МИНИСТЕРСТВО ОБРАЗОВАНИЯ РЕСПУБЛИКИ БЕЛАРУСЬ

БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ФАКУЛЬТЕТ ПРИКЛАДНОЙ МАТЕМАТИКИ И ИНФОРМАТИКИ

РЕШЕНИЕ СМЕШАННЫХ ЗАДАЧ ДЛЯ НЕОДНОРОДНЫХ УРАВНЕНИЙ МЕТОДОМ РАЗДЕЛЕНИЯ ПЕРЕМЕННЫХ

Лабораторная работа 3

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Минск, 2024

Вариант 4

Условие:

$$u_{tt} - u_{xx} = e^t \cos x, \quad 0 < x < l, \quad t > 0$$

$$\begin{cases} u_x(0, t) + hu(0,t) = 2t\,; \quad u_x(l,\ t) - hu(l,\ t) = t - 1 \\ u(x, 0) = \cos x\,; \quad u_t(x, 0) = 2x \end{cases}$$

Решение:

Ищем в виде $u(x,t) = V(x,t) + w(x,t)$.

$$w(x,t) = \frac{(1-2lh)t + 1}{(2-lh)h} + \frac{3t-1}{2-lh}x$$

$$V_{tt} - V_{xx} = e^t \cos x, \quad 0 < x < l, \quad t > 0$$

$$\begin{cases} V_x(0,t) + hV(0, t) = 0\,; \quad V_x(l,\ t) - hV(l,\ t) = 0 \\ V(x, 0) = \cos x - \frac{1}{(2-lh)h} + \frac{x}{2-lh}\,; \quad V_t(x, 0) = 2x - \frac{1-2lh}{(2-lh)h} - \frac{3x}{2-lh} \end{cases}$$

$$V(x,t) = P(x,t) + Q(x,t)$$

1) $P_{tt} = P_{xx}, \quad 0 < x < l, \quad t > 0$

$$\begin{cases} P_x(0,t) + hP(0, t) = 0\,; \ P_x(l,\ t) - hP(l,\ t) = 0 \\ P(x, 0) = \cos x - \frac{1}{(2-lh)h} + \frac{x}{2-lh}\,; \ P_t(x, 0) = 2x - \frac{1-2lh}{(2-lh)h} - \frac{3x}{2-lh} \end{cases}$$

2) $Q_{tt} - Q_{xx} = e^t \cos x, \quad 0 < x < l, \quad t > 0$

$$\begin{cases} Q(0, t) = 2 + t\,; \ Q_x(l,\ t) - hQ(l,\ t) = t \\ Q(x, 0) = 0\,; \ Q_t(x, 0) = 0 \end{cases}$$

1) $P(x,t) = X(x)T(t)$:

$$XT'' = X''T$$

$$\frac{T''}{T} = \frac{X''}{X} = -\lambda^2$$

$$T'' + \lambda^2 T = 0$$
$$\begin{cases} X'' + \lambda^2 X = 0 \\ X'(0) + hX(0) = 0\ ;\ X'(l) - hX(l) = 0 \end{cases}$$

При $\lambda = 0, X(x) = 0$. Поэтому $\lambda > 0$.
$X(x) = C_1\cos\lambda x + C_2\sin\lambda x$
$X'(0) + hX(0) = \lambda C_2 + hC_1 = 0$
Отсюда получаем:
$$C_2 = -\frac{hC_1}{\lambda}$$
$X'(l) - hX(l) = -\lambda C_1\sin\lambda l + \lambda C_2\cos\lambda l - hC_1\cos\lambda l - hC_2\sin\lambda l = 0,$
Получаем:
$$C_1\left(\left(-\lambda + \frac{h^2}{\lambda}\right)\sin\lambda l - 2h\cos\lambda l\right) = 0\ , C_1 = \lambda,\ C_2 = -h$$
$$c\mathrm{tg}\lambda l = \frac{h^2 - \lambda^2}{2\lambda h}, \qquad \lambda = \lambda_n, \quad n = 1, 2, 3, \ldots$$

$$X_n(x) = \lambda_n\cos\lambda_n x - h\sin\lambda_n x$$

$$T'' + {\lambda_n}^2 T = 0$$
$$T_n(t) = A_n\cos\lambda_n t + B_n\sin\lambda_n t$$
$$P(x,t) = \sum_{n=1}^{\infty}(A_n\cos\lambda_n t + B_n\sin\lambda_n t)(\lambda_n\cos\lambda_n x - h\sin\lambda_n x)$$

$$P(x,0) = \sum_{n=1}^{\infty} A_n\left(\lambda_n\cos\lambda_n x - h\sin\lambda_n x\right) = \cos x - \frac{1}{(2 - lh)h} + \frac{x}{2 - lh}$$

$$P_t(x,0) = \sum_{n=1}^{\infty} \lambda_n B_n(\lambda_n\cos\lambda_n x - h\sin\lambda_n x) = 2x - \frac{1 - 2lh}{(2 - lh)h} - \frac{3x}{2 - lh}$$

$$\left\|X_n\right\|^2 = \int_0^l (\lambda_n\cos\lambda_n x - h\sin\lambda_n x)^2 dx$$

$$A_n = \frac{1}{\|X_n\|^2}\int_0^l (\cos x - \frac{1}{(2-lh)h} + \frac{x}{2-lh})(\lambda_n\cos\lambda_n x - h\sin\lambda_n x)dx$$

$$B_n = \frac{1}{\lambda_n\|X_n\|^2}\int_0^l (2x - \frac{1-2lh}{(2-lh)h} - \frac{3x}{2-lh})(\lambda_n\cos\lambda_n x - h\sin\lambda_n x)dx$$

$$P(x,t) = \sum_{n=1}^{\infty}(A_n\cos\lambda_n t + B_n\sin\lambda_n t)(\lambda_n\cos\lambda_n x - h\sin\lambda_n x)$$

2) $Q(x,t) = \sum_{n=1}^{\infty} T_n(t)X_n(x) = \sum_{n=1}^{\infty} T_n(t)(\lambda_n\cos\lambda_n x - h\sin\lambda_n x)$

$$\sum_{n=1}^{\infty} T_n''(t)(\lambda_n\cos\lambda_n x - h\sin\lambda_n x) + \lambda_n{}^2 T_n(t)(\lambda_n\cos\lambda_n x - h\sin\lambda_n x) = e^t\cos x$$

$$\sum_{n=1}^{\infty}(T_n''(t) + \lambda_n{}^2 T_n(t))(\lambda_n\cos\lambda_n x - h\sin\lambda_n x) = e^t\cos x$$

$$T_n''(t) + \lambda_n{}^2 T_n(t) = e^t * E_n$$

$$E_n = \frac{1}{\|X_n\|^2}\int_0^l \cos x(\lambda_n\cos\lambda_n x - h\sin\lambda_n x)dx$$

$$T_n''(t) + \lambda_n{}^2 T_n(t) = e^t * E_n = e^t * \frac{1}{\|X_n\|^2}\int_0^l \cos x(\lambda_n\cos\lambda_n x - h\sin\lambda_n x)dxdx$$

$$T_n(t) = G_n\cos\lambda_n t + F_n\sin\lambda_n t + \frac{e^t * E_n}{\lambda_n{}^2}$$

$$\begin{cases} T_n(0) = 0 \\ T_{n_t}(0) = 0 \end{cases}$$

$$T_n(t) = \frac{E_n}{\lambda_n{}^2} * \cos\lambda_n t - \frac{E_n}{\lambda_n{}^3} * \sin\lambda_n t + e^t * \frac{E_n}{\lambda_n{}^2}$$

h = 1900, l = 0.05

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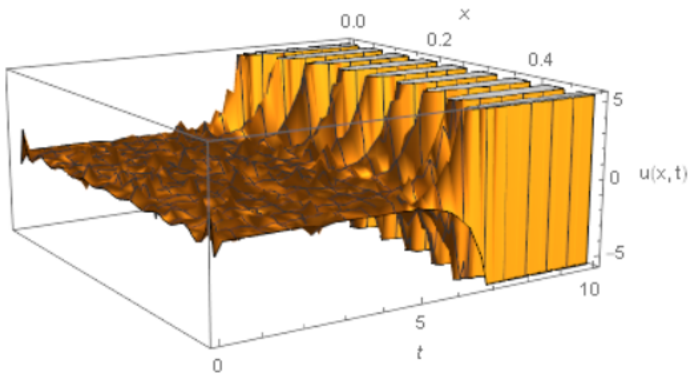

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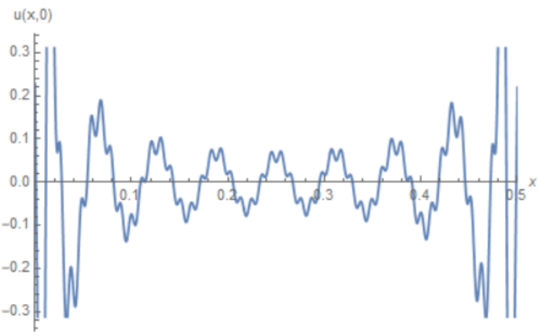

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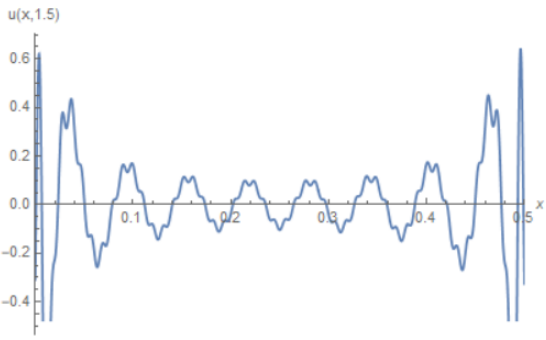

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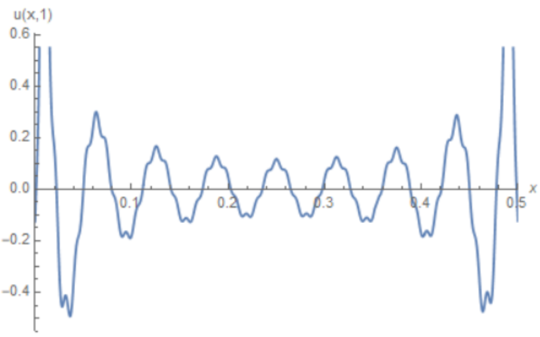

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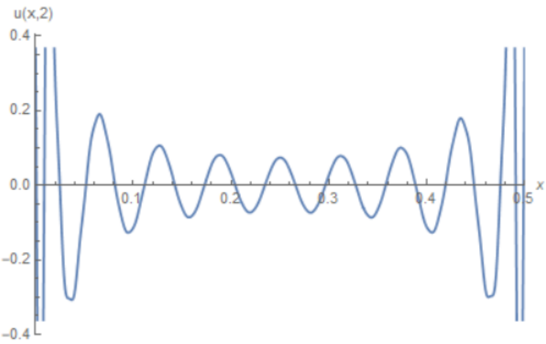

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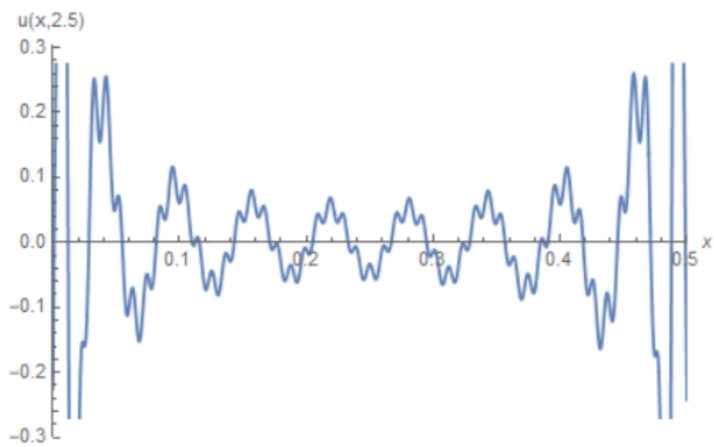

h = 100, l = 5