№ 34 $(3(6+5) \bmod 44)+1$

$$A=\begin{pmatrix}0&0&-1&2&3\\1&0&0&0&1\\-2&1&0&4&0\end{pmatrix};\ b=\begin{pmatrix}7\\2\\6\end{pmatrix};\ c=\begin{pmatrix}-4\\2\\1\\12\\10\end{pmatrix};\ d_*=\begin{pmatrix}0\\-1\\-1\\-5\\-6\end{pmatrix};\ d^*=\begin{pmatrix}4\\3\\4\\2\\1\end{pmatrix}$$

$$-4x_1+2x_2+x_3+12x_4+10x_5 \longrightarrow \max$$
$$-x_3+2x_4+3x_5=7$$
$$x_1+x_5=2$$
$$-2x_1+x_2+4x_4=6$$
$$0\le x_1\le 4;\ -1\le x_2\le 3$$
$$-1\le x_3\le 4;\ -5\le x_4\le 2$$
$$-6\le x_5\le 1$$

Решение

1) Первая фаза:

$$-x_6-x_7-x_8\longrightarrow \max$$
$$-x_3+2x_4+3x_5+x_6=7$$
$$x_1+x_5+x_7=2$$
$$-2x_1+x_2+4x_4+x_8=6$$
$$0\le x_1\le 4;\ -1\le x_2\le 3$$
$$-1\le x_3\le 4;\ -5\le x_4\le 2$$
$$-6\le x_5\le 1;\ 0\le x_6\le 34$$
$$0\le x_7\le 8;\ 0\le x_8\le 27$$

$$\tilde{x}=(0;-1;-1;-5;-6;34;8;27)$$
$$J_Б=\{6;7;8\}$$

① $u=(-1;-1;-1)$

② $\Delta_1=0-1=-1;\ \Delta_2=0+1=1;\ \Delta_3=0-1=-1;\ \Delta_4=0+6=6;\ \Delta_5=0+4=4$

③ $j_*=2$

④ $l_H:\ l_1=0;\ l_2=1;\ l_3=0;\ l_4=0;\ l_5=0$

$l_Б:\ \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}l_6\\l_7\\l_8\end{pmatrix}=\begin{pmatrix}0\\0\\-1\end{pmatrix};\ l_6=0;\ l_7=0;\ l_8=-1 \Rightarrow l=(0;1;0;0;0;0;0;-1)$

⑤ $\Theta_1=\Theta_3=\Theta_4=\Theta_5=\Theta_6=\Theta_7=\infty;\ \Theta_2=\frac{3+1}{1}=4;\ \Theta_8=\frac{0-27}{-1}=27 \Rightarrow \Theta=4;\ j_0=2$

$$\bar{x}=(0;3;-1;-5;-6;34;8;23);\ J_Б=\{6;7;8\}$$

1. $u = (-1; -1; -1)$
2. $\Delta_1 = -1;\ \Delta_2 = 1;\ \Delta_3 = -1;\ \Delta_4 = 6;\ \Delta_5 = 4$
3. $j_* = 4$
4. $\ell_H:\ \ell_1 = 0;\ \ell_2 = 0;\ \ell_3 = 0;\ \ell_4 = 1;\ \ell_5 = 0$

$\ell_Б:\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \ell_6 \\ \ell_7 \\ \ell_8 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -4 \end{pmatrix};\ \ell_6 = -2;\ \ell_7 = 0;\ \ell_8 = -4 \Big| \Rightarrow \ell = (0; 0; 0; 1; 0; -2; 0; -4)$

5. $\Theta_1 = \Theta_2 = \Theta_3 = \Theta_5 = \Theta_7 = \infty;\ \Theta_4 = \frac{2+5}{1} = 7;\ \Theta_6 = \frac{0-34}{-2} = 17;\ \Theta_8 = \frac{0-23}{-4} = \frac{23}{4}$

$\Theta = \frac{23}{4};\ j_0 = 8$

$\overline{X} = (0; 3; -1; \frac{3}{4}; -6; \frac{55}{2}; 8; 0);\ \overline{J_Б} = \{4; 6; 7\}$

1. $\begin{pmatrix} 2 & 0 & 4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} u = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix};\ u = (-1; -1; \frac{1}{2})$
2. $\Delta_1 = 0 + 2 = 2;\ \Delta_2 = 0 - \frac{1}{2} = -\frac{1}{2};\ \Delta_3 = 0 - 1 = -1;\ \Delta_5 = 0 + 4 = 4;\ \Delta_8 = -1 - \frac{1}{2} = -\frac{3}{2}$
3. $j_* = 1$
4. $\ell_H:\ \ell_1 = 1;\ \ell_2 = 0;\ \ell_3 = 0,\ \ell_5 = 0,\ \ell_8 = 0$

$\ell_Б:\ \begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_4 \\ \ell_6 \\ \ell_7 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix};\ \ell_4 = \frac{1}{2};\ \ell_6 = -1;\ \ell_7 = -1 \Big| \Rightarrow \ell = (1; 0; 0; \frac{1}{2}; 0; -1; -1; 0)$

5. $\Theta_2 = \Theta_3 = \Theta_5 = \Theta_8 = \infty;\ \Theta_1 = \frac{4-0}{1} = 4;\ \Theta_4 = \frac{2-\frac{3}{4}}{\frac{1}{2}} = \frac{5}{2};\ \Theta_6 = \frac{0-\frac{55}{2}}{-1} = \frac{55}{2};\ \Theta_7 = \frac{0-8}{-1} = 8;\ \Theta = \frac{5}{2};\ j_0 = 4$

$\overline{X} = (\frac{5}{2}; 3; -1; 2; -6; 20; \frac{11}{2}; 0);\ \overline{J_Б} = \{1; 6; 7\}$

1. $\begin{pmatrix} 0 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} u = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix};\ u = (-1; -1; -\frac{1}{2})$
2. $\Delta_2 = 0 + \frac{1}{2} = \frac{1}{2};\ \Delta_3 = 0 - 1 = -1;\ \Delta_4 = 0 + 4 = 4;\ \Delta_5 = 0 + 4 = 4;\ \Delta_8 = -1 + \frac{1}{2} = -\frac{1}{2}$
3. $j_* = 5$
4. $\ell_H:\ \ell_2 = 0;\ \ell_3 = 0;\ \ell_4 = 0;\ \ell_5 = 1;\ \ell_8 = 0$

$\ell_Б:\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 \\ \ell_6 \\ \ell_7 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix};\ \ell_1 = 0;\ \ell_6 = -3;\ \ell_7 = -1 \Big| \Rightarrow \ell = (0; 0; 0; 0; 1; -3; -1; 0)$

5. $\Theta_1 = \Theta_2 = \Theta_3 = \Theta_4 = \Theta_8 = \infty;\ \Theta_5 = \frac{1+6}{1} = 7;\ \Theta_6 = \frac{0-20}{-3} = \frac{20}{3};\ \Theta_7 = \frac{0-\frac{11}{2}}{-1} = \frac{11}{2}$

$\Theta = \frac{11}{2};\ j_0 = 7$

$\overline{X} = (\frac{5}{2}; 3; -1; 2; -\frac{1}{2}; \frac{7}{2}; 0; 0),\ \overline{J_Б} = \{1; 5; 6\}$

1. $\begin{pmatrix} 0 & 1 & -2 \\ 3 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} u = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix};\ u = (-1; 3; \frac{3}{2})$

$\Delta_3 = 0 - 1 = -1;\ \Delta_4 = 0 - 4 = -4;\ \Delta_7 = -1 - 3 = -4;\ \Delta_8 = -1 - \frac{3}{2} = -\frac{5}{2}$

2. $\Delta_2 = 0 - \frac{3}{2} = -\frac{3}{2};$
3. $j_* = 2$
4. $\ell_H:\ \ell_2 = -1;\ \ell_3 = 0;\ \ell_4 = 0;\ \ell_7 = 0;\ \ell_8 = 0$

$\ell_Б:\ \begin{pmatrix} 0 & 3 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 \\ \ell_5 \\ \ell_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};\ \ell_1 = -\frac{1}{2};\ \ell_5 = \frac{1}{2};\ \ell_6 = -\frac{3}{2} \Big| \Rightarrow \ell = (-\frac{1}{2}; -1; 0; 0; \frac{1}{2}; -\frac{3}{2}; 0; 0)$

$\Theta_1 = \frac{0-\frac{5}{2}}{-\frac{1}{2}} = 5;\ \Theta_2 = \frac{-1-3}{-1} = 4;\ \Theta_5 = \frac{1+2\frac{1}{2}}{\frac{1}{2}} = 3;\ \Theta_6 = \frac{0-\frac{7}{2}}{-\frac{3}{2}} = \frac{7}{3}$

5. $\Theta_3 = \Theta_4 = \Theta_7 = \Theta_8 = \infty;$

$\Theta = \frac{7}{3};\ j_0 = 6$

$\overline{X} = (\frac{4}{3}; \frac{2}{3}; -1; 2; \frac{2}{3}; 0; 0; 0);\ \overline{J_Б} = \{1; 2; 5\}$

1. $\begin{pmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} u = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix};\ u = (0; 0; 0)$

② $\Delta_3 = 0-0=0$; $\Delta_4 = 0-0=0$; $\Delta_6 = -1$; $\Delta_7 = -1$; $\Delta_8 = -1 \Rightarrow$ выполняется $\Rightarrow$
$\Rightarrow$ получаем стартовый план: $x = \left(\frac{4}{3}; \frac{2}{3}; -1; 2; \frac{2}{3}\right)$ и базис $J_Б = \{1;2;5\}$

2) Вторая фаза: $x = \left(\frac{4}{3}; \frac{2}{3}; -1; 2; \frac{2}{3}\right)$; $J_Б = \{1;2;5\}$

① $\begin{pmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} u = \begin{pmatrix} -4 \\ 2 \\ 10 \end{pmatrix}$; $u = \left(\frac{10}{3}; 0; 2\right)$

② $\Delta_3 = 1 + \frac{10}{3} = \frac{13}{3}$; $\Delta_4 = 12 - \frac{44}{3} = -\frac{8}{3}$

③ $j_* = 3$

④ $l_н$: $l_3 = 1$; $l_4 = 0$

$l_Б$: $\begin{pmatrix} 0 & 0 & 3 \\ 1 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} l_1 \\ l_2 \\ l_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$; $l_1 = -\frac{1}{3}$; $l_2 = -\frac{2}{3}$; $l_5 = \frac{1}{3}$ $\Rightarrow l = \left(-\frac{1}{3}; -\frac{2}{3}; 1; 0; \frac{1}{3}\right)$

⑤ $\theta_4 = \infty$; $\theta_1 = \frac{0 - \frac{4}{3}}{-\frac{1}{3}} = 4$; $\theta_2 = \frac{-1-\frac{2}{3}}{-\frac{2}{3}} = \frac{5}{2}$; $\theta_3 = \frac{4+1}{1} = 5$; $\theta_5 = \frac{1-\frac{2}{3}}{\frac{1}{3}} = 1$

$\theta = 1$; $j_0 = 5$

$\bar{x} = (1; 0; 0; 2; 1)$; $\bar{J}_Б = \{1;2;3\}$

① $\begin{pmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} u = \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix}$; $u = (-1; 0; 2)$

② $\Delta_4 = 12 - 6 = 6$; $\Delta_5 = 10 + 3 = 13 \Rightarrow$ все выполняется

Ответ: $x = (1; 0; 0; 2; 1)$; $J_Б = \{1;2;3\}$