Induced Voltage: Faraday's Law

To understand what happens when the magnetic field surounding a wound component changes, you need to know about Faraday's Law of Induction. Every inductor used with AC obeys it. There are four variations on a theme by Faraday -

- Flux tells you volts
- Current tells you volts
- Volts tells you flux
- Volts tells you current
- Postscript: Time to wave goodbye

About your browser: if this character '×' does not look like a multiplication sign, or you see lots of question marks '?' or symbols like '' or sequences like '&cannot;' then please accept my apologies.

See also ...

[↑ Producing wound components] [Air coils] [A guide to the terminology of magnetism] [Power loss in wound components]

Flux tells you volts

Michael Faraday's greatest contribution to physics was to show that an electric potential, v, is generated by a coil of wire when the magnetic flux, Φ, enclosed by it changes -

$$v = N \times d\Phi/dt$$ Faraday's Law (differential)

Frequently, a minus sign follows the '=', depending on the sign conventions used for flux and potential. The flux may change because -

- A nearby permanent magnet is moving about.
- The coil rotates with respect to the magnetic field.
- The coil is wound on a core whose effective permeability changes.
- The coil is the secondary winding on a transformer where the primary current is changing.

In electric motors and generators you will usually have more than one of these causes at the same time. It doesn't matter what causes the change; the result is an induced voltage, and the faster the flux changes the greater the voltage.

Example: In the standard example toroid core the flux, in webers, varies according to -

$$\Phi = 5\times10^{-6} \sin(8\times10^{5}\ t)$$ Equation FYS

What potential is induced in its winding of two turns?

Substituting equation FYS into Faraday's law -

$$v = 2 \times d(5\times10^{-6} \sin(8\times10^{5}\ t))/dt$$ Equation FYC

$v = 2 \times 5{\times}10^{-6} \times 8{\times}10^{5} \cos(8{\times}10^{5}\ t)$ Equation FYA

$v = 8 \cos(8{\times}10^{5}\ t)$ Equation FYB

So this winding will have a peak voltage of 8 volts. You can easily generalize this for any flux varying sinusoidally at a frequency f to show

$V = \sqrt{2}\ \pi\ N \times f \times \Phi_{pk} = 4.44\ N \times f \times \Phi_{pk}$ The transformer equation

Where V/volts is the root mean squared potential across the coil.

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Current tells you volts

OK, that's all clear enough, but there is one other reason for alteration to the flux: current flow in the coil. Hans Christian Oersted discovered that an electric current can produce a magnetic field. The more current you have the more flux you generate. That, too, is easy enough to grasp. What needs a firm intellectual grip is to appreciate that Faraday's Law does not stop operating just because you have current flowing in the coil. When the coil current varies then that will alter the flux and, says Faraday, if the flux changes then you get an induced voltage. This merry-go-round between current, flux and voltage lies at the heart of electromagnetism.

"Very interesting", you yawn, "but what use is all this if I don't know anything about the flux in my inductor?" Well, you often do know what inductance you have (henries). From its definition you know that -

$d\Phi = L \times di / N$ Equation FYN

Substituting this into Faraday's Law -

$v = N\ (L\ di / N) / dt$ Equation FYO

Therefore -

$v = L\ di/dt$ Equation FYU

As the French say, this is *très utile* because you no longer worry about flux or the number of turns. Equation FYU doesn't really say anything different from Faraday's Law - it's just that all the nasty magnetics stuff has been abstracted away in the magic parameter we call inductance. Once you know the current flowing through L you then simply multiply its value in henries by the differential of current with respect to time *et voilà* you have the voltage across that inductor.

Again, a minus sign is sometimes included to express the fact that the induced voltage opposes an external potential driving the current. Fair enough, but the same is true of the voltage drop in a resistor, and who ever says V = - I × R?

Example: In the standard example toroid core current increases linearly by 3 amps per second. What potential is induced in the winding?

The two turn winding gives an inductance of 8.8 μH (derived immediately on the terms page). The current is given by i = 3 t. Substituting into equation FYU -

$v = 8.8\times10^{-6} \times d(3\ t)/dt = 8.8\times10^{-6} \times 3 = 26.4\ \mu V$ Equation FYZ

This aspect to inductance comes into play with the ignition coil in automotive engines. When the contact breaker goes open, the current in the coil drops almost instantaneously. Its rate of change (di/dt) is so great that an enormously high voltage is induced - generating a spark to ignite the fuel mixture.

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Volts tells you flux

Now let's spin our fairground ride in the other direction. Instead of getting an induced voltage by putting in a current we'll put a voltage across the coil and see what happens. Normally, if you put several volts across any randomly arranged bit of wire then what will happen will be a flash and a bang; the current will follow Ohm's Law and (unless the wire is very long and thin) there won't be enough resistance to prevent fireworks. It's a different story when the wire is wound into a coil. If the current increases then we get flux build up which induces a voltage of its own. The sign of this induced voltage is always such that the voltage will be positive if the current into the coil increases. We say that the induced voltage will *oppose* the externally applied voltage which made the current change (Lenz's law). This creates a limit to the rate of rise of the current and prevents (at least temporarily) the melt-down we get without coiling.

In this situation it is helpful to build a mental picture by imagining that, rather than having an idealised inductor (Figure FSA a), you replace it with a model which includes a very small series resistance - as all practical inductors must have (Figure FSA b). Pretend that R_S is one μΩ, if you wish. In addition, replace the inductance with a voltage source whose potential difference is

$v_L = N\ d\Phi/dt$ Equation FSE

This is what Faraday's Law insists the 'back emf' of the inductor must be. It should now be clear that the value of v_L must be only the tiniest bit less than v_G. If it were significantly less then the PD across R_S would increase and (because R_S is only a μΩ) the current would rapidly increase, the flux in the core would increase and thereby compensate through increased v_G - rather like a 'feed-back' system.

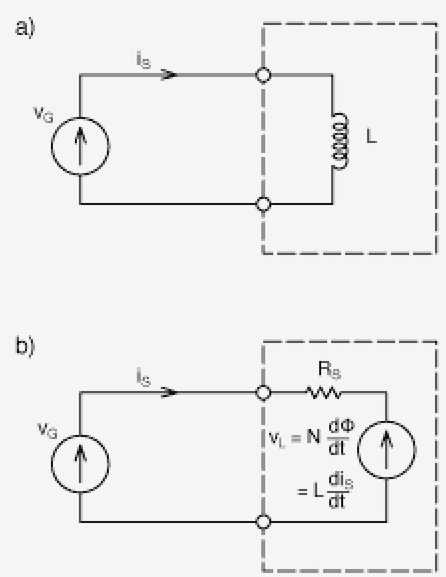

Fig. FSA
An inductor circuit and an equivalent model

You seek the actual value of flux, rather than its differential, so you integrate -

$\Phi = (1 / N)\int v.dt$ Faraday's law (integral)

Example: In the standard example toroid core assume you have no flux to start with. After applying twenty volts for three microseconds (figure FYV). have you then sufficient flux to saturate the core?

The value of the potential, v, is 20 volts. So you integrate that with respect to time -

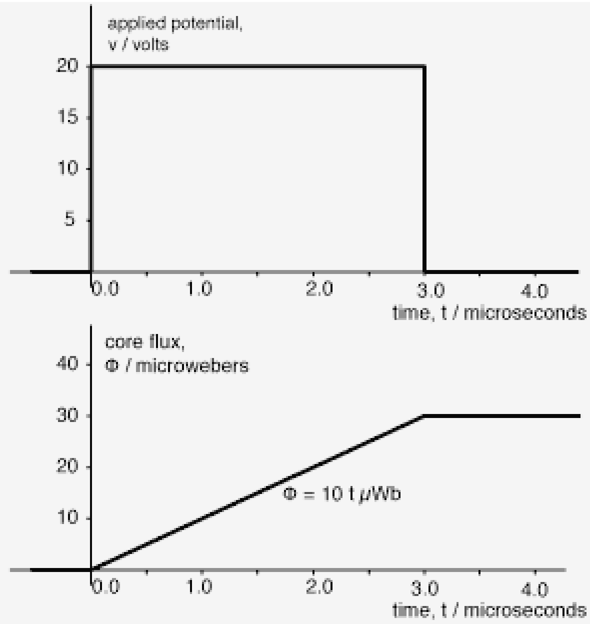

Fig. FYV: Flux build-up in an inductor

$\Phi = (1 / N) \int 20.dt = (1 / 2)\, 20\, t = 10\, t$ Equation FYW

You sustain this potential for 3 μs, so substituting this time you obtain an actual flux value

$\Phi = 10 \times 3.0{\times}10^{-6} = 3.0{\times}10^{-5}$ Wb Equation FYX

From the core datasheet you know that the saturation flux density is 0.36 tesla and the effective core area is 19.4 mm^2. The maximum allowable peak flux is therefore -

$\Phi_{max} = B_{sat} \times A_e = 0.36 \times 19.4{\times}10^{-6} = 6.98{\times}10^{-6}$ Wb Equation FYY

This is several times less than the actual flux, so the core is saturated. To correct the problem you should either apply fewer volts (4.66), apply them for less time (698 ns) or use more turns (9).

Satisfy yourself that the above result is consistent with our original formulation of Faraday's law: voltage is proportional to the rate of change of flux. No change of flux no induced voltage. Flux increasing linearly constant positive induced voltage. It's still Faraday's Law, just seen from a different perspective.

Example: If a sinusoidal waveform of RMS amplitude V = 230 volts at frequency f = 50 Hz is applied to an inductor having 200 turns then what is the peak value of magnetic flux, Φ?

$v = (\sqrt{2})\, V \sin(2\pi.f.t)$ Equation FYE

Substituting this into the integral form of Faraday -

$\Phi = (1 / N) \int (\sqrt{2})\, V \sin(2\pi.f.t).dt$ Equation FYF

$\Phi = ((\sqrt{2})\, V / N) \int \sin(2\pi.f.t)\, .dt$ Equation FYG

$\Phi = (-(\sqrt{2})\, V / (2\pi.f\, N)) \cos(2.\pi.f.t)$ Equation FYH

The peak value of flux is then given by

$\Phi_{pk} = \sqrt{2}\, V / (2\pi\, f\, N)$ Equation FYJ

$\Phi_{pk} = V / (4.44\, f\, N)$ Equation FYK

Compare this with the transformer equation above.

$\Phi_{pk} = 230 / (4.44 \times 50 \times 200)$ Equation FYL

Φ_{pk} = 5.18 mWb

Equation FYM

Consider, then, how useful this integration method is in practical inductor design; if you know the number of turns on a winding and the voltage waveform on it then you integrate wrt time and then you have found the amount of flux. What's more is that you found it without knowing about the inductance or the core: its permeabilty, size, shape, or even whether there was a core at all!

Nice work Mike!

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Volts tells you current

Often, you know what voltage you wish to apply, v_G, and want to know the resulting current. Can Faraday's Law still help you understand what happens? Refer again to figure FSA.

$$i_S = (v_G - v_L) / R_S$$

Equation FSH

Because $v_G \approx v_L$ you therefore know by equation FYU that -

$$di_S/dt = v_G / L$$

Equation FSG

So you know what the differential of i_S must be. However, what you actually need to know is i_S itself. Well, to convert any differential quantity into its plain, undifferentiated equivalent you must perform an integration -

$$i_S = (1/L)\int v.dt$$

Equation FYR

where v is the externally applied voltage.

In the special case where v is a constant then this formula for the current simplifies to -

$$i = V \times t / L + I_0$$

Equation FYQ

Here, I_0 may be thought of as a constant of integration representing the current which flowed in the inductor at the start (t = 0).

One important general point: if your winding has to cope with a given signal amplitude then the core flux is proportional to the **inverse of the frequency**. This means, for example, that mains transformers operating at 50 or 60 Hz are larger than transformers in switching supplies (capable of handling the same power) working at, say, 50 kHz.

Example v_G is a sine wave (Figure FSBa) of RMS amplitude, V_G = 230 volts at a frequency f = 50 hertz - standard values for the UK mains supply. You apply this to a 3.5 henry inductance. What current will flow?

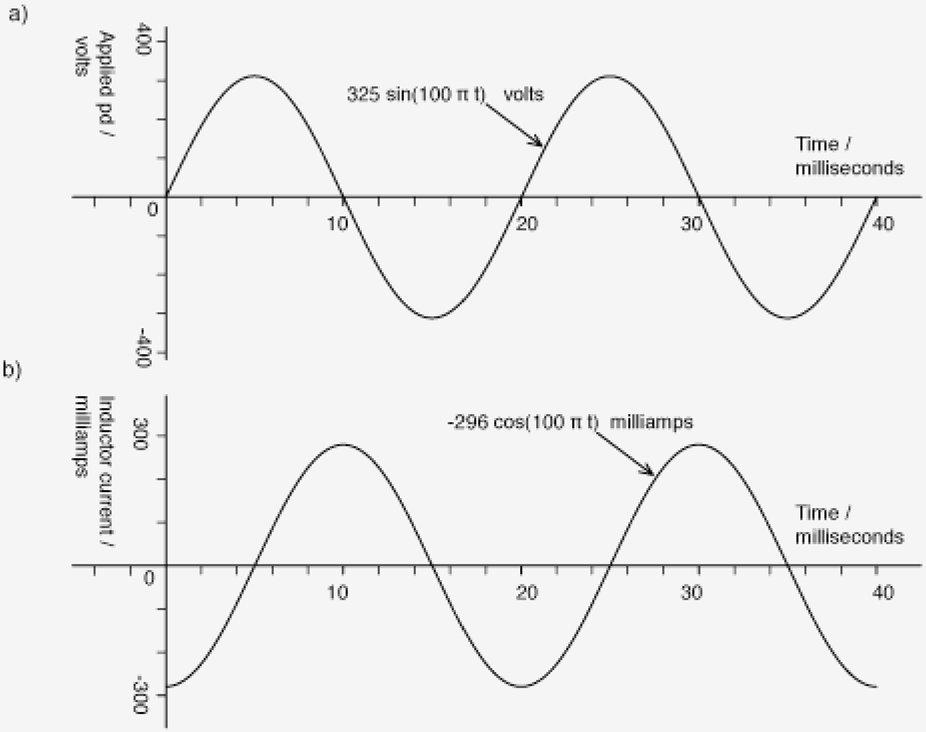

Fig. FSB Voltage and current waveforms for an inductor

$$v_G = \sqrt{2} \times V_G \sin(2 \pi f \times t) = 1.41 \times 230 \times \sin(2 \times 3.14 \times 50 t)$$ Equation FSI

Therefore

$$v_G = 325 \sin(314 t)$$ Equation FSJ

Putting this into equation FYR -

$$i_S = (1/L)\int v_G.dt = (325/3.5)\int \sin(314 t).dt$$ Equation FSK

therefore

$$i_S = 92.9 \times -\cos(314 t)/314 + I_0 = -0.296 \cos(314 t) + I_0$$ Equation FSL

where I_0 is the arbitrary constant of integration which, in this case, may be taken as zero. The RMS value of this current is $0.296/\sqrt{2}$ = 209 milliamperes. Does this cosine current fulfill the requirements of Faraday's Law?

As far as the general shape and phase of the applied potential goes, you can easily see that it is the same as the differential of the current. At t = 0 you have the maximum (negative) value of current but its slope or time variation is zero so the induced voltage is zero. At t = 5ms the current slope is at its most positive value so the induced voltage is also at its most positive. Mickey is a happy man.

The amplitude of the current can be verified using an ordinary circuit analysis approach -

$$I_S = V_G / X_L = 230 / (2 \pi 50 \times 3.5) = 0.209 \text{ amperes}$$ Equation FSM

From this equation it is clear that the amplitude of the current will fall as the frequency is increased. The current, however, will always stay 90 degrees behind the voltage. At 50 hertz current reaches its peak value 5 milliseconds after the voltage does.

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Postscript: Time to wave goodbye

Caveat (there's always one): The coils described here are idealised. See the paragraph on flux linkage for a more complete picture. We have also kept things simple by pretending that the series resistance and parallel capacitance are both negligible. Real world inductors sometimes approach the ideal behaviour closely, and sometimes not.

These web pages concentrate on the theory required to design coils. Please don't leave with the idea that Faraday's law is just about bits of wire - they are there simply to 'sample' the electric field produced by a shifting magnetic field. The E-field is there even without a coil.

It was deduced in 1862 by James Clerk Maxwell that the converse happens: that a changing electric field produces a magnetic field. Put the two together, that a change in one gives the other and vice-versa, and you might wonder where it all ends. In fact it ends with a dance between the two forms of energy in which E and M continually rub shoulders within what is called an *electromagnetic wave* - of which light, radio and X-rays are all examples. Maxwell expresses Faraday's law in a more general vector field equation -

$$\text{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t \qquad \text{Maxwell's equation for E}$$

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Cup cores.

More on magnetics.

The force due to a field.

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