NATIONAL UNIVERSITY OF SCIENCES AND TECHNOLOGY (NUST)
SCHOOL OF MECHANICAL AND MANUFACTURING ENGINEERING (SMME)

Fluid Mechanics I

Assignment I

Design of a Modular Rainwater Harvesting System with Flood Gate and Irrigation Jet

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ME-16, Section C

1. Introduction:

This report consists of the design and theoretical analysis of a modular rainwater harvesting basin intended for urban application. The primary objectives of this design are to ensure structural integrity under maximum hydrostatic loading and to achieve a targeted irrigation range through a controlled free jet. The analysis is based on the fundamental principles of Fluid Statics for the retention gate and Fluid Dynamics (Bernoulli's Principle) for the orifice's discharge.

The modular system is designed according to the unique boundary conditions assigned in Table 1. These parameters define the geometric limits and performance targets for the basin and gate assembly:

Width, W (m)	0.70
Height, H (m)	2.37
Height of Gate, H_G (m)	1.3
Gate submersion depth, d (m)	0.87
Orifice Height, z_O (m)	0.21
Target Range, R (m)	3.4
Diameter of Orifice (mm)	29~49

Assumptions:

- Fluid flow is steady.
- Fluid is assumed to be incompressible fresh water.
- Fluid is assumed to be inviscid.
- Density (ρ) is 998 kg/m^3.
- Temperature of water is 20^{o}C.

The analysis is divided into two critical phases:

Hydrostatic Analysis: The flood gate is analysed under worst-case conditions (**fully filled basin**). Calculations indicated that a substantial hydrostatic force (**13.54 kN**) would be acting on the gate. To address this, a dual pulley system has been integrated into the design, alongside a siphoning tube system to act as a fail-safe for overflow regulation to provide the necessary mechanical advantage for operation. **Aluminum 6061-T6** will be used as the flood gate's material, providing ample strength, all while being eco friendly and cost effective, it also avoids rusting.

Free Jet Analysis: The irrigation jet is analysed assuming steady, inviscid flow. Due to the geometric constraint of a low orifice height (**z_o=0.21 m**) relative to the required range (**R=3.4 m**), the design includes a stand to elevate the basin. This ensures the trajectory meets the target distance.

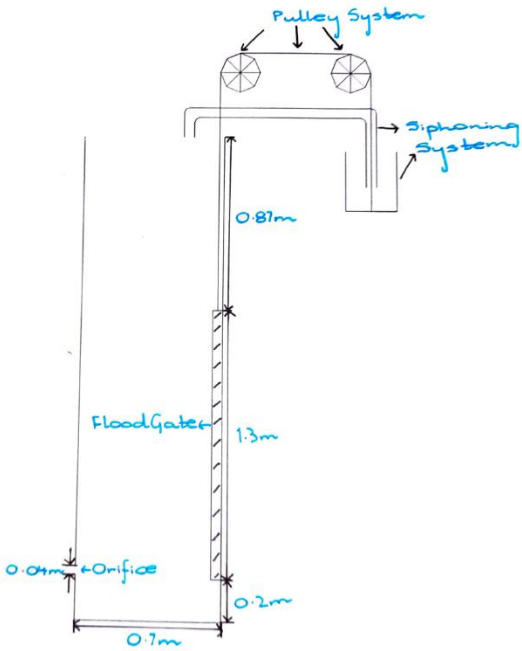


Figure 1.1: Modular Rainwater Harvesting System Design

This report includes details of the mathematical calculations of these forces, the resulting trajectory profiles, and the optimization of the orifice diameter (**selected as 40 mm**) along with the use of Python for Graphical representation.

2. Hydrostatic Force Analysis:

To accurately model the relationship between water depth and hydrostatic load, the analysis was performed computationally using the **Python** programming language. The scientific computing library **NumPy** was utilized to calculate the Hydrostatic Force and Moment across a continuous range of water depths. The resulting data was visualized using **Matplotlib**. This analysis was done after the completion of the mathematical calculations to verify them and to visualize the results.

Step 1: Calculate Area (A)

$$\mathrm{A} = \mathrm{W} \times \mathrm{h_g}$$

$$\mathrm{A} = 0.70\ \mathrm{m} \times 1.30\ \mathrm{m}$$

$$\mathbf{A = 0.91\ m^2}$$

Step 2: Calculate Depth to Centroid ($\bar{h}$)

The centroid is at the geometric centre of the gate.

$$\bar{\mathrm{h}} = \mathrm{d} + \frac{\mathrm{h_g}}{2}$$

$$\bar{\mathrm{h}} = 0.87 + \frac{1.30}{2}$$

$$\bar{\mathbf{h}} = \mathbf{1.52}$$

Step 3: Calculate Hydrostatic Force (F)

Using Equation 4.1:

$$\mathrm{F} = \rho \mathrm{g} \mathrm{A} \bar{\mathrm{h}}$$

$$\mathrm{F} = (998)(9.81)(0.91)(1.52)$$

$$\mathbf{F = 13,541\ N = 13.54\ kN}$$

Step 4: Calculate Center of Pressure (y_{cp})

To find the moment, we need the exact location of the force.

$$\mathbf{y_{cp} = \bar{h} + \frac{\bar{h}A}{I_{xc}}}$$

Where:

$$I_{xc} = \frac{1}{12} W h_g^3$$

$$I_{xc} = \frac{1}{12}(0.7)(1.3^3)$$

$$\mathbf{I_{xc} = 0.128}$$

$$y_{cp} = 1.52 + \frac{1.52 \times 0.91}{0.128}$$

$$y_{cp} = 1.52 + 0.093$$

$$\mathbf{y_{cp} = 1.613\ m}\ \text{(for surface)}$$

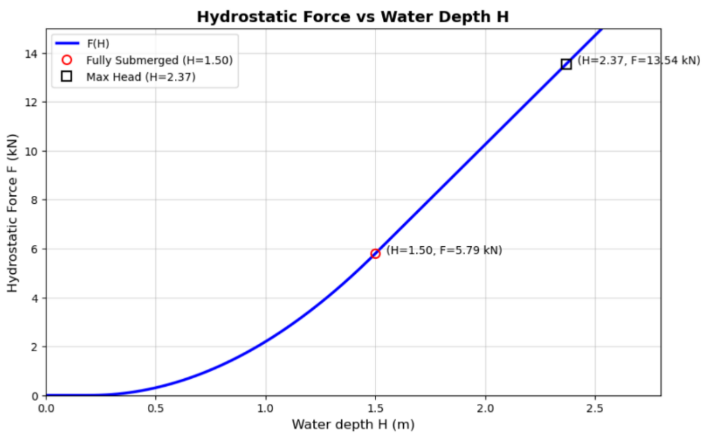
Step 5: Moment about Top Hinge (M)

The point at **d = 0.87 m** is assumed to be a hinge.

$$\text{Moment Arm} = y_{cp} - d = 1.613 - 0.87 = 0.743\ \text{m}$$

$$M = F \times \text{Arm} = 13.54\ \text{kN} \times 0.743\ \text{m}$$

$$\mathbf{M = 10.06\ k} \qquad \textbf{N/m}$$



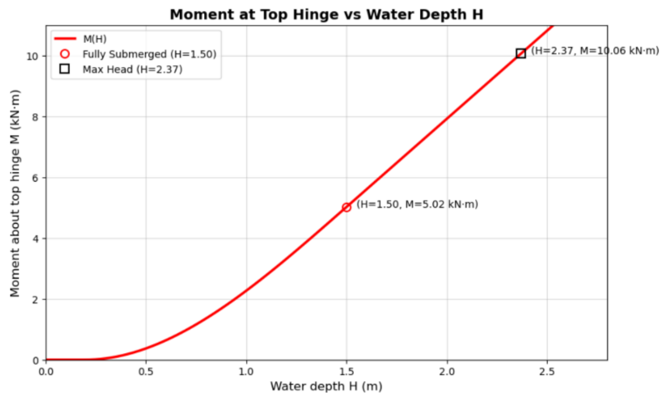


Figure 2.1: Hydrostatic Force (F) vs Water Depth (H)

Figure 2.2: Moment about Top Hinge (M) vs Water Depth (H)

- **Maximum Hydrostatic Force:** 13.54 kN (**at full head**)
- **Maximum Moment at Top Hinge:** 10.05 kN.m (**at full head**)

3. Free Jet Design using Bernoulli:

To achieve the targeted irrigation range of $R = 3.4$ m, the system was analysed using Bernoulli's Principle and kinematic equations for projectile motion. The analysis assumes steady, inviscid flow with negligible losses at the orifice exit.

Exit Velocity Calculation:

Applying Bernoulli's equation between the free surface of the basin and the orifice exit:

$$\boldsymbol{P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2}$$

Boundary Conditions:

- $P_1 = P_2 = P_{\text{atm}}$(Atmospheric pressure cancels out)
- $V_1 \approx 0$(Velocity at the basin surface is negligible)
- $z_1 - z_2 = H - z_o$(Hydrostatic head driving the flow)

Substituting design parameters $H = 2.37$ m, $z_o = 0.21$ m:

$$\mathrm{V_{jet}} = \sqrt{2\mathrm{g}(\mathrm{H} - \mathrm{z_o})}$$

$$\mathrm{V_{Jet}} = 2(9.81)(2.37 - 0.21)$$

$$\mathbf{V_{Jet} = 6.51\ m/s}$$

Trajectory Analysis and Design Optimization:

The trajectory of the horizontal free jet is given by:

$$y = x\tan(\theta) - \frac{gx^2}{2V_{\text{jet}}^2\cos^2(\theta)}$$

For a *horizontal* jet ($\theta = 0°$):

$$y = -\frac{gx^2}{2V_{\text{jet}}^2}$$

Initial Feasibility Check (No Elevation):

If the basin sits on the ground, the available vertical drop is the orifice height ($z_o = 0.21$ m). Solving for range:

$$x = \sqrt{\frac{2V_{\text{jet}}^2 y}{g}}$$

Optimization (Structural Stand):

To meet the requirement $R = 3.4$ m, the system must be elevated.

Required vertical drop:

$$y_{\text{req}} = \frac{gR^2}{2V_{\text{jet}}^2}$$

$$y_{\text{req}} = \frac{9.81(3.4)^2}{2(6.51)^2}$$

$$\boldsymbol{y_{\text{req}} \approx 1.34\ \text{m}}$$

Since the orifice is already at 0.21 m:

$$h_{\text{stand}} = y_{\text{req}} - z_o = 1.34 - 0.21 = 1.13\ \text{m}$$

Design Decision:

A **1.15 m** structural stand is selected to ensure the jet reaches the target distance with margin.

Volumetric Flow Rate:

Orifice diameter: $D = 40 \text{ mm} = 0.04 \text{ m}$

$$A_{\text{orifice}} = \frac{\pi D^2}{4}$$

$$A_{\text{orifice}} = \frac{\pi (0.04)^2}{4}$$

$$\boldsymbol{A_{\text{orifice}} = 1.257 \times 10^{-3}\, m^2}$$

$$Q = AV_{\text{jet}} = (1.257 \times 10^{-3})(6.51)$$

$$\boldsymbol{Q = 0.0082 \frac{m^3}{s} = 8.2 \frac{L}{s}}$$

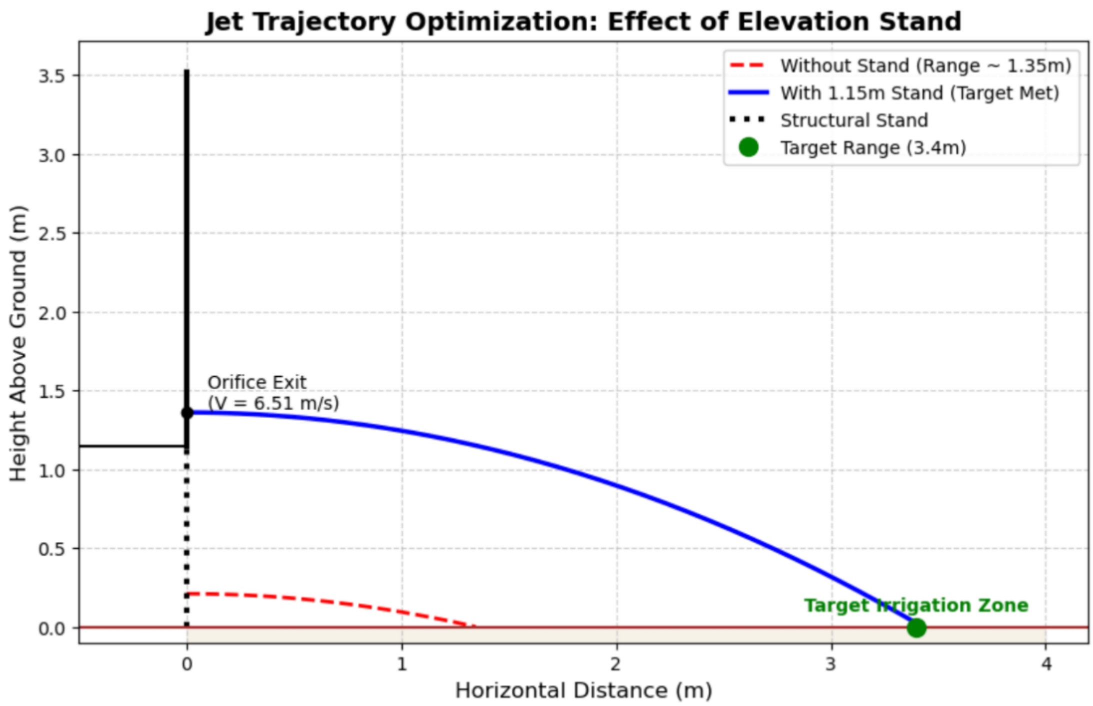


Figure 3.1: Height above Ground vs Horizontal Distance

4. Integration and Optimization:

Integration and Optimization:

The design of the rainwater harvesting system represents an engineering problem where a single variable, the water depth, dictates two performance metrics:

- Structural load on the flood gate.
- Range of the irrigation jet.

Siphoning System:

To regulate the maximum hydraulic head within the basin, a siphoning mechanism was integrated as a fail-safe measure. This system was selected primarily for its reliability, unlike active electromechanical valves or sensors, the siphon operates entirely on fluid physics and requires no external power source ensuring that overflow protection remains fully functional even during severe storm conditions where power outages are common.

The critical trade-off introduced by this subsystem is Safety versus Retention Capacity. By automatically decreasing the water level once it exceeds the design limit, the system deliberately sacrifices potential water storage volume to guarantee that the hydrostatic forces never escalate to a point that would compromise the structural integrity of the flood gate.

Geometric Optimization: The Structural Stand:

To resolve the conflict between high force and required range, the design utilizes **Potential Energy (Elevation)** rather than the **Pressure Energy**.

By mounting the basin on a **1.15 m structural stand**, we achieve the target range without increasing the water depth.

- **Without Stand:** Range depends solely on hydrostatic pressure.
- **With Stand:** Range utilizes gravitational acceleration during the extended drop.

This approach prioritizes safety without sacrificing performance by optimizing the system to keep the hydrostatic force constant while extending the kinematic trajectory.

Operational Optimization: Orifice Diameter Selection:

The assigned constraint for the orifice diameter was 29 to 49 mm. While the diameter does not theoretically affect the exit velocity, it directly controls the **Flow Rate** and the **Irrigation Duration**.

Hence optimization was done for a diameter of **40 mm**.

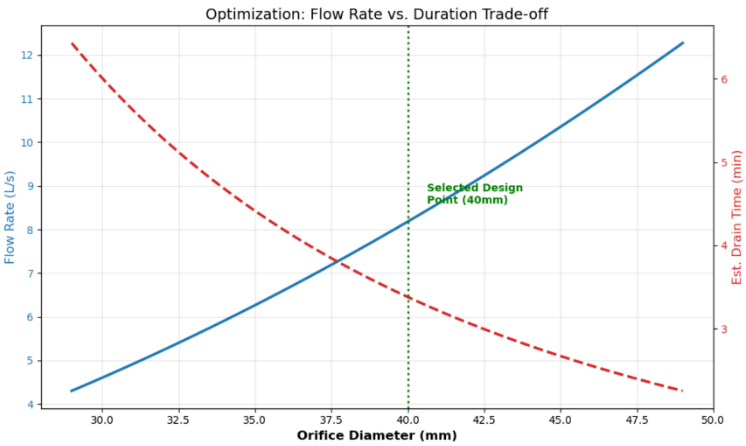
Trade-off Analysis:

1. **Maximum Diameter (49 mm):** High flow rate but the basin drains too quickly, causing the head to drop rapidly. This results in the jet range decreasing from the target zone in a short span of time
2. **Minimum Diameter (29 mm):** Low flow rate but the jet is able to sustain the target range for longer, but the irrigation volume may be insufficient for the vegetation.
3. **Selected (40 mm):** Flow rate is nor too high, neither too low, it creates the perfect balance between the two metrics. It provides a robust stream that resists wind deflection (**unlike smaller jets**) while maintaining the necessary head for a practical duration.

Energy Grade Line (EGL) and Hydraulic Grade Line (HGL):

The fluid energy transformation is visualized in the EGL/HGL plot (Figure 4.2).

- **EGL (Energy Grade Line):** Remains constant at m (assuming negligible friction), representing the total available energy.
- **HGL (Hydraulic Grade Line):** Tracks the potential pressure energy. Inside the tank, it coincides with the water surface. As the fluid accelerates through the orifice, pressure energy is converted into kinetic energy, causing the HGL to drop sharply to the centre line of the orifice.



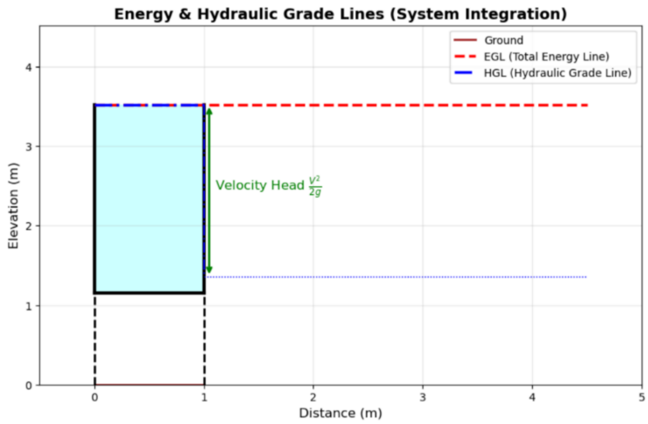


Figure 4.1: Optimization Trade-Off Graph

Figure 4.2: Energy and Hydraulic Grade Lines

5. Discussion and Limitations:

Limitations of Theoretical Assumptions:

The design analysis relied on the Bernoulli equation and projectile motion kinematics, which introduce several idealizations that may cause the actual system performance to deviate from the calculated values:

- **Viscosity and Friction Losses:** The analysis assumed the fluid is inviscid (viscosity). In reality, water at has dynamic viscosity. Friction at the orifice edges and internal turbulence will generate minor head losses, slightly reducing the exit velocity.
- **Aerodynamic Drag:** The trajectory calculation ignored air resistance. As the jet travels through the air, aerodynamic drag will oppose motion, causing the jet to break up and hitting the ground slightly earlier than the predicted.
- **Unsteady Flow:** The calculations assumed steady flow at maximum head. In reality, as water discharges, the head decreases, causing a continuous drop in exit velocity. The target range is only valid while the tank is near full capacity, as the level drops, the jet range will recede due to decreasing velocity.

Operational Factors and Errors:

- **Vena Contracta Effect:** The discharge through a sharp-edged orifice undergoes contraction, forming a ***Vena Contracta*** where the jet diameter is smaller than the orifice diameter. While we optimized for a 40 mm physical opening, the effective flow area will be smaller. This reduces the volumetric flow rate, though it does not significantly reduce the velocity.

Proposed Improvements and Optimization:

To bridge the gap between the theoretical model and real-world application, the following improvements are recommended:

1. **Convergent Nozzle Implementation:** Replacing the sharp-edged orifice with a rounded, convergent nozzle would minimize the ***Vena Contracta*** effect. This would maintain a continuous jet stream for a longer distance, countering the effects of air resistance.
2. **Automated Actuation:** The current design uses a manual dual pulley system to overcome the 13.54 kN load. A linear hydraulic actuator or a lead-screw mechanism could be integrated to allow for automated opening when water levels trigger a float sensor.
3. **Diffuser Plate:** To prevent soil erosion at the impact point (3.4 m away), a splash pad or diffuser stones should be installed in the garden bed to dissipate the jet's kinetic energy upon landing.
4. **Corrosion and Material Durability:** The hydrostatic analysis confirmed a significant load of **13.54 kN** acting on the gate. To withstand this force while minimizing maintenance, **Aluminum Alloy 6061-T6** was selected instead of coated steel. Unlike steel, which relies on galvanization or toxic marine-grade epoxy to prevent degradation, aluminum naturally forms a microscopic **passive oxide layer** upon exposure to oxygen. This barrier makes the gate immune to freshwater corrosion and rust, ensuring it maintains the necessary structural rigidity to resist the calculated hydrostatic load over its entire service life without requiring re-coating.

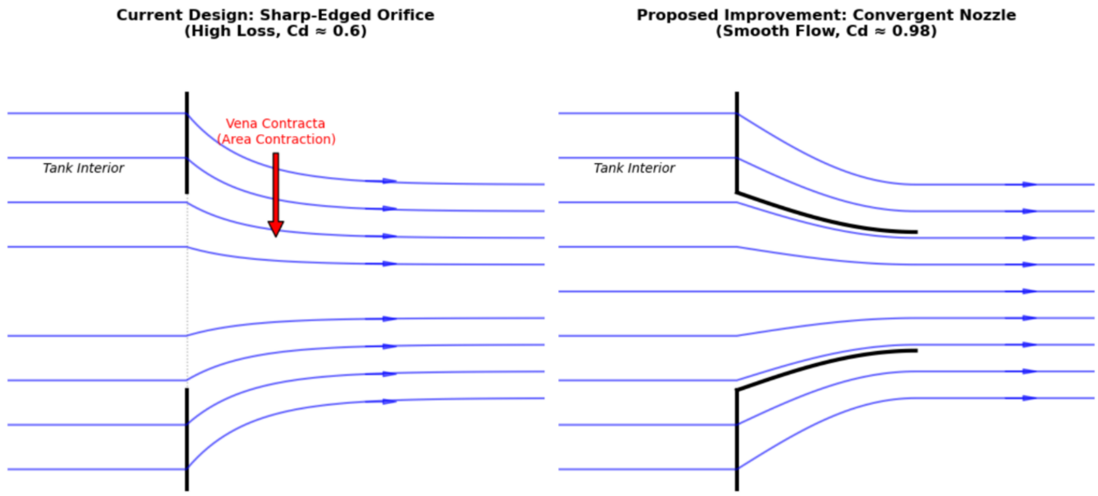


Figure 5.1: Vena Contracta vs. Nozzle Schematic

6. References:

1. Munson's Fluid Mechanics Global Edition
2. Assignment Guidelines