Introduction to Nuclear reactions

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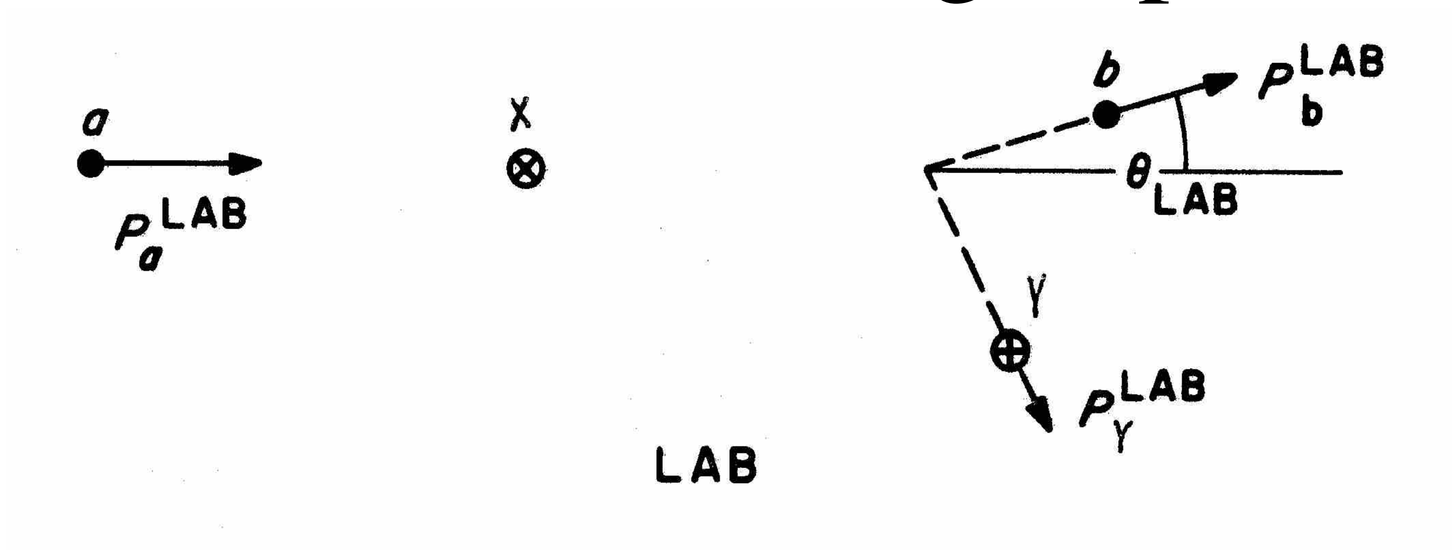
Hodgson, Gadioli, and Erba, Introductory Nuclear Physics (1997)
N.A. Jelley, Fundamentals of Nuclear Physics (1990)
K.S. Krane, Introductory Nuclear Physics (1988)
G.R. Satchler Introduction to Nuclear Reactions (1990)

Notation for scattering experiment



$$a + X \text{ --> } b + Y$$

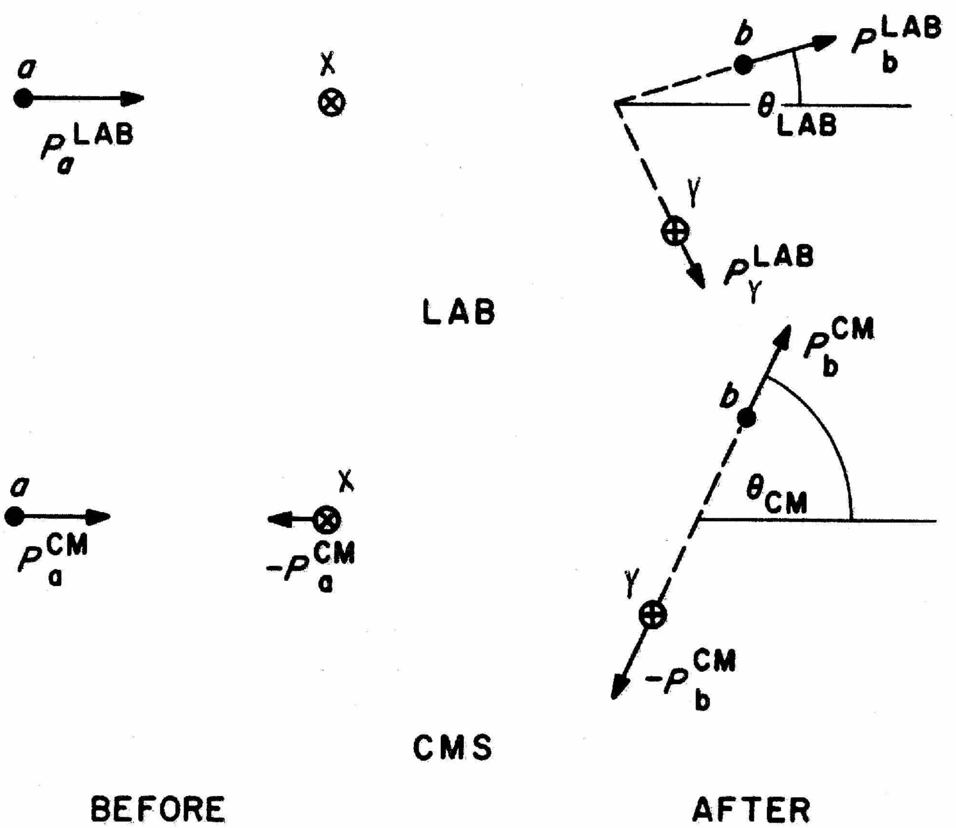
$$X(a,b)Y$$

a	incoming particle (from source)
X	target nucleus
b	outgoing particle (detected)
Y	residual nucleus

Examples of nuclear reactions:

Fission	$n + {}^{235}U \dashrightarrow 2n + {}^{93}Rb + {}^{141}Cs$
Fusion	${}^{3}H(d,n){}^{4}He$
Elastic	${}^{208}Pb(n,n){}^{208}Pb$
Inelastic	${}^{208}Pb(n,n'){}^{208}Pb$
Pickup	${}^{208}Pb(p,d){}^{207}Pb$
Stripping	${}^{208}Pb(d,p){}^{209}Pb$
Capture	${}^{3}He(\alpha,\gamma){}^{7}Be$
Photodisintegration	${}^{3}He(\gamma,p){}^{2}H$
$(n,2n\gamma)$ reactions	${}^{179}Hf(n,2n\gamma){}^{178}Hf$

Scattering experiment is done in lab frame but data is reported in center of mass frame



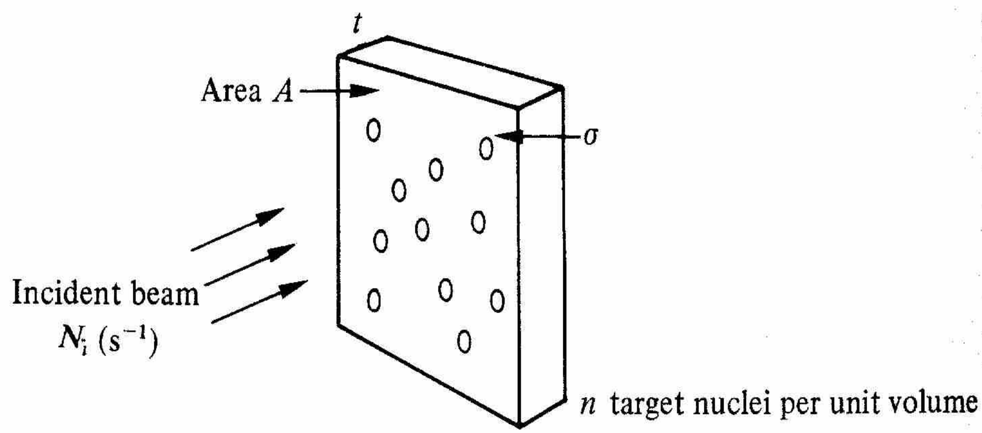
Lab system

Classically:

$v_{cm} = (m_a v_a)/(m_a + m_x)$

Center of Mass system

Cross Section, σ



σ = "effective area" of a target nucleus for a certain reaction

Units are barns, b = $10^{-28}m^2$ (or mb = $10^{-31}m^2$)

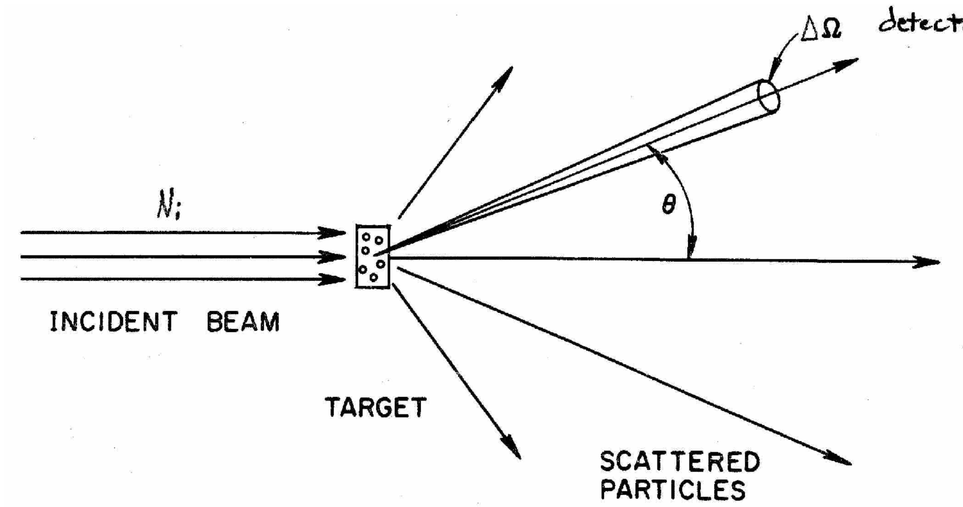
The total effective area of all nuclei in target = σ (nAt)

Rate of reaction

= (incoming beam rate) x (total effective area) / (area of the target)

$$R_{reaction} = N_i \, \sigma \, (nAt)/A = N_i \, \sigma \, nt$$

Differential cross section, $\sigma(\theta)$



A cross section per solid angle, $\sigma(\theta)$, or $d\sigma/d\Omega$, with units of mb/sr

$\Delta\Omega$ = solid angle of detector (sr)

Total steradians in a sphere = 4π

ΔA = area of detector face

$\Delta\Omega = 4\pi\, \Delta A/(4\pi r^2) = \Delta A/r^2$

$$R_{detector} = N_i\, [\sigma(\theta)\, \Delta\Omega]\, (nAt)/A = N_i\, [\sigma(\theta)\, \Delta\Omega]\, nt$$

Q Value

X(a,b)Y

Define Q value as a "mass difference":

$$Q = (m_Y c^2 + m_b c^2) - (m_X c^2 + m_a c^2)$$

From energy conservation (target X initially at rest):

$$m_X c^2 + m_a c^2 + K_a = m_Y c^2 + m_b c^2 + K_Y + K_b$$

Therefore, we also can think of Q as change in K_{total}:

$$Q = (K_Y + K_b) - K_a$$

For positive Q values, energy is released (exothermic)

Fission $n + {}^{235}U \rightarrow 2n + {}^{93}Rb + {}^{141}Cs$

$Q = 181$ MeV

Fusion ${}^{3}H(d,n){}^{4}He$

$Q = 17.6$ MeV

For negative Q values, reaction threshold = -Q (endothermic)

Pickup ${}^{208}Pb(p,d){}^{207}Pb$

Threshold = 5.17 MeV

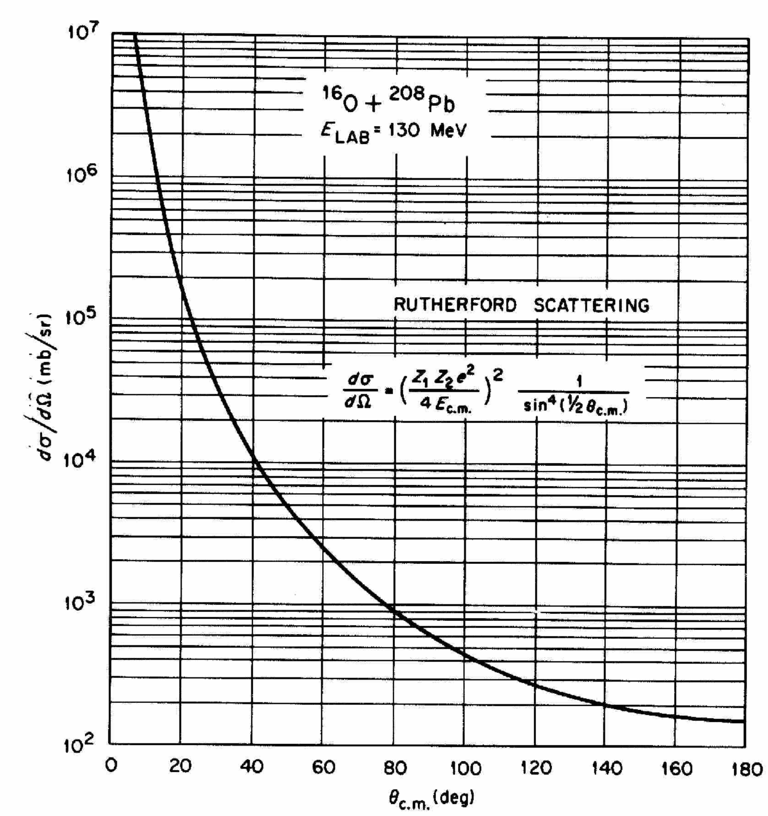
(n,2n) ${}^{179}Hf(n,2n){}^{180}Hf$

Threshold = 6.09 MeV

Rutherford Scattering

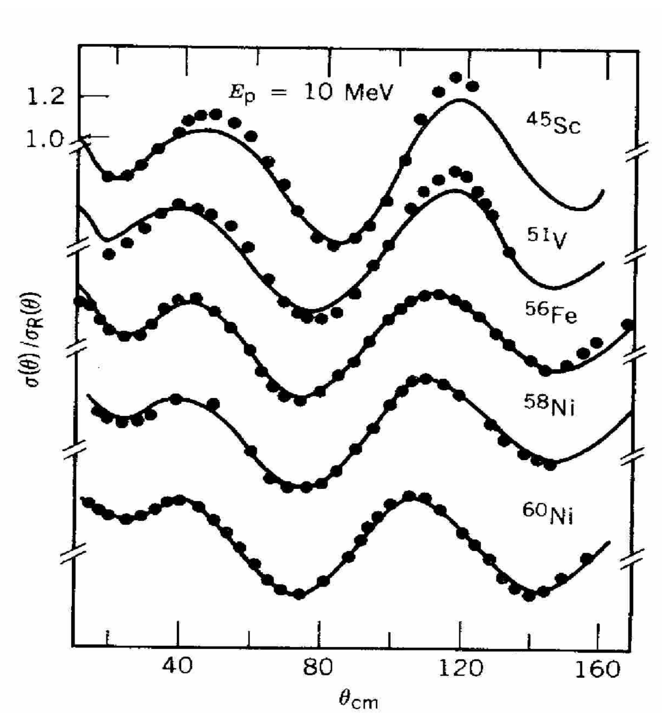
Scattering from Coulomb field
Differential cross section, $\sigma_R(\theta)$, is forward peaked.

Rutherford worked on $^{197}Au(\alpha,\alpha)^{197}Au$.
Similar results for large enough charge or low enough energies.
$^{208}Pb(^{16}O,^{16}O)^{208}Pb$



Nuclear elastic scattering

Using higher-energy and/or smaller-charged projectiles, we see nuclear scattering.
Divide the measured differential cross section by the theoretical Rutherford cross section.



$\sigma(\theta)/\sigma_R(\theta)$ for (p,p) elastic scattering shows diffraction-like pattern!

Incoming "wave" of protons of de Broglie wavelength $\lambda = h/p$
At E_p = 10 MeV, λ = 9 fm

Partial Wave Analysis of nuclear scattering

Represent incoming particle beam as a plane wave with de Broglie wavelength $\lambda = h/p$.

Assume solution to Schroedinger equation depends only on r and θ.
For a central force, assume the form of the solution at infinity is:
(Solution) = (Plane wave) + (Scattered spherical wave)

$$\text{(Scattered wave)} = \lambda/(4\pi i) \sum_{\ell} (2\ell+1)\, P_{\ell}(\cos\theta)\, (\exp(2i\delta_{\ell}) - 1),$$

where $P_{\ell}(\cos\theta)$ are “Legendre polynomials.”

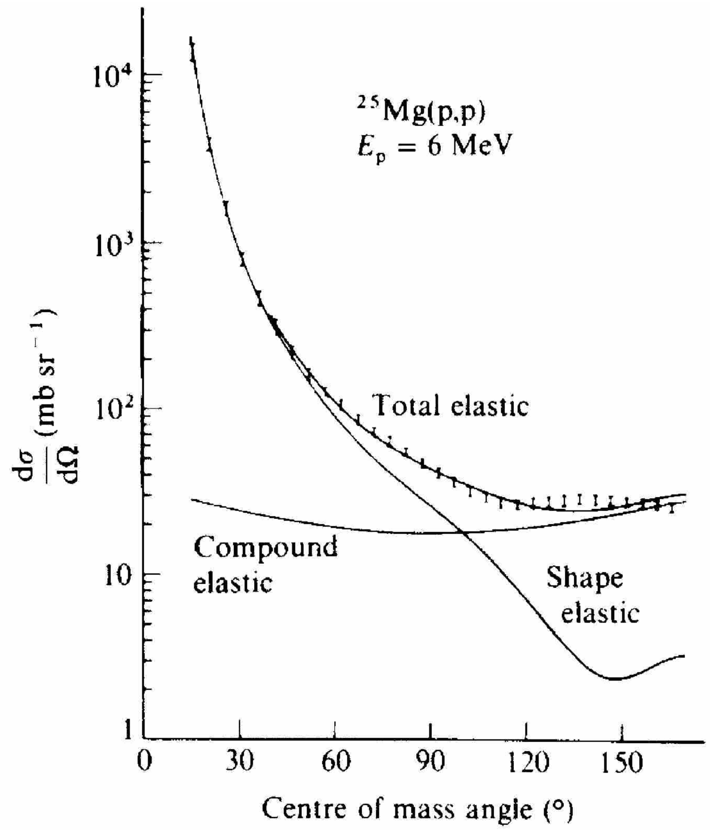
The δ_{ℓ}, or “phase shifts,” are determined via curve fitting or modeling.

The lower the energy, the fewer ℓ values are needed.

Direct vs. Compound nuclear reactions

<u>Direct</u>: Incoming particle either scatters elastically (shape elastic) or only "grazes" target, interacting with nucleons at surface.
Interaction time around 10^{-22} s.

<u>Compound</u>: A two-step process. First, incoming particle enters target and forms a compound nucleus. Then, compound nucleus ejects particle(s).
Interaction time around 10^{-16} s.



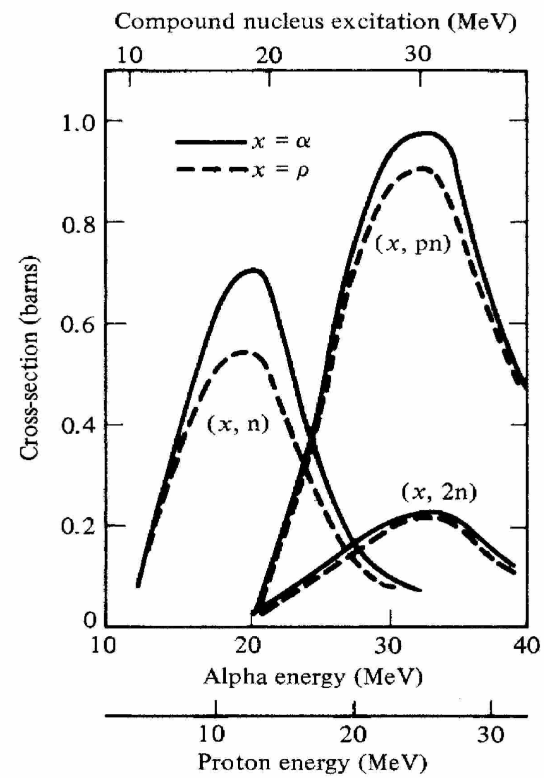
Direct scattering "remembers" beam direction; its differential cross section is forward peaked

Compound Reactions

a + X --> C* --> b + Y

Independence hypothesis:
The decaying compound nucleus, C* has no "memory" of how it was formed.

Excitation functions; graphs of σ vs. incoming particle energy.



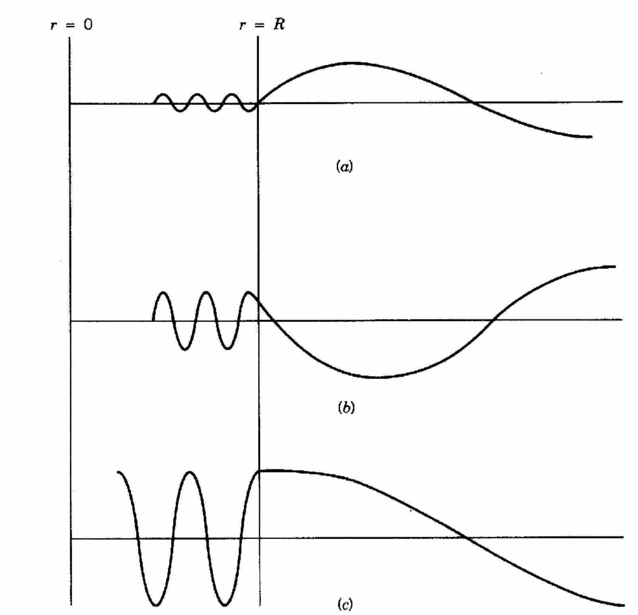
dotted line: $^{63}Cu(p,n)^{63}Zn$
$^{63}Cu(p,2n)^{62}Zn$
$^{63}Cu(p,pn)^{62}Cu$

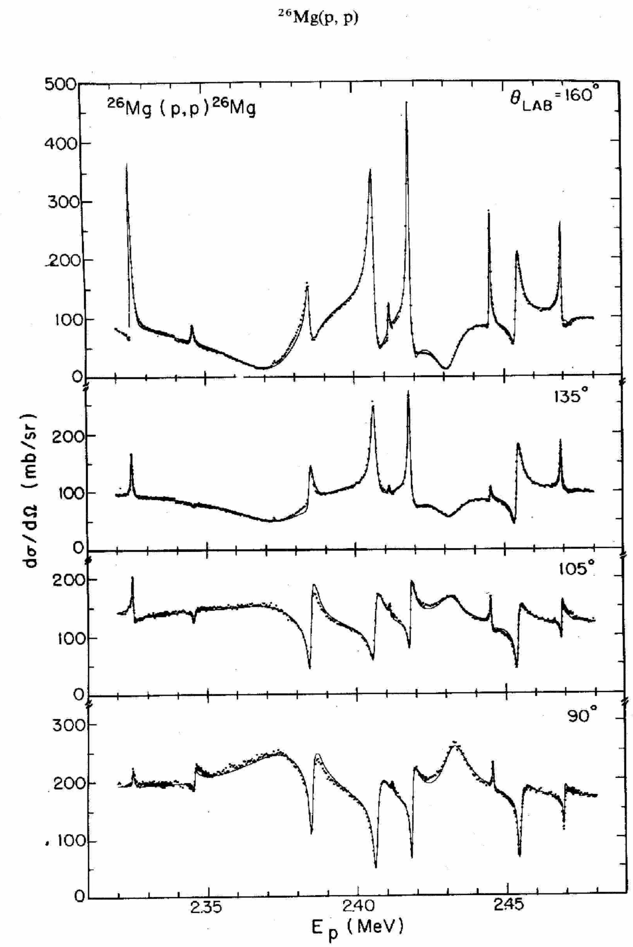
solid line: $^{60}Ni(\alpha,n)^{63}Zn$
$^{60}Ni(\alpha,2n)^{62}Zn$
$^{60}Ni(\alpha,pn)^{62}Cu$

For both sets of experiments, we see the same resonances, reflecting states of the compound nucleus ^{64}Zn ("nuclear structure")

Individual resonances of compound nucleus

R-matrix (reaction matrix) analysis
Match internal and external waves at boundary of nuclear potential to determine angular momentum and parity of excited states of compound nucleus.





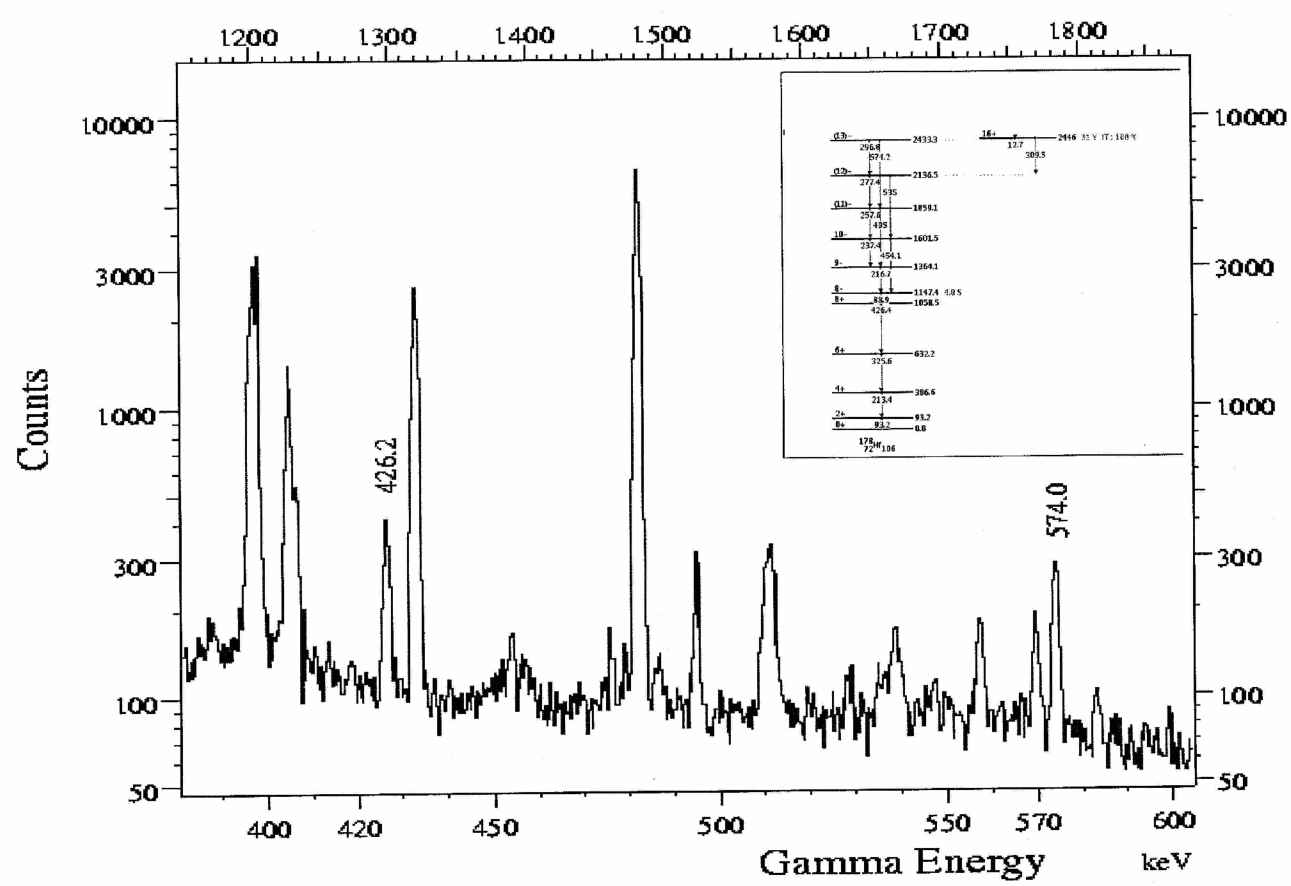
p + ^{26}Mg --> ^{27}Al --> p + ^{26}Mg
Excitation function of $\sigma(\theta)$ shows resonances for states in ^{27}Al

(n,n'γ) and (n,2nγ) reactions

Neutron is captured to form compound nucleus. Then, the compound nucleus ejects neutron(s), leaving residual nucleus.
Measure the energy spectrum of the gammas to identify excited states of the residual nucleus and determine the cross sections.

Example:
^{179}Hf(n,2nγ)^{178}Hf

Reactions of interest to national security (bomb design)



Direct Reaction

Optical model of nuclear elastic scattering

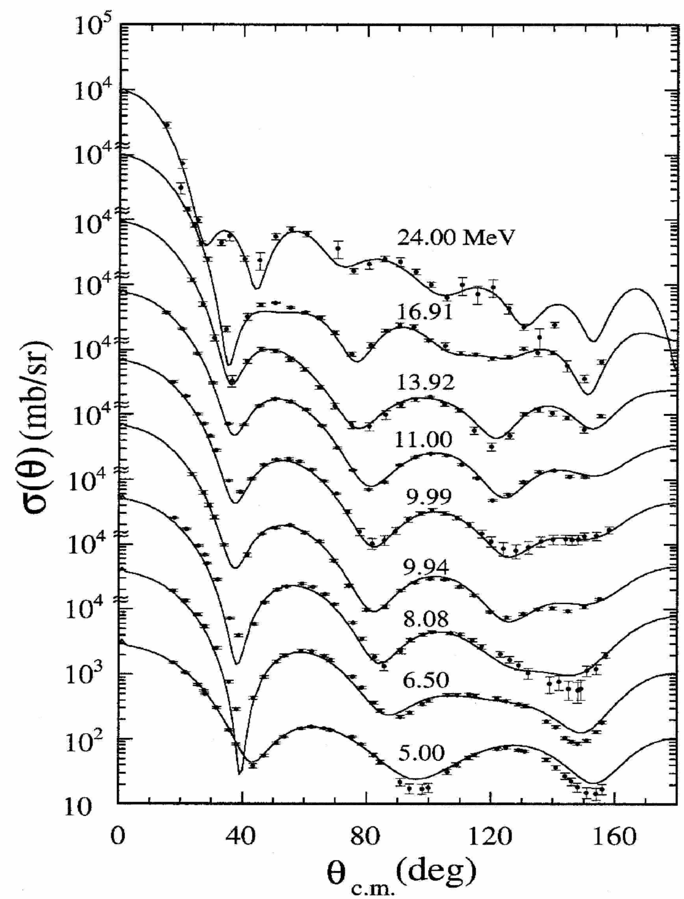
Analogy between optical scattering (complex index of refraction) and nuclear scattering (complex nuclear potential)

The imaginary part of the nuclear potential acts as a sort of "sink hole" for all nuclear reactions. It gives σ_R predictions that are smooth with energy.

Therefore, the optical model cannot see nuclear structure. It is best used when the compound-nucleus channels form a continuum of reactions (above 10 MeV or so).

$^{120}Sn\ (n,n)^{120}Sn$

Note diffraction-like patterns



Nuclear Optical Model Potential

$$V(E) = (V_v(E) + iW_v(E)) + (V_s(r) + iW_s(E))$$

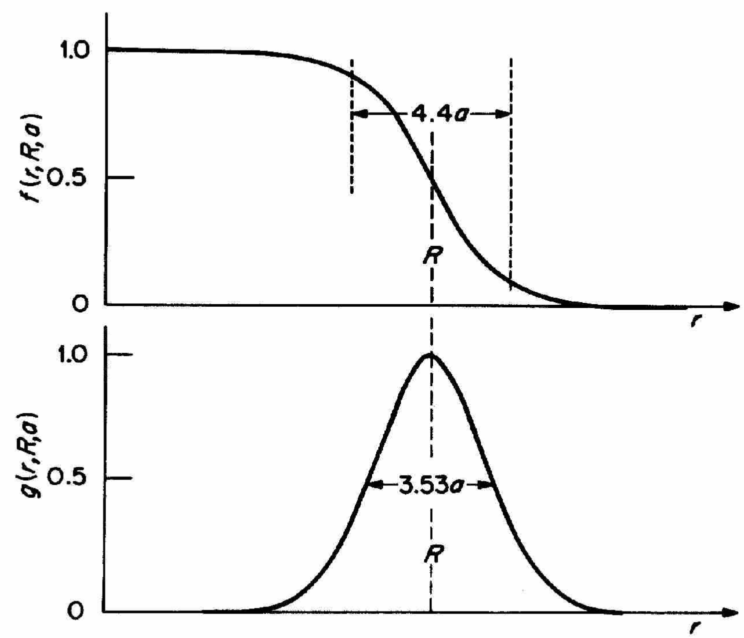
volume surface

Real terms, V_v and V_s, model elastic scattering

Imaginary terms, W_v and W_s, are the "sink hole" for reactions

At relatively high energies (above 40 MeV), W_v dominates

At relatively low energies , W_s is also important (projectiles do not penetrate nucleus as much).



Direct reaction
Stripping Reaction

Incoming projectile leaves a particle in well-defined state of the residual nucleus. From $\sigma(\theta)$ of scattered protons, deduce properties of single-particle states of residual nucleus.

"Distorted Wave Born approximation" (DWBA):
Approximate the incoming and outgoing waves with optical model representations, from analysis of elastic scattering.

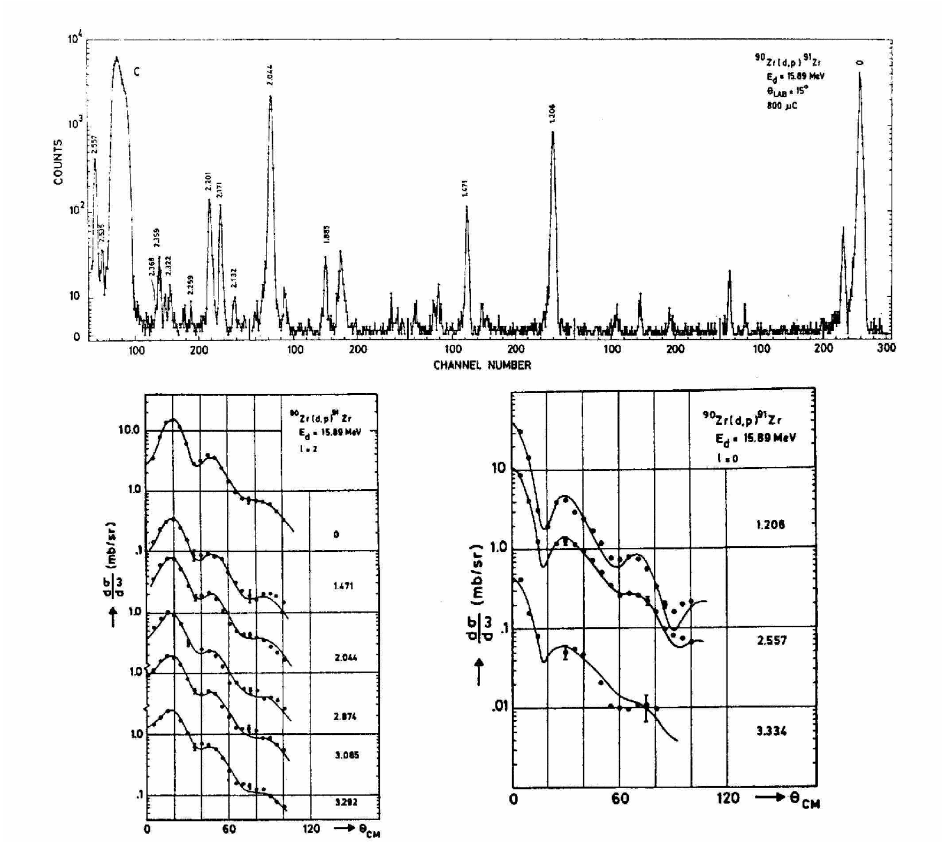
The "transition amplitude" is

$$T = \int [\text{outgoing wave}]\ F(r)\ [\text{incoming wave}]\ dv,$$

where F(r), the "form factor," contains the structure information.

Differential cross section, $\sigma(\theta)$, is proportional to $|T|^2$. Modify F(r) until it reproduces $\sigma(\theta)$for each state of the residual nucleus.

Example of stripping reaction: $^{90}Zr(d,p)^{91}Zr$



Spectrum of protons (for each angle) show peaks related to states of ^{91}Zr

Data is sorted by gating counts for each proton peak when graphing differential cross sections.

Deduce single-particle structure information from the $\sigma(\theta)$ curves.

Reactions involving gamma rays

Radiative Capture

Target absorbs incident particle and
at the same time a gamma ray is released

Photodisintegration

Incident gamma ray breaks up target
Inverse reaction to radiative capture

Example:

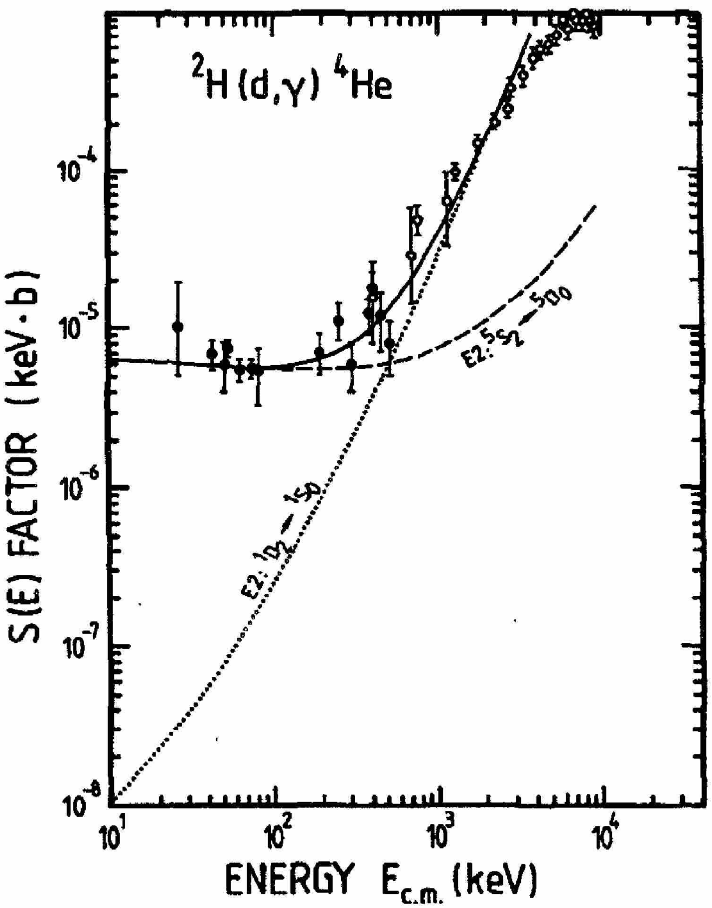
$H(n,\gamma)^2H$	Deuteron formation
	Q = 2.225 MeV (exothermic)
$^2H(\gamma,n)p$	Deuteron breakup
	Threshold = 2.225 MeV (endothermic)

Radiative Capture

Deduce electromagnetic transitions:
from scattering state to state of residual nucleus. Many such reactions are important in nucleosynthesis. "Nuclear astrophysics"

At low energies (near Coulomb barrier), it is convenient to multiply the cross section by the energy:
S factor = $\sigma E e^{-2\pi\eta}$.
(The exponential is the "penetration factor.")

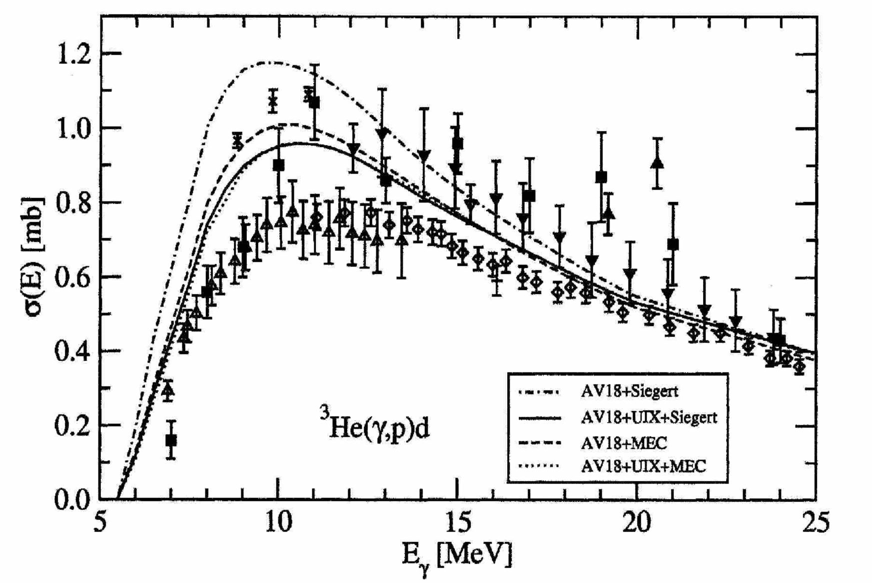
Example: $^2H(d,\gamma)^4He$
Excitation function for S factor fit using two E2 transitions
(electric transitions with $\Delta L = 2$)



Photodisintegation

Example: $^3He(\gamma,p)d$

Excitation function
for total cross section, σ



Model incoming channel with electromagnetic interaction.
For outgoing channel, do a three-body (Faddeev) calculation,
based on a model of the NN interaction (Bonn, AV18, etc.)

Precision σ data places constraints on the three-body computations
and on the NN models.