1.

a)

set $p_1 = (2,3)$ $p_2 = (1,2)$ $p_3 = (3,-1)$

Then the corresponding matrix should be $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

set $p_4 = (10,8)$ $p_5 = (8,-4)$ $p_6 = (2,0)$

Then the corresponding matrix should be $\begin{bmatrix} 10 & 8 & 2 \\ 8 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Then we set the 2D affine transformation M = $\begin{bmatrix} x & y & z \\ u & m & n \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} x & y & z \\ u & m & n \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 8 & 2 \\ 8 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3y+z & x+2y+z & 3x-y+z \\ 2u+3m+n & u+2m+n & 3u-m+n \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 8 & 2 \\ 8 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

After solving six equations, we can get x = 0, y = 2, z = 4, u = 8, m = 4, n = -20

So the transformation matrix = $\begin{bmatrix} 0 & 2 & 4 \\ 8 & 4 & -20 \\ 0 & 0 & 1 \end{bmatrix}$

b)

4 points to determined a 2D Homography.

2 points to determined a 2D Rigid transform

c)

Centroid: preserved

Proof:

Set the vertex of the triangle be A(x_1, y_1), B(x_2, y_2), A(x_3, y_3)

The centroid of the triangle should be(1/3*($x_1 + x_2 + x_3$), 1/3*($y_1 + y_2 + y_3$))

Then we set the 2D affine transformation M = $\begin{bmatrix} x & y & z \\ u & m & n \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} x & y & z \\ u & m & n \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} x_1x + y_1y + z & xx_2 + y_2y + z & x_3x + y_3y + z \\ x_1u + y_1m + n & x_2u + y_2m + n & x_3u + y_3m + n \\ 1 & 1 & 1 \end{bmatrix}$$

The centroid of the triangle should be

(1/3*($x_1x + y_1y + z + xx_2 + y_2y + z + x_3x + y_3y + z$),

1/3*($x_1u + y_1m + n + x_2u + y_2m + n + x_3u + y_3m + n$))

MC = $\begin{bmatrix} x & y & z \\ u & m & n \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} * (x_1 + x_2 + x_3) \\ \frac{1}{3} * (y_1 + y_2 + y_3) \\ 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{3} * (x_1 x + y_1 y + z + x x_2 + y_2 y + z + x_3 x + y_3 y + z) \\ \frac{1}{3} * (x_1 u + y_1 m + n + x_2 u + y_2 m + n + x_3 u + y_3 m + n) \\ 1 \end{bmatrix}$ = centroid of the triangle after transform

So, it is preserved

Ortho-center:

Not preserved

Given A(0,0) B(0,2), C(1,$\sqrt{3}$)

Then the ortho-center is (1, $\sqrt{3}/3$)

Suppose M = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & \sqrt{3} \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1+\sqrt{3} \\ 1 & 3 & 1+\sqrt{3} \\ 1 & 1 & 1 \end{bmatrix}$

Then the ortho-center is (3$\sqrt{3}$ - 4, 6-$\sqrt{3}$) ≠ (1, $\sqrt{3}/3$)

So, it is not preserved.

2.

a)

lens: reflect the incoming light and converge it on the image plane.

Focal length: it will affect the size of the object displayed on the image.

Aperture: it will affect the light coming to the image, So, it will affect the image brightness, and the depth of field.

b)

(-1,2,1) – (2,1,3) = (-3,1,-2)

so w = $\frac{-(-3,1,-2)}{\sqrt{9+4+1}} = (\frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}})$

as given in the camera model (lecture notes,37)

suppose the up vector t = (0,1,0)

then $\frac{w \times t}{||w \times t||}$ = (-$\frac{2}{\sqrt{13}}$,0, $\frac{3}{\sqrt{13}}$) = u

v = w×u = (-$\frac{3}{\sqrt{182}}$, -$\sqrt{\frac{13}{14}}$, -$\sqrt{\frac{2}{91}}$)

$M_{cw} = [\mathrm{u}, \mathrm{v}, \mathrm{w}] = \begin{bmatrix} -\frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{182}} & \frac{3}{\sqrt{14}} \\ 0 & -\sqrt{\frac{13}{14}} & \frac{-1}{\sqrt{14}} \\ \frac{3}{\sqrt{13}} & -\sqrt{\frac{2}{91}} & \frac{2}{\sqrt{14}} \end{bmatrix}$

so $M_{wc} = \begin{bmatrix} -\frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{182}} & -\sqrt{\frac{13}{14}} & -\sqrt{\frac{2}{91}} \\ \frac{3}{\sqrt{14}} & \frac{-1}{\sqrt{14}} & \frac{2}{\sqrt{14}} \end{bmatrix}$

c)
as given in the camera model (lecture notes,40)
since view direction is (0,0,1), so w = (0,0,-1)
then u = (-1,0,0) and v = (0,1,0)

lens equation: $\frac{1}{f} = \frac{1}{pz-d} + \frac{1}{d}$ => $\frac{1}{f} = \frac{pz}{d(pz-d)}$ => f = $\frac{d(pz-d)}{pz}$

$$M_{wc} = M_{cw} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$M_{wc}P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} -p_x \\ p_y \\ -p_z \end{bmatrix}$$

$$x = M_p M_{wc} P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} -p_x \\ p_y \\ -p_z \\ 1 \end{bmatrix} = (-p_x, p_y, -p_z, -p_z/f)$$

let the last element become 1
then x = $(p_x * \frac{f}{p_z}, -p_y * f/p_z, f, 1)$ = $(p_x \frac{d(pz-d)}{{p_z}^2}$, $-p_y \frac{d(pz-d)}{{p_z}^2})$
So, the 2D point should be($p_x \frac{d(pz-d)}{{p_z}^2}$, $-p_y \frac{d(pz-d)}{{p_z}^2}$)

d)
Sometime they are parallel and sometime they are not
If the direction vector b (bx, by, bz) is orthogonal to the view direction, then the two lines are parallel.
So, we can get that (bx, by, bz) · (0, 0, 1) = 0
The direction vector should be (bx, by, 0)
If they are not parallel, they will converge at the vanishing point.

3.

a)

we need to find the gradient of f(x,y,z)

$\nabla f(x,y,z) = (2x - 2x * R\left(\frac{1}{\sqrt{x^2+y^2}}\right),\ 2y - 2y * R\left(\frac{1}{\sqrt{x^2+y^2}}\right), 2z)$

b)

The tangent plane at $p(x_1, y_1, z_1)$ should be

$\nabla f(x_1, y_1, z_1) * (x - x_1, y - y_1, z - z_1) = 0$

where $\nabla f(x_1, y_1, z_1) = (2x_1 - 2x_1 * R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right),\ 2y_1 - 2y_1 * R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right), 2z_1)$

then we can get

$$\left(2x_1 - 2x_1 * R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right)\right) * (x - x_1) + \left(2y_1 - 2y_1 * R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right)\right) * (y - y_1) + 2z_1 * (z - z_1) = 0$$

c)

put $q(\lambda) = (R\cos\lambda, R\sin\lambda, r)$ into that equation

$f(q(\lambda)) = (R - \sqrt{R\cos\lambda^2 + R\sin\lambda^2})^2 + r^2 - r^2 = (R - R)^2 + 0 = 0 = 0$

So, $q(\lambda)$ lies on the surface

d)

Tangent vector = (dx(λ)/ dλ , dy(λ)/ dλ , dz(λ)/ dλ)

$(R\cos\lambda)' = -R\sin\lambda$

$(R\sin\lambda)' = R\cos\lambda$

$(\lambda)' = 0$

So, the tangent vector = $(-R\sin\lambda, R\cos\lambda, 0)$

e)

$$\left(2x_1 - 2x_1 * R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right)\right) * (-R\sin\lambda) + R\cos\lambda * \left(2y_1 - 2y_1 * R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right)\right) + 0 * \quad 2z_1$$

$= -2x_1 R\sin\lambda + 2x_1 R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right) * R\sin\lambda + 2y_1 R\cos\lambda - 2y_1 R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right) * R\cos\lambda$

$= 2R(x_1\sin\lambda + x_1 R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right) * \sin\lambda + y_1\cos\lambda - y_1 R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right) * \cos\lambda)$

$= 2R(x_1\sin\lambda(1 + R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right)) + y_1\cos\lambda(1 - R\left(\frac{1}{\sqrt{{x_1}^2+{y_1}^2}}\right)))$

$= 2R * 0$

$= 0$

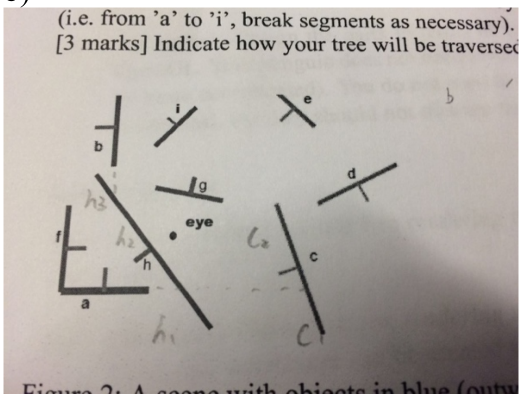
So, the tangent vector of $q(\lambda)$ lie on the implicit equation of tangent plane.

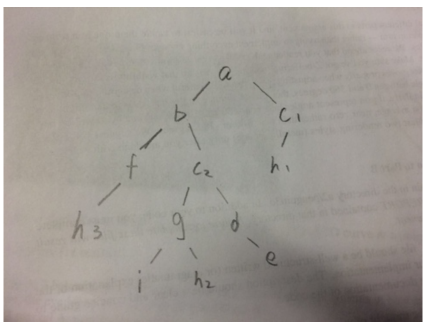
4.

a)

Yes, if we use the perspective projection, we do not need to render a, d, f, I,e, since they were blocked from the eye view.

b)





c)

Since e is inside of a, so we render [right subtree of a, a, left subtree of a]

Then comes to c_1, since e is inside of c_1, so we render [right subtree of c_1, c_1, left subtree of c_1]

Then comes to b, since e is outside of b, so we render [left subtree of b, b, right subtree of b]

Then comes to f, since e is inside of f, so we render [right subtree of f, f, left subtree of f]

Then comes to c_2, since e is inside of c_2 , so we render [right subtree of c_2, c_2, left subtree of c_2]

Then comes to d, since e is outside of d, so we render [left subtree of d, d, right subtree of d]

Then comes to g, since e is outside of g, so we render [left subtree of g, g, right subtree of g]

The order should be c_1, h_1, a, f, h_3, b, d, e, c_2, I, g, h_2