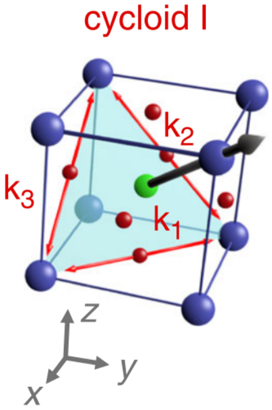
Analytic formulas BFO cycloids

BFO 001, cycloid type 1

Geometry



- $\vec{P}$ is along [111].
- $\vec{k_1}$ is in the surface plane, along $[1\bar{1}0]$.
- $\vec{k_2}$ and $\vec{k_3}$ are inside the film, along $[\bar{1}01]$ and $[0\bar{1}1]$.
- The layers containing the Fe atoms are vertically separated by a.

BFO 001, cycloid type 1 along $\vec{k_1}$, N layers

$$\begin{cases} B_x = \dfrac{\mu_0 \, m_{\mathrm{DM}}}{\sqrt{3}V} e^{-kz} \dfrac{1 - e^{-kaN}}{1 - e^{-ka}} \sinh\left(\dfrac{ka}{2}\right) \sin\left(\dfrac{k}{\sqrt{2}}(x - y)\right) \\ B_y = -\dfrac{\mu_0 \, m_{\mathrm{DM}}}{\sqrt{3}V} e^{-kz} \dfrac{1 - e^{-kaN}}{1 - e^{-ka}} \sinh\left(\dfrac{ka}{2}\right) \sin\left(\dfrac{k}{\sqrt{2}}(x - y)\right) \\ B_z = \sqrt{\dfrac{2}{3}} \dfrac{\mu_0 \, m_{\mathrm{DM}}}{V} e^{-kz} \dfrac{1 - e^{-kaN}}{1 - e^{-ka}} \sinh\left(\dfrac{ka}{2}\right) \cos\left(\dfrac{k}{\sqrt{2}}(x - y)\right) \end{cases}$$

BFO 001, cycloid type 1 along $\vec{k_2}$, N layers

$$\begin{cases} B_x = -\dfrac{\mathcal{A}}{\sqrt{2}} \left(\mathrm{Re}\{\mathcal{S}\} - \mathrm{Im}\{\mathcal{S}\}\right) \\ B_y = 0 \\ B_z = \sqrt{2}\mathcal{A}\,\mathrm{Re}\{\mathcal{S}\} \end{cases}$$

$$\mathcal{A} = \frac{\mu_0 m_{\mathrm{DM}}}{\sqrt{3}V} \sinh\left(\frac{ka}{2\sqrt{2}}\right)$$

$$\mathcal{S} = e^{-kz/\sqrt{2}} e^{ik(x-z)/\sqrt{2}} \, \frac{1-e^{-kNa(1+i)/\sqrt{2}}}{1-e^{-ka(1+i)/\sqrt{2}}}$$

BFO 001, cycloid type 1 along $\vec{k_3}$, N layers

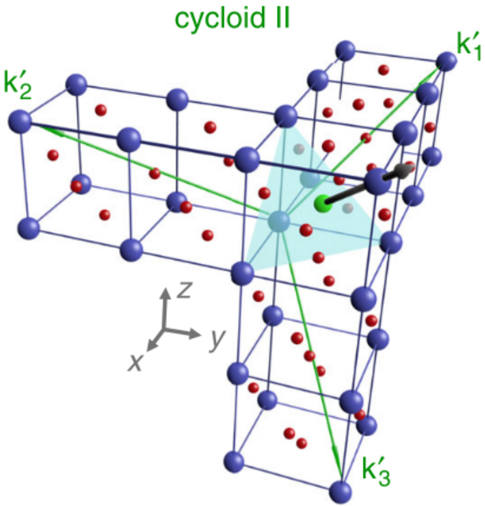
$$\begin{cases} B_x = 0 \\ B_y = -\dfrac{\mathcal{A}}{\sqrt{2}} \left(\mathrm{Re}\{\mathcal{S}\} - \mathrm{Im}\{\mathcal{S}\} \right) \\ B_z = \sqrt{2}\mathcal{A}\, \mathrm{Re}\{\mathcal{S}\} \end{cases}$$

$$\mathcal{A} = \frac{\mu_0 m_{\mathrm{DM}}}{\sqrt{3}V} \sinh\left(\frac{ka}{2\sqrt{2}}\right)$$

$$\mathcal{S} = e^{-kz/\sqrt{2}} e^{ik(y-z)/\sqrt{2}}\ \frac{1-e^{-kNa(1+i)/\sqrt{2}}}{1-e^{-ka(1+i)/\sqrt{2}}}$$

BFO 001, cycloid type 2

Geometry



- $\vec{P}$ is along [111].
- $\vec{k}'_1$ is along [$\bar{2}$11], $\vec{k}'_2$ is along [1$\bar{2}$1], they are equivalent.
- $\vec{k}'_3$ is along [11$\bar{2}$], mostly out-of-plane.
- The layers containing the Fe atoms are vertically separated by a.

BFO 001, cycloid type 2 along $\vec{k_1'}$, N layers

$$\begin{cases} B_x = \sqrt{\frac{2}{5}}\mathcal{A}\left(\frac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_y = \frac{\mathcal{A}}{\sqrt{10}}\left(\frac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_z = \frac{\mathcal{A}}{\sqrt{10}}\left((\frac{1}{\sqrt{5}} + \sqrt{5})\,\mathrm{Re}\{\mathcal{S}\}\right) \end{cases}$$

$$\mathcal{A} = \frac{\mu_0 m_{\mathrm{DM}}}{V}\sinh\left(\sqrt{\frac{5}{6}}\frac{ka}{2}\right)$$

$$\mathcal{S} = e^{-\sqrt{\frac{5}{6}}kz} e^{\frac{ik}{\sqrt{6}}(-2x+y+z)}\ \frac{1-e^{-\frac{kNa}{\sqrt{6}}(\sqrt{5}-i)}}{1-e^{-\frac{ka}{\sqrt{6}}(\sqrt{5}-i)}}$$

BFO 001, cycloid type 2 along $\vec{k'_2}$, N layers

$$\begin{cases} B_x = \dfrac{\mathcal{A}}{\sqrt{10}}\left(\dfrac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_y = \sqrt{\dfrac{2}{5}}\mathcal{A}\left(\dfrac{1}{\sqrt{5}}\,\mathrm{Re}\{\mathcal{S}\} + \mathrm{Im}\{\mathcal{S}\}\right) \\ B_z = -\dfrac{\mathcal{A}}{\sqrt{10}}\left((\dfrac{1}{\sqrt{5}} + \sqrt{5})\,\mathrm{Re}\{\mathcal{S}\}\right) \end{cases}$$

$$\mathcal{A} = \frac{\mu_0 m_{\mathrm{DM}}}{V}\sinh\left(\sqrt{\frac{5}{6}}\frac{ka}{2}\right)$$

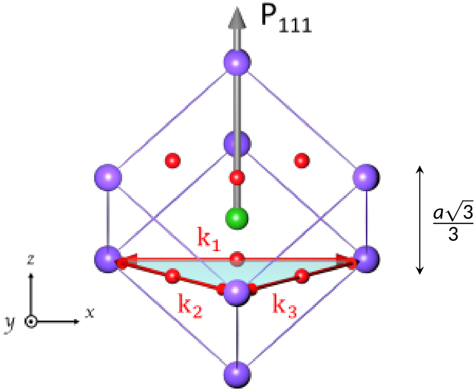
$$\mathcal{S} = e^{-\sqrt{\frac{5}{6}}kz}e^{\frac{ik}{\sqrt{6}}(x-2y+z)}\ \frac{1-e^{-\frac{kNa}{\sqrt{6}}(\sqrt{5}-i)}}{1-e^{-\frac{ka}{\sqrt{6}}(\sqrt{5}-i)}}$$

BFO 001, cycloid type 2 along $\vec{k_3'}$

The SDW is in-plane, no stray field!

BFO 111, cycloid type 1

Geometry



- $\vec{P}$ is normal to the surface, along $\hat{e}_z$.
- $\vec{k_1}$, $\vec{k_2}$ and $\vec{k_3}$ are in the surface plane and geometrically equivalent, we only need to compute one case. We take $\vec{k_1}$ along $\hat{e}_x$.
- The layers containing the Fe atoms are vertically separated by $\frac{a\sqrt{3}}{3}$.

BFO 111, cycloid type 1 along $\hat{e_x}$, N layers

$$\begin{cases} B_x = 2\mu_0 \frac{m_s}{V} e^{-kz} \sinh\left(\frac{ka\sqrt{3}}{6}\right) \left(\frac{1-(-e^{-\frac{ka\sqrt{3}}{3}})^N}{1+e^{-\frac{ka\sqrt{3}}{3}}}\right) \cos(kx) \\ B_y = 0 \\ B_z = -2\mu_0 \frac{m_s}{V} e^{-kz} \sinh\left(\frac{ka\sqrt{3}}{6}\right) \left(\frac{1-(-e^{-\frac{ka\sqrt{3}}{3}})^N}{1+e^{-\frac{ka\sqrt{3}}{3x}}}\right) \sin(kx) \end{cases}$$