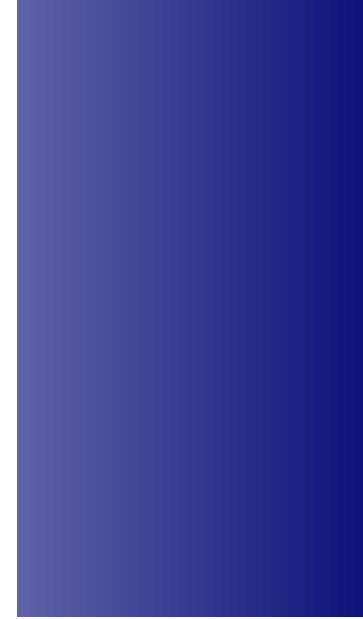




The Hunt for High-T_c Superconductivity: from Discovery to Breakthrough



Christopher Lane

Physics of Condensed Matter and Complex Systems, Theoretical Division
Los Alamos National Laboratory
Los Alamos, NM

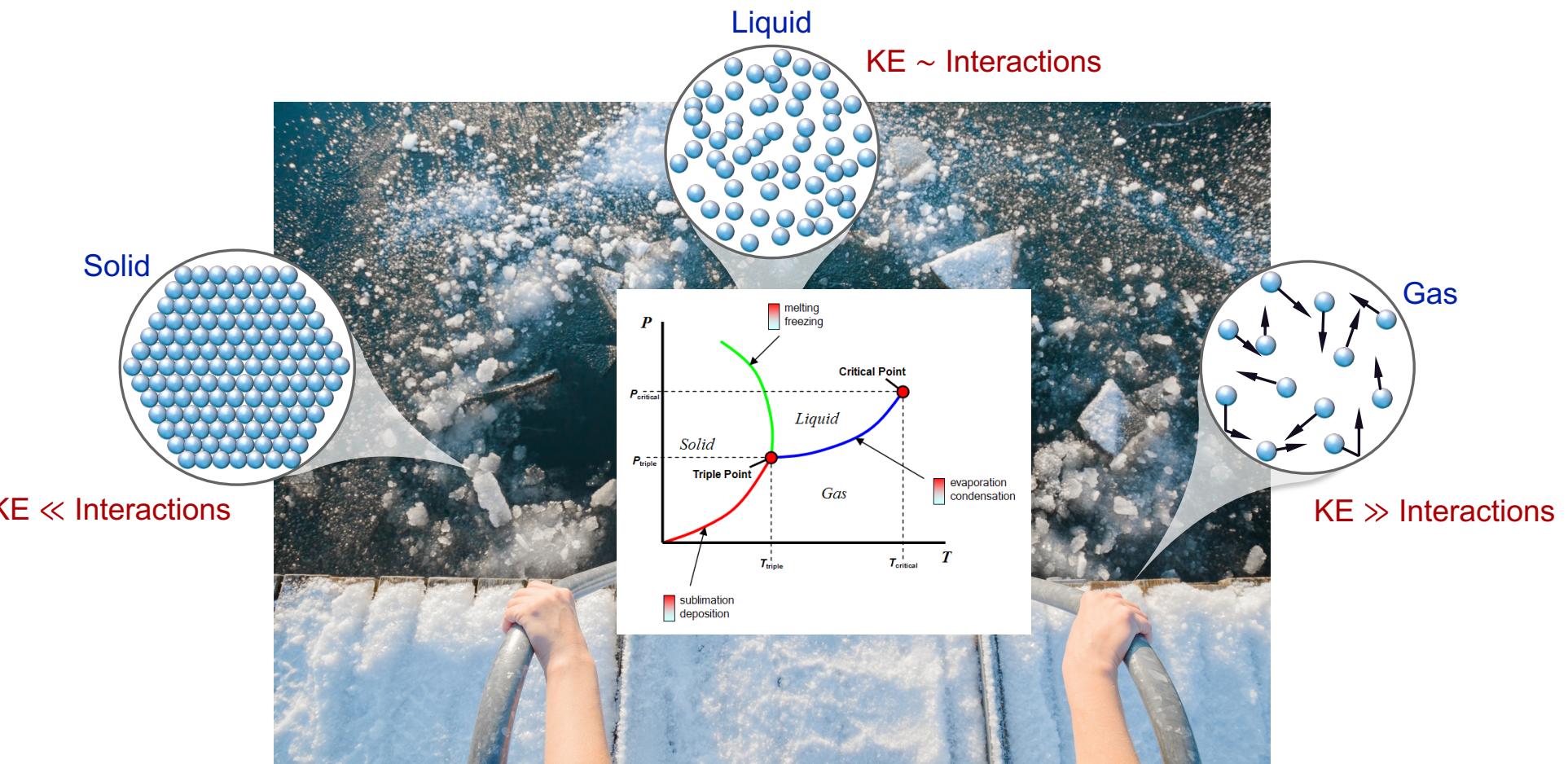
LA-UR-23-30665



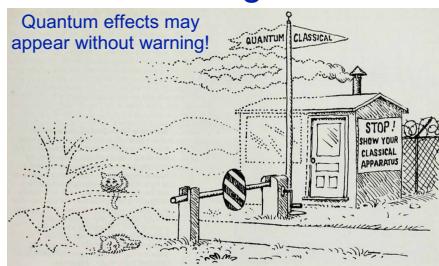
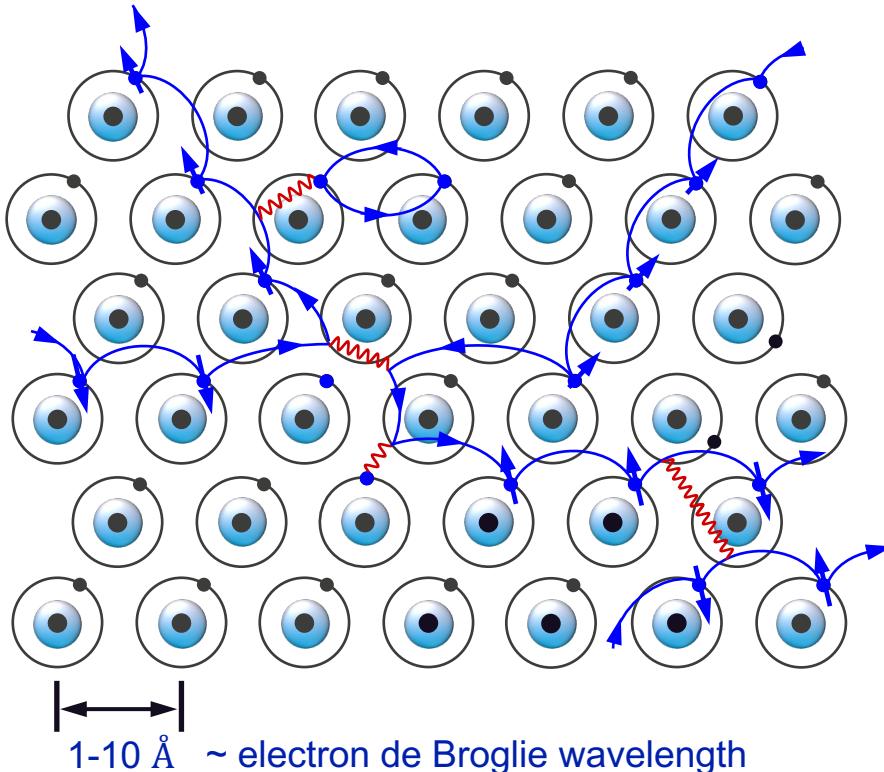
Los Alamos Computational Condensed Matter Summer School 2025
June 24, 2025



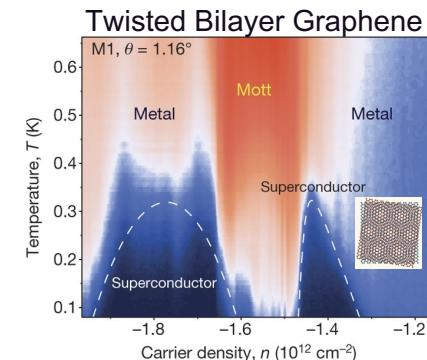
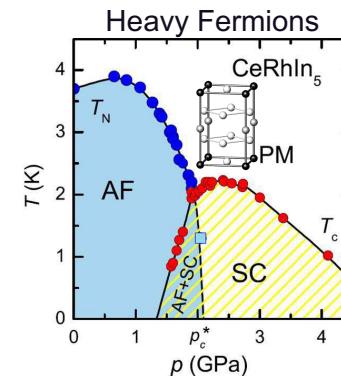
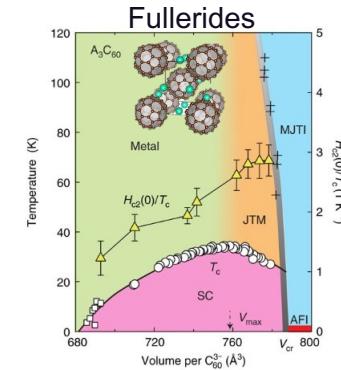
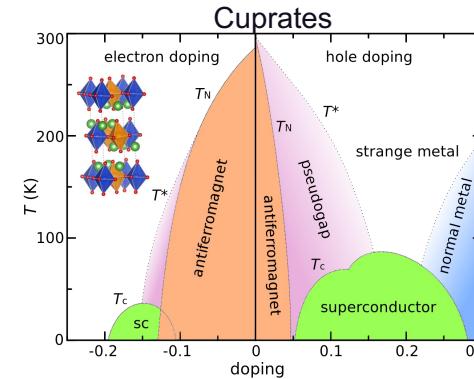
Phases of Matter



Microscopic View of a Solid



Wojciech H. Zurek.
Physics Today, 44:36-44 (1991)

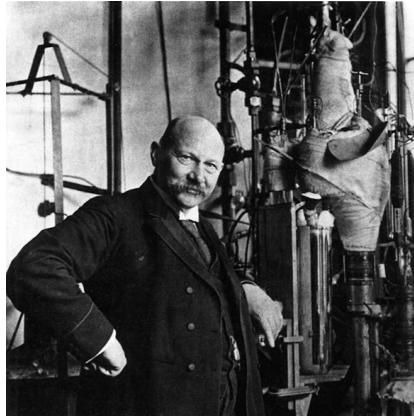
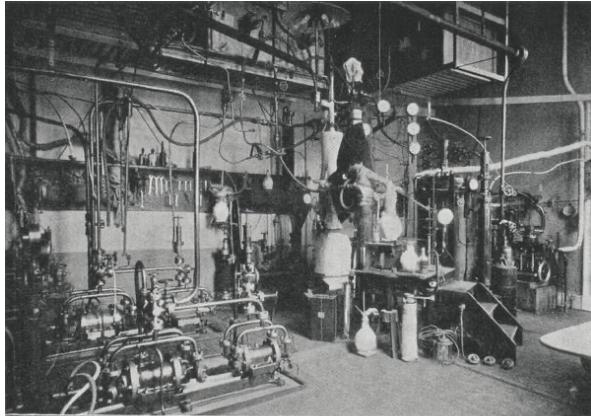


Richness of phases come from the competition of electron kinetic energy and Coulomb interactions, along with electron wave function geometry

Superconductivity

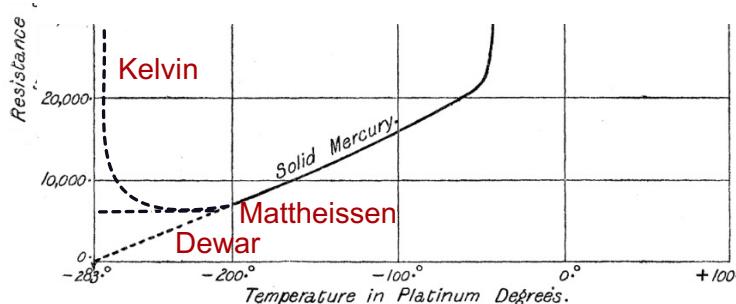


The Discovery of Superconductivity

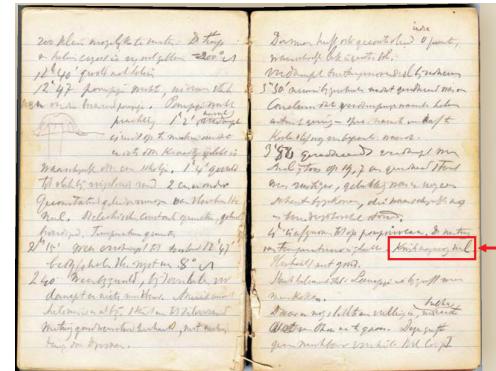


Heike Kamerlingh Onnes

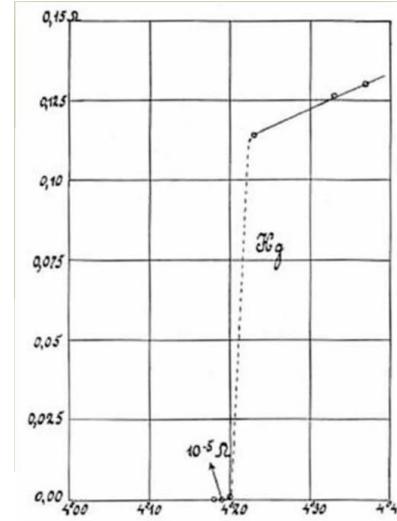
- 1908 Liquification of helium
- 1910 Interest in the low temperature of solids growing



Note: Theories Pre-dates Quantum Mechanics



A terse entry for 8 April 1911 in Heike Kamerling Onnes's notebook 56 records the first observation of superconductivity, "Mercury's resistance] practically zero [at 3 K.]".



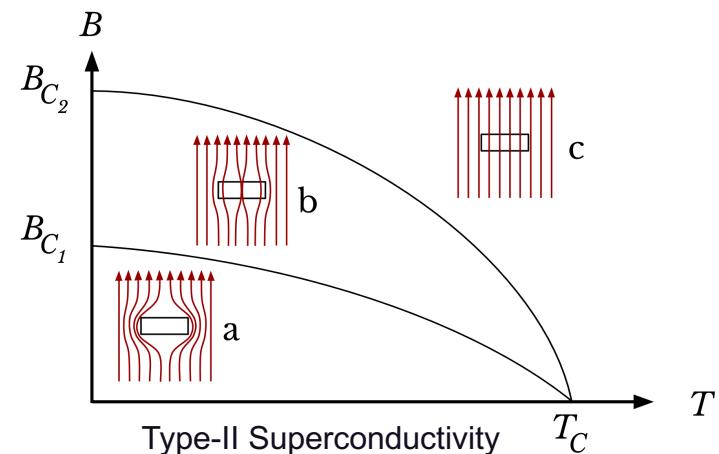
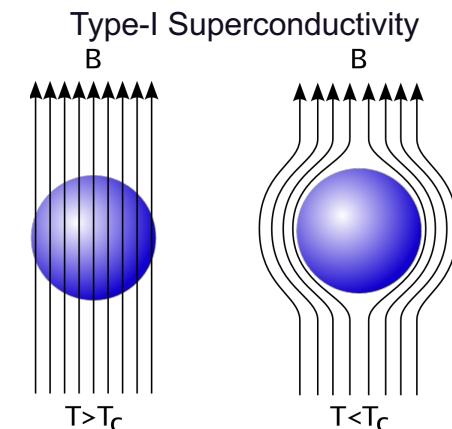
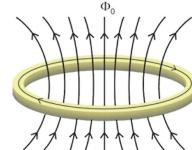
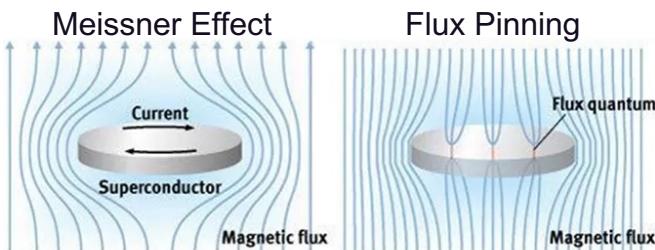
- 1911 “Mercury Practically Zero!”, The discovery of superconductivity in Hg

When you find a metal cool it and graph it!

K. Gavroglu. Ann. Phys. (Berlin) 524, No. 3–4, A61–A64 (2012)
Comm. Leiden. April 28, 1911; Comm. Leiden. May 27, 1911; Comm. Leiden. November 25, 1911.
Dirk van Delft and Peter Kes. Physics Today 63 (9), 38–43 (2010)

Other Superconducting Properties

- 1911 Heike Kamerlingh Onnes discovers superconductivity in Hg.
- 1913 Nobel Prize in Physics 1913 “for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium”
- 1914 Observation of Persistent Currents
- 1933 Walther Meißner and Robert Ochsenfeld finds a superconductor completely expels magnetic fields (Perfect Diamagnetism)
- 1935 Phenological Model by F. London and H. London
- 1935 Discovery of Partial expulsion at higher fields in alloys J.N. Rjabinin and L. Shubnikov



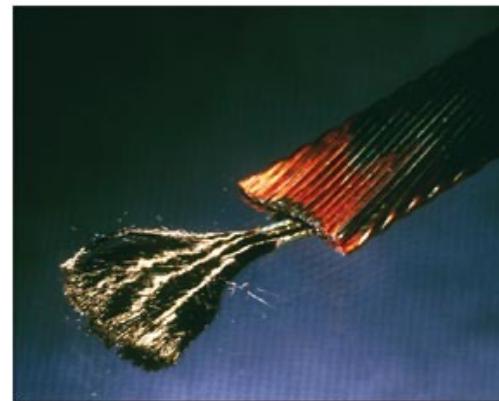
F. London and H. London. Proc. R. Soc. Lond. A14971–88 (1935)
Rjabinin, J. N. and Schubnikow, L.W. Physikalische Zeitschrift der Sowjetunion, 7(1), 122–125 (1935)
Rjabinin, J. N.; Shubnikow, L. W. Nature. 135 (3415): 581 (1935)

A. Shepelev and D. Larbalestier. The discovery of type II superconductors Cern Courier 25 October (2011)

Application of Superconductors



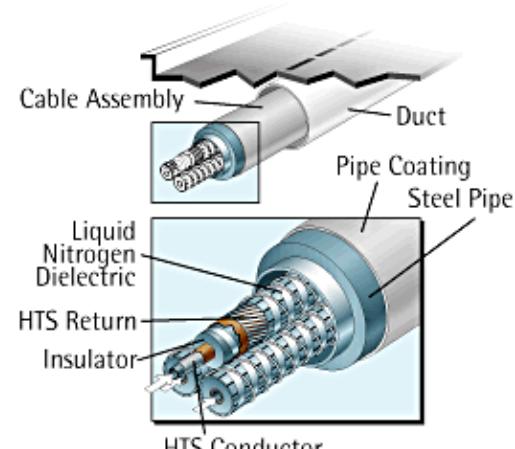
MRI



Superconducting cable (left) the heart of the magnets for the LHC at CERN (right), where experiments found the Higgs boson.



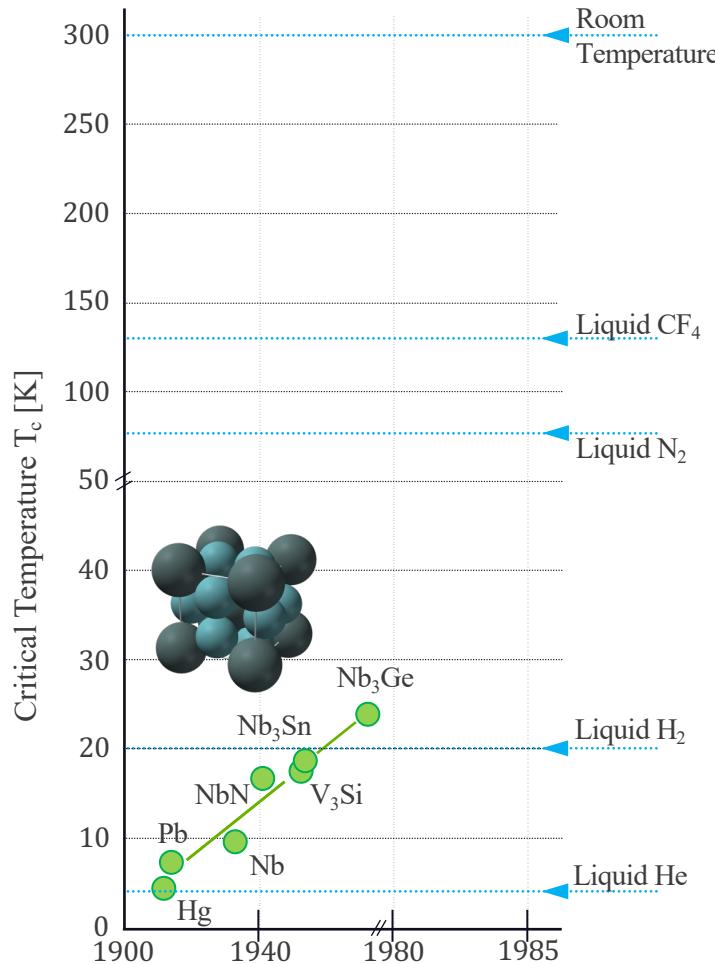
Superconducting Maglev trains



Transmission Lines

Currently used in: Holbrook, Long Island;
Essen, Germany; Albany, New York

Finding New Superconducting Materials



John Hulm



Bernd Matthias

'Rules' For High-Tc:

- Metals. Must have d-electrons (not just s, p, not f)
- High symmetry is good, cubic is best.
- Look for the right filling -- peak in the density of states at the Fermi level
- Stay away from oxides
- Stay away from magnetism
- Stay away from Theorists!

Attempts to Understand the Origin of Superconductivity

Failed Theories of Superconductivity

Theory invoked
molecular chains
reminiscent of solitons

Postulated: the theory
of SC cannot be
based on the theory
of noninteracting
electrons



Albert Einstein
(1879-1955)



Niels Bohr
(1885-1962)



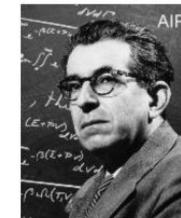
Ralph Kronig
(1905-1995)



John Bardeen
(1908-1991)



Werner Heisenberg
(1901-1976)



Fritz London
(1900-1954)

Work led to the
Landau theory of
phase transitions
and a successful
phenomenological model
of SC



Lev D. Landau
(1908-1968)



Felix Bloch
(1905-1983)



Léon Brillouin
(1889 -1969)



Max Born
(1882-1970)



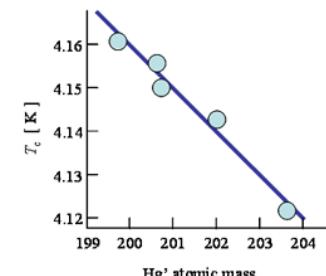
Herbert Fröhlich
(1905-1991)



Richard Feynman
(1918-1988)

Concluded that the
solution must go
beyond the scope of
perturbation theory

Hints to the problem: Zero Resistance, Perfect Diamagnetism, Isotope Effect



J. Schmalian. Modern Physics Letters B, 24(27), 2679-2691 (2012) [arXiv:1008.0447]
C. A. Reynolds, B. Serin, W. H. Wright, and L. B. Nesbitt, Physical Review 78, 487 (1950)
E. Maxwell, Physical Review 78, 477 (1950); ibid. 79, 173 (1950).

Theoretical Explanation of Superconductivity

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARdeen, L. N. COOPER, AND J. R. SCHRIEFFER
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

A theory of superconductivity is presented based on the fact that the interaction between electrons resulting from virtual exchange of phonons is attractive when the energy difference between the two electrons is less than the pair binding energy, Δ . It is favorable to form a superconducting phase when the electron density is large enough so that the average Coulomb interaction. The normal phase is described by the Bloch individual-particle model. The ground state of a superconductor, formed from the condensation of a number of Cooper pairs, in which electrons are virtually excited in pairs of opposite spin and momentum, is described by a wave function whose amplitude is proportional to an average $(\langle k \rangle^2)$, consisting of excited states in

one-to-one correspondence with those of the normal phase is obtained by specifying occupation of certain Bloch states and by using the theory to form a linear combination of wave pair functions. The theory predicts the Meissner effect, the London penetration depth, the temperature dependence of the magnetic susceptibility, the temperature variation of the resistivity, and the temperature dependence of the Hall effect. The theory also predicts the Meissner effect, the London penetration depth, and the temperature dependence of the resistivity, and the temperature dependence of the Hall effect.

INTRODUCTION
THE main facts which a theory of superconductivity must account for are (1) a second-order phase transition at the critical temperature T_c , (2) an electronic heat varying as $\exp(-T_c/T)$ near $T=0^\circ\text{K}$ and other evidence for an energy gap for individual particles, (3) the Meissner effect, (4) the London-Oschwinger effect ($B=0$), (5) effects of finite temperature with infinite conductivity ($R=0$), and (6) the dependence of T_c on isotope mass, $T_c/M = \text{const}$. We present here a theory which accounts for all of these, and in addition gives quantitative predictions for the magnetic fields, penetration depths and their variation with temperature when evaluated from experimentally determined values of the parameters.

When superconductivity was discovered by Onnes (1911), and for many years afterwards, it was thought to consist simply of a vanishing of all electrical resistance in metals. A major advance was the discovery of the Meissner effect (1933), which showed that a superconductor is a perfect diamagnet; magnetic flux is excluded from all but a thin penetrative region near the surface. It was very long after 1933 (London 1936, London 1937) that a phenomenological theory of the electromagnetic properties in which the diamagnetic aspects were assumed

* This work was supported in part by the Office of Defense Research, U. S. Army. One of the authors (J. R. Schrieffer) was aided by grants from the National Science Foundation and the Office of Naval Research. The results are based on a thesis submitted by Dr. Schrieffer for the requirements for a Ph.D. degree in Physics, University of Illinois, 1957.

† Present address: Department of Physics and Astronomy, The Ohio State University.

‡ Present address: Department of Theoretical Physics, University of Illinois, Urbana, Illinois.

§ Present address: Commissariat à l'Energie Atomique, Paris, France.

** W. Meissner and R. Ochsenfeld, Naturwiss. 21, 707 (1933).

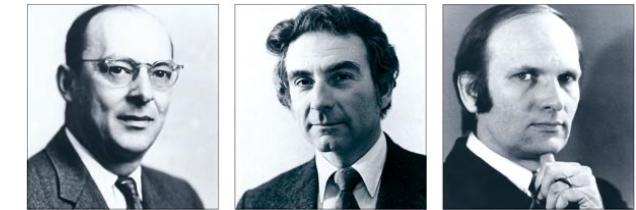
†† F. London, Proc. Roy. Soc. (London) A149, 71 (1955); Physica 2, 341 (1955).

1175

The BCS paper, published in Physical Review on 1 December 1957.

Steven Weinberg From BCS to the LHC Cern Courier 21 January (2008)
J. Bardeen, L. N. Cooper, and J. R. Schrieffer. Phys. Rev. 108, 1175 (1957)
<https://www.insidescience.org/news/superconducting-dance-electron-pairs>

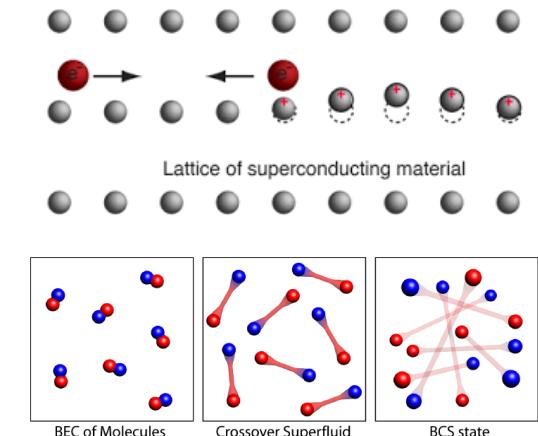
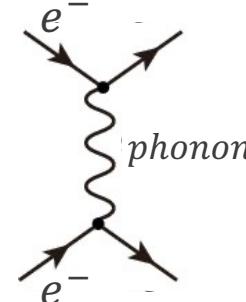
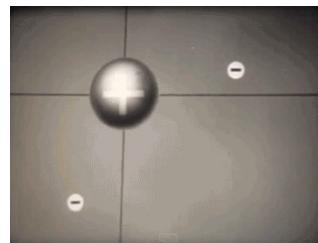
BCS Theory of Superconductivity



John Bardeen
Leon Cooper
Robert Schrieffer
Nobel Prize in Physics 1972

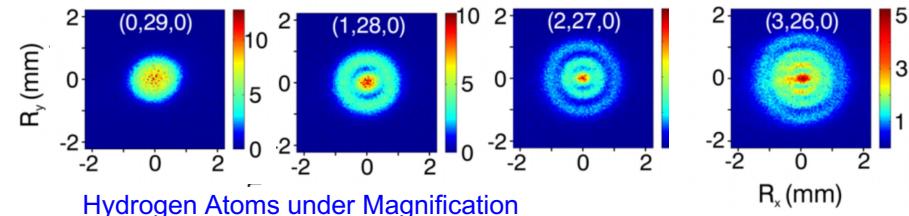
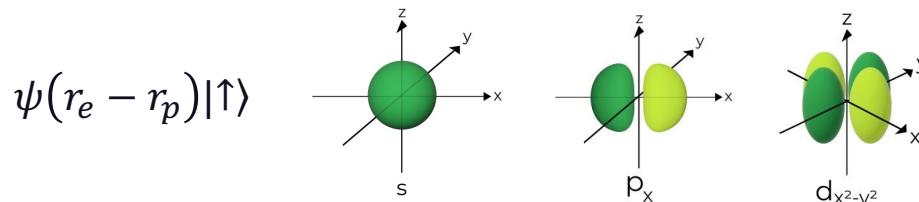


- Theory builds on: Sommerfeld model of electrons in solids. Fermi Surface. Landau Fermi Liquid Theory.
- In spite of the repulsive Coulomb interaction, electrons of opposite momenta bind in pairs, because electrons polarize the crystal lattice



The Pair Wave Function

Atomic Wave functions



Cooper pair Wave functions

In a superconductor electrons pair and all pairs occupy the same quantum state.

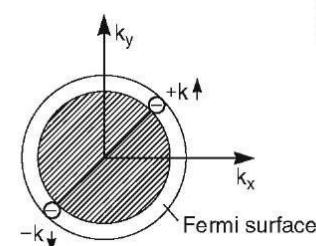
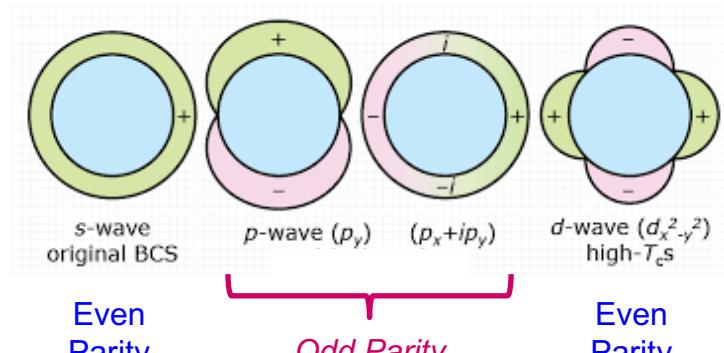
$$\Psi(\mathbf{k}, i; -\mathbf{k}, j) \frac{(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)}{\sqrt{2}}$$

Singlet

$$\Psi(\mathbf{k}, i; -\mathbf{k}, j)|\uparrow\rangle|\uparrow\rangle$$

Triplet

Symmetries of the superconducting order parameter



The relative phase of the wavefunction of two superconductors can be measured!

Josephson Effect

Nobel Prize in Physics 1973



Brian Josephson

Superconducting Gap Function

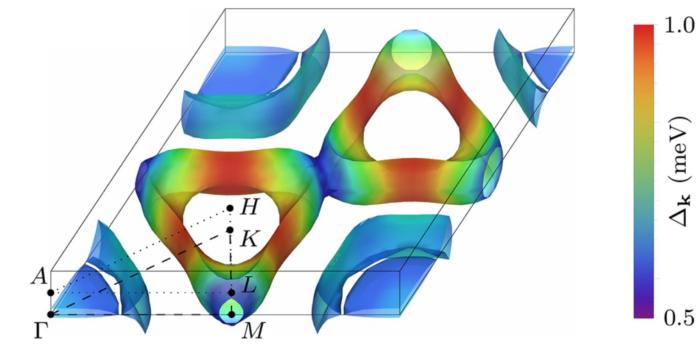
A superconductor has a gap $\Delta_i(\mathbf{k})$, which is simple related to the superconducting order parameter

$$\Delta_i(\mathbf{k}) = - \sum_{i,k'} V_{ik,jk'} \frac{\Delta_j(\mathbf{k}')}{2\sqrt{\varepsilon_{jk'}^2 + \Delta_j^2(\mathbf{k}')}} \tanh \left[\frac{\sqrt{\varepsilon_{jk'}^2 + \Delta_j^2(\mathbf{k}')}}{2k_B T} \right]$$

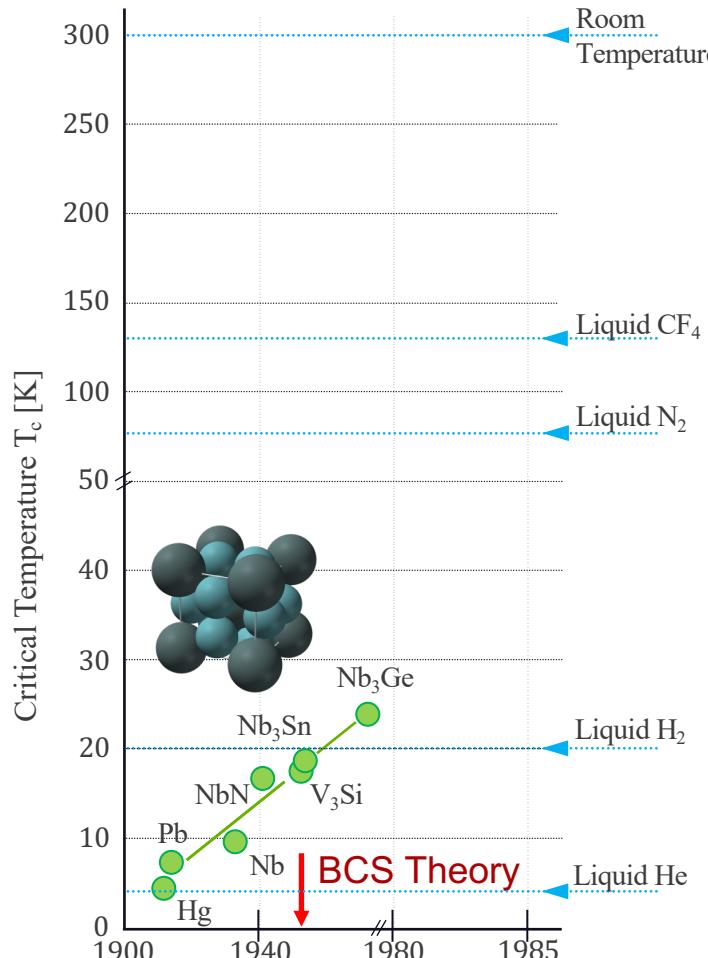
- One band case (one Fermi surface): pairing instability requires effective attractive interaction, i.e. phonons
- Multiple bands (multiple Fermi Sheets): pairing can result from attraction or repulsion among the bands
- Theory was almost universally accepted. Properties that were measured the theory could explain, and it could predict many experiments using only a small set of parameters.
- It is rigorous and builds on top of a successful theory of the normal state.

$$T_c = 1.13\omega_D e^{-1/N(0)V}$$

↑
DOS at
Fermi level



Slow Down in the 70s and Early 80s...



13



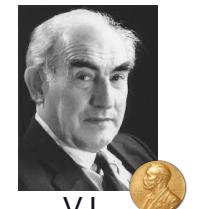
Bernd Matthias

“BCS tells us everything but finds us nothing”

V.L. Ginzburg and D.A. Kirzhnits, *On the problem of high temperature superconductivity* 345

1. Introduction

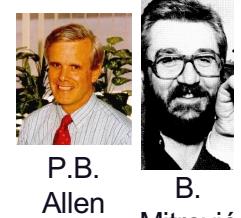
The critical temperature T_c of known superconductors does not exceed 20–21°K. Using traditional ways for choosing new alloys one can hope to raise this temperature by 5–10°. This, however, does not solve the problem of a radical increase of T_c up to liquid air temperature (about 80–100°K) or, even more radically, up to room temperatures (about 300°K). The importance of this problem and also the scepticism expressed sometimes with respect to the possibility of its solution (see, particularly below) induced us to consider it once again.



V.L.
Ginzburg
Nobel Prize in
Physics 2003

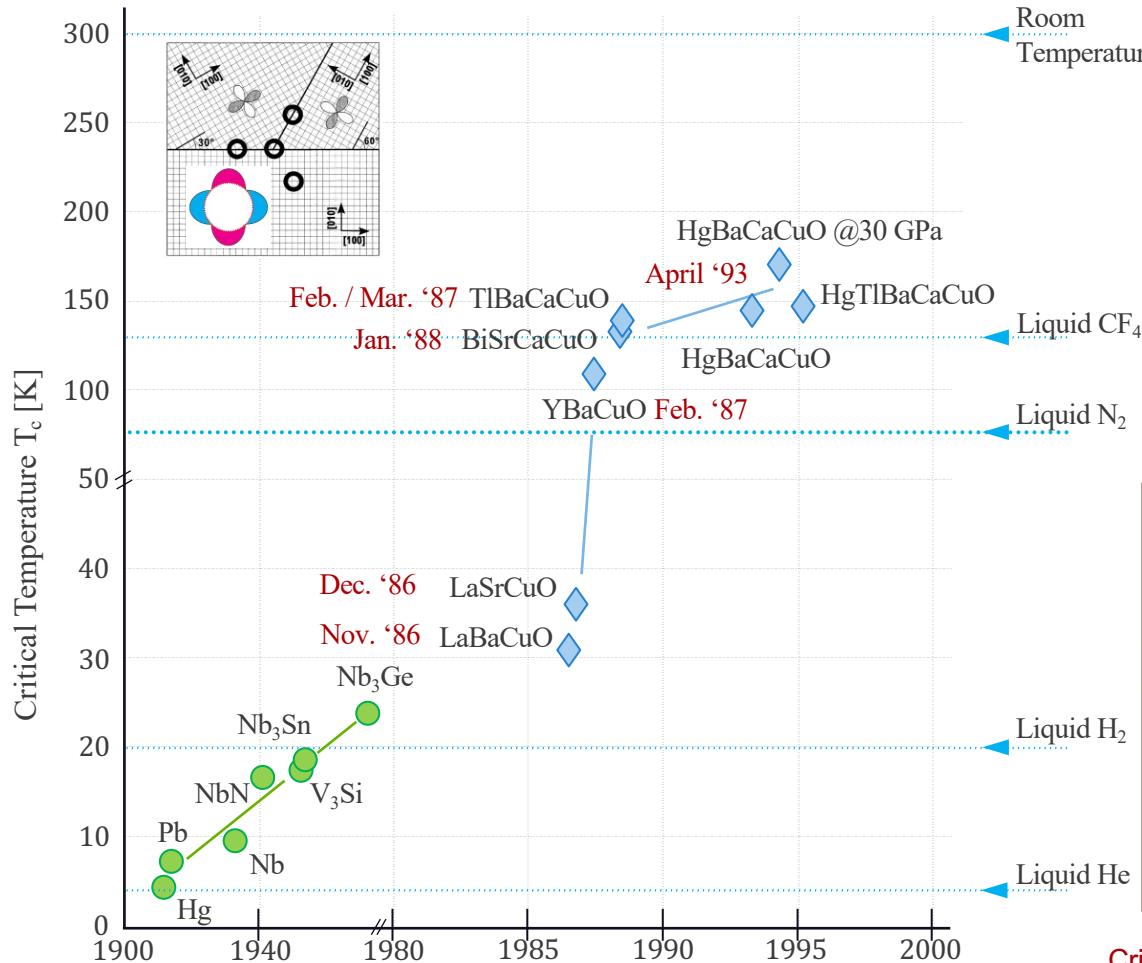
21. Is THERE A MAXIMUM T_c ?

As of January 1982, there has been a maximum T_c of ~23°K for the last 8 years.¹⁸⁴ This represents a normal fluctuation in the steady trend of the 3°K increase of T_c per decade¹⁸⁵ that has occurred since 1911. However, the investment of manpower and money in the last decade has been large and the results disappointing. Nevertheless, it is clearly dangerous to assert¹⁸⁶ that T_c is saturating at a maximum. Two different sensible arguments were advanced in the past^{15,187} to set a limit for T_c , and each was later shown to be wrong.^{76,188} Meanwhile the maximum T_c jumped 3°K.



P.B.
Allen
B.
Mitrović

A Sudden Breakthrough...



J. Georg
Bednorz



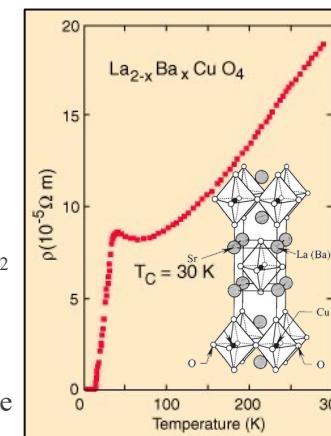
K. Alexander
Müller



Nobel Prize in Physics 1987
One of the fastest awards on record!

'Violates all of Matthias' Rules:

- Near a (Mott) insulator
- Layered Perovskite Structure
- Is an oxide
- Near an AFM magnet

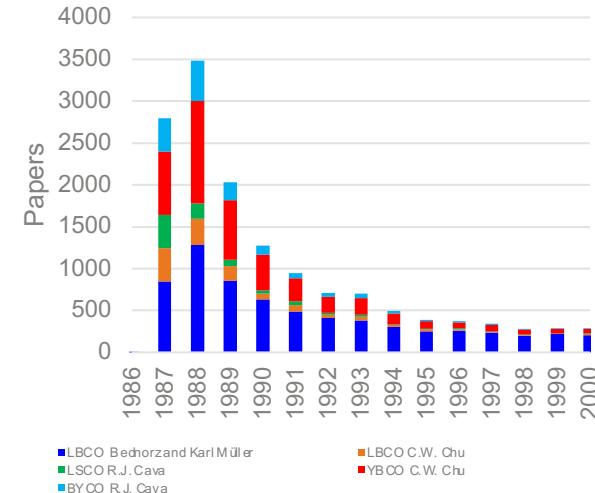


Critically: These materials do not fall within the BCS framework.

March 18, 1987: Woodstock of Physics



- The discoveries were so recent that no papers on them had been submitted by the deadline. However, a last-minute session was added to discuss the new research.
- Session started at 7:30pm with lines forming at 5:30pm and finished at 3:30am
- Nearly 2,000 scientists tried to squeeze into the ballroom, with more watching outside the room on television monitors.
- The session consisted of a marathon of talks, given by about 50 speakers



<https://www.aps.org/publications/apsnews/updates/woodstock.cfm>

Possible Ingredients of High-T_c in the Cuprates

➤ Spin-Fluctuations

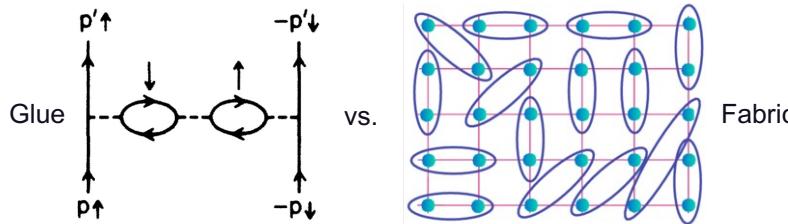
- d-wave pairing near a spin-density-wave instability D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch. Phys. Rev. B 34, 8190(R) (1986)
- Spin-fluctuation-induced superconductivity in the copper oxides: A strong coupling calculation. P. Monthoux and D. Pines. Phys. Rev. Lett. 69, 961 (1992)

➤ Plasmons / Excitons

- A Cu d-d excitation model for the pairing in the high-T_c cuprates. W. Weber Zeitschrift für Physik B Cond. Matt. 70, 323–329 (1988)
- Landscape of coexisting excitonic states in the insulating single-layer cuprates and nickelates. C. Lane and J.-X. Zhu. Physical Review B 101, 155135 (2020)
- Acoustic plasmons and conducting carriers in hole-doped cuprate superconductors. A. Singh, H. Y. Huang, C. Lane, J. H. Li, J. Okamoto, S. Komiya, R.S. Markiewicz, A. Bansil, T. K. Lee, A. Fujimori, C. T. Chen, and D. J. Huang. Phys. Rev. B 105, 235105 (2022)

➤ Resonating Valence Bond State

- The Resonating Valence Bond State in La₂CuO₄ and Superconductivity. P. W. Anderson Science 235, 1196–1198 (1987)
- A renormalised Hamiltonian approach to a resonant valence bond wavefunction. F C Zhang, C Gros, T M Rice and H Shiba. Supercond. Sci. Technol. 1, 36 (1988)
- A Unified Theory Based on SO(5) Symmetry of Superconductivity and Antiferromagnetism. S.-C. Zhang. Science 275, 1089 (1997)



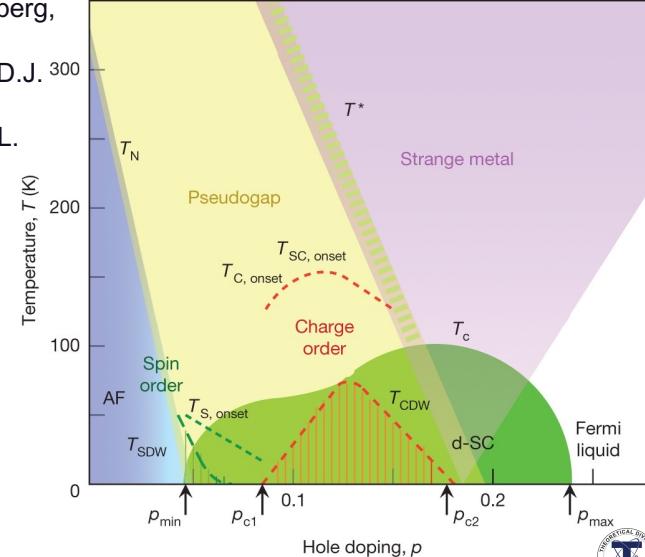
➤ CDW / Phonon Softening

- CDW and SDW mediated pairing interactions. N.E. Bickers, D.J. Scalapino and R.T. Scalettar. Int. J. Mod. Phys. B 1, 687–695 (1987)
- Vibronic mechanism of high-T_c superconductivity. M. Tachiki, M. Machida, and T. Egami. Phys. Rev. B 67, 174506 (2003)
- Competing stripe and magnetic phases in the cuprates from first-principles. Y. Zhang, C. Lane, J.W. Furness, B. Barbiellini, J.P. Perdew, R.S. Markiewicz, A. Bansil, and J. Sun. PNAS, 117, 68 (2020)

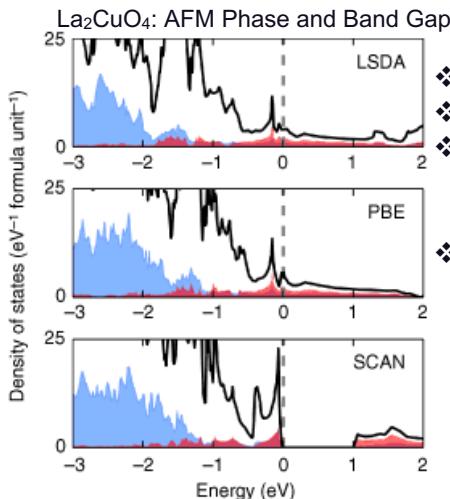
➤ Stripes and Intertwined orders

- Are Stripes a Universal Feature of High-T_c Superconductors? Barbara Goss Levi Physics Today 51 (6), 19–22 (1998)
- Colloquium: Theory of intertwined orders in high temperature superconductors. Eduardo Fradkin, Steven A. Kivelson, and John M. Tranquada. Rev. Mod. Phys. 87, 457 (2015)
- The Physics of Pair-Density Waves: Cuprate Superconductors and Beyond. D.F. Agterberg, J.C.S. Davis, S.D. Edkins, E. Fradkin, D.J. Van Harlingen, S.A. Kivelson, P.A. Lee, L. Radzihovsky, J.M. Tranquada, and Y. Wang.

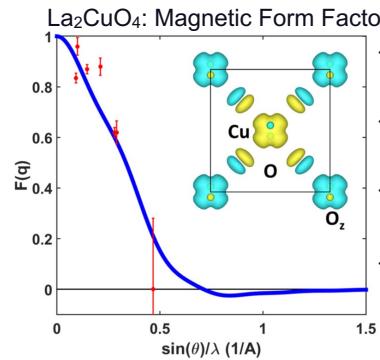
Still Many Open Questions!



First-Principles Ground State and Excitation Properties

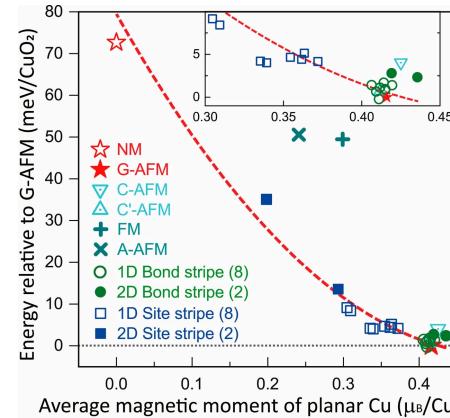


- ❖ LSDA and PBE: Metal
- ❖ SCAN: AFM Insulator (**no U**)
- ❖ Band Gap
 - Theory: 0.98 eV
 - Expt. (Optics): ~ 1.0 eV
- ❖ Generalize Kohn-Sham gives fundamental gap (no excitonic effects)

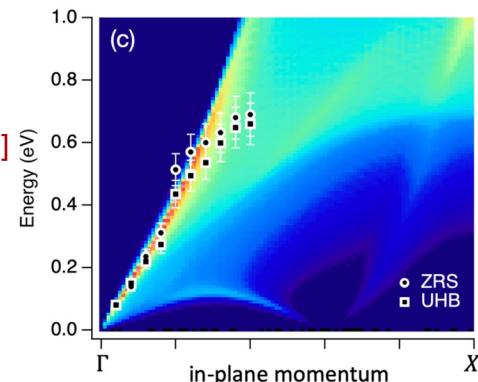


- ❖ AFM state yields moments on Cu and O_z sites.
- ❖ The predicted moment on Cu: **0.495 μB [Exp. $0.48 \pm 0.15 \mu\text{B}$]**
- ❖ Cu-O hybridization effects intrinsically included.
- ❖ In-plane magnetization has quadropole form.

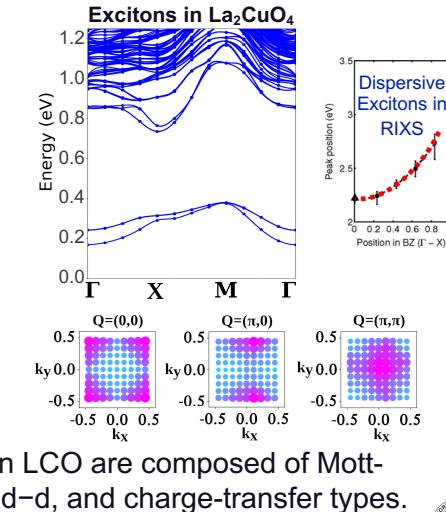
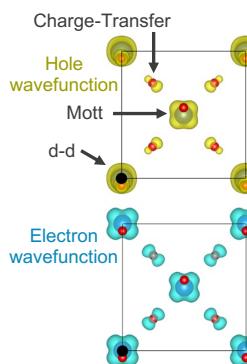
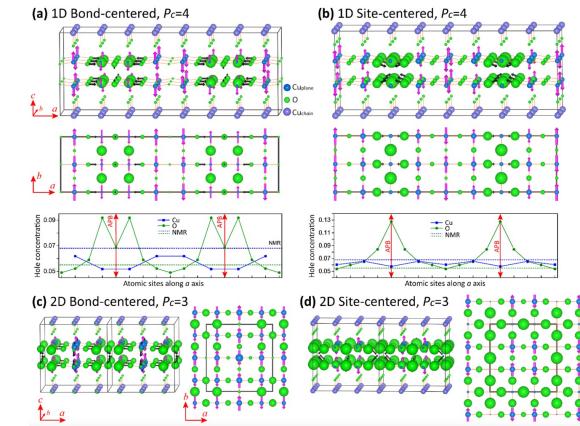
Y. Zhang, C. Lane, et al. PNAS, 117, 68 (2020)
 C. Lane, et al. Physical Review B 98, 125140 (2018)
 J.W. Furness, et al. Communications Physics 1, 11 (2018)
 C. Lane and J.-X. Zhu Physical Review B 101, 155135 (2020)
 A. Singh, H. Y. Huang, C. Lane, et al. Physical Review B 105, 235105 (2022)



- ❖ Ground state has many competing phases; role in superconducting glue, models of pseudogap, nematicity, temperature effects, etc.

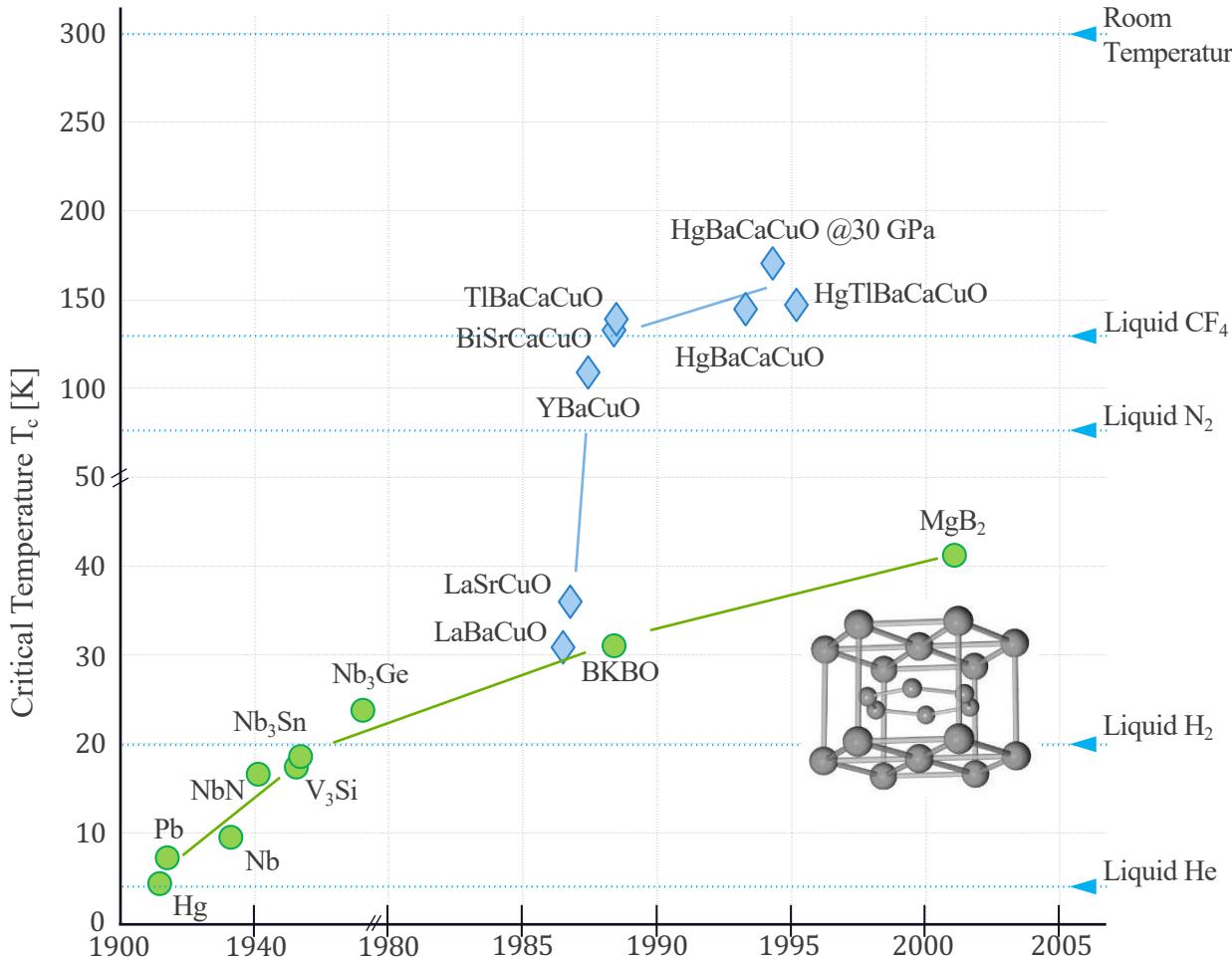


- ❖ Agreement between theoretical loss function for LSCO and RIXS spectra

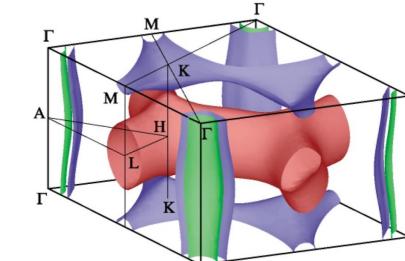


- ❖ Excitons in LCO are composed of Mott-Hubbard, d-d, and charge-transfer types.

A Conventional Surprise



Jun Nagamatsu, Norimasa Nakagawa, Takahiro Muranaka, Yuji Zenitani & Jun Akimitsu Nature 410, 63–64 (2001)
P.C. Canfield and G.W. Crabtree. Physics Today 56 (3), 34–40 (2003)



- Quasi-two-dimensional layered system, violate all of Matthias' Rules
- Discovered by accident (searching for Ferromagnets)
- Conventional, phonon mediated superconductor
- Violates $T_c < 23$ K
- Multicomponent order parameter, multiple active Fermi sheets
- Workhorse material for MRIs and the LHC

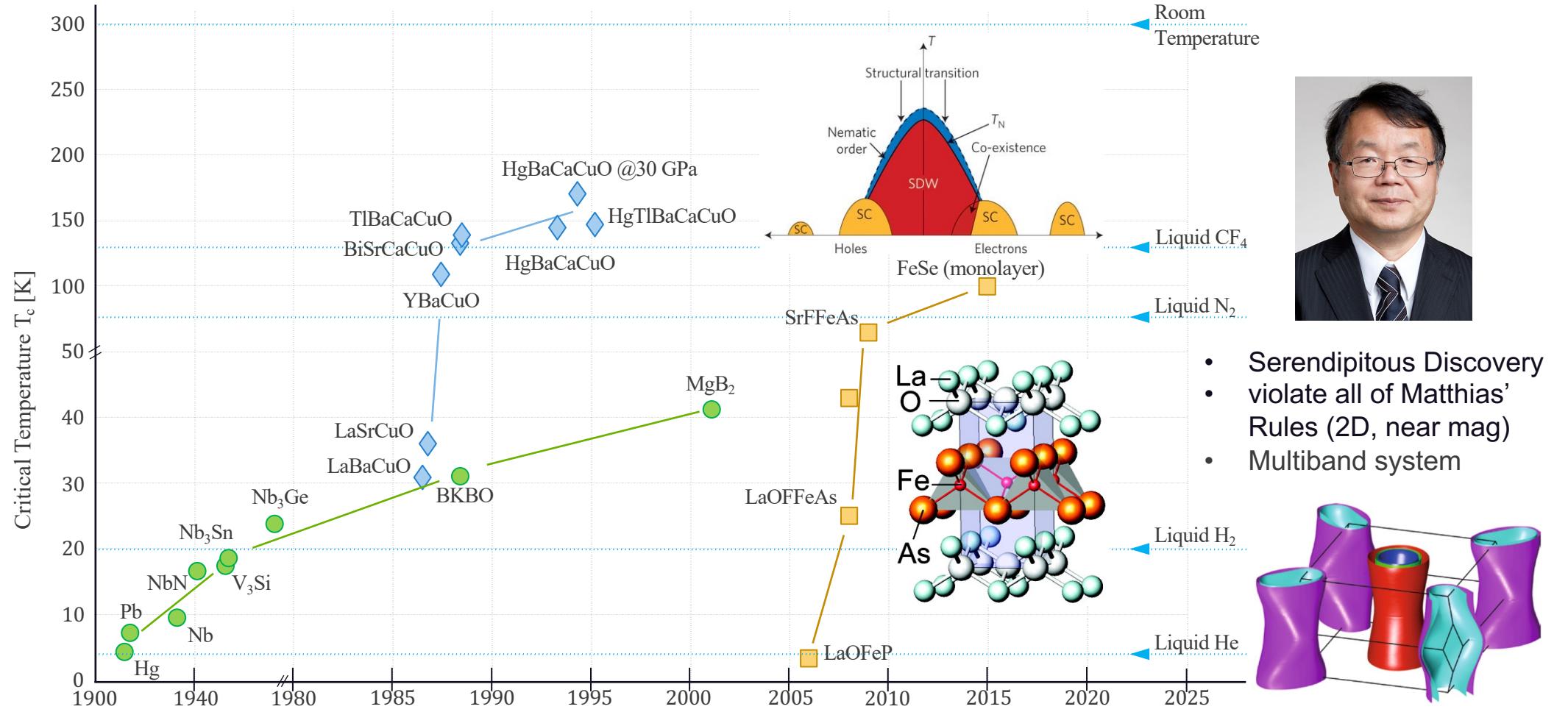
Low Temperature Heat Capacities of Magnesium Diboride (MgB_2) and Magnesium Tetraboride (MgB_4)

BY ROBINSON M. SWIFT AND DAVID WHITE¹
RECEIVED FEBRUARY 14, 1957

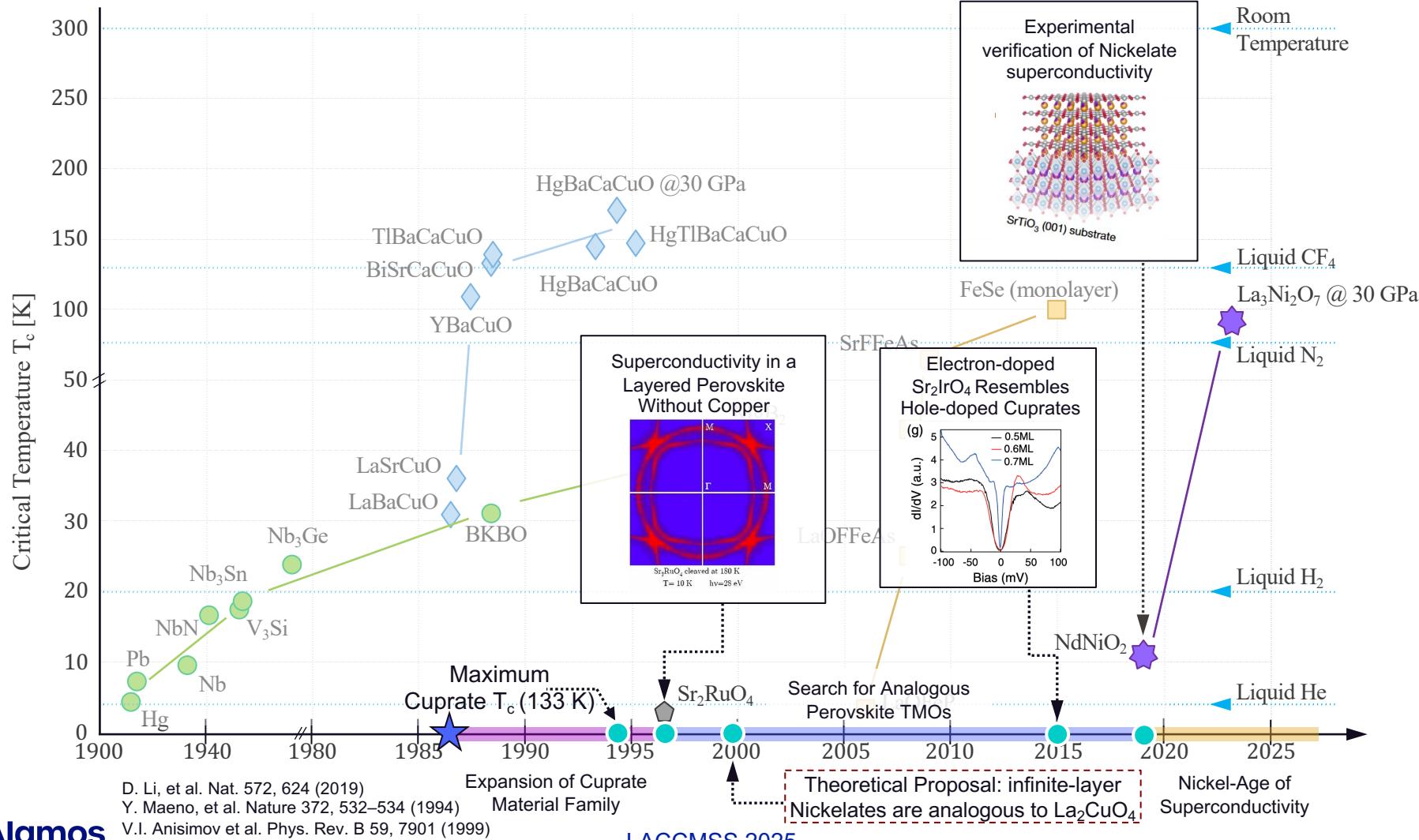
The heat capacities of magnesium diboride (MgB_2) and magnesium tetraboride (MgB_4) were measured in the temperature range 1.8°C to 300°K. The values of the entropy and free energy at low temperatures have been calculated at integral values of temperature. The entropy at 298.16°K. of MgB_2 is 8.60 ± 0.04 cal. deg.⁻¹ mole⁻¹, that of MgB_4 is 12.41 ± 0.09 cal. deg.⁻¹ mole⁻¹. The heat capacity of these compounds at the lowest temperatures measured do not exhibit at T^2 relationship characteristic of some substances having a layer structure.

Should have been discovered in 1957!

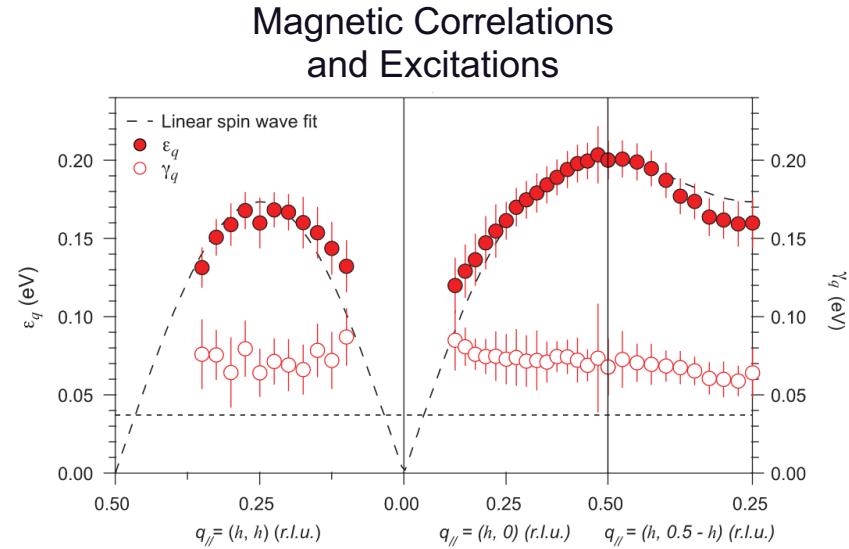
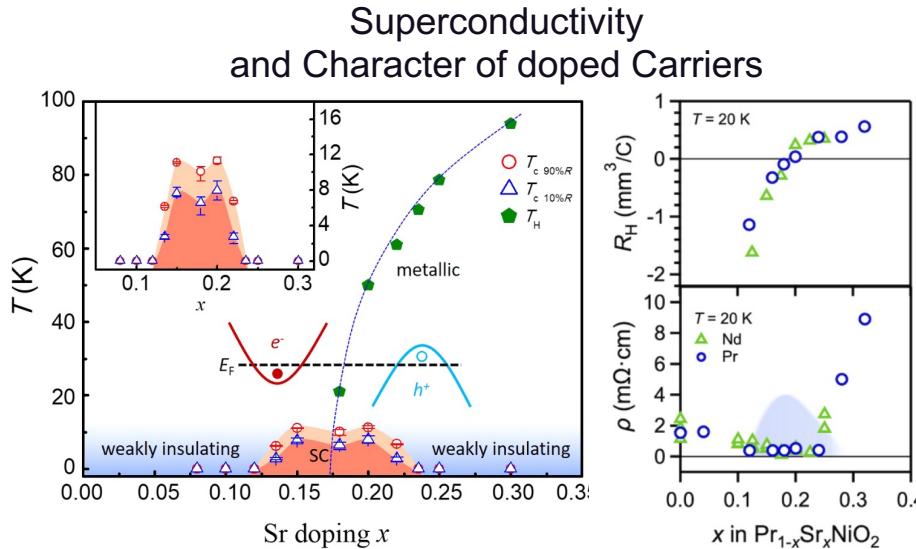
Iron Age of Superconductivity



Birth of a New Age...



Electronic and Magnetic Properties of the Infinite-Layer Nickelates



Transport

- SC in $(\text{La},\text{Nd},\text{Pr})\text{NiO}_2$, $T_c \sim 12 - 14$ K
- Dip in SC dome of $(\text{La},\text{Nd})\text{NiO}_2$, indicates possible stripes
- Under- and over-doped regime weakly insulating
- R_H crosses changes sign at optimal doping

XAS / RIXS

- Hole resides on $\text{Ni}-d_{x^2-y^2}$
- dd transition distinct from Cuprates
- Hole might be forming singlets
- Possible minor $5d$ doping

RE Substitution

- Role of f -electron probably minimal
- $5d/3d$ hybridization might be important

Neutron Scattering

- No AFM order, but strong non-local correlations

μ SR / $\chi(H,T)$

- Intrinsic magnetism
- Strong non-local magnetic correlations
- Glassy short-range behavior
- Weak to intermediate spin-cluster interactions

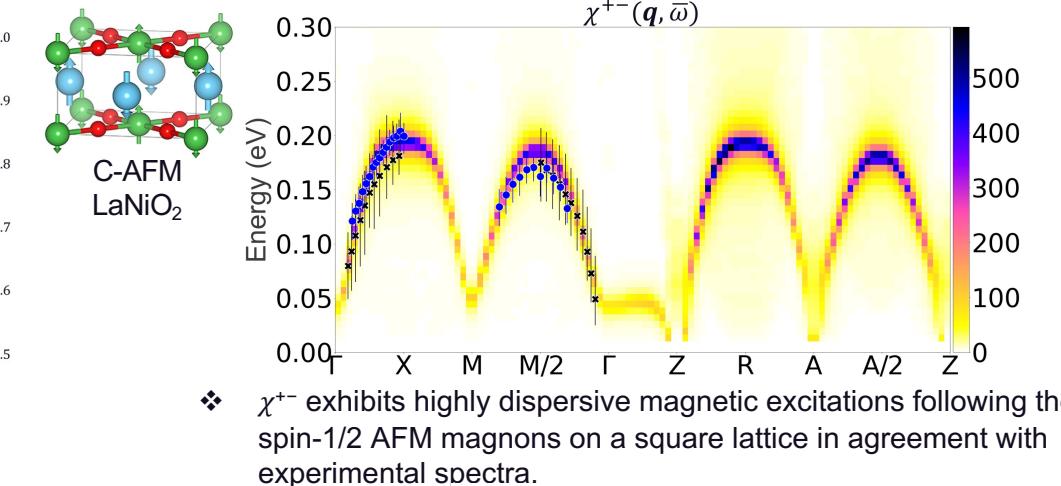
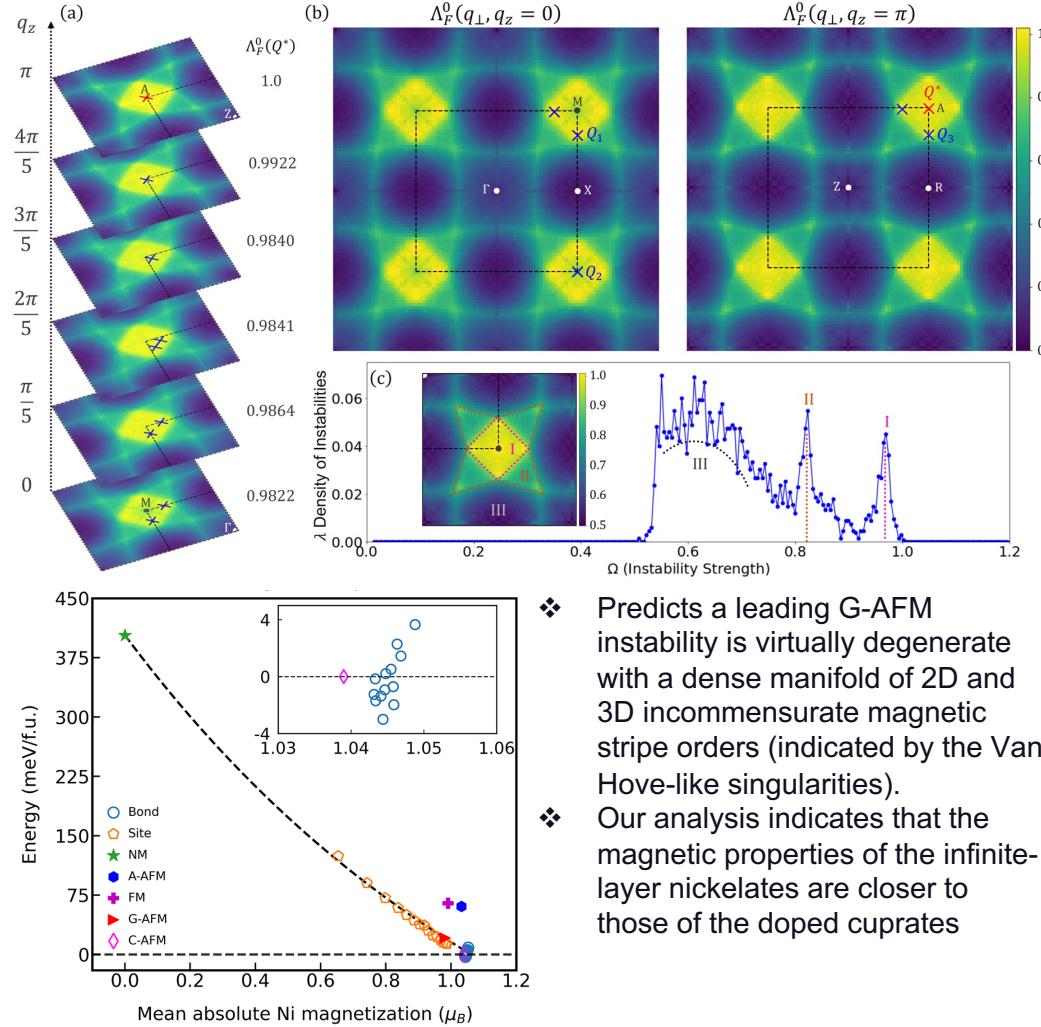
RIXS

- 2D AFM spin wave dispersion
- $J_{ex} = 63.6 \pm 3.3$ meV \sim cuprates
- Damping appears to be constant in zone

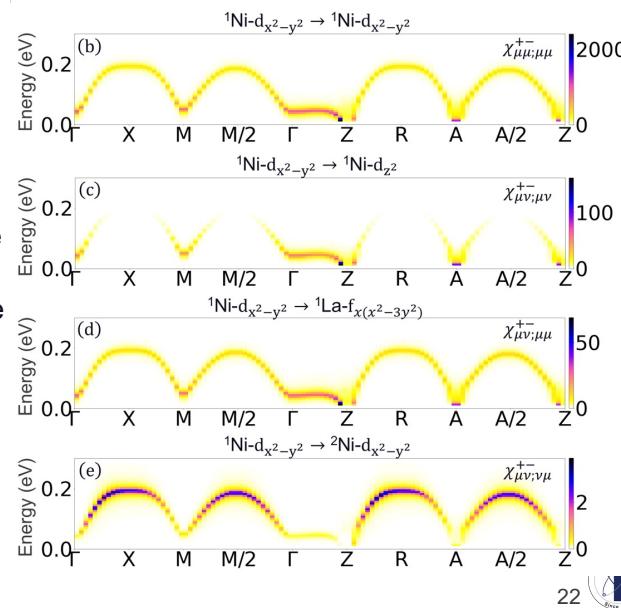
S. Zeng, et al. *Phys. Rev. Lett.* 125, 147003 (2020)
 M. Osada, et al. *Nano Lett.* 20, 8, 5735–5740 (2020)
 M. Osada, et al. *Phys. Rev. Materials* 4, 121801 (2020)
 M. Hepting, Nat. Mater. 19, 381–385 (2020)
 D. Li, et al. *Nat.* 572, 624–627 (2019)
 M. Hayward, et al. *Solid state sciences* 5(6) 839–850 (2003)
 M. Hayward, et al. *JACS*, 121(38), 8843–8854 (1999)

K. Lee, et al. *arXiv:2203.02580*
 J. Fowlie, et al. *Nature Physics (in press)* (2022). *arXiv:2201.11943*
 R. A. Ortiz, et al. *Phys. Rev. Research* 4, 023093 (2022)
 S. Zeng, et al. *Sci. Adv.* 8, eaab1927 (2022)
 H. Lu, et al. *Science* 373, 213–216 (2021)
 M. Osada, et al. *Adv. Mater.* 33, 2104083 (2021)
 M. Rossi, et al. *Phys Rev B* 104, L220505 (2021)

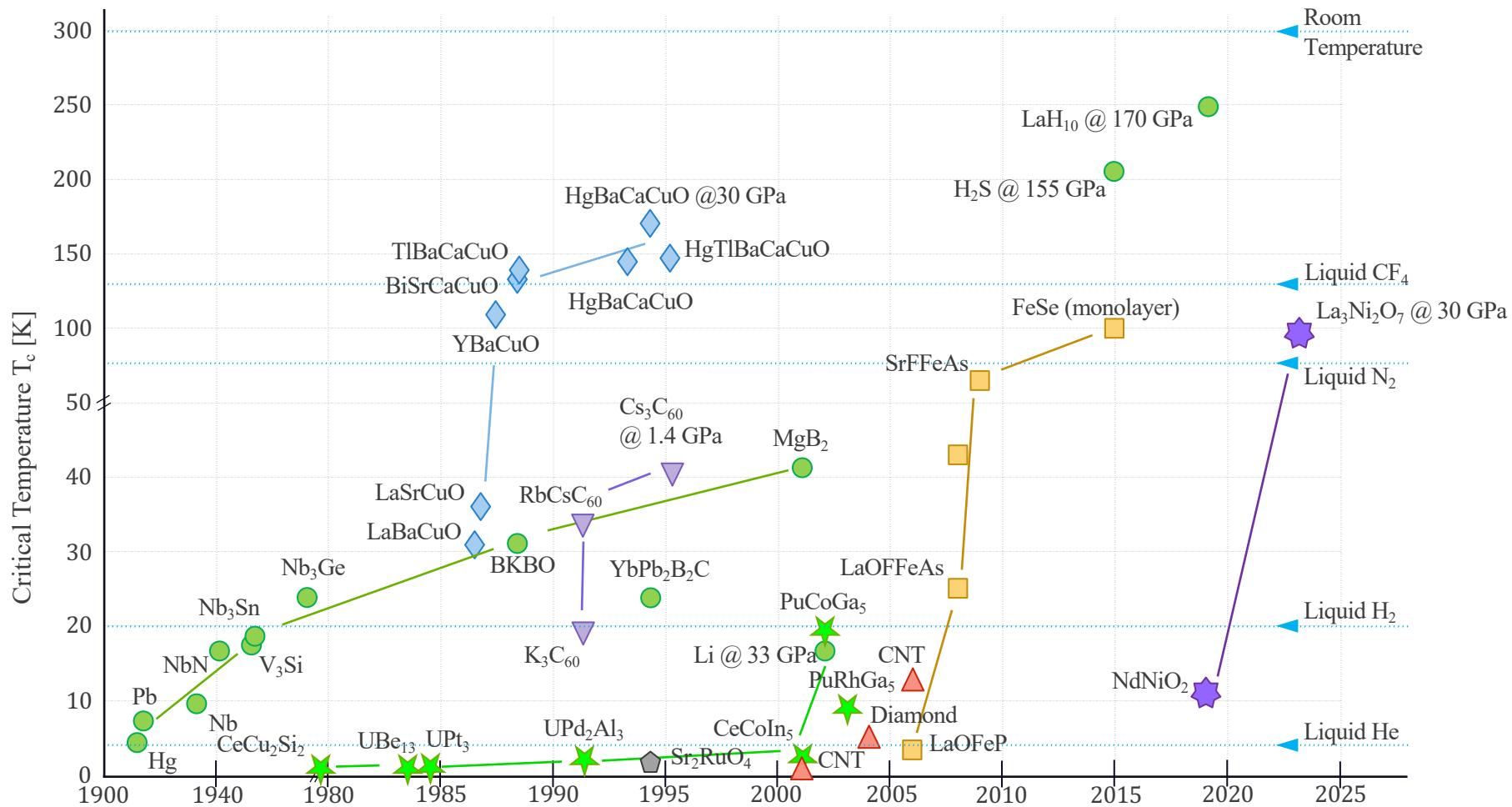
Magnetic Instabilities and Excitations in LaNiO_2



- ◆ non-trivial Ni-La hybridization could contribute to the long-range behavior of the Heisenberg exchange parameters
- ◆ To reproduce both the charge and magnetic fluctuations, a minimum of two orbitals is required.



Present State of Superconducting Materials



Concluding Remarks

Summary/Outlook

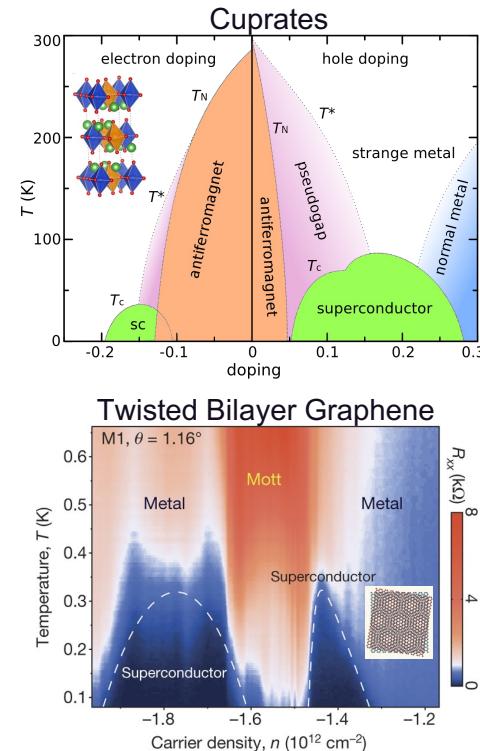
- BCS pairing theory of superconductivity is a prototypical example of a condensed matter physics problem. It has inspired the Weinberg Salam model for Electroweak interactions.
- So far there is no quantitative theory of superconductivity in strongly correlated materials. The limits on T_c are still unknown
- Almost all known superconducting materials were found without theoretical guidance.
- The next-generation of first-principles approaches is beginning to captures a more holistic picture of correlated materials, giving way to a more quantitative theory of materials.

Superconductivity Illustrates the Process of Scientific Discovery

- Non-linear, convoluted, different from the linearity to courses and books
- Knowledge builds overtime on top of previous discoveries – “There are decades where nothing happens; and there are weeks where decades happen”.
- Interplay of technical advances and scientific discovery

Open Challenges:

- What is the origin of these phases of matter?
- Why and how does the transition temperature depend on specific material properties?
- Can we predict/find new materials with even higher transition temperatures?
- How can we move from serendipitous discovery to theoretical design of materials?



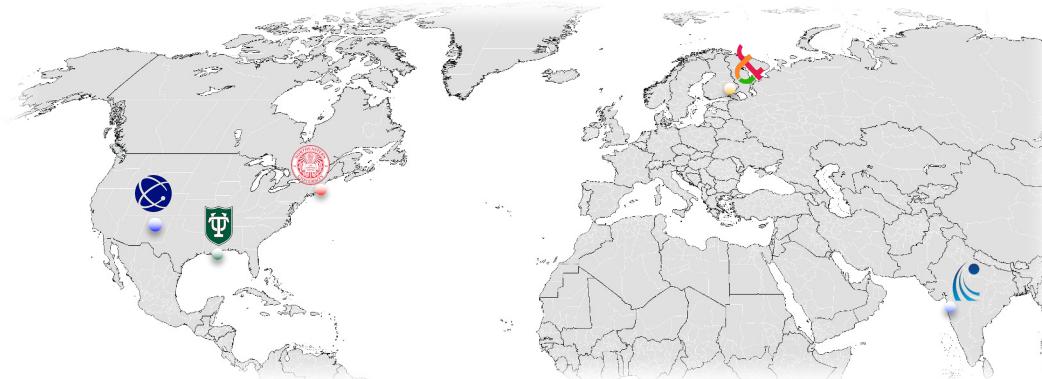
Acknowledgments / Collaborators



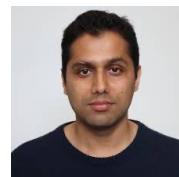
Dr. Christopher
Lane



Dr. Jian-Xin
Zhu



Prof. Jianwei
Sun



Dr. Kanun
Pokharel



Dr. James
Furness



Dr. Ruiqi
Zhang



Prof. Bahadur
Singh



Prof. Arun
Bansil



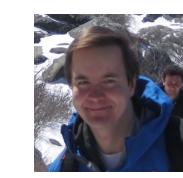
Prof. Robert
Markiewicz



Dr. Matt
Matzelle



Prof. Bernardo
Barbiellini



Dr. Johannes
Nokelainen

