Robot Localization

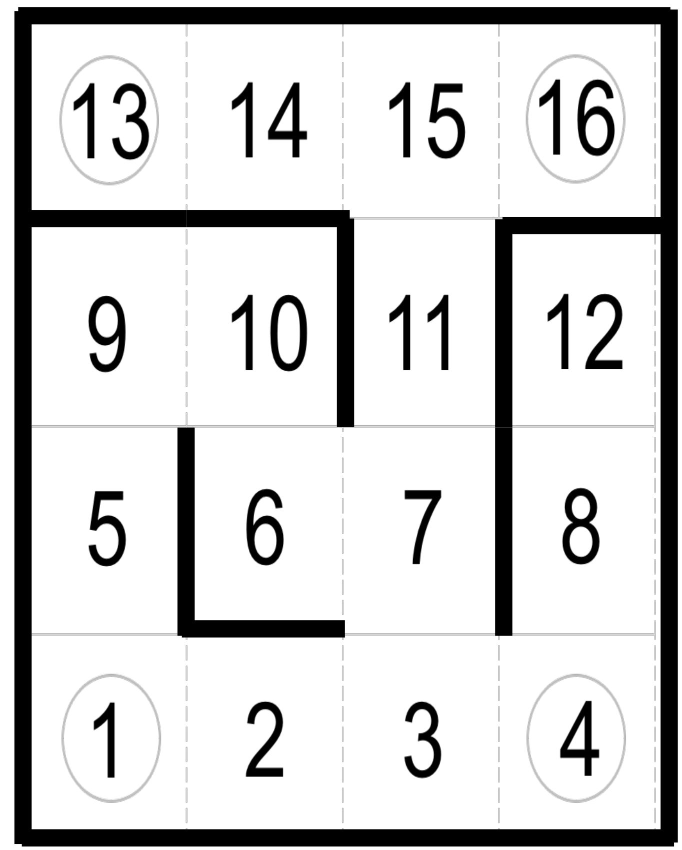
Alfredo Weitzenfeld

Markov Localization

- **Markov Localization** uses a discrete representation of the state space. The environment is represented by a finite number of states. At each iteration, the probability of each state of the entire space is updated.

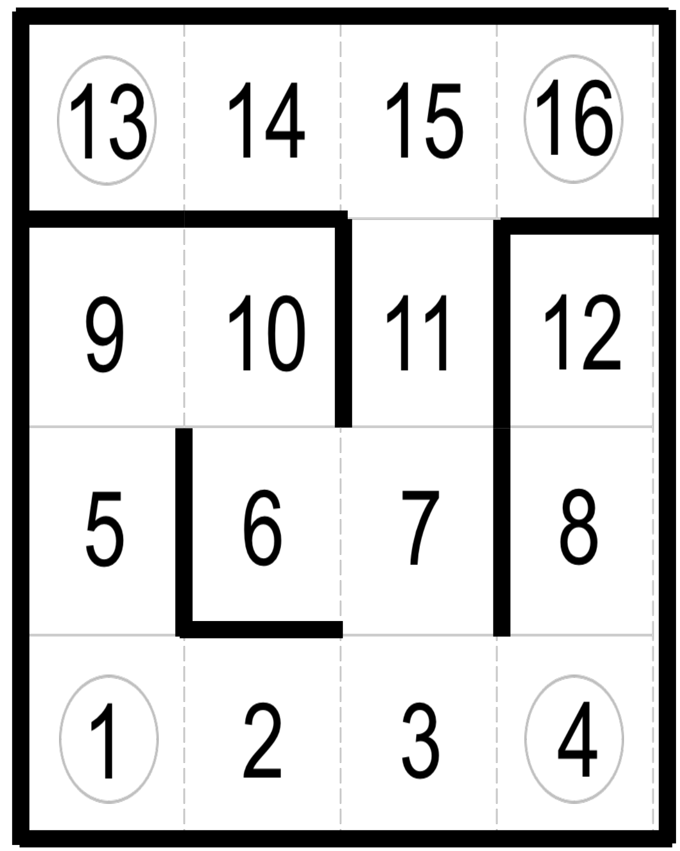
Robot Localization

- Consider a robot equipped with encoders and a perfect compass moving in a square room that is discretized into a map of 16 cells.
- Assume pose is represented by the number of the cell, disregarding orientation.
- Where is the robot located in this 4x4 maze?



Robot Localization

- Robot pose (state) *s*, has probability p(*s*) of being in one particular grid cell in the maze.
- Without further knowledge, p(s) = 1/16 for any grid cell.
- This probability is assigned without knowledge of initial pose, robot control (motion) or measurements (sensor readings).



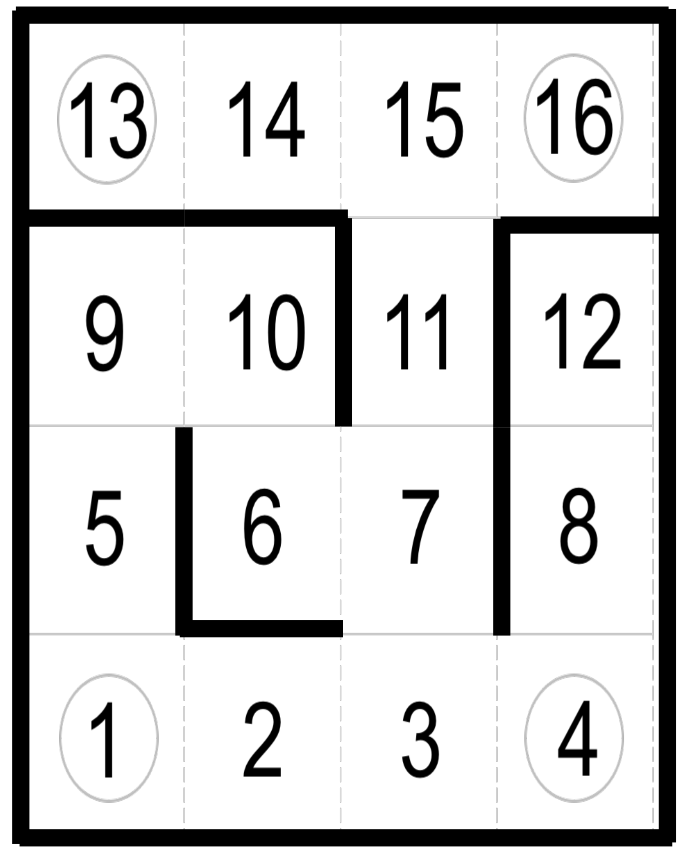
Robot Localization

- According to Bayes Theorem, robot pose (state) s is given by sensor measurements z:

  $$p(s|z) = p(z|s)\, p(s) \,/\, p(z)$$

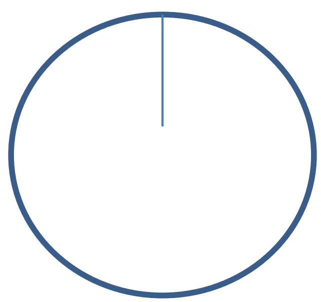
- The *posterior* probabilities $p(s|z)$ are proportional to $p(z|s)\, p(s)$:

  $$p(s|z) \propto p(z|s)\, p(s)$$



Robot Localization

- Assume robot does not rotate
- Assume robot always points up

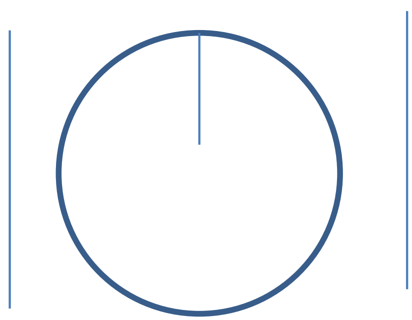


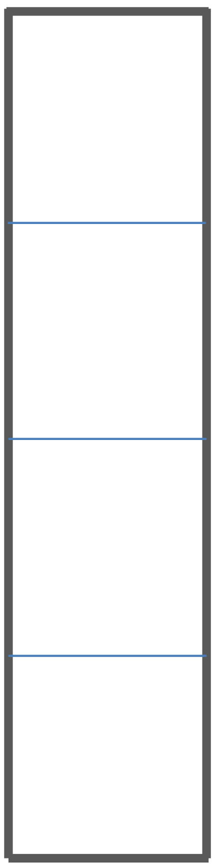
Robot Localization

- Assume three distance sensors (left, right, front)
- Assume binary sensors: free or occupied

Robot Localization

- Assume robot "perfect" sensors
- Where is the robot located if it reads a left and right wall but not a front wall?





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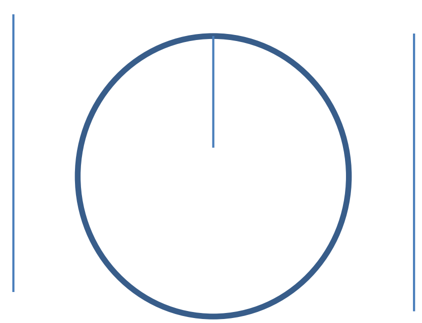

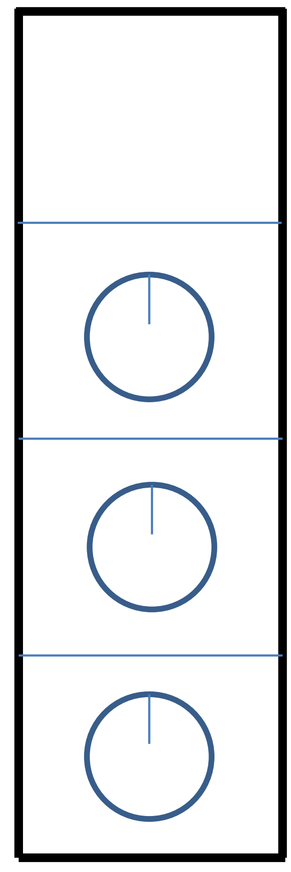

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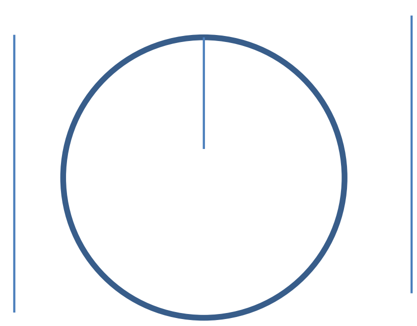

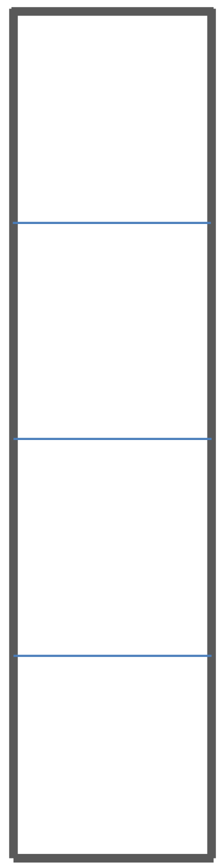

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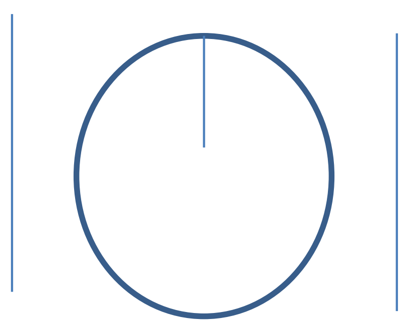

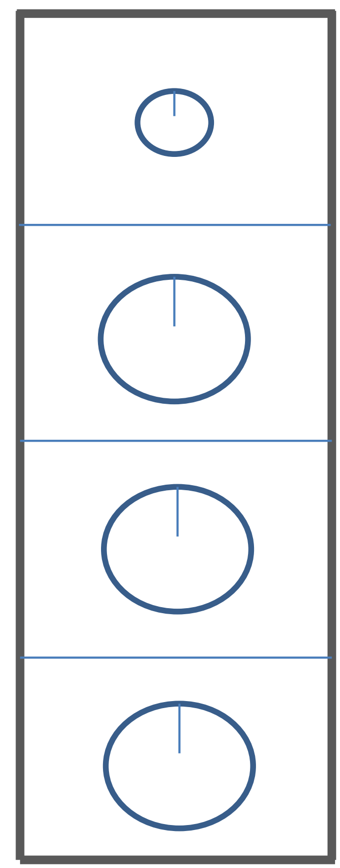

Robot Localization

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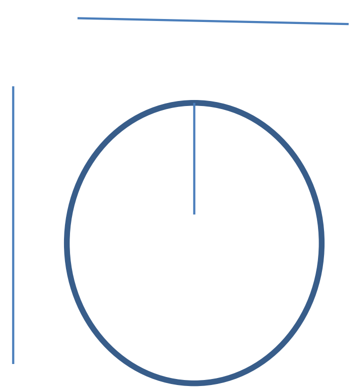

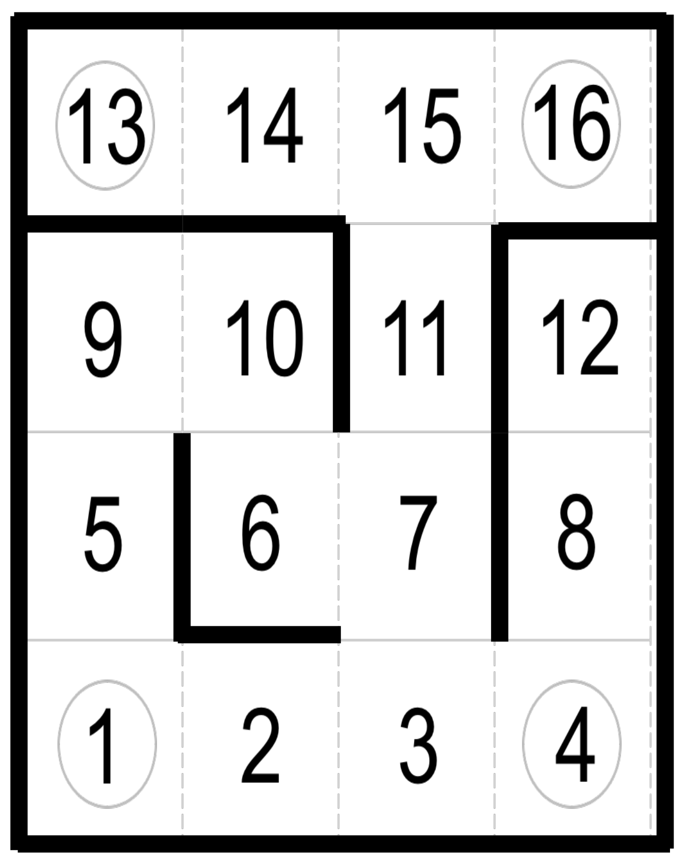

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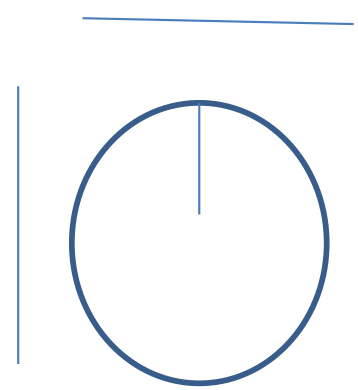

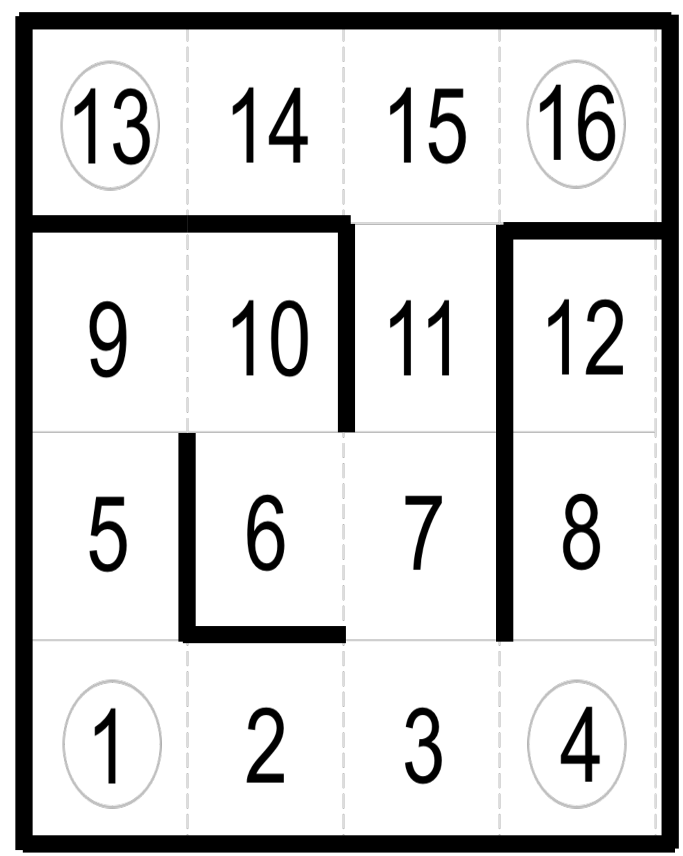

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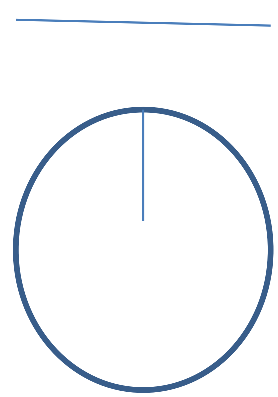

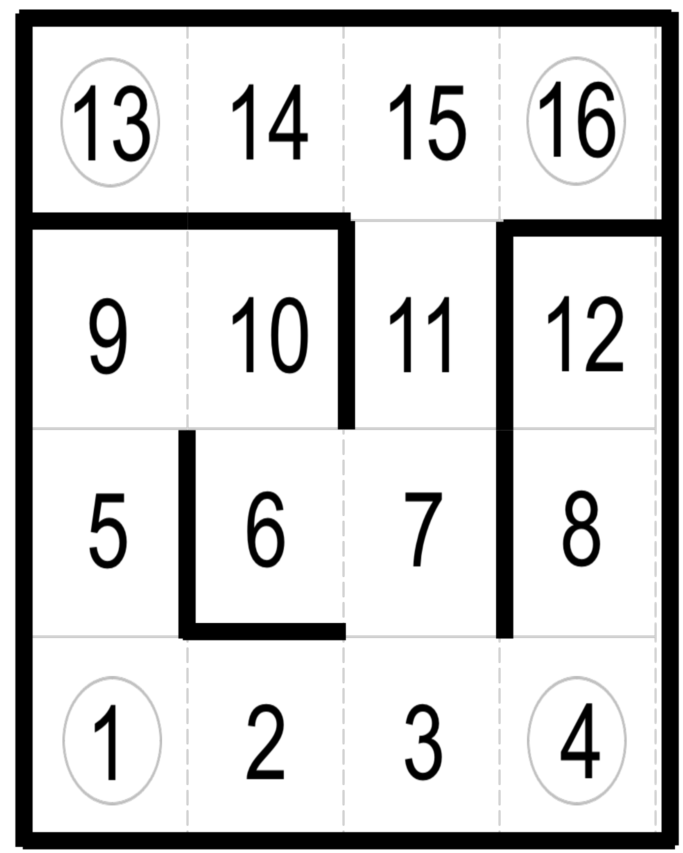

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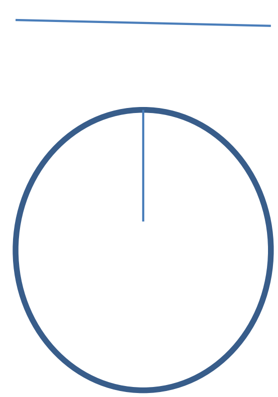
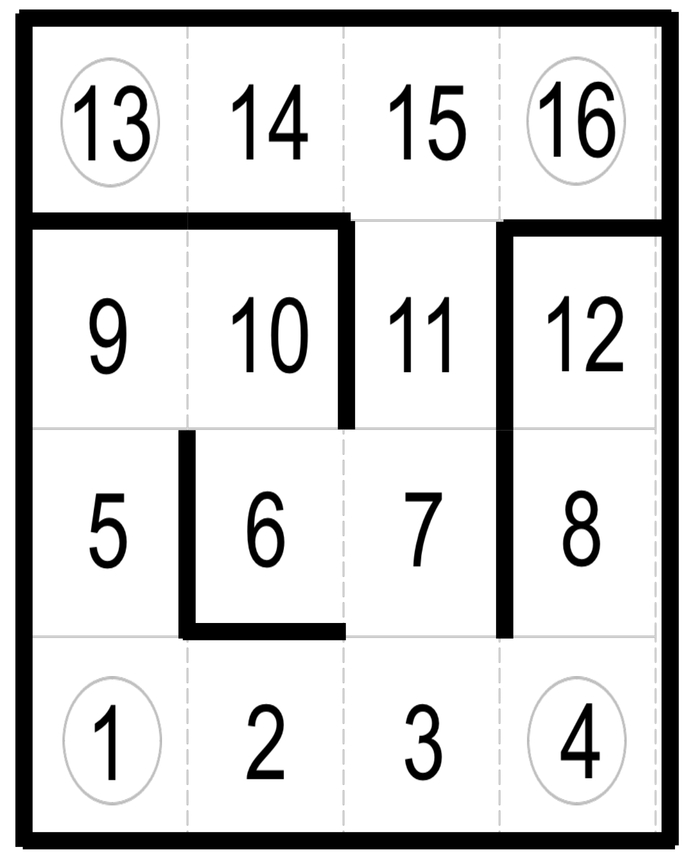
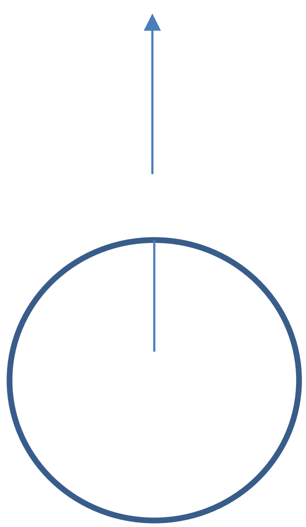


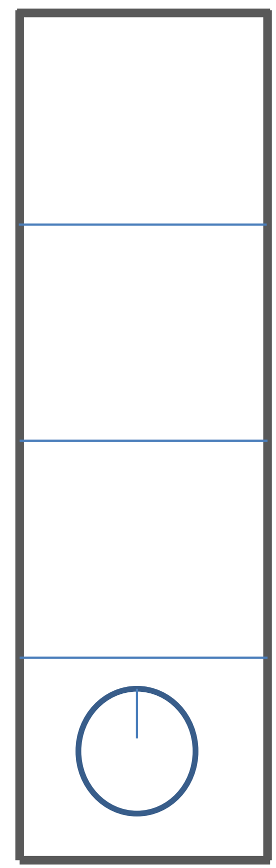


Robot Localization

- Assume robot “perfect” control
- Assume robot can move at most on square at a time
- Where is the robot located if it moves forwards?



forward



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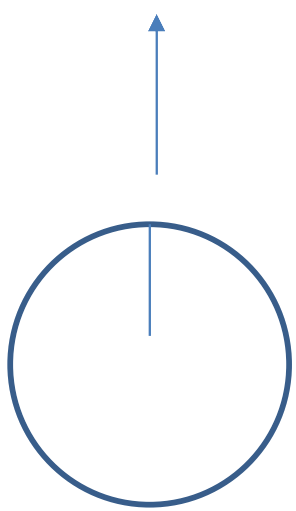
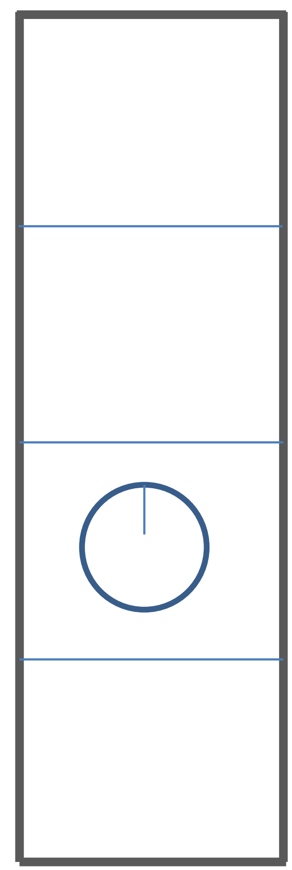


forward



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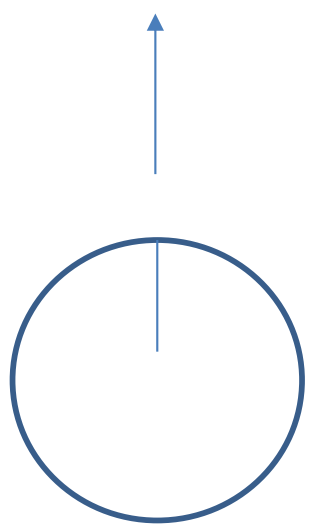

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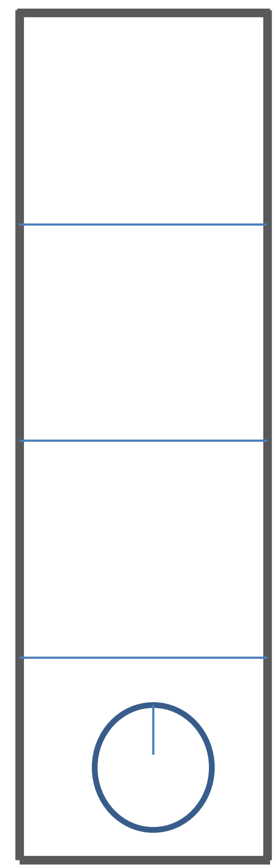

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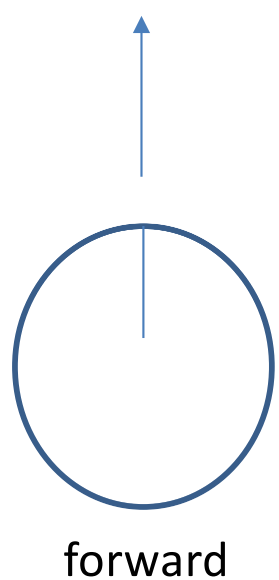
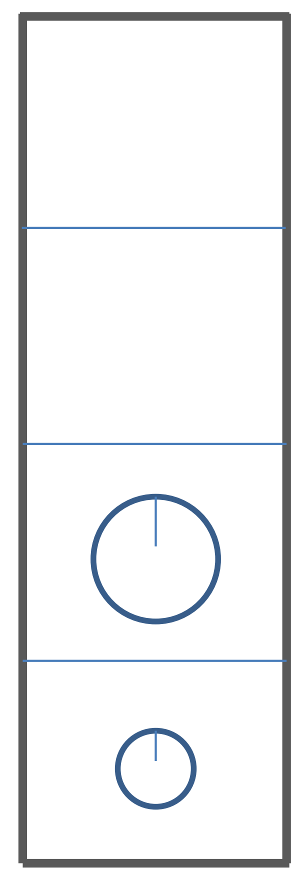
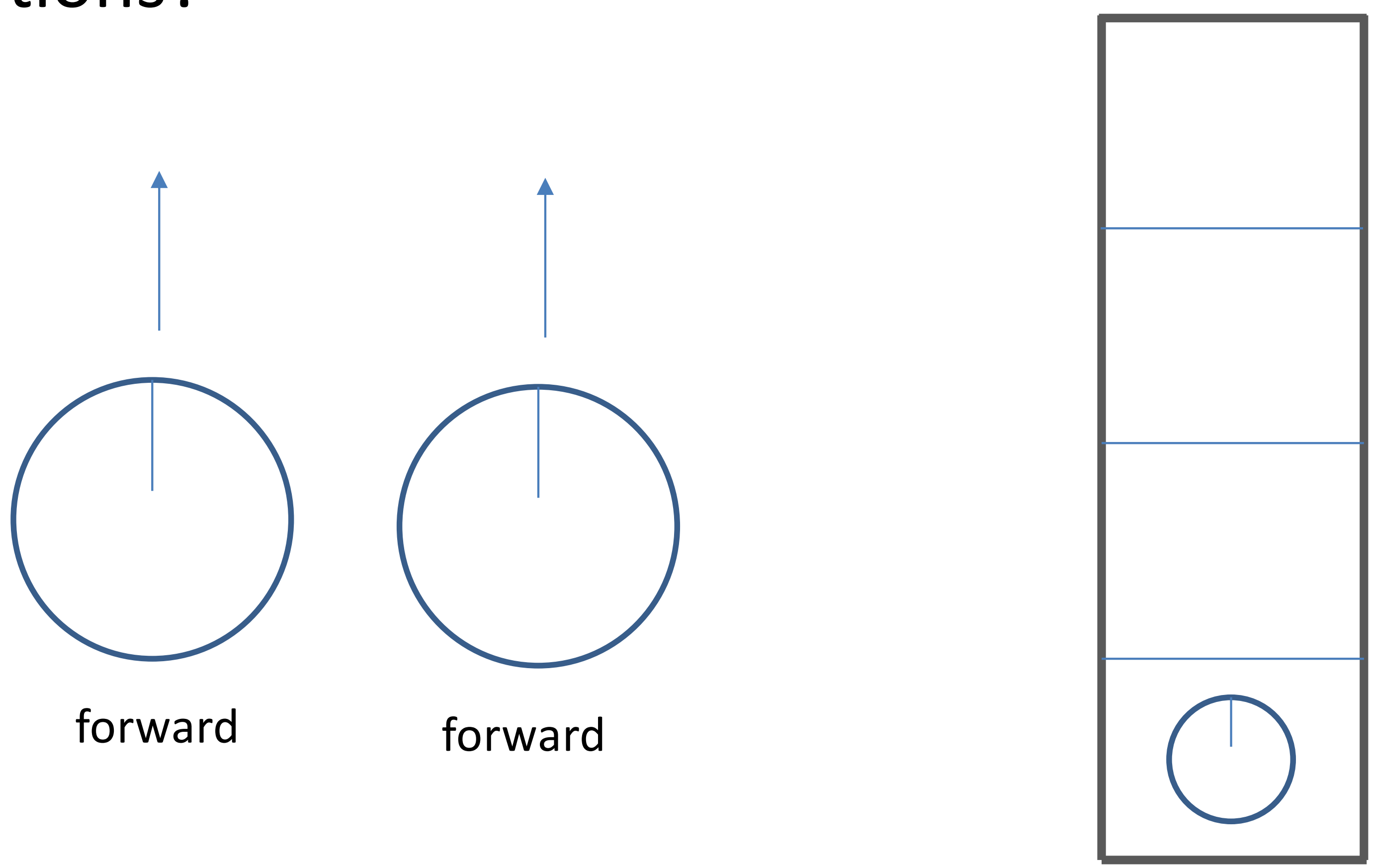




Robot Localization

- Assume robot "perfect" control
- Assume robot can move at most on square at a time
- Where is the robot located after two forward motions?



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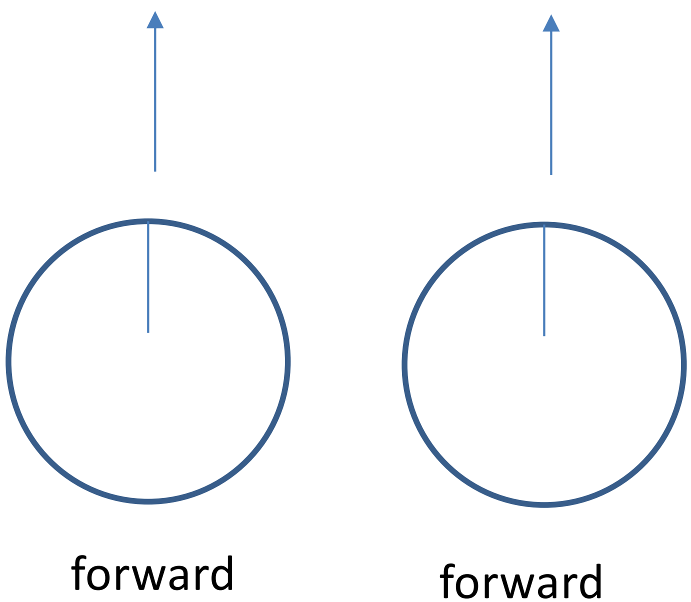

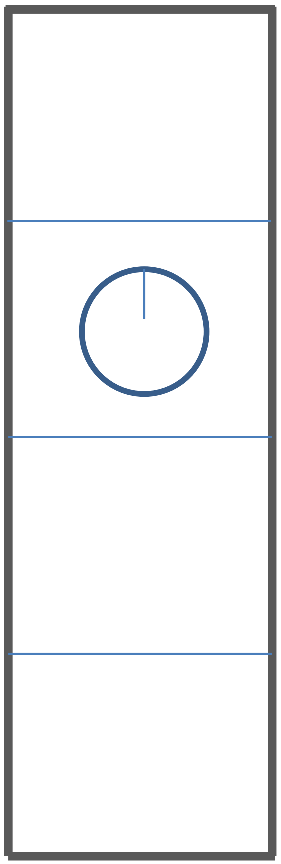

Robot Localization

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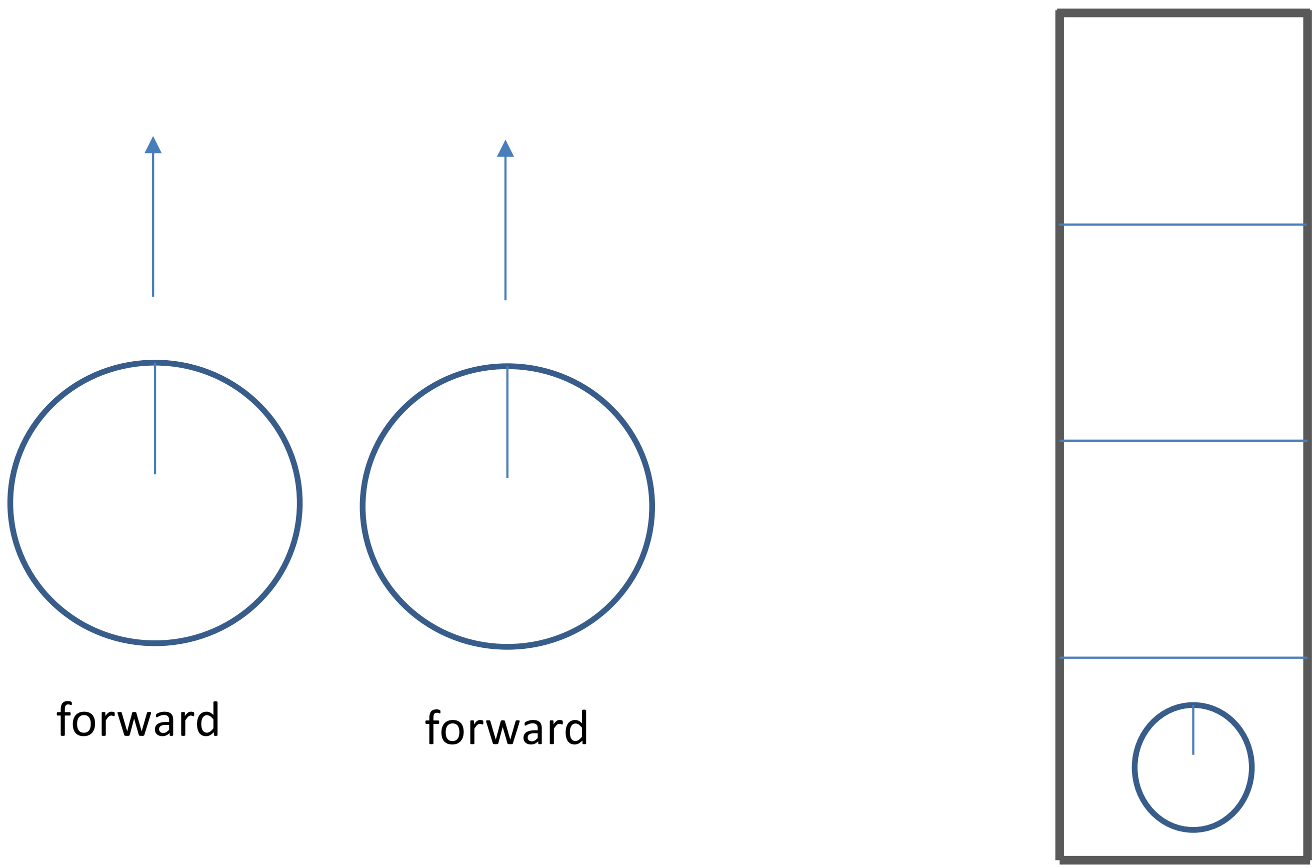
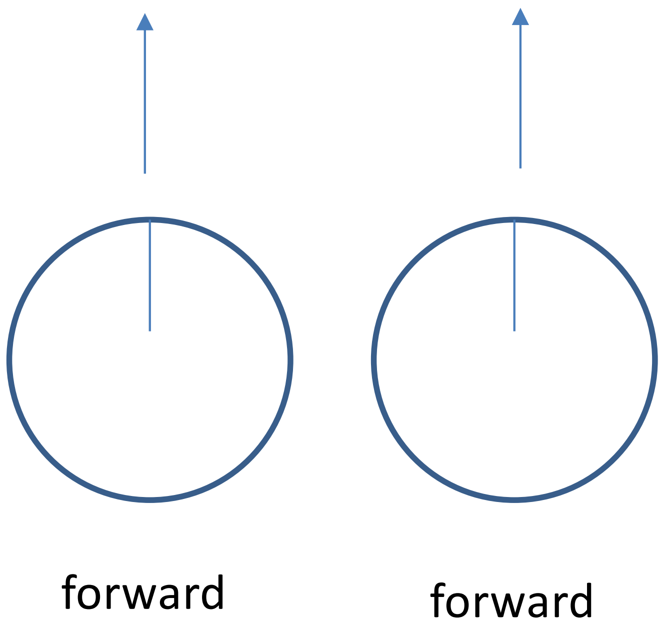
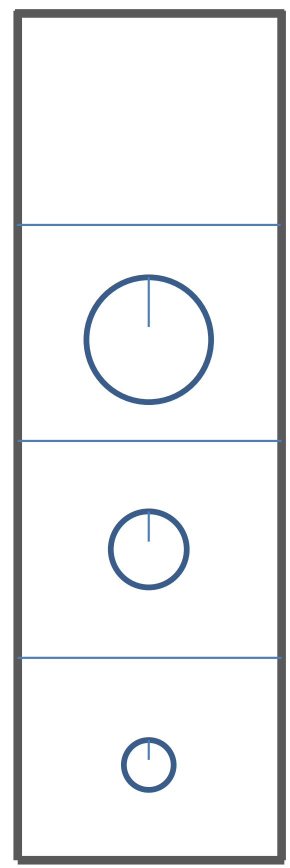
- Where is the robot located if it moves forwards?



Robot Localization

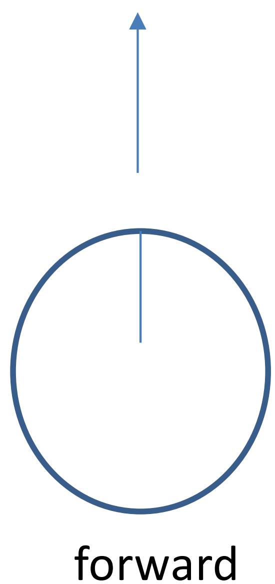
- Assume robot "noisy" control
- Assume robot can move at most on square at a time
- Where is the robot located if it does two consecutive forward motions?

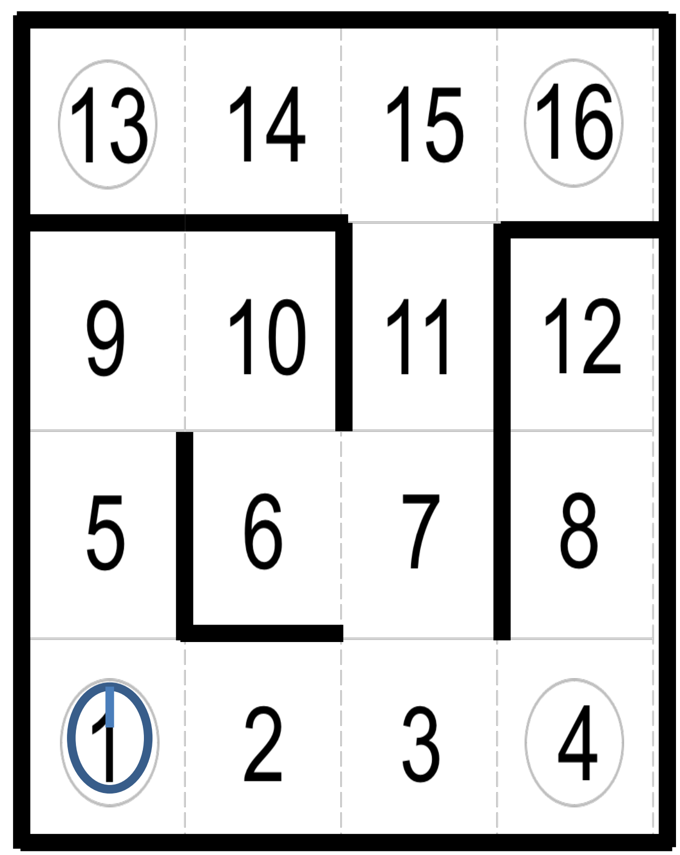




Robot Localization

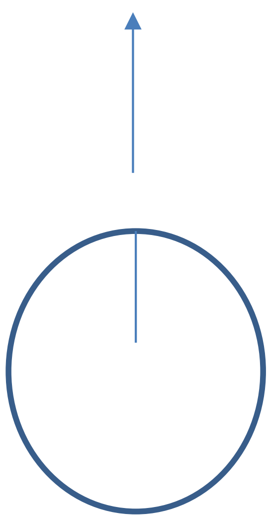
- Assume robot "noisy" control
- Assume robot can move at most on square at a time
- Assume robot initial position is at "1"
- Where is the robot located if it moves forwards?

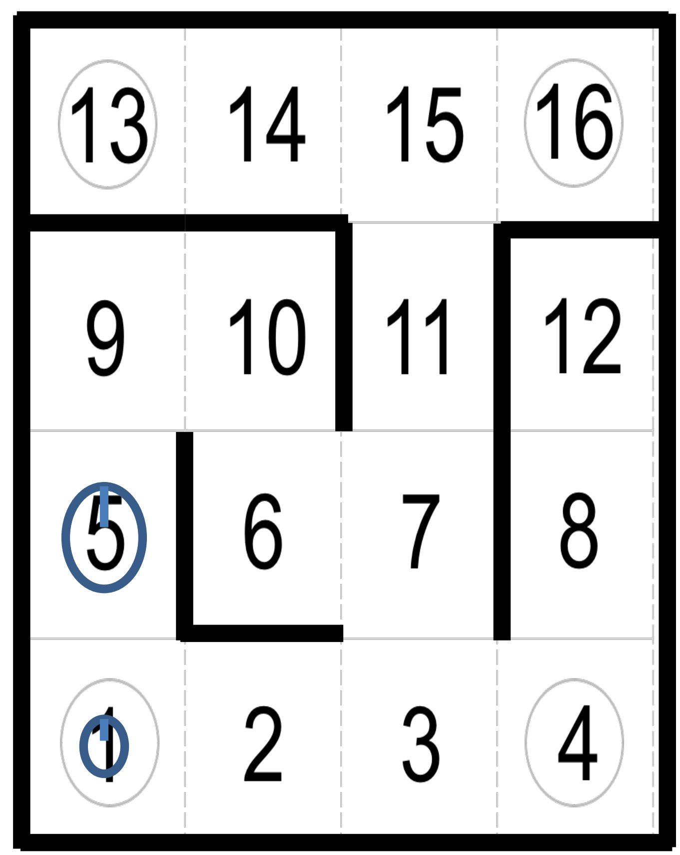




Robot Localization

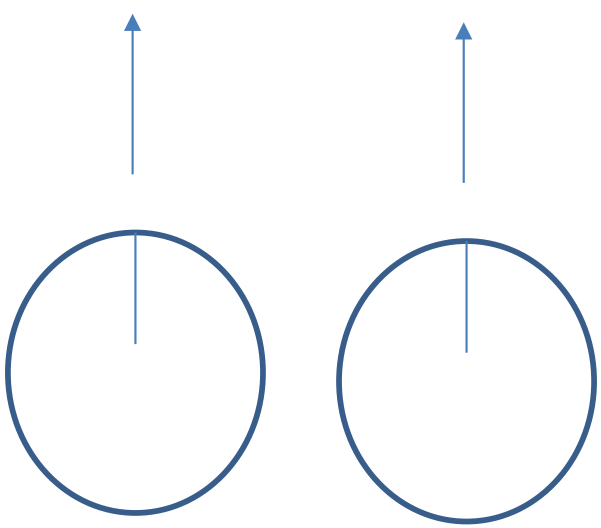
- Assume robot "noisy" control
- Assume robot can move at most on square at a time
- Assume robot initial position is at "1"
- Robot may stay in "1" or move "up" to "5"



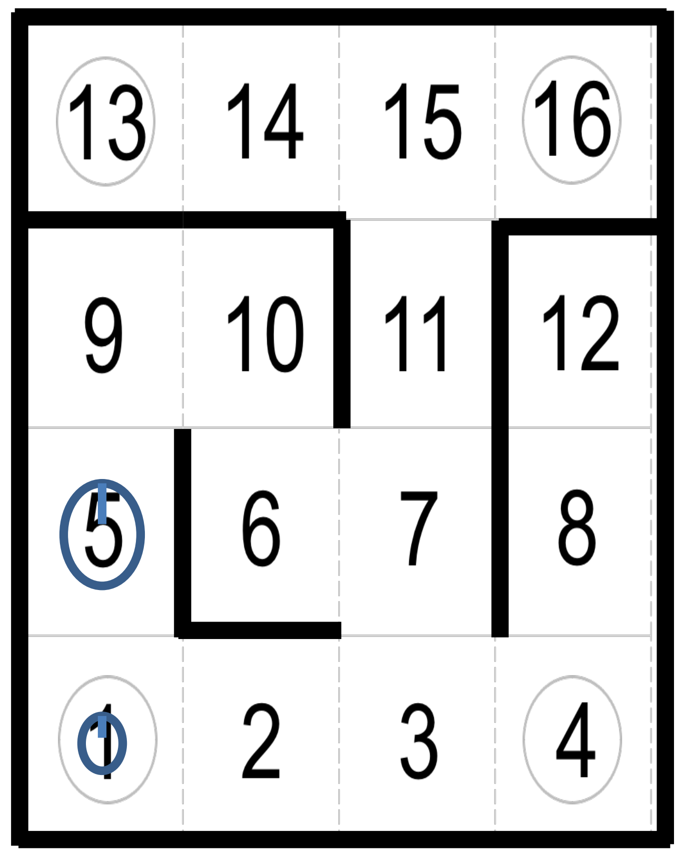


Robot Localization

- Assume robot "noisy" control
- Assume robot can move at most on square at a time
- Assume robot initial position is at "1"
- Where is the robot located after two forward motions?



forward forward



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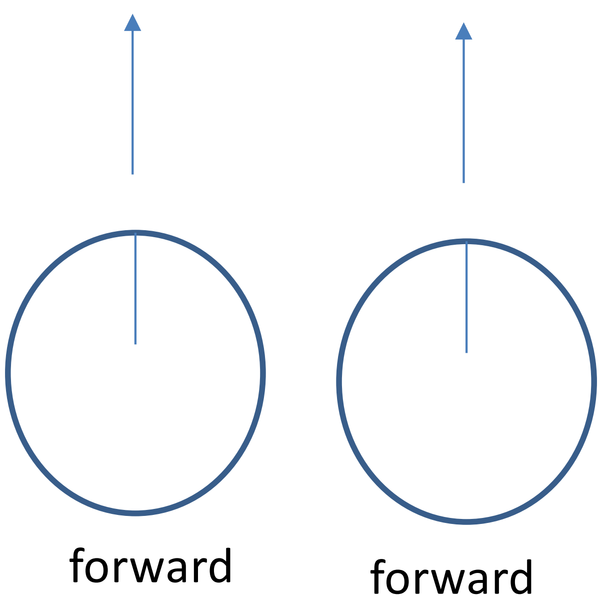

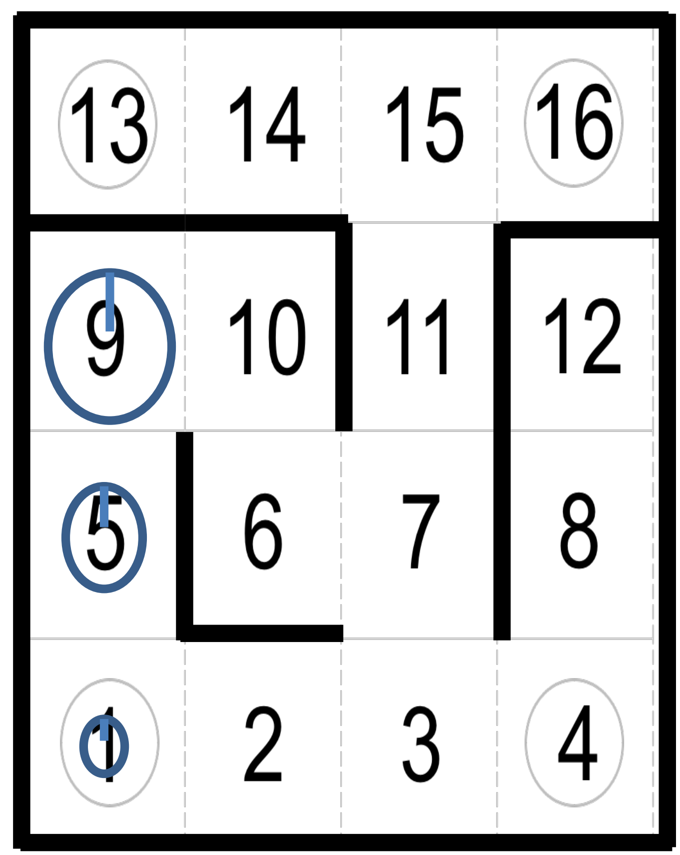

Robot Localization

- Assume robot "noisy" control
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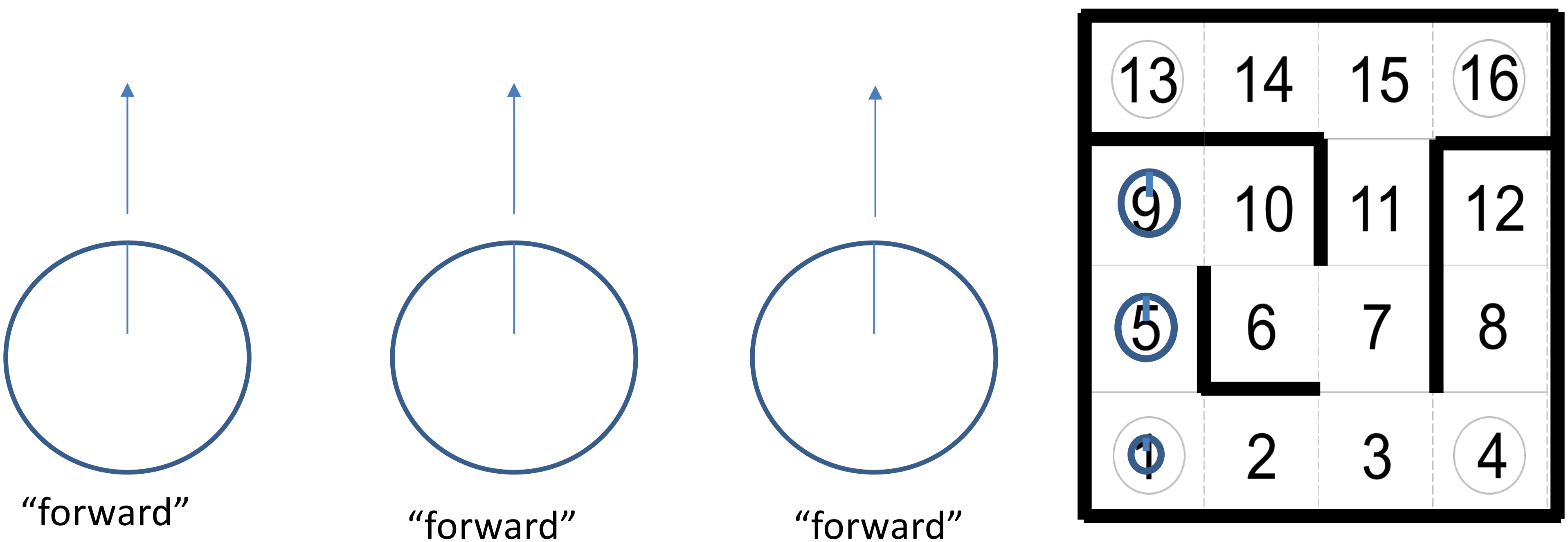
- Assume robot initial position is at "1"
- Where is the robot located after three forward motions?



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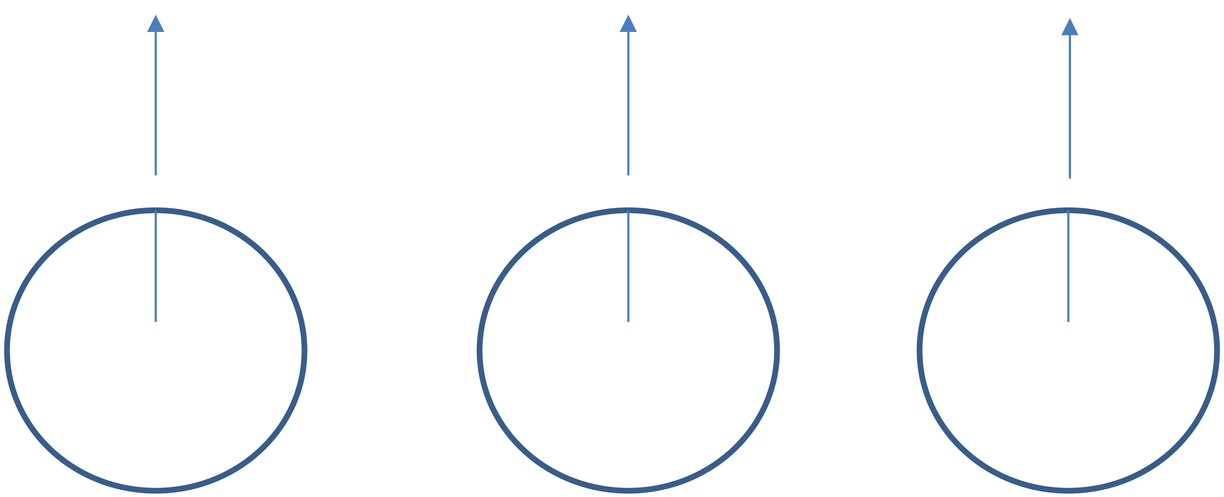
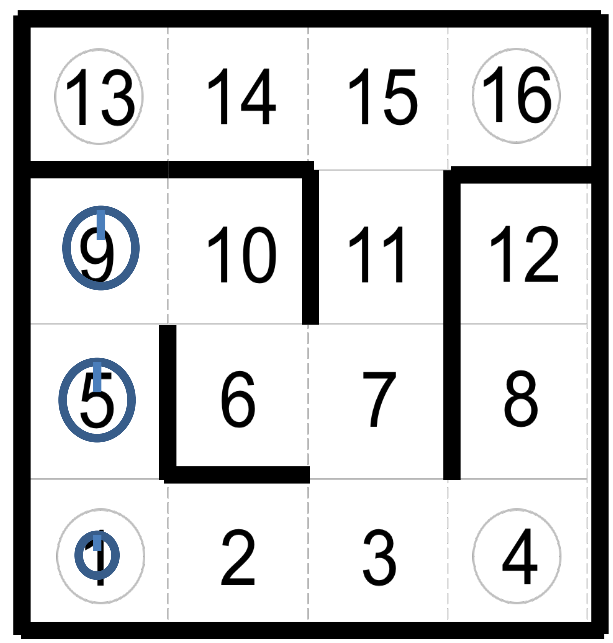
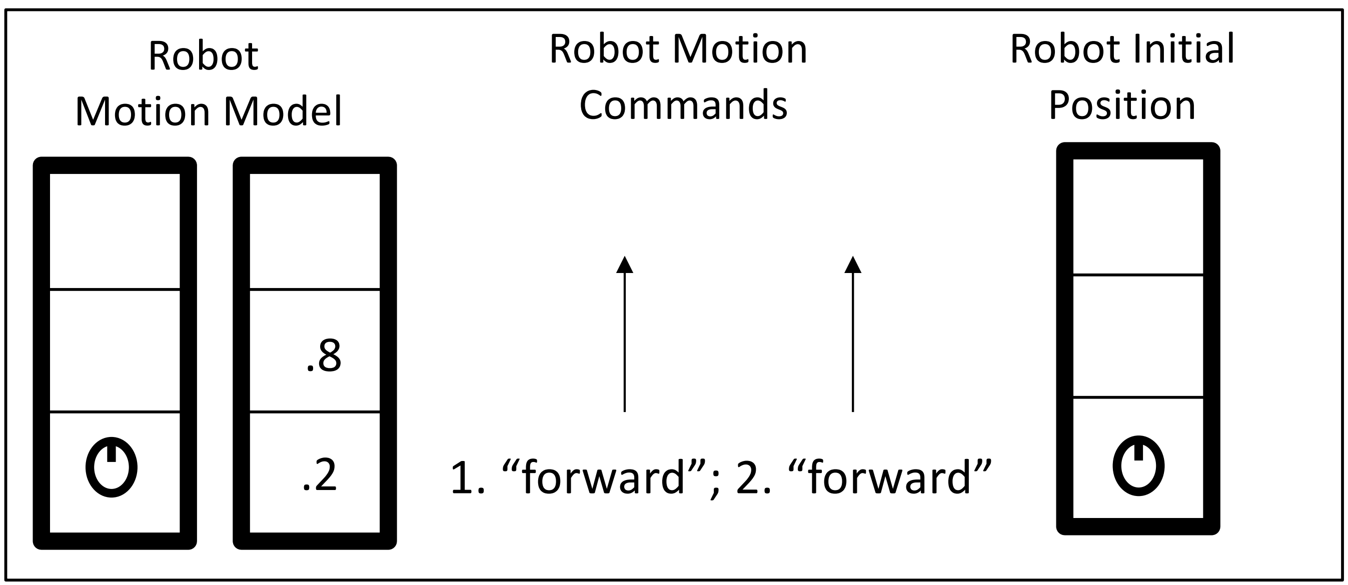


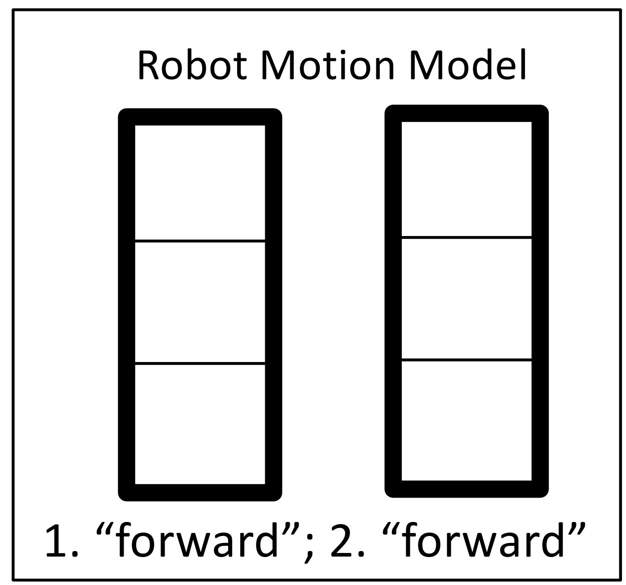
"forward" "forward" "forward"



Robot State from Motion Control

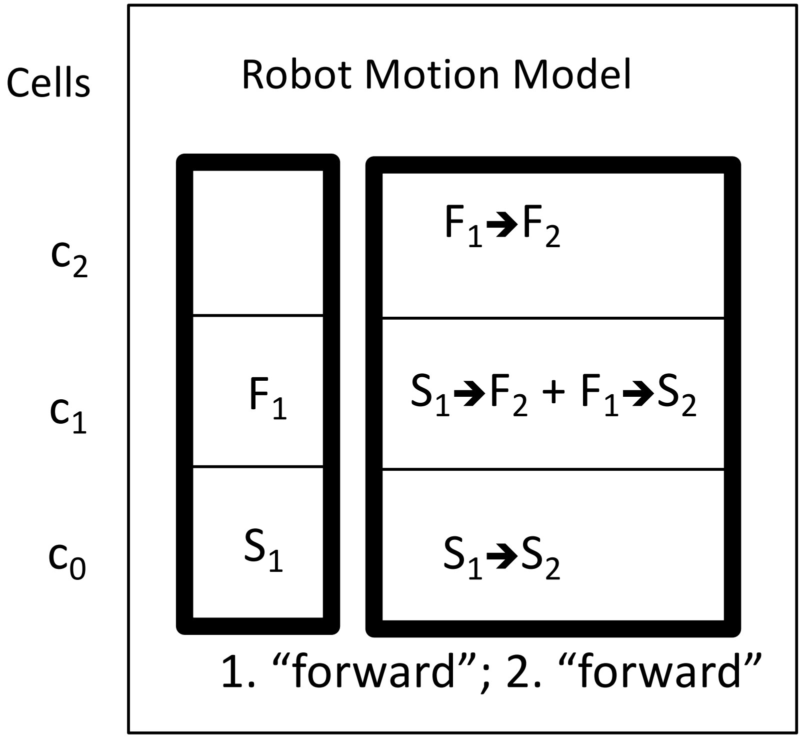
- Robot initial state is at the bottom of a vertical corridor and motion control u = "forward".
- When moving forward the robot has 0.2 probability of staying in the same place, and 0.8 of moving up one square.
- Where can the robot be after two "forward" motions and with what probabilities?
- NOTE: Sum of all states probabilities must be equal to 1.0.

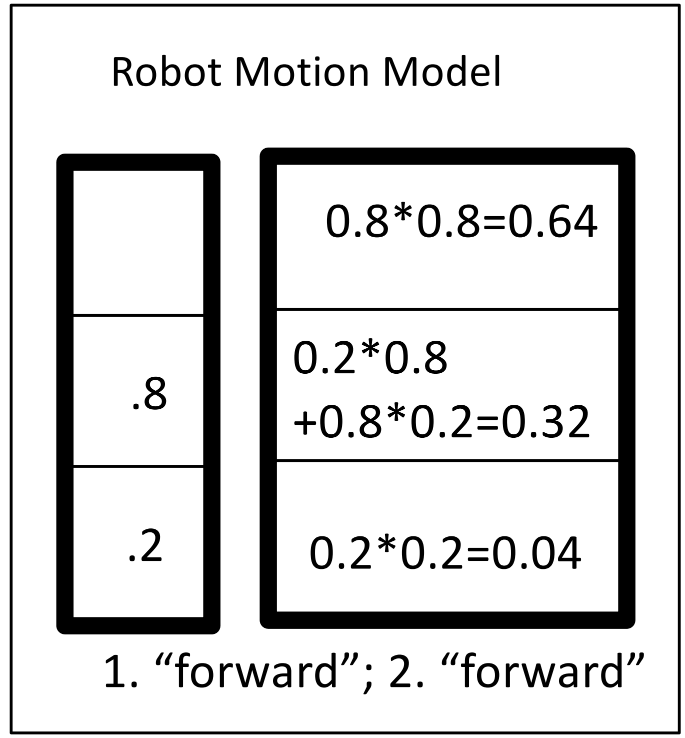




Robot State from Motion Control

- How many possible robot motion combinations?





S – Stay ; F – Move Forward

Law of Total Probability

- <u>Law of total probability</u>: The sum of all disjoint probabilities:

  $$p(A) = \Sigma_i \, p(A|B_i) \, p(B_i)$$

- The law expresses the total probability as the summation of several disjoint events.
- Robot state is given by s_t:

  $$p(s_t) = \Sigma_{st} \, p(s_t \mid s_{t-1}, u_t) \, p(s_{t-1})$$

- A new robot state is obtained by a single control from a previous state involving disjoint events.

Robot State from Motion Control

Disjoint probabilities at t=0 and t=1 forward motion

- s_t (state s at time t), c_n(cell c)
- $s_{t=0}$ Initial state $p(s_{t=0} = c_0) = 1.0$
- $s_{t=1} = s_{t=1}(c_0) + s_{t=1}(c_1)$
- $s_{t=1}(c_0)$:

  $p(s_{t=1} = c_0) = p(s_{t=1} = c_0 | s_{t=0} = c_0, u_{t=1}=\text{“forward”})\ p(s_{t=0} = c_0)$

  $p(s_{t=1} = c_0) = 0.2*1.0 = 0.2$
- $s_{t=1}(c_1)$:

  $p(s_{t=1} = c_1) = p(s_1 = c_1 | s_{t=0} = c_0, u_{t=1}=\text{“forward”})\ p(s_{t=0} = c_0)$

  $p(s_{t=1} = c_1) = 0.8*1.0 = 0.8$

Robot State from Motion Control

Disjoint probabilities at t=2 forward motion

- $s_{t=2} = s_{t=2}(c_0) + s_{t=2}(c_1) + s_{t=2}(c_2)$
- $s_{t=2}(c_0)$:

  $p(s_{t=2} = c_0) = p(s_{t=2} = c_0 | s_{t=1} = c_0, u_{t=2}=\text{“forward”})\ p(s_{t=2} = c_0)$

  $p(s_{t=2} = c_0) = 0.2*0.2 = 0.4$
- $s_{t=2}(c_1)$:

  $p(s_{t=2}= c_1) = p(s_{t=2}= c_1 | s_{t=1}= c_0, u_{t=2}=\text{“forward”})\ p(s_{t=2}= c_0) +$

  $p(s_{t=2}= c_1) = p(s_{t=2}= c_1 | s_{t=1}= c_1, u_{t=2}=\text{“forward”})\ p(s_{t=2}= c_1)$

  $p(s_{t=2}= c_1) = 0.8*0.2 + 0.2*0.8 = 0.32$
- $s_{t=2}(c_2)$:

  $p(s_{t=2}= c_2) = p(s_{t=2}= c_2 | s_{t=1}= c_1, u_{t=2}=\text{“forward”})\ p(s_{t=2}= c_1)$
  $p(s_{t=2}= c_2) = 0.8*0.8 = 0.64$

Robot State from Measurements

- Robot new state s_t, given prior state $s_{t\text{-}1}$, prior measurement $z_{t\text{-}1}$, and latest control u_t,

  $$\mathrm{p}(s_t | s_{t\text{-}1}, u_t, z_{t\text{-}1}) \propto \mathrm{p}(\mathrm{z}_t | s_t)\ \Sigma_{st}\ \mathrm{p}(s_t | s_{t\text{-}1}, u_t)\ \mathrm{p}(s_{t\text{-}1})$$

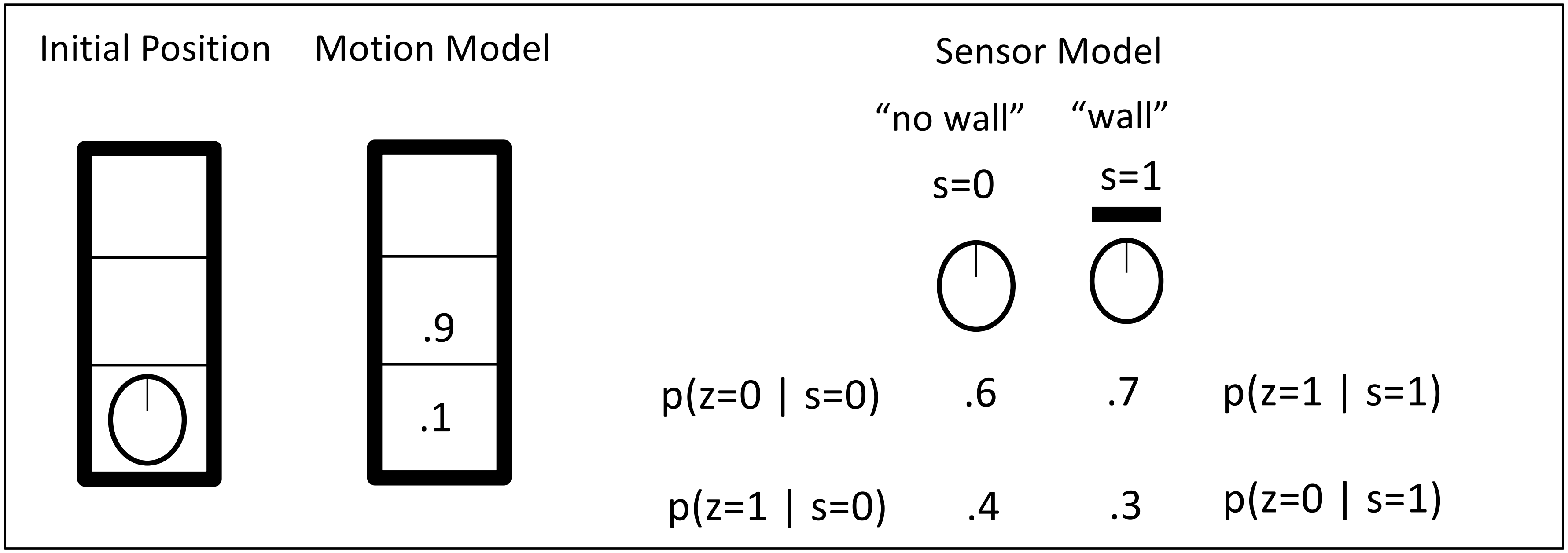
  where $\mathrm{p}(z_t | s_t)$ will tell us how likely is the sensed measurement z_t given the current robot state.
- First recalculate the previous estimate using the motion model given control u_t:

  $$\mathrm{p}(s_t) = \Sigma_{st}\ \mathrm{p}(s_t \mid s_{t\text{-}1}, u_t)\ \mathrm{p}(s_{t\text{-}1})$$
- Then update the new estimate using the sensor model given measurements z_t

  $$\mathrm{p}(s | z) = \mathrm{p}(z | s)\ \mathrm{p}(s)\ /\ \mathrm{p}(z)$$

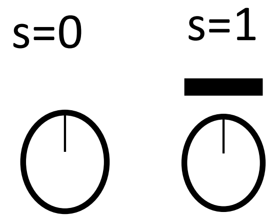
  $$\mathrm{p}(s | z) \propto \mathrm{p}(\mathrm{z} | s)\ \mathrm{p}(s)$$

Robot Measurement



- Assume z=1 corresponds to the sensor reading “wall”.
- Assume z=0 corresponds to the sensor reading “no wall”.
- Assume s=1 corresponds to the state having a “wall”.
- Assume s=1 corresponds to the state having a “no wall”.
- Assume p(z=1 | s=1) = 0.7 corresponds to the probability of the sensor reading “wall” in a state where there is “wall”.
- Assume p(z=0 | s=1) = 0.3 corresponds to the probability of the sensor reading “no wall” in a state where there is “wall”.
- Assume p(z=0 | s=0) = 0.6 corresponds to the probability of the sensor reading “no wall” in a state where there is “no wall”.
- Assume p(z=1 | s=0) = 0.4 corresponds to the probability of the sensor reading “wall” in a state where there is “no wall”.

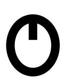
Robot Measurement



p(z="no wall" | s="not in front of a wall") .6

.7 p(z="wall" | s="in front of a wall")

Predict, Measure, Update

Robot Initial Position	<u>Predict</u> Motion Model	<u>Measure</u> Measurement Model	<u>Update</u> (before normalization)	<u>Update</u> (after normalization)
		.7	.0x.7=0	0/.6=0
	.9	.6	.9x.6=.54	.54/.6=.9
	.1	.6	.1x.6=.06	.06/.6=.1

Total: 0.54+0.06=0.6

*Note that thick lines represent walls

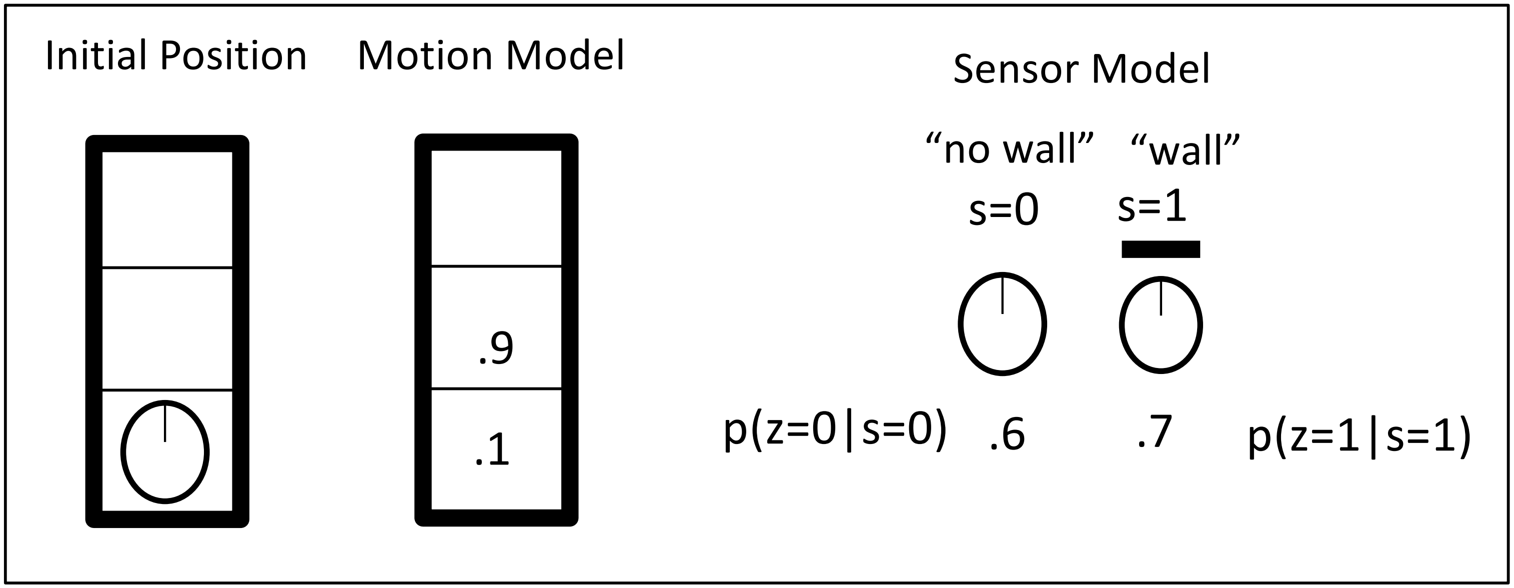
Robot State

- Given the previous robot motion model and robot sensor model, compute the probability distribution p(z|s) for the robot after two (2) forward motions for "predict, measure, and update" cycles.

(1) Motion Probability Distribution	(1) Measure Probability Distribution	(1) Normalized Update Probability Distribution	(2) Motion Probability Distribution	(2) Measure Probability Distribution	(2) Normalized Update Probability Distribution

Robot State

Compute the probability distribution for robot localization



(1) Motion Probability Distribution	(1) Measure Probability Distribution	(1) Normalized Update Probability Distribution	(2) Motion Probability Distribution	(2) Measure Probability Distribution	(2) Normalized Update Probability Distribution
0	0*0.7=0	0	0.81	0	0.832
0.9	0.9*0.6 =0.54	0.9	0.18	0.18*0.6 =0.108	0.159
0.1	0.1*0.6 =0.06	0.1	0.01	0.01*0.6 =0.006	0.009

Robot State from Motion

- What is the localization probability distribution after two “forward motions.
- Assume that after a “forward” motion the robot has a 0.1 probability of staying in the same place, 0.7 of moving “forward”, 0.1 of moving “right” and 0.1 probability of moving “left” one square.
- The robot initial location is grid cell “3” in the map.

Initial Probability Distribution

		1		

“Forward” Motion Probability Distribution

		.7		
	.1	.1	.1	

Grid Cell Numbering

21	22	23	24	25
16	17	18	19	20
11	12	13	14	15
6	7	8	9	10
1	2	3	4	5

Robot State from Motion

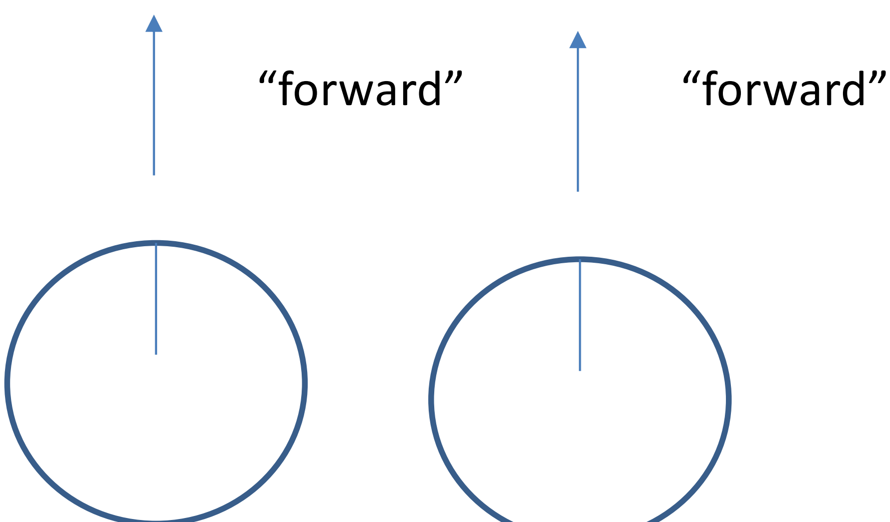
- Localization probability distribution after 2nd "Forward" motion, where "forward" motion the robot has a 0.1 probability of staying in the same place, 0.7 of moving "forward", 0.1 of moving "right" and 0.1 probability of moving "left" one square.

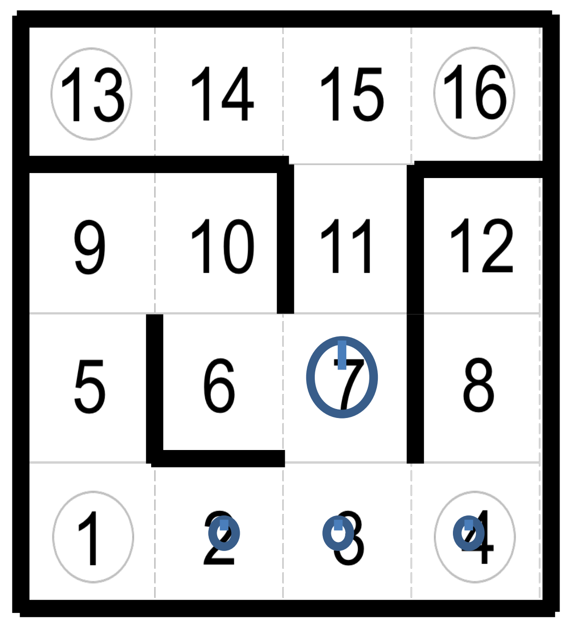
		$F_1 \rightarrow F_2$		
	$L_1 \rightarrow F_2$ $F_1 \rightarrow L_2$	$S_1 \rightarrow F_2$ $F_1 \rightarrow S_2$	$R_1 \rightarrow F_2$ $F_1 \rightarrow R_2$	
$L_1 \rightarrow L_2$	$S_1 \rightarrow L_2$ $L_1 \rightarrow S_2$	$S_1 \rightarrow S_2$ $R_1 \rightarrow L_2$ $L_1 \rightarrow R_2$	$S_1 \rightarrow R_2$ $R_1 \rightarrow S_2$	$R_1 \rightarrow R_2$

		0.49		
	0.14	0.14	0.14	
0.01	0.02	0.03	0.02	0.01

Robot State from Motion

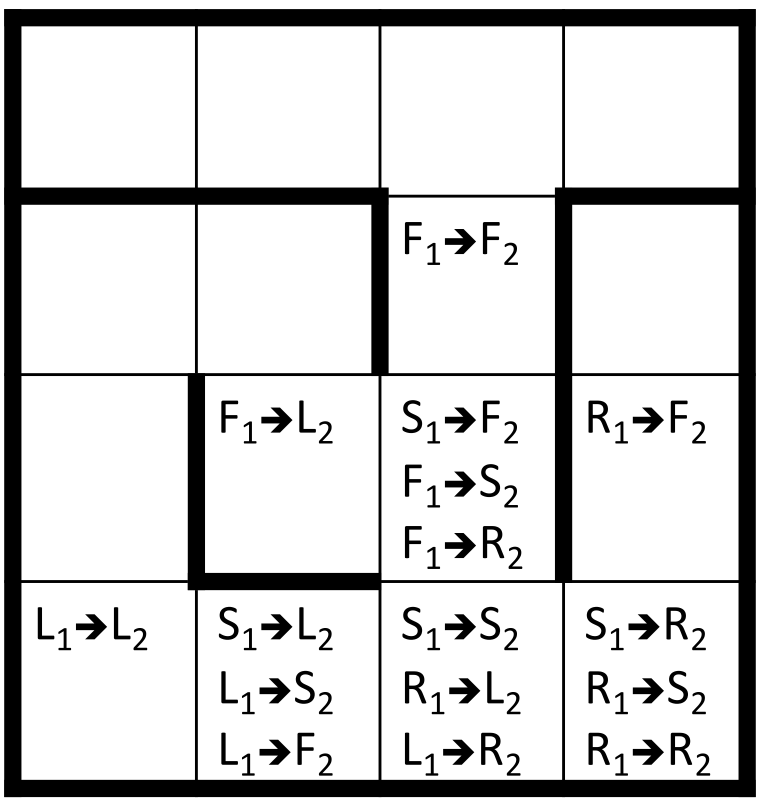
- Assume a "forward" motion the robot has a 0.1 probability of staying in the same place, 0.7 of moving "forward", 0.1 of moving "right", and 0.1 probability of moving "left" one square.
- What would be the possible robot locations and with what probability if the robot started in location "3" in the map: (i) initial probability distribution; (ii) after the first "forward" motion, (iii) after the second "forward" motion.

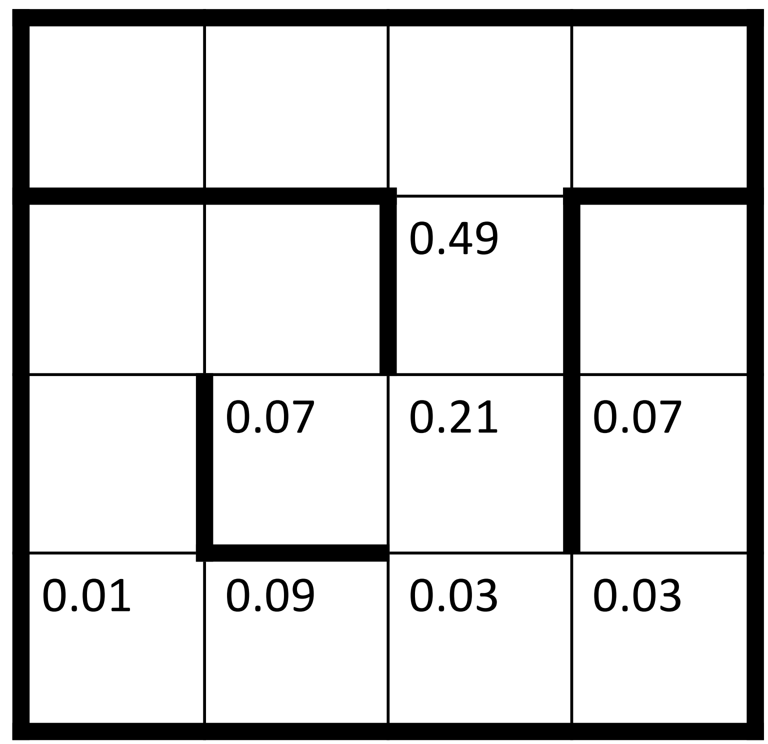




Robot State from Motion

- Localization probability distribution after 2nd "Forward" motion, where "forward" motion the robot has a 0.1 probability of staying in the same place, 0.7 of moving "forward", 0.1 of moving "right" and 0.1 probability of moving "left" one square.

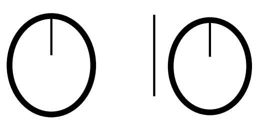




Robot State After Measurement

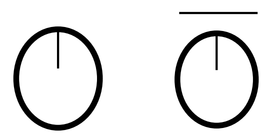
- Assume the following sensor model distribution for wall measurements, for either “front”, “left”, and “right” walls.

Robot Left Sensor Measure Distribution



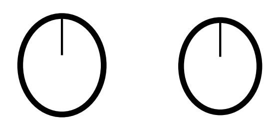
	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.8	.4
$p(z = 1 \mid s)$	.2	.6

Robot Front Sensor Measure Distribution



	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.9	.3
$p(z = 1 \mid s)$	.1	.7

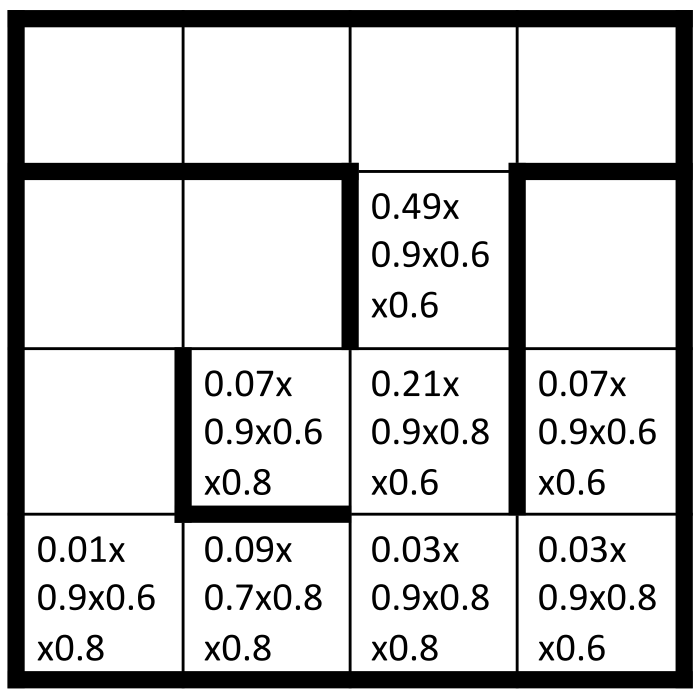
Robot Right Sensor Measure Distribution



	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.8	.4
$p(z = 1 \mid s)$	.2	.6

Robot State After Measurement

- Assume the robot has a 0.1 probability of staying in the same place, 0.7 of moving “forward”, 0.1 of moving “right” and 0.1 probability of moving “left”.
- Assume the previous sensor model distribution.
- Compute the sensor measurement distribution after two “forward” motions. Note that robot always points up.
- Figure shows motion distribution x front sensor x left sensor x right sensor

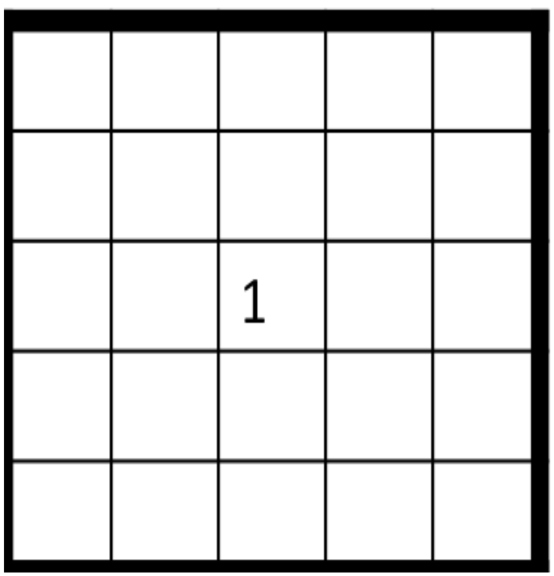


Robot State

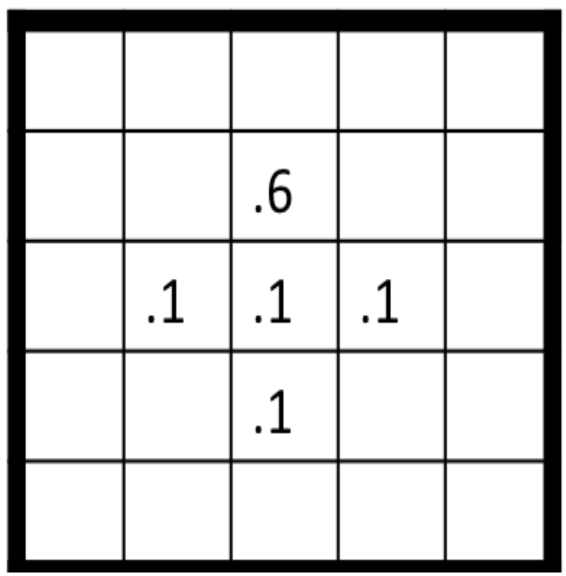
Spring 2017

- Assume we have a robot that always points forwards, never reorients, and moves forwards one square at a time.
- Use the following motion distribution: 0.6 “forward” (“F”), 0.1 “staying” (“S”), 0.1 “right” (“R”), 0.1 “left” (“L”), and 0.1 “backward” (“B”).
- Consider the robot is requested to do two “forward” motions starting at grid cell number “13” in the map as shown in the maze (right figure below).

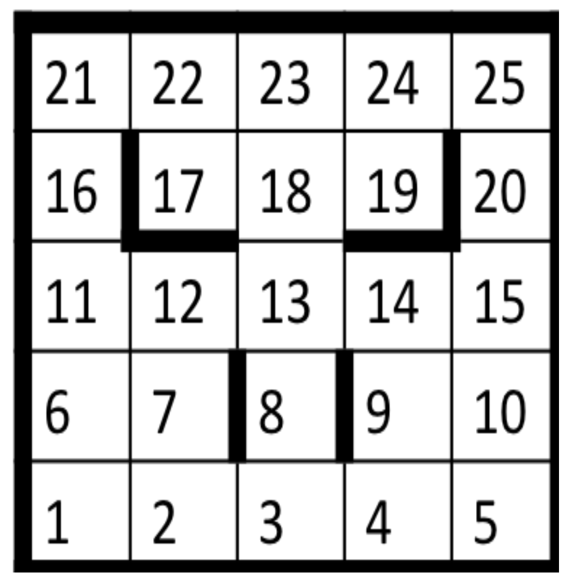
Initial Probability Distribution



“Forward” Motion Probability Distribution



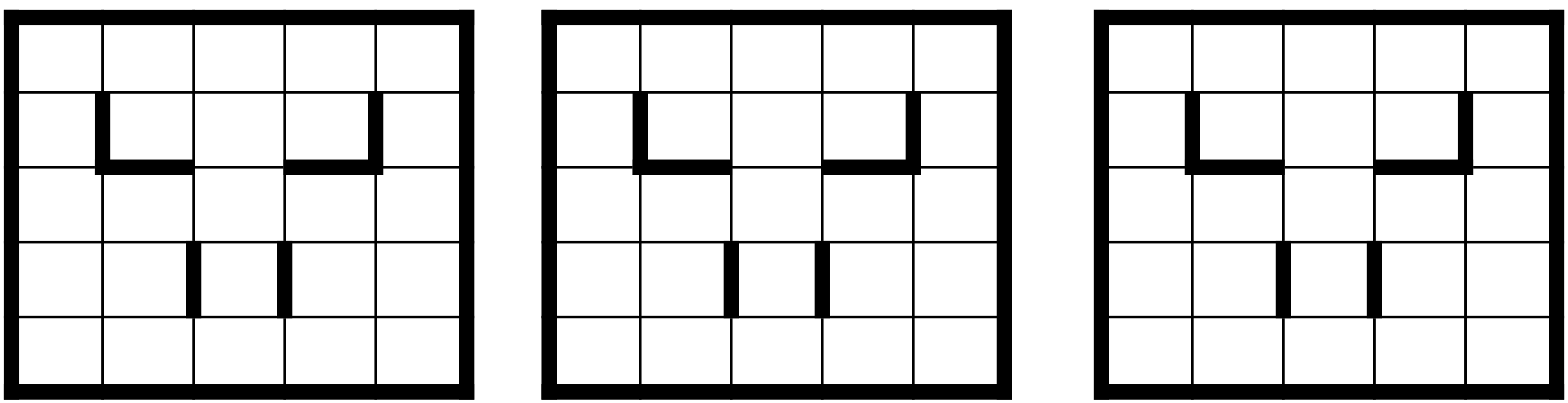
Grid Cell Numbering and Wall Configuration



Robot State from Motion

Spring 2017

1. Provide all possible grid cell number locations that may be reached after the two “forward” motions starting at grid cell “13”.
2. Provide all possible motion combinations (“S”, “L”, “R”, “F”, “B”) to reach all possible grid cells in (1) after the two “forward” motions starting at “13”.
3. Provide final probabilities to reach all possible grid cells in (1) after the two “forward” motions starting at grid cell “13”.



Robot State after Measurement

Spring 2017

- Consider all the previous assumptions including motion probability distributions, map, and starting in the middle grid cell location.
- Consider the robot has front, left and right sensors, as summarized in the table below.

Robot Left Sensor Measure Distribution

	"no wall" $s = 0$	"wall" $s = 1$
$p(z = 0 \mid s)$	.8	.4
$p(z = 1 \mid s)$	.2	.6

Robot Front Sensor Measure Distribution

	"no wall" $s = 0$	"wall" $s = 1$
$p(z = 0 \mid s)$	.9	.3
$p(z = 1 \mid s)$	.1	.7

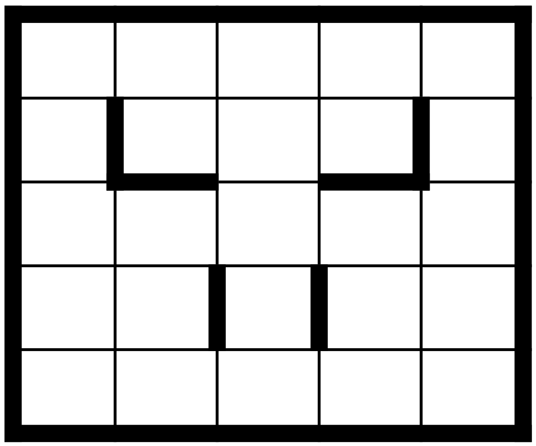
Robot Right Sensor Measure Distribution

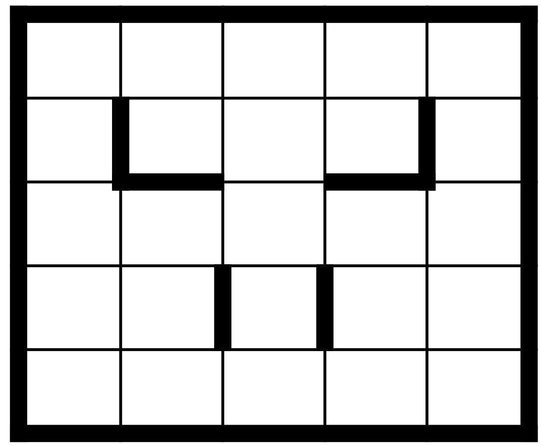
	"no wall" $s = 0$	"wall" $s = 1$
$p(z = 0 \mid s)$	.8	.4
$p(z = 1 \mid s)$	.2	.6

Robot State

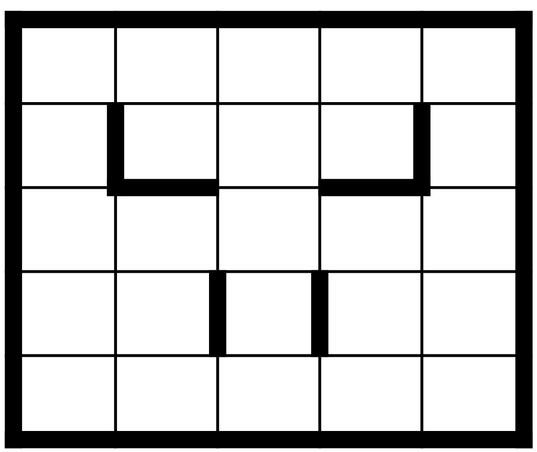
Spring 2017

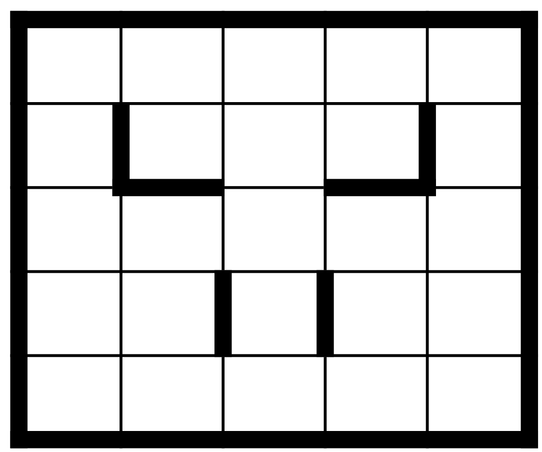
- Compute the “value” and “normalized” sensor probability distribution for all possible grid cell locations after the FIRST “forward” motion starting at grid cell “13” in the map. Consider the robot has a front, left, and right sensors.





- Compute the “value” and “normalized” sensor probability distribution for all possible grid cell locations after the SECOND “forward” motion starting at grid cell “13” in the map. Consider the robot has a front, left, and right sensors.





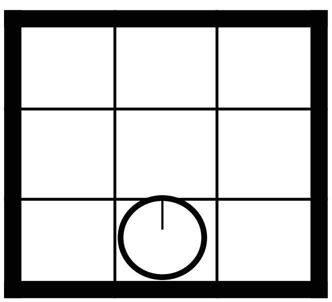
Robot State

Fall 2017

Consider a robot navigating in a maze as follows:

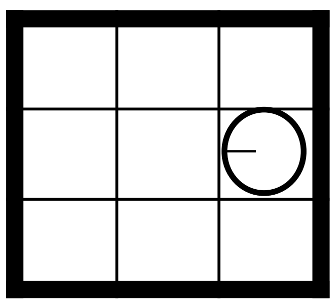
- In the maze, thick lines represent “walls” and thin lines represent “no walls”.
- Robot “forward” motion probabilities are given in the figure below.
- Robot “forward” motion depends on current robot orientation.
- Robot can move a maximum of one square at a time.
- Robot may reorient 90 degrees resulting in an update in its relative “forward” motion direction (“up”, “left”, “right”, “down”).
- Robot reorientation does not change current localization probabilities.

Robot “Forward” Motion Probability (“up”)



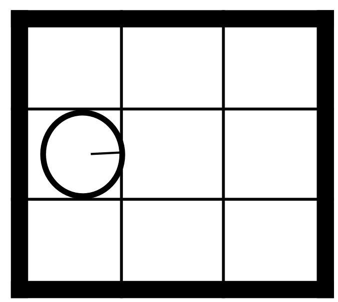
	.7	
.1	.1	.1

Robot “Forward” Motion Probability (“left”)



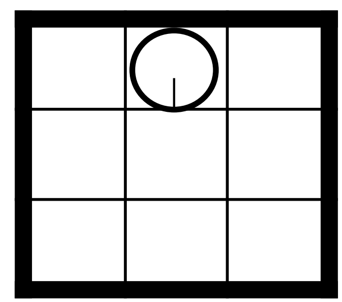
		.1
	.7	.1
		.1

Robot “Forward” Motion Probability (“right”)



.1		
.1	.7	
.1		

Robot “Forward” Motion Probability (“down”)



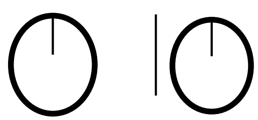
.1	.1	.1
	.7	

Robot State

Fall 2017

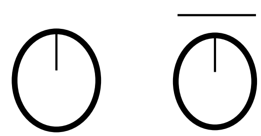
- The robot has 3 distance sensors relative to its “forward” motion (“front”, “left”, “right”).
- Sensor measurement probability distributions are shown in the figure below.

Robot Left Sensor Measure Distribution



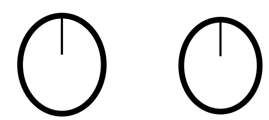
	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.8	.4
$p(z = 1 \mid s)$	.2	.6

Robot Front Sensor Measure Distribution



	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.9	.3
$p(z = 1 \mid s)$	.1	.7

Robot Right Sensor Measure Distribution

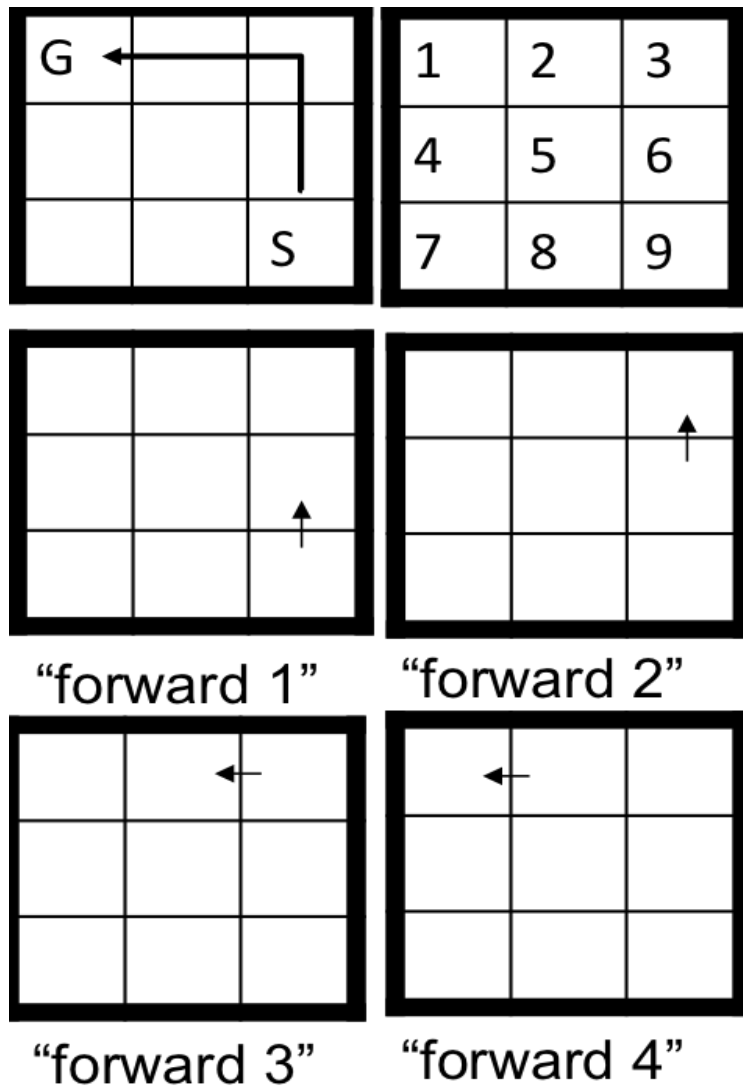


	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.8	.4
$p(z = 1 \mid s)$	.2	.6

Robot State

Fall 2017

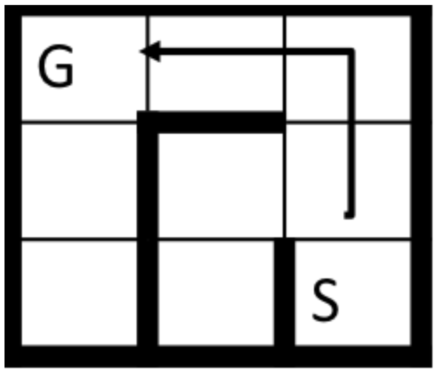
- Fill the empty mazes below with corresponding numerical values by applying ONLY motion control distributions as the robot moves from “S” to “G” as follows: “forward 1”, “forward 2”, “reorient left”, “forward 3”, “forward 4”.

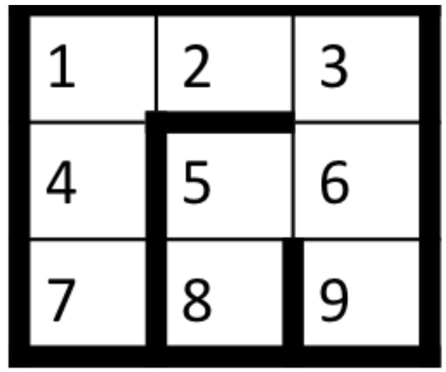


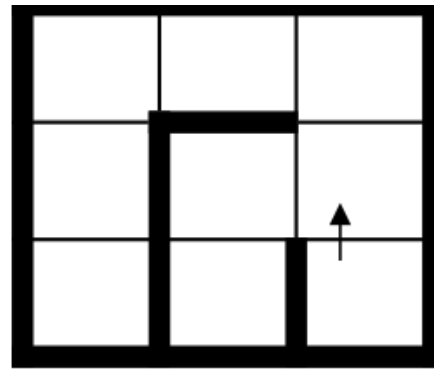
Robot State

Fall 2017

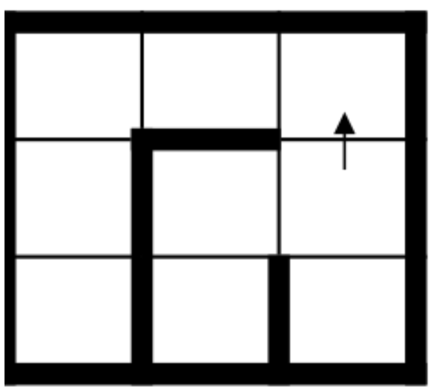
- Fill the empty mazes below using ONLY sensor measurement distributions as the robot moves from “S” to “G” as follows: “forward 1”, “forward 2”, “reorient left”, “forward 3”, “forward 4”.



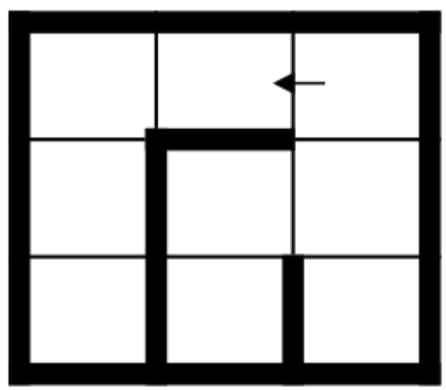




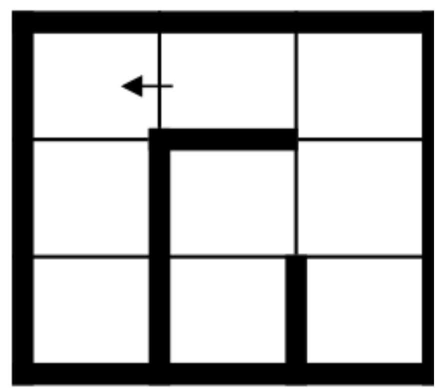
“forward 1”



“forward 2”



“forward 3”



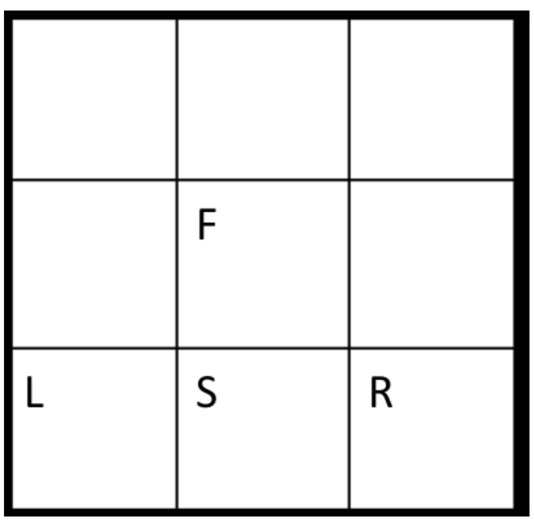
“forward 4”

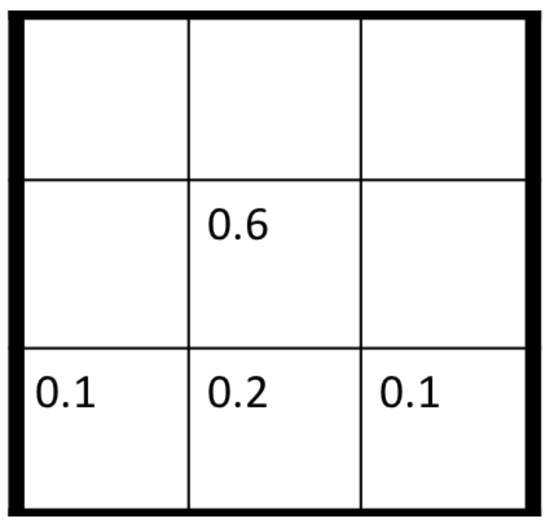
Robot State

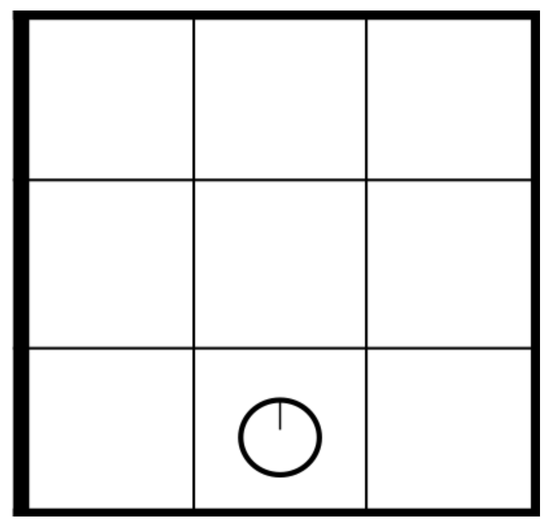
Spring 2018

Consider a robot with the following motion control model and assumptions:

- Thick lines in the maze represent “walls”, while thin lines represent “no walls”.
- Robot can move a maximum of one square at a time.
- Robot possible motions are shown in the left figure below: Forward (“F”), Stay (“S”), Left (“L”), and Right (“R”).
- Robot motion distribution probabilities are shown in the middle figure below: Forward – 0.6, Stay – 0.2, Left – 0.1, and Right – 0.1.
- Robot always points up as shown in the right figure below. Figure also shows robot starting location.







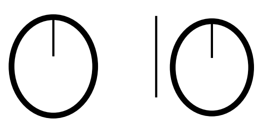
Robot State

Spring 2018

Additionally, consider the robot with the following sensor model:

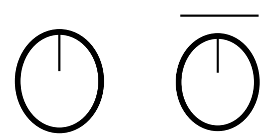
- The robot has 3 binary distance sensors (“front”, “left”, “right”), represented by the corresponding lines next to the robot as shown in the figure below.
- Sensor measurement probability distributions are shown in the figure below.

Robot Left Sensor Measure Distribution



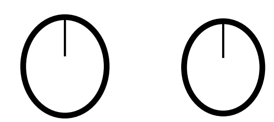
	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.8	.1
$p(z = 1 \mid s)$	.2	.9

Robot Front Sensor Measure Distribution



	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.6	.3
$p(z = 1 \mid s)$	.4	.7

Robot Right Sensor Measure Distribution

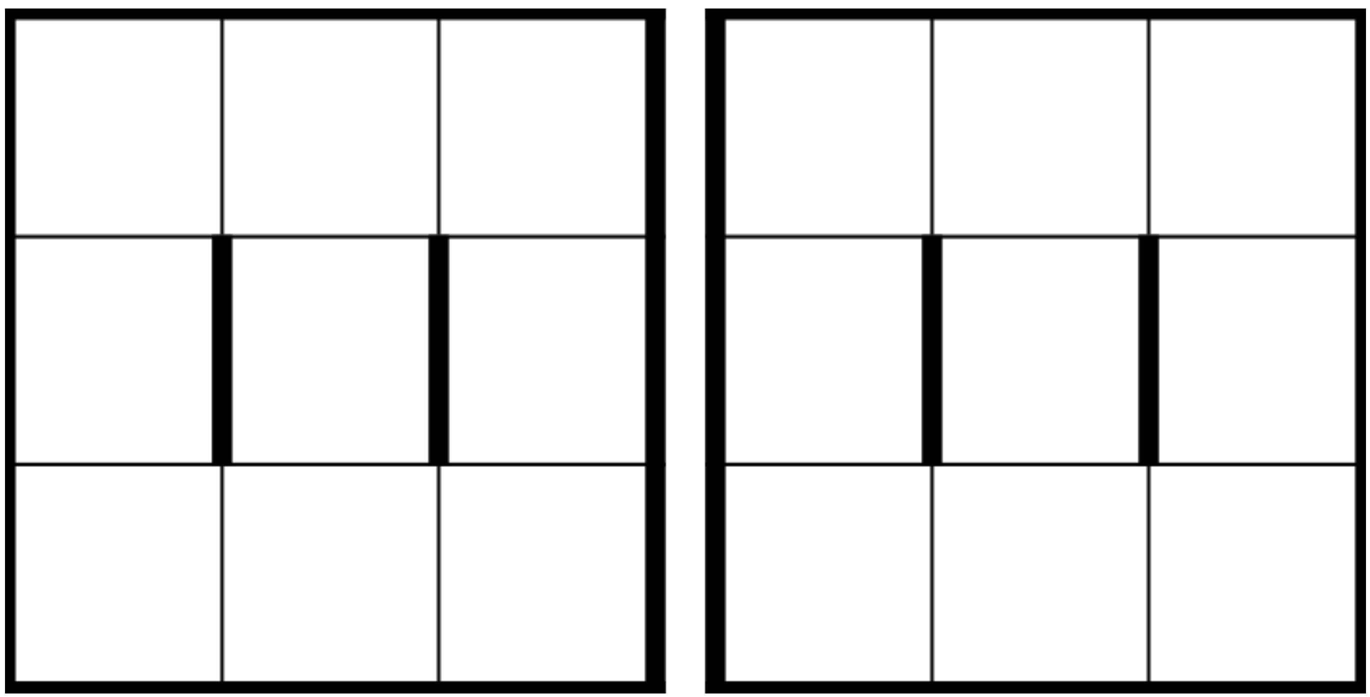


	“no wall” $s = 0$	“wall” $s = 1$
$p(z = 0 \mid s)$	.8	.1
$p(z = 1 \mid s)$	.2	.9

Robot State

Fill the empty mazes below by applying ONLY the motion control model AFTER the robot performs 2 forward movements:

- (left) show motion combinations after “forward 2” (e.g. F1-> F2)
- (right) show probabilities distributions after “forward 2” (e.g. 0.6x0.6=0.36) corresponding to your previous answer.



Robot State

Fill the empty mazes below by applying ONLY the motion control model AFTER the robot performs 2 forward movements:

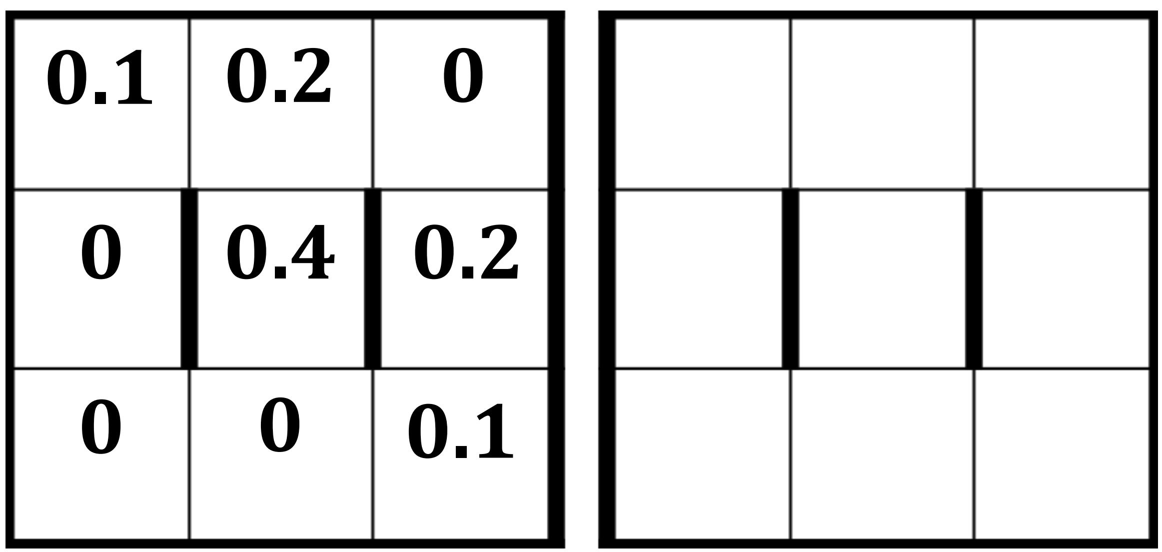
- (Left) Motion combinations after "forward 2" (e.g. F1-> F2)
- (Right) Probabilities distributions after "forward 2" (e.g. 0.6x0.6=0.36)

	$F_1 \rightarrow F_2$	
$L_1 \rightarrow F_2$	$S_1 \rightarrow F_2$ $F_1 \rightarrow S_2$ $F_1 \rightarrow L_2$ $F_1 \rightarrow R_2$	$R_1 \rightarrow F_2$
$S_1 \rightarrow L_2$ $L_1 \rightarrow S_2$ $L_1 \rightarrow L_2$	$S_1 \rightarrow S_2$ $R_1 \rightarrow L_2$ $L_1 \rightarrow R_2$	$S_1 \rightarrow R_2$ $R_1 \rightarrow S_2$ $R_1 \rightarrow R_2$

	0.6x0.6=0.36	
0.1x0.6=0.06	0.2x0.6=0.12 0.6x0.2=0.12 0.6x0.1=0.06 0.6x0.1=0.06 Total=0.36	0.1x0.6=0.06
0.2x0.1=0.02 0.1x0.2=0.02 0.1x0.1=0.01 Total=0.05	0.2x0.2=0.04 0.1x0.1=0.01 0.1x0.1=0.01 Total=0.06	0.2x0.1=0.02 0.1x0.2=0.02 0.1x0.1=0.01 Total=0.05

Robot State

- After a series of motions, assume that the resulting motion probability distribution shown in the left image below. What will the new state probability distribution be after applying the sensor measurement model?
- Write the results in the maze on the right and circle the cell where you believe the robot will be located.



Robot State

- After a series of motions, assume that the resulting motion probability distribution shown in the left image below. What will the new state probability distribution be after applying the sensor measurement model? Write the results in the maze on the right and circle the cell where you believe the robot will be located. (Measurements are shown as LxFxR sensors)

0.1	0.2	0
0	0.4	0.2
0	0	0.1

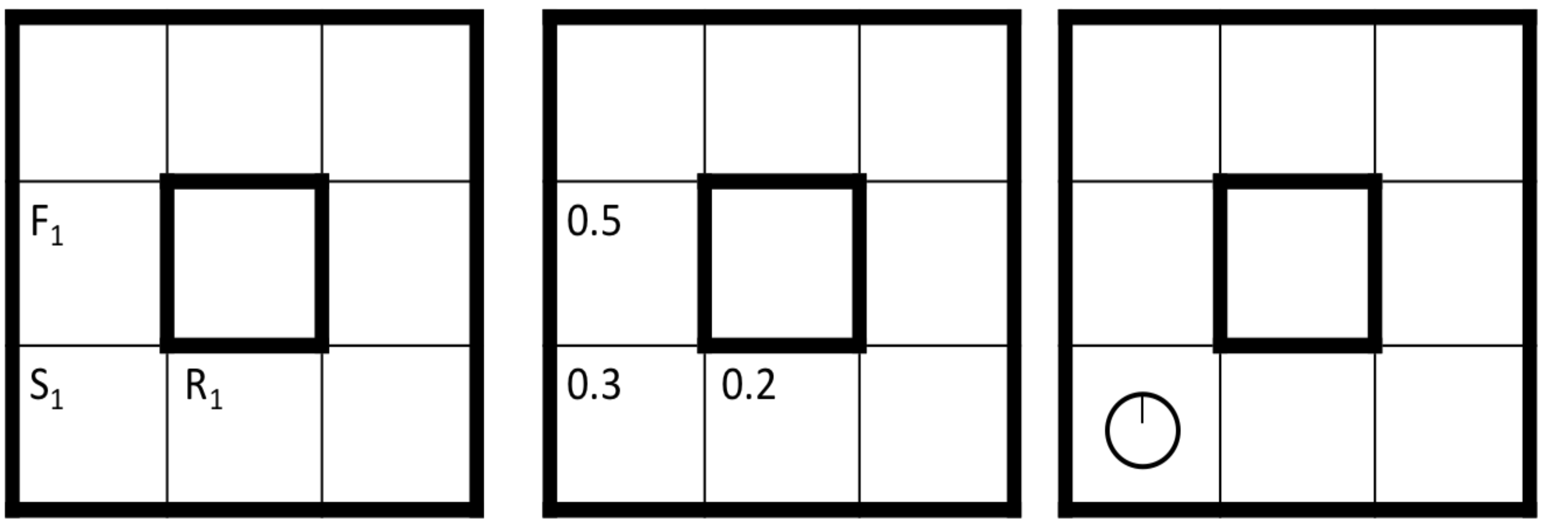
0.1x0.9x0.7 x0.8	0.2x0.8x0.7 x0.8	0
0	0.4x0.8x0.6 x0.8	0.2x0.8x0.6 x0.9
0	0	0.1x0.8x0.6 x0.9

Robot State

Fall 2018

Consider a robot with the following motion control model and assumptions:

- Thick lines represent “walls”.
- Robot can move a maximum of one square at a time.
- Robot motion probabilities: Forward (“F”): 0.5, Stay (“S”): 0.3, Right (“R”): 0.2.
- Robot starts from bottom left corner.



Robot State

Fall 2018

Fill the empty cells in the diagram by applying the motion model only, AFTER the robot performs the 2nd motion:

- Show in the left box the possible movement combinations AFTER the 2nd motion (e.g. F1-> F2)
- Show in the right box the probability distribution for the location of robot AFTER the 2nd motion (e.g. 0.6x0.6=0.36).
- Mark in the right box the cell where the robot has the highest localization probability.

$F_1 \rightarrow F_2$		
$S_1 \rightarrow F_2$ $F_1 \rightarrow S_2$ $F_1 \rightarrow R_2$		
$S_1 \rightarrow S_2$	$R_1 \rightarrow S_2$ $R_1 \rightarrow F_2$ $S_1 \rightarrow R_2$	$R_1 \rightarrow R_2$

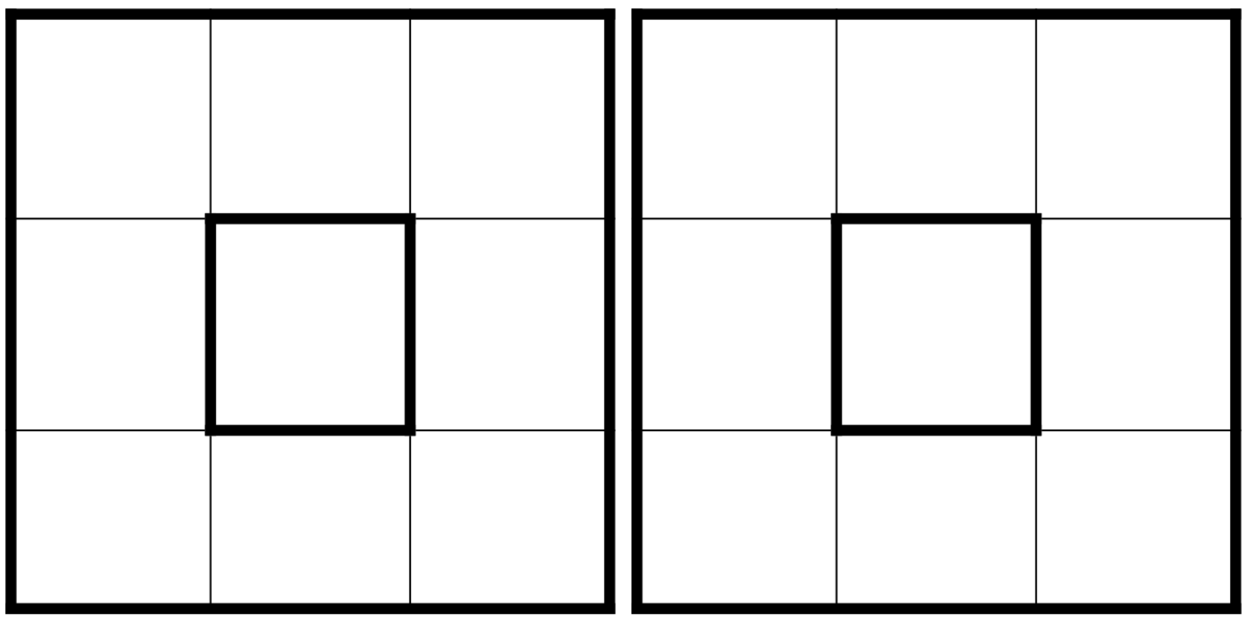
0.5x0.5=0.25 Total=0.25		
0.3x0.5=0.15 0.5x0.3=0.15 0.5x0.2=0.10 Total=0.40 Highest Probability		
0.3x0.3=0.09 Total=0.09	0.2x0.3=0.06 0.2x0.5=0.10 0.3x0.2=0.06 Total=0.22	0.2x0.2=0.04 Total=0.04

Robot State

Fall 2018

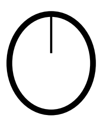
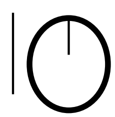
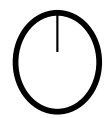
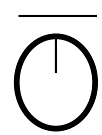
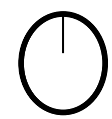
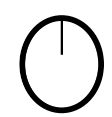
Fill the empty cells in the diagram by applying the measurement model only.

- Show in the left box wall readings for each of the 3 sensors using: “L(0)”, “F(0)”, “R(0)” for “no wall” readings, “L(1)” “F(1)”, “R(1)” for “wall” readings (e.g. L(0)xF(1)xR(0)).
- Show in the right box the probability distribution for the robot sensor readings (e.g. 0.8x0.7x0.8).
- Mark in the right box the cell where the robot has the highest localization probability based exclusively on the measurement model.



Robot State

- Assume sensor measurement probability distributions in the table below:

	Robot Left Sensor Measure Distribution		Robot Front Sensor Measure Distribution		Robot Right Sensor Measure Distribution	
	"no wall"	"wall"	"no wall"	"wall"	"no wall"	"wall"
	s = 0	s = 1	s = 0	s = 1	s = 0	s = 1
						
p(z = 0|s)	.8	.1	.6	.3	.8	.1
p(z = 1|s)	.2	.9	.4	.7	.2	.9

Robot State

Fall 2018

Fill the empty cells in the diagram by applying the measurement model only.

- Show in the left box wall readings for each of the 3 sensors using: “L(0)”, “F(0)”, “R(0)” for “no wall” readings, “L(1)” “F(1)”, “R(1)” for “wall” readings (e.g. L(0)xF(1)xR(0)).
- Show in the right box the probability distribution for the robot sensor readings (e.g. 0.8x0.7x0.8).
- Mark in the right box the cell where the robot has the highest localization probability based exclusively on the measurement model.

L(1)xF(1)xR(0)	L(0)xF(1)xR(0)	L(0)xF(1)xR(1)
L(1)xF(0)xR(1)	L(1)xF(1)xR(1)	L(1)xF(0)xR(1)
L(1)xF(0)xR(0)	L(0)xF(1)xR(0)	L(0)xF(0)xR(1)

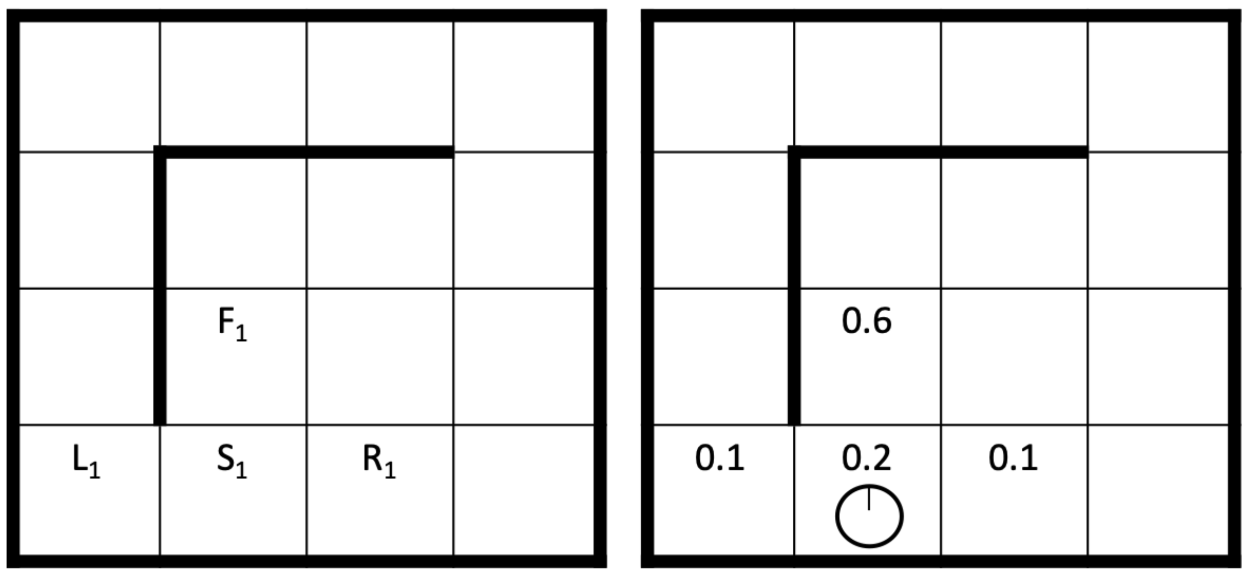
0.9x0.7x0.8	0.8x0.7x0.8	0.8x0.7x0.9
0.9x0.6x0.9	0.9x0.7x0.9 Highest Probability	0.9x0.6x0.9
0.9x0.6x0.8	0.8x0.7x0.8	0.8x0.6x0.9

Robot State

Spring 2019

Consider a robot with the following motion model:

- Thick lines represent “walls”.
- Robot can move a maximum of one square at a time.
- Assume robot ALWAYS points up.
- Assume starting location as illustrated by the circular robot.
- Assume the following robot motion distribution probabilities:
- Forward (“F”): 0.6, Stay (“S”): 0.2, Right (“R”): 0.1, Left (“L”): 0.1



Robot State

Spring 2019

- Show in the left diagram the robot possible motion combinations, AFTER the 2nd motion (e.g. F1-> F2)
- Show in the right diagram the robot motion numerical probability distribution AFTER the 2nd motion (e.g. 0.6x0.6)
- Mark in the right diagram the cells where the robot has the TWO highest localization probabilities AFTER the 2nd motion

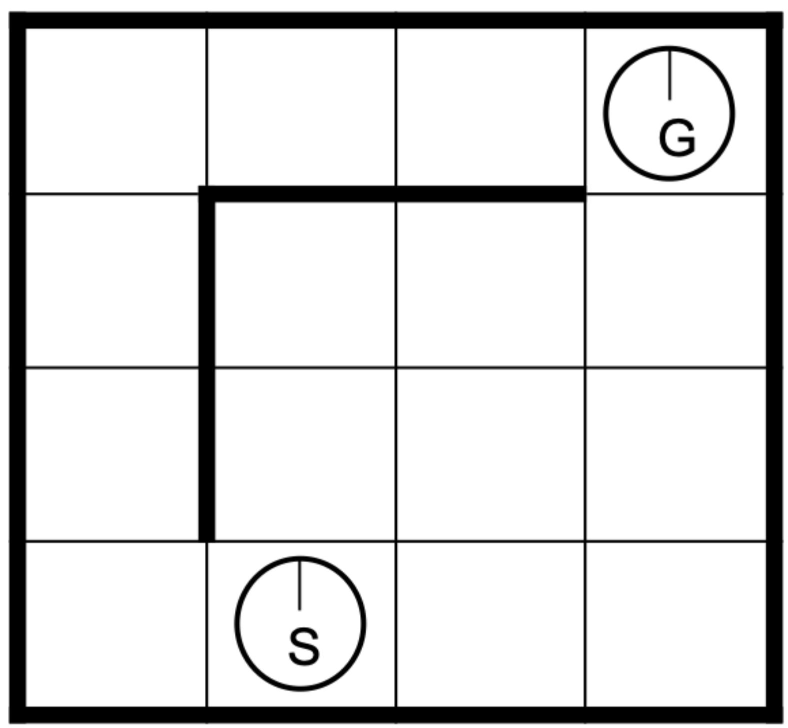
	$F_1 \rightarrow F_2$		
$L_1 \rightarrow F_2$	$S_1 \rightarrow F_2$ $F_1 \rightarrow S_2$ $F_1 \rightarrow L_2$	$F_1 \rightarrow R_2$ $R_1 \rightarrow F_2$	
$L_1 \rightarrow L_2$ $S_1 \rightarrow L_2$ $L_1 \rightarrow S_2$	$S_1 \rightarrow S_2$ $R_1 \rightarrow L_2$ $L_1 \rightarrow R_2$	$S_1 \rightarrow R_2$ $R_1 \rightarrow S_2$	$R_1 \rightarrow R_2$

	0.6x0.6 1st Highest		
0.1x0.6	0.2x0.6 0.6x0.2 0.6x0.1 2nd Highest	0.6x0.1 0.1x0.6	
0.1x0.1 0.2x0.1 0.1x0.2	0.2x0.2 0.1x0.1 0.1x0.1	0.2x0.1 0.1x0.2	0.1x0.1

Robot State

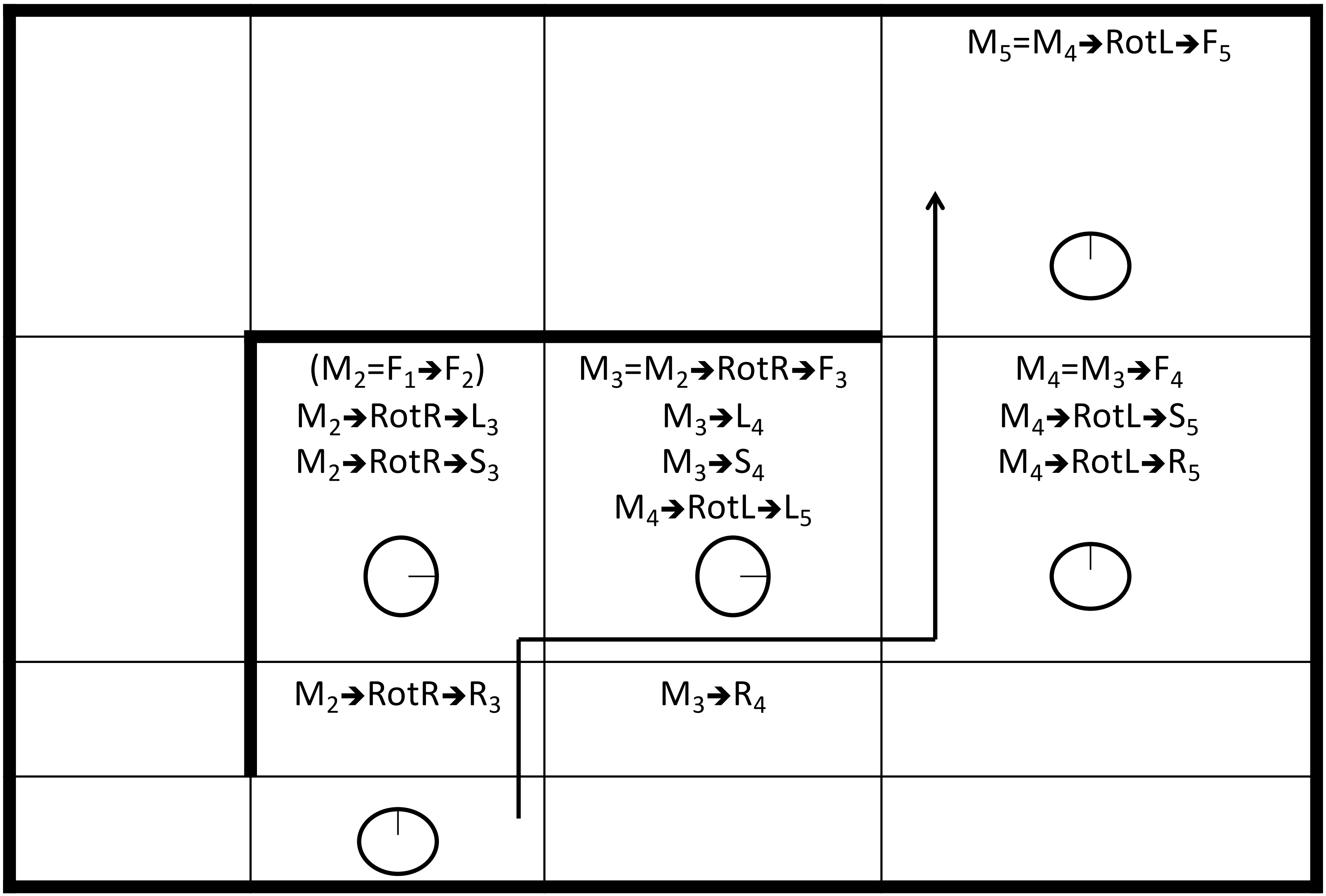
Spring 2019

- Assume the robot can rotate either left ("RotL") or right ("RotR") when encountering a front wall, where all motions and sensing are relative to the robot local frame.
- Highlight in the diagram the shortest path from "S" to "G" using the initial robot orientation at "S".
- Consider that M_t is the state at time t with the highest localization probability, represented as s_{t-1}^{max} , such that $p(M_t)=p(s_t^{max})$.
- A new probability distribution $p(s_t)$ can be computed based exclusively on M_{t-1}, i.e. $p(s_t|M_{t-1})p(M_{t-1})$.
- Fill the boxes to show the shortest path according to $p(M_t)$ for $t \geq 2$.



Robot State from Motion

Spring 2019



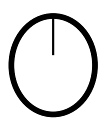
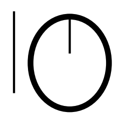
Motion Predictions based on Max States $p(s_t^{max})$ for $t \geq 2$

Robot State

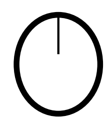
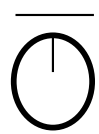
Spring 2019

- Assume sensor measurement probability distributions in the table below.

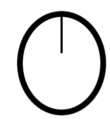
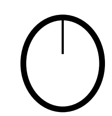
Robot Left Sensor Measure Distribution

	"no wall"	"wall"
	s = 0	s = 1
		
p(z = 0|s)	.7	.1
p(z = 1|s)	.3	.9

Robot Front Sensor Measure Distribution

	"no wall"	"wall"
	s = 0	s = 1
		
p(z = 0|s)	.7	.1
p(z = 1|s)	.3	.9

Robot Right Sensor Measure Distribution

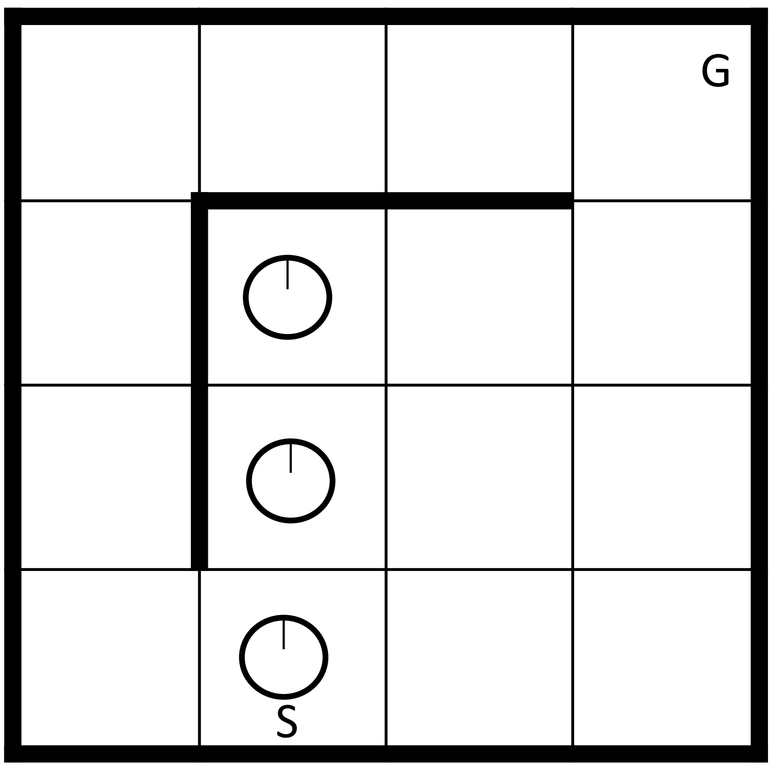
	"no wall"	"wall"
	s = 0	s = 1
		
p(z = 0|s)	.7	.1
p(z = 1|s)	.3	.9

Robot State

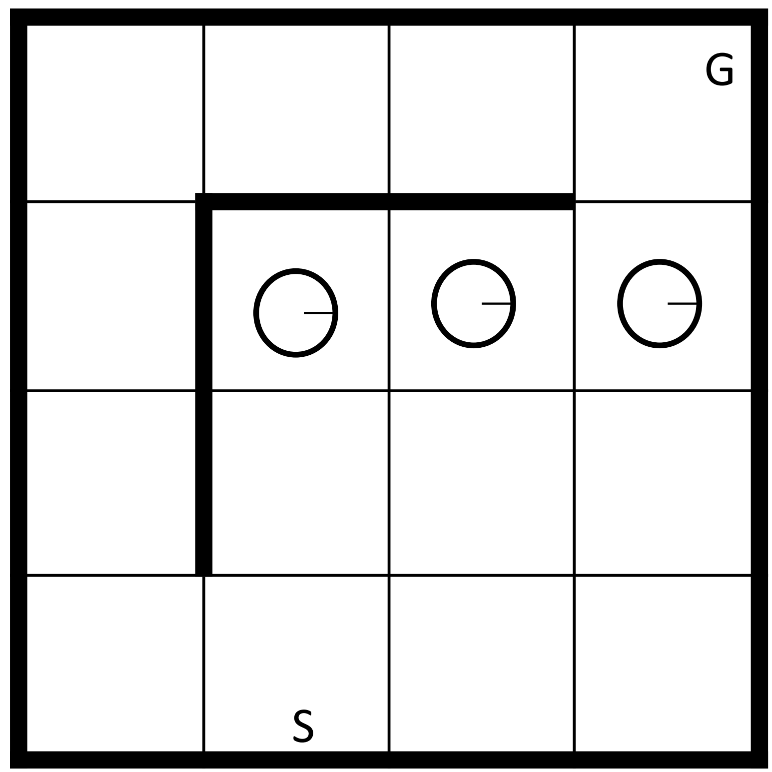
Spring 2019

For the PATH followed by the robot from “S” to “G” before and after rotations:

- Show in the sensor readings for each of the 3 sensors using: “L(0)”, “F(0)”, “R(0)” for “no wall” readings, “L(1)” “F(1)”, “R(1)” for “wall” readings (e.g. L(0)xF(1)xR(0)).
- Show in the probability distribution for the robot sensor readings (e.g. 0.7x0.9x0.7).
- Highlight the locations with highest probability on that path based exclusively on the measurement model.



before rotations



after 1st rotation rotation

Robot State

Spring 2019

For the PATH followed by the robot from “S” to “G” before and after rotations:

- Left diagram shows sensor readings for each of the 3 sensors using: “L(0)”, “F(0)”, “R(0)” for “no wall” readings, “L(1)” “F(1)”, “R(1)” for “wall” readings (e.g. L(0)xF(1)xR(0)).
- Right diagram shows the probability distribution for the robot sensor readings (e.g. 0.7x0.9x0.7).