Fields and Galois theory

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Fields and Galois theory

Richting	Wiskunde
Jaar	3BWIS

Examenvragen

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1. (12 points) Determine for four of the following five statements whether they are true or false:
    1. For every $\alpha\in\mathbb{R}$ transcendental over $\mathbb{Q}$, also $\alpha+\sqrt{2}$ is transcendental over $\mathbb{Q}$.
    2. Every algebraic extension of $\mathbb{F}_{131}$ is normal and seperable.
    3. $\mathbb{F}_{25}$ has a Galois extension whose Galois group is isomorphic to A_4.
    4. The Galois group of X^5-6X+3 over $\mathbb{Q}$ contains an element of order 4.
    5. Every separable polynomial in $\mathbb{R}[X]$ of degree 10 has an even number of roots in $\mathbb{R}$.
2. (10 points) Let L/K be a field extension such that $[L:K]=\infty$. Show that L/K has infinitely many intermediate fields.
3. (8 points) Let f and g be two irreducible polynomials in $K[X]$. Let K_f and K_g denote their respective root fields over K. Show that the following are equivalent:
    1. f and g have roots in precisely the same field extensions of K.
    2. f has a root in K_g and g has a root in K_f.
    3. $K_f\simeq_K K_g$.
4. (8 points) Let $L/\mathbb{Q}$ be a finite field extension. Show that the number of automorphisms of L divides $[L:\mathbb{Q}]$. (**Hint:** Argue via a certain subfield of L.)
5. (12 points) Let $\alpha=\sqrt[3]{3}+\sqrt{2}\in\mathbb{R}$ and let f be the minimal polynomial of α over $\mathbb{Q}$. Let L denote the splitting field of f over $\mathbb{Q}$. Show the following:
    1. $\deg(f)=6$.
    2. $\mathrm{Gal}(L/\mathbb{Q})\simeq S_3\times C_2$.
    3. L has a unique subfield K with $[K:\mathbb{Q}]=4$.

Categorieën:

- Wiskunde
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