

# Category Theory - Encyclopedia Academia

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## Category Theory

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## Examenvragen

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#### Theorie

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1. (2pt) Define the following concepts: full functor, faithful functor, limit-reflecting functor.
2. (4pt) Show that a full and faithful functor reflects small limits.

#### Oefeningen

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De oefeningen waren open-boek.

1. (3pt) Let  $\mathcal{C}$  be an arbitrary category. Construct a functor  $F: \mathcal{C} \rightarrow \mathbf{Set}$  which is not representable.
2. (3pt) Let  $\mathcal{C}$  be a category such that any two objects have a product. Show that any non-empty finite family of objects in  $\mathcal{C}$  has a product.
3. (4pt)
  1. Prove that there is a functor from  $\mathbf{Grp}$  to  $\mathbf{Ab}$  which assigns to every group  $G$  the quotient group  $G/[G, G]$ , where  $[G, G]$  denotes the commutator subgroup of  $G$ . (Recall that  $[G, G]$  is the subgroup of  $G$  generated by all the commutators  $[x, y] = xyx^{-1}y^{-1}$ , where  $x, y \in G$ .)
  2. Show that  $\mathbf{Ab}$  is a reflective subcategory of  $\mathbf{Grp}$ .
4. (4pt) Let  $F: \mathcal{C} \rightarrow \mathcal{D}$ ,  $G: \mathcal{D} \rightarrow \mathcal{C}$  be two functors such that  $F \dashv G$  and consider  $\eta: 1_{\mathcal{C}} \rightarrow GF$  to be the unit of the adjunction. Show that:
  1.  $F$  is faithful if and only if  $\eta_C$  is a monomorphism for all  $C \in \mathcal{C}$ ;
  2.  $F$  is full if and only if  $\eta_C$  has a right inverse for all  $C \in \mathcal{C}$ . (That is, there exists a morphism  $u_C \in \text{Hom}(GF(C), C)$  such that  $\eta_C \circ u_C = 1_{GF(C)}$ .)

Categorieën:

- Wiskunde
- MWIS