Forest Fire Simulation Model

Model Composition

The simulation area is divided into a two-dimensional grid G, where each cell $G_{i,j}$ can have different states and attributes. Each cell's state $s_{i,j}$ can be UNBURNED, BURNING, or BURNT. Each cell also contains environmental attributes such as humidity $h_{i,j}$, temperature $T_{i,j}$, rainfall $R_{i,j}$, slope $S_{i,j}$, vegetation type $V_{i,j}$, and is affected by wind direction W_d and wind speed W_s.

Fire Spread Rules

Fire spread is influenced by multiple factors including the state of adjacent cells, environmental attributes, and wind. A fire spread function F is defined as:

$$F(G_{i,j}, G_{\text{neighbors}}, W_d, W_s) \to G'_{i,j}$$

where $G_{\text{neighbors}}$ represents the set of neighboring cells of $G_{i,j}$, and $G'_{i,j}$ is the state of the cell at the next time step.

Environmental Impact

An integrated environmental impact function E calculates the ignition probability under given environmental conditions:

$$E(h_{i,j}, T_{i,j}, R_{i,j}, S_{i,j}, V_{i,j}, W_d, W_s) \to p_{\text{ignition}}$$

where p_{ignition} is the probability of a cell transitioning from an UNBURNED state to a BURNING state under the current environmental conditions.

Detailed Forest Fire Simulation Model

Environmental Factors

The impact of environmental factors on fire ignition and spread includes:

- Humidity (h): Higher humidity reduces flammability.
- Temperature (T): Higher temperatures increase the likelihood of fire.
- Rainfall (R): Rainfall significantly reduces fire risk.
- Slope (S): Fire spreads faster uphill due to rising heat.
- Vegetation Type (V): Different vegetation types have different flammability.
- Wind (W_d, W_s): Wind direction and speed directly affect fire spread.

Fire Spread Rules

Fire spread is determined by:

$$F(G_{i,j}, G_{\text{neighbors}}, W_d, W_s) \to G'_{i,j}$$

where $G_{\text{neighbors}}$ are the neighboring cells, and $G'_{i,j}$ is the state at the next time step.

Mathematical Model

The ignition probability under given environmental conditions is calculated by:

$$E(h, T, R, S, V, W_d, W_s) \to p_{\text{ignition}}$$

The fire spread function can be modeled as:

$$F(G_{i,j}, G_{\text{neighbors}}, W_d, W_s) = \text{some complex function}$$

Forest Fire Simulation Model Functions

Environmental Impact Function E

The environmental impact function E calculates the ignition probability p_{ignition} based on various environmental factors such as humidity (h), temperature (T), rainfall (R), slope (S), vegetation type (V), wind direction (W_d), and wind speed (W_s). The function can be represented as:

$$E(h, T, R, S, V, W_d, W_s) = \sigma(a \cdot h + b \cdot T + c \cdot R + d \cdot S + e \cdot V + f \cdot W_d + g \cdot W_s + C)$$

where $\sigma(x) = \frac{1}{1+e^{-x}}$ is the logistic function, and a, b, c, d, e, f, g, and C are model parameters that need to be determined based on data.

Fire Spread Function F

The fire spread function F determines the state of a cell at the next time step based on its current state, the states of its neighbors, and environmental conditions such as wind direction and speed. A simplified representation is:

$$F(G_{i,j}, G_{\text{neighbors}}, W_d, W_s) = \begin{cases} \text{BURNING}, & \text{if } \sum_{\text{neighbors}} \text{BURNING} \geq T \text{ and } E(h, T, R, S, V, W_d, W_s) > p_{\text{threshold}} \\ \text{UNBURNED}, & \text{otherwise} \end{cases}$$

Here, T is a threshold indicating the minimum number of BURNING neighbors required for a cell to catch fire, and $p_{\text{threshold}}$ is a probability threshold based on the environmental impact function E above which a cell transitions from UNBURNED to BURNING state.

1 Introduction

This document presents a mathematical model for optimizing the number and distribution of relay drones to ensure effective communication coverage for SSA (Situational Awareness) UAVs monitoring a wildfire.

2 Model Formulation

Let's consider a wildfire monitoring scenario where N SSA UAVs are deployed around a wildfire perimeter. The objective is to determine the minimum number of relay drones required and their optimal positions to ensure all SSA UAVs can communicate effectively either directly with the control center or through one or more relay drones.

2.1 Notations

- N: The total number of SSA UAVs deployed.
- M: The number of relay drones (to be optimized).
- D: The maximum communication distance for UAVs.
- P_i: The position of the i-th SSA UAV, $i \in \{1, 2, \ldots, N\}$.
- Q_j: The position of the j-th relay drone, $j \in \{1, 2, \ldots, M\}$.
- C: The control center location.

2.2 Objective

$$\min M \tag{1}$$

subject to the constraint that every SSA UAV must be within communication range of at least one relay drone or the control center.

2.3 Constraints

For each SSA UAV, there must exist at least one relay drone such that the distance between them is less than or equal to D. This can be mathematically represented as:

$$\forall i \in \{1, \ldots, N\}, \exists j \in \{1, \ldots, M\} : \|P_i - Q_j\| \leq D \quad \text{or} \quad \|P_i - C\| \leq D \tag{2}$$

2.4 Distance Function

The Euclidean distance is used to measure the distance between two points, which for two-dimensional space, is defined as:

$$\|P_i - Q_j\| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{3}$$

where (x_i, y_i) and (x_j, y_j) are the coordinates of P_i and Q_j, respectively.

3 Solution Approach

Solving this optimization problem can be approached through various methods, including genetic algorithms, particle swarm optimization (PSO), or other heuristic methods that are well-suited for solving complex optimization problems in multi-dimensional spaces.

4 Conclusion

This model provides a foundational framework for optimizing the distribution of relay drones to support SSA UAVs in wildfire monitoring scenarios. Further research could include dynamic positioning based on real-time data and integrating more complex constraints.

5 Model Definition

Considering the relay drone layout optimization problem, we define the following mathematical model to minimize the number of required relay drones while ensuring effective communication between all SSA drones and the EOC.

5.1 Variable Definitions

- x_i: Indicates whether a relay drone is placed at location i, with $x_i = 1$ for placement and $x_i = 0$ otherwise.
- d_{ij}: The distance between locations i and j.
- N: The total number of candidate locations.
- R: The communication range of the drone.
- C_{ij}: If the distance between locations i and j is less than or equal to the drone's communication range R, then $C_{ij} = 1$; otherwise, $C_{ij} = 0$.

5.2 Objective Function

$$\text{minimize} \quad \sum_{i=1}^{N} x_i \tag{4}$$

5.3 Constraints

- Communication coverage constraint: Ensure that each SSA drone communicates with the EOC through at least one relay drone.

$$\sum_{j \in S_i} x_j \geq 1, \quad \forall i \in \text{SSA drones} \tag{5}$$

  where S_i is the set of candidate relay drone locations within the communication range of SSA drone i.

- Distance constraint: Two locations can communicate directly if the distance between them is less than or equal to the drone's communication range.

$$C_{ij} = \begin{cases} 1, & \text{if } d_{ij} \leq R \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

5.4 Model Integration

Integrating the above definitions and constraints, we obtain the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{N} x_i \\ \text{subject to} \quad & \sum_{j \in S_i} x_j \geq 1, \quad \forall i \in \text{SSA drones}, \\ & C_{ij} = \begin{cases} 1, & \text{if } d_{ij} \leq R, \\ 0, & \text{otherwise}, \end{cases} \\ & x_i \in \{0, 1\}, \quad \forall i = 1, \ldots, N. \end{aligned}$$