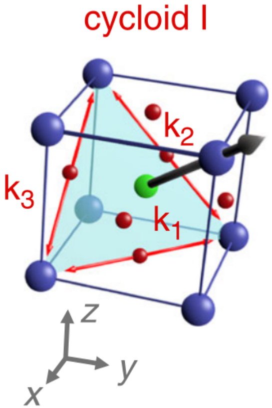
Analytic formulas BFO cycloids

BFO 001, cycloid type 1

Geometry



- $\vec{P}$ is along [111].
- $\vec{k_1}$ is in the surface plane, along $[1\bar{1}0]$.
- $\vec{k_2}$ and $\vec{k_3}$ are inside the film, along $[\bar{1}01]$ and $[0\bar{1}1]$.
- The layers containing the Fe atoms are vertically separated by a.

BFO 001, cycloid type 1 along $\vec{k_1}$, N layers

$$\begin{cases} B_x = \dfrac{\mu_0\, m_{\mathrm{DM}}}{\sqrt{3}V} e^{-kz} \dfrac{1-e^{-kaN}}{1-e^{-ka}} \sinh\left(\dfrac{ka}{2}\right) \sin\left(\dfrac{k}{\sqrt{2}}(x-y)\right) \\ B_y = -\dfrac{\mu_0\, m_{\mathrm{DM}}}{\sqrt{3}V} e^{-kz} \dfrac{1-e^{-kaN}}{1-e^{-ka}} \sinh\left(\dfrac{ka}{2}\right) \sin\left(\dfrac{k}{\sqrt{2}}(x-y)\right) \\ B_z = \sqrt{\dfrac{2}{3}} \dfrac{\mu_0\, m_{\mathrm{DM}}}{V} e^{-kz} \dfrac{1-e^{-kaN}}{1-e^{-ka}} \sinh\left(\dfrac{ka}{2}\right) \cos\left(\dfrac{k}{\sqrt{2}}(x-y)\right) \end{cases}$$

BFO 001, cycloid type 1 along $\vec{k_2}$, N layers

$$\begin{cases} B_x = -\dfrac{\mathcal{A}}{\sqrt{2}}\left(\mathrm{Re}\{\mathcal{S}\} - \mathrm{Im}\{\mathcal{S}\}\right) \\ B_y = 0 \\ B_z = \sqrt{2}\mathcal{A}\,\mathrm{Re}\{\mathcal{S}\} \end{cases}$$

$$\mathcal{A} = \frac{\mu_0 m_{\mathrm{DM}}}{\sqrt{3}V}\sinh\left(\frac{ka}{2\sqrt{2}}\right)$$

$$\mathcal{S} = e^{-kz/\sqrt{2}} e^{ik(x-z)/\sqrt{2}}\,\frac{1-e^{-kNa(1+i)/\sqrt{2}}}{1-e^{-ka(1+i)/\sqrt{2}}}$$

BFO 001, cycloid type 1 along $\vec{k_3}$, N layers

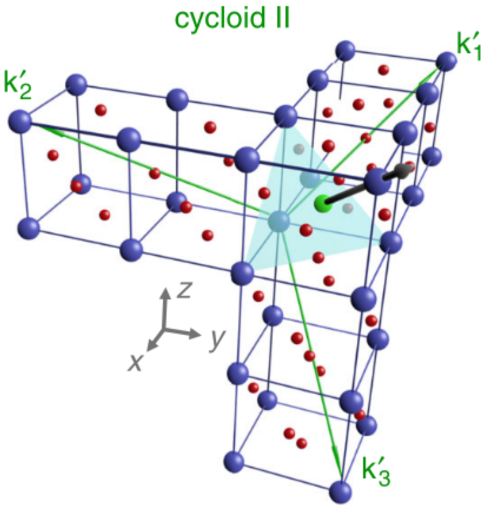
$$\begin{cases} B_x = 0 \\ B_y = -\dfrac{\mathcal{A}}{\sqrt{2}} \left(\mathrm{Re}\{\mathcal{S}\} - \mathrm{Im}\{\mathcal{S}\} \right) \\ B_z = \sqrt{2}\mathcal{A}\,\mathrm{Re}\{\mathcal{S}\} \end{cases}$$

$$\mathcal{A} = \frac{\mu_0 m_{\mathrm{DM}}}{\sqrt{3}V} \sinh\left(\frac{ka}{2\sqrt{2}} \right)$$

$$\mathcal{S} = e^{-kz/\sqrt{2}} e^{ik(y-z)/\sqrt{2}} \; \frac{1-e^{-kNa(1+i)/\sqrt{2}}}{1-e^{-ka(1+i)/\sqrt{2}}}$$

BFO 001, cycloid type 2

Geometry



- $\vec{P}$ is along [111].
- $\vec{k'_1}$ is along $[\bar{2}11]$, $\vec{k'_2}$ is along $[1\bar{2}1]$, they are equivalent.
- $\vec{k'_3}$ is along $[11\bar{2}]$, mostly out-of-plane.
- The layers containing the Fe atoms are vertically separated by a.

BFO 001, cycloid type 2 along $\vec{k_1'}$, N layers

$$\begin{cases} B_x = \sqrt{\frac{2}{5}}\mathcal{A}\left(\frac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_y = \frac{\mathcal{A}}{\sqrt{10}}\left(\frac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_z = \frac{\mathcal{A}}{\sqrt{10}}\left((\frac{1}{\sqrt{5}} + \sqrt{5})\,\mathrm{Re}\{\mathcal{S}\}\right) \end{cases}$$

$$\mathcal{A} = \tfrac{\mu_0 m_{\mathrm{DM}}}{V}\sinh\left(\sqrt{\tfrac{5}{6}}\tfrac{ka}{2}\right)$$

$$\mathcal{S} = e^{-\sqrt{\frac{5}{6}}kz} e^{\frac{ik}{\sqrt{6}}(-2x+y+z)}\ \frac{1-e^{-\frac{kNa}{\sqrt{6}}(\sqrt{5}-i)}}{1-e^{-\frac{ka}{\sqrt{6}}(\sqrt{5}-i)}}$$

BFO 001, cycloid type 2 along $\vec{k'_2}$, N layers

$$\begin{cases} B_x = \dfrac{\mathcal{A}}{\sqrt{10}}\left(\dfrac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_y = \sqrt{\dfrac{2}{5}}\mathcal{A}\left(\dfrac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_z = -\dfrac{\mathcal{A}}{\sqrt{10}}\left((\dfrac{1}{\sqrt{5}} + \sqrt{5})\,\mathrm{Re}\{\mathcal{S}\}\right) \end{cases}$$

$$\mathcal{A} = \frac{\mu_0 m_{\mathrm{DM}}}{V}\sinh\left(\sqrt{\frac{5}{6}}\frac{ka}{2}\right)$$

$$\mathcal{S} = e^{-\sqrt{\frac{5}{6}}kz}e^{\frac{ik}{\sqrt{6}}(x-2y+z)}\ \frac{1-e^{-\frac{kNa}{\sqrt{6}}(\sqrt{5}-i)}}{1-e^{-\frac{ka}{\sqrt{6}}(\sqrt{5}-i)}}$$

BFO 001, cycloid type 2 along $\vec{k}_3'$

The SDW is in-plane, no stray field!

BFO 001, cycloid type 2 along $\vec{k_3'}(\alpha)$

$$\begin{cases} B_x = \dfrac{\mathcal{A}}{\gamma}\left(\cos\alpha - \sqrt{3}\sin\alpha\right)\ \mathrm{Im}\{\mathcal{S}\} \\ B_y = \dfrac{\mathcal{A}}{\gamma}\left(\cos\alpha + \sqrt{3}\sin\alpha\right)\ \mathrm{Im}\{\mathcal{S}\} \\ B_z = \mathcal{A}\left(\mathrm{Re}\{\mathcal{S}\} - \dfrac{2\cos\alpha}{\gamma}\,\mathrm{Im}\{\mathcal{S}\}\right) \end{cases}$$

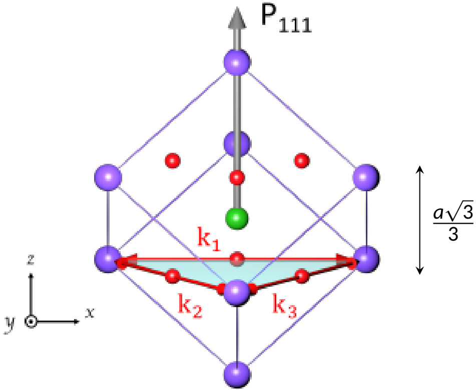
$$\vec{k_3'}(\alpha) = \frac{2\pi}{\lambda\sqrt{6}}\left[(\cos\alpha - \sqrt{3}\sin\alpha)\,\hat{e}_x + (\cos\alpha + \sqrt{3}\sin\alpha)\,\hat{e}_y - 2\cos\alpha\,\hat{e}_z\right]$$

$$\mathcal{A} = \frac{2\mu_0 m_{\mathrm{DM}}}{\sqrt{6}V}\ \sin\alpha\ \sinh\left(\frac{\gamma ka}{2\sqrt{6}}\right)e^{-\frac{\gamma k d_{\mathrm{NV}}}{\sqrt{6}}},\ \gamma = \sqrt{\cos^2\alpha + 3\sin^2\alpha}$$

$$\mathcal{S} = e^{i\vec{k_3'}(\alpha)\cdot\vec{r}}\ \frac{1 - e^{-\frac{kNa}{\sqrt{6}}(\gamma + 2i\cos\alpha)}}{1 - e^{-\frac{ka}{\sqrt{6}}(\gamma + 2i\cos\alpha)}}$$

BFO 111, cycloid type 1, $\vec{P}$ out-of-plane

Geometry



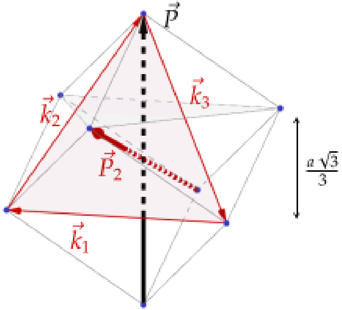
- $\vec{P}$ is normal to the surface, along $\hat{e}_z$.
- $\vec{k_1}$, $\vec{k_2}$ and $\vec{k_3}$ are in the surface plane and geometrically equivalent, we only need to compute one case. We take $\vec{k_1}$ along $\hat{e}_x$.
- The layers containing the Fe atoms are vertically separated by $\frac{a\sqrt{3}}{3}$.

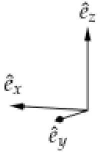
BFO 111, cycloid type 1 along $\hat{e_x}$, N layers, $\vec{P}$ out-of-plane

$$\begin{cases} B_x = 2\mu_0 \frac{m_s}{V} e^{-kz} \sinh\left(\frac{ka\sqrt{3}}{6}\right)\left(\frac{1-(-e^{-\frac{ka\sqrt{3}}{3}})^N}{1+e^{-\frac{ka\sqrt{3}}{3}}}\right)\cos(kx) \\ B_y = 0 \\ B_z = -2\mu_0 \frac{m_s}{V} e^{-kz} \sinh\left(\frac{ka\sqrt{3}}{6}\right)\left(\frac{1-(-e^{-\frac{ka\sqrt{3}}{3}})^N}{1+e^{-\frac{ka\sqrt{3}}{3x}}}\right)\sin(kx) \end{cases}$$

BFO 111, cycloid type 1, $\vec{P}$ mostly in-plane

Geometry





- $\vec{P}_2$ is along $\frac{2\sqrt{2}}{3}\hat{e}_y + \frac{\hat{e}_z}{3}$.
- We take $\vec{k}_1$ along $\hat{e}_x$.
- $\vec{k}_2$ is along $-\frac{\hat{e}_x}{2} - \frac{\hat{e}_y}{2\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{3}}\hat{e}_z$
- $\vec{k}_3$ is along $-\frac{\hat{e}_x}{2} + \frac{\hat{e}_y}{2\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}}\hat{e}_z$
- The layers containing the Fe atoms are vertically separated by $\frac{a\sqrt{3}}{3}$.

BFO 111, cycloid type 1 along $\vec{k_1}$, $\vec{P_2}$, N layers

$$
\begin{cases}
B_x = \dfrac{\mu_0}{V} e^{-kz} \sinh\left(\dfrac{ka\sqrt{3}}{6}\right) \left[m_s\left(1 - \dfrac{\varepsilon}{3}\right) \left(\dfrac{1 - \left(-e^{-\frac{ka\sqrt{3}}{3}}\right)^N}{1 + e^{-\frac{ka\sqrt{3}}{3}}}\right) \cos(kx) \right. \\
\qquad \left. + \dfrac{2\sqrt{2}}{3} m_{\mathrm{DM}} \left(\dfrac{1 - e^{-\frac{kaN\sqrt{3}}{3}}}{1 - e^{-\frac{ka\sqrt{3}}{3}}}\right) \sin(kx + \varphi) \right] \\
B_y = 0 \\
B_z = -\dfrac{\mu_0}{V} e^{-kz} \sinh\left(\dfrac{ka\sqrt{3}}{6}\right) \left[m_s\left(1 - \dfrac{\varepsilon}{3}\right) \left(\dfrac{1 - \left(-e^{-\frac{ka\sqrt{3}}{3}}\right)^N}{1 + e^{-\frac{ka\sqrt{3}}{3}}}\right) \sin(kx) \right. \\
\qquad \left. - \dfrac{2\sqrt{2}}{3} m_{\mathrm{DM}} \left(\dfrac{1 - e^{-\frac{kaN\sqrt{3}}{3}}}{1 - e^{-\frac{ka\sqrt{3}}{3}}}\right) \cos(kx + \varphi) \right]
\end{cases}
$$

BFO 111, cycloid type 1 along $\vec{k_2}$, $\vec{P_2}$, N layers

$$\begin{cases} B_x = \dfrac{\sqrt{3}\,\mathcal{A}}{2}\left[m_s(\sqrt{3}+\varepsilon)\left(\mathrm{Re}\{\mathcal{S}_1\} - \sqrt{2}\,\mathrm{Im}\{\mathcal{S}_1\}\right) + \sqrt{2}\,m_{\mathrm{DM}}\left(\sqrt{2}\,\mathrm{Re}\{\mathcal{S}_2\} + \mathrm{Im}\{\mathcal{S}_2)\}\right)\right] \\ B_y = \dfrac{\mathcal{A}}{2}\left[m_s(\sqrt{3}+\varepsilon)\left(\mathrm{Re}\{\mathcal{S}_1\} - \sqrt{2}\,\mathrm{Im}\{\mathcal{S}_1\}\right) + \sqrt{2}\,m_{\mathrm{DM}}\left(\sqrt{2}\,\mathrm{Re}\{\mathcal{S}_2\} + \mathrm{Im}\{\mathcal{S}_2)\}\right)\right] \\ B_z = \mathcal{A}\left[3\,m_s(\sqrt{3}+\varepsilon)\,\mathrm{Im}\{\mathcal{S}_1\} - \sqrt{2}\,m_{\mathrm{DM}}(2\sqrt{2}\,\mathrm{Im}\{\mathcal{S}_2\} + \mathrm{Re}\{\mathcal{S}_2\}\right] \end{cases}$$

$$\mathcal{A} = \frac{\mu_0}{3V}\sinh\left(\frac{ka}{6}\right)e^{-\frac{kz}{\sqrt{3}}}$$

$$\mathcal{S}_1 = e^{i\vec{k}_2\cdot\vec{r}}\,\frac{1-(-1)^N e^{-\frac{kNa}{3}(1-i\sqrt{2})}}{1+e^{-\frac{ka}{3}(1-i\sqrt{2})}},\ \ \mathcal{S}_2 = e^{i(\vec{k}_2\cdot\vec{r}+\varphi)}\,\frac{1-e^{-\frac{kNa}{3}(1-i\sqrt{2})}}{1-e^{-\frac{ka}{3}(1-i\sqrt{2})}}$$

BFO 111, cycloid type 1 along $\vec{k_3}$, $\vec{P_2}$, N layers

$$
\begin{cases}
B_x = \dfrac{\sqrt{3}\,\mathcal{A}}{2}\left[m_s(\sqrt{3}+\varepsilon)\left(\mathrm{Re}\{\mathcal{S}_1\}+\sqrt{2}\,\mathrm{Im}\{\mathcal{S}_1\}\right)-\sqrt{2}\,m_{\mathrm{DM}}\left(\sqrt{2}\,\mathrm{Re}\{\mathcal{S}_2\}-\mathrm{Im}\{\mathcal{S}_2)\}\right)\right]\\
B_y = -\dfrac{\mathcal{A}}{2}\left[m_s(\sqrt{3}+\varepsilon)\left(\mathrm{Re}\{\mathcal{S}_1\}+\sqrt{2}\,\mathrm{Im}\{\mathcal{S}_1\}\right)-\sqrt{2}\,m_{\mathrm{DM}}\left(\sqrt{2}\,\mathrm{Re}\{\mathcal{S}_2\}-\mathrm{Im}\{\mathcal{S}_2)\}\right)\right]\\
B_z = \mathcal{A}\left[3\,m_s(\sqrt{3}+\varepsilon)\,\mathrm{Im}\{\mathcal{S}_1\}-\sqrt{2}\,m_{\mathrm{DM}}(2\sqrt{2}\,\mathrm{Im}\{\mathcal{S}_2\}-\mathrm{Re}\{\mathcal{S}_2\})\right]
\end{cases}
$$

$$
\mathcal{A} = \frac{\mu_0}{3V}\sinh\left(\frac{ka}{6}\right)e^{-\frac{kz}{\sqrt{3}}}
$$

$$
\mathcal{S}_1 = e^{i\vec{k}_3\cdot\vec{r}}\,\frac{1-(-1)^N e^{-\frac{kNa}{3}(1+i\sqrt{2})}}{1+e^{-\frac{ka}{3}(1+i\sqrt{2})}},\quad \mathcal{S}_2 = e^{i(\vec{k}_3\cdot\vec{r}+\varphi)}\,\frac{1-e^{-\frac{kNa}{3}(1+i\sqrt{2})}}{1-e^{-\frac{ka}{3}(1+i\sqrt{2})}}
$$