Optimisation - Encyclopedia Academia

Optimisation

Optimisation	
Richting	Wiskunde
Jaar	3BWIS

Algemeen

Dit vak bestaat uit hoorcolleges door prof. Ahookhosh waar de theorie aan bod komt en practicumsessies gegeven door de assistent Jeffrey Cornelis. In de practicumsessies implementeer je de algoritmes die in de theorie aan bod komen in Matlab om meer inzicht te verwerven. Deze oefeningensessies zijn op zich niet belangrijk voor het examen, maar het kan zijn dat er een taak komt waarbij je deze oefening kan gebruiken. (Masoud had dit beloofd, maar er is uiteindelijk toch geen taak gekomen.)

Probeer bij het studeren te zorgen dat je goed overweg kan met de basisbegrippen uit de cursus zoals: sublevelset, minimizers, sufficient descent conditions, en convergentie van algoritmes. Het examen zelf valt beter mee dan de cursus doet uitschijnen.

Examenvragen

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1. Let $f:\mathbb{R}^n\to\mathbb{R}$ be a continuous on a compact sublevelset $S(f,r)$ ($r\in\mathbb{R}$).
   (a) Show that: f has a global minimizer and maximizer over $S(f,r)$.

   (b) Show that: f has a global minimizer over $\mathbb{R}^n$

   (c) Let g be a continuous coercive function over $\mathbb{R}^n$. Show that g has atleast one global minimizer.

2. Prove the Second-order sufficient optimality conditions: Let $f:\mathbb{R}^n\to\mathbb{R}$ a twice continuous differentiable function over an open neighbourhood $B(x^*,r)$ for a point x^* and $r>0$ and let $\nabla f(x^*)=0$ and $\nabla^2 f(x^*)$ be positive definite. Then, x^* is a strict local minimizer of f.

3. Let $f:Rn \to Rf:Rn \to R$ be continuously differentiable with Lipschitz continuous gradients ($f \in C1,1Lf \in CL1,1$).
   (a) Show that if $dkdk$ satisfies the sufficiënt descent condition ($\langle \nabla f(xk),dk \rangle \leq -c1||\nabla f(xk)||2\langle \nabla f(xk),dk \rangle \leq -c1||\nabla f(xk)||2$, $||dk|| \leq c2||\nabla f(xk)||||dk|| \leq c2||\nabla f(xk)||$), it follows that: $f(xk+\alpha kdk) \leq f(xk)-\alpha(c1-c22L2\alpha)||\nabla f(xk)||2f(xk+\alpha kdk) \leq f(xk)-\alpha(c1-c22L2\alpha)||\nabla f(xk)||2$.

   (b) Show that if $\alpha \in (0,2c1Lc22)\alpha \in (0,2c1Lc22)$ that $f(xk+1) \leq f(xk)f(xk+1) \leq f(xk)$.

   (c) Show that the maximum decrease is attained by $\hat{}\alpha=c1Lc22\alpha\hat{}=c1Lc22$.

4. Let $f:Rn \to Rf:Rn \to R$ be a twice continuously differentiable function, $x0x0$ a point close enough to the minimizer $x*x*$ satisfying $\nabla f(x*)=0\nabla f(x*)=0$. If the Hessian $\nabla 2f(x*)\nabla 2f(x*)$ is positive definite and $\nabla 2f(x)\nabla 2f(x)$ satisfies the Lipschitz condition $||\nabla 2f(x)-\nabla 2f(y)|| \leq L||x-y||||\nabla 2f(x)-\nabla 2f(y)|| \leq L||x-y||$.
   (a) Show that the Newton iteration $xk+1=xk-(\nabla 2f(xk))-1\nabla f(xk)xk+1=xk-(\nabla 2f(xk))-1\nabla f(xk)$ is well-defined.

   (b) Show that the series $\{xk\}k \geq 0\{xk\}k \geq 0$ converges to $x*x*$ QQ-quadratically, i.e. $||xk+1-x*||||xk-x*|| \leq R||xk+1-x*||||xk-x*|| \leq R$ for some $R>0R>0$.

5. Let $f(x)=100(x2-x21)2+(1-x1)2f(x)=100(x2-x12)2+(1-x1)2$ and $g(x)=x41-4x1x2+x42g(x)=x14-4x1x2+x24$.
   (a) Compute the gradients and Hessians of ff and gg.

   (b) Find the critical points of ff and gg.

   (c) Identify the minimizers, maximizers and saddle points of ff and gg.

   (d) Determine whether they are global or local. (provide reasoning)

6. Consider the quadratic minimization funtion $f:Rn \to Rf:Rn \to R$ given by $f(x)=\langle p,x \rangle +12\langle x,Qx \rangle f(x)=\langle p,x \rangle +12\langle x,Qx \rangle$ for the vector $p \in Rnp \in Rn$ and the positive definite matrix $Q \in Rn \times nQ \in Rn \times n$.
   (a) Derive the formula for exact line search step-size $\alpha k\alpha k$ in the Conjugate Gradient Algorithm.

   (b) Let $d0=-\nabla f(x0)d0=-\nabla f(x0)$, $xk+1=xk+\alpha kdkxk+1=xk+\alpha kdk$, $dk=-\nabla f(xk)+\beta k-1dk-1dk=-\nabla f(xk)+\beta k-1dk-1$. Derive $\beta 0\beta 0$ such that $d0d0$ and $d1d1$ are GG-conjugate.

   (c) Let $x0=[00]x0=[00]$, $p=[10]p=[10]$, $G=[1002]G=[1002]$ Compute the first two iterations $x1,x2x1,x2$.