Wiskundige modellen in de economie

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Richting	Wiskunde
Jaar	Bachelor Wiskunde Keuzevakken

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1. We assume that an insurance company takes iets decisions using exceptional utility function $u(x)=-\alpha e^{-\alpha x}, \alpha>0$. The initial wealth of one company is denoted bij R (R>0). The company had the possibility to insure a rise S (S positive random var corresponding to the claim amaint of a contract on a 1 year period).
   - Give, in this context, the definitions of:
     - The net premium associated to rise S.
     - The premium obtained using the variance principle with some parameter γ
     - The premium obtained using the zero-utility principle.
   - In this context (aka exponential utility), does the zero-utility premium statisfy the additivity property? Does it have positive security loading? Justify your answers.
2. Same framework as Q1. You have the choice between the next 2 questions:
   - We assume now that the rise S follows an exponential distribution with the parameter $\beta>0 (P[S\leq x]=1-e^{-\beta x})$, where we assume $\beta>\alpha$. Compute in function of parameters β,α the net premium α zero-utility premium.
   - Show that the premium obtained using the zero-utility principle with exponential utility is an increasing function of the rise aversion coefficient α. (Hint: Use Jensen inequality and apply this inequality by choosing for Y an exponential function of the rise index interest with a coefficient linked to the rise aversion coefficient).

3. We assume that the first 3 moments of S have been estimated and take the values μS=σ2S=γS=1

   $$μS=σS2=γS=1$$

   . Explain how to preform a translated camma-approse of the destribution of S and use that approse in order to compute the probability P[S≤4]P[S≤4] (up to the evaluation in order to compute the probability at some point). Hint fΓ(x,α,β)=βαΓ(α)xα−1e−αβIx≥0,α,β>0,x∼Γ(α,β)⇒E[X]=αβ,Var[X]=αβ2,γX=2α−−√

   $$fΓ(x,α,β)=βαΓ(α)xα−1e−αβIx≥0,α,β>0,x∼Γ(α,β)⇒E[X]=αβ,Var[X]=αβ2,γX=2α$$

4. We assume that a company models one total claim amount of given insurance contracts portfolio within a collective model S=∑i=1NXi

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   , N = total number of claims on a year period, XiXi = ith claim amount, N⊥Xi,Xiiid∼FN⊥Xi,Xiiid∼F
   - Compute E[S] in function of the moments of the distributions of the X′isXi′s and N.
   - Choose between following 2 questions:
     - Show that in case where N∼P(λ)N∼P(λ) (Poissar distribution with parameter λλ); the moment generating fn of S, ms(t)=E[est]ms(t)=E[est] is given by ms(t)=eλ(mx1(t),...)ms(t)=eλ(mx1(t),...), where mx1(t)=E[etx]mx1(t)=E[etx] is the mgf of distribution F of the claim amounts.
     - We suppose that the distribution of the number of claims, N, has a discrete density denoted by (pn)n∈N(pn)n∈N. What are the conditions to impose to that distribution, in order that N belongs to the Panjer fam? Show that if N∼P(λ)N∼P(λ), then N belongs to the Panjer fam.

Categorieën:
- Wiskunde
- BWIS Keuzevakken