Algebraic Number Theory

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Richting	Wiskunde
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1. Prove or give a counterexample:
   - An integrally closed domain is also factorial.
   - Assume A a Dedekind domain. Every nonzero element of A is contained in only finitely many ideals of A.
   - A $\mathbb{Z}$-submodule of $\mathbb{Q}$ is also a fractional ideal of $\mathbb{Z}$.
2. Which of the following rings are Dedekind domains?
   $$\mathbb{Z}[X], \mathbb{C}, \mathbb{Z}[\sqrt{5}], \mathbb{Z}[\sqrt{6}], \mathbb{R}[X^2, X^3]$$
3. Consider the equation $x^2+y^2=7z^2$. Show that it has no nontrivial integer solution.
4. Assume $\omega \in \mathbb{C}$ such that $\omega^5 = 1 (\neq \omega)$ and let $K=\mathbb{Q}(\omega)$. What is the discriminant of K? Furthermore, prove that $(1, \omega, \omega^2, \omega^3)$ is an integral basis of K.

Categorieën:

- Wiskunde
- MWIS