DECISION-MAKING IN FUZZY SEQUENCING VIA NANOGONAL FUZZY NUMBER

A project report submitted to the Department of Mathematics (SF),

PSG College of Arts & Science (Autonomous)
In partial fulfilment of the requirements for the award of the degree of

MASTER OF SCIENCE

IN

MATHEMATICS

Affiliated to Bharathiar University

Submitted by

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DECLARATION

DECLARATION

We, **LAVANYA. N(23MMA112), LEGA SRI. K(23MMA113)** hereby declare that the project report, entitled " **DECISION-MAKING IN FUZZY SEQUENCING VIA NANOGONAL FUZZY NUMBER** ", submitted to the Department of Mathematics (SF), PSG College of Arts & Science, Coimbatore, in partial fulfilment of the requirements for the award of degree of **Master of Science in Mathematics** of Bharathiar University is a record of original project work done by us during 2024-2025 under the supervision and guidance of **Dr. R. KAVITHA,** Assistant Professor, Department of Mathematics (SF), PSG College of Arts & Science, Coimbatore and it has not formed the basis for the award of any Degree / Diploma / Associateship / Fellowship or other similar titles to any candidate of any University.

Name of the Candidates	**Signature of the Candidates**
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CERTIFICATE

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This is to certify that the project entitled " **DECISION-MAKING IN FUZZY SEQUENCING VIA NANOGONAL FUZZY NUMBER** ", submitted to the Department of Mathematics (SF), PSG College of Arts & Science, Coimbatore, in partial fulfilment of the requirements for the award of degree of **Master of Science in Mathematics**, of Bharathiar University, Coimbatore, is a record of original project work done by the following candidates

1. **LAVANYA N (23MMA112)**
2. **LEGA SRI K (23MMA113)**

during the period of 2024-2025 of their study in the Department of Mathematics (SF), PSG College of Arts & Science, Coimbatore, under the supervision and guidance and the project has not formed the basis for the award of any Degree / Diploma / Associateship / Fellowship or other similar title to any candidate of any University.

Head of the Department **Signature of the Guide**

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ABSTRACT

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The proposed methodology integrates the weighted average method to defuzzify the Nanogonal fuzzy numbers enabling a systematic ranking and sequencing process. We develop a mathematical model that incorporates various operational constraints and objectives, such as minimizing the total flow time and untimely arrival, while considering the fuzzy nature of processing times and due dates. Numerical experiments demonstrate the effectiveness of our approach in comparison to existing fuzzy sequencing methods. The results show improved robustness and adaptability to uncertainty due to vagueness, leading to more reliable scheduling decisions. Our project advances the area of fuzzy optimization and gives qualified individuals an effective tool for handling sequencing problems in unpredictable environments. We have used MATLAB Tool to provide the solutions for various Fuzzy Sequencing Problem.

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CHAPTER 1

CHAPTER-1
INTRODUCTION

In this chapter, we will look at the history and fundamental notions of the Fuzzy set, Fuzzy numbers and their types, the Sequencing problem and Fuzzy sequencing problems.

1.1 HISTORY OF FUZZY SET

Fuzzy set theory has advanced significantly since its formal introduction by L.A. Zadeh in his landmark paper 'Fuzzy Sets' published in the Journal 'Information and Control' in 1965. He introduced the fuzzy set, which is defined using grades of membership. In many ways, fuzzy sets behave similarly to ordinary sets, although they are more general. It offers a natural approach to dealing with issues in which the source of imprecision is the lack of precisely defined membership criteria in the sense of two-valued logic or the law of excluded middle, rather than the presence of random variables.

Zadeh's concept of fuzzy subsets provided a suitable foundation for generalizing algebraic and topological concepts, as well as other fields such as logic, measure theory, probability, number theory, and so on. Mathematicians, physicists, and social scientists, as well as computer and management scientists and engineers from around the world, have conducted numerous research studies on the subject's theory and applications. The use of fuzzy logic and fuzzy set theory in decision making, pattern recognition, image processing, control systems, neural networks, evolutionary algorithms, and a variety of other fields has produced major advances.

1.2. FUZZY SET

Let X denotes a universal set. Then, the membership function μ_A by which a fuzzy set A is usually defined has the form

$$\mu_A: X \to [0,1]$$

where [0,1] denotes the interval of real numbers from 0 to 1.

For example, we can define a possible membership function for the fuzzy set of real numbers close to 0 as follows:

$$\mu_A(x) = \frac{1}{10 + x^2}$$

Using this function, we can determine the membership grade of each real number in this fuzzy set, which signifies the degree to which that number close to 0. For instance, the number 3 is assigned a grade of 0.01, the number 1 a grade of 0.09, the number 0.25 a grade of 0.62 and the number 0 a grade of 1. We might intuitively expect that by performing some operation on the function corresponding to the set of numbers very close to 0. One possible way of accomplishing this is to square the function, that is

$$\mu_A(x) = \left(\frac{1}{1 + 10x^2}\right)^2$$

We could also generalize this function to a family of functions representing the set of real numbers close to any given number a as follows

$$\mu_A(x) = \left(\frac{1}{1 + 10(x - a)^2}\right)$$

Although the range of values between 0 and 1, inclusive, is the one most commonly used representing membership grades, any arbitrary set with some natural full or partial ordering can, in fact, be used. Elements of this set are not required to be numbers as long as the ordering among them can be interpreted as representing various strengths of membership degree. This generalised membership function has the form

$$\mu_A(x): X \to L$$

Where L denotes any set that is at least partially ordered. Since L is most frequently a lattice, fuzzy sets defined by this generalized membership grade function are called L-fuzzy sets, where L is intended as an abbreviation for lattice.

1.3. BASIC CONCEPTS OF FUZZY SET

The Support of a Fuzzy set $\widetilde{A}$ is the set of elements in $\widetilde{A}$ whose membership function is non-zero. Let a Fuzzy set $\widetilde{A}$ be defined on a universe of discourse U. Then we define support of a Fuzzy set $\widetilde{A}$ as

$$Supp\,\widetilde{A} = \{x \in U \mid \mu_A(x) > 0\}$$

An empty fuzzy set has an empty support; that is, the membership function assigns 0 to all elements of the universal set. The height of a fuzzy set is the largest membership grade attained by any element in that set. A fuzzy set is called normalized when at least one of its elements attains the maximum possible membership grade. If membership grades range in the closed interval between 0 and 1, for instance, then at least one element must have a membership grade of 1 for the fuzzy set to be considered normalized. An α-cut of a fuzzy set A is a crisp set A_α that contains all the elements of the universal set X that have a membership grade in A greater than or equal to the specified value of α. This definition can be written as

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$$

The value of α can be chosen arbitrarily but is often designated at the values of the membership grades appearing in the set. The set of all levels α ∈ [0,1] that represents distinct α-cuts of a given fuzzy set A is called a level set of A. Formally,

$$\overline{\overline{A}} = \{\alpha \mid \mu_A(x) = \alpha \text{ for some } x \in X\}$$

A fuzzy set is convex if and only if each of its α-cuts is a convex set. Equivalently we may say that a fuzzy set A is convex if and only if

$$\mu_A(\lambda r + (1-\lambda)\, s) \geq min\, [\mu_A(r), \mu_A\,(s)].$$

1.4. FUZZY LOGIC

The basic assumption upon which classical logic is based that every proposition is either true or false-has been questioned since Aristotle. In this treatise on Interpretation, Aristotle discusses the problematic truth status of matters that are future-contingent. Propositions about future events, he maintains, are neither actually true nor actually false but are potentially either; hence, their truth value is undetermined, at least prior to the event.

It is now well understood that propositions whose truth status is problematic are not restricted to future events. As a consequence of the Heisenberg principle of uncertainty, for example, it is known that truth values of certain propositions in quantum mechanics are inherently indeterminate due to fundamental limitations of measurement. In order to deal with such propositions, we must relax the true or false dichotomy of classical two valued logic by allowing a third truth value, which may be called indeterminate

The classical two-valued logic can be extended into three-valued logic in various ways. Several three-valued logics, each with its own rationale, are now well established. It is common in these logics to denote the truth, falsity and indeterminacy by 1, 0, and $\frac{1}{2}$ respectively. It is common to define the negation $\overline{a}$ of a proposition a as $1-a$; that is,

$\overline{1} = 0$, $\overline{0} = 1$ and $\overline{\frac{1}{2}} = \frac{1}{2}$. Other primitives such as ∧ , ∨ , ⇒ , ⇔ .Some basic properties of the primitives are:

$$\bar{a} = 1 - a$$

$$a \wedge b = min\,(a, b)$$

$$a \vee b = max\,(a, b)$$

$$a \Rightarrow b = min\,(1, 1 + b - a)$$

$$a \Leftrightarrow b = 1 - |a - b|$$

In fact, used only negation and implication as primitives and defined the other logic operations in terms of these two primitives, as follows:

$$a \vee b = (a \Rightarrow b) \Rightarrow b$$

$$a \wedge b = \overline{\overline{a} \vee \overline{b}}$$

$$a \Leftrightarrow b = (a \Rightarrow b) \wedge (b \Rightarrow a)$$

1.5. OPERATIONS IN FUZZY SET

1.5.1. FUZZY COMPLEMENT

A complement of a fuzzy set A is specified by a function c: [0,1]→[0,1] which assigns a value $c(\mu_A(x))$ to each membership grade $\mu_A(x)$. This assigned value is interpreted as the membership grade of the element x in the fuzzy set representing the negation of the concept represented by A.

1.5.2. FUZZY UNION

The union of two fuzzy sets A and B is specified in general by a function of the form

$$u: [0,1] \times [0,1] \rightarrow [0,1]$$

For each element x in the universal set, this function takes as its argument the pair consisting of the element's membership grades in set A and in set B and yields the membership grade of the element in the set constituting the union of A and B. Thus,

$$\mu_{A \cup B}(x) = u([\mu_A(x), \mu_B(x)]$$

1.5.3. FUZZY INTERSECTION

Fuzzy intersection closely parallels that of fuzzy union. Like fuzzy union, the general fuzzy intersection of two fuzzy sets A and B is specified by a function

$$i: [0,1] \times [0,1] \rightarrow [0,1]$$

The argument to this function is the pair consisting of the membership grade of some element x in fuzzy set A and the membership grade of that same element in fuzzy set B. The function returns the membership grade of the element in the set A∩B.Thus,

$$\mu_{A \cup B}(x) = i\,[\mu_A\,(x), \mu_B\,(x)]$$

1.6. FUZZY NUMBER

A fuzzy set $\overline{A}$ on R must possess the following three properties,

1.$\overline{A}$ must be a normal fuzzy set

2.Each of its α-level sets, $\overline{A_\alpha}$, $\alpha \in (0,1)$ must be a closed interval

3.The support $\overline{A_0}$ of $\overline{A}$, must be bounded

1.7 MEMBERSHIP FUNCTION

A membership function is a mathematical function that defines the degree of membership of an element to a fuzzy set. Unlike classical set theory, where an element either belongs to a set (membership value 1) or does not (membership value 0), fuzzy sets allow for partial membership. The membership function maps each element of the universe of discourse to a real number in the interval [0, 1], representing the degree of membership.

1.7.1 TYPES OF MEMBERSHIP FUNCTION

Several types of membership functions are commonly used in fuzzy systems:

1. Triangular Membership Function
2. Trapezoidal Membership Function
3. Gaussian Membership Function
4. Sigmoidal Membership Function
5. Bell-shaped Membership Function

Each type has its own characteristics and is suitable for different applications.

1.7.2 PROPERTIES OF MEMBERSHIP FUNCTION

1. Support: The set of elements with non-zero membership values.
2. Core: The set of elements with membership value 1.
3. Boundaries: Elements with membership values between 0 and 1.
4. Height: The maximum value of the membership function.
5. Normality: A membership function is normal if its height is 1.

1.7.3 PRACTICAL APPLICATIONS OF MEMBERSHIP FUNCTION

Control Systems

In fuzzy control systems, membership functions play a crucial role in mapping system inputs to linguistic variables and vice versa. For example, in a temperature control system, input temperatures might be mapped to linguistic terms like "cold," "warm," and "hot" using appropriate membership functions.

Risk Assessment

Membership functions are used in risk assessment to model uncertainties and subjective judgments. For instance, in financial risk analysis, linguistic terms like "low risk," "medium risk," and "high risk" can be represented by membership functions over a range of numerical risk scores.

Image Processing And Computer Vision

In image processing, membership functions can be used for tasks such as edge detection, image segmentation, and feature extraction. They help in dealing with imprecise boundaries and gradual transitions in image characteristics.

Natural Language Processing

Membership functions are utilized in NLP for tasks like sentiment analysis, where the degree of positivity or negativity of words or phrases can be modelled using fuzzy sets.

Alpha-Cuts And The Resolution Identity

Alpha-cuts are a powerful tool in fuzzy set theory, allowing the transformation of fuzzy sets into crisp sets. The α-cut of a fuzzy set A is defined as:

$$A_\alpha = \{x \in X | \mu_A(x)\} \geq \alpha\}$$

Extension Principle

Zadeh's extension principle is a fundamental concept in fuzzy set theory that allows the extension of crisp mathematical concepts to fuzzy domains. It's particularly useful in defining operations on fuzzy numbers and intervals.

1.7.4 OPTIMIZATION OF MEMBERSHIP FUNCTIONS

Genetic Algorithms For Membership Function Optimization

Genetic algorithms can be used to optimize the parameters of membership functions, such as the center, width, and shape. This approach is particularly useful when designing fuzzy systems for complex problems where manual tuning is impractical.

Clustering Techniques

Data clustering techniques, such as Fuzzy C-Means or subtractive clustering, can be employed to automatically generate membership functions from data. This is particularly useful in data-driven fuzzy modelling.

1.8 FUZZIFICATION

Fuzzification is the process of converting crisp input values into fuzzy values using membership functions. It is typically the first step in a fuzzy inference system, transforming precise numerical inputs into linguistic variables with associated degrees of membership in fuzzy sets.

1.8.1 PROCESS OF FUZZIFICATION

1. Define input variables and their ranges
2. Define fuzzy sets and their linguistic labels for each input variable
3. Construct membership functions for each fuzzy set
4. Map crisp input values to fuzzy set memberships

1.8.2 TYPES OF MEMBERSHIP FUNCTIONS USED IN FUZZIFICATION

1. Triangular Membership Functions
2. Trapezoidal Membership Functions
3. Gaussian Membership Functions
4. Sigmoidal Membership Functions
5. Bell-shaped Membership Functions

1.8.3 EXAMPLE OF FUZZIFICATION

Consider a temperature control system with input range [0°C, 40°C] and linguistic variables "Cold", "Comfortable", and "Hot". A crisp input of 25°C might be fuzzified as:

- μ(Cold) = 0.1
- μ(Comfortable) = 0.7
- μ(Hot) = 0.2

1.8.4 CHALLENGES IN FUZZIFICATION

1. Selecting appropriate membership function shapes
2. Determining the number of fuzzy sets
3. Handling outliers and extreme values

1.9 DEFUZZIFICATION

Defuzzification is the process of converting fuzzy output back to crisp values. It is typically the final step in a fuzzy inference system, producing a precise numerical output that can be used for decision-making or control actions.

1.9.1 COMMON DEFUZZIFICATION METHODS

1. Centroid Method (Center of Gravity)
2. Mean of Maximum (MOM)
3. First of Maximum (FOM) or Smallest of Maximum (SOM)
4. Last of Maximum (LOM) or Largest of Maximum (LOM)
5. Bisector Method
6. Weighted Average Method

1. CENTROID METHOD

The most commonly used defuzzification method, the centroid method calculates the center of gravity of the aggregated fuzzy set. Mathematically, it is expressed as:

$$Z^* = \frac{\int \mu_A(z) * zdz}{\int \mu_A(z)dz}$$

where Z^* is the defuzzified output, $\mu_A(z)$ is the aggregated membership function, and Z is the output variable.

2. MEAN OF MAXIMUM (MOM)

This method takes the average of all points with the highest degree of membership. It is particularly useful when the output fuzzy set has multiple peaks.

1.9.2 COMPARISON OF DEFUZZIFICATION METHODS

Method	Advantages	Disadvantages
Centroid	Considers all active rules, smooth output	Computationally intensive
MOM	Simple, fast computation	May not consider all active rules
FOM/SOM	Fast, good for time-critical applications	May lead to abrupt changes in output
LOM	Fast, good for time-critical applications	May lead to abrupt changes in output

Bisector	Similar to centroid, considers shape of fuzzy set	Computationally intensive
Weighted Average	Simple, fast for certain membership functions	Only applicable to symmetrical output membership function

1.9.3 CHOOSING THE RIGHT DEFUZZIFICATION METHOD

The choice of defuzzification method depends on various factors:

1. Nature of the application (e.g., control systems vs. decision support)
2. Computational resources available
3. Desired properties of the output (e.g., smoothness, responsiveness)
4. Shape and distribution of the output fuzzy sets

1.9.4 IMPORTANCE IN FUZZY SYSTEMS

Fuzzification and defuzzification are crucial for:

1. Interfacing fuzzy systems with real-world inputs and outputs
2. Handling uncertainty and imprecision in data
3. Enabling linguistic interpretation of numerical values
4. Facilitating rule-based reasoning in fuzzy inference systems

1.9.5 APPLICATIONS

1. Control Systems: temperature control, automotive systems, industrial processes
2. Decision Support Systems: risk assessment, medical diagnosis
3. Pattern Recognition: image processing, speech recognition
4. Robotics: navigation, obstacle avoidance
5. Consumer Electronics: washing machines, air conditioners

1.10 TYPES OF FUZZY NUMBERS

Fuzzy numbers are classified based on their membership functions or shapes, which determine how they represent uncertainty or imprecision. The classification of fuzzy number is primarily based on the shape of their membership function and the number of parameters that are used to define them. The common classification include:

- Based on Parameters - An inaccurate or uncertain quantity can be mathematically represented as a fuzzy number based on parameters. It is distinguished by a membership function that is defined using certain numerical values (parameters).
    - Triangular Fuzzy Numbers
    - Trapezoidal Fuzzy Numbers
    - Pentagonal Fuzzy Numbers
    - Hexagonal Fuzzy Numbers
    - Heptagonal Fuzzy Numbers
    - Octagonal Fuzzy Numbers
    - Nanogonal Fuzzy Numbers
    - Decagonal Fuzzy Numbers
- Gaussian Fuzzy Numbers
- Singelton Fuzzy Numbers
- Based on the shape - Fuzzy numbers based on their shape are defined by the specific form of their membership functions, which illustrate how different values relate to the concept of "fuzziness."
    - Pi - shaped fuzzy Numbers
    - Z - shaped fuzzy Numbers
    - S - shaped fuzzy Numbers
- L-R Fuzzy numbers
- Generalized Fuzzy Numbers
- Based on its type - Fuzzy numbers based on the type of fuzzy number are classified according to their mathematical representation and characteristics.
    - Normal Fuzzy Numbers
    - Non-Normal Fuzzy Numbers
- Based on its support - Fuzzy numbers based on their support are defined by the range of values over which they have non-zero membership, which reflects the extent of uncertainty they represent.
    - Bounded Fuzzy Numbers
    - Unbounded Fuzzy Numbers
- Based on its convexity - Fuzzy numbers based on convexity are classified according to the shape of their membership functions, particularly whether they exhibit convex or non-convex characteristics.
    - Convex Fuzzy Numbers
    - Non-Convex Fuzzy Numbers
- Based on its arithmetic operators - Fuzzy numbers based on arithmetic operations are defined by how they can be combined using standard mathematical operations while preserving their fuzzy characteristics.
    - Linguistic Fuzzy Numbers
    - Numerical Fuzzy Numbers

1.11. SEQUENCING PROBLEM

If there are n jobs to be performed, one at a time, on each of m machines, the sequence of the machines in which each job should be performed is given, and the time required by the jobs on each machine is also given, the problem is to find the sequence (order) of the jobs that minimizes the total time taken from the start of the first job on the first machine to the completion of the last job on the final machine. This is known as the Sequencing Problem.

The processing times on separate machines are precisely known and independent of the order of processing. The time spent by the jobs to switch from one computer to another is insignificant. When a work is started on a machine, it must be finished before another project can start on the same machine. Only one job can be processed on a single machine at a time. The order in which jobs are completed is not determined by their sequence.

In computer science, the sequencing problem is essential for effective job scheduling in operating systems; in logistics, it is used to plan delivery routes for prompt service; and in manufacturing, it optimizes task order on assembly lines to reduce production time and costs. In project management, it is used to organize tasks to improve timeliness and resource utilization. It also has a big impact on game theory in decision-making, supply chain management by optimizing inventory restocking and distribution, genomics by analyzing DNA sequences for medical insights, and data analysis by simplifying data processing processes. All things considered, efficient sequencing raises productivity, reduces expenses, and produces better results in a range of tasks.

1.12. FUZZY SEQUENCING PROBLEM

A Fuzzy Sequencing Problem is a type of sequencing problem where the elements, constraints, or objectives are represented by fuzzy sets, allowing for the modelling of uncertainty, vagueness, or imprecision in decision-making processes. In a typical sequencing problem, tasks or jobs need to be ordered in a way that optimizes a specific objective, like minimizing time or cost. However, in a fuzzy sequencing problem, the processing times, deadlines, or priorities of these tasks are not precisely known. Instead, they are described using fuzzy numbers or fuzzy sets, which account for uncertainty and ambiguity.

Due to its ability to reduce uncertainty and imprecision in decision-making, fuzzy sequencing problems find applications in a wide range of industries. They support production line scheduling in manufacturing by taking variable processing times and resource availability into account. Fuzzy sequencing in project management can improve project deadlines by optimizing work orders in the face of erratic durations. It helps with delivery vehicle routing in logistics by taking varying demand and traffic circumstances into account. Furthermore, by prioritizing activities based on erratic arrival times and service durations, fuzzy sequencing might enhance customer service in the service sector. All things considered, fuzzy sequencing improves operational effectiveness and decision-making in uncertain circumstances.

1.13. RANKING IN FUZZY SEQUENCING PROBLEM

Ranking in the context of fuzzy logic is the process of allocating preferences or suitability—often expressed as fuzzy numbers or linguistic variables—to things, alternatives, or solutions in order of choice. Fuzzy ranking takes into consideration the inherent uncertainty and vagueness of the data, in contrast to traditional ranking techniques that rely on exact numerical values. This enables a more sophisticated evaluation of possibilities by taking into account fuzzy terms (e.g., "high," "medium," and "low") that are expressed in relation to variables like importance, quality, or performance. Fuzzy numbers, membership functions, and defuzzification approaches are common fuzzy ranking strategies that are used to extract a distinct order from fuzzy inputs.

In fuzzy sequencing problems ranking plays a crucial role in decision-making optimization for a variety of applications. By assessing jobs based on fuzzy processing times and resource availability, it facilitates task prioritization in manufacturing and project management, guaranteeing that vital tasks are completed first. Fuzzy ranking is a logistical tool that facilitates efficient task scheduling and resource allocation based on fuzzy criteria, such as demand variability. When criteria are ambiguous, it helps analyze and choose the best choices, improving multi-criteria decision-making. It also enables more accurate performance assessments by taking qualitative elements into account. Fuzzy ranking also helps with supply chain optimization by using fuzzy criteria to evaluate inventory products and suppliers, which eventually improves operational effectiveness and efficiency in unpredictable circumstances.

1.14. CONCLUSION

The basis for exploring the fuzzy logic and fuzzy sequencing problem has been established in this chapter. We began by studying the fundamental concepts and terminology essential to understand this concept. We examined important theories and real-world viewpoints that have influenced its growth. We explore fuzzy in further detail in the upcoming chapters to give readers a thorough grasp of the subject.

CHAPTER 2

CHAPTER – 2

LITERATURE REVIEW

2.1 INTRODUCTION

Fuzzy sequencing problem continues to be an important topic of current the study, due to the development of new connections with other branches of mathematics, interactions with other fields of study, fascinating reinterpretations of basic concepts and theories over time, new discoveries in the 20th century, and other factors. In this chapter, we present literature on our project activity from an international and national perspective. Some publications are pertinent to our topic; similar works were done by many authors before us. This literature review provides insights into advancements in numerical solutions for fuzzy sequencing problems.

2.2 AUTHORS AND THEIR RESEARCH WORKS

DECISION – MAKING IN A FUZZY ENVIRONMENT by Bellman. R. E and Zadeh. L. A (1970) [3]

In this work, examples of multistage decision processes with either deterministic or stochastic systems under control are used to demonstrate the applicability of these principles. Using dynamic programming, determining a maximizing decision is reduced to solving a set of functional equations. This article describes a reverse-flow technique for solving a functional equation related to a decision process. The termination time is implicitly defined as the system entering a specified set of states in its state space.

FUZZY SET THEORY AND ITS APPLICATIONS by Zimmermann. H. J (1991) [23]

This book concentrates on decision making and expert systems, introducing fuzzy set theory only where necessary. It provides a didactically produced material that requires minimal mathematical background for readers. It attempts to introduce fuzzy set theory as thoroughly as possible, without delving into highly theoretical topics or offering any mathematical proofs that do not help to a deeper understanding. It provides numerical examples whenever possible.

FUZZY JOB SEQUENCING FOR A FLOW SHOP by Mc Cahon. S and Lee. E. S (1992) [12]

This study examines work sequencing in a flow shop, processing times are often unknown and the precise and only estimated intervals are provided. Fuzzy numbers are ideally suited for representing these intervals. Campbell, Dudek, and Smith's (CDS) jobs the sequencing algorithm that has been modified to support Trapezoidal fuzzy processing times. Deterministic sequences exist, however the sequence performance measures of make span and job mean flow time are uncertain and had been determined using fuzzy arithmetic. The application of possibility theory and the fuzzy integral enables the scheduler to interpret the fuzzy results in a reasonable manner. Deterministic estimations for this fuzzy approach were also investigated.

FUZZY LOGIC SYSTEMS FOR ENGINEERING A TUTORIAL PROCEEDINGS OF THE IEEE by Mendel. J. M (1995) [14]

This tutorial post will lead you through the aspects of fuzzy sets and fuzzy logic that are essential to create a FLS. It accomplishes this by starting with crisp set theory and dual logic and then demonstrating how these can be extended to their fuzzy versions. Because engineering systems are primarily causal, we employ causality as a constraint on FLS development. This allows us to guide down a specific and regularly utilized tributary of the FL literature, which is helpful for engineering usage but may not be as valuable for non-technical applications.

RANKING OF FUZZY NUMBERS BY SIGN DISTANCE by Abbasbandy. S and Asady. B (2006) [1]

This study, suggests a version of the distance-based strategy known as the sign distance, which is both efficient to analyse and capable of overcoming the limitations of earlier techniques. These include approaches that use the coefficient of variation (CV index), distance between fuzzy sets, centroid point and original point, and weighted mean value. Each of these strategies has been proved to deliver unexpected effects in specific situations. The proposed method has a simplified calculation compared to existing ways.

RANKING OF GENERALIZED TRAPEZOIDAL FUZZY NUMBERS BASED ON RANK, MODE, DIVERGENCE AND SPREAD by Kumar. A, Singh. P, Kaur. A and Kaur. P (2010) [11]

This study proposes a new approach for ranking generalized Trapezoidal fuzzy numbers. The proposed method relies on rank, mode, divergence, and spread. The key advantage of the proposed approach is that it provides the correct ordering of generalized and normal Trapezoidal fuzzy numbers, as well as being very simple and easy to apply to real-world issues. For validation, the suggested approach's results are compared to those of other current methodologies.

FUZZY SEQUENCING PROBLEM by Kripa. K and Govindarajan. R (2016) [10]

This study presents many ways for solving fuzzy sequencing problems using technology values such as Triangular fuzzy numbers. Additionally, a numerical example is provided that does not employ non-negative fuzzy values. The technique followed was that fuzzy sequencing problems were defuzzified using ranking functions, and thus solving the crisp sequencing problem by conventional sequencing algorithm for finding the optimum sequence and the shortest completion time in terms of fuzzy values, shown with numerical examples and solutions.

UNCERTAIN RULE-BASED FUZZY SYSTEMS: INTRODUCTIONA AND NEW DIRECTIONS by Mendel, J. M (2017) [13]

This textbook offers a complete updated approach to fuzzy sets and systems capable of modeling uncertainty due to vagueness. The author shows how to overcome the constraints of conventional fuzzy sets and systems, allowing for a wide range of applications, including time-series forecasting, knowledge mining, and control. This revised version takes a bottom-up approach, introducing classical fuzzy sets and systems before explaining how they can be changed to deal with uncertainty that arises due to vagueness.

AN APPROACH FOR SOLVING FUZZY SEQUENCING PROBLEMS WITH OCTAGONAL FUZZY NUMBERS USING ROBUST RANKING TECHNIQUES by Selvakumari. K and Santhi. S (2018) [17]

This study, addresses the fuzzy Sequencing problem, where processing time is represented by Octagonal fuzzy integers. The Robust Ranking approach is used to tackle fuzzy sequencing problems, while Johnson's Algorithm is addressed for crisp valued sequence problems.

SOLVING FLO-SHOP SCHEDULING PROBLEM TO MINIMIZE TOTAL ELAPSED TIME USING FUZZY APPROACH by Jadhav. V. S and Jadhav. O. S (2019) [8]

This study addresses job-shop scheduling in a fuzzy context to optimize total elapsed time. The Triangular fuzzy membership function represents uncertainty in task processing time. Job sequences are created using the average high-ranking algorithm and the branch and bound methodology, which is based on fuzzy processing time. A numerical illustration is performed to assess the efficacy of the recommended methodologies.

SOLVING SEQUENCING PROBLEMS UNDER FUZZY ENVIRONMENT by Srimathi. S and Prabakaran. K (2019) [19]

This article assists in determining the sequence in which critical tasks should be completed in order to reduce the total amount of time spent on all jobs. In general, job sequencing problems have precise processing timeframes. However, it has been discovered that processing times during work performance are endless. As a result, the concept of fuzzy work sequencing problem provides an effective blueprint that can be applied to real-world scenarios with imprecise processing times. Here, we propose a new method for solving fuzzy sequencing problems involving Triangular fuzzy numbers without converting them into comparable crisp sequencing problems.

FUZZY SEQUENCING PROBLEMS WITH PENTAGONAL FUZZY NUMBER USING RANKING TECHNIQUE by Priya. M and Dr. Elumalai. P (2021) [16]

In this study, we explore a fuzzy sequencing problem in which processing time is represented as a pentagonal fuzzy integer. To get the solution, use the Ranking approach and the fuzzy sequencing problem. Can be transformed into a crisp-valued sequencing issue. Also, identify the best remedy.

FUZZY SEQUENCING PROBLEM USING GRAPHICAL METHOD by S. Senthil and E. Baskarprabhu (2022) [18]

This paper proposes a solution for a fuzzy sequencing problem involving '2' jobs and 'm' machines. The ranking index method is used to convert the fuzzy processing time into crisp sequencing problems that can be solved using existing methods. A numerical case was explored and solved for illustration purposes.

OPTIMIZATION OF FUZZY SEQUENCING PROBLEMS WITH HEPTAGONAL FUZZY NUMBERS by M. Ananthanarayanan and R. Mahalakshmi (2023) [2]

Consider the issue of planning 5 occupations to 4 machines, and the handling time as Heptagonal fuzzy numbers. In this publication the fuzzy sequencing numbers are converted into crisp values using a Python application. As a result, the optimal succession of positions is determined using Johnson's Bellmans Algorithm, which calculates total elapsed time and idle time for each machine.

2.3 CONCLUSION

In this chapter, we read several articles and books for reference purposes. It serves as the foundation for our study, offering a complete review of current knowledge and research on our issue. It is a critical component of our project. It has assisted us in identifying gaps, developing a theoretical framework, selecting acceptable approaches, and positioning our study within a larger context. This chapter not only sets the stage for our study, but also illustrates our commitment to expanding on current knowledge and contributing to the growth of our area.

CHAPTER 3

CHAPTER – 3

FUZZY SEQUENCING PROBLEM INVOLVING OCTAGONAL FUZZY NUMBER

3.1 INTRODUCTION

A key idea in operations research is optimization, which is the process of determining the optimal way to solve a problem while adhering to predetermined parameters. Numerous fields, including engineering, finance, logistics, transportation, and manufacturing, use optimization techniques. In an economic setting, fuzzy numbers are ranked to aid in decision-making. Various tasks, including planning, carrying out, and other processes, are ongoing in a company. This calls for close attention to a number of variables, all of which are erratic in nature because of the highly competitive global business climate.

Ranking fuzzy numbers is an essential phase in the decision-making process in a fuzzy environment. Bellman first introduced the concept of a fuzzy set as a way to deal with uncertainty that results from imprecision rather than randomness. Researchers and practitioners from a wide range of fields are becoming more and more interested in fuzzy systems. In this system, the output defined by a fuzzy set is transformed into scalar values. This technique is known a defuzzification. Fuzzy amounts lack a natural order, as contrast to real numbers. Consequently, there are numerous fuzzy number ranking techniques, each of which has some flaws of their own, such as a lack of uniqueness.

As a result, no single technique can accurately rank the fuzzy numbers. Certain approaches appear to be more appropriate than others, depending on the application's environment. Because fuzzy numbers have many applications in fuzzy decision-making theory, risk analysis, data analysis, optimization, etc., ranking fuzzy numbers has garnered special academic attention. Since 1976, numerous authors have studied a variety of ranking techniques in an effort to increase ranking results' efficiency and accuracy.

When we encounter scenarios in which many tasks are to be accomplished, job sequencing problems arise. In the world of computers, job sequencing problems have emerged as the main concern. One of the most significant and traditional uses of operations research is sequencing problem. The primary goal of the classical sequencing problem is to determine the best order for the jobs to be completed on machines in order to reduce the overall time needed to do all of the jobs.

Johnson's 1954 method for production scheduling, which minimized both the total idle time of the machines and the overall production times of the project, is one of the most well-known pieces of work in the field to this day.

In this chapter, we discuss about the prelimineries, the Robust ranking technique and a numerical example for solving the fuzzy sequencing problem involving octagonal fuzzy number.

3.2 PRELIMINERIES

In this section, we introduce some fundamental definitions that have been investigated in fuzzy numbers.

3.2.1 FUZZY SET

If X is a collection of objects denoted generically by x, then the fuzzy set A is defined to be set of ordered pairs, $A = \left(x, \mu_A(x)\right)$. Where $\mu_A(x)$ is called the membership function for the fuzzy set A . The membership function maps each element of to x a membership grade or membership value between 0 and 1.

3.2.2 FUZZY NUMBER

A Fuzzy number A in the real line $\mathbb{R}$ is a fuzzy set $\mu_A(x): \mathbb{R} \to (0,1)$ that satisfies the following characteristics:

(1) There exists at least one $x \in \mathbb{R}$ with $\mu_A(x) = 1$
(2) $\mu_A(x)$ is piece wise continuous

REMARK 1

Membership function $\mu_A(x)$ are continuous functions

3.2.3 OCTAGONAL FUZZY NUMBER

A fuzzy number is said to be an octagonal fuzzy number if it is denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$, where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8 \le a_9$ are real numbers and its membership function $\mu_A(x)$ is given by

$$\mu_A(x) = \begin{cases} 0 & ,x \le a_1; \\ k\left(\dfrac{x - a_1}{a_2 - a_1}\right) & ,a_1 \le x \le a_2; \\ k & ,a_2 \le x \le a_3; \\ k + (1-k)\left(\dfrac{x - a_3}{a_4 - a_3}\right) & ,a_3 \le x \le a_4; \\ 1 & ,a_4 \le x \le a_5; \\ k + (1-k)\left(\dfrac{a_6 - x}{a_6 - a_5}\right) & ,a_5 \le x \le a_6; \\ k & ,a_6 \le x \le a_7; \\ k\left(\dfrac{a_8 - x}{a_8 - a_7}\right) & ,a_7 \le x \le a_8; \\ 0 & ,x \ge a_8 \end{cases}$$

Where $0 \le k \le 1$

REMARK 2

If $K = 0$, the octagonal fuzzy number reduces to trapezoidal fuzzy number (a_3, a_4, a_5, a_6) and if $K = 1$ reduces to trapezoidal fuzzy number (a_1, a_4, a_5, a_8).

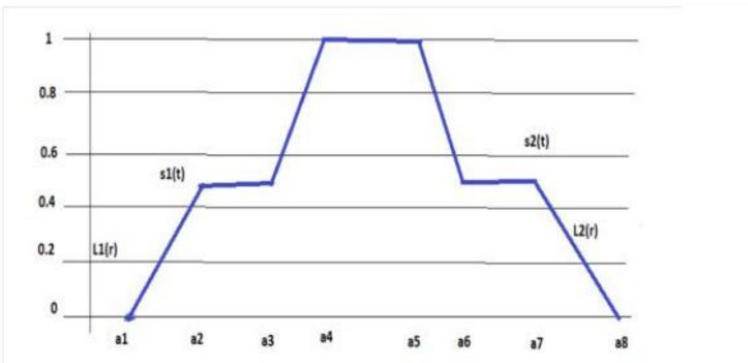


Figure 1. Graphical representation of an Octagonal Fuzzy Number

3.3 ROBUST RANKING TECHNIQUE

To provide results which are consistent with human intuition, robust ranking technique is used and it satisfies compensation, linearity and additive properties. If a is a convex fuzzy number, the robust ranking index is defined by

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)d\alpha$$

Where,

$$(a_\alpha^L, a_\alpha^U) = [\{(b-a)\alpha + a, d-(d-c)\alpha\}, \{(f-e)\alpha + e, h-(h-g)\alpha\}]$$

is the $\alpha -$level cut of a fuzzy number $\tilde{a}$. Here this method is proposed for ranking the objective values. The representation value of fuzzy number $\tilde{a}$ is given by robust ranking index $R(\tilde{a})$.

3.4 PROPOSED METHOD FOR SOLVING FUZZY SEQUENCING PROBLEM

In this study, we proposed a new solving method for fuzzy sequencing problem by using robust ranking technique. The proposed method must operate the following steps:

Step 1: Using Robust ranking technique, the fuzzy sequencing problem can be converted into crisp sequencing problem

Step 2: The optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.

Step 3: After finding the optimal sequence, determine the total elapsed fuzzy time and also the fuzzy idle time on machines

3.5 NUMERICAL EXAMPLE

In this section, a numerical example has been considered to illustrate the proposed solution procedure. Consider the following fuzzy sequencing problem.

Global export house has to process 5 items through 3 stages of production viz, cutting, sewing & processing times are given in the following table

Items	Cutting M_1	Sewing M_2	Pressing M_3
A_1	(-2, -1, 0, 1, 4, 5, 6, 7)	(-3, -2, -1, 0, 1, 2, 3, 4)	(7, 8, 10, 11, 14, 15, 17, 18)
A_2	(1, 2, 3, 4, 5, 6, 9, 10)	(-3, -2, -1, 0, 4, 5, 6, 7)	(6, 8, 9, 10, 12, 13, 14, 16)
A_3	(1, 2, 4, 5, 8, 10, 12, 14)	(-4, -3, 0, 1, 2, 3, 4, 5)	(-1, 0, 1, 2, 4, 5, 6, 7)
A_4	(2, 3, 6, 7, 8, 10, 11, 13)	(-2, -1, 0, 1, 2, 3, 4, 5)	(5, 6, 7, 8, 12, 13, 14, 15)
A_5	(3, 5, 6, 8, 10, 12, 13, 15)	(-1, 0, 1, 2, 3, 4, 5, 6)	(1, 2, 3, 4, 5, 6, 7, 8)

Determine an order in which these items should be processed so as to minimize the total processing time.

Step 1: Using robust ranking technique for octagonal fuzzy number, the fuzzy times can be converted in to crisp items.

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)da$$

R (-2, -1, 0, 1, 4, 5, 6, 7) = 5

R (1, 2, 3, 4, 5, 6, 9, 10) = 10

R (1, 2, 4, 5, 8, 10, 12, 14) =14

R (2, 3, 6, 7, 8, 10, 11, 13) = 15

R (2, 3, 6, 7, 8, 10, 11, 13) = 18

R (-3, -2, -1, 0, 1, 2, 3, 4) = 1

R (-3, -2, -1, 0, 4, 5, 6, 7) = 4

R (-4, -3, 0, 1, 2, 3, 4, 5) = 2

R (-2, -1, 0, 1, 2, 3, 4, 5) = 3

R (-1, 0, 1, 2, 3, 4, 5, 6) = 5

R (7, 8, 10, 11, 14, 15, 17, 18) = 25

R (6, 8, 9, 10, 12, 13, 14, 16) = 17

R (-1, 0, 1, 2, 4, 5, 6, 7) = 6

R (5, 6, 7, 8, 12, 13, 14, 15) = 20

R (1, 2, 3, 4, 5, 6, 7, 8) = 9

Items	**Cutting M_1**	**Sewing M_2**	**Pressing M_3**
A_1	5	1	25
A_2	10	4	17
A_3	14	2	6
A_4	15	3	20
A_5	18	5	9

Step 2: Three machine problem can be converted into two machine problem

Items	Machine M_1	Machine M_2
A_1	6	26
A_2	14	21
A_3	16	8
A_4	18	23
A_5	23	14

Optimum Sequence: A_1, A_2, A_4, A_5, A_3

Step 3: Total Elapsed time and Idle time

Jobs	Machine M_1		Machine M_2		Machine M_3	
	Time in	Time out	Time in	Time out	Time in	Time out
J_1	0	5	5	6	6	31
J_2	5	15	15	19	31	48
J_4	15	30	30	33	48	68
J_5	30	48	48	53	68	77
J_3	48	62	62	64	77	83

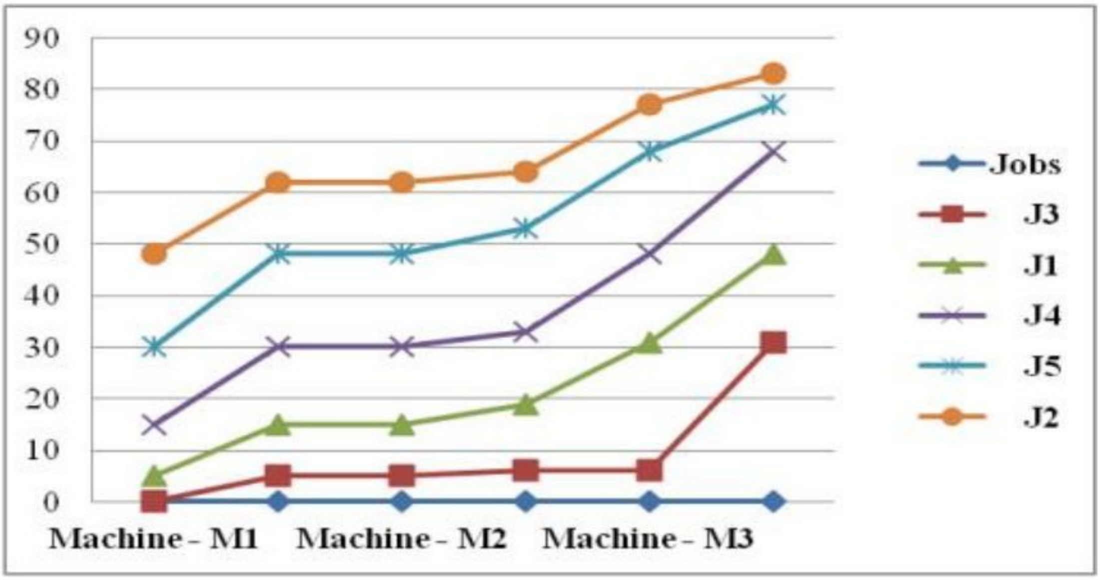


Figure 2. Graphical representation of total elapsed time and idle time.

Using above procedure, the problem can be reduced as following optimal solution

Total Elapsed time = 83 hrs

Idle time for Machine M_1 = 21 hrs

Idle time for Machine M_2 = 68 hrs

Idle time for Machine $M_3 = 6$ hrs

Total Idle time for all the Machines = $M_1 + M_2 + M_3$= 21+68+6

= 95 hrs

3.6 CONCLUSION

This chapter has been presented the successful implementation of Octagonal Fuzzy Numbers. This method has been provided simple and powerful ranking criteria of solving fuzzy sequencing problem. In this problem the Octagonal Fuzzy Numbers are the representation of processing times which has been transformed into crisp sequencing problem using robust ranking technique. In order to minimize the overall elapsed time, we extended the well-known Johnson's approach to find the exclusive order of the sequencing problem with fuzzy numbers. Using this approach, we were able to determine the best overall cost in crisp nature and the best solution in fuzzy nature. This new method is illustrated by solving numerical example which is very easy to understand and to apply for solving fuzzy sequencing problems in real life situation. This concept of ranking method can be used to all types of sequencing problems which would give effective solutions for any uncertain data.

CHAPTER 4

CHAPTER – 4

FUZZY SEQUENCING PROBLEM INVOLVING NANOGONAL FUZZY NUMBER

4.1 INTRODUCTION

Fuzzy sequencing problem is an extension of classical sequencing problems in operations research and scheduling theory. It deals with situations where the parameters involved in sequencing tasks or jobs are not precisely known, but can be represented using fuzzy numbers. When we incorporate Nanogonal fuzzy numbers into this framework, we add an extra layer of sophistication and flexibility to model complex uncertainties due to vagueness. We have a set of jobs to be sequenced where each job has associated parameters (like processing time, due date, release time) represented as Nanogonal fuzzy numbers. The objective is to find an optimal or near-optimal sequence of jobs that optimizes one or more fuzzy criteria. This chapter discusses the fundamental definition of the weighted average approach, numerous fuzzy numbers and their membership functions, numerical examples of various fuzzy sequencing problems involving different fuzzy numbers, and the solving method for the fuzzy sequencing problem with Nanogonal fuzzy number by using MATLAB Program.

4.2 WEIGHTED AVERAGE METHOD

The Weighted Average Method is a technique used in various fields such as statistics, decision-making, inventory valuation, and data analysis. It involves calculating an average where each value is multiplied by a weight that reflects its relative importance or frequency before being summed up. This method allows you to emphasize certain values more than others, depending on their significance.

4.3 TRIANGULAR FUZZY NUMBER

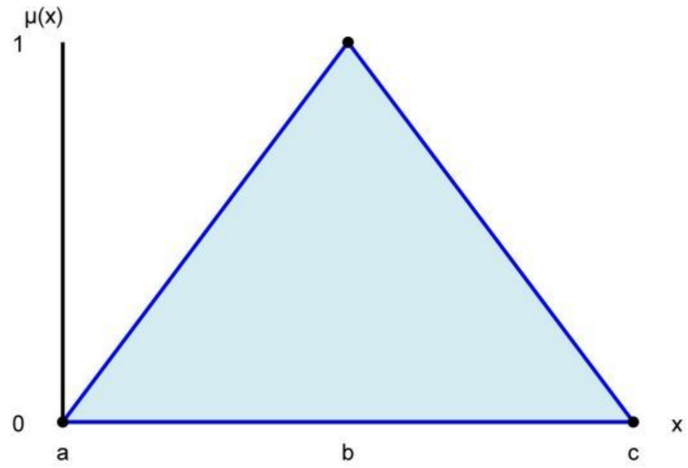
A Triangular fuzzy number is a simple and widely used type of fuzzy number characterized by a Triangular -shaped membership function. It is defined by three parameters: (a_1, a_2, a_3),

where,

- a_1 - (lower bound) represents the smallest possible value of the fuzzy number.
- a_2 - (middle value) represents the most likely or expected value.
- a_3 - (upper bound) represents the largest possible value.

The membership function $\mu_A(x)$ of a Triangular fuzzy number is given by

$$\mu_A(x) = \begin{cases} 0 & for\, x < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_3}\right) & for\, a_1 \le x \le a_2 \\ \left(\dfrac{a_2 - x}{a_3 - a_2}\right) & for\, a_2 \le x \le a_3 \\ 0 & for\, x > a_3 \end{cases}$$



4.4 NUMERICAL EXAMPLE FOR TRIANGULAR FUZZY NUMBER

Sequencing problems containing 2 machines and 5 jobs, the processing time are Triangular fuzzy numbers.

Jobs	Machine M_1	Machine M_2
A_1	(6, 7, 8)	(10, 11, 13)
A_2	(4, 5, 6)	(6, 7, 11)
A_3	(2, 4, 5)	(10, 12, 14)
A_4	(6, 8, 12)	(0, 6, 7)
A_5	(0, 2, 4)	(0, 1, 2)

SOLUTION:

By using Weighted Average method,

$$R_A(x) = \frac{(a_1 + 2a_2 + 3a_3)}{4}$$

We have converted fuzzy values into crisp values

$R_{11}(6, 7, 8) = 7$

$R_{12}(10, 11, 13) = 11.25$

$R_{21}(4, 5, 6) = 5$

$R_{22}(6, 7, 11) = 7.75$

$R_{31}(2, 4, 5) = 3.75$

$R_{32}(10, 12, 14) = 24$

$R_{41}(6, 8, 12) = 8.5$

$R_{42}(0, 6, 7) = 4.75$

$R_{51}(0, 2, 4) = 2$

$R_{52}(0, 1, 2) = 1$

Jobs	**Machine M_1**	**Machine M_2**
A_1	7	11.25
A_2	5	7.75
A_3	3.75	24
A_4	8.5	4.75
A_5	2	1

The order of jobs as follows:

A_3	A_2	A_1	A_4	A_5

Time calculation utilizing the job order is as follows:

Jobs	**Machine M_1**		**Machine M_2**	
	In Time	**Out Time**	**In Time**	**Out Time**
A_3	0	3.75	3.75	27.75
A_2	3.75	8.75	27.75	35.5
A_1	8.75	15.75	35.5	46.75
A_4	15.75	24.25	46.75	51.5
A_5	24.25	26.25	51.5	52.5

Total Elapsed time $= 52.5$

Idle Time for Machine $M_1 = 26.25$

Idle Time for Machine $M_2 = 3.75$

Total Idle Time $= M_1 + M_2$

$= 26.25 + 3.75$

Total Idle Time $= 30$

4.5 TRAPEZOIDAL FUZZY NUMBER

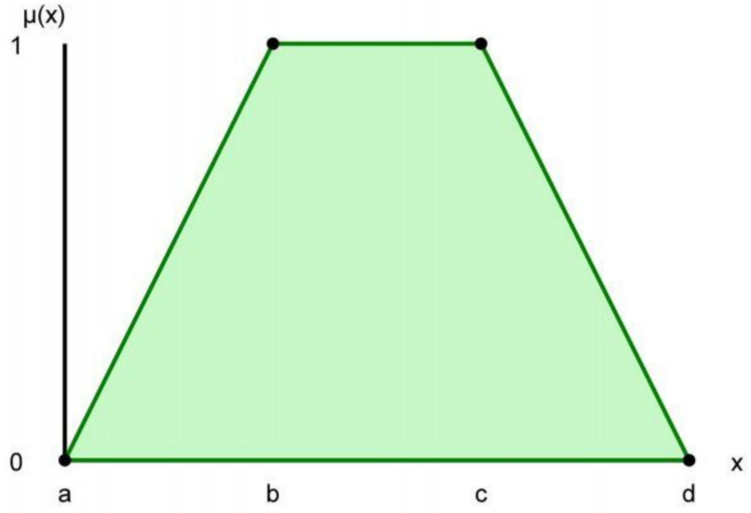
A Trapezoidal fuzzy number is a type of fuzzy number represented by a Trapezoidal-shaped membership function. It is defined by four parameters: (a_1, a_2, a_3, a_4),

where,

- a_1 (lower bound) is the smallest possible value of the fuzzy number.
- a_2 and a_3 (middle values) represent the range where the fuzzy number has the highest degree of membership (usually 1).
- a_4 (upper bound) is the largest possible value.

The membership function $\mu_A(x)$ for a Trapezoidal fuzzy number is described as

$$\mu_A(x) = \begin{cases} 0 & for\, x < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_1}\right) & for\, a_1 \leq x \leq a_2 \\ 1 & for\, a_2 \leq x \leq a_3 \\ \left(\dfrac{a_4 - x}{a_4 - a_3}\right) & for\, a_3 \leq x \leq a_4 \\ 0 & for\, x > a_4 \end{cases}$$



4.6 NUMERICAL EXAMPLE FOR TRAPEZOIDAL FUZZY NUMBER

Sequencing problems containing 2 machines and 5 jobs, the processing time are Trapezoidal fuzzy numbers.

Jobs	Machine M_1	Machine M_2
A_1	(1, 2, 4, 5)	(0, 2, 6, 8)
A_2	(4, 7, 9, 12)	(5, 9, 11, 15)
A_3	(3, 4, 6, 7)	(4, 5, 7, 8)
A_4	(1, 6, 8, 13)	(3, 4, 6, 7)
A_5	(0, 2, 6, 8)	(4, 7, 9, 12)

Obtain the optimal sequence and also determine the minimum total elapsed time and idle time for each of the machines

SOLUTION:

By using Weighted Average method,

$$R_A(x) = \frac{(a_1 + a_2 + a_3 + a_4)}{4}$$

We have converted fuzzy values into crisp values

$R_{11}(1, 2, 4, 5) = 3$

$R_{12}(0, 2, 6, 8) = 4$

$R_{21}(4, 7, 9, 12) = 8$

$R_{22}(5, 9, 11, 15) = 10$

$R_{31}(3, 4, 6, 7) = 5$

$R_{32}(4, 5, 7, 8) = 6$

$R_{41}(1, 6, 8, 13) = 7$

$R_{42}(3, 4, 6, 7) = 5$

$R_{51}(0, 2, 6, 8) = 4$

$R_{52}(4, 7, 9, 12) = 8$

Jobs	**Machine M_1**	**Machine M_2**
A_1	3	4
A_2	8	10
A_3	5	6
A_4	7	5
A_5	4	8

The order of jobs as follows:

A_1	A_5	A_3	A_2	A_4

Time calculation utilizing the job order is as follows:

Jobs	**Machine M_1**		**Machine M_2**	
	Time in	**Time Out**	**Time in**	**Time Out**
A_1	0	3	3	7
A_5	3	7	7	15
A_3	7	12	15	21
A_2	12	20	21	26
A_4	20	35	35	40

Total Elapsed time $= 40$

Idle Time for Machine $M_1 = 5$

Idle Time for Machine $M_2 = 12$

Total Idle Time $= M_1 + M_2$

$= 5 + 12$

Total Idle Time $= 17$

4.7 PENTAGONAL FUZZY NUMBER

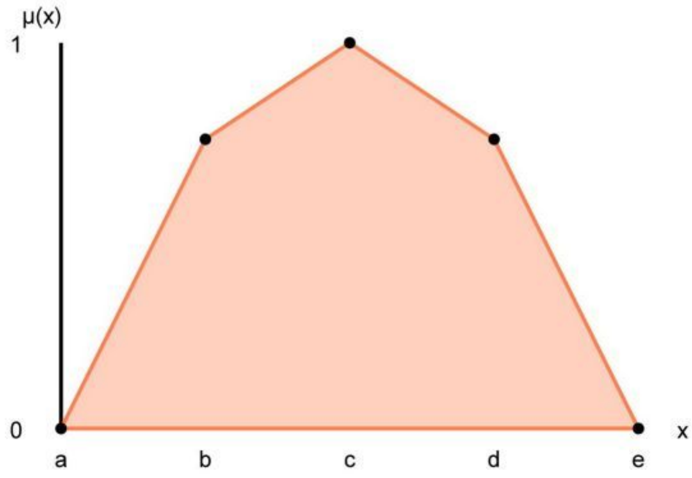
A Pentagonal fuzzy number is a more complex type of fuzzy number characterized by a Pentagon-shaped membership function. It is defined by five parameters: $(a_1, a_2, a_3, a_4, a_5)$,

where,

- a_1 (lower bound) represents the smallest possible value, with zero membership.
- a_3 and a_4 are points where the membership function transitions between increasing and decreasing.
- a_3 (central point) is the value with the highest membership degree, usually 1.
- a_5 (upper bound) is the largest possible value, also with zero membership.

The membership function $\mu_A(x)$ of a Pentagonal fuzzy number is given by

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_1}\right) & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \left(\dfrac{a_5 - x}{a_5 - a_4}\right) & for\ a_4 \le x \le a_5 \\ 0 & for\ x > a_5 \end{cases}$$



4.8 NUMERICAL EXAMPLE FOR PENTAGONAL FUZZY NUMBER

Sequencing problems containing 2 machines and 5 jobs, the processing time are Pentagonal fuzzy numbers.

Jobs	Machine M_1	Machine M_2
A_1	(34, 36, 38, 40, 42)	(9, 11, 13, 15, 17)
A_2	(24, 26, 28, 30, 32)	(3, 4, 5, 6, 7)
A_3	(9, 11, 13, 15, 17)	(18, 19, 20, 21, 22)
A_4	(3, 4, 5, 6, 7)	(34, 36, 38, 40, 42)
A_5	(18, 19, 20, 21, 22)	(24, 26, 28, 30, 32)

SOLUTION:

By using Weighted Average method,

$$R_A(x) = \frac{(a_1 + a_2 + a_3 + a_4 + a_5)}{5}$$

We have converted fuzzy values into crisp values

$R_{11}(34, 36, 38, 40, 42) = 38$

$R_{12}(9, 11, 13, 15, 17) = 13$

$R_{21}(24, 26, 28, 30, 32) = 28$

$R_{22}(3, 4, 5, 6, 7) = 5$

$R_{31}(9, 11, 13, 15, 17) = 13$

$R_{32}(18, 19, 20, 21, 22) = 20$

$R_{41}(3, 4, 5, 6, 7) = 5$

$R_{42}(34, 36, 38, 40, 42) = 38$

$R_{51}(18, 19, 20, 21, 22) = 20$

$R_{52}(24, 26, 28, 30, 32) = 28$

Jobs	**Machine M_1**	**Machine M_2**
1	38	13
2	28	5
3	13	20
4	5	38
5	20	28

The order of jobs as follows:

A_4	A_3	A_5	A_1	A_2

Time calculation utilizing the job order is as follows:

Jobs	**Machine M_1**		**Machine M_2**	
	Time In	**Time Out**	**Time In**	**Time Out**
A_4	0	5	5	43
A_3	5	18	43	63
A_5	18	38	63	91
A_1	38	76	91	104
A_2	76	104	104	109

Total Elapsed time $= 109$

Idle Time for Machine $M_1 = 5$

Idle Time for Machine $M_2 = 5$

Total Idle Time $= M_1 + M_2$

$= 5 + 5$

Total Idle Time =10

4.9 HEXAGONAL FUZZY NUMBER

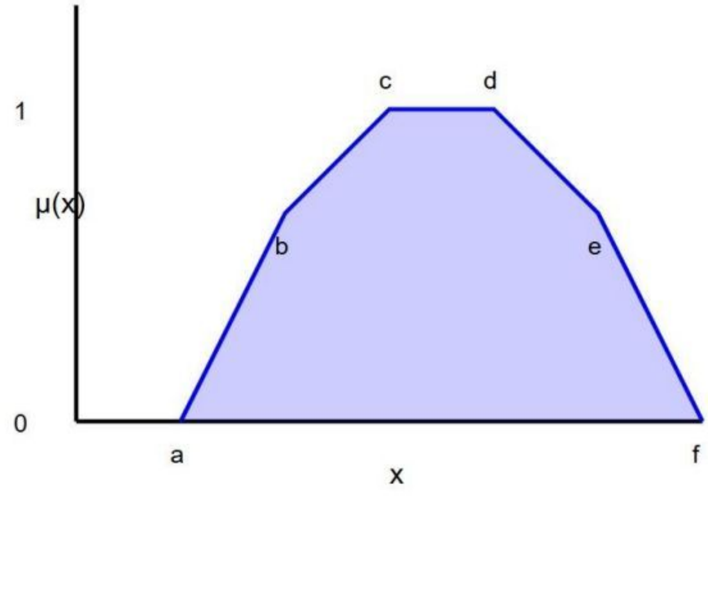
A Hexagonal fuzzy number is a type of fuzzy number characterized by a Hexagonal shape in its membership function. Unlike traditional Triangular or Trapezoidal fuzzy numbers, Hexagonal fuzzy numbers provide a more flexible representation of uncertainty due to vagueness. It is defined by six parameters: $(a_1, a_2, a_3, a_4, a_{5,}a_6)$,

where,

- a_1 and a_6 represent the lower and upper bounds of the fuzzy number. These are points where the membership function is equal to zero.
- a_2 and a_5 represent the points where the membership function begins to increase from zero or decrease back to zero, respectively.
- a_4 is the intermediate points that are closer to the peak, where the membership function is higher but not equal to 1.
- a_3 is the peak or modal value, where the membership function reaches its maximum value of 1.

The membership function $\mu_A(x)$ of a Hexagonal fuzzy number is given by

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_1}\right) & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \left(\dfrac{a_5 - x}{a_5 - a_4}\right) & for\ a_4 \le x \le a_5 \\ 0 & for\ x > a_6 \end{cases}$$



4.10 NUMERICAL EXAMPLE FOR HEXAGONAL FUZZY NUMBER

Sequencing problems containing 2 machines and 5 jobs, the processing time are Hexagonal fuzzy numbers.

Jobs	Machine M_1	Machine M_2	Machine M_3
A_1	(2, 3, 4, 5, 6, 7)	(3, 4, 5, 6, 7, 8)	(4, 5, 6, 7, 8, 9)
A_2	(1, 2, 3, 4, 5, 6)	(2, 3, 4, 5, 6, 7)	(3, 4, 5, 6, 7, 8)
A_3	(4, 5, 6, 7, 8, 9)	(5, 6, 7, 8, 9, 10)	(6, 7, 8, 9, 10, 11)
A_4	(3, 4, 5, 6, 7, 8)	(4, 5, 6, 7, 8, 9)	(5, 6, 7, 8, 9, 10)
A_5	(2, 3, 4, 5, 6, 7)	(3, 4, 5, 6, 7, 8)	(4, 5, 6, 7, 8, 9)

SOLUTION:

By using Weighted Average method

$$R_A(x) = \frac{(a_1 + a_2 + a_3 + a_4 + a_5 + a_6)}{6}$$

We have converted fuzzy values into crisp values

$R_{11}(2, 3, 4, 5\ , 6, 7) = 4.5$

$R_{12}(3, 4, 5, 6, 7, 8) = 5.5$

$R_{13}(4, 5, 6, 7, 8, 9) = 6.5$

$R_{21}(1, 2, 3, 4, 5, 6) = 3.5$

$R_{22}(2, 3, 4, 5, 6, 7) = 4.5$

$R_{23}(3, 4, 5, 6, 7, 8) = 5.5$

$R_{31}(4, 5, 6, 7, 8, 9) = 6.5$

$R_{32}(5, 6, 7, 8, 9, 10) = 7.5$

$R_{33}(6, 7, 8, 9, 10, 11) = 8.5$

$R_{41}(3, 4, 5, 6, 7, 8) = 5.5$

$R_{42}(4, 5, 6, 7, 8, 9) = 6.5$

$R_{43}(5, 6, 7, 8, 9, 10) = 7.5$

$R_{51}(2, 3, 4, 5, 6, 7) = 4.5$

$R_{52}(3, 4, 5, 6, 7, 8) = 5.5$

$R_{53}(4, 5, 6, 7, 8, 9) = 6.5$

JOBS	**Machine M_1**	**Machine M_2**	**Machine M_3**
A_1	4.5	5.5	6.5
A_2	3.5	4.5	5.5
A_3	6.5	7.5	8.5
A_4	5.5	6.5	7.5
A_5	4.5	5.5	6.5

Converting 3 machines into 2 machines,

JOBS	A_1	A_2	A_3	A_4	A_5
M_1	10	8	14	12	10
M_2	12	10	16	14	12

The order of jobs as follows:

A_2	A_1	A_5	A_4	A_3

Time calculation utilizing the job order is as follows:

Jobs	**Machine M_1**		**Machine M_2**		**Machine M_3**	
	In Time	**Out Time**	**In Time**	**Out Time**	**In Time**	**Out Time**
A_2	0	3.5	3.5	4.5	4.5	5.5
A_1	3.5	8	8	13.5	13.5	20
A_5	8	12.5	13.5	19	20	26.5
A_4	12.5	18	19	25.5	26.5	34
A_3	18	24.5	25.5	33	34	42.5

Total Elapsed time $= 42.5$

Idle Time for Machine $M_1 = 18$

Idle Time for Machine $M_2 = 16.5$

Idle Time for Machine $M_3 = 12.5$

Total Idle Time $= M_1 + M_2 + M_3$

$= 18 + 16.5 + 12.5$

Total Idle Time $= 47$

4.11 HEPTAGONAL FUZZY NUMBER

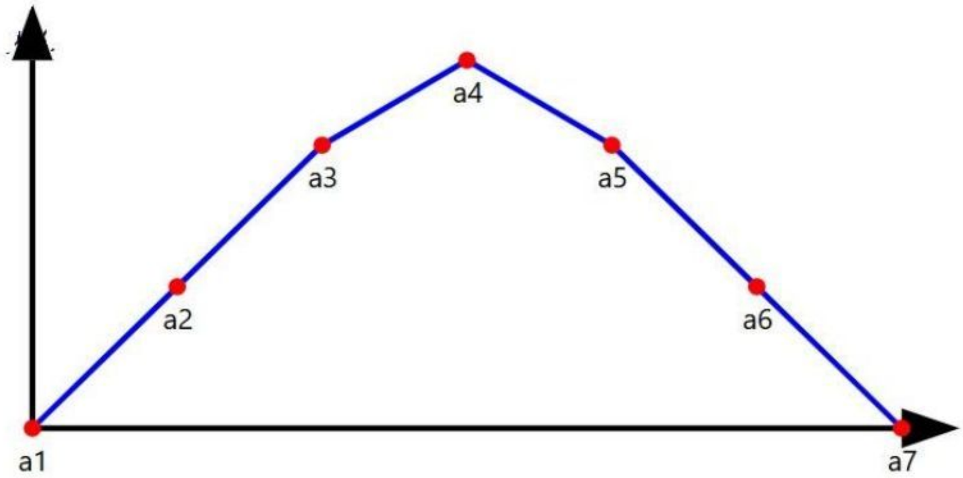
A Heptagonal Fuzzy Number is a type of fuzzy number that is defined by seven distinct points, providing more flexibility than the commonly used Triangular or Heptagonal fuzzy numbers. Fuzzy numbers are a way to represent uncertainty due to vagueness in the data, and they are especially useful in situations where values aren't crisp or precise. It is defined by seven parameters: $(a_1, a_2, a_3, a_4, a_{5,}a_6, a_7)$,

where,

- a_1 and a_7 represent the lower and upper bounds of the fuzzy number. These are points where the membership function is equal to zero.
- a_2 and a_6 represent the points where the membership function begins to increase from zero or decrease back to zero, respectively.
- a_3 and a_5 is the intermediate points that are closer to the peak, where the membership function is higher but not equal to 1.
- a_4 is the peak or modal value, where the membership function reaches its maximum value of 1.

The membership function $\mu_A(x)$ of a Heptagonal fuzzy number is given by

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_1}\right) & for\ a_1 \leq x \leq a_2 \\ 1 & for\ a_3 \leq x \leq a_4 \\ \left(\dfrac{a_5 - x}{a_5 - a_4}\right) & for\ a_4 \leq x \leq a_5 \\ 0 & for\ x > a_7 \end{cases}$$



4.12. NUMERICAL EXAMPLE FOR HEPTAGONAL FUZZY NUMBER

There are five tasks A_1, A_2, A_3, A_4 and A_5 which must go through three machines 1, 2, 3. The fuzzy processing times for all the tasks on two machines are given below.

Jobs	Machine M_1	Machine M_2	Machine M_3
A_1	(9, 13, 15, 18, 20, 22, 23)	(15, 18, 20, 22, 24, 27, 28)	(1, 2, 3, 8, 9, 11, 13)
A_2	(8, 9, 10, 13, 15, 17, 18)	(6, 9, 10, 12, 13, 14, 15)	(2, 3, 4, 8, 13, 14, 15)
A_3	(1, 4, 6, 8, 10, 14, 16)	(6, 8, 9, 10, 11, 12, 13)	(3, 4, 5, 7, 9, 10, 11)
A_4	(5, 7, 8, 9, 10, 11, 12)	(1, 2, 3, 4, 5, 6, 7)	(3, 4, 5, 6, 7, 8, 9)
A_5	(11, 14, 16, 18, 22, 24, 26)	(10, 12, 14, 15, 18, 20, 22)	(7, 10, 12, 14, 16, 18, 20)

Determine a sequence for the jobs

SOLUTION:

By using Weighted Average method

$$R_A(x) = \frac{(a_1 + 2a_2 + 3a_3 + 4a_4 + 3a_5 + 2a_6 + a_7)}{16}$$

We have converted fuzzy values into crisp values

$R_{11}(9, 13, 15, 18, 20, 22, 23) = 17.43$

$R_{12}(15, 18, 20, 22, 24, 27, 28) = 8.37$

$R_{13}(1, 2, 3, 8, 9, 11, 13) = 6.75$

$R_{21}(8, 9, 10, 13, 15, 17, 18) = 12.81$

$R_{22}(6, 9, 10, 12, 13, 14, 15) = 11.50$

$R_{23}(2, 3, 4, 8, 13, 14, 15) = 22.06$

$R_{31}(1, 4, 6, 8, 10, 14, 16) = 13.93$

$R_{32}(6, 8, 9, 10, 11, 12, 13) = 9.93$

$R_{33}(3, 4, 5, 7, 9, 10, 11) = 7.00$

$R_{41}(5, 7, 8, 9, 10, 11, 12) = 8.93$

$R_{42}(1, 2, 3, 4, 5, 6, 7) = 4.00$

$R_{43}(3, 4, 5, 6, 7, 8, 9) = 6.00$

$R_{51}(11, 14, 16, 18, 22, 24, 26) = 18.68$

$R_{52}(10, 12, 14, 15, 18, 20, 22) = 15.75$

$R_{53}(7, 10, 12, 14, 16, 18, 20) = 8.31$

JOBS	Machine M_1	Machine M_2	Machine M_3
A_1	17.43	8.37	6.75
A_2	12.81	11.50	22.06
A_3	13.93	9.93	7
A_4	8.93	4	6
A_5	18.68	15.75	8.31

Converting 3 machines into 2 machines,

JOBS	A_1	A_2	A_3	A_4	A_5
M_1	25.41	24.31	23.86	12.93	34.43
M_2	15.12	33.56	16.93	10	24.06

The order of jobs as follows:

A_2	A_5	A_3	A_1	A_4

Time calculation utilizing the job order is as follows:

Jobs	Machine M_1		Machine M_2		Machine M_3	
	In Time	Out Time	In Time	Out Time	In Time	Out Time
A_2	0	12.81	12.81	24.31	24.31	46.37
A_5	12.81	31.49	31.49	47.24	47.24	55.55
A_3	31.49	45.42	47.24	57.17	57.17	64.17
A_1	45.42	62.85	62.85	71.22	71.22	77.97
A_4	62.85	71.78	71.78	77.97	77.97	83.97

Total Elapsed time $= 83.97$

Idle Time for Machine $M_1 = 12.19$

Idle Time for Machine $M_2 = 31.67$

Idle Time for Machine $M_3 = 33.85$

Total Idle Time $= M_1 + M_2 + M_3$

$= 12.19 + 31.67 + 33.85$

Total Idle Time $= 77.71$

4.13 OCTAGONAL FUZZY NUMBER

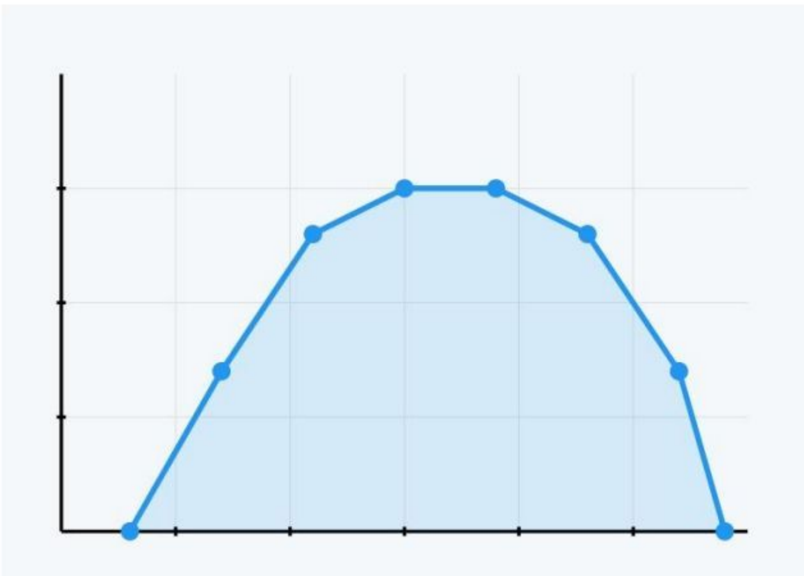
An Octagonal fuzzy number is a type of fuzzy number characterized by its membership function, which is typically represented in an Octagonal shape. This shape allows for more flexibility in modelling uncertainty compared to traditional Triangular or Trapezoidal fuzzy numbers. It is defined by eight parameters: $(a_1, a_2, a_3, a_4, a_{5,} a_6, a_7, a_8)$,

where,

- a_1 (lower bound) represents the smallest possible value, with zero membership
- a_2, a_3, a_4, a_6, a_7 are points where the membership function transitions between increasing and decreasing
- a_8 (upper bound) is the largest possible value, also with zero membership.

The membership function $\mu_A(x)$ of an Octagonal fuzzy number is given by

$$\mu_A(x) = \begin{cases} 0 & for\, x < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_1}\right) & for\, a_1 \leq x \leq a_2 \\ 1 & for\, a_2 \leq x \leq a_5 \\ \left(\dfrac{a_5 - x}{a_5 - a_4}\right) & for\, a_5 \leq x \leq a_8 \\ 0 & for\, x > a_8 \end{cases}$$



4.14 NUMERICAL EXAMPLE FOR OCTAGONAL FUZZY NUMBER

Sequencing problems containing 3 machines and 5 jobs, the processing time are Octagonal fuzzy numbers.

Jobs	Machine M_1	Machine M_2	Machine M_3
A_1	(-2, -1, 0, 1, 4, 5, 6, 7)	(-3, -2, -1, 0, 1, 2, 3, 4)	(7, 8, 10, 11, 14, 15, 17, 18)
A_2	(1, 2, 3, 4, 5, 6, 9, 10)	(-3, -2, -1, 0, 4, 5, 6, 7)	(6, 8, 9, 10, 12, 13, 14, 16)
A_3	(1, 2, 4, 5, 8, 10, 12, 14)	(-4, -3, 0, 1, 2, 3, 4, 5)	(-1, 0, 1, 2, 4, 5, 6, 7)
A_4	(2, 3, 6, 7, 8, 10, 11, 13)	(-2, -1, 0, 1, 2, 3, 4, 5)	(5, 6, 7, 8, 12, 13, 14 15)
A_5	(3, 5, 6, 8, 10, 12, 13,	(-1, 0, 1, 2, 3, 4, 5, 6)	(1, 2, 3, 4, 5, 6, 7, 8)

SOLUTION:

By using Weighted Average method

$$R_A(x) = \frac{(a_1 + 2a_2 + 3a_3 + 4a_4 + 4a_5 + 3a_6 + 2a_7 + a_8)}{12}$$

We have converted fuzzy values into crisp values

$R_{11}(-2, -1, 0, 1, 4, 5, 6, 7) = 4.16$

$\mathrm{R}_{12}(-3, -2, -1, 0, 1, 2, 3, 4) = 0.83$

$R_{13}(7, 8, 10, 11, 14, 15, 17, 18) = 20.83$

$R_{21}(1, 2, 3, 4, 5, 6, 9, 10) = 8$

$R_{22}(-3, -2, -1, 0, 4, 5, 6, 7) = 3.33$

$R_{23}(6, 8, 9, 10, 12, 13, 14, 16) = 18.33$

$R_{31}(1, 2, 4, 5, 8, 10, 12, 14) = 11.41$

$R_{32}(-4, -3, 0, 1, 2, 3, 4, 5) = 2$

$R_{33}(-1, 0, 1, 2, 4, 5, 6, 7) = 2$

$R_{41}(2, 3, 6, 7, 8, 10, 11, 13) = 12.58$

$R_{42}(-2, -1, 0, 1, 2, 3, 4, 5) = 2.5$

$R_{43}(5, 6, 7, 8, 12, 13, 14\ 15) = 16.66$

$R_{51}(3, 5, 6, 8, 10, 12, 13, 15) = 15$

$R_{52}(-1, 0, 1, 2, 3, 4, 5, 6) = 4.16$

$R_{53}(1, 2, 3, 4, 5, 6, 7, 8) = 7.50$

Jobs	**Machine M_1**	**Machine M_2**	**Machine M_3**
A_1	4.16	0.83	20.83
A_2	8	3.33	18.33
A_3	11.41	2	5
A_4	12.58	2.50	16.66
A_5	15	4.16	7.50

Converting 3 machines into 2 machines,

JOBS	A_1	A_2	A_3	A_4	A_5
$\boldsymbol{M_1}$	4.99	11.33	13.41	15.08	19.16
$\boldsymbol{M_2}$	12.66	21.66	7	19.16	11.66

The order of jobs as follows:

A_1	A_2	A_4	A_5	A_3

Time calculation utilizing the job order is as follows:

Jobs	**Machine M_1**		**Machine M_2**		**Machine M_3**	
	In Time	Out Time	In Time	Out Time	In Time	Out Time
A_1	0	4.16	4.6	4.99	4.99	25.82
A_2	4.16	12.16	12.10	15.49	25.82	44.15
A_4	12.16	24.74	24.74	27.24	44.15	60.81
A_5	24.74	39.74	39.74	43.90	60.81	68.31

A_3	39.74	51.15	51.15	53.15	68.31	73.31

Total Elapsed time $= 73.31$

Idle Time for Machine $M_1 = 22.16$

Idle Time for Machine $M_2 = 60.87$

Idle Time for Machine $M_3 = 4.99$

Total Idle Time $= M_1 + M_2 + M_3$

$= 22.16 + 60.87 + 4.99$

Total Idle Time $= 88.02$

4.15 NANOGONAL FUZZY NUMBER

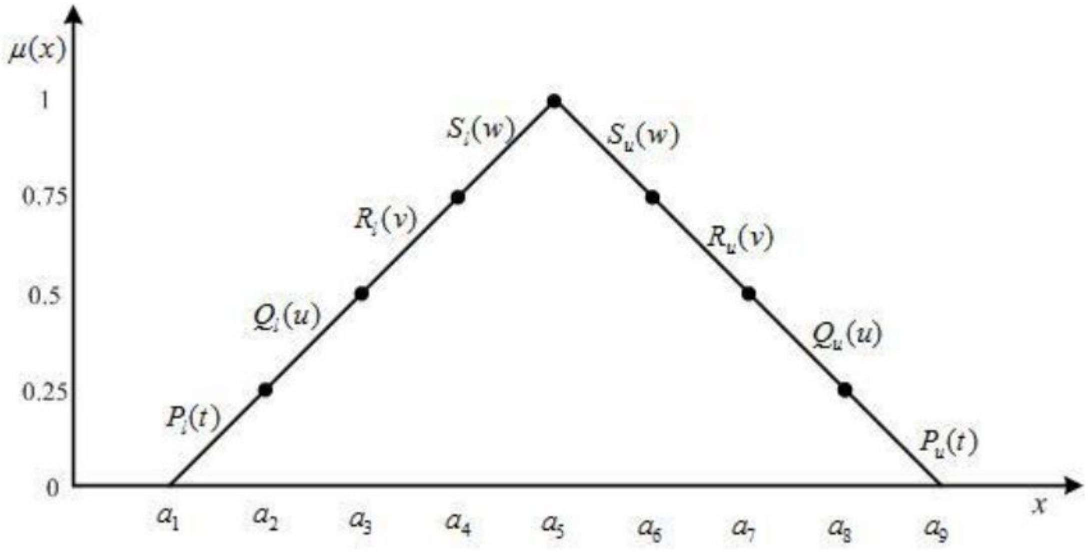
A Nanogonal fuzzy number refers to a type of fuzzy number whose membership function is typically defined by nine points or parameters. Fuzzy numbers are extensions of real numbers used in fuzzy set theory to represent quantities that are not precisely defined but can range over a spectrum of possible values. In the case of a Nanogonal fuzzy number, the membership function might be characterized by a shape with nine defining points, such as a piecewise linear function that increases and decreases across certain intervals. It is defined by nine parameters: $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$,

where,

- a_1 (lower bound) represents the smallest possible value, with zero membership
- $a_2, a_3, a_4, a_6, a_7, a_8$ are points where the membership function transitions between increasing and decreasing
- a_5 (central point) is the value with the highest membership degree, usually 1
- a_9 (upper bound) is the largest possible value, also with zero membership.

The membership function $\mu_A(x)$ of a Nanogonal fuzzy number is given by

$$\mu_A(x) = \begin{cases} 0 & for\, x < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_1}\right) & for\, a_1 \le x \le a_2 \\ 1 & for\;\; a_5 \le x \le a_6 \\ \left(\dfrac{a_5 - x}{a_5 - a_4}\right) & for\, a_4 \le x \le a_5 \\ 0 & for\, x < a_9 \end{cases}$$



4.16 NUMERICAL EXAMPLE FOR NANOGONAL FUZZY NUMBER

A garment manufacturing company produces 5 different clothing items ($A_1$to A_5) that must go through three sequential processes: Cutting (M_1), Sewing (M_2), and Pressing (M_3). The processing times for each item at each stage are represented by Nanogonal fuzzy numbers, as shown in the table. Find the optimal sequence and total elapsed time.

Items	**Cutting M_1**	**Sewing M_2**	**Pressing M_3**
A_1	(-2, -1, 0, 1, 4, 5, 6, 7, 8)	(-3, -2, -1, 0, 1, 2, 3, 4, 5)	(7, 8, 10, 11, 14, 15, 17, 18, 19)
A_2	(1, 2, 3, 4, 5, 6, 7, 9, 10, 11)	(-3, -2, -1, 0, 4, 5, 6, 7, 8)	(6, 8, 9, 10, 12, 13, 14, 16, 18)
A_3	(1, 2, 4, 5, 8, 10, 12, 14, 16)	(-4, -3 ,0, 1, 2, 3, 4, 5, 6)	(-1, 0 ,1 ,2, 4, 5 ,6 ,7 ,8)
A_4	(2, 3, 6, 7, 8, 10, 11, 13, 15)	(-2, -1, 0, 1, 2, 3, 4, 5, 6)	(5, 6, 7, 8, 12, 13, 14, 15, 16)
A_5	(3, 5, 6, 8, 10, 12, 13, 15, 17)	(-1, 0, 1, 2, 3, 4, 5, 6, 7)	(1, 2, 3, 4, 5, 6, 7, 8, 9)

SOLUTION:

By using Weighted Average method,

$$R_A(x) = \frac{(a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 4a_6 + 3a_7 + 2a_8 + a_9)}{25}$$

We have converted fuzzy values into crisp values

$R_{11}(-2, -1, 0, 1, 4, 5, 6, 7, 8) = 3.2$

$R_{12}(-3, -2, -1, 0, 1, 2, 3, 4, 5) = 1$

$R_{13}(7, 8, 10, 11, 14, 15, 17, 18, 19) = 13.32$

$R_{21}(1, 2, 3, 4, 5, 6, 7, 9, 10, 11) = 5.48$

$R_{22}(-3, -2, -1, 0, 4, 5, 6, 7, 8) = 2.8$

$R_{23}(6, 8, 9, 10, 12, 13, 14, 16, 18) = 11.72$

$R_{31}(1, 2, 4, 5, 8, 10, 12, 14, 16) = 7.88$

$R_{32}(-4, -3, 0, 1, 2, 3, 4, 5, 6) = 1.76$

$R_{33}(-1, 0, 1, 2, 4, 5, 6, 7, 8) = 3.6$

$R_{41}(2, 3, 6, 7, 8, 10, 11, 13, 15) = 8.32$

$R_{42}(-2, -1, 0, 1, 2, 3, 4, 5, 6) = 2$

$R_{43}(5, 6, 7, 8, 12, 13, 14, 15, 16) = 10.8$

$R_{51}(3, 5, 6, 8, 10, 12, 13, 15, 17) = 9.88$

$R_{52}(-1, 0, 1, 2, 3, 4, 5, 6, 7) = 3$

$R_{53}(1, 2, 3, 4, 5, 6, 7, 8, 9) = 5.08$

Items	**Cutting M_1**	**Sewing M_2**	**Pressing M_3**
A_1	3.2	1	14.08
A_2	5.48	2.8	11.72
A_3	7.88	1.76	3.6
A_4	8.32	2	10.8
A_5	9.88	3	5.08

Converting 3 machines into 2 machines,

Items	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$	$\boldsymbol{A_4}$	$\boldsymbol{A_5}$
M_1	4.2	8.28	9.64	10.35	12.88
M_2	15.08	14.52	5.36	12.8	8.08

The order of jobs as follows:

A_1	A_2	A_4	A_5	A_3

Time calculation utilizing the job order is as follows:

Jobs	**Cutting $\boldsymbol{M_1}$**		**Sewing $\boldsymbol{M_2}$**		**Pressing $\boldsymbol{M_3}$**	
	In Time	Out Time	In Time	Out Time	In Time	Out Time
A_1	0	3.2	3.2	4.2	4.2	18.28
A_2	3.2	8.68	8.68	11.48	18.28	30
A_4	8.68	17	17	19	30	40.8
A_5	17	26.88	26.88	29.77	40.8	45.88
A_3	34.76	42.64	34.76	36.52	45.88	49.48

Total Elapsed time $= 49.48$

Idle Time for Machine $M_1 = 6.84$

Idle Time for Machine $M_2 = 39.03$

Idle Time for Machine $M_3 = 4.2$

Total Idle Time $= M_1 + M_2 + M_3$

$= 6.84 + 39.03 + 4.2$

Total Idle Time $= 50.07$

4.17 CONCLUSION

The Weighted Average approach has been introduced. The Weighted Average method serves as an effective tool for working with Nanogonal fuzzy numbers in the context of sequencing problems. It provides a means to convert nanogonal fuzzy numbers into crisp values, facilitating decision-making and comparison of different scheduling options and the ranking of Nanogonal fuzzy numbers, which is crucial for determining the optimal sequence of jobs or tasks. We applied the Weighted Average method to solve fuzzy sequencing problems with different fuzzy numbers and we have used MATLAB Tool to provide the solutions. Additionally, numerical examples involving different fuzzy numbers were studied through, and the solutions were found. As scheduling problems in various industries continue to grow in complexity and face increasing uncertainties due to vagueness, this approach provides a robust framework for developing more effective and adaptable scheduling solutions. Hence, Decision-makers can better account for complicated uncertainties due to vagueness in processing times, due dates and other scheduling characteristics, making more informed decisions based on a more accurate representation.

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