CSC418 A1

MINGYE WANG
999573938
cdf: g3nicky

Part A:

1.

1) x(t) = 4 cos(2πt) + 1/16 * cos(32πt)

$\frac{d}{dt}$(x(t) = 4 cos(2πt) + 1/16 * cos(32πt)) = -2π(4sin(2πt) + sin(32πt))

$\frac{d}{dt}$(y(t) = 2 sin(2πt) + 1/16 * sin(32πt)) = 2 π (2 cos (2 π t) + cos (32 π t))

So, the tangent vector is (-2π(4sin(2πt) + sin(32πt)), 2 π (2 cos (2 π t) + cos (32 π t)))

The normal vector is (2 π (2 cos (2 π t) + cos (32 π t)), 2π(4sin(2πt) + sin(32πt))

2) the curve is symmetric around the X-axis

Since cosθ is even, so x(t) is even.

Since sinθ is odd, so y(t) is odd

Let a = -t, then x(a) = x(t) and y(a) = -y(t)

We can see that the curve is symmetric around the X-axis

The curve is not symmetric around Y-axis

Counter Example:

Let t_1 = 1, t_2 = 1.5

Y(t_1) = 0 and y(t_2) = 0

However, x(t_1) = 4 + 1/16, and x(t_2) = -3 – 15/16.

Obviously, x(t_1) $\neq$ -x(t_2)

We can see that the curve is not symmetric around Y-axis.

3)

Formula of the length of the perimeter should be

$\int_b^a \sqrt{x'(t)^2 + y'(t)^2}$,

Since the curve above x-axis are the opposite direction of the curve under x-axis. We only need to calculate the curve above x-axis and multiply it by 2

So, L =2* $\int_{0.5}^{0} \sqrt{x'(t)^2 + y'(t)^2}$

4)

suppose there exits t* such that [0, t*] determined ¼ of the curve.

Then the ¼ curve length should be $\int_{t*}^{0} \sqrt{x'(t)^2 + y'(t)^2}$

The perimeter should be 4*$\int_{t*}^{0} \sqrt{x'(t)^2 + y'(t)^2}$

2.

1) The area should be $\pi {r_2}^2 - \pi {r_1}^2$
2) The number can be 0 or 1 or 2 or 3 or 4
3) When a line intersects with a circle, we can get the equation such that

$||p_0 + \lambda\vec{d} - p_1||^2 = {r_1}^2$ or $||p_0 + \lambda\vec{d} - p_1||^2 = {r_2}^2$

$||\lambda\vec{d} + (p_0 - p_1)||^2 = r^2$

$(\lambda\vec{d})^2 + (p_0 - p_1)^2 + 2\lambda\vec{d}(p_0 - p_1) = r^2$

$(\lambda\vec{d})^2 + (p_0 - p_1)^2 + 2\lambda\vec{d}(p_0 - p_1) - r^2 = 0$

since $\Delta = B^2 - 4AC$

Δ_2 for larger circle is $(2\vec{d}(p_0 - p_1))^2 - 4 * (\vec{d})^2((p_0 - p_1)^2 - {r_2}^2)$

Δ_1 for larger circle is $(2\vec{d}(p_0 - p_1))^2 - 4 * (\vec{d})^2((p_0 - p_1)^2 - {r_1}^2)$

if $\Delta < 0$, we can get that there is no solution for this equation, there is no intersection

if $\Delta = 0$, we can get that there is one solution for this equation, there is one intersection

if $\Delta > 0$, we can get that there are two solutions for this equation, there are two intersection points.

So,

if $\Delta_2 < 0$, there is no intersection point

if $\Delta_2 = 0$, there is only one intersection point

point p = $p_0 + \vec{d} * \frac{-2\vec{d}(p_0 - p_1)}{2 * (\vec{d})^2}$

if $\Delta_1 < 0$ and $\Delta_2 > 0$, then there are two intersection points p = $p_0 + \vec{d} * \frac{-2\vec{d}(p_0 - p_1) \mp \sqrt{\Delta_2}}{2*(\vec{d})^2}$

if Δ_1 = 0 and $\Delta_2 > 0$, then there are three intersection points

p = $p_0 + \vec{d} * \frac{-2\vec{d}(p_0 - p_1) \mp \sqrt{\Delta_2}}{2*(\vec{d})^2}$

p_3 = $p_0 + \vec{d} * \frac{-2\vec{d}(p_0 - p_1)}{2*(\vec{d})^2}$

if Δ_1 > 0 and $\Delta_2 > 0$, then there are three intersection points

p = $p_0 + \vec{d} * \frac{-2\vec{d}(p_0 - p_1) \mp \sqrt{\Delta_2}}{2*(\vec{d})^2}$

p = $p_0 + \vec{d} * \frac{-2\vec{d}(p_0 - p_1) \mp \sqrt{\Delta_2}}{2*(\vec{d})^2}$

4)

If the line and donut are both transformed by a non-uniform scale (S_x, S_y) around the origin, the location will transformed by (S_x, S_y), and the number of intersections will not change.

5)

we can first apply the scale on the donut, it becomes an ellipse, then we can use the method in 3), and get the location and the number of the intersections both of them will change.

3.

a) Not commute

Counter Example:

Let uniform scaling matrix be S = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let translate matrix be T = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$ST = \begin{matrix} 3 & 0 & 6 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{matrix} \qquad TS = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that ST≠TS, So, translate and uniform scaling are not equal

b) Not commute

Counter Example:

Let non-uniform scaling matrix be $S = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let translate matrix be $T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$ST = \begin{bmatrix} 3 & 0 & 6 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad TS = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that ST≠TS, So, translate and non-uniform scaling are not equal

c) there exists 2 cases:

1) uniform scaling: commute

Let uniform scaling matrix be $S = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let rotation matrix be $R = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$SR = \begin{bmatrix} mcos\theta & -msin\theta & 0 \\ msin\theta & mcos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad RS = \begin{bmatrix} mcos\theta & -msin\theta & 0 \\ msin\theta & mcos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that SR = RS, so scaling and rotation, both having the same fixed points are commute when it is uniform scaling

2) non-uniform scaling: not commute

Let non-uniform scaling matrix be $S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let rotation matrix be R = $\begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

SR = $\begin{bmatrix} 2cos\theta & -2sin\theta & 0 \\ 3sin\theta & 3cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ RS = $\begin{bmatrix} 2cos\theta & -3sin\theta & 0 \\ 2sin\theta & 3cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

We can see that whatever θ is, $-3sin\theta \neq -2sin\theta$, then SR ≠ RS. So scaling and rotation, both having the same fixed points(0,0) are not commute when it is non-uniform scaling

d) not commute

counter example:

Let the different fixed points be (1,0) and (0,1)

Let the scale matrix $S_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let the scale matrix $S_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$S_1 S_2 = \begin{bmatrix} 8 & 0 & -1 \\ 0 & 9 & -6 \\ 0 & 0 & 1 \end{bmatrix}$ $S_2 S_1 = \begin{bmatrix} 8 & 0 & -4 \\ 0 & 9 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

We can see that $S_1 S_2 \neq S_2 S_1$, so, scaling and scaling with different fixed point are not commute.

e) not commute

counter example:

Let translate matrix be T = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Let shear matrix be S = $\begin{bmatrix} 1 & 0 & 0 \\ m & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ST = $\begin{bmatrix} 1 & 0 & 2 \\ m & 1 & 2m+1 \\ 0 & 0 & 1 \end{bmatrix}$ TS = $\begin{bmatrix} 1 & 0 & 2 \\ m & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

We can see that ST≠TS, so translation and shearing are not commute

4.

a) choose a point q, if q satisfied three conditions:

1. the distance from v_0 to q is less than the distance from v_0 to v_1v_2
2. the distance from v_1 to q is less than the distance from v_1 to v_0v_2
3. the distance from v_2 to q is less than the distance from v_2 to v_1v_0

then q should inside the triangle

if q satisfied three conditions:

1. the segment from v_0 to q has intersection point with the segment from v_1 to v_2

OR

2. the segment from v_1 to q has intersection point with the segment from v_0 to v_2

OR

3. the segment from v_2 to q has intersection point with the segment from v_1 to v_0

Then q is outside the triangle

b)

based on part a, if q satisfied:

1. (the distance from q to v_1 plus the distance from q to v_2 equals to the length of v_1v_2)

OR

2. (the distance from q to v_0 plus the distance from q to v_2 equals to the length of v_0v_2)

OR

3. (the distance from q to v_1 plus the distance from q to v_0 equals to the length of $v_0 v_1$)

Then q should on the edge of triangle

c)

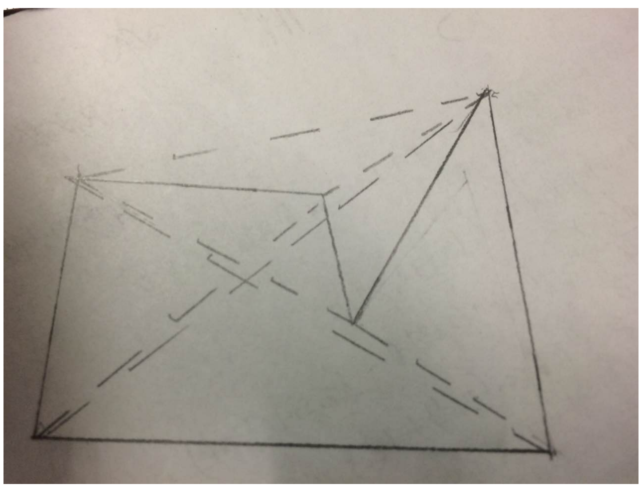
choose any two non-adjacent vertexes, connect them with each other, there exists two conditions:

1. if the new line is outside the quadrilateral, reject it.
2. if the new line is inside the quadrilateral, then we can see that the quadrilateral is combine of two triangles.

d)

for any vertex v, connect v with any other non-adjacent vertexes, then we can see that the polygon is combine of many triangles

e)



we can see that the method in d will failed, since many lines will appear outside, and the polygon cannot be triangular.

f)

Outside:

for segment s_1 between any two adjacent vertexes, if there exists a segment between q and other vertexes that intersects s_1, then q is outside of the polygon.

In:

For any two adjacent vertexes v_n and v_m, if the distance from v_n to q plus the distance form v_m to q equals to the distance from v_m to v_n, then q is on triangle

Inside:

Suppose for any v_m, v_{m-1}, they are adjacent vertexes.

Set $d_{v_a \to v_m v_{m-1}}$ means the distance from vertex v_a to segment $v_m v_{m-1}$

Let $S_{v_\mathrm{m}} = \{d_{v_m \to v_1 v_2}, d_{v_m \to v_2 v_3}, \ldots\ldots.. d_{v_m \to v_{m-1} v_{m-2}}, d_{v_m \to v_{m+1} v_{m+2}}, \ldots\ldots d_{v_m \to v_{n-1} v_n}\}$

Choose a point q, if q satisfied all the conditions:

1. for $d_{v0,q}$ (the distance from v_0 to q), $\max(S_{v_0}) > d_{v0,q}$
2. for $d_{v1,q}$ (the distance from v_1 to q), $\max(S_{v_1}) > d_{v1,q}$
3. for $d_{v2,q}$ (the distance from v_2 to q), $\max(S_{v_2}) > d_{v2,q}$

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n. for $d_{vn,q}$ (the distance from v_n to q), $\max(S_{v_\mathrm{n}}) > d_{vn,q}$