Quadratic Forms - Encyclopedia Academia

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Richting	<u>Wiskunde</u>
Jaar	<u>MWIS</u>

Examenvragen/Test

2023-2024

Test

Let K always be a field of characteristic different from 2 . Let φ and ψ be regular quadratic forms over K.

1. (5 points) Let $\varphi=(V,q)$ and $n=\dim(\varphi)$. Let $\sigma,\sigma'\in O(\varphi)$ be such that $\det(\sigma')=\det(\sigma)$, Let $v_1,\ldots,v_{n-1}\in V$ be such that $\sigma(v_i)=\sigma'(v_i)$ for $1 \leq i \leq n-1$. Show that $\sigma=\sigma'$ or $v_1,\ldots,v_{n-1}$ are K-linearly dependent.
2. (5 points) Assume that φ and ψ are anisotropic and $\dim(\psi)=2$. Show that the Witt index of $\psi\otimes\varphi$ is even.
3. (5 points) Assume that $\dim(\varphi)=\dim(\psi)$ and $\text{sign}_p(\varphi)=\text{sign}_p(\psi)$ for every ordering P of K. Show that $2^r\times\varphi\simeq 2^r\times\psi$ for some $r\in\mathbb{N}$.
4. (5 points) Assume that $P=\square_K$ is an ordering of K. Show that φ is isotropic if and only if $|\text{sign}_P(\varphi)|<\dim(\varphi)$.

Examen

Let K always be a field of characteristic different from 2 .

1. (10 points) Show that $I^2K=0$ if and only if every 3-dimensional quadratic form over K is isotropic.
2. (10 points) Assume that $|K^\times/K^{\times 2}|=2$ and $-1\in K^{\times 2}$. Show that $WK\simeq\mathbb{Z}/2\mathbb{Z}[X]/(X^2)$.
3. (4 points) Let φ be regular quadratic form over K. Assume that $n\times\varphi$ is isotropic over K for some $n\in\mathbb{N}$. Show that, for every ordering P of K, we have $|\text{sign}_P(\varphi)|<\dim(\varphi)$.

4. (6 points) Assume that $K = \mathbb{Q}(\theta)$ for $\theta \in \mathbb{C}$ with $\theta^4 = 3\theta + 3$. Let $s: K \to \mathbb{Q}$ be the $\mathbb{Q}$-linear form given by $1, \theta, \theta^2 \mapsto 0$ and $\theta^3 \mapsto 1$. Consider the quadratic form $q: K \to \mathbb{Q}, x \mapsto s(x^2)$ over $\mathbb{Q}$. Show that q is hyperbolic.

5. (10 points) Let $n \in \mathbb{N}$. Let φ and ψ be n-fold Pfister forms over K. Assume that we have $[\varphi] \equiv [\psi] \bmod I^{n+1} K$. Show that $\varphi \simeq \psi$. (**Hint**: Consider $\varphi \perp -\psi$.)

6. (10 points) Let L/K be a finite field extension of odd degree and let φ be a quadratic form over K. Assume that φ_L is a Pfister form. Show that φ is a Pfister form over K. (**Hint:** Consider $L(\varphi_L)/K(\varphi)$.)

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