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Multiple Lineal Regression

Minimize the error $\beta_0, \beta_1, \beta_2$

$$1: \frac{\partial}{\beta_0} \sum_{i=1}^{n} \left[y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}) \right]^2 = 0$$

$$\rightarrow 2 \sum_{i=1}^{n} \left[y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}) \right] (-1) = 0$$

$$\rightarrow 2 \sum_{i=1}^{n} \left[y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2} \right) (-1) = 0$$

$$\rightarrow \frac{2}{2} \sum_{i=1}^{n} \left(-y_i + \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} \right) = \frac{0}{2}$$

$$\rightarrow \sum_{i=1}^{n} \left(-y_i + \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} \right) = 0$$

$$\rightarrow -\sum_{i=1}^{n} y_i + \beta_0 \sum_{i=1}^{n} + \beta_1 \sum_{i=1}^{n} x_{i1} + \beta_2 \sum_{i=1}^{n} x_{i2} = 0$$

$$\rightarrow n\beta_0 + \beta_1 \sum_{i=1}^{n} x_{i1} + \beta_2 \sum_{i=1}^{n} x_{i2} = \sum_{i=1}^{n} y_i$$

$$\frac{\partial}{\beta_1}\sum_{i=1}^{n}\left[y_i-(\beta_0+\beta_1 x_{i1}+\beta_2 x_{i2})\right]^2=0$$

$$\rightarrow 2\sum_{i=1}^{n}\left[y_i-(\beta_0+\beta_1 x_{i1}+\beta_2 x_{i2})\right](-x_{i1})=0$$

$$\rightarrow \sum_{i=1}^{n}\left(y_i-\beta_0-\beta_1 x_{i1}+\beta_2 x_{i2}\right)(-x_{i1})=0$$

$$\rightarrow \frac{2}{2}\sum_{i=1}^{n}\left(-x_{i1}y_i+\beta_0 x_{i1}+\beta_1 x_{i1}^2+\beta_2 x_{i1}x_{i2}\right)=0/2$$

$$\rightarrow \sum_{i=1}^{n}\left(-x_{i1}y_i+\beta_0 x_{i1}+\beta_1 x_{i1}^2+\beta_2 x_{i1}x_{i2}\right)=0$$

$$\rightarrow -\sum_{i=1}^{n}x_{i1}y_i+\beta_0\sum_{i=1}^{n}x_{i1}+\beta_1\sum_{i=1}^{n}x_{i1}^2+\beta_2\sum_{i=1}^{n}x_{i1}x_{i2}=0$$

$$\rightarrow \beta_0\sum_{i=1}^{n}x_{i1}+\beta_1\sum_{i=1}^{n}x_{i1}^2+\beta_2\sum_{i=1}^{n}x_{i1}x_{i2}=\sum_{i=1}^{n}x_{i1}y_i$$

$$3: \frac{\partial}{\beta_2} \sum_{i=1}^{n} \left[y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}) \right]^2 = 0$$

$$\rightarrow 2 \sum_{i=1}^{n} \left[y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}) \right] (-x_{i2}) = 0$$

$$\rightarrow 2 \sum_{i=1}^{n} \left(y_i - \beta_0 - \beta_1 x_{i1} + \beta_2 x_{i2} \right) \left(-x_{i2} \right) = 0$$

$$\rightarrow \frac{2}{2} \sum_{i=1}^{n} \left(-x_{i2} y_i + \beta_0 x_{i2} + \beta_1 x_{i1} x_{i2} + \beta_2 x_{i2}^2 \right) = \frac{0}{2}$$

$$\rightarrow \sum_{i=1}^{n} \left(-x_{i2} y_i + \beta_0 x_{i2} + \beta_1 x_{i1} x_{i2} + \beta_2 x_{i2}^2 \right) = 0$$

$$\rightarrow -\sum_{i=1}^{n} x_{i2} y_i + \beta_0 \sum_{i=1}^{n} x_{i2} + \beta_1 \sum_{i=1}^{n} x_{i1} x_{i2} + \beta_2 \sum_{i=1}^{n} x_{i2}^2 = 0$$

$$\rightarrow \beta_0 \sum_{i=1}^{n} x_{i2} + \beta_1 \sum_{i=1}^{n} x_{i1} x_{i2} + \beta_2 \sum_{i=1}^{n} x_{i2}^2 = \sum_{i=1}^{n} x_{i2} y_i$$

Minimized

$$1: n\beta_0 + \beta_1 \sum_{i=1}^{n} x_{i1} + \beta_2 \sum_{i=1}^{n} x_{i2} = \sum_{i=1}^{n} y_i$$

$$2: \beta_0 \sum_{i=1}^{n} x_{i1} + \beta_1 \sum_{i=1}^{n} x_{i1}^2 + \beta_2 \sum_{i=1}^{n} x_{i1} x_{i2} = \sum_{i=1}^{n} x_{i1} y_i$$

$$3: \beta_0 \sum_{i=1}^{n} x_{i2} + \beta_1 \sum_{i=1}^{n} x_{i1} x_{i2} + \beta_2 \sum_{i=1}^{n} x_{i2}^2 = \sum_{i=1}^{n} x_{i2} y_i$$

Matrix Notation

1: $n\beta_0 + \beta_1 \sum_{i=1}^{n} x_{i1} + \beta_2 \sum_{i=1}^{n} x_{i2} = \sum_{i=1}^{n} y_i$

2: $\beta_0 \sum_{i=1}^{n} x_{i1} + \beta_1 \sum_{i=1}^{n} x_{i1}^2 + \beta_2 \sum_{i=1}^{n} x_{i1} x_{i2} = \sum_{i=1}^{n} x_{i1} y_i$

3: $\beta_0 \sum_{i=1}^{n} x_{i2} + \beta_1 \sum_{i=1}^{n} x_{i1} x_{i2} + \beta_2 \sum_{i=1}^{n} x_{i2}^2 = \sum_{i=1}^{n} x_{i2} y_i$

$$\begin{bmatrix} n & \sum x_{i1} & \sum x_{i2} \\ \sum x_{i1} & \sum x_{i1}^2 & \sum x_{i1} x_{i2} \\ \sum x_{i2} & \sum x_{i1} x_{i2} & \sum x_{i2}^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_{i1} y_i \\ \sum x_{i2} y_i \end{bmatrix}$$

Solve the SoLEq $\beta_0, \beta_1, \beta_2$.

Cramer: 3x3

1: D_S

2: $\beta_0 = \frac{D_{\beta_0}}{D_S}$

3: $\beta_1 = \frac{D_{\beta_1}}{D_S}$

4: $\beta_2 = \frac{D_{\beta_2}}{D_S}$

D_S

$$\begin{bmatrix} n & \sum x_{i1} & \sum x_{i2} & n & \sum x_{i1} \\ \sum x_{i1} & \sum x_{i1}^2 & \sum x_{i1} x_{i2} & \sum x_{i1} & \sum x_{i1}^2 \\ \sum x_{i2} & \sum x_{i1} x_{i2} & \sum x_{i2}^2 & \sum x_{i2} & \sum x_{i1} x_{i2} \end{bmatrix}$$

$$D_5 = n\sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2}^2 + \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i2} + \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} x_{i2} - \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2}$$

$$- \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i1} x_{i2}\, n - \sum_{i=1}^{n} x_{i2}^2 \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1}$$

$$D_{\beta 0} \begin{vmatrix} \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i2} & \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} x_{i1} \\ \sum_{i=1}^{n} x_{i1} y_i & \sum_{i=1}^{n} x_{i1}^2 & \sum_{i=1}^{n} x_{i1} x_{i2} & \sum_{i=1}^{n} x_{i1} y_i & \sum_{i=1}^{n} x_{i1}^2 \\ \sum_{i=1}^{n} x_{i2} y_i & \sum_{i=1}^{n} x_{i1} x_{i2} & \sum_{i=1}^{n} x_{i2}^2 & \sum_{i=1}^{n} x_{i2} y_i & \sum_{i=1}^{n} x_{i1} x_{i2} \end{vmatrix}$$

$$D_{\beta 0} = \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2}^2 + \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i2} y_i + \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1} y_i \sum_{i=1}^{n} x_{i1} x_{i2}$$

$$- \sum_{i=1}^{n} x_{i2} y_i \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2} - \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_{i2}^2 \sum_{i=1}^{n} x_{i1} y_i \sum_{i=1}^{n} x_{i1}$$

$$D_{B1}\begin{bmatrix} n & \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} x_{i2} & n & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}y_i & \sum_{i=1}^{n} x_{i1}x_{i2} & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}y_i \\ \sum_{i=1}^{n} x_{i2} & \sum_{i=1}^{n} x_{i2}y_i & \sum_{i=1}^{n} x_{i2}^2 & \sum_{i=1}^{n} x_{i2} & \sum_{i=1}^{n} x_{i2}y_i \end{bmatrix}$$

$$D_{B1} = n\sum_{i=1}^{n} x_{i1}y_i \sum_{i=1}^{n} x_{i2}^2 + \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_{i1}x_{i2} \sum_{i=1}^{n} x_{i2} + \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i2}y_i - \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1}y_i \sum_{i=1}^{n} x_{i2} - \sum_{i=1}^{n} x_{i2}y_i \sum_{i=1}^{n} x_{i1}x_{i2} n - \sum_{i=1}^{n} x_{i2}^2 \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} y_i$$

$$D_{B2}\begin{bmatrix} n & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} y_i & n & \sum_{i=1}^{n} x_{i1} \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^2 & \sum_{i=1}^{n} x_{i1}y_i & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^2 \\ \sum_{i=1}^{n} x_{i2} & \sum_{i=1}^{n} x_{i1}x_{i2} & \sum_{i=1}^{n} x_{i2}y_i & \sum_{i=1}^{n} x_{i2} & \sum_{i=1}^{n} x_{i1}x_{i2} \end{bmatrix}$$

$$D_{B2} = n\sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2}y_i + \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1}y_i \sum_{i=1}^{n} x_{i2} + \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1}x_{i2} - \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1}x_{i2} \sum_{i=1}^{n} x_{i1}y_i \; n - \sum_{i=1}^{n} x_{i2}y_i \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1}$$

$$D_s = n\sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2}^2 + \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i2} + \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} x_{i2} - \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2}$$
$$- \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i1} x_{i2}\, n - \sum_{i=1}^{n} x_{i2}^2 \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1}$$

$$\beta_0 = \frac{\left(\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2}^2 + \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i2} y_i + \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1} y_i \sum_{i=1}^{n} x_{i1} x_{i2} \right.}{D_s}$$
$$\left. - \sum_{i=1}^{n} x_{i2} y_i \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2} - \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_{i2}^2 \sum_{i=1}^{n} x_{i1} y_i \sum_{i=1}^{n} x_{i1} \right)$$

$$\beta_1 = \frac{\left(n \sum_{i=1}^{n} x_{i1} y_i \sum_{i=1}^{n} x_{i2}^2 + \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i2} + \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i2} y_i \right.}{D_s}$$
$$\left. - \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1} y_i \sum_{i=1}^{n} x_{i2} - \sum_{i=1}^{n} x_{i2} y_i \sum_{i=1}^{n} x_{i1} x_{i2}\, n - \sum_{i=1}^{n} x_{i2}^2 \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} y_i \right)$$

$$\beta_2 = \frac{\left(n \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} x_{i2} y_i + \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} y_i \sum_{i=1}^{n} x_{i2} + \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} x_{i2} \right.}{D_s}$$
$$\left. - \sum_{i=1}^{n} x_{i2} \sum_{i=1}^{n} x_{i1}^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_{i1} x_{i2} \sum_{i=1}^{n} x_{i1} y_i\, n - \sum_{i=1}^{n} x_{i2} y_i \sum_{i=1}^{n} x_{i1} \sum_{i=1}^{n} x_{i1} \right)$$