

Multiple Linear Regression

Minimize the error $\beta_0, \beta_1, \beta_2$

$$1: \frac{\partial}{\partial \beta_0} \sum_{i=1}^n [y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2})]^2 = 0$$

$$\rightarrow 2 \sum_{i=1}^n [y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2})] (-1) = 0$$

$$\rightarrow 2 \sum_{i=1}^n [(y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2})] (-1) = 0$$

$$\rightarrow 2 \sum_{i=1}^n (-y_i + \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}) = 0$$

$$\rightarrow \sum_{i=1}^n (-y_i + \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}) = 0$$

$$\rightarrow - \sum_{i=1}^n y_i + \beta_0 \sum_{i=1}^n + \beta_1 \sum_{i=1}^n x_{i1} + \beta_2 \sum_{i=1}^n x_{i2} = 0$$

$$\rightarrow n \beta_0 + \beta_1 \sum_{i=1}^n x_{i1} + \beta_2 \sum_{i=1}^n x_{i2} = \sum_{i=1}^n y_i$$

$$2 \cdot \frac{\partial}{\beta_1} \sum_{e=1}^n [y_e - (\beta_0 + \beta_1 x_{e1} + \beta_2 x_{e2})]^2 = 0$$

$$\rightarrow 2 \sum_{e=1}^n [y_e - (\beta_0 + \beta_1 x_{e1} + \beta_2 x_{e2})] (-x_{e1}) = 0$$

$$\rightarrow 2 \sum_{e=1}^n (y_e - \beta_0 - \beta_1 x_{e1} + \beta_2 x_{e2})(-x_{e1}) = 0$$

$$\rightarrow \frac{2}{2} \sum_{e=1}^n (-x_{e1} y_e + \beta_0 x_{e1} + \beta_1 x_{e1}^2 + \beta_2 x_{e1} x_{e2}) = 0 / 2$$

$$\rightarrow \sum_{e=1}^n (-x_{e1} y_e + \beta_0 x_{e1} + \beta_1 x_{e1}^2 + \beta_2 x_{e1} x_{e2}) = 0$$

$$\rightarrow - \sum_{e=1}^n x_{e1} y_e + \beta_0 \sum_{e=1}^n x_{e1} + \beta_1 \sum_{e=1}^n x_{e1}^2 + \beta_2 \sum_{e=1}^n x_{e1} x_{e2} = 0$$

$$\rightarrow \beta_0 \sum_{e=1}^n x_{e1} + \beta_1 \sum_{e=1}^n x_{e1}^2 + \beta_2 \sum_{e=1}^n x_{e1} x_{e2} - \sum_{e=1}^n x_{e1} y_e$$

$$3: \frac{\partial}{\beta_2} \sum_{e=1}^n [y_e - (\beta_0 + \beta_1 x_{e1} + \beta_2 x_{e2})]^2 = 0$$

$$\rightarrow 2 \sum_{e=1}^n [y_e - (\beta_0 + \beta_1 x_{e1} + \beta_2 x_{e2})] (-x_{e2}) = 0$$

$$\rightarrow 2 \sum_{e=1}^n (y_e - \beta_0 - \beta_1 x_{e1} - \beta_2 x_{e2}) (-x_{e2}) = 0$$

$$\rightarrow \frac{2}{2} \sum_{e=1}^n (-x_{e2} y_e + \beta_0 x_{e2} + \beta_1 x_{e1} x_{e2} + \beta_2 x_{e2}^2) = 0$$

$$\rightarrow \sum_{e=1}^n (-x_{e2} y_e + \beta_0 x_{e2} + \beta_1 x_{e1} x_{e2} + \beta_2 x_{e2}^2) = 0$$

$$\rightarrow - \sum_{e=1}^n x_{e2} y_e + \beta_0 \sum_{e=1}^n x_{e2} + \beta_1 \sum_{e=1}^n x_{e1} x_{e2} + \beta_2 \sum_{e=1}^n x_{e2}^2 = 0$$

$$\rightarrow \beta_0 \sum_{e=1}^n x_{e2} + \beta_1 \sum_{e=1}^n x_{e1} x_{e2} + \beta_2 \sum_{e=1}^n x_{e2}^2 = \sum_{e=1}^n x_{e2} y_e$$

Minimized

$$1: n\beta_0 + \beta_1 \sum_{e=1}^n x_{e1} + \beta_2 \sum_{e=1}^n x_{e2} = \sum_{e=1}^n y_e$$

$$2: \beta_0 \sum_{e=1}^n x_{e1} + \beta_1 \sum_{e=1}^n x_{e1}^2 + \beta_2 \sum_{e=1}^n x_{e1} x_{e2} = \sum_{e=1}^n x_{e1} y_e$$

$$3: \beta_0 \sum_{e=1}^n x_{e2} + \beta_1 \sum_{e=1}^n x_{e1} x_{e2} + \beta_2 \sum_{e=1}^n x_{e2}^2 - \sum_{e=1}^n x_{e2} y_e$$

Matrix Notation

$$1: n\beta_0 + \beta_1 \sum_{e=1}^n x_{e1} + \beta_2 \sum_{e=1}^n x_{e2} = \sum_{e=1}^n y_e$$

$$2: \beta_0 \sum_{e=1}^n x_{e1} + \beta_1 \sum_{e=1}^n x_{e1}^2 + \beta_2 \sum_{e=1}^n x_{e1} x_{e2} = \sum_{e=1}^n x_{e1} y_e$$

$$3: \beta_0 \sum_{e=1}^n x_{e2} + \beta_1 \sum_{e=1}^n x_{e1} x_{e2} + \beta_2 \sum_{e=1}^n x_{e2}^2 = \sum_{e=1}^n x_{e2} y_e$$

$$\begin{bmatrix} n & \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e2} \\ \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e1}^2 & \sum_{e=1}^n x_{e1} x_{e2} \\ \sum_{e=1}^n x_{e2} & \sum_{e=1}^n x_{e1} x_{e2} & \sum_{e=1}^n x_{e2}^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} \sum_{e=1}^n y_e \\ \sum_{e=1}^n x_{e1} y_e \\ \sum_{e=1}^n x_{e2} y_e \end{bmatrix}$$

Solve the SLEq. $\beta_0, \beta_1, \beta_2$.

Cramer: 3×3

$$1: D_s$$

$$2: \beta_0 = \frac{D_{\beta_0}}{D_s}$$

$$3: \beta_1 = \frac{D_{\beta_1}}{D_s}$$

$$4: \beta_2 = \frac{D_{\beta_2}}{D_s}$$

$$D_s = \begin{bmatrix} n & \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e2} & n & \sum_{e=1}^n x_{e1} \\ \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e1}^2 & \sum_{e=1}^n x_{e1} x_{e2} & \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e1}^2 \\ \sum_{e=1}^n x_{e2} & \sum_{e=1}^n x_{e1} x_{e2} & \sum_{e=1}^n x_{e2}^2 & \sum_{e=1}^n x_{e2} & \sum_{e=1}^n x_{e1} x_{e2} \end{bmatrix}$$

$$D_5 = n \sum_{e=1}^n \sum_{c=1}^n x_{e1}^2 + \sum_{e=1}^n x_{e1} \sum_{c=1}^n x_{e1} x_{e2} \sum_{c=1}^n x_{e2} + \sum_{e=1}^n x_{e2} \sum_{c=1}^n x_{e1} \sum_{c=1}^n x_{e1} x_{e2} - \sum_{e=1}^n x_{e2} \sum_{c=1}^n x_{e1}^2 \sum_{c=1}^n x_{e2}$$

$$- \sum_{e=1}^n x_{e1} x_{e2} \sum_{e=1}^n x_{e1} x_{e2} n - \sum_{e=1}^n x_{e2}^2 \sum_{c=1}^n x_{e1} \sum_{c=1}^n x_{e1}$$

$$D_{\beta_0} = \begin{bmatrix} \sum_{c=1}^n y_c \\ \sum_{e=1}^n x_{e1} \\ \sum_{e=1}^n x_{e2} \\ \sum_{e=1}^n y_e \\ \sum_{e=1}^n x_{e1} \\ \sum_{e=1}^n x_{e1} y_e \\ \sum_{e=1}^n x_{e1}^2 \\ \sum_{e=1}^n x_{e1} x_{e2} \\ \sum_{e=1}^n x_{e2} y_e \\ \sum_{e=1}^n x_{e2}^2 \\ \sum_{e=1}^n x_{e1} y_e \\ \sum_{e=1}^n x_{e1}^2 \\ \sum_{e=1}^n x_{e1} x_{e2} \end{bmatrix}$$

$$D_{\beta_0} = \sum_{e=1}^n y_e \sum_{c=1}^n x_{e1}^2 \sum_{c=1}^n x_{e2}^2 + \sum_{e=1}^n x_{e1} \sum_{c=1}^n x_{e1} x_{e2} \sum_{c=1}^n x_{e2} y_e + \sum_{e=1}^n x_{e2} \sum_{c=1}^n x_{e1} y_e \sum_{c=1}^n x_{e1} x_{e2}$$

$$- \sum_{c=1}^n x_{e2} y_e \sum_{e=1}^n x_{e1}^2 \sum_{e=1}^n x_{e2} - \sum_{e=1}^n x_{e1} x_{e2} \sum_{c=1}^n x_{e1} x_{e2} \sum_{e=1}^n y_e - \sum_{c=1}^n x_{e2}^2 \sum_{e=1}^n x_{e1} y_e \sum_{e=1}^n x_{e1}$$

$D_{\beta 1}$

$$\begin{bmatrix} n & \sum_{e=1}^n y_e & \sum_{e=1}^n x_{e2} & n & \sum_{e=1}^n y_e \\ \vdots & \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e1} y_e & \sum_{e=1}^n x_{e1} x_{e2} & \sum_{e=1}^n x_{e1} \\ \sum_{e=1}^n x_{e2} & \sum_{e=1}^n x_{e2} y_e & \sum_{e=1}^n x_{e2}^2 & \sum_{e=1}^n x_{e2} & \sum_{e=1}^n x_{e2} y_e \end{bmatrix}$$

$$D_{\beta 1} = n \sum_{e=1}^n x_{e1} y_e \sum_{e=1}^n x_{e2}^2 + \sum_{e=1}^n y_e \sum_{e=1}^n x_{e1} x_{e2} \sum_{e=1}^n x_{e2} + \sum_{e=1}^n x_{e2} \sum_{e=1}^n x_{e1} \sum_{e=1}^n x_{e2} y_e$$

$$- \sum_{e=1}^n x_{e2} \sum_{e=1}^n x_{e1} y_e \sum_{e=1}^n x_{e2} - \sum_{e=1}^n x_{e2} y_e \sum_{e=1}^n x_{e1} x_{e2} n - \sum_{e=1}^n x_{e2} \sum_{e=1}^n x_{e1} \sum_{e=1}^n y_e$$

 $D_{\beta 2}$

$$\begin{bmatrix} n & \sum_{e=1}^n x_{e1} & \sum_{e=1}^n y_e & n & \sum_{e=1}^n x_{e1} \\ \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e1}^2 & \sum_{e=1}^n x_{e1} y_e & \sum_{e=1}^n x_{e1} & \sum_{e=1}^n x_{e1}^2 \\ \sum_{e=1}^n x_{e2} & \sum_{e=1}^n x_{e1} x_{e2} & \sum_{e=1}^n x_{e2} y_e & \sum_{e=1}^n x_{e2} & \sum_{e=1}^n x_{e1} x_{e2} \end{bmatrix}$$

$$D_{\beta 2} = n \sum_{e=1}^n x_{e1}^2 \sum_{e=1}^n x_{e2} y_e + \sum_{e=1}^n x_{e1} \sum_{e=1}^n x_{e1} y_e \sum_{e=1}^n x_{e2} + \sum_{e=1}^n y_e \sum_{e=1}^n x_{e1} \sum_{e=1}^n x_{e1} x_{e2}$$

$$- \sum_{e=1}^n x_{e2} \sum_{e=1}^n x_{e1}^2 \sum_{e=1}^n y_e - \sum_{e=1}^n x_{e2} x_{e1} \sum_{e=1}^n x_{e1} y_e n - \sum_{e=1}^n x_{e2} y_e \sum_{e=1}^n x_{e1} \sum_{e=1}^n x_{e1}$$

$$D_s = n \sum_{e=1}^n \sum_{e=1}^n X_{e1}^2 + \sum_{e=1}^n \sum_{e=1}^n X_{e1} X_{e2} \sum_{e=1}^n X_{e2} + \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} \sum_{e=1}^n X_{e1} X_{e2} - \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} \sum_{e=1}^n X_{e2}$$

$$- \sum_{e=1}^n \sum_{e=1}^n X_{e1} X_{e2} \sum_{e=1}^n X_{e1} X_{e2} n - \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} \sum_{e=1}^n X_{e1}$$

$$P_0 = \left(\sum_{e=1}^n \sum_{e=1}^n X_{e1}^2 \sum_{e=1}^n X_{e2} + \sum_{e=1}^n \sum_{e=1}^n X_{e1} X_{e2} \sum_{e=1}^n X_{e2} Y_e + \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} Y_e \sum_{e=1}^n X_{e1} X_{e2} \right.$$

$$\left. - \sum_{e=1}^n \sum_{e=1}^n X_{e2} Y_e \sum_{e=1}^n X_{e1}^2 - \sum_{e=1}^n \sum_{e=1}^n X_{e1} X_{e2} \sum_{e=1}^n X_{e1} X_{e2} \sum_{e=1}^n Y_e - \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} Y_e \sum_{e=1}^n X_{e1} \right)$$

D_s

$$P_1 = \left(n \sum_{e=1}^n \sum_{e=1}^n X_{e1} Y_e \sum_{e=1}^n X_{e2} + \sum_{e=1}^n \sum_{e=1}^n X_{e1} X_{e2} \sum_{e=1}^n X_{e2} + \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} \sum_{e=1}^n X_{e2} Y_e \right.$$

$$\left. - \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} Y_e \sum_{e=1}^n X_{e2} - \sum_{e=1}^n \sum_{e=1}^n X_{e2} Y_e \sum_{e=1}^n X_{e1} X_{e2} n - \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1} \sum_{e=1}^n Y_e \right)$$

D_s

$$P_2 = \left(n \sum_{e=1}^n \sum_{e=1}^n X_{e1}^2 \sum_{e=1}^n X_{e2} Y_e + \sum_{e=1}^n \sum_{e=1}^n X_{e1} \sum_{e=1}^n X_{e1} Y_e \sum_{e=1}^n X_{e2} + \sum_{e=1}^n \sum_{e=1}^n Y_e \sum_{e=1}^n X_{e1} \sum_{e=1}^n X_{e1} X_{e2} \right.$$

$$\left. - \sum_{e=1}^n \sum_{e=1}^n X_{e2} \sum_{e=1}^n X_{e1}^2 \sum_{e=1}^n Y_e - \sum_{e=1}^n \sum_{e=1}^n X_{e1} X_{e2} \sum_{e=1}^n X_{e1} Y_e n - \sum_{e=1}^n \sum_{e=1}^n X_{e2} Y_e \sum_{e=1}^n X_{e1} \sum_{e=1}^n X_{e1} \right)$$

D_s