Lichamen en klassieke Galoistheorie

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Galoistheorie	
Richting	Wiskunde
Jaar	3BWIS

Academiejaar 2021-2022

De lessen werden dit jaar uitzonderlijk overgenomen door prof. Fred van Oystaeyen, omdat iemand Boris moest vervangen. De leerstof van dit examen kan daarom afwijken van die van andere jaren.

Examen juni 2021-2022

Theorie

Het theorie-eamen was mondeling. Je mocht één vraag zelf kiezen, en je kon ook een werkje maken dat als tweede vraag telde. De rest van de vragen koos Fred. Vooral vragen uit latere delen van de cursus kwamen aan bod. Ondanks dat Fred moeilijke vragen stelde, was hij wel gul in het geven van punten.

Oefeningen (Marco)

1. Prove or disprove the following statements:
    1. {1,21/3,21/2,22/3,25/6,27/6}{1,21/3,21/2,22/3,25/6,27/6} is a QQ-basis of a normal closure of Q(21/3,21/2)/QQ(21/3,21/2)/Q.
    2. Q[X]/(X4+1)/QQ[X]/(X4+1)/Q is radical and cyclic.
    3. Let KK be a field and let AA be a KK-algebra. For a∈Aa∈A the map A→AA→A that sends b∈Ab∈A to ab−baab−ba is a KK-derivation.
2. Let f(X)=X4−5X2+9∈Q[X]f(X)=X4−5X2+9∈Q[X], and let L⊆CL⊆C be the splitting field of ff. Determine a QQ-basis for LL, the degree and Galois group of L/QL/Q. Compute how many intermediate fields extensions of degree 2 between QQ and LL there are up to isomorfism.
3. Let x,y,z∈N0x,y,z∈N0 pairwise coprime, such that x2+y2=z2x2+y2=z2
    1. Let NN be the norm of Q(i)/QQ(i)/Q. Show that N(xz+iyz)=1N(xz+iyz)=1.
    2. Show that there exist m,n∈Nm,n∈N such that {x,y}={m2−n2,2mn}{x,y}={m2−n2,2mn}.
4. For any k∈N0k∈N0, set ζk=e2πik∈Cζk=e2πik∈C, and Kk=Q(ζk)Kk=Q(ζk). Let m,n∈Nm,n∈N coprime. Show that KmKm and KnKn are linearly disjoint over QQ, that Q(ζm,ζn)=KmnQ(ζm,ζn)=Kmn and that Km∩Kn=QKm∩Kn=Q. Furthermore, find m′,n′∈Nm′,n′∈N such that Km′Km′ and Kn′Kn′ are not linearly disjoint.
5. Let FF be a finite field and with characteristic p∈Np∈N. Let LL be an algebraic closure of F(t)F(t) and let K=⋃n∈NF(tp−n)⊆LK=⋃n∈NF(tp−n)⊆L. Show that K/FK/F is separable but separably generated. Conclude that KK is not finitely generated over FF.

Academiejaar 2017-2018

Examen januari 2018

Oefeningen

1. Do the following situations exist? If so, give an example, if not, explain why.
   - A Principal Ideal Domain that contains a prime ideal that is not maximal.
   - A normal field extension $K/\mathbb{Q}$ with $[K:\mathbb{Q}]=4$.
   - A field with 36 elements.
2. We consider the following polynomial in $\mathbb{Q}[X]: f(X)=X^7-52X^6+39X^5+26X^4-26X^3+13X+13$. Let θ be one of the roots of f in $\mathbb{C}$. Let $K=\mathbb{Q}(\theta)$. Consider the following two elements of the field K
   $\gamma=2+3\theta+6\theta^2-7\theta^4, \delta=4+7\theta+2\theta^2-4\theta^5$
   $$\gamma=2+3\theta+6\theta^2-7\theta^4, \delta=4+7\theta+2\theta^2-4\theta^5$$
   Prove the following statements:
   - $[K:\mathbb{Q}]=7$
   - $K=\mathbb{Q}(\gamma)$
   - There exists a polynomial $g(X)$ in $\mathbb{Q}[X]$ such that $g(\gamma)=\delta$.
   - Without determining $g(X)$, what is the best upper bound for the degree of g and why?
3. Let p be a prime number. In this exercise we ultimately want to show the following statement: For any $n\in\mathbb{N}$, the polynomial X^4+1 is reducible over $\mathbb{F}_{p^n}$ To do this prove the following steps:
   - X^4+1 splits completely over $\mathbb{F}_{2^n}$ for any $n\in\mathbb{N}$
   - For the rest of the exercise, we assume that p is odd. $p^2\equiv 1 \bmod 4$
   - The splitting field of X^4+1 over $\mathbb{F}_p$ is contained in $\mathbb{F}_{p^2}$
   - For any $n\in\mathbb{N}$, the polynomial X^4+1 is reducible over $\mathbb{F}_{p^n}$
4. Let $n\in\mathbb{N}$. We denote Φ_n for the nth cyclotomic polynomial and set $\zeta_n=e^{\frac{2\pi i}{n}}=\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}$
   - Show that if n is an odd number we have $\Phi_{2n}(X)=\Phi_n(-X)$
   - Describe $\mathrm{Gal}(\mathbb{Q}(\zeta_{14})/\mathbb{Q})$ and show that it is isomorphic to C_6
   - Show that the minimal polynomial of the element $\zeta_{14}+\zeta_{14}^{-3}+\zeta_{14}^{-5}$ is X^2-X+2.
   - Prove that $\mathbb{Q}(\sqrt{-7})\subseteq\mathbb{Q}(\zeta_{14})$

Theorie

1.
   - Define an irreducible polynomial with coefficients in a ring.
   - Define a primitive polynomial in $\mathbb{Z}[x]$. Prove the Gauss lemma for polynomials in $\mathbb{Z}[x]$, saying that if $f,g\in\mathbb{Z}[x]$ are primitive $f\cdot g$ is also primitive
   - Assume $f(x)\in\mathbb{Z}[x]$ is an irreducible polynomial. Prove that it remains irreducible in $\mathbb{Q}[x]$.
2.
   - Let $k\subset K$ be a field extension. Define the degree $\deg_k K$
   - Formulate and prove the Tower Law
   - Let $k\subset K$ be a field extension such that $\deg_k K=p$ is a prime number. Show that any intermediate field $k_1, k\subset k_1\subset K$ coincides with either k or K.

3.
   - Define a splitting field of a polynomial and a normal field extension $k\subset K$.
   - Define the p-th cyclotomic polynomial $\Phi_p(x)\in\mathbb{Q}[x]$. Assume (without prove) that it is irreducible. Prove that the elementary field extension $K=\mathbb{Q}[x]/(\Phi_p)$ is the splitting field of Φ_p
   - Describe all field automorphisms $K/\mathbb{Q}$ and find the Galois group $Gal(K/\mathbb{Q})$

Examen januari 2018 (versie 2)

Oefeningen

1. Do the following situations exist? If so, give an example, if not, explain why.
   - A finite trancendental extension
   - Two finite normal field extensions L/K and M/L such that M/K is not normal.
   - A Galois extension of $\mathbb{F}_{11}$ with Galois group S_3
2. Let $n\in\mathbb{N}$. We denote Φ_n for the nth cyclotomic polynomial and set $\zeta_n=e^{\frac{2\pi}{n}}=\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}$ Show the following statements:
   - For any prime number p and integer m with $p\nmid m: \Phi_{mp}(X)=\frac{\Phi_m(X^p)}{\Phi_m(X)}$
   - For $n\geq 3: [\mathbb{Q}(\zeta_n):\mathbb{Q}(\zeta_n+\zeta_n^{-1})]=2$
   - $[\mathbb{Q}(\zeta_{15}):\mathbb{Q}(\zeta_5+\zeta_5^{-1})]=4$
3. Let p be a prime number. Show the following:
   - If $p\equiv 1 \pmod{17}$, then $X^{16}+X^{15}+\ldots+X^2+X+1$ splits completely in $\mathbb{F}_p$.
   - If $p\equiv 2 \pmod{17}$, then $X^{16}+X^{15}+\ldots+X^2+X+1$ factors in $\mathbb{F}_p$ as a product of two irreducible polynomials of degree 8.
4. Let $f(X)=X^3-12X-34$ be a polynomial in $\mathbb{Q}[X]$
   - Show that f is irreducible over $\mathbb{Q}$
   - Determine $f(x)$ and $f'(x)$ for $x=-2, x=2, x=\xi$ and conclude that f has only one root α and two complex roots β and γ.
   - Let K be the splitting field of f over $\mathbb{Q}$. Show that $[K:\mathbb{Q}]=6$.
   - Describe $Gal(K/\mathbb{Q})$ and show that it is isomorphic to S_3

Theorie

1.
   - Define an irreducible polynomial with coefficients in a ring
   - Formulate and prove the Eisenstein criterium for irreducibility of a polynomial in $\mathbb{Z}[x]$.
   - Is it true that any irreducible polynomial in $\mathbb{Z}[x]$ with the highest coefficient 1 fulfils the Eisenstein criterium for some prime p?
2.
   - Define finite, finitely-generated, and algebraic field extensions $k\subset K$.
   - Prove that an algebraic field extension generated by a single element is finite
   - Prove that finite $\Leftrightarrow$ finitely-generated and algebraic (you may use the Tower Law without proof).

3.
   - Define a splitting field of a polynomial and a normal field extension k⊂Kk⊂K
   - Let k be a field which contains all primitive p-th roots of 1, and let a∈ka∈k be an element such that a≠bpa≠bp for any b∈kb∈k. Recall that f(x)=xp−af(x)=xp−a is irreducible in k[x]k[x] (without proof). Prove that K=k[x]/(xp−a)K=k[x]/(xp−a) is a splitting field of f(x)f(x).
   - Describe all field automorphisms K/kK/k (with proof) and compute the Galois group Gal(K/k)Gal(K/k) (with proof).

Academiejaar 2016-2017

Test oktober 2016

1. Show that the following rings are pairwise not isomorphic \
   F3[X],Q[X],Z[−13−−−−√],C[[X]],R[X]/(X2−2X+1)

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2. Which of the following statements are correct? Justify your answer.
   1. If RR is a principal ideal domain, then R[X]R[X] is a factorial domain (UFD).
   2. If KK is a field and f∈K[X]f∈K[X] such that f(α)=0f(α)=0 for all α∈Kα∈K, then f=0f=0.
   3. Any subring of a field is a domain.
   4. In the domain Z[X,Y]Z[X,Y], every irreducible element is prime.
   5. Z[X]Z[X] is a euclidean domain.
3. Let R={a+X2f|a∈Q,f∈Q[X]}R={a+X2f|a∈Q,f∈Q[X]}. Show the following:
   1. RR is a subring of Q[X]Q[X] with R×=Q×R×=Q×.
   2. The elements X2X2 and X3X3 are irreducible in RR but not prime.
   3. The ideal generated bij X2X2 and X3X3 in RR is not principal.
4. Which of the following polynomials are irreducible over QQ?
   1. 5X−255X−25
   2. X3+6X+1X3+6X+1
   3. X4+X2+1X4+X2+1
   4. X5+15X+54X5+15X+54
   5. 2X6+42X6+4

Oefeningenexamen januari 2017

1. Geef voorbeelden van de volgende:
   - Een separabele veelterm over QQ van graad 4 die geen wortel in RR heeft.
   - Een f∈F3[X]f∈F3[X] zodanig dat F3[X]/(f)F3[X]/(f) een lichaam met 81 elementen is.
   - Een oneindig domein met characteristiek 7.
   - Een basis van Q(5–√3)Q(53) als QQ-vectorruimte.
   - Een priemelement van Z[2–√]Z[2].
2. Zij pp een priemgetal. Beschouw de verzameling R={ab∈Q|a∈Z,b∈Z\pZ}R={ab∈Q|a∈Z,b∈Z\pZ}.
   - Toon dat RR een deelring is van QQ.
   - Bepaal R×R× en vind een priemelement in RR.
   - Toon aan dat RR een uniek factorisatie domein is.
   - Ga na of RR een Euclidisch domein is t.o.v. abpn↦n(a,b∈Z\pZ,n∈N)abpn↦n(a,b∈Z\pZ,n∈N).

3. Zij K het splijtlichaam van de veelterm $f(X)=X^{156}-2$ over $\mathbb{F}_5$.
    - Toon aan dat $[K:\mathbb{F}_5]=4$.
    - Toon aan dat $\alpha\in\mathbb{F}_{625}\setminus\mathbb{F}_{25}$ voor elke wortel $\alpha\in K$ van f.
    - Bepaal het aantal factoren in de priemfactorisatie van f in $\mathbb{F}_5[X]$.
4. Stel K een perfect lichaam en $a,b\in K$. Zij L/K het splijtlichaam van $f(X)=X^4+aX^3+bX^2+aX+1$ over K. Toon volgende beweringen aan:
    - Voor elke wortel $\alpha\in L$ van f is ook α^{-1} een wortel van f.
    - $[L:K]$ deelt 8.
5. Stel M/K een cyclische lichaamsuitbreiding en L een tussenlichaam van M/K. Toon aan dat ook L/K en M/L cyclisch zijn.
6. Stel K het splijtlichaam van X^4-6X^2+16 over $\mathbb{Q}$. Toon dat $[K:\mathbb{Q}]=4$ en bepaal de Galois-groep en alle tussenlichamen van $K/\mathbb{Q}$.

Oefeningenexamen augustus 2017

1. Bewijs of ontkracht de volgende beweringen.
    - De karakteristiek van de ring $\mathbb{Z}[i]/(1+2i)$ is 5.
    - $\mathbb{C}[X,Y]$ is een hoofdideaaldomein.
    - Er bestaat een domein met exact 15 elementen.
    - Als K en L deellichamen van $\mathbb{C}$ zijn met $K\subseteq L$ en $[L:K]<\infty$, dan is $L=K(\alpha)$ voor een $\alpha\in L$.
2. Stel $A=\mathbb{Z}[\omega]$ met $\omega\in\mathbb{C}$ zodanig dat $\omega^3=1\neq\omega$. (We mogen $\omega=e^{\frac{2\pi i}{3}}$ kiezen.) Toon de volgende beweringen aan:
    - A is een euclidisch domein met de normaalafbeelding als euclidische graadfunctie.
    - A heeft precies 6 inverteerbare elementen.
3. Stel L/K een eindige lichaamsuitbreiding en $f\in K[X]$ irreducibel en zodanig dat met $\deg(f)$ en $[L:K]$ copriem zijn. Toon aan dat f ook in $L[X]$ irreducibel is.
4. Zij p een priemgetal en $K/\mathbb{F}_p$ een eindige lichaamsuitbreiding. Zij $\beta,\gamma\in K$. Toon aan dat $\beta^p-\beta=\gamma^p-\gamma$ als en slechts als $\beta-\gamma\in\mathbb{F}_p$.
5. Zij $f\in\mathbb{Q}[X]$ met $\deg(f)=4$ en zij $K/\mathbb{Q}$ het splijtlichaam van f. Stel dat $\mathrm{Gal}(K/\mathbb{Q})\cong S_4$. Toon de volgende beweringen aan:
    - f is irreducibel over $\mathbb{Q}$ en heeft 4 verschillende wortels $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ in K.
    - Het element $\beta=\alpha_1\alpha_2+\alpha_3\alpha_4$ ligt in $K\setminus\mathbb{Q}$.
    - $K/\mathbb{Q}(\beta)$ is een Galois-uitbreiding met $\mathrm{Gal}(f/\mathbb{Q}(\beta))\cong D_4$.
6. Stel K het splijtlichaam van X^4-3. Toon aan dat $[K:\mathbb{Q}]=8$ en bepaal de Galois-groep en alle tussenlichamen van $K/\mathbb{Q}$.

Academiejaar 2014-2015

Examen augustus 2015 (2de zit)

1. Show that the following rings are not pairwise isomorphic

$$\mathbb{F}_3[T],\mathbb{Q},\mathbb{Z}[\sqrt{-14}],\mathbb{C}[[T]],\mathbb{R}[T]/(T^3)$$

2. Define $R=\{f\in\mathbb{Q}[T]\mid f(0)\in\mathbb{Z}\}$. Show that...
    - ... R is a domain with two invertible elements.
    - ... T is not a product of irreducible elements.
    - ... R is not a principal ideal domain.

3. Prove or give a counter-example:
    - Every field extansion of degree five of $\mathbb{Q}$ is normal.
    - The degree of any finite inseparable field extension L/K is a multiple of $\mathrm{char}(K)$.
    - Every polynomial over $\mathbb{R}$ of degree five is a product of linear factors in $\mathbb{R}[T]$.
4. Let $f(T)=T^6+T^3+1\in\mathbb{Q}[T]$. Let $\xi\in\mathbb{C}$ be a root of f and $K=\mathbb{Q}[\xi]$. Show that...
    - ... $\xi^9=1\neq x^3$ and every root of f in $\mathbb{C}$ is a power of ξ.
    - ... f is irreducible over $\mathbb{Q}$ and splits over K.
    - ... $K/\mathbb{Q}$ is a Galois extension of degree six.
    - ... K contains a quadratic extension of $\mathbb{Q}$.
5. Consider a finite Galois extension M/K with an abelian Galois group. Show that every field extension L/K with $L\subseteq M$ is normal.
6. Let K be the splitting field of T^4-8T^2+8 over $\mathbb{Q}$. Show that ...
    - ... $[K:\mathbb{Q}]=4$
    - ... there exists a unique intermediate field $K/\mathbb{Q}$ other than K and $\mathbb{Q}$.

Examen januari 2015

1. Toon aan dat volgende ringen niet (2 aan 2) isomorf zijn:
    - $\mathbb{Z}$
    - $\mathbb{Z}[\sqrt{-5}]$
    - $\mathbb{F}_7[T]$
    - $\mathbb{R}[T]$
    - $\mathbb{C}[T]/(T^3)$
2. Stel $R=\{f\in\mathbb{Q}[T] \mid f(\mathbb{Z})\subseteq\mathbb{Z}\}$ en $\xi=\frac{T(T-1)}{2}$, toon aan:
    - R is een deelring van $\mathbb{Q}[T]$.
    - de enige inverteerbare elementen in R zijn -1 en 1.
    - ξ irreducibel in R.
    - ξ niet priem in R.
3. Toon aan of geef een tegenvoorbeeld:
    - elke irreducibele veelterm in $\mathbb{Q}[T]$ is separabel.
    - Stel $f\in K[T]$ met L het splijtlichaam van f. De galoisgroep $\mathrm{Gal}(L/K)$ is isomorf met een deelgroep van S_n.
    - Voor elk tussenlichaam L van een eindige galois extensie M/K is L/K een galois extensie.
    - Elke veelterm van graad 4 in $\mathbb{R}[T]$ is reducibel.
4. Stel p een priemgetal.
    - Stel $\mathrm{Char}(K)=p$, als voor een extensie L/K geldt dat p geen deler is van $[L:K]$ dan is L/K separabel.
    - Stel $K=\mathbb{Q}[T]/(T^{p-1}+...+T+1)$, dan is $K/\mathbb{Q}$ normaal.
5. Toon aan:
    - Voor elk eindig lichaam geldt dat het product van alle van nul verschillende elementen -1 is.
    - Stel p een priemgetal, dan is $(p-1)!+1$ deelbaar door p.
6. Als $K=\mathbb{Q}[T]/(T^4+12T^2+18)$, toon aan dat $[K:\mathbb{Q}]=4$. Toon ook aan dat er een tussenlichaam L van $K/\mathbb{Q}$ bestaat.

Test November 2014

1. Decide which of the following rings are factorial domains:
    - R[[T]]R[[T]]
    - Z[−14−−−−√]Z[−14]
    - M2(Z/2Z)M2(Z/2Z)
    - Z[X,Y,Z]Z[X,Y,Z]
2. Let *A* be a finite commutative ring. Show the following:
    - The characteristic of *A* is different from 0.
    - Every prime ideal of *A* is maximal.
    - There exists no ring homomorphism A→ZA→Z.
3. Decide which of the following polynomials are irreducible in Q[T]Q[T]:
    - T9−10T9−10
    - T6−T5+T4−T3+T2−T+1T6−T5+T4−T3+T2−T+1
    - 7T4+T3−17T4+T3−1
    - T5−4T5−4
4. Let R={a−T2f|a∈Q,f∈Q[T]}R={a−T2f|a∈Q,f∈Q[T]}. Show the following:
    - *R* is a subring of Q[T]Q[T] with R×=Q×R×=Q×.
    - The elements T2T2 and T3T3 are irreducible in *R* but not prime.
    - The ideal (T2,T3)(T2,T3) is not principal in *R*.
5. Let *K* be a field and f,g∈K[T]f,g∈K[T] irreducible polynomials. Assume that ff has a root in K[T]/(g)K[T]/(g). Show that deg(ff) divides deg(gg).

Academiejaar 2013-2014

Examen 2014

1. Exercise 1. Let R={f∈R[T]|f′(0)=0}R={f∈R[T]|f′(0)=0}. Show the following:
    - RR is a subring of R[T]R[T].
    - Every polynomial f∈R[T]f∈R[T] of degree 2 or 3 and with f′(0)=0f′(0)=0 is an irreducible element of RR.
    - RR is not a factorial domain.
2. Exercise 2. Decide by giving an argument or counter-example whether the follwing statements are true or false:
    - Every irreducible polynomial in Q[T]Q[T] is also irreducible in R[T]R[T].
    - Every irreducible polynomial over Q[T]Q[T] is separable.
    - Every seperable algebraic field extension is normal.
    - For any field KK, there are infinitely many monic irreducible polynomials in K[T]K[T].
3. Exercise 3. Let K/QK/Q be a field extension and α∈Kα∈K.
    - Show that [Q(α):Q(α2)]≤2[Q(α):Q(α2)]≤2.
    - If K/QK/Q is algebraic of odd degree, conclude that Q(α)=Q(α2)Q(α)=Q(α2).
4. Exercise 4. Consider the polynomial f=T4−5T2+9∈Q[T]f=T4−5T2+9∈Q[T] and let LL be its splitting field over QQ.
    - Show that ff is irreducible in Q[T]Q[T].
    - Find the roots of ff in CC and show that [L:Q]=4[L:Q]=4.
    - Find the subfields of LL using Galois correspondence.
5. Exercise 5.
    - Show that K=F2[T]/(t4+t+1)K=F2[T]/(t4+t+1) is a field.
    - Does the field KK contain an element α≠1α≠1 with α5=1α5=1?
    - Give field with 113113 elements, as an extension over F11F11.
    - Does there exist a field with 1024 elements?

6. Exercise 6. Show that every polynomial in R[T]R[T] can be factorized into a product of polynomials of degree 1 and 2. (Use that CC is algebraically closed.)

Test November 2013

1. Which of the following polynomials in Q[T]Q[T] are irreducible?
    - T4−T3+T2−T+1T4−T3+T2−T+1;
    - 3T6+9T3−183T6+9T3−18;
    - T4+3T2+2T4+3T2+2;
    - T+4T+4.
    - Justify your answers.
2. Show the following:
    - In the domain Z[−14−−−−√]Z[−14], the element 5 is irreducible but not prime.
    - For any n≥1n≥1 there exists a field extention Q→KQ→K with [K:Q]=n[K:Q]=n.
3. Let KK be a field and n∈Nn∈N. Explain why a polynomial f∈K[T]f∈K[T] of degree *n* has at most *n* roots in KK.
4. Prove that in a principal ideal domain every nonzero prime ideal is maximal. (Don't use statements from the course, but only definitions.)
5. Let R={f∈R[T]|f(0)∈Q}R={f∈R[T]|f(0)∈Q}. Show the following:
    - RR is a subring of R[T]R[T] and Rx=QxRx=Qx.
    - The ideal {f∈R[T]|f(0)=0}{f∈R[T]|f(0)=0} of RR is not finitely generated.
    - Every nonzero element of R\RxR\Rx is a product of irreducible elements of RR.
    - The element TT is irreducible but not prime in RR.

Examen 2013

1. Decide which of the following statements are correct. Provide a proof or a counter-example:
    - If RR is a principal ideal domain, then so is R[T]R[T].
    - Any euclidean domain is a principal ideal domain.
    - If KK is a field and f∈K[T]f∈K[T] is such that f(α)=0f(α)=0 for all α∈Kα∈K, then f=0f=0.
2. Let KK be a field and n∈Nn∈N with n≥2n≥2. Show that the polynomial Tn−T+1Tn−T+1 is separable over KK unless the characteristic of KK divides nn−(n−1)n−1nn−(n−1)n−1.
3. Let L/KL/K be a finite separable field extension of degree n. Show that the number of KK-automorphisms of LL is at most n and not equal to n - 1. Do **not** make use of any degree inequalities from the course.
4. Let L/KL/K be a finite Galois extension with cyclic Galois group and α,β∈Lα,β∈L. Show that K(α)=K(β)K(α)=K(β) if and only if the minimal polynomials of αα and ββ over KK have the same degree.
5. Determine the minimal polynomials over QQ of the following algebraic complex numbers:
    - ζ=e2πipζ=e2πip where p is a prime number (using that ζp=1≠ζζp=1≠ζ);
    - α∈Rα∈R with α>0α>0 such that α8=6α8=6;
    - 2–√+3–√2+3.
6. Consider the polynomial f=T4+4T2+9f=T4+4T2+9 over QQ. Show that it is irreducible and that its Galois group is Z/2Z×Z/2ZZ/2Z×Z/2Z. Determine the subfields of the splitting field of ff using Galois correspondence.

Categorieën:

- Wiskunde
- 3BWIS