# Thermodynamics III Cheatsheet

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## 1 Conduction

### 1.1 Definition

Heat transfer in a body due to temperature difference

### 1.2 Fouriers Law

$$\boxed{\begin{aligned} \vec{\dot{q}}'' &= -\lambda \nabla T \\ \text{1D:} \quad \dot{q_x}'' &= \frac{\dot{Q}}{A} = -\lambda \frac{dT}{dx} \end{aligned}}$$

$\vec{\dot{q}}'' =$ Heat flux, $\left[\frac{W}{m^2}\right]$

$\lambda =$ thermal conductivity, $\left[\frac{W}{mK}\right]$

3-Dimensional equations follow the fouriers and fick's law:

$$\vec{\dot{q}}'' = \underbrace{-\lambda \frac{\partial T}{\partial x}\hat{x}}_{\dot{q}_x''} \underbrace{-\lambda \frac{\partial T}{\partial y}\hat{y}}_{\dot{q}_y''} \underbrace{-\lambda \frac{\partial T}{\partial z}\hat{z}}_{\dot{q}_z''}$$

$$\vec{\dot{q}}'' = \underbrace{-\lambda \frac{\partial T}{\partial r}\hat{r}}_{\dot{q}_r''} \underbrace{-\lambda \frac{\partial T}{r\partial \phi}\hat{\phi}}_{\dot{q}_\phi''} \underbrace{-\lambda \frac{\partial T}{\partial z}\hat{z}}_{\dot{q}_z''}$$

$$\vec{\dot{q}}'' = \underbrace{-\lambda \frac{\partial T}{\partial r}\hat{r}}_{\dot{q}_r''} \underbrace{-\lambda \frac{\partial T}{r\partial \theta}\hat{\theta}}_{\dot{q}_\theta''} \underbrace{-\lambda \frac{\partial T}{r\sin(\theta)\partial \phi}\hat{\phi}}_{\dot{q}_\phi''}$$

**Koordinatensysteme Zylindrisch und Sphärisch**

**Some common thermal conductivities**

| Material | thermal conductivity $\lambda\left[\frac{W}{mK}\right]$ |
|---|---|
| Aluminium: | 240 |
| Copper: | 400 |
| Carbon steels: | 40-60 |
| Stainless steels: | 15 |
| Helium: | 0.152 |
| Air or $N_2$: | 0.026 |
| Steam (100°C): | 0.025 |
| Plastic: | 0.2 |
| Rubber: | 0.16 |

### 1.3 Heat Conduction Equation

Remember: $\nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}$

$$= \underbrace{\vec{\nabla}\left(\lambda \vec{\nabla} T\right)}_{\text{Change in thermal energy storage}} + \underbrace{\dot{Q}'''_{\text{source}}}_{\text{Thermal energy generation}}$$

w

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$$\frac{\partial T}{\partial x}\Big) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \dot{Q}'''_{\text{source}}$$

$$\lambda r \frac{\partial T}{\partial r}\Big) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda \frac{\partial T}{\partial \varphi}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \dot{Q}'''_{\text{source}}$$

$$\Big(\lambda r^2 \frac{\partial T}{\partial r}\Big) + \frac{1}{r^2 \sin^\theta}\frac{\partial}{\partial \varphi}\left(\lambda \frac{\partial T}{\partial \varphi}\right) + \frac{1}{r^2 \sin(\theta)}\frac{\partial}{\partial \theta}\left(\lambda \sin(\theta) \frac{\partial T}{\partial \theta}\right) + \dot{Q}'''_{\text{source}}$$

#### 1.3.1 Solving the Heat Conduction Equation: Boundary conditions (Noch machen, Lecture 2 oder 3)

- Surface Temperature known
- Heat Flux known
- Convection known

## 2 Convection

### 2.1 Heat transfer by convection

$$\boxed{\dot{q}'' = \frac{\dot{Q}}{A} = \alpha(T_\infty - T_W)}$$

$\vec{\dot{q}}'' =$ Heat flux, $\left[\frac{W}{m^2}\right]$

$\alpha =$ heat transfer coefficient, $\left[\frac{W}{mK}\right]$

## 3 Thermal Resistances

### 3.1 Thermal resistance model

assumption: $\lambda = const$

$$\dot{Q_x} = \dot{Q_x}'' A = \frac{1}{R_{th}} \Delta T$$

| **Geometry** | **Conduction** $R_{th}$ | **Convection** $R_{th}$ |
|---|---|---|
| Planar | $\frac{L}{\lambda A}$ | $\frac{1}{\alpha A}$ |
| Cylinder | $\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda L}$ | $\frac{1}{2\pi r_1 L\alpha}$ |
| Sphere | $\frac{\frac{1}{R_1} - \frac{1}{R_2}}{4\pi\lambda}$ | $\frac{1}{4\pi r^2 \alpha}$ |

Note that for some pipes, there exists a critical thickness of the pipe insulation, for which the thermal insulation of a thin insulation layer is lower than without insulation layer.

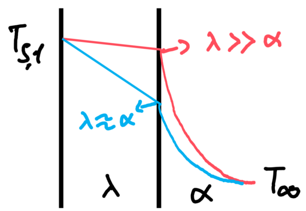
$R_{crit} : \frac{d}{dr_2}(R_{tot}) = 0, R_{tot} = R_{th,ins} + R_{th,conv}$

$\Rightarrow r_{2,crit} = \frac{\lambda}{\alpha}$

## 4 Lumped capacitance method

### 4.1 Lumped capacitance

Idea: The temperature in a body is almost uniform, so we can assume it to be uniform. Temperature within body will now be $T(t)$ instead of $T(t,x,y,z)$. That means, the temperature difference inside the body $\Delta T_i = T_{s,1} - T_{s,2}$ must be much smaller than the temperature difference outside the body $\Delta T_o = T_{s,2} - T_\infty$.




$$\dot{q} = \frac{\lambda A}{L}(T_{s,1} - T_{s,2}) = \alpha A(T_{s,2} - T_\infty)$$

$$\Rightarrow \frac{(T_{s,1} - T_{s,2})}{(T_{s,2} - T_\infty)} = \frac{\alpha L}{\lambda} = Bi$$

$$L_{\text{cylinder}} = \frac{r}{2} \qquad L_{\text{sphere}} = \frac{r}{3}$$