EE

Feuille 1

Exercice 1.1

α est non nulle, donc, $\dim \mathrm{Im}(\alpha) \geq \cancel{0}\, 1$, $Rg(\alpha) \geq 1$.

Il existe $v \in V$ avec $\alpha(v) \neq 0$.
Soit $\lambda \in K$, alors $\alpha\left(\frac{\lambda}{\alpha(v)} v\right) = \lambda$. Donc α est surjective :
$\Rightarrow \mathrm{Im}(\alpha) = K$.
~~Rang~~ $\mathrm{Rang}(\alpha) = 1$.

($\Rightarrow$ $\mathrm{Ker}(\alpha)$ est hyperplan).

D'après le théorème du Rang : $\dim \mathrm{Ker}(\alpha) = n-1$.

Exercice 1.2

On cherche $C^* = \{\varphi_1, \varphi_2, \varphi_3\}$.
avec $C = \{f_1, f_2, f_3\}$.

a/ φ_1 est de $\mathbb{R}^3 \to \mathbb{R}$. $\varphi_1(f_i) = \begin{cases} 1 & i = 1 \\ 0 & i \neq 1 \end{cases}$

$$\begin{cases} \alpha + \gamma = 1 \\ \alpha + \beta = 0 \\ -\beta + 2\gamma = 0 \end{cases}$$

$$\varphi_1(x_1, x_2, x_3) = \alpha x_1 + \beta x_2 + \gamma x_3.$$

$$\Rightarrow \begin{cases} \alpha = 1 - \gamma \\ 1 + \beta = \gamma \\ \beta + 2 = 0 \end{cases} \Rightarrow \begin{cases} \alpha = 2 \\ \gamma = -1 \\ \beta = -2 \end{cases}$$

$$\boxed{\varphi_1(x_1, x_2, x_3) = \cancel{2x_1 + (-2)}\; 2x_1 - 2x_2 - x_3.}$$

$$\varphi_2(x_1, x_2, x_3) = \alpha x_1 + \beta x_2 + \gamma x_3. \qquad \begin{cases} \alpha + \gamma = 0 \\ \alpha + \beta = 1 \\ -\beta + 2\gamma = 0 \end{cases}$$

$$\begin{cases} \alpha = -\gamma \\ -\gamma + \beta = 1 \\ -\beta + 2\gamma = 0 \end{cases} \Rightarrow \begin{cases} \alpha = -\gamma \\ \beta = 1 + \gamma \\ -1 + \gamma = 0 \end{cases} \Rightarrow \begin{cases} \alpha = -1 \\ \beta = \cancel{0}\, 2 \\ \gamma = 1 \end{cases}$$

$$\boxed{\varphi_2(x_1, x_2, x_3) = -x_1 + x_3 + 2x_2.}$$

$$\varphi_3(x_1, x_2, x_3) = \alpha x_1 + \beta x_2 + \gamma x_3 \Rightarrow \begin{cases} \alpha + \overset{\gamma}{\beta} = 0 \\ \alpha + \beta = 0 \\ -\beta + 2\gamma = 1 \end{cases}$$

$$\Rightarrow \begin{cases} \alpha = -\gamma \\ -\gamma + \beta = 0 \\ -\beta + 2\gamma = 1 \end{cases} \Rightarrow \begin{cases} \alpha = -\gamma \\ \beta = \gamma \\ 2\gamma = 1 + \beta. \end{cases} \Rightarrow \begin{cases} \alpha = -\gamma \\ \beta = \gamma \\ 2\beta = 1 + \beta. \end{cases}$$

$$\Rightarrow \begin{cases} \alpha = -\gamma \\ \beta = \gamma \\ \beta = 1 \end{cases} \Rightarrow \begin{cases} \alpha = -1 \\ \beta = 1 \\ \gamma = 1 \end{cases}$$

$$\boxed{\varphi_3(x_1, x_2, x_3) = -x_1 + x_2 + x_3.}$$

b/ $\{\phi\}_{\mathcal{B}^*} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ $\qquad \phi = \varepsilon_2 + 2\varepsilon_3.$

$$\{\phi\}_{\mathcal{C}^*} = \{id_{\mathbb{R}^{3*}}\}_{\mathcal{C}^*}^{\mathcal{B}^*} \{\phi\}_{\mathcal{B}^*}$$

$$= \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$\boxed{\phi = 2\varphi_1 + \varphi_2 + 3\varphi_3.}$$