

# Today

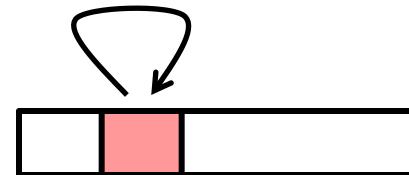
- Cache memory organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# Recall: Locality

- **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently

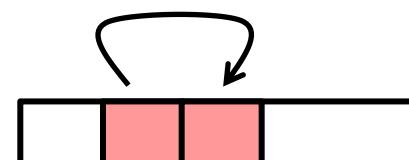
- **Temporal locality:**

- Recently referenced items are likely to be referenced again in the near future



- **Spatial locality:**

- Items with nearby addresses tend to be referenced close together in time



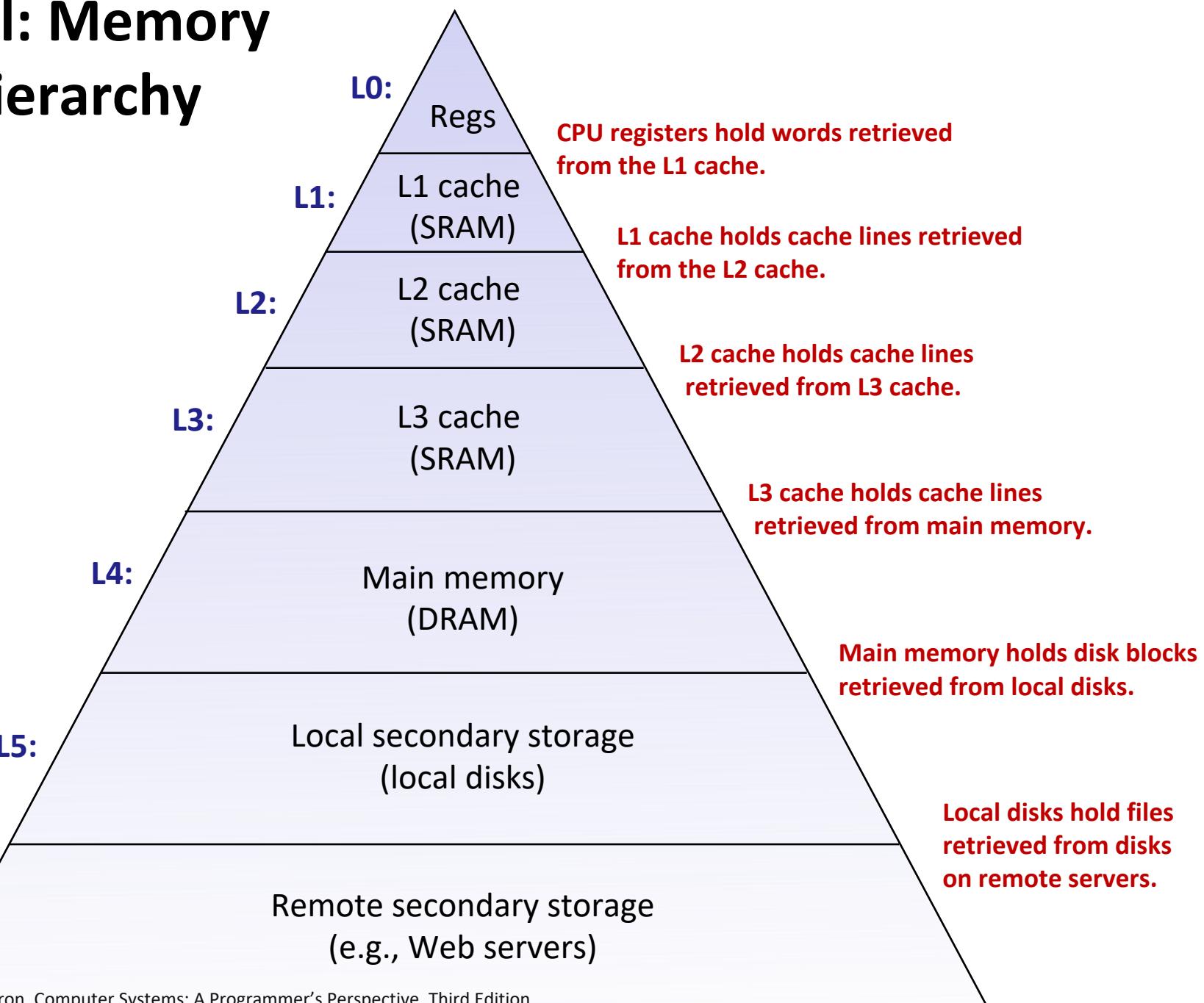
# Recall: Memory Hierarchy

Smaller,  
faster,  
and  
costlier  
(per byte)  
storage  
devices

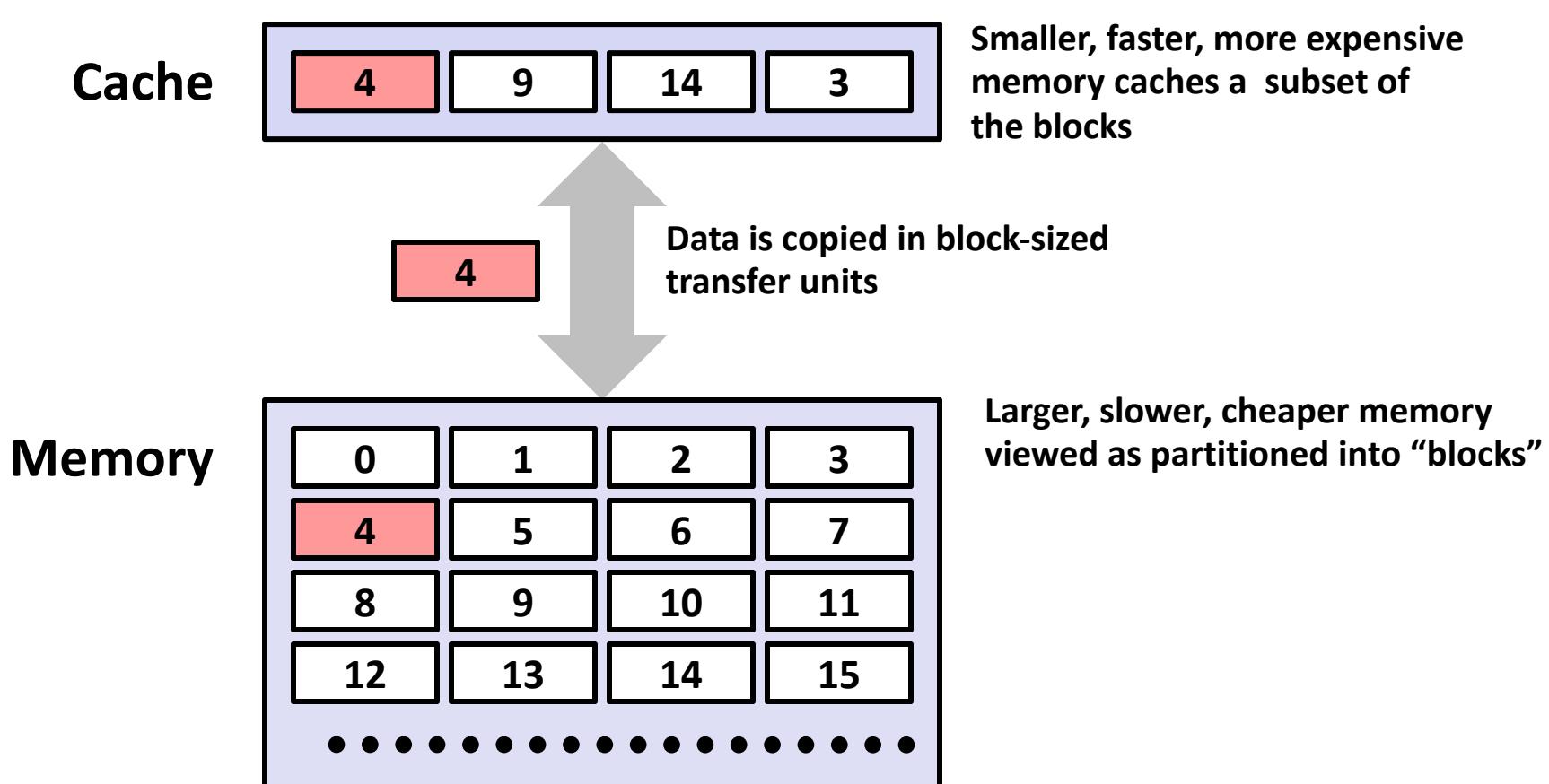
Larger,  
slower,  
and  
cheaper  
(per byte)  
storage  
devices

L6:

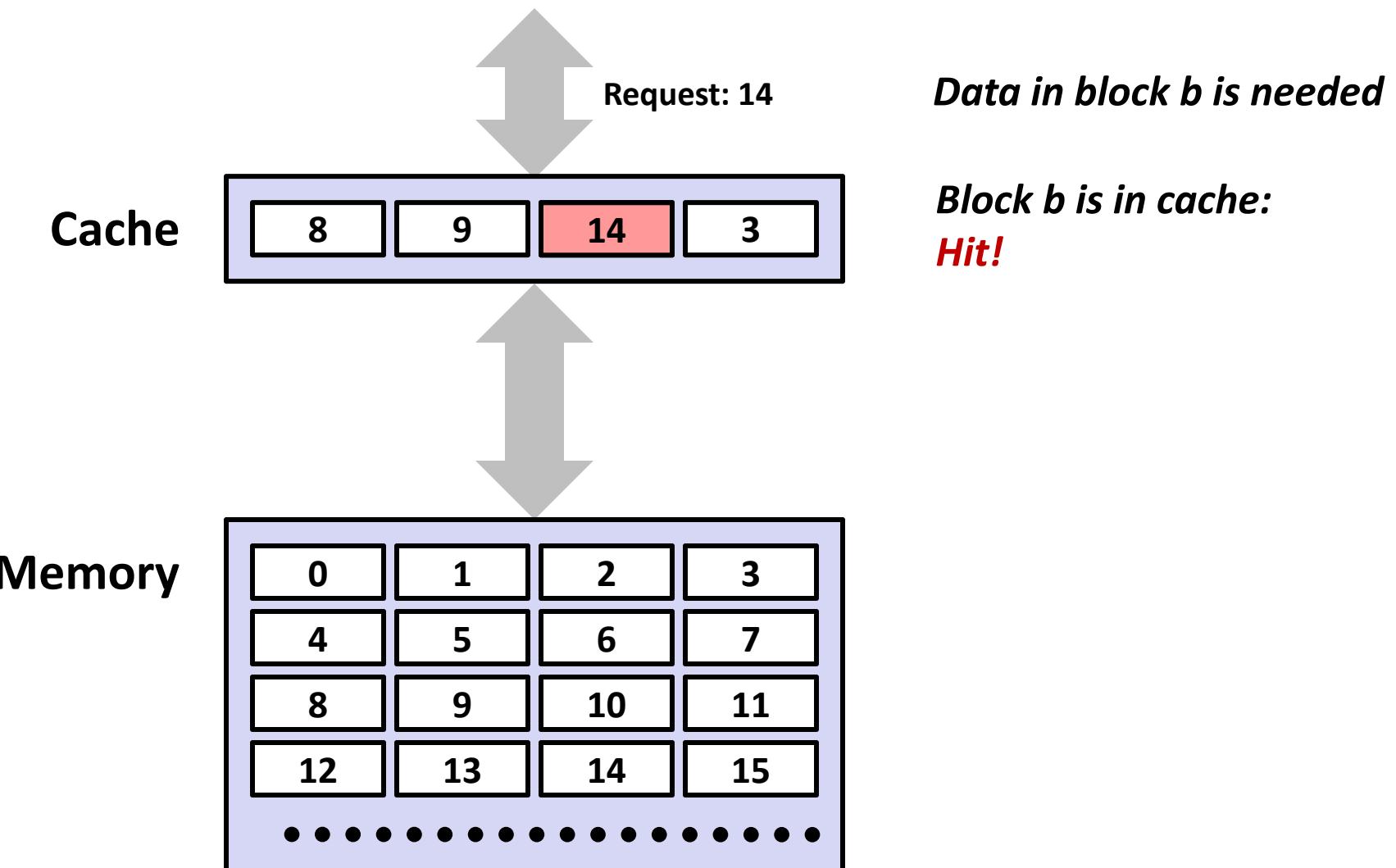
Remote secondary storage  
(e.g., Web servers)



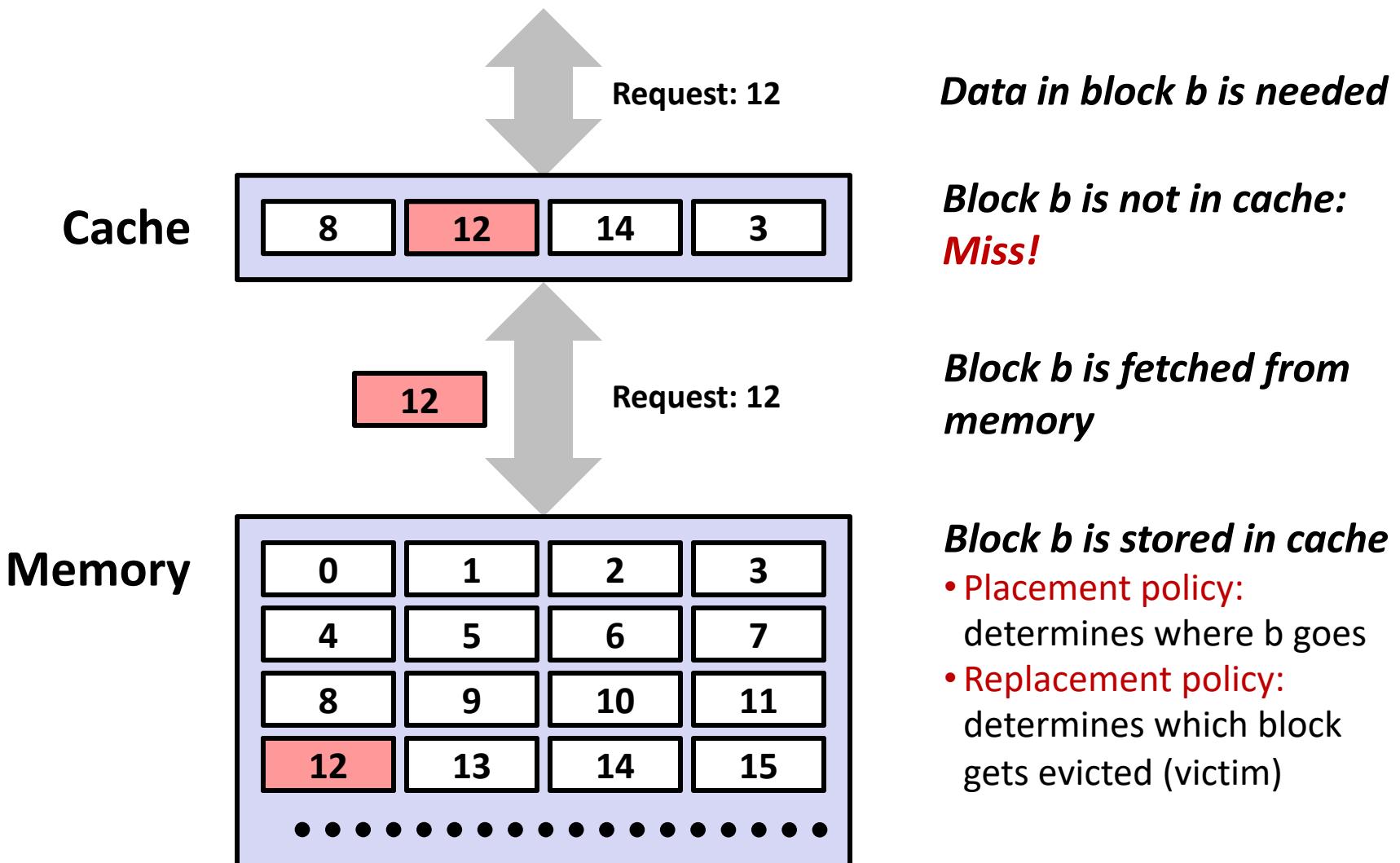
# Recall: General Cache Concepts



# General Cache Concepts: Hit



# General Cache Concepts: Miss



# Recall: General Caching Concepts:

## 3 Types of Cache Misses

### ■ Cold (compulsory) miss

- Cold misses occur because the cache starts empty and this is the first reference to the block.

### ■ Capacity miss

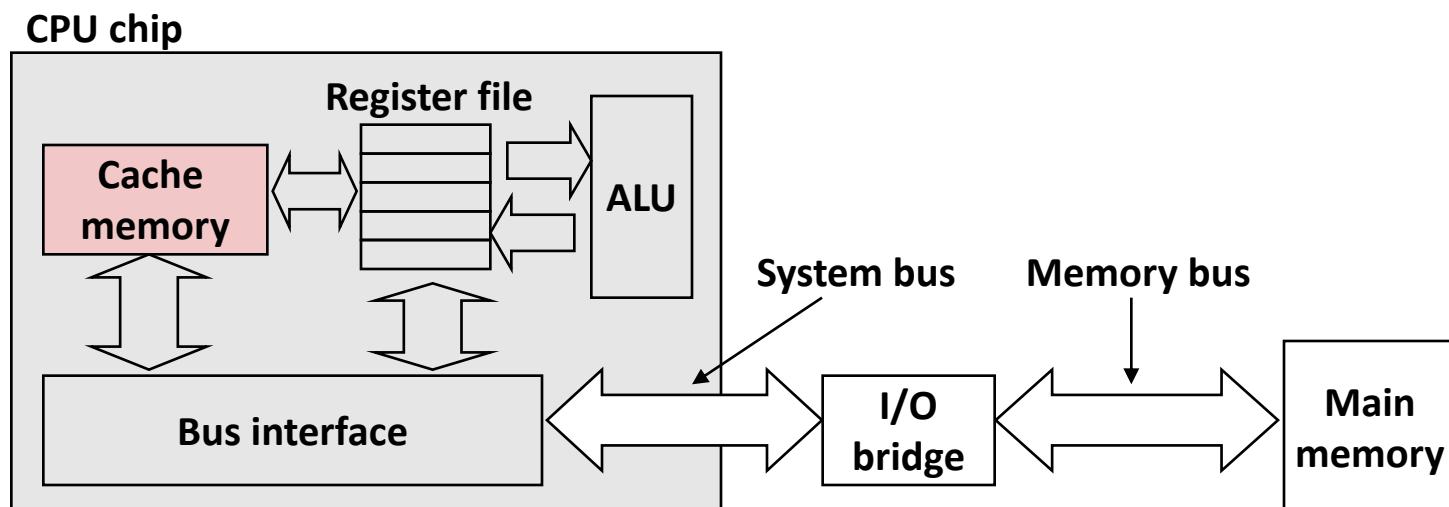
- Occurs when the set of active cache blocks (**working set**) is larger than the cache.

### ■ Conflict miss

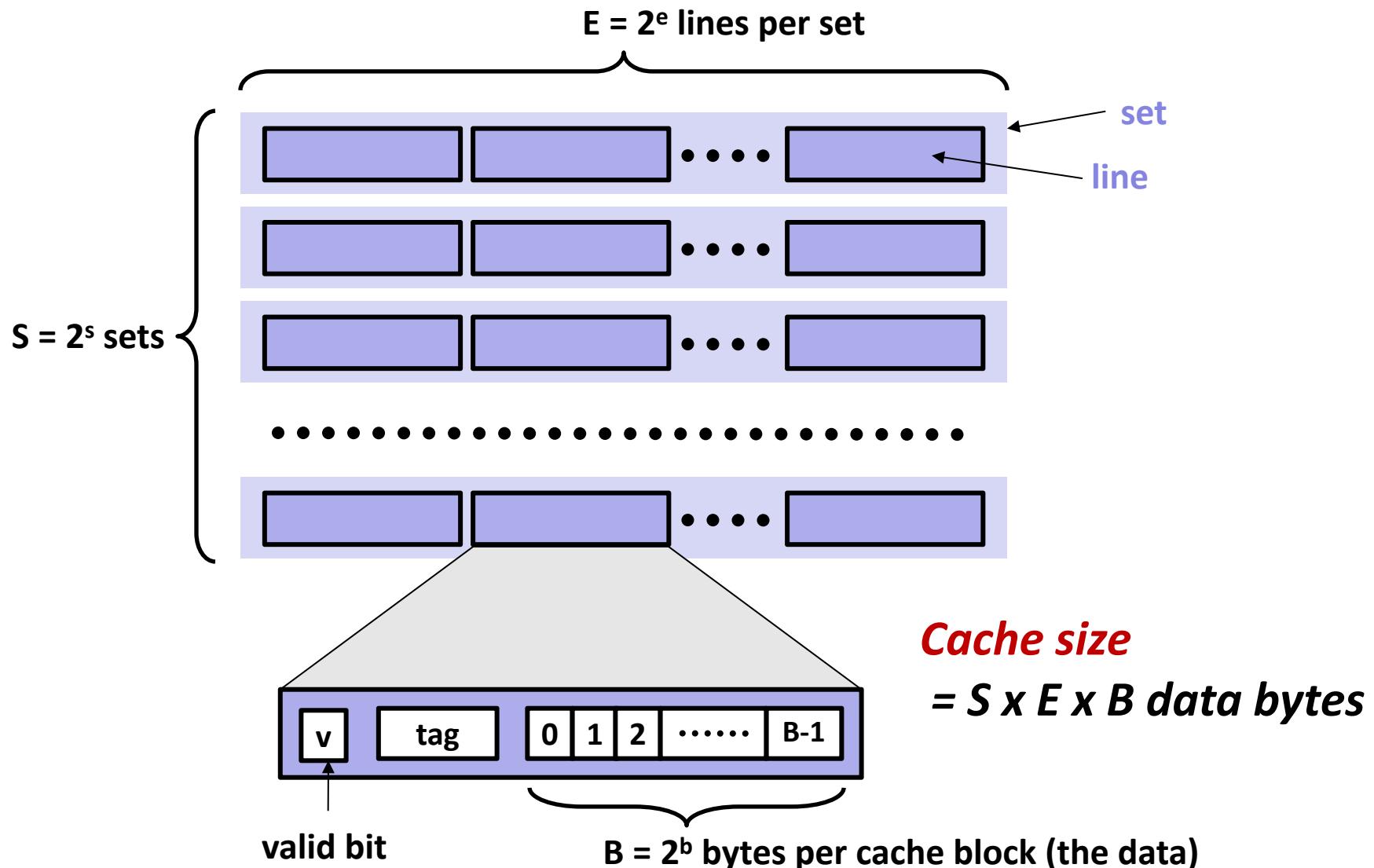
- Most caches limit blocks at level  $k+1$  to a small subset (sometimes a singleton) of the block positions at level  $k$ .
  - E.g. Block  $i$  at level  $k+1$  must be placed in block  $(i \bmod 4)$  at level  $k$ .
- Conflict misses occur when the level  $k$  cache is large enough, but multiple data objects all map to the same level  $k$  block.
  - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.

# Cache Memories

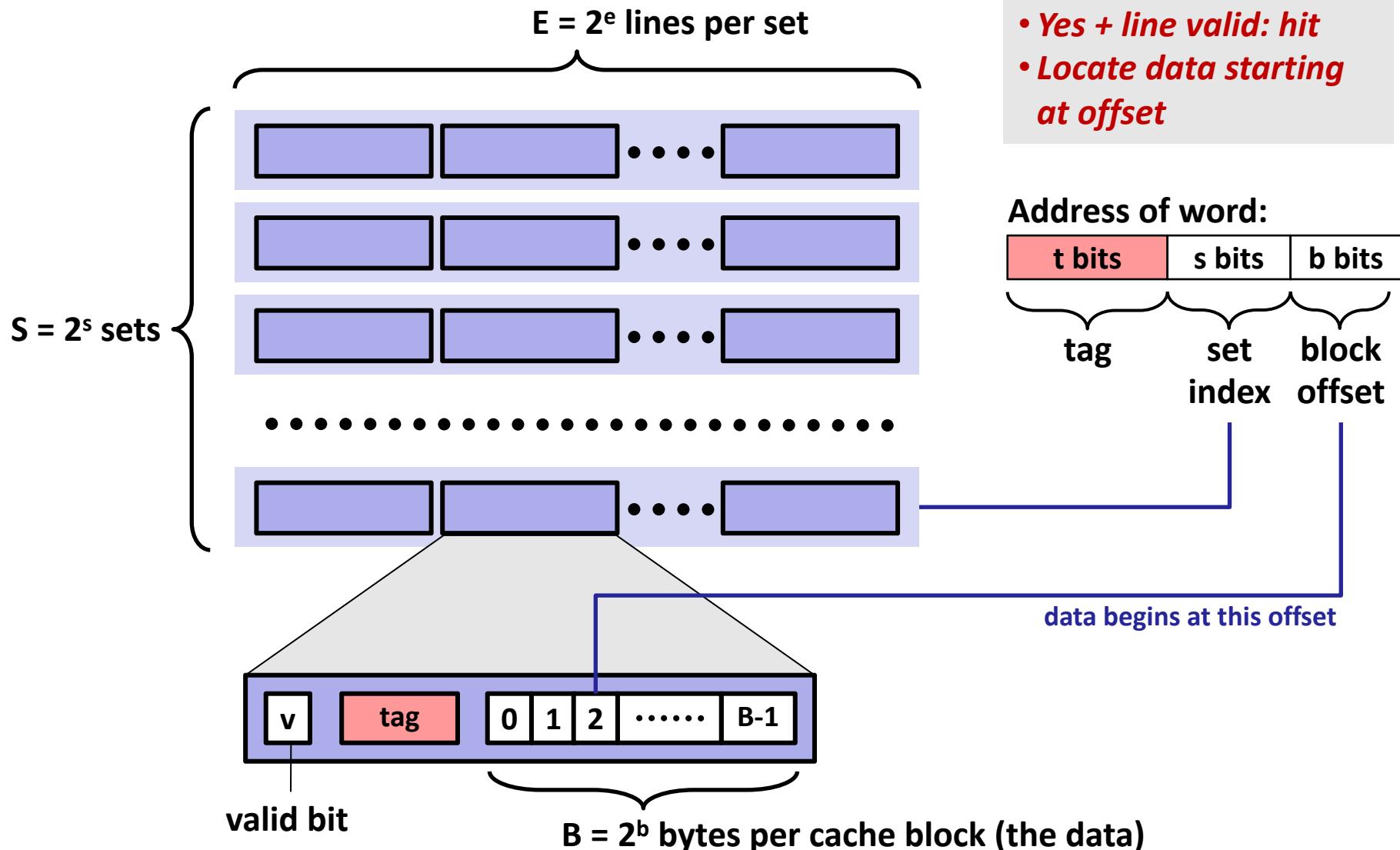
- Cache memories are small, fast SRAM-based memories managed automatically in hardware
  - Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



# General Cache Organization (S, E, B)



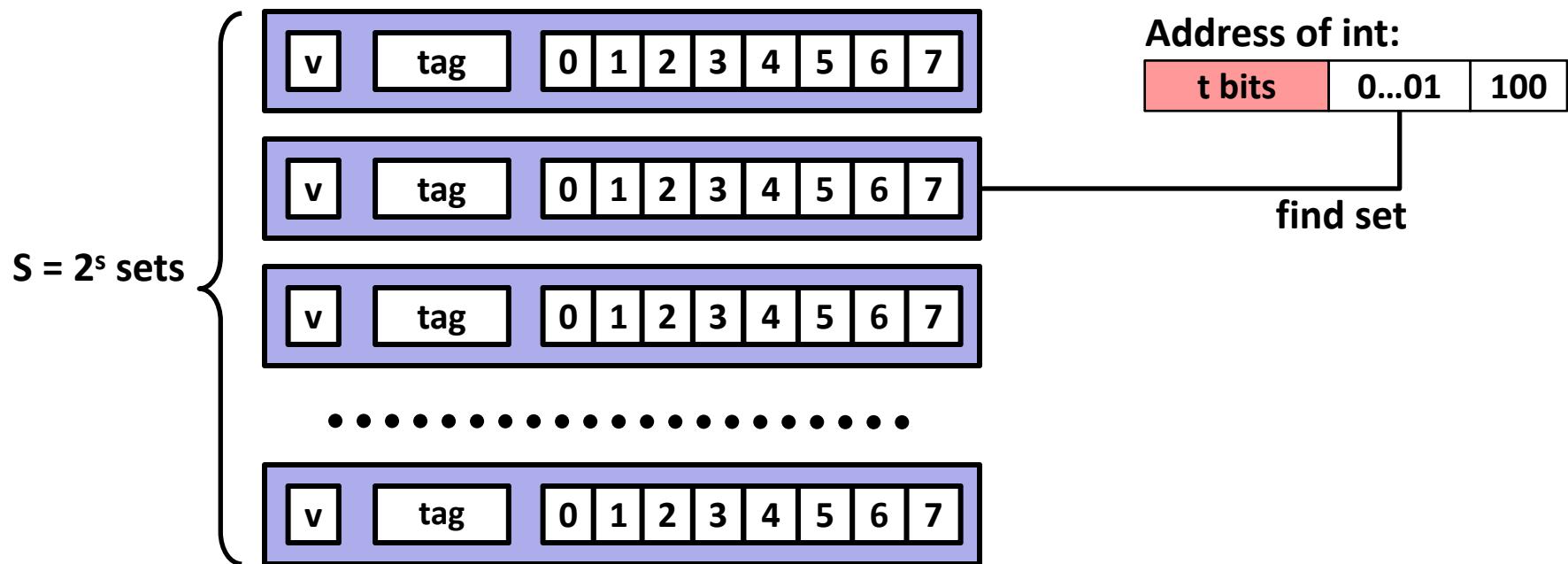
# Cache Read



# Example: Direct Mapped Cache ( $E = 1$ )

Direct mapped: One line per set

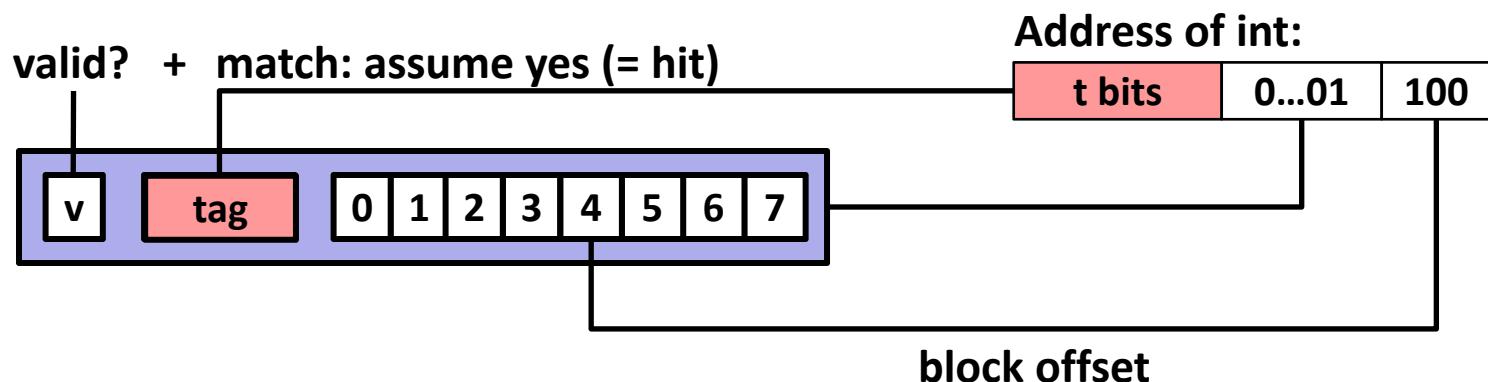
Assume: cache block size  $B=8$  bytes



# Example: Direct Mapped Cache ( $E = 1$ )

Direct mapped: One line per set

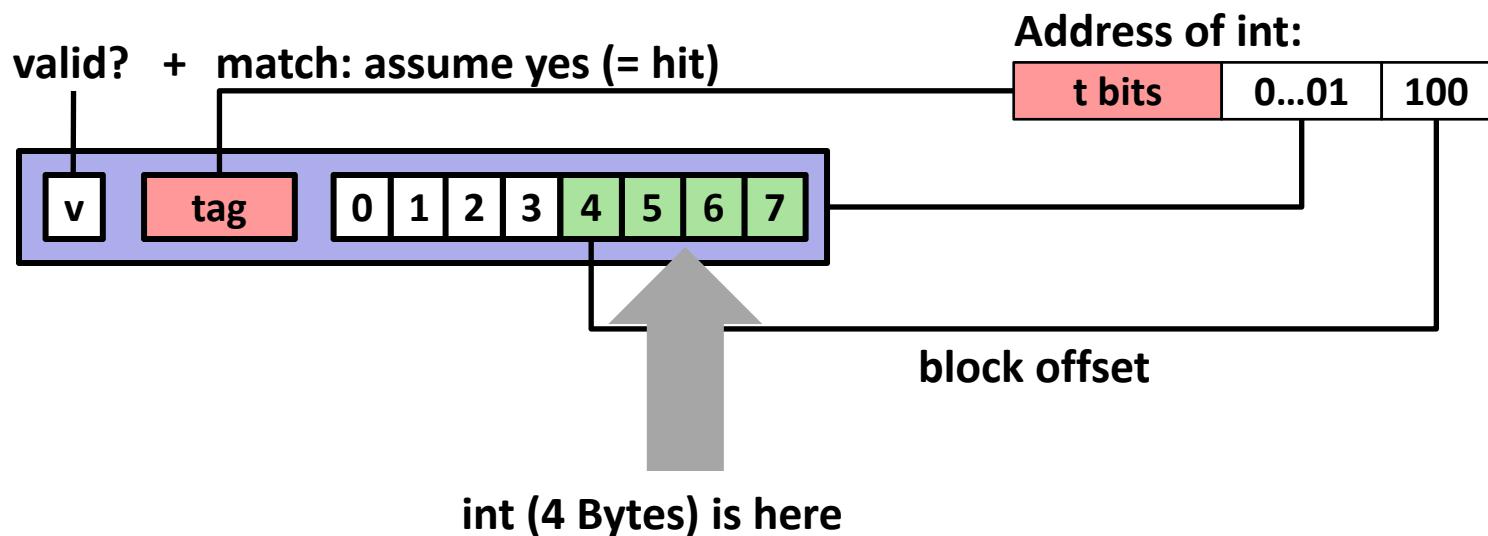
Assume: cache block size  $B=8$  bytes



# Example: Direct Mapped Cache ( $E = 1$ )

Direct mapped: One line per set

Assume: cache block size  $B=8$  bytes



If tag doesn't match (= miss): old line is evicted and replaced

# Direct-Mapped Cache Simulation

t=1    s=2    b=1

X	XX	X
---	----	---

4-bit addresses (address space size M=16 bytes)  
 S=4 sets, E=1 Blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

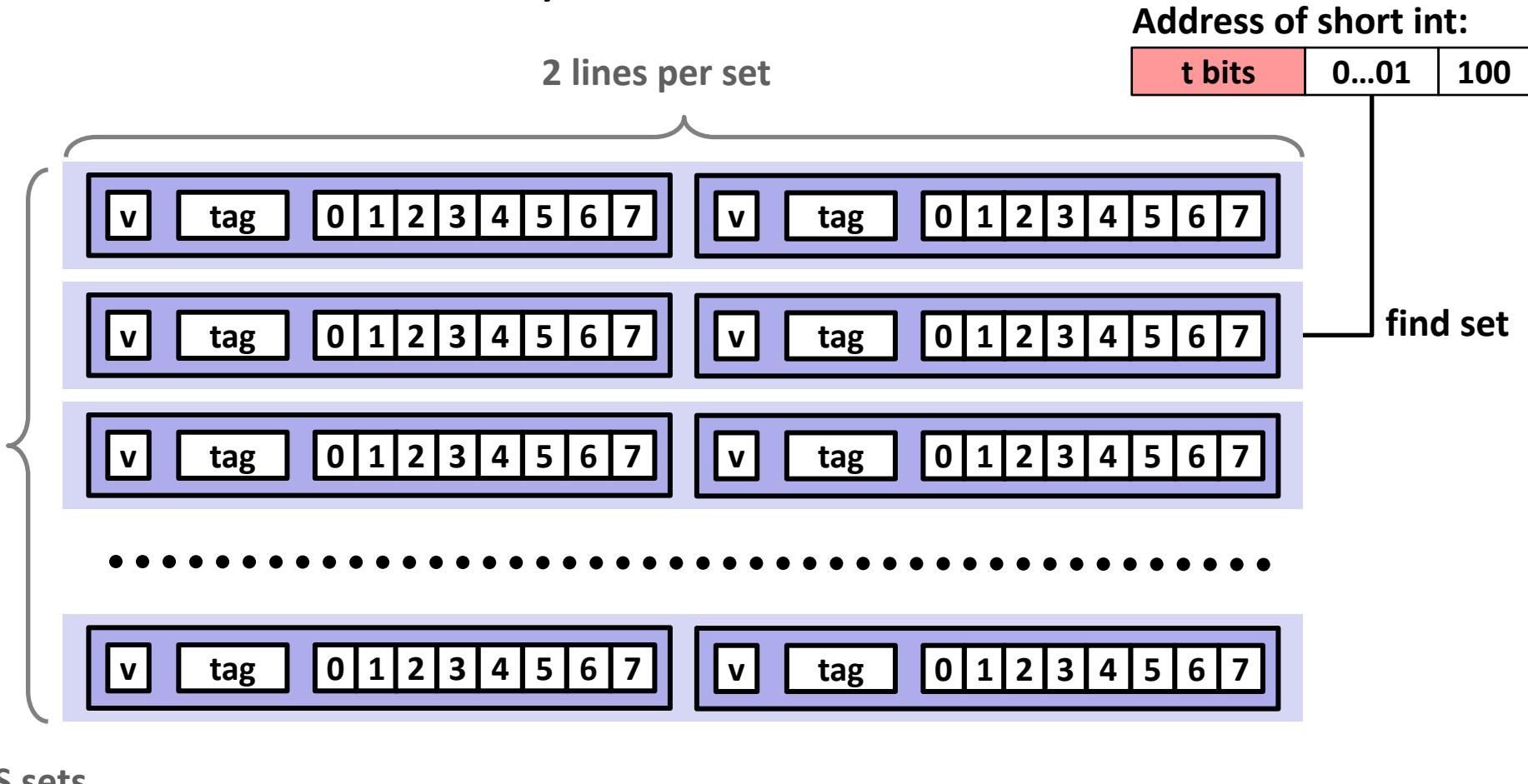
0	$[0\underline{00}0_2]$	miss
1	$[0\underline{00}1_2]$	hit
7	$[0\underline{11}1_2]$	miss
8	$[1\underline{00}0_2]$	miss
0	$[0\underline{00}0_2]$	miss

	v	Tag	Block
<b>Set 0</b>	1	0	M[0-1]
<b>Set 1</b>	0		
<b>Set 2</b>	0		
<b>Set 3</b>	1	0	M[6-7]

# E-way Set Associative Cache (Here: E = 2)

$E = 2$ : Two lines per set

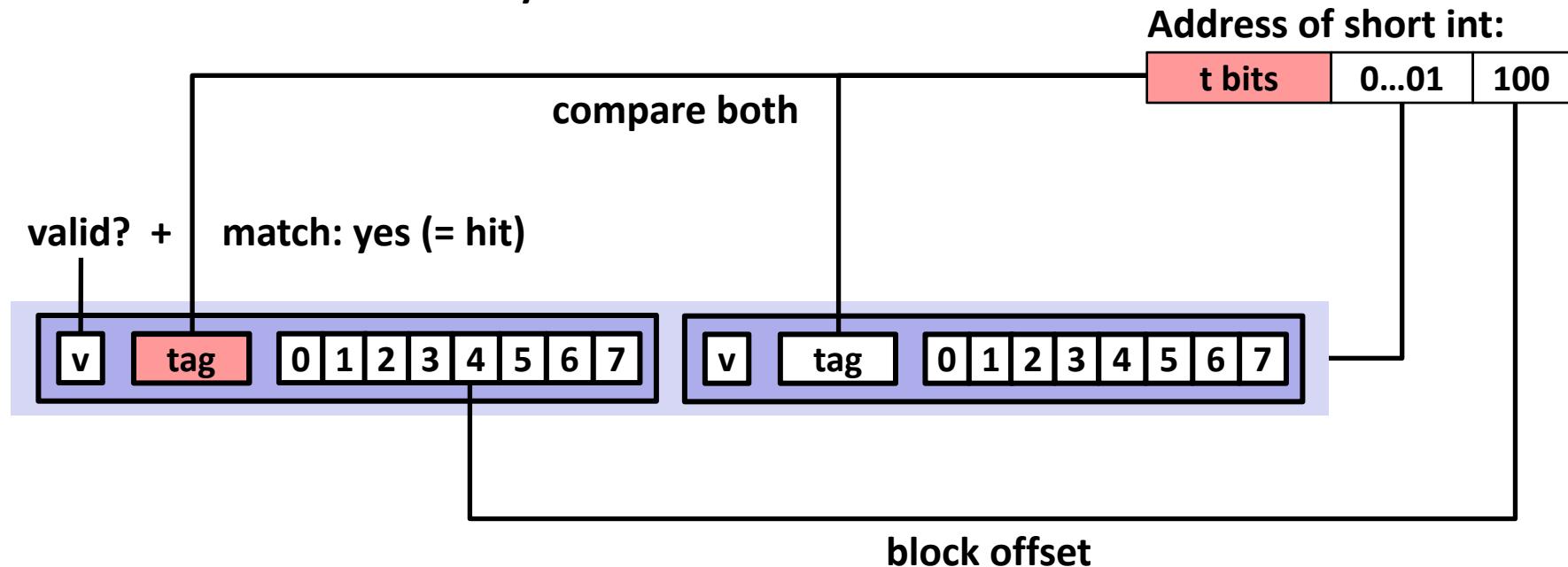
Assume: cache block size  $B=8$  bytes



# E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

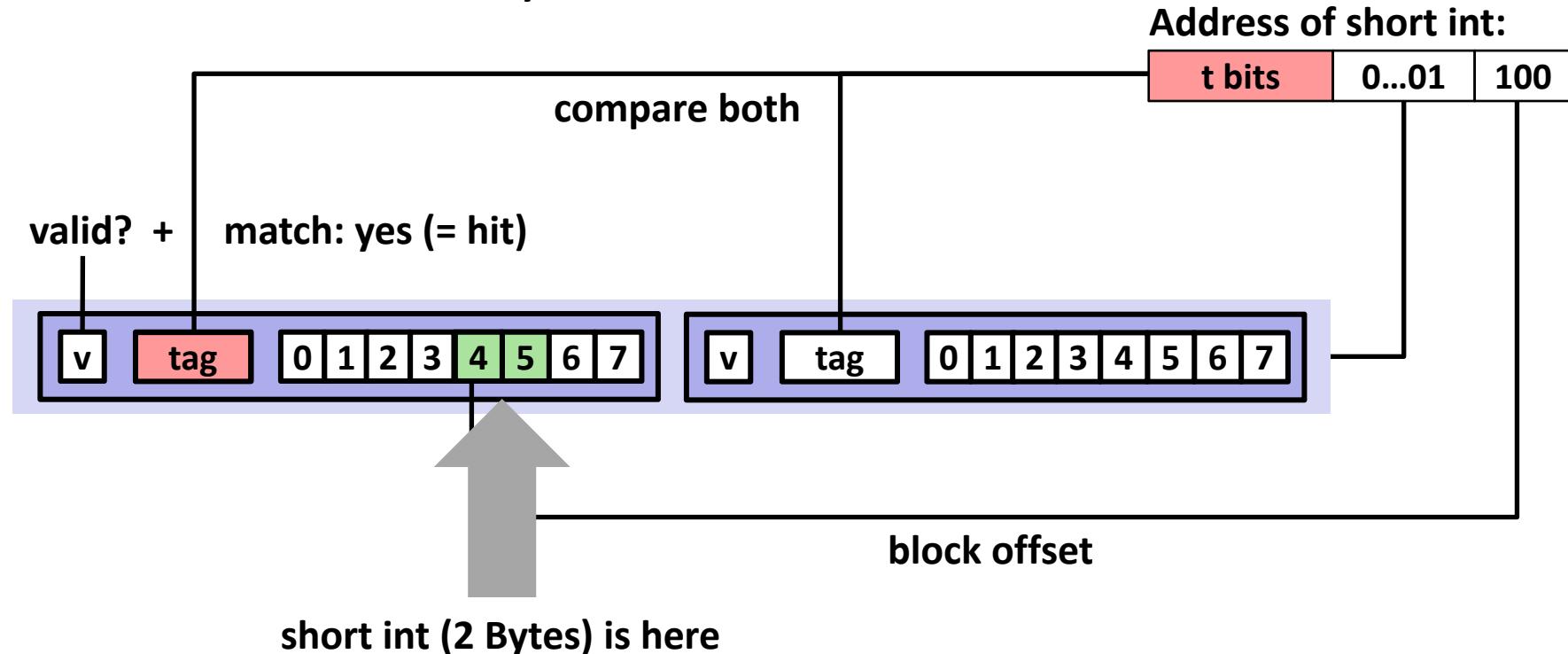
Assume: cache block size B=8 bytes



# E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes



## No match or not valid (= miss):

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

# 2-Way Set Associative Cache Simulation

$t=2$     $s=1$     $b=1$



4-bit addresses ( $M=16$  bytes)

$S=2$  sets,  $E=2$  blocks/set,  $B=2$  bytes/block

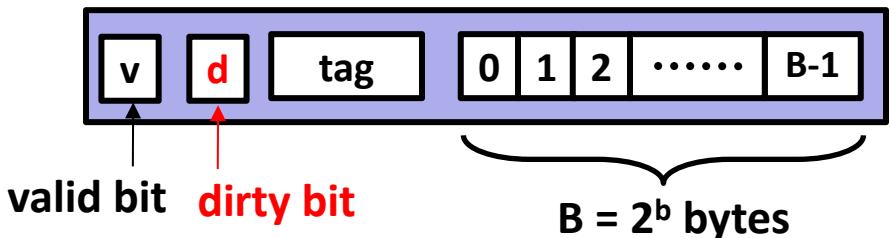
Address trace (reads, one byte per read):

0	$[0000_2]$	miss
1	$[0001_2]$	hit
7	$[0111_2]$	miss
8	$[1000_2]$	miss
0	$[0000_2]$	hit

	v	Tag	Block
Set 0	1	00	$M[0-1]$
	1	10	$M[8-9]$
Set 1	1	01	$M[6-7]$
	0		

# What about writes?

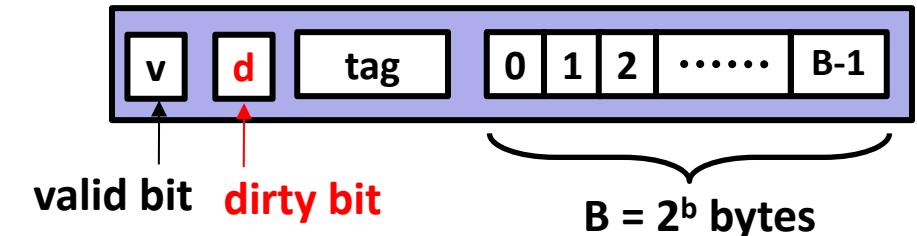
- **Multiple copies of data exist:**
  - L1, L2, L3, Main Memory, Disk
- **What to do on a write-hit?**
  - **Write-through** (write immediately to memory)
  - **Write-back** (defer write to memory until replacement of line)
    - Each cache line needs a dirty bit (set if data has been written to)
- **What to do on a write-miss?**
  - **Write-allocate** (load into cache, update line in cache)
    - Good if more writes to the location will follow
  - **No-write-allocate** (writes straight to memory, does not load into cache)
- **Typical**
  - Write-through + No-write-allocate
  - **Write-back + Write-allocate**



# Practical Write-back Write-allocate

- A write to address X is issued

- If it is a hit
  - Update the contents of block
  - Set dirty bit to 1 (bit is sticky and only cleared on eviction)



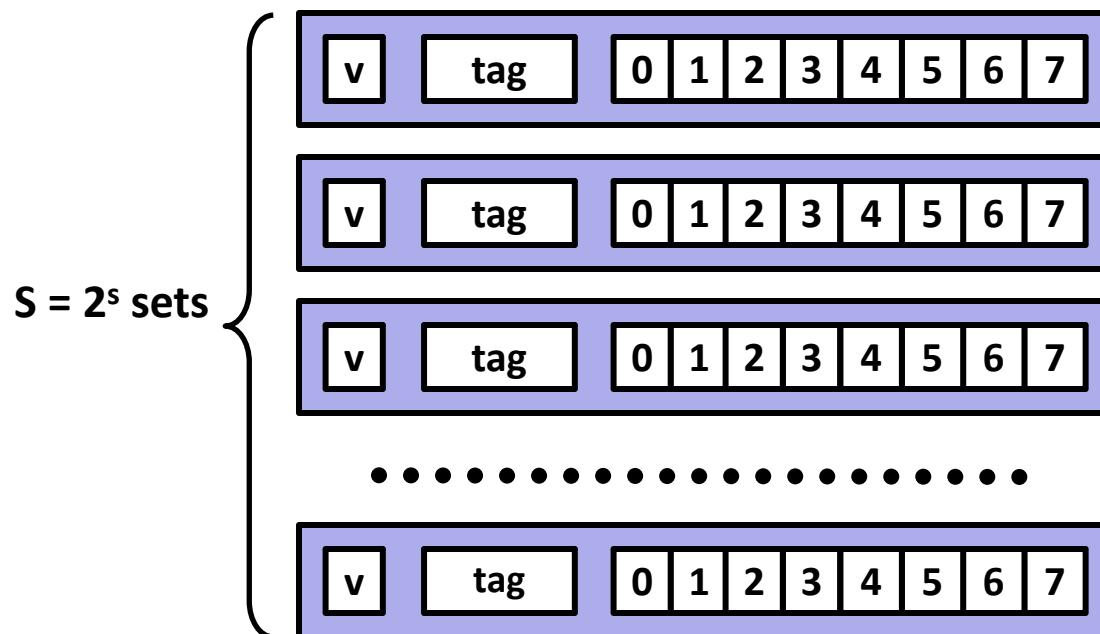
- If it is a miss
  - Fetch block from memory (per a read miss)
  - Then perform the write operations (per a write hit)

- If a line is evicted and dirty bit is set to 1
  - The entire block of  $2^b$  bytes are written back to memory
  - Dirty bit is cleared (set to 0)
  - Line is replaced by new contents

# Why Index Using Middle Bits?

Direct mapped: One line per set

Assume: cache block size 8 bytes



## Standard Method: Middle bit indexing

Address of int:



find set

## Alternative Method: High bit indexing

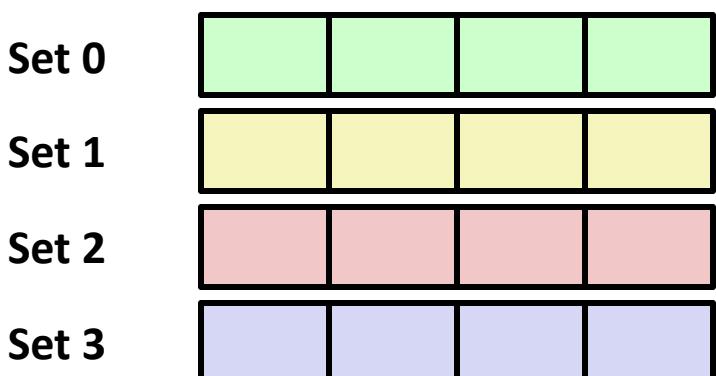
Address of int:



find set

# Illustration of Indexing Approaches

- 64-byte memory
  - 6-bit addresses
- 16 byte, direct-mapped cache
- Block size = 4. (Thus, 4 sets; why?)
- 2 bits tag, 2 bits index, 2 bits offset



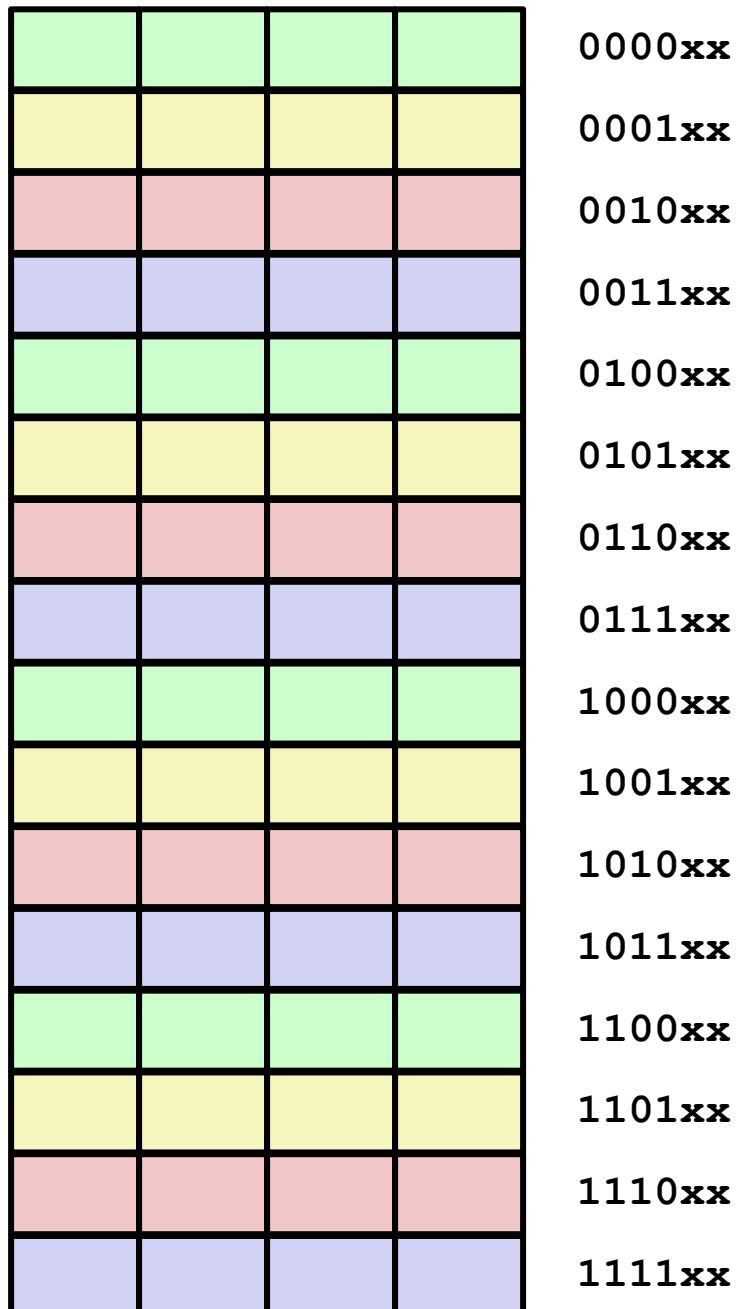
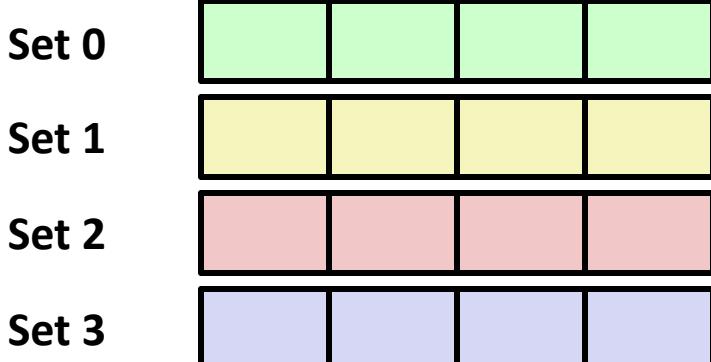
				0000xx
				0001xx
				0010xx
				0011xx
				0100xx
				0101xx
				0110xx
				0111xx
				1000xx
				1001xx
				1010xx
				1011xx
				1100xx
				1101xx
				1110xx
				1111xx

# Middle Bit Indexing

- Addresses of form **TTSSBB**

- **TT** Tag bits
- **SS** Set index bits
- **BB** Offset bits

- Makes good use of spatial locality

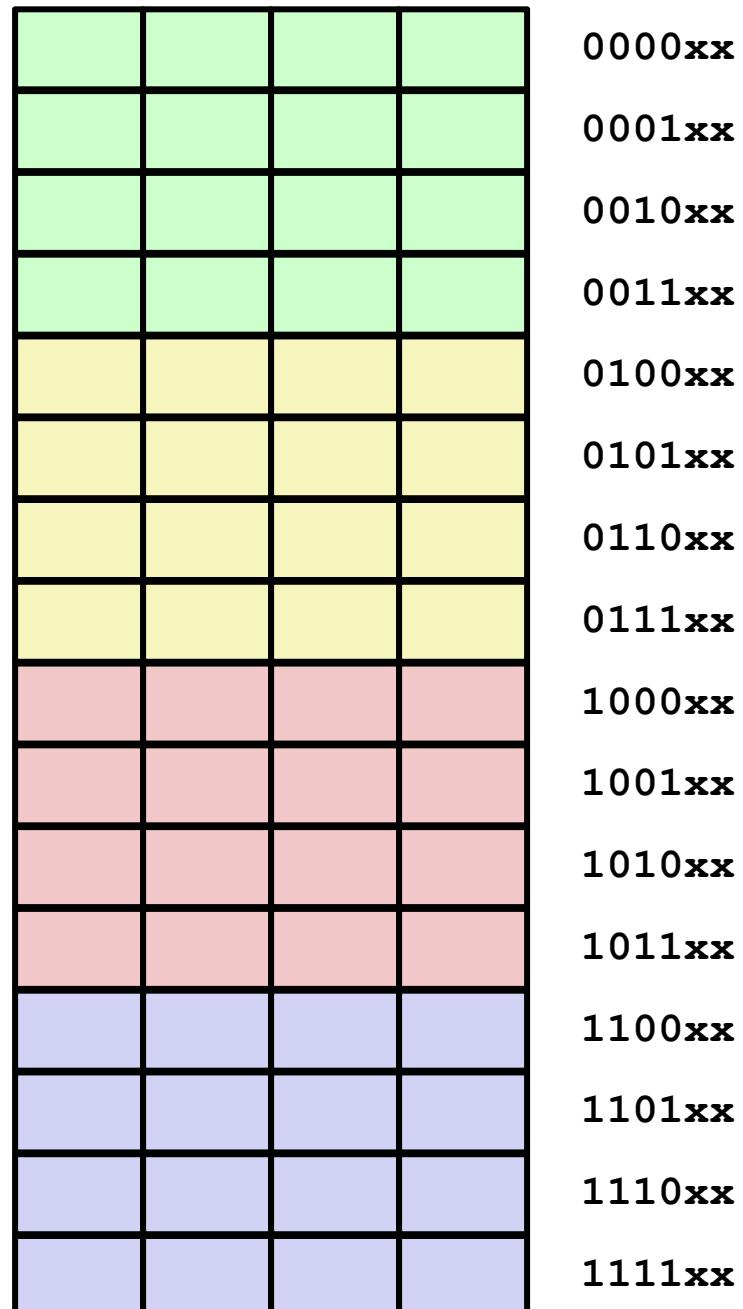
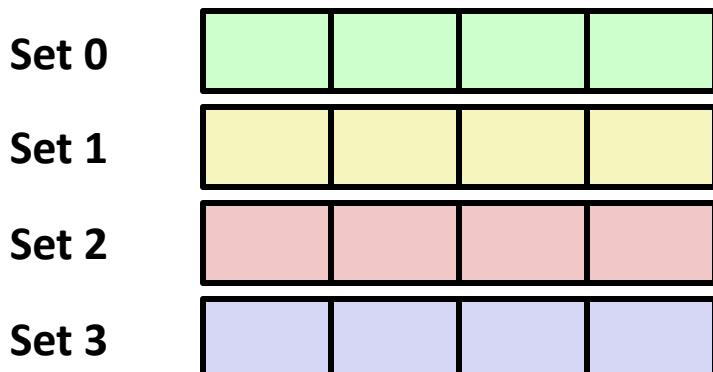


# High Bit Indexing

## ■ Addresses of form **SSTTBB**

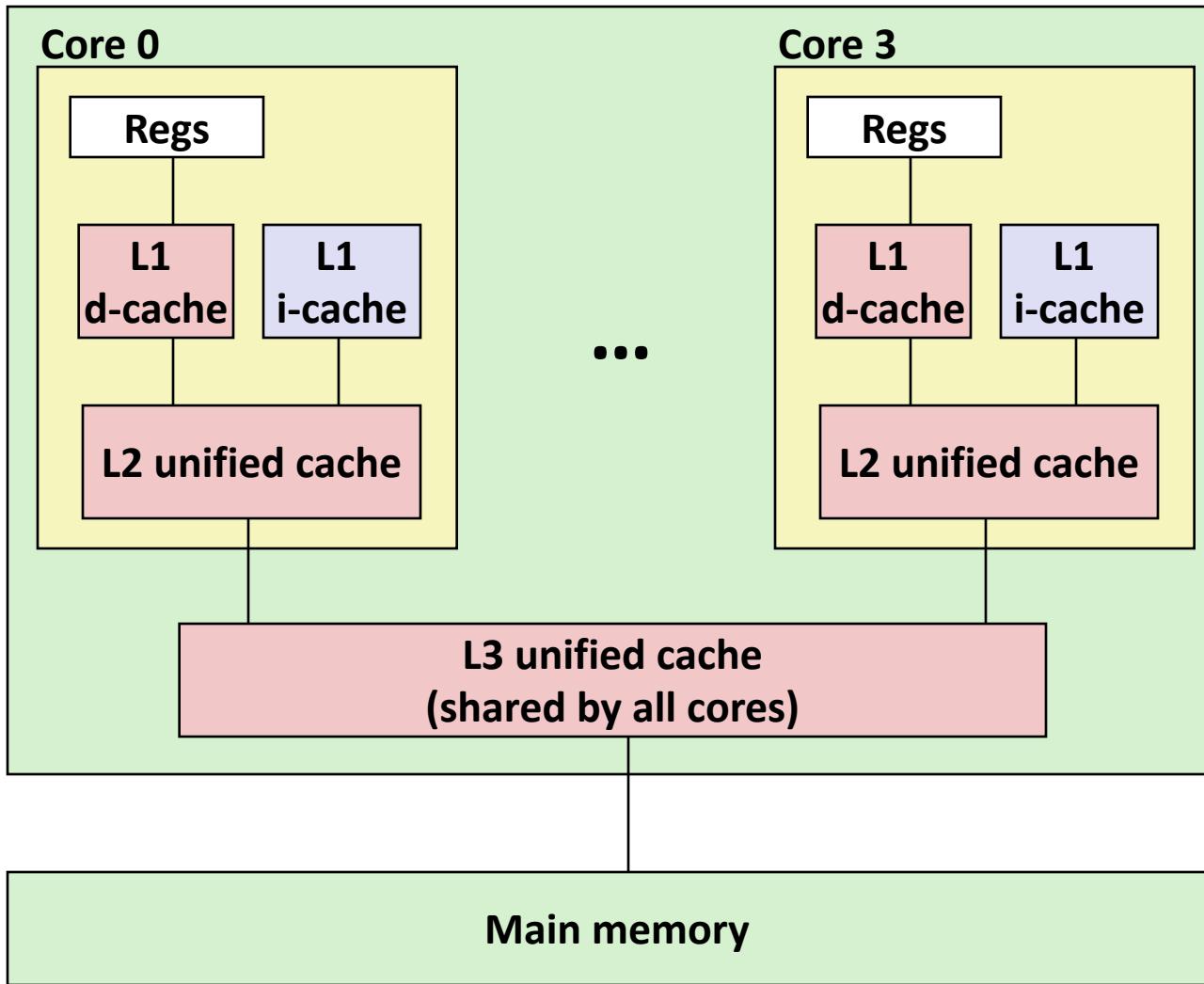
- **SS** Set index bits
- **TT** Tag bits
- **BB** Offset bits

## ■ Program with high spatial locality would generate lots of conflicts



# Intel Core i7 Cache Hierarchy

Processor package



**L1 i-cache and d-cache:**  
32 KB, 8-way,  
Access: 4 cycles

**L2 unified cache:**  
256 KB, 8-way,  
Access: 10 cycles

**L3 unified cache:**  
8 MB, 16-way,  
Access: 40-75 cycles

**Block size:** 64 bytes for  
all caches.

# Cache Performance Metrics

## ■ Miss Rate

- Fraction of memory references not found in cache (misses / accesses)  
=  $1 - \text{hit rate}$
- Typical numbers (in percentages):
  - 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.

## ■ Hit Time

- Time to deliver a line in the cache to the processor
  - includes time to determine whether the line is in the cache
- Typical numbers:
  - 4 clock cycle for L1
  - 10 clock cycles for L2

## ■ Miss Penalty

- Additional time required because of a miss
  - typically 50-200 cycles for main memory (Trend: increasing!)

# Let's think about those numbers

- Huge difference between a hit and a miss
  - Could be 100x, if just L1 and main memory
- Would you believe 99% hits is twice as good as 97%?
  - Consider this simplified example:  
cache hit time of 1 cycle  
miss penalty of 100 cycles
  - Average access time:  
97% hits: 1 cycle +  $0.03 \times 100$  cycles = **4 cycles**  
99% hits: 1 cycle +  $0.01 \times 100$  cycles = **2 cycles**
- This is why “miss rate” is used instead of “hit rate”

# Writing Cache Friendly Code

- **Make the common case go fast**
  - Focus on the inner loops of the core functions
- **Minimize the misses in the inner loops**
  - Repeated references to variables are good (**temporal locality**)
  - Stride-1 reference patterns are good (**spatial locality**)

**Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories**

# Quiz

<https://canvas.cmu.edu/courses/24383/quizzes/67224>

# Today

- Cache organization and operation
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# The Memory Mountain

- **Read throughput (read bandwidth)**
  - Number of bytes read from memory per second (MB/s)
- **Memory mountain: Measured read throughput as a function of spatial and temporal locality.**
  - Compact way to characterize memory system performance.

# Memory Mountain Test Function

```

long data[MAXELEMS]; /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *         array "data" with stride of "stride",
 *         using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}

```

*mountain/mountain.c*

Call `test()` with many combinations of `elems` and `stride`.

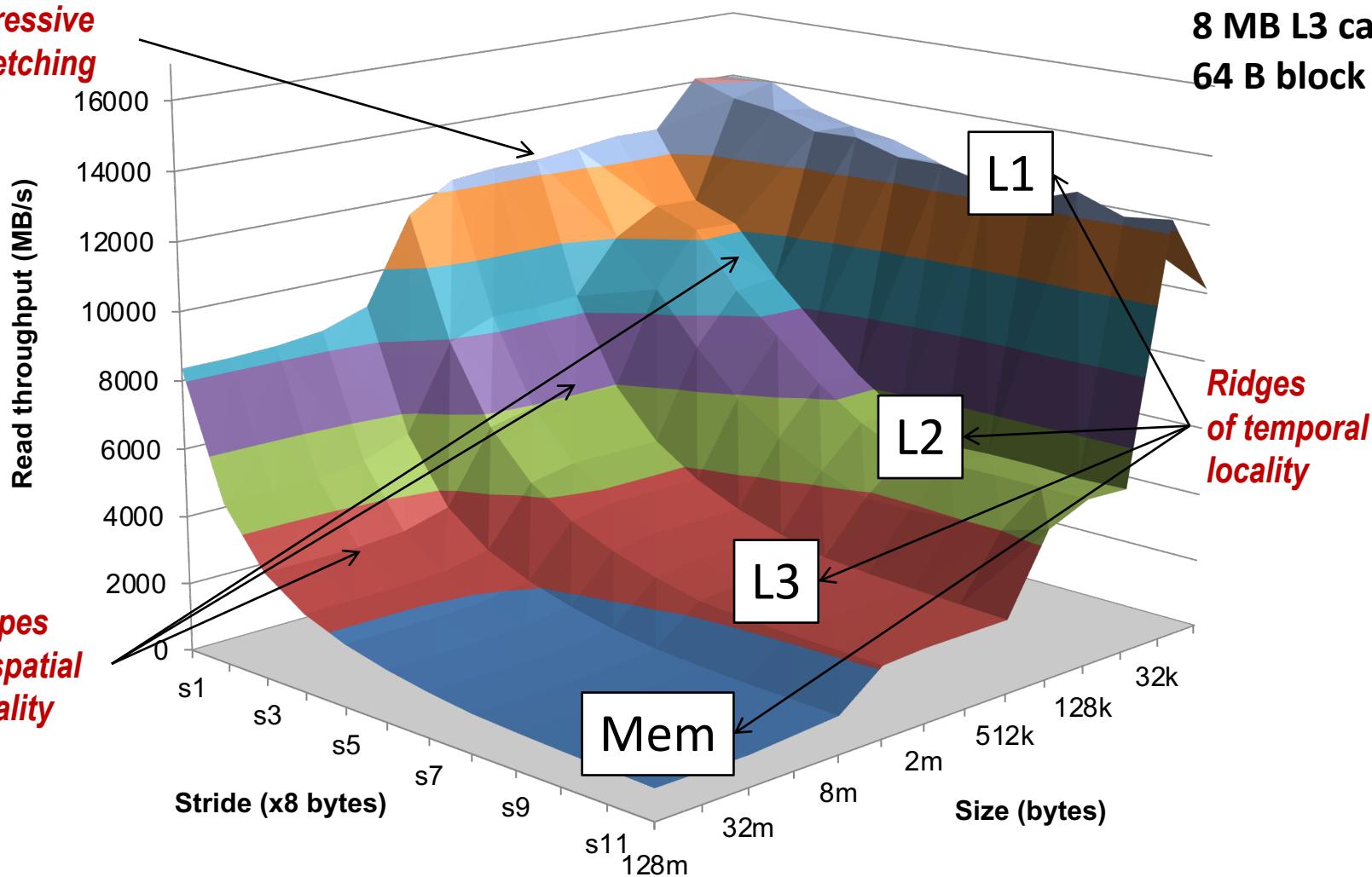
For each `elems` and `stride`:

1. Call `test()` once to warm up the caches.

2. Call `test()` again and measure the read throughput(MB/s)

# The Memory Mountain

Aggressive prefetching

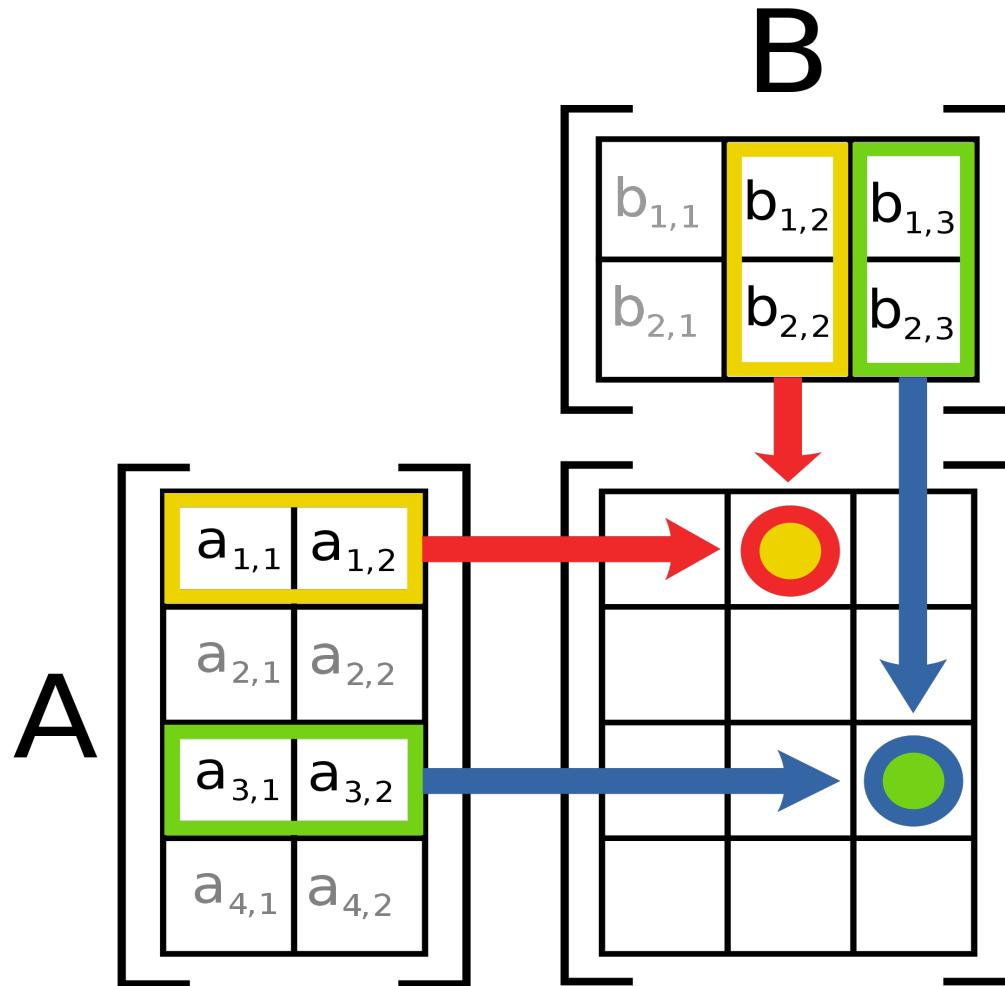


Core i7 Haswell  
2.1 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

# Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# Remember matrix multiplication



```
Out[i, j] =  
    dot product(A[i, ..], B[..,j])  
= sum (  
    a[i, 0] * b[0, j],  
    a[i, 1] * b[1, j]  
)
```

# Matrix Multiplication Example

## ■ Description:

- Multiply  $N \times N$  matrices
- Matrix elements are doubles (8 bytes)
- $O(N^3)$  total operations
- $N$  reads per source element
- $N$  values summed per destination
  - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*matmult/mm.c*

*Variable sum  
held in register*

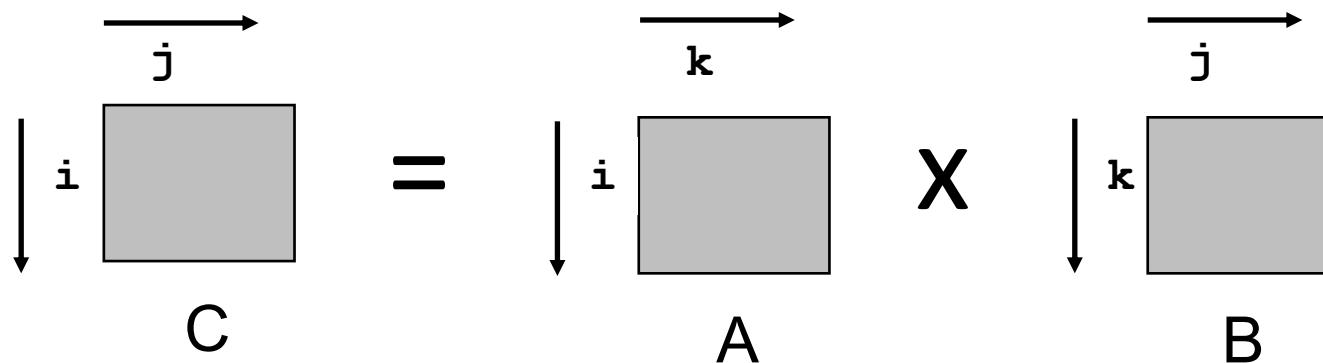
# Miss Rate Analysis for Matrix Multiply

## ■ Assume:

- Block size =  $32B$  (big enough for four doubles)
- Matrix dimension ( $N$ ) is very large
  - Approximate  $1/N$  as 0.0
- Cache is not even big enough to hold multiple rows

## ■ Analysis Method:

- Look at access pattern of inner loop



# Layout of C Arrays in Memory (review)

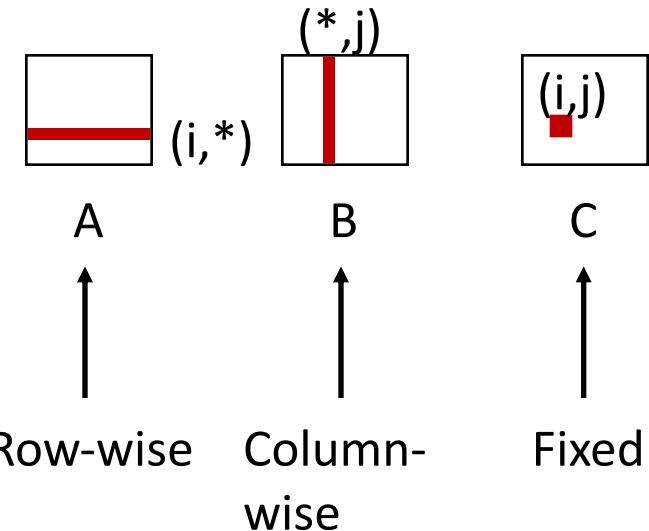
- C arrays allocated in row-major order
  - each row in contiguous memory locations
  - $a[i][j] = a[i*N + j]$  where N is the number of columns
- Stepping through columns in one row:
  - ```
for (i = 0; i < N; i++)  
    sum += a[0][i];
```
  - accesses successive elements
  - if block size (B) > sizeof( $a_{ij}$ ) bytes, exploit spatial locality
    - miss rate =  $\text{sizeof}(a_{ij}) / B$
- Stepping through rows in one column:
  - ```
for (i = 0; i < n; i++)  
    sum += a[i][0];
```
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

# Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

A

B

C

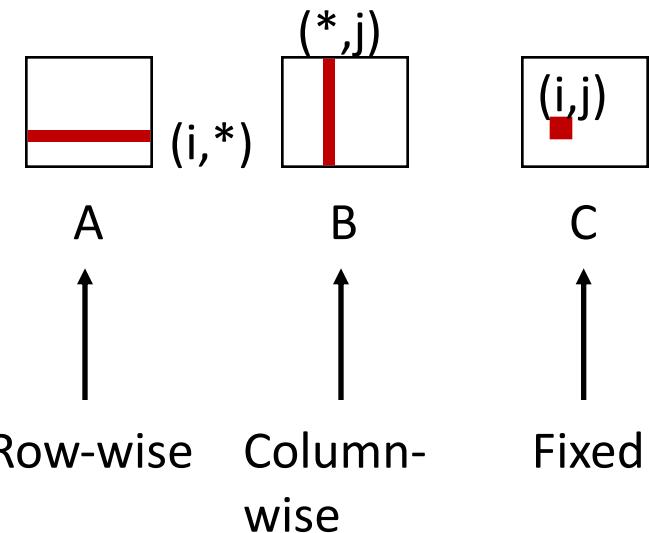
**Block size = 32B (four doubles)**

# Matrix Multiplication (ijk)

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/* ijk */
for (i=0; i<n; i++) {
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        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

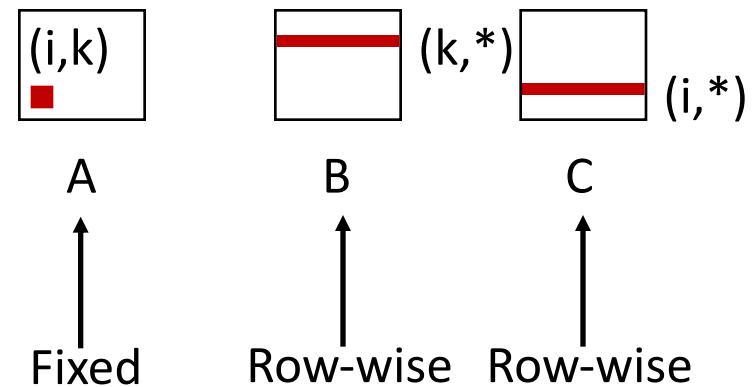
**Block size = 32B (four doubles)**

# Matrix Multiplication ( $kij$ )

```
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

A

B

C

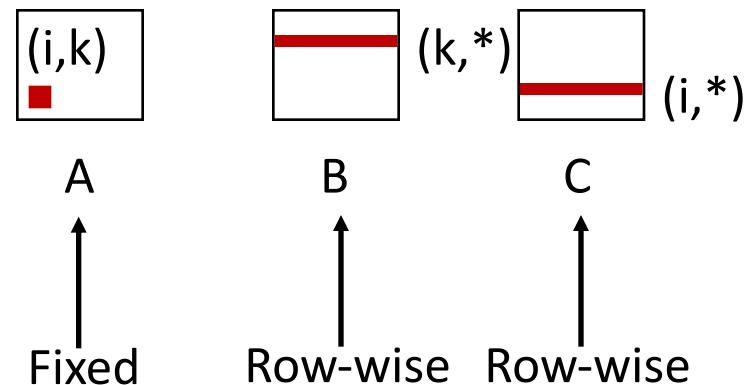
**Block size = 32B (four doubles)**

# Matrix Multiplication ( $kij$ )

```
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

A  
0.0

B  
0.25

C  
0.25

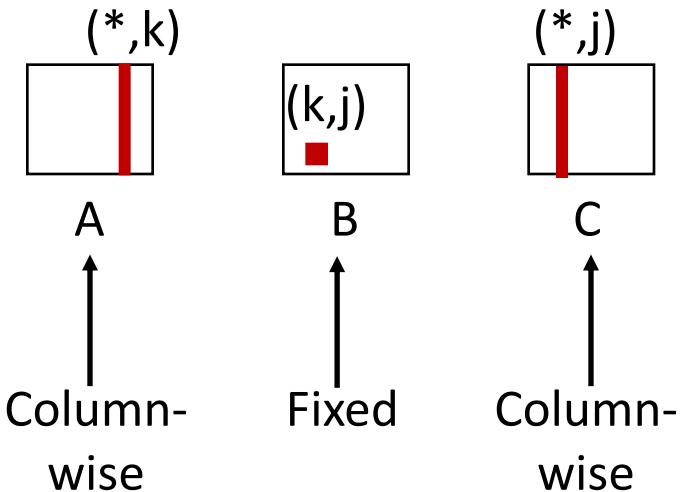
**Block size = 32B (four doubles)**

# Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

A

B

C

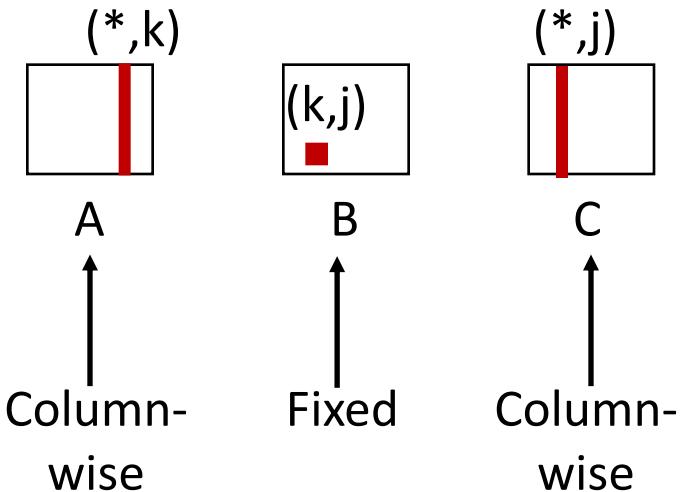
**Block size = 32B (four doubles)**

# Matrix Multiplication (jki)

```
/* jki */
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    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

*matmult/mm.c*

Inner loop:



Miss rate for inner loop iterations:

A  
1.0

B  
0.0

C  
1.0

**Block size = 32B (four doubles)**

# Summary of Matrix Multiplication

```

for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}

```

```

for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}

```

```

for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}

```

## **ijk (& jik):**

- 2 loads, 0 stores
- avg misses/iter = **1.25**

## **kij (& ikj):**

- 2 loads, 1 store
- avg misses/iter = **0.5**

## **jki (& kji):**

- 2 loads, 1 store
- avg misses/iter = **2.0**

# Core i7 Matrix Multiply Performance

Cycles per inner loop iteration

100

jki / kji (2.0)

jki

kji

ijk

jik

kij

ikj

10

ijk / jik (1.25)

iij

ijk

jik

kij

ikj

1

kij / ikj (0.5)

50 100 150 200 250 300 350 400 450 500 550 600 650 700

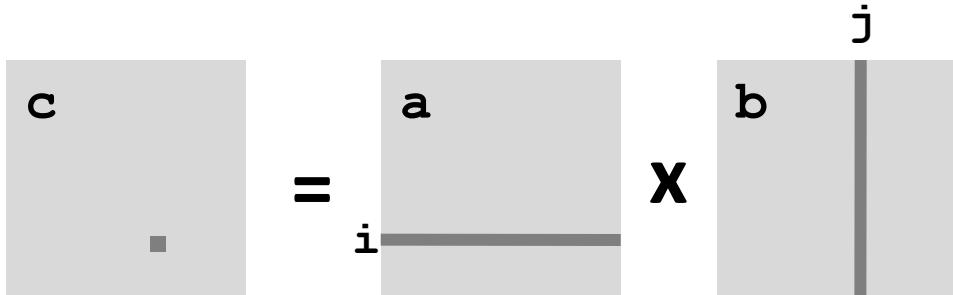
Array size (n)

# Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);  
  
/* Multiply n x n matrices a and b */  
void mmm(double *a, double *b, double *c, int n) {  
    int i, j, k;  
    for (i = 0; i < n; i++)  
        for (j = 0; j < n; j++)  
            for (k = 0; k < n; k++)  
                c[i*n + j] += a[i*n + k] * b[k*n + j];  
}
```



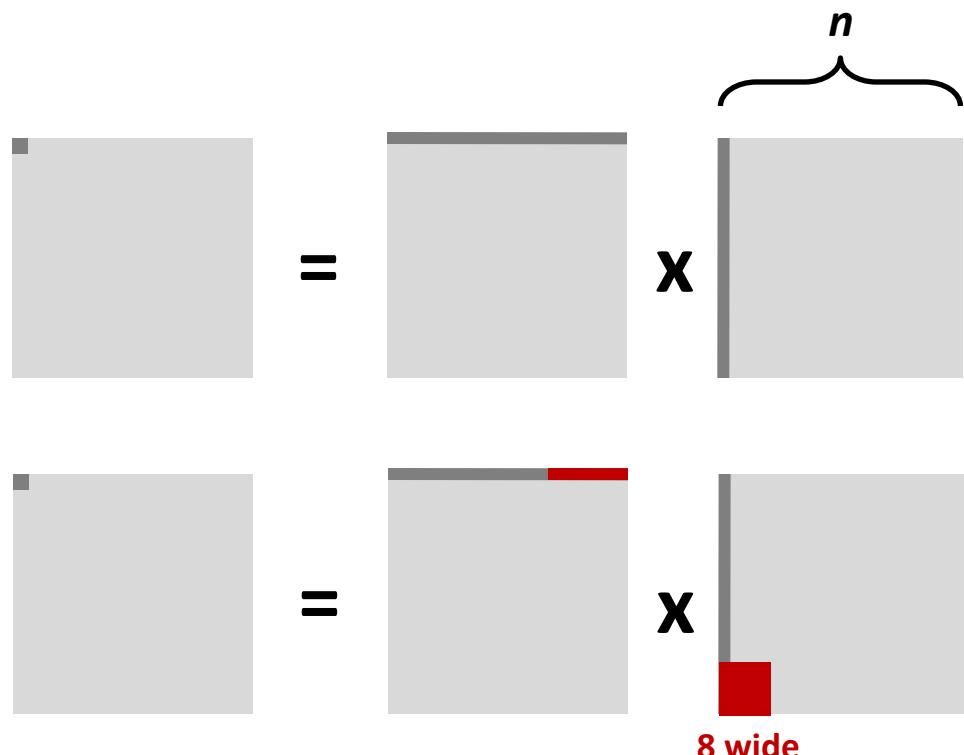
# Cache Miss Analysis

## ■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )

## ■ First iteration:

- $n/8 + n = 9n/8$  misses



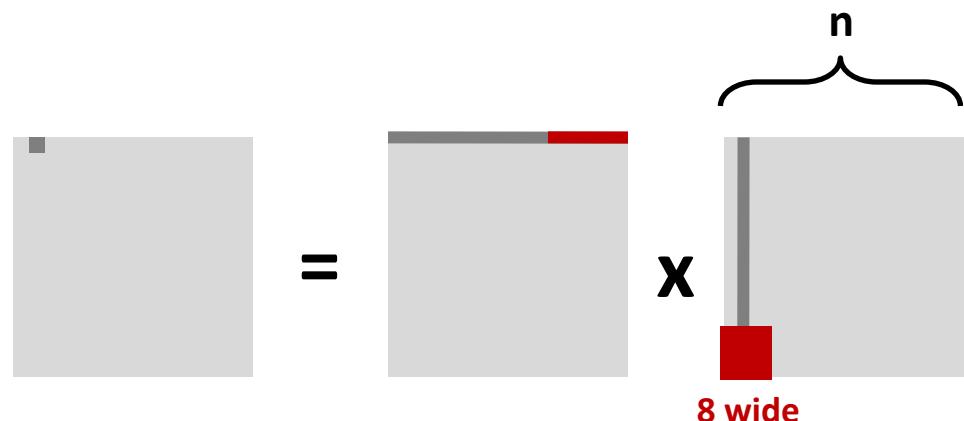
# Cache Miss Analysis

## ■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )

## ■ Second iteration:

- Again:  
 $n/8 + n = 9n/8$  misses



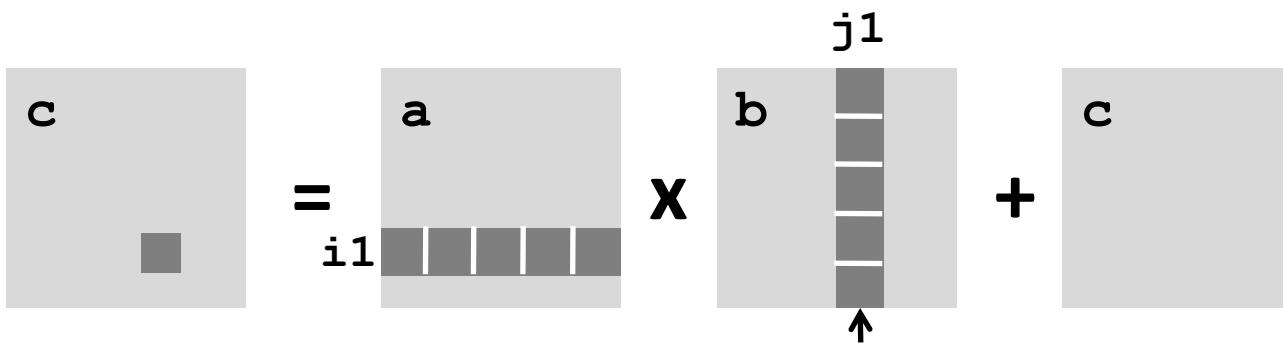
## ■ Total misses:

- $9n/8 n^2 = (9/8) n^3$

# Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i1++)
                    for (j1 = j; j1 < j+B; j1++)
                        for (k1 = k; k1 < k+B; k1++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
                                            matmult/bmm.c
```



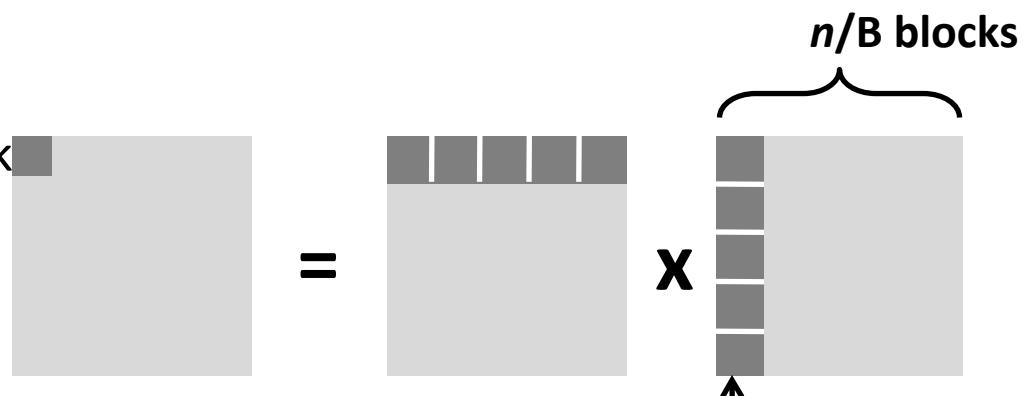
# Cache Miss Analysis

## ■ Assume:

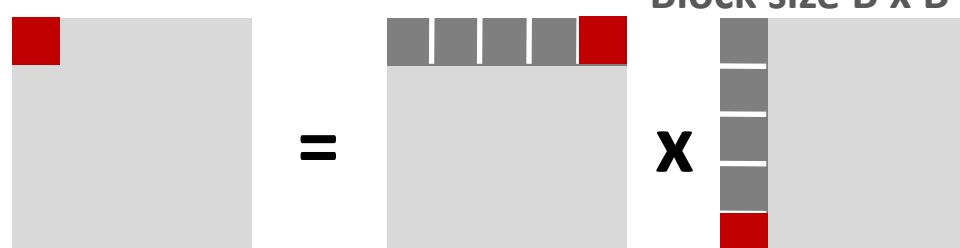
- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks ■ fit into cache:  $3B^2 < C$

## ■ First (block) iteration:

- $B^2/8$  misses for each block
- $2n/B \times B^2/8 = nB/4$   
(omitting matrix c)



- Afterwards in cache  
(schematic)



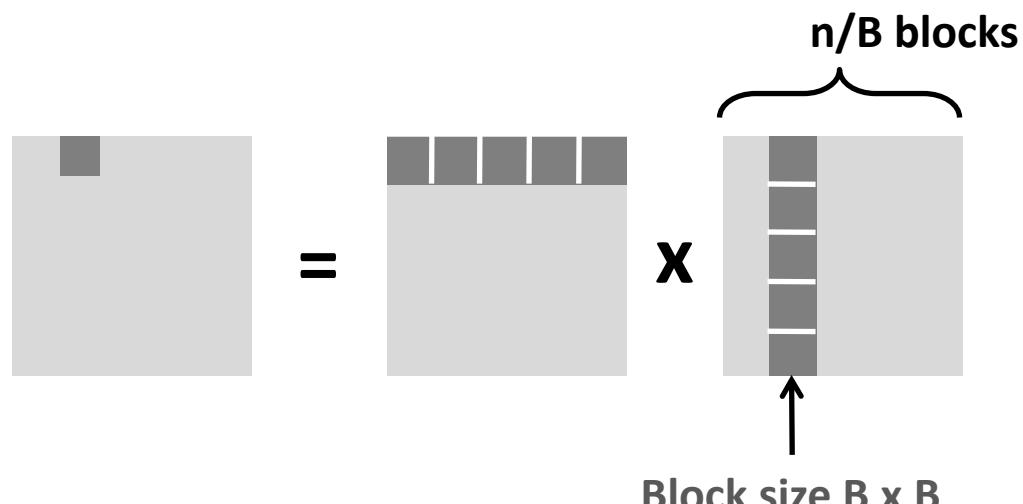
# Cache Miss Analysis

## ■ Assume:

- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks ■ fit into cache:  $3B^2 < C$

## ■ Second (block) iteration:

- Same as first iteration
- $2n/B \times B^2/8 = nB/4$



## ■ Total misses:

- $nB/4 * (n/B)^2 = n^3/(4B)$

# Blocking Summary

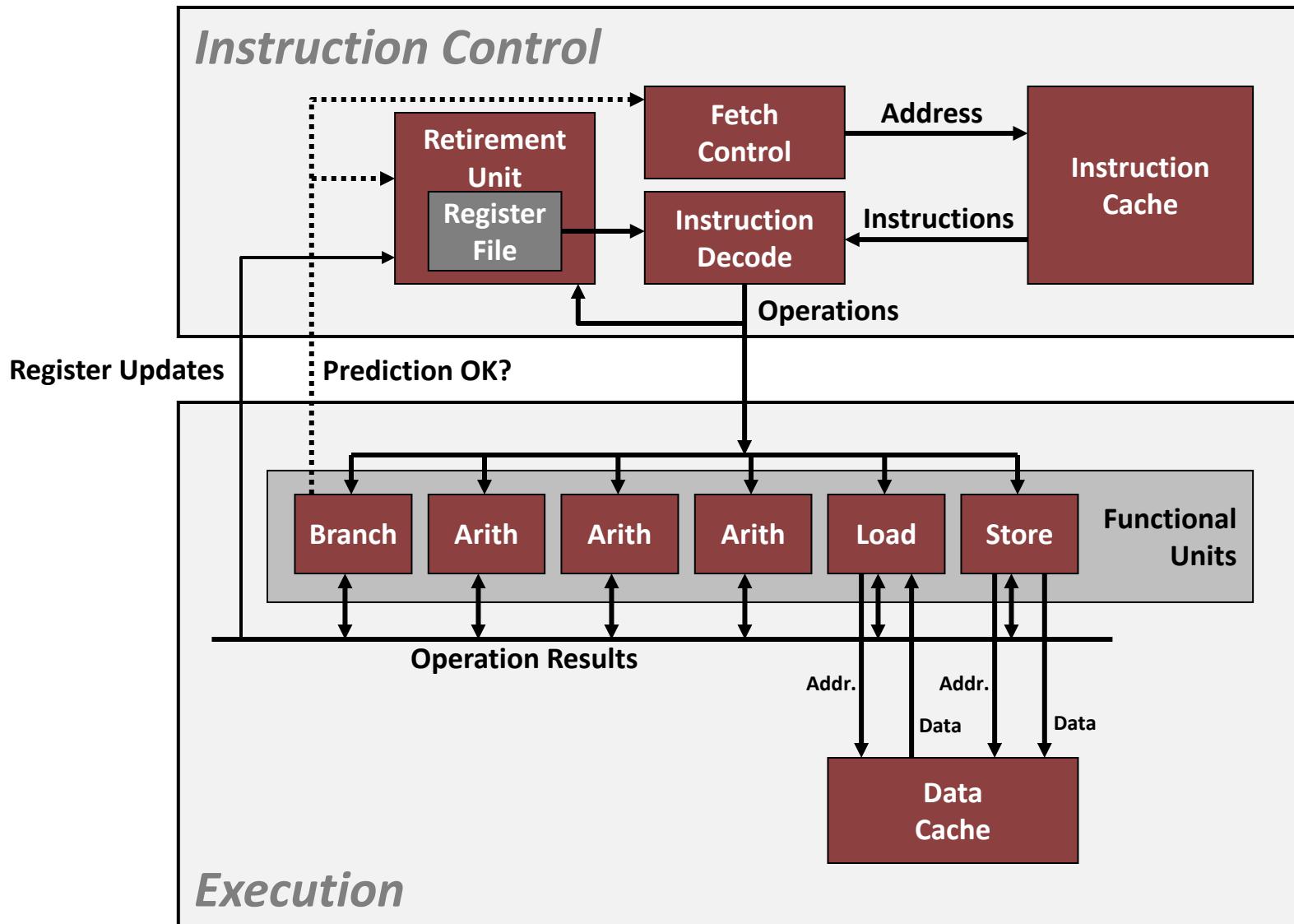
- No blocking:  $(9/8) n^3$  misses
- Blocking:  $(1/(4B)) n^3$  misses
  
- Use largest block size  $B$ , such that  $B$  satisfies  $3B^2 < C$ 
  - Fit three blocks in cache! Two input, one output.
  
- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data:  $3n^2$ , computation  $2n^3$
    - Every array elements used  $O(n)$  times!
  - But program has to be written properly

# Cache Summary

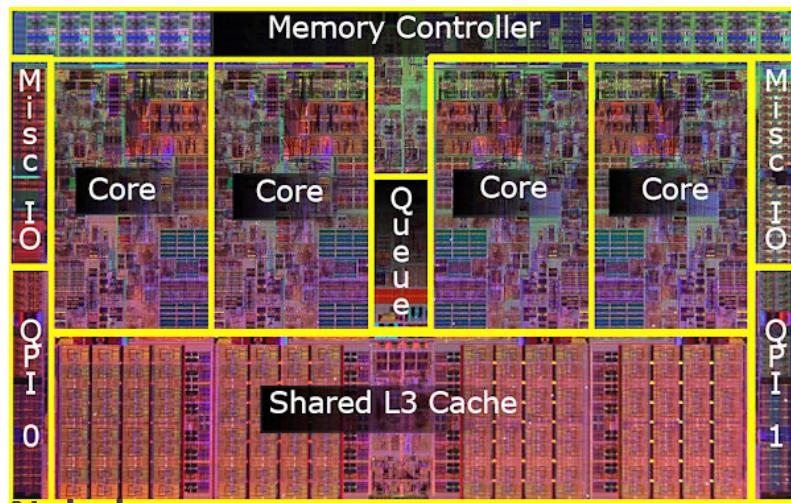
- Cache memories can have significant performance impact
- You can write your programs to exploit this!
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

# Supplemental slides

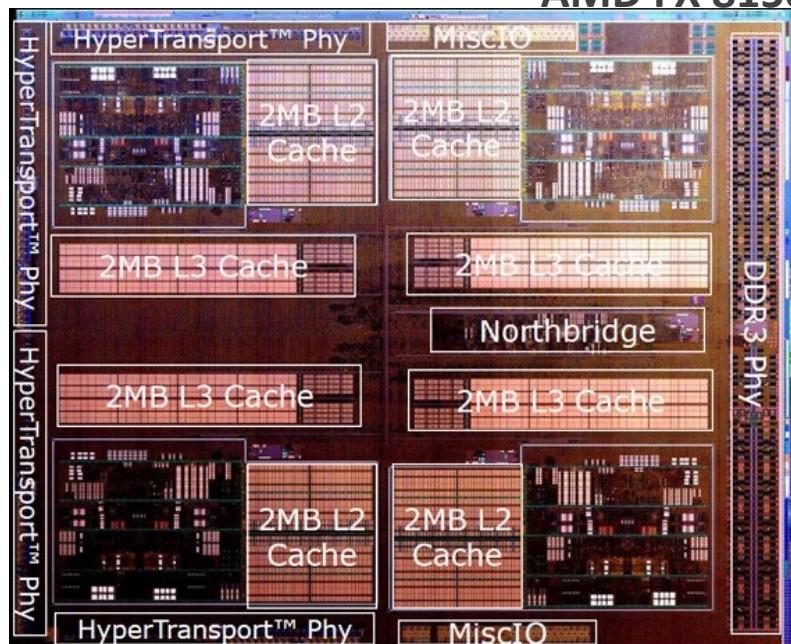
# Recall: Modern CPU Design



# What it Really Looks Like

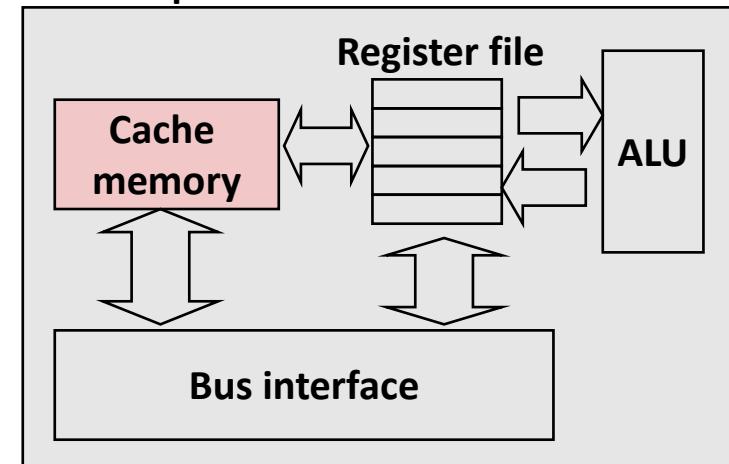


Nehalem

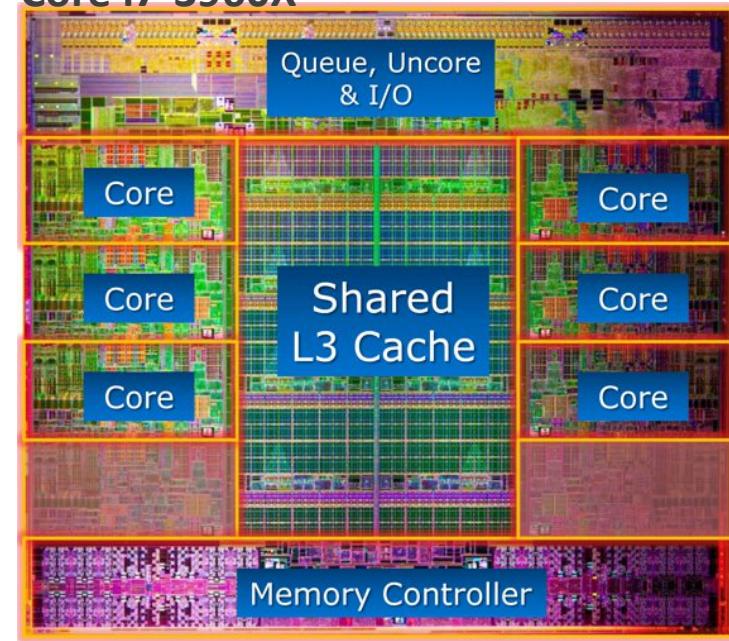


AMD FX 8150

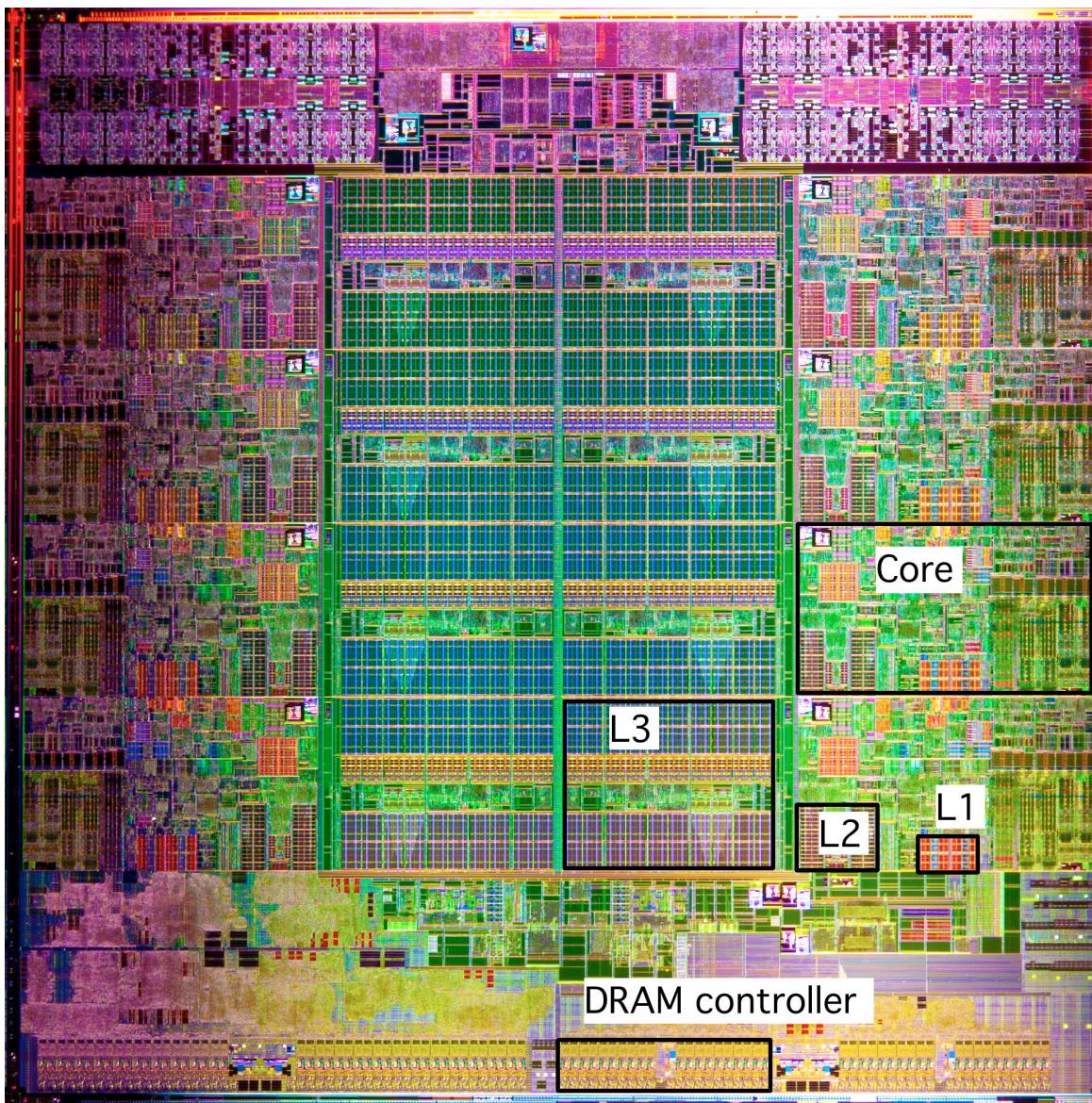
CPU chip



Core i7-3960X



# What it Really Looks Like (Cont.)



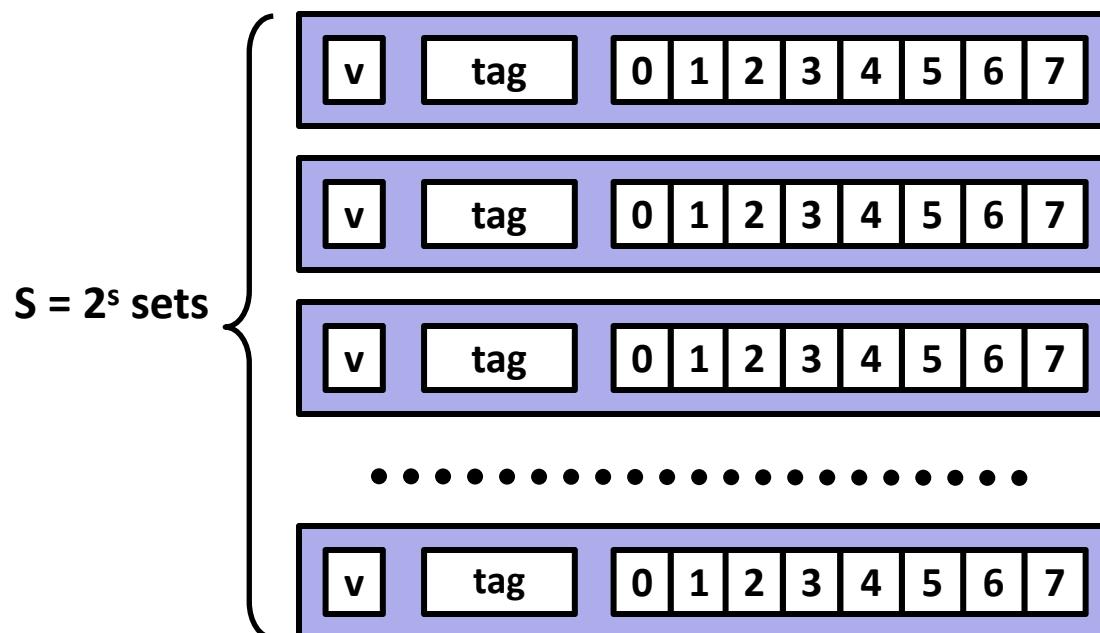
Intel Sandy Bridge  
Processor Die

- L1: 32KB Instruction + 32KB Data
- L2: 256KB
- L3: 3–20MB

# Why Index Using Middle Bits?

Direct mapped: One line per set

Assume: cache block size 8 bytes



## Standard Method: Middle bit indexing

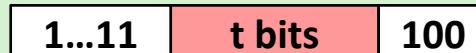
Address of int:



find set

## Alternative Method: High bit indexing

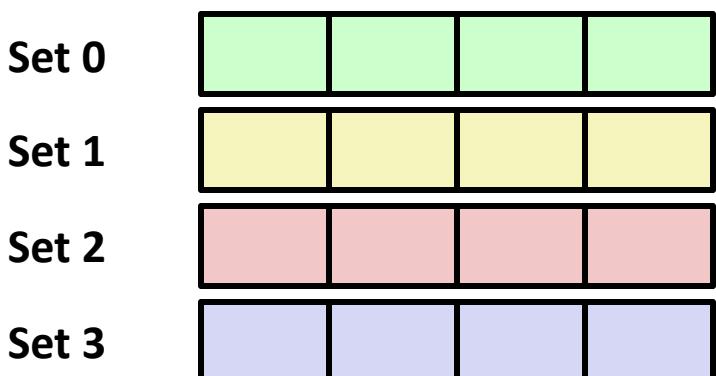
Address of int:



find set

# Illustration of Indexing Approaches

- **64-byte memory**
  - 6-bit addresses
- **16 byte, direct-mapped cache**
- **Block size = 4. (Thus, 4 sets; why?)**
- **2 bits tag, 2 bits index, 2 bits offset**



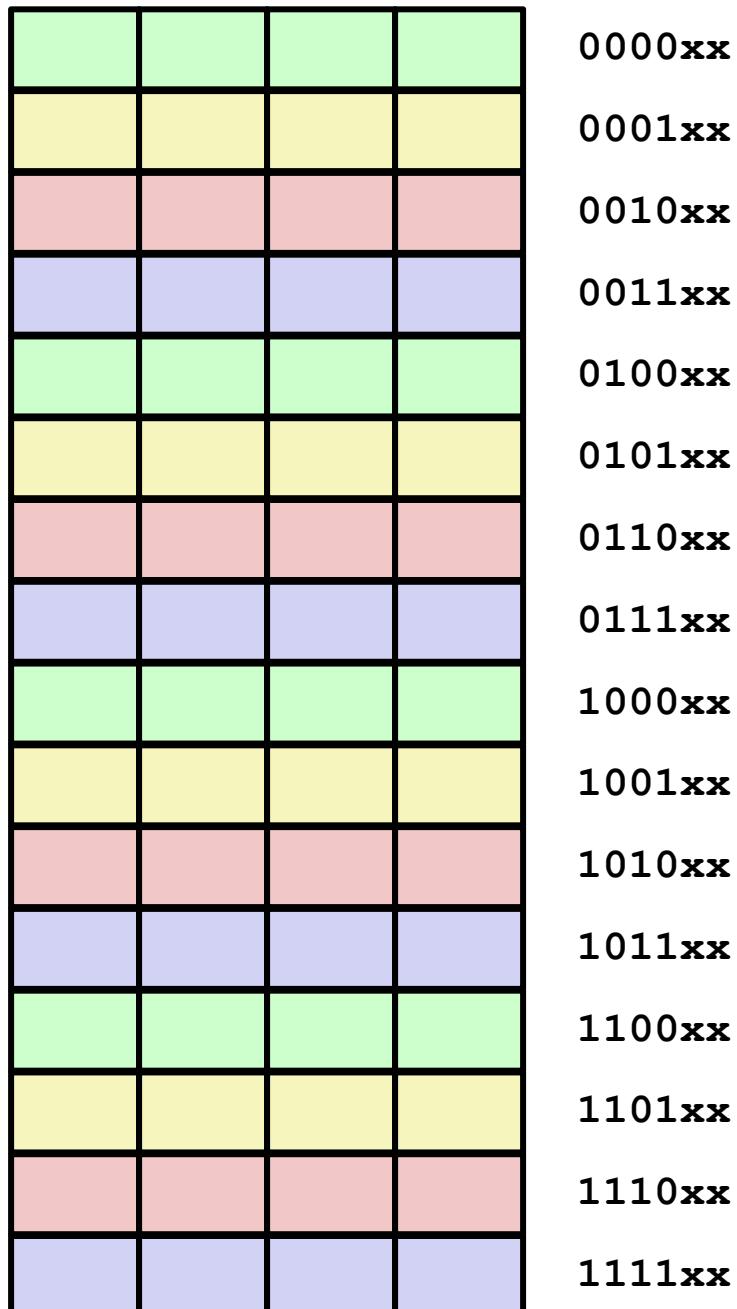
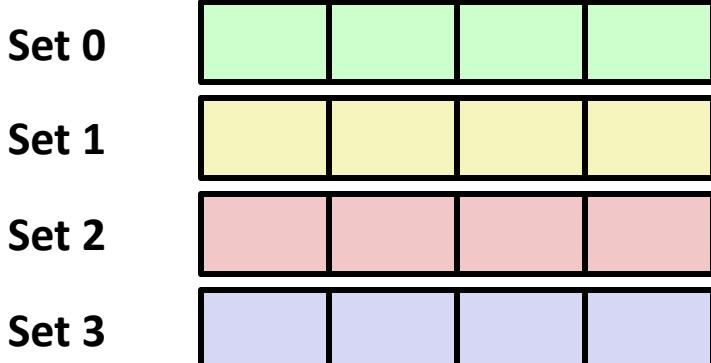
				0000xx
				0001xx
				0010xx
				0011xx
				0100xx
				0101xx
				0110xx
				0111xx
				1000xx
				1001xx
				1010xx
				1011xx
				1100xx
				1101xx
				1110xx
				1111xx

# Middle Bit Indexing

- Addresses of form **TTSSBB**

- **TT** Tag bits
- **SS** Set index bits
- **BB** Offset bits

- Makes good use of spatial locality

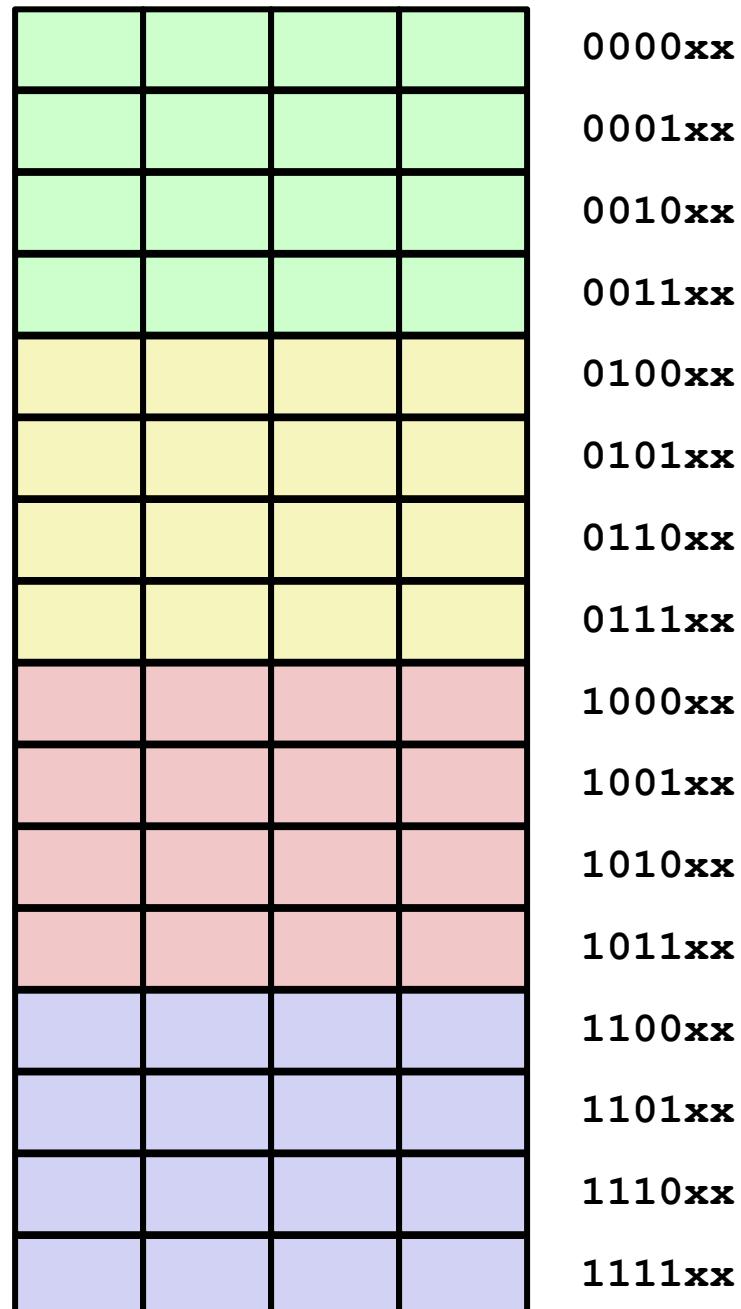
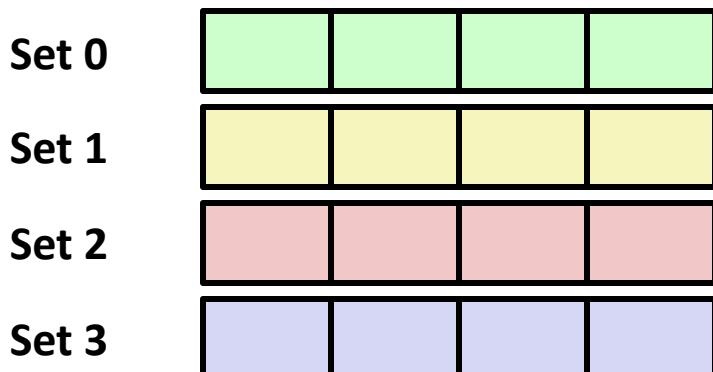


# High Bit Indexing

## ■ Addresses of form **SSTTBB**

- **SS** Set index bits
- **TT** Tag bits
- **BB** Offset bits

## ■ Program with high spatial locality would generate lots of conflicts



# Example: Core i7 L1 Data Cache

32 kB 8-way set associative

64 bytes/block

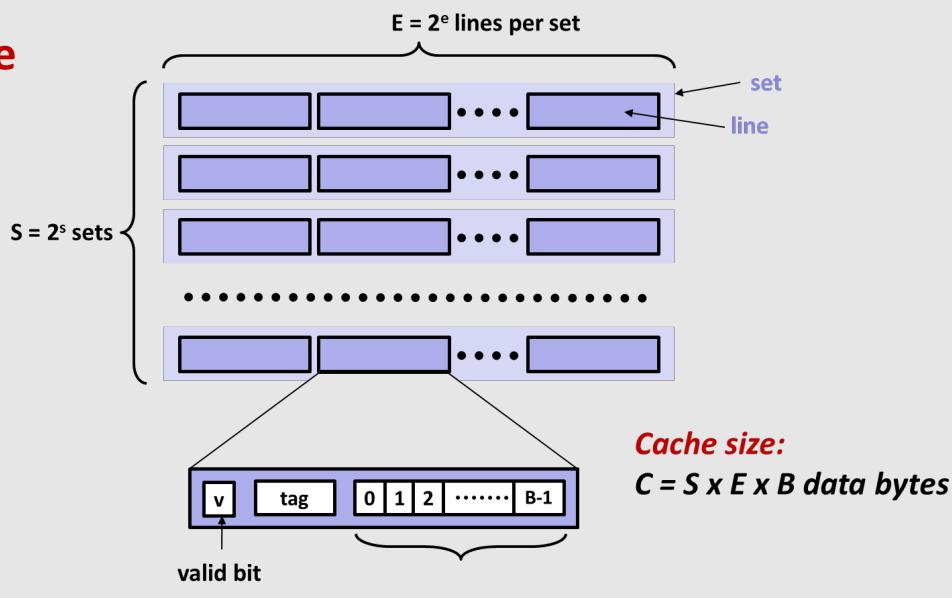
47 bit address range

B =

S = , s =

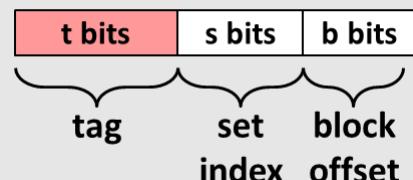
E = , e =

C =



	Hex	Decimal	Binary
0	0	0000	
1	1	0001	
2	2	0010	
3	3	0011	
4	4	0100	
5	5	0101	
6	6	0110	
7	7	0111	
8	8	1000	
9	9	1001	
A	10	1010	
B	11	1011	
C	12	1100	
D	13	1101	
E	14	1110	
F	15	1111	

Address of word:



Block offset: . bits

Set index: . bits

Tag: . bits

Stack Address:  
0x00007f7262a1e010

Block offset: 0x??  
Set index: 0x??  
Tag: 0x??

# Example: Core i7 L1 Data Cache

**32 kB 8-way set associative**

**64 bytes/block**

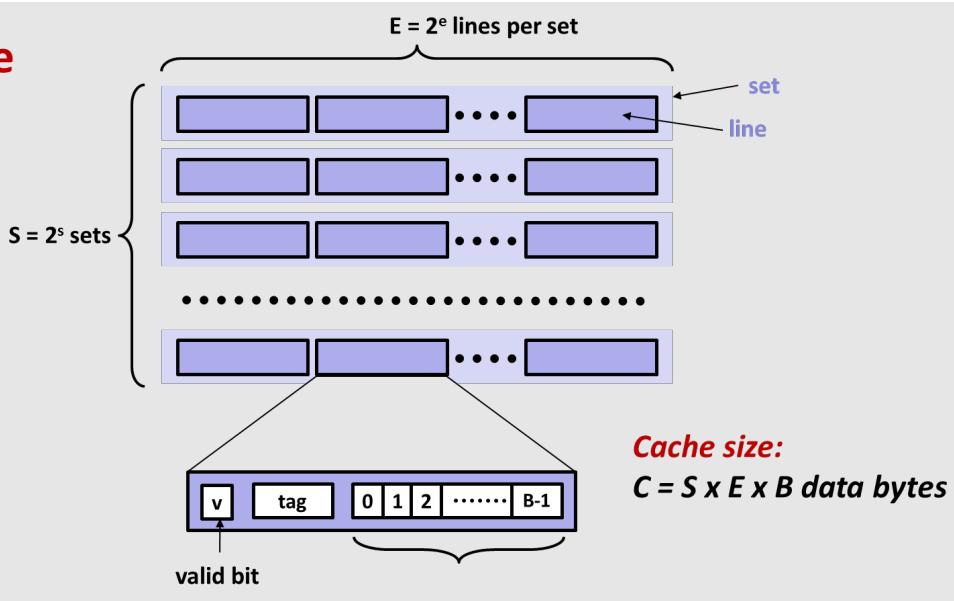
**47 bit address range**

$$B = 64$$

$$S = 64, s = 6$$

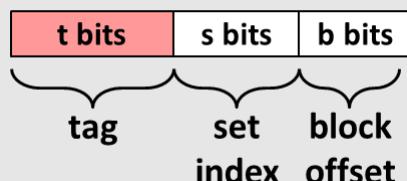
$$E = 8, e = 3$$

$$C = 64 \times 64 \times 8 = 32,768$$



Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

**Address of word:**



**Block offset:** 6 bits

**Set index:** 6 bits

**Tag:** 35 bits

**Stack Address:**

**0x00007f7262a1e010**

0000 0001 0000

**Block offset:** 0x10

**Set index:** 0x0

**Tag:** 0x7f7262a1e

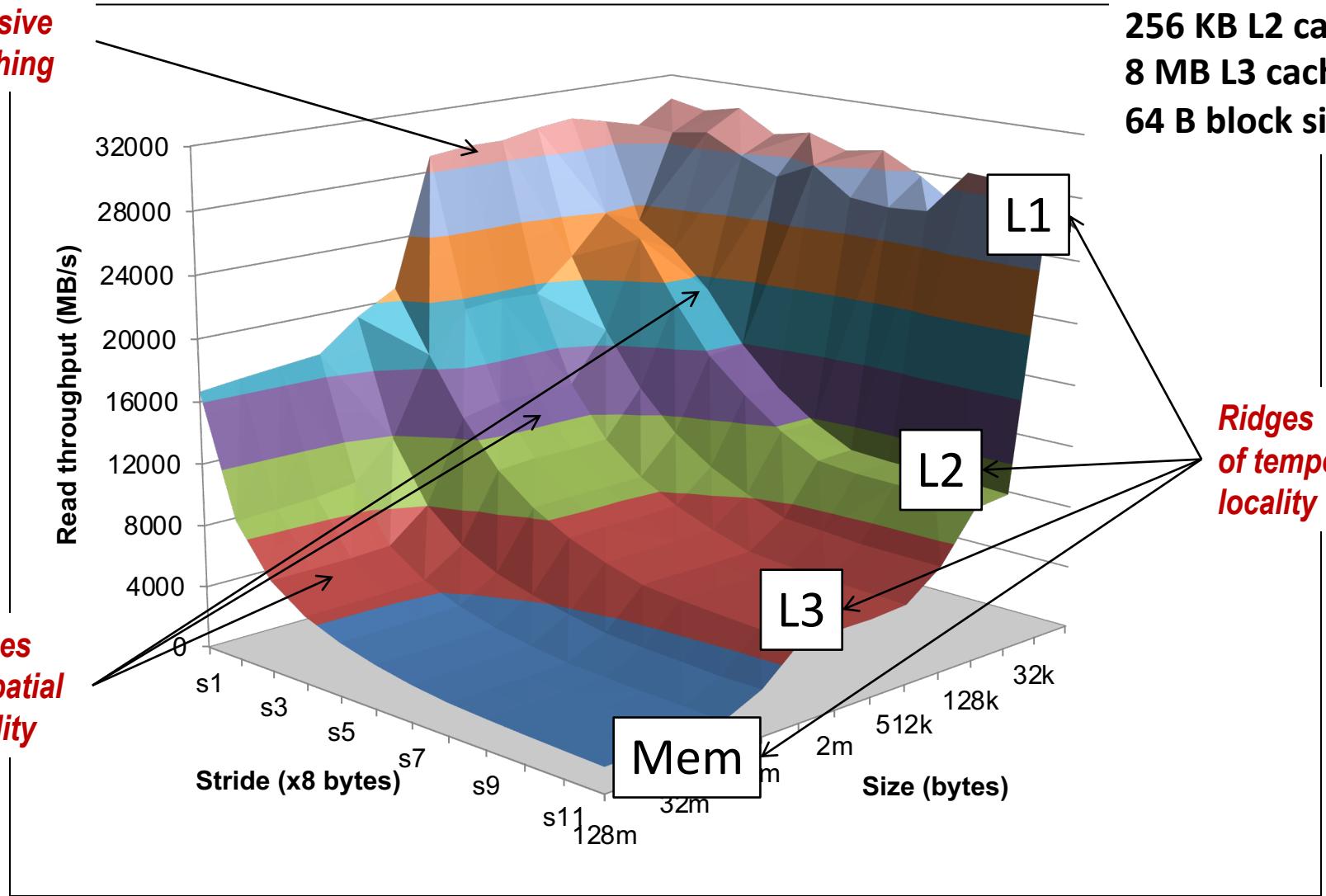
# The Memory Mountain

Core i5 Haswell  
3.1 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

Aggressive  
prefetching

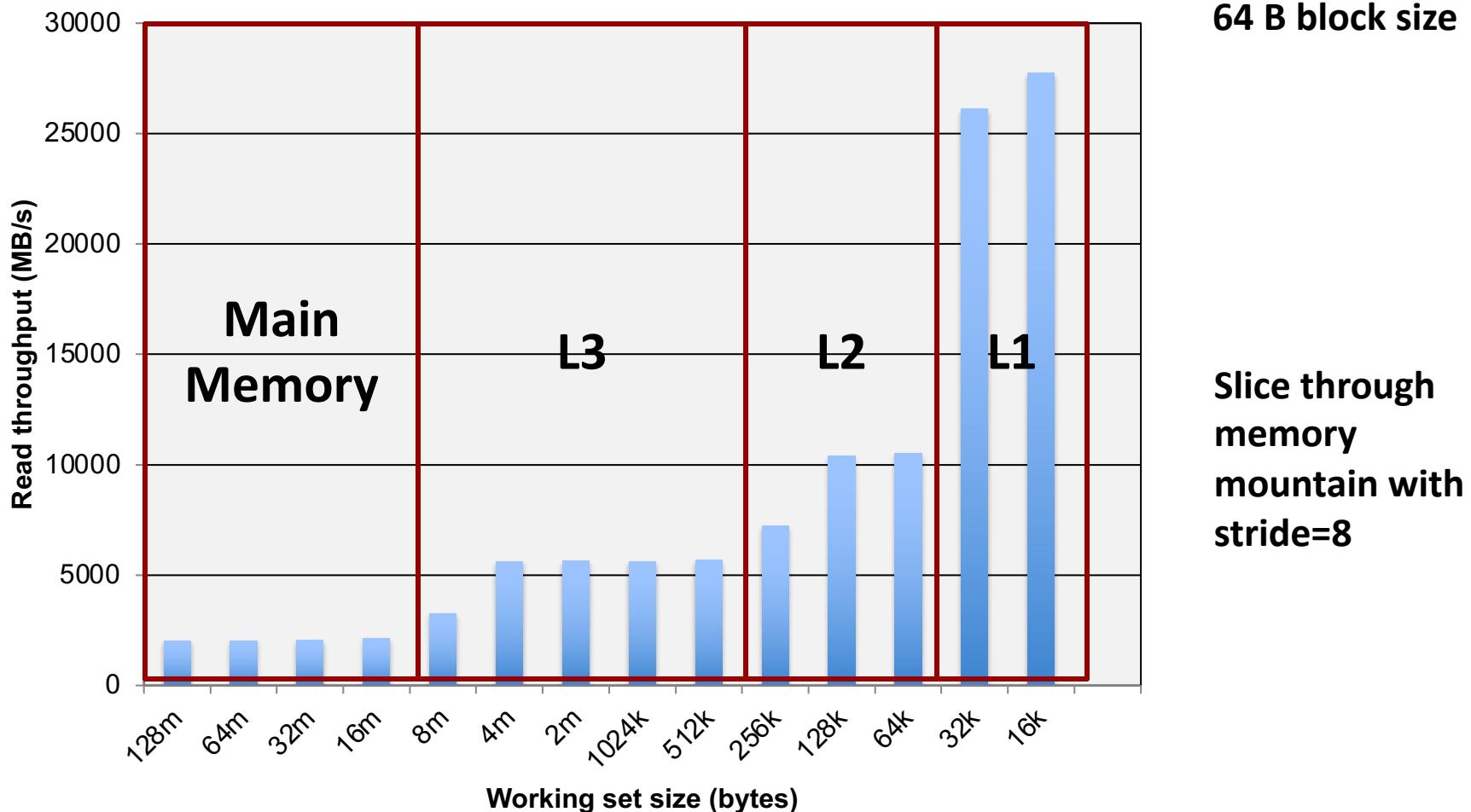
Slopes  
of spatial  
locality

Ridges  
of temporal  
locality



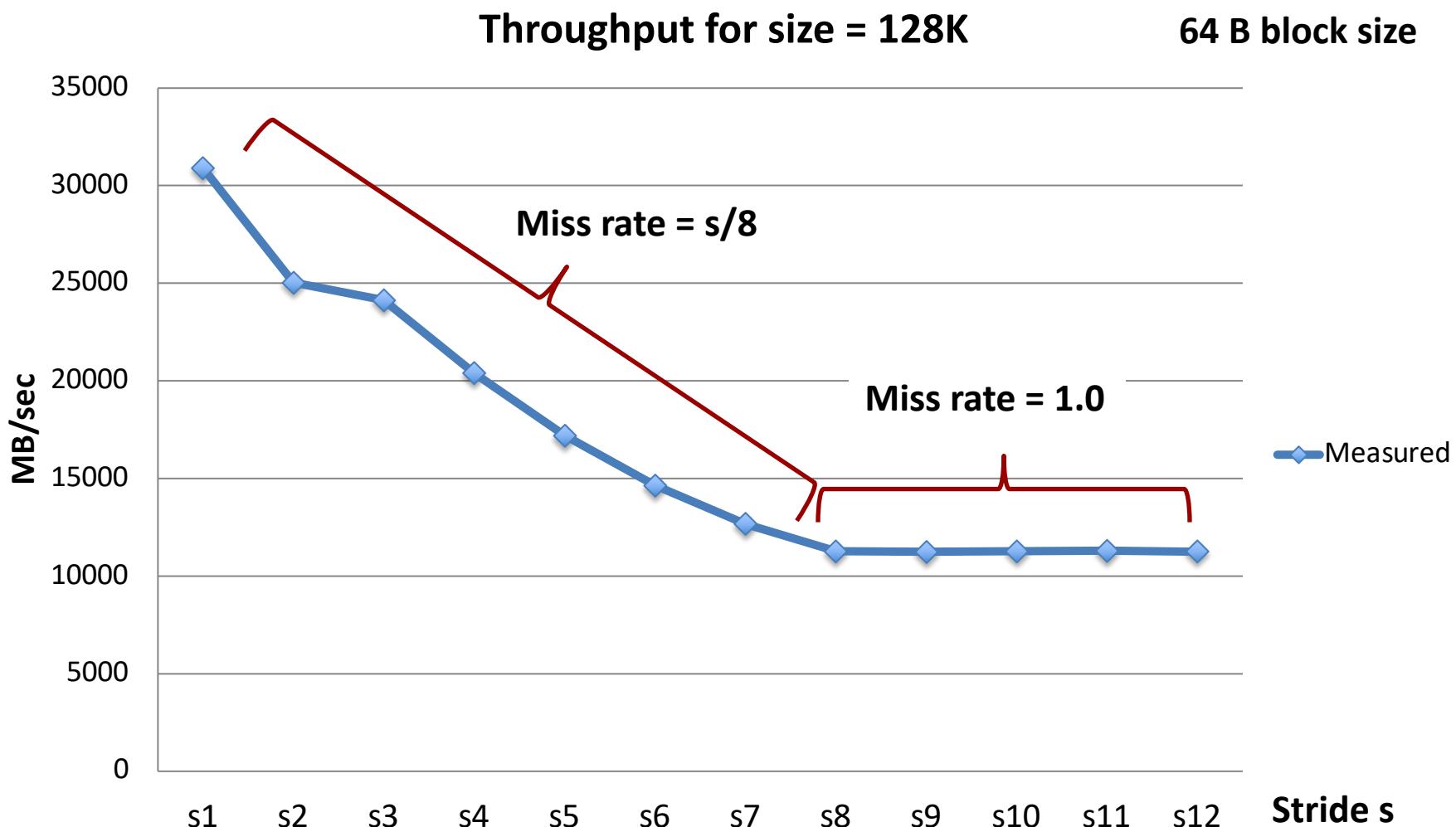
# Cache Capacity Effects from Memory Mountain

Core i7 Haswell  
3.1 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size



# Cache Block Size Effects from Memory Mountain

Core i7 Haswell  
2.26 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

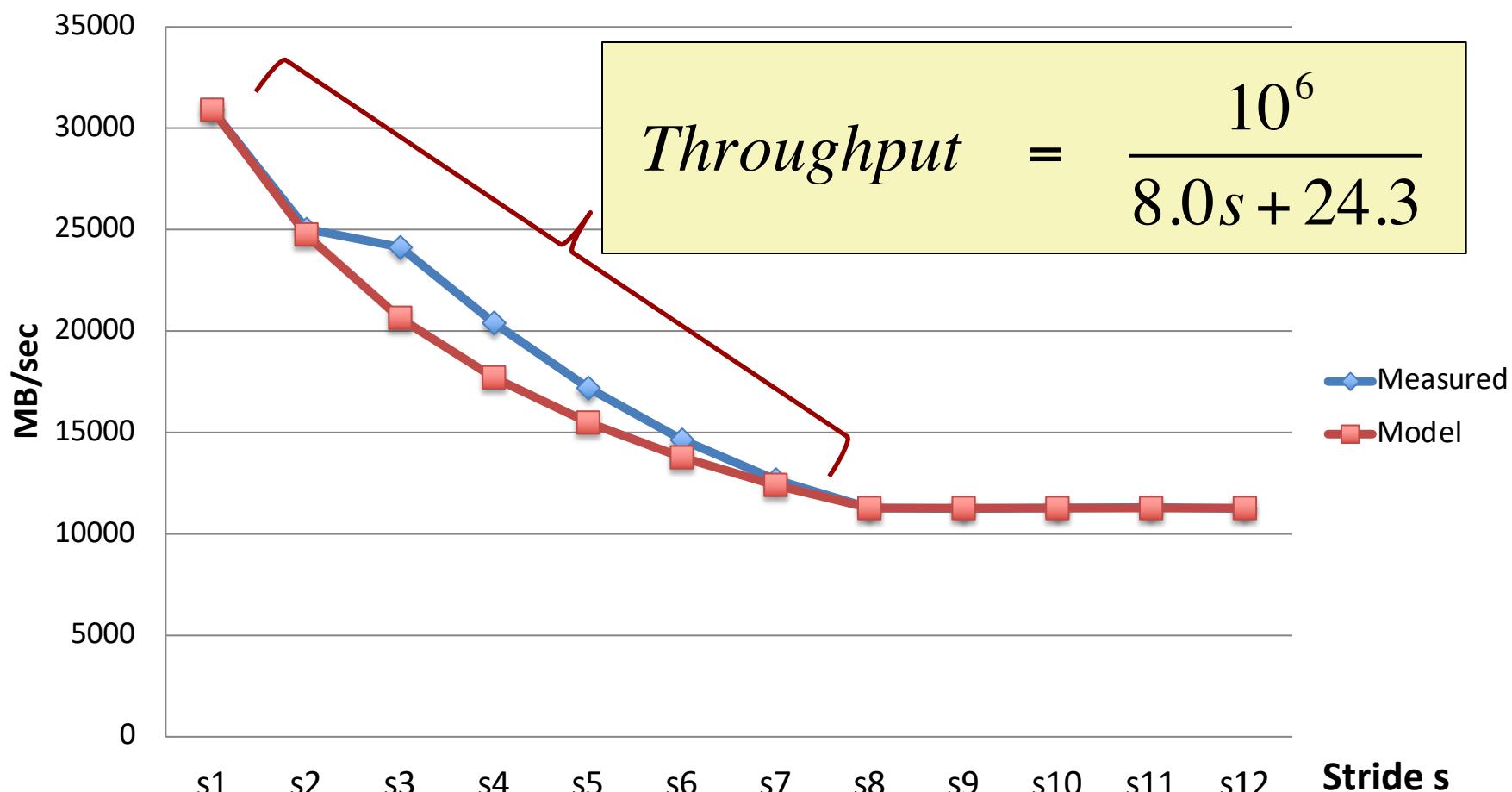


# Modeling Block Size Effects from Memory Mountain

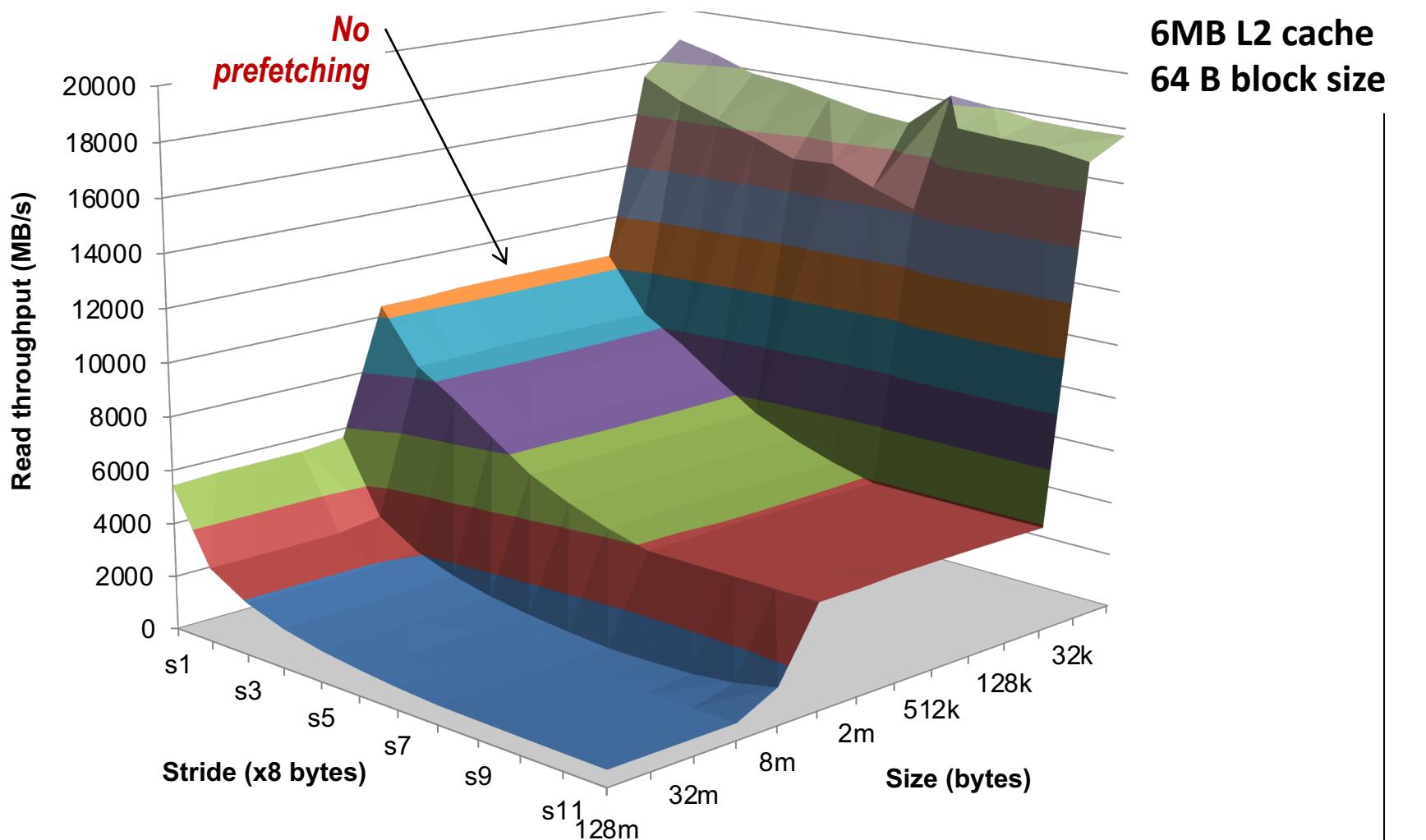
Core i7 Haswell  
2.26 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size

Throughput for size = 128K

$$\text{Throughput} = \frac{10^6}{8.0s + 24.3}$$



# 2008 Memory Mountain

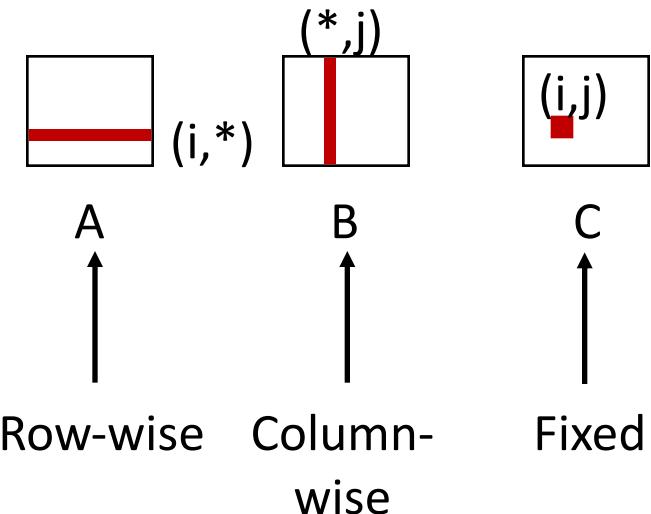


# Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
    for (i=0; i<n; i++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum
    }
}
```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

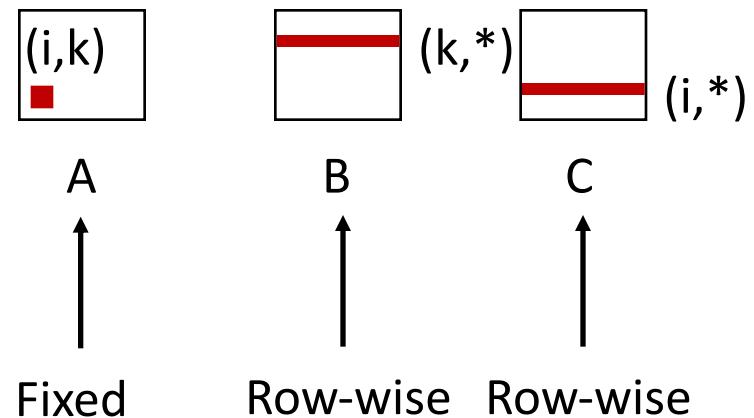
**Block size = 32B (four doubles)**

# Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

A  
0.0

B  
0.25

C  
0.25

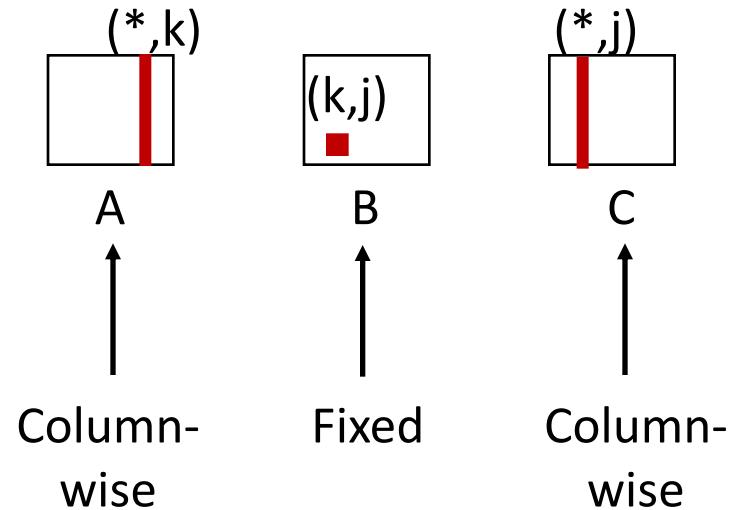
**Block size = 32B (four doubles)**

# Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
    for (j=0; j<n; j++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

A  
1.0

B  
0.0

C  
1.0

**Block size = 32B (four doubles)**