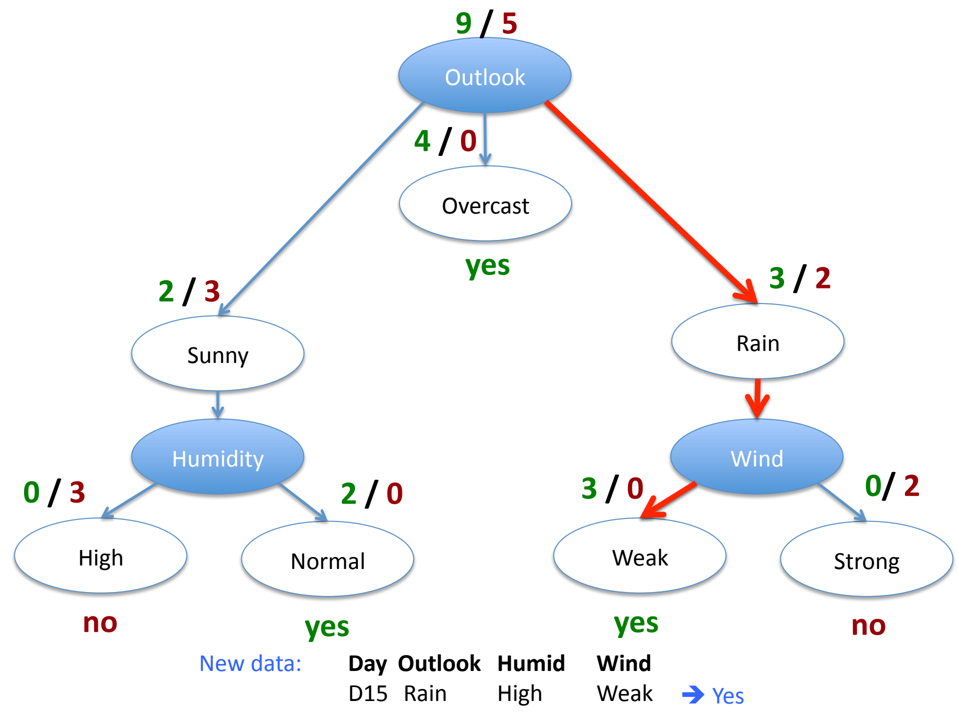
New data:

Day	Outlook	Humidity	Wind	Play
D15	Rain	High	Weak	?





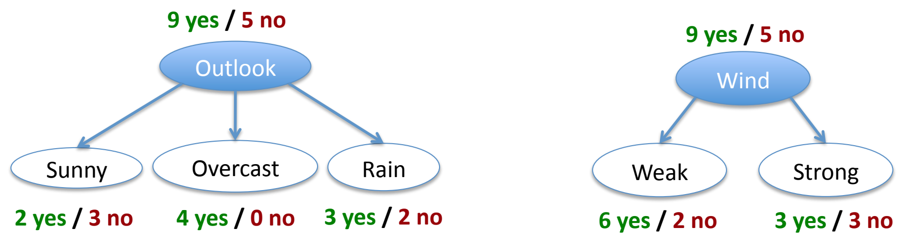




ID3 algorithm

- Split (node, {examples}):
  1. A ← the best attribute for splitting the {examples}
  2. Decision attribute for this node ← A
  3. For each value of A, create new child node
  4. Split training {examples} to child nodes
  5. If examples perfectly classified: STOP
     else: iterate over new child nodes
     Split (child_node, {subset of examples})
- Ross Quinlan (ID3: 1986), (C4.5: 1993)
- Breimanetal (CaRT: 1984) from statistics

Which attribute to split on?



- Want to measure "purity" of the split
  - more certain about Yes/No after the split
    - pure set (4 yes / 0 no) => completely certain (100%)
    - impure (3 yes / 3 no) => completely uncertain (50%)
  - can't use P("yes" | set):
    - must be symmetric: 4 yes / 0 no as pure as 0 yes / 4 no

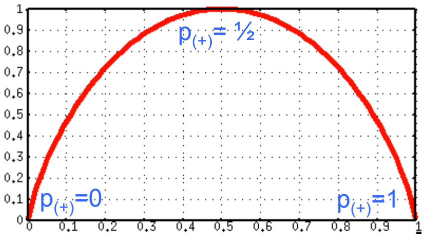
Entropy

- Entropy: $H(S) = - p_{(+)} \log_2 p_{(+)} - p_{(-)} \log_2 p_{(-)}$ bits
  - S … subset of training examples
  - $p_{(+)}$ / $p_{(-)}$ … % of positive / negative examples in S
- Interpretation: assume item X belongs to S
  - how many bits need to tell if X positive or negative
- impure (3 yes / 3 no):

$$H(S) = -\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{6} = 1 \text{ bits}$$

- pure set (4 yes / 0 no):

$$H(S) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0 \text{ bits}$$



Information Gain

- Want many items in pure sets
- Expected drop in entropy after split:

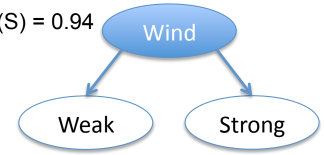
$$Gain(S,A) = H(S) - \sum_{V \in Values(A)} \frac{|S_V|}{|S|} H(S_V)$$

V … possible values of A
S … set of examples {X}
S_V … subset where X_A = V

- Mutual Information
  - between attribute A and class labels of S

$-\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14}$ 9 yes / 5 no

H(S) = 0.94



6 yes / 2 no — $-\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8}$ — $H(S_{weak}) = 0.81$

3 yes / 3 no — $-\frac{3}{6}\log_2\frac{3}{6} - \frac{3}{6}\log_2\frac{3}{6}$ — $H(S_{strong}) = 1.0$

Gain (S, Wind)
$= H(S) - {}^{8}/_{14}\, H(S_{weak}) - {}^{6}/_{14}\, H(S_{weak})$
$= 0.94 - {}^{8}/_{14} * 0.81 - {}^{6}/_{14} * 1.0$
$= 0.049$

Overfitting in Decision Trees

- Can always classify training examples perfectly
  - keep splitting until each node contains 1 example
  - singleton = pure
- Doesn't work on new data

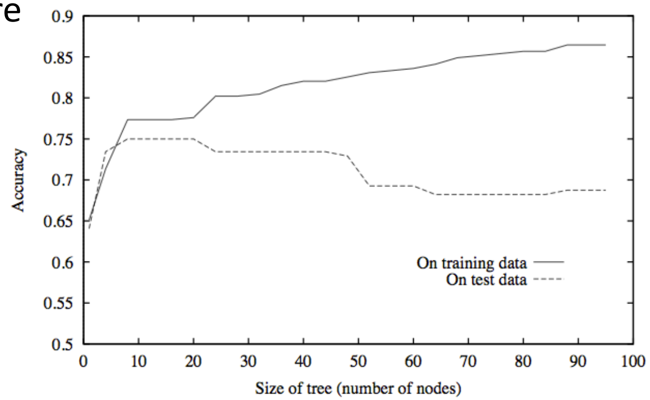


Figure credit: Tom Mitchell, 1997

Avoid overfitting

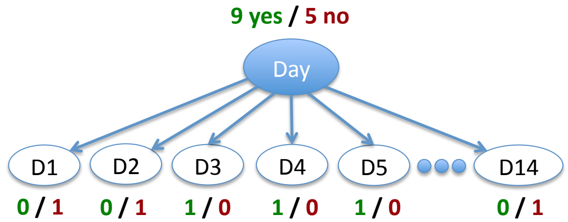
- Stop splitting when not statistically significant
- Grow, then post-prune
  - based on validation set
- Sub-tree replacement pruning (WF 6.1)
  - for each node:
    - pretend remove node + all children from the tree
    - measure performance on validation set
  - remove node that results in greatest improvement
  - repeat until further pruning is harmful

General Structure

- **Task:** classification, discriminative
- **Model structure:** decision tree
- **Score function**
  - information gain at each node
  - preference for short trees
  - preference for high-gain attributes near the root
- **Optimization / search** method
  - greedy search from simple to complex
  - guided by information gain

Problems with Information Gain

- Biased towards attributes with many values



all subsets perfectly pure => optimal split

- Won't work for new data: D15 Rain High Weak
- Use GainRatio:

$$SplitEntropy(S,A) = -\sum_{V \in Values(A)} \frac{|S_V|}{|S|} \log \frac{|S_V|}{|S|}$$

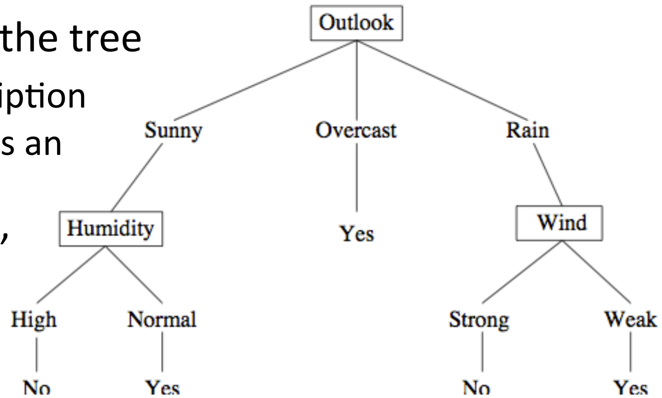
A … candidate attribute
V … possible values of A
S … set of examples {X}
S_V … subset where $X_A = V$

$$GainRatio(S,A) = \frac{Gain(S,A)}{SplitEntropy(S,A)}$$

penalizes attributes with many values

Trees are interpretable

- Read rules off the tree
  - concise description of what makes an item positive
- No "black box"
  - important for users



Rule: (Outlook = Overcast) ∨
(Outlook = Rain ∧ Wind = Weak) ∨
(Outlook = Sunny ∧ Humidity = Normal)

Figure credit: Tom Mitchell, 1997

Continuous Attributes

- Dealing with continuous-valued attributes:
  - create a split: (Temperature > 72.3) = True,False
- Threshold can be optimized (WF 6.1)

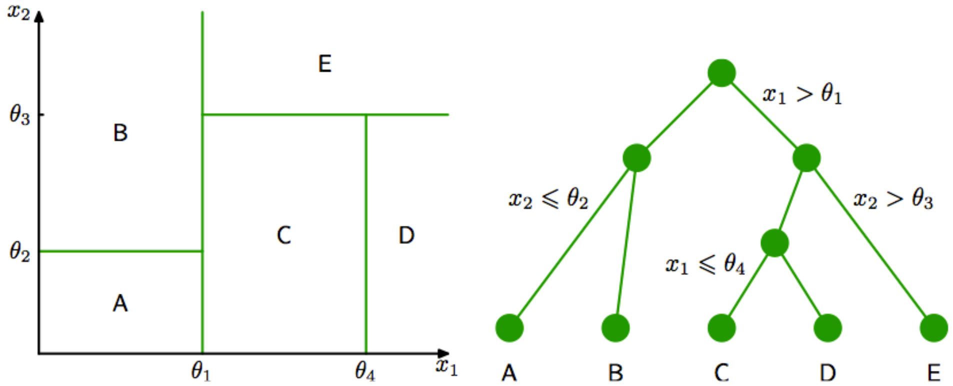


Figure credit: Chris Bishop, PRML

Multi-class and Regression

- Multi-class classification:
  - predict most frequent class in the subset
  - entropy: $H(S) = - \Sigma_c \, p_{(c)} \log_2 p_{(c)}$
  - $p_{(c)}$ … % of examples of class c in S
- Regression:
  - predicted output = mean of the training examples in the subset
  - requires a different definition of entropy
  - can use linear regression at the leaves (WF 6.5)

Random Decision Forest

- Grow K different decision trees:
  - pick a random subset S_r of training examples
  - grow a full ID3 tree (no prunning):
    - when splitting: pick from $d << D$ random attributes
    - compute gain based on S_r instead of full set
  - repeat for r = 1 … K
- Given a new data point X:
  - classify X using each of the K trees
  - use majority vote: class predicted most often
- Fast, scalable, state-of-the-art performance

Summary

- ID3: grows decision tree from the root down
  - greedily selects next best attribute (Gain)
- Searches a complete hypothesis space
  - prefers smaller trees, high gain at the root
- Overfitting addressed by post-prunning
  - prune nodes, while accuracy ⇧ on validation set
- Can handle missing data (see WF 6.1)
- Easily handles irrelevant variables
  - Information Gain = 0 => will not be selected