Chapter 2: Electrostatics and Coulomb's Law

2.1 Introduction

Our knowledge of electric charge is not new. The earliest known experiments on electrostatics were performed by Thales of Miletus (640-546 BC) around 600 BC, when he found that amber, when rubbed with fur, was able to attract light objects. Today, it is said that the amber possesses an electrical charge. (The word electric is derived from the Greek word, elektron, meaning amber.) The study of electric charges at rest under the action of electric forces is called electrostatics.

It has been found that there are two and only two types of electric charges in nature - negative charge and positive charge. As a result of experiments, *The Fundamental Principle of Electrostatics can be stated: Like electric charges repel each other while unlike electric charges attract each other.* To give a more modern description of electrostatics it is necessary at this time to discuss atomic structure.

2.2 Atomic Structure

In an attempt to find simplicity in nature, the Greek philosophers Leucippus and Democritus suggested in the fifth century B.C. that matter is composed of very small particles called atoms. The word atom comes from a Greek word that means that which is indivisible. However, it was not until the early nineteenth century that John Dalton (1766-1844), an English chemist, proposed that to every known chemical element there corresponds an atom of matter. Every material in the world is just some combination of these indivisible atoms. However, in 1897 J. J. Thompson (1856-1940), an English physicist, discovered the electron, a negatively charged particle having a mass m_e = 9.1095 x 10^{-31} kg. Although it had been known that there was such a thing as negative electrical charge, it was not known what the carrier of that negative charge was. This newly discovered electron, however, was the basic or elementary particle carrying the smallest amount of negative charge. All other negative charge that occur in electrostatic experiments are multiples of the electronic charge.

The finding of the electron, however, presented a rather difficult problem. Where did it come from? The only place it could come from is the interior of the indivisible atom, which could not then be indivisible. The indivisible atom must have some structure. Because the atom is generally neutral, there must be some positive charge within the atom to neutralize the negative electron. In the early 1900's Ernest Rutherford (1871-1937), a British physicist, bombarded atoms with alpha particles (positively charged particles) and by observing the effects of the collision, developed the nuclear model of the atom. His model of the atom consisted of a small, dense, positively charged nucleus with negative electrons orbiting about it, somewhat in the manner of the planets orbiting about the sun in the solar system. Rutherford found this positive particle of the nucleus and named it the proton in 1919. The proton has a positive charge equal in magnitude to the charge

on the electron. The mass of the proton, m_p = 1.6726 x 10^{-27} kg, is about 1836 times greater than the electron mass.

In 1920 Rutherford suggested that there is probably another particle within the nucleus, a neutral one, to which he gave the name the neutron. The neutron was discovered some twelve years later in 1932 by the English physicist, James Chadwick . In terms of these particles, or building blocks, the different atoms are formed. The difference between one chemical element and another is in the number of protons, and electrons within it. As seen in figure 2.1(a), the chemical element hydrogen contains a nucleus which consists of one positive proton about which orbits the lighter, negative electron.

Sometimes the electron is to the right and sometimes to the left of the nucleus. Sometimes it is above and sometimes below the nucleus. By symmetry, the electron's mean position coincides with the position of the positive nucleus. Therefore, the atom as a whole acts as though it were electrically neutral. The next chemical element helium is formed by the addition of another proton to the nucleus, and another electron to the orbit, figure 2.1(b). Two neutrons are also found in the
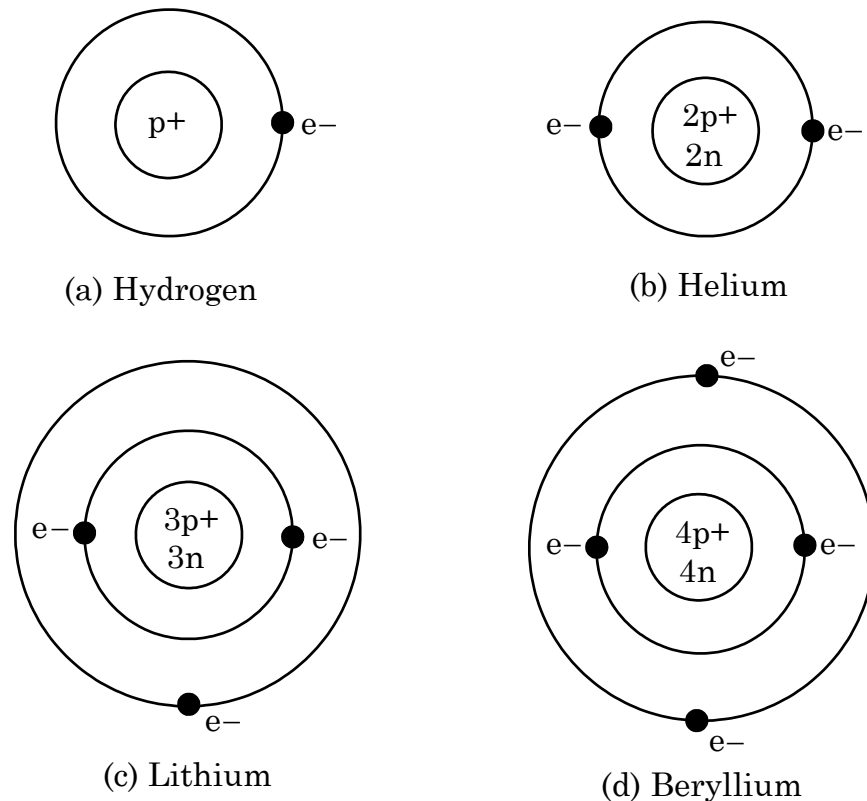

(a) Hydrogen

(b) Helium

(c) Lithium

(d) Beryllium

Figure 2.1 Atomic structure.

nucleus of helium. The next chemical element, lithium is formed by the addition of another proton, another electron, and another neutron, figure 2.1(c). In this way all of the chemical elements are formed, although in the higher elements there are

usually more neutrons than protons. Because each element contains the same number of electrons and protons, each element is electrically neutral.

Although this model of the atom is quite useful, it is not completely correct. An electron moving in a circle is an accelerated charge, and it has been found that whenever a charge is accelerated, it radiates energy. Therefore the radiating electron should lose energy and spiral into the nucleus and the atom should cease to exist. The world, which is made up of atoms, should also cease to exist. Since the world continues to exist, the above model of the atom can not be completely correct.

Also since like charges repel each other, the protons in the nucleus should also repel each other and the nucleus should blow itself apart. Hence, the whole world should blow itself apart. But it does not. Therefore, there must be some other force within the nucleus holding the protons together. This new force is called the strong nuclear force. *Because the electron is the smallest unit of charge ever found, the fundamental unit of charge, the Coulomb, named after the French physicist Charles A de Coulomb (1736-1806), is defined in terms of a certain number of these electronic charges. That is, 1 Coulomb of charge is equal to 6.242 $\times 10^{18}$ electronic charges, and the charge on one electron is 1.60219 $\times 10^{-19}$ Coulomb.* The unit for electrical charge, the Coulomb, will be abbreviated as a capital *C*, in keeping with the SI convention that units named after a person are symbolized by capital letters. *Because electric charge only comes in multiples of the electronic charge, it is said that electric charge is quantized. Also, the total net charge of any system is constant, a result known as the law of conservation of electric charge.*

Although no electric charges have ever been found carrying fractional portions of the electronic charge, the latest hypothesis in elementary particle physics is that protons and neutrons are made up of more elementary particles called quarks. (Murray Gell-Mann, 1964) It is presently proposed that there are six quarks: the (1) up (u), (2) down (d), (3) strange (s), (4) charm (c), (5) bottom (b), and (6) top (t) quark. The charges on these quarks are fractional as shown in table 2.1.

QUARK	CHARGE (Fraction of Electron Charge)
u	2/3
d	-1/3
s	-1/3
c	2/3
b	-1/3
t	2/3

**Table 2.1** Quarks and their charges.

A proton is assumed to be made up of two "up" quarks and one "down" quark, as shown in figure 2.2. The neutron is assumed to be made up of one "up" quark and two "down" quarks as shown in figure 2.3. Of course, individual quarks have not yet

been found, and indeed most theories of particle physics predict that an isolated single quark cannot exist. There is, however, strong indirect evidence from scattering experiments that quarks do indeed exist. The quark model has also predicted the existence of other elementary particles which have been found, indicating that the quark hypothesis is on very good experimental ground. Of course, if the existence of quarks are definitely confirmed, the next question that would then have to be asked is, "What are quarks made of?"
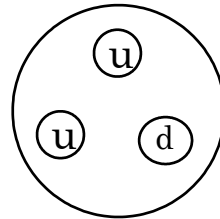
charge
2/3 +2/3 - 1/3 = 1

Figure 2.2 The quark configuration of a proton.
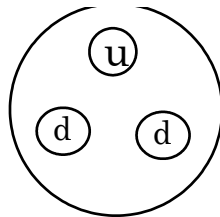
charge
2/3 - 1/3 - 1/3 = 0

Figure 2.3 The quark configuration of a neutron.

In general, most substances are either good conductors of electricity or poor conductors. *Materials that permit the free flow of electric charge through them are called conductors. Materials that do not permit the free flow of electric charge through them are called insulators or dielectrics.* Most metals are good conductors of electric charge, while most nonmetals are insulators. (There are a few materials called semiconductors, whose characteristics lie between those of conductors and those of insulators.)

An interesting characteristic of all conductors is that whenever an electric charge is placed on a conducting body, that charge will redistribute itself until all of the charge is on the outside of the body. As an example if electric charges are placed on a solid metallic sphere, the charges exert forces of repulsion on one another and the charges try to move as far apart as they can. The greatest separation they can achieve is when they are on the outside of the sphere.

2.3 Coulomb's Law

As has just been seen, electric charges exert forces on each other. But what are the magnitudes of these forces? Charles Augustin de Coulomb (1736-1806) invented a torsion balance in 1777 in which a quantity of force could be measured by the amount of twist it produced in a thin wire. In 1785 he used this torsion balance to measure the force between electrical charges. If very small spherical charges, q_1 and q_2, called point charges, are separated by a distance r between their centers, figure 2.4, then Coulomb found that the force between the charges could
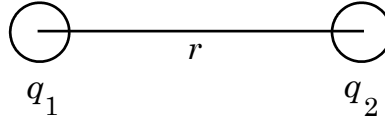
Figure 2.4 Coulomb's Law.

be stated as: *The force between the point charges q_1 and q_2 is directly proportional to the product of their charges and inversely proportional to the square of the distance separating them.* The direction of this force lies along the line separating the charges. *This result is known as Coulomb's law.* Coulomb's law can be stated mathematically as

$$F = \frac{kq_1q_2}{r^2} \qquad (2.1)$$

where k is a constant depending upon the units employed and on the medium in which the charges are located. For a vacuum

$$k = 8.9876 \times 10^9 \text{ N m}^2 / \text{C}^2$$

For air the value of k is so close to the value of k for a vacuum that the same value will be used for both. To simplify the solution of problems in this text the value of k used will be rounded off to the value

$$k = 9.00 \times 10^9 \text{ N m}^2/\text{C}^2$$

To simplify more advanced theories of electromagnetism, Coulomb's law is also written in the form

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r^2} \qquad (2.2)$$

where

$$\frac{1}{4\pi\varepsilon_0} = k \qquad (2.3)$$

and $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2/N m^2 and is called the permittivity of free space. If the charges are placed in a medium other than air or vacuum, then there will be a

different value for the permittivity ε for that medium and hence a different value of k. In this text only the simple form, equation 2.1, of Coulomb's law will be used.

If the charges are much larger than point charges, Coulomb's law can still be used if the distance separating the charges is quite large compared to the size of the electric charge. Under these circumstances the charges approximate point charges. Let us consider some examples of the use of Coulomb's law.

Example 2.1

Coulombs law. A point charge, q_1 = 2.00 μC is placed 0.500 m from another point charge q_2 = −5.00 μC. Calculate the magnitude and direction of the force on each charge.
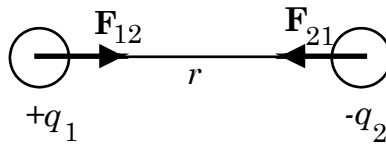
Diagram for example 2.1

Solution

The force acting on charge q_1, is a force of attraction caused by the negative charge of q_2. This force will be called F_{12} (force on charge 1 caused by charge 2). Since this force is in the positive x-direction it can be written as

$$\mathbf{F}_{12} = \mathbf{i}F_{12}$$

The magnitude of the force F_{12} is found by equation 2.1 as

$$F_{12} = \frac{kq_1q_2}{r^2}$$
$$= \frac{\left(9.00\times10^9 \text{ N m}^2/\text{C}^2\right)\left(2.00\times10^{-6} \text{ C}\right)\left(5.00\times10^{-6} \text{ C}\right)}{\left(0.500 \text{ m}\right)^2}$$
$$F_{12} = 0.360 \text{ N}$$

and the force is

$$\mathbf{F}_{12} = (0.360 \text{ N})\mathbf{i}$$

The force acting on charge q_2, $\mathbf{F}_{21}$, is a force of attraction caused by charge q_1. Since this force points in the negative x-direction, it can be written as

$$\mathbf{F}_{21} = -F_{21}\mathbf{i}$$

The magnitude of the force F_{21} is found from Coulomb's law and is given by

$$F_{21} = \frac{kq_2 q_1}{r^2}$$

$$= \frac{\left(9.00 \times 10^9 \ \text{N m}^2/\text{C}^2\right)\left(5.00 \times 10^{-6} \ \text{C}\right)\left(2.00 \times 10^{-6} \ \text{C}\right)}{\left(0.500 \ \text{m}\right)^2}$$

$$F_{21} = 0.360 \ \text{N}$$

and the force is

$$\mathbf{F}_{21} = -\ (0.360 \ \text{N})\mathbf{i}$$

Note that the magnitudes of the forces on q_1 and q_2 are identical, but their directions are opposite. This could have been deduced immediately from Newton's third law, for if charge 1 exerts a force on charge 2, then charge 2 must exert an equal but opposite force on charge 1.

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Example 2.2

Transfer of charges and Coulomb's law. Two identical metal spheres are placed 0.200 m apart. A charge q_1 of 9.00 μC is placed on one sphere while a charge q_2 of −3.00 μC is placed upon the other. (a) What is the force on each of the spheres? (b) If the two spheres are brought together and touched and then returned to their original positions, what will be the force on each sphere?
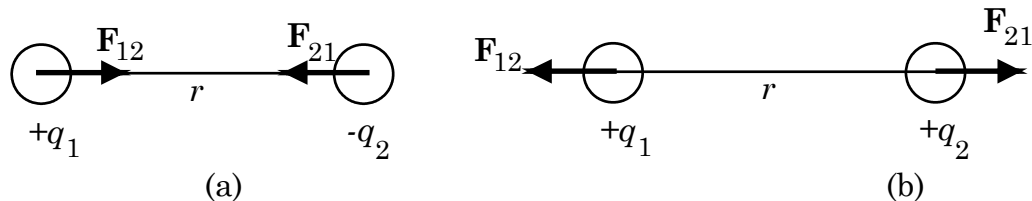

Diagram for example 2.2

Solution

(a) The magnitude of the attractive force on sphere 1 is found from Coulomb's law as

$$F_{12} = \frac{kq_1 q_2}{r^2}$$

$$= \frac{\left(9.00 \times 10^9 \ \text{N m}^2/\text{C}^2\right)\left(9.00 \times 10^{-6} \ \text{C}\right)\left(3.00 \times 10^{-6} \ \text{C}\right)}{\left(0.200 \ \text{m}\right)^2}$$

$$F_{12} = 6.08 \ \text{N}$$

and the force is found as

$$\mathbf{F}_{12} = (6.08 \ \text{N})\mathbf{i}$$

The force on sphere 2 is equal and opposite to this.

(b) When the spheres are touched together, the -3.00 µC of charge q_2, neutralizes $+3.00$ µC of charge q_1, leaving a net charge of

$$9.00 \times 10^{-6} \text{ C} - 3.00 \times 10^{-6} \text{ C} = 6.00 \times 10^{-6} \text{ C}$$

This charge of 6.00×10^{-6} C is then equally distributed between sphere one and sphere two, giving $q_1' = q_2' = 3.00 \times 10^{-6}$ C. Since the charges on both spheres are now positive, the force on each sphere is now repulsive. When the spheres are removed to the original distance the magnitude of the force on each sphere is now

$$F = \frac{kq_2'q_1'}{r^2}$$
$$= \frac{\left(9.00 \times 10^9 \text{ N m}^2/\text{C}^2\right)\left(3.00 \times 10^{-6} \text{ C}\right)^2}{\left(0.200 \text{ m}\right)^2}$$
$$F = 2.03 \text{ N}$$

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2.4 Multiple Discrete Charges

If there are three or more charges present, then the force on any one charge is found by the vector addition of the forces associated with the other charges. That is, the resultant force on any one charge is given by

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + ... \tag{2.4}$$

Example 2.3

Multiple discrete charges. Three charges are placed on the line as shown. The separation of the charges are r_{12}, $= 0.500$ m and $r_{23} = 0.500$ m. If $q_1 = 1.00$ µC, $q_2 = -2.00$ µC, and $q_3 = 3.00$ µC, find: (a) the resultant force on charge q_1, (b) the resultant force on charge q_2, and (c) the resultant force on charge q_3.
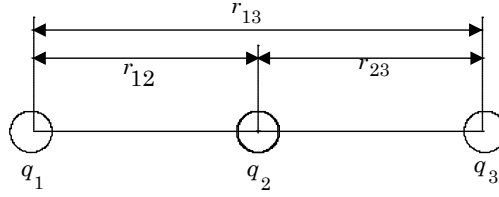
Diagram for example 2.3.

Solution

The resultant force on each charge is equal to the vector sum of all the forces acting on that charge. (a) As can be seen in diagram 2.3(a), the force on charge 1 is

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13}$$

where $\mathbf{F}_{12}$ is the force on charge 1 caused by charge 2, and $\mathbf{F}_{13}$ is the force on charge 1 caused by charge 3.
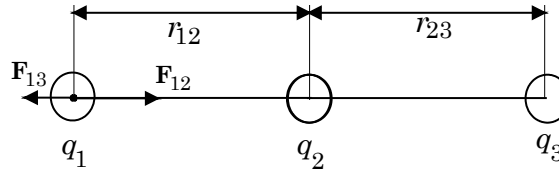

Diagram for example 2.3(a).

Because q_2 is negative, $\mathbf{F}_{12}$ is a force of attraction to the right and is given by

$$\mathbf{F}_{12} = \mathbf{i}F_{12}$$

Since q_3 is positive, $\mathbf{F}_{13}$ is a force of repulsion to the left and is given by

$$\mathbf{F}_{13} = -\mathbf{i}F_{13}$$

The total force on charge 1 is therefore

$$\mathbf{F}_1 = \mathbf{i}F_{12} - \mathbf{i}F_{13}$$

where

$$F_{12} = \frac{kq_1q_2}{r_{12}^2}$$

$$= \frac{\left(9.00\times10^9 \text{ N m}^2/\text{C}^2\right)\left(1.00\times10^{-6} \text{ C}\right)\left(2.00\times10^{-6} \text{ C}\right)}{\left(0.500 \text{ m}\right)^2}$$

$$F_{12} = 0.0720 \text{ N}$$

while

$$F_{13} = \frac{kq_1q_3}{r_{13}^2}$$

$$= \frac{\left(9.00\times10^9 \text{ N m}^2/\text{C}^2\right)\left(1.00\times10^{-6} \text{ C}\right)\left(3.00\times10^{-6} \text{ C}\right)}{\left(1.00 \text{ m}\right)^2}$$

$$F_{13} = 0.0270 \text{ N}$$

Therefore,

$$\mathbf{F}_1 = \mathbf{i}F_{12} - \mathbf{i}F_{13} = (0.0720 \text{ N})\mathbf{i} - (0.0270 \text{ N})\mathbf{i}$$
$$\mathbf{F}_1 = (0.0450 \text{ N})\mathbf{i}$$

That is, the resultant force on charge 1 is a force of 0.0450 N to the right.

(b) The resultant force on charge 2 is

$$\mathbf{F}_2 = \mathbf{F}_{21} + \mathbf{F}_{23}$$

or as can be seen from diagram 2.3b

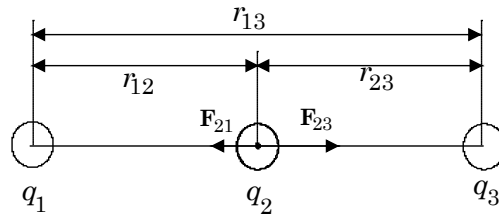$$\mathbf{F}_2 = \mathbf{i}F_{23} - \mathbf{i}F_{21}$$

Diagram for example 2.3(b).

From Newton's third law

$$\mathbf{F}_{21} = -\mathbf{F}_{12} = -(0.0720 \text{ N})\mathbf{i}$$

and

$$F_{23} = \frac{kq_2q_3}{r_{23}^2}$$

$$= \frac{\left(9.00\times10^9 \text{ N m}^2/\text{C}^2\right)\left(2.00\times10^{-6} \text{ C}\right)\left(3.00\times10^{-6} \text{ C}\right)}{\left(0.500 \text{ m}\right)^2}$$

$$F_{23} = 0.216 \text{ N}$$

Therefore, the net force on charge 2 is

$$\mathbf{F}_2 = \mathbf{i}F_{23} - \mathbf{i}F_{21} = (0.216 \text{ N})\mathbf{i} - (0.0720 \text{ N})\mathbf{i}$$
$$\mathbf{F}_2 = (0.144 \text{ N})\mathbf{i}$$

to the right.

(c) The resultant force on charge 3 is

$$\mathbf{F}_3 = \mathbf{F}_{31} + \mathbf{F}_{32}$$

and as can be seen in diagram 2.3(c) this can be written as

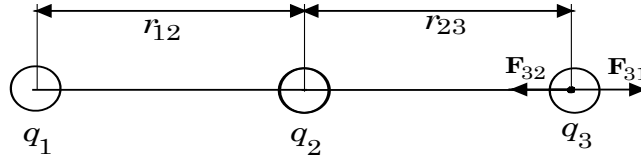$$\mathbf{F}_3 = \mathbf{i}F_{31} - \mathbf{i}F_{32}$$

Diagram for example 2.3(c).

But by Newton's third law

$$\mathbf{F}_{31} = -\mathbf{F}_{13} = (0.0270 \text{ N})\mathbf{i}$$

and

$$\mathbf{F}_{32} = -\mathbf{F}_{23} = - (0.2160 \text{ N})\mathbf{i}$$

The net force on charge 3 is therefore

$$\mathbf{F}_3 = (0.0270 \text{ N})\mathbf{i} - (0.2160 \text{ N})\mathbf{i}$$
$$\mathbf{F}_3 = -(0.189 \text{ N})\mathbf{i}$$

to the left.

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Example 2.4

Charges not on a line. Find the resultant force on charge q_2 in the diagram if $q_1 = 13.0$ μC, $q_2 = 4.00$ μC, $q_3 = 5.00$ μC, $r_{13} = 0.500$ m, and $r_{23} = 0.800$ m.

Solution

The resultant force on charge q_2 is found as

$$\mathbf{F}_2 = \mathbf{F}_{21} + \mathbf{F}_{23}$$

where $\mathbf{F}_{21}$ is the force on charge 2 caused by charge 1, and $\mathbf{F}_{23}$ is the force on charge 2 caused by charge 3. The distance r_{12} is found from the diagram and the Pythagorean theorem as
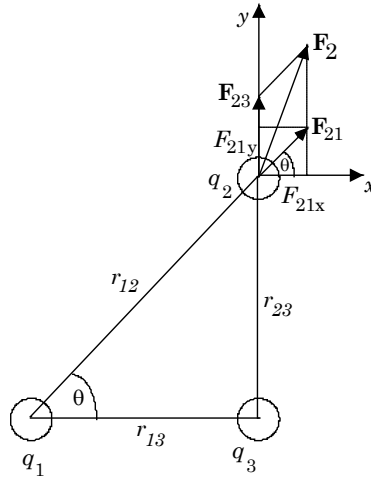
Diagram for example 2.4

$$r_{12} = \sqrt{r_{13}^2 + r_{23}^2}$$
$$r_{12} = \sqrt{(0.500 \text{ m})^2 + (0.800 \text{ m})^2}$$
$$r_{12} = 0.940 \text{ m}$$

The magnitude F_{21} is found from Coulomb's law as

$$F_{21} = \frac{kq_2 q_1}{r_{12}^2}$$
$$F_{21} = \left(9.00 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}\right) \frac{(4.00 \times 10^{-6} \text{ C})(13.00 \times 10^{-6} \text{ C})}{(0.940 \text{ m})^2}$$
$$F_{21} = 0.530 \text{ N}$$

while the magnitude of F_{23} is

$$F_{23} = \frac{kq_2 q_3}{r_{23}^2}$$
$$F_{23} = \left(9.00 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}\right) \frac{(4.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})}{(0.800 \text{ m})^2}$$
$$F_{23} = 0.281 \text{ N}$$

The addition of the two vectors is an example of the addition of vectors discussed in chapter 1. The resultant vector is given by

$$\mathbf{F}_2 = \mathbf{i}F_{2x} + \mathbf{j}F_{2y}$$

The magnitude of the resultant vector is found, as before, as

$$F_2 = \sqrt{(F_{2x})^2 + (F_{2y})^2}$$

The force $\mathbf{F}_{21}$ is given by
$$\mathbf{F}_{21} = \mathbf{i}F_{21x} + \mathbf{j}F_{21y}$$

The vector $\mathbf{F}_{21}$ has an x-component given by

$$F_{21x} = F_{21} \cos\theta$$

The angle θ is found from the geometry of the diagram as

$$\theta = \tan^{-1}\left(\frac{0.800}{0.500}\right) = 58.0^0$$

Therefore, the x-component of $\mathbf{F}_{21}$ is

$$F_{21x} = F_{21} \cos\theta = (0.530 \text{ N})\cos 58.0^0 = 0.281 \text{ N}$$

Similarly, the y-component of $\mathbf{F}_{21}$ is

$$F_{21y} = F_{21} \sin\theta = (0.530 \text{ N})\sin 58.0^0 = 0.449 \text{ N}$$

The vector $\mathbf{F}_{23}$ is in the y-direction and is given by

$$\mathbf{F}_{23} = \mathbf{j}F_{23}$$

and has no x-component. Therefore, the x-component of the resultant vector is

$$F_{2x} = F_{21x} = 0.281 \text{ N}$$

and as can be seen from the diagram, the y-component of the resultant vector is

$$F_{2y} = F_{23} + F_{21y} = 0.281 \text{ N} + 0.449 \text{ N} = 0.730 \text{ N}$$

The resultant force on charge 2 is

$$\mathbf{F}_2 = \mathbf{i}F_{2x} + \mathbf{j}F_{2y} = (0.281 \text{ N})\mathbf{i} + (0.730 \text{ N})\mathbf{j}$$

The magnitude of the resultant force on charge 2 is therefore

$$F_2 = \sqrt{(F_{2x})^2 + (F_{2y})^2} = \sqrt{(0.281 \text{ N})^2 + (0.730 \text{ N})^2}$$
$$F_2 = 0.782 \text{ N}$$

The angle ϕ that $\mathbf{F}_2$ makes with the x-axis is found from

$$\phi = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\left(\frac{0.730}{0.281}\right)$$

$$\phi = 68.9^0$$

To go to this Interactive Example click on this sentence.

Example 2.5

More charges not on a line. Find the resultant force on charge q_2 in the diagram if $q_1 = 3.00\ \mu C$, $q_2 = 4.00\ \mu C$, $q_3 = 5.00\ \mu C$, $r_{13} = 0.500$ m and $r_{23} = 0.500$.
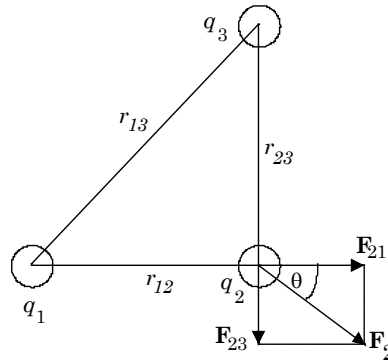

Diagram for example 2.5

Solution

The resultant force on charge 2 is found from the vector sum

$$\mathbf{F}_2 = \mathbf{F}_{21} + \mathbf{F}_{23}$$

Because $\mathbf{F}_{21}$ is perpendicular to $\mathbf{F}_{23}$, the magnitude of the resultant force is

$$F_2 = \sqrt{\left(F_{21}\right)^2 + \left(F_{23}\right)^2}$$

where

$$F_{21} = \frac{kq_2q_1}{r_{21}^2}$$

$$= \frac{\left(9.00\times10^9\ \text{N m}^2/\text{C}^2\right)\left(5.00\times10^{-6}\ \text{C}\right)\left(3.00\times10^{-6}\ \text{C}\right)}{\left(0.500\ \text{m}\right)^2}$$

$$F_{21} = 0.540\ \text{N}$$

and

$$F_{23} = \frac{kq_2q_3}{r_{23}^2}$$

$$= \frac{\left(9.00 \times 10^9 \text{ N m}^2/\text{C}^2\right)\left(5.00 \times 10^{-6} \text{ C}\right)\left(4.00 \times 10^{-6} \text{ C}\right)}{\left(0.500 \text{ m}\right)^2}$$

$$F_{23} = 0.720 \text{ N}$$

As can be seen from the diagram, the resultant force on charge 2 is

$$\mathbf{F}_2 = \mathbf{i}F_{21} - \mathbf{j}F_{23}$$
$$\mathbf{F}_2 = (0.540 \text{ N})\mathbf{i} - (0.720)\mathbf{j}$$

The magnitude of the resultant force on charge 2 is therefore

$$F_2 = \sqrt{\left(F_{21}\right)^2 + \left(F_{23}\right)^2} = \sqrt{\left(0.540 \text{ N}\right)^2 + \left(0.720 \text{ N}\right)^2}$$
$$F_2 = 0.900 \text{ N}$$

The direction of the resultant force is determined by

$$\theta = \tan^{-1}\frac{F_{23}}{F_{21}} = \tan^{-1}\left(\frac{-0.720}{0.540}\right)$$
$$\theta = -53.1^0$$

To go to this Interactive Example click on this sentence.

In section 2.3, and in particular in equation 2.1, we wrote Coulomb's law in terms of its magnitude only. In this section we have been using the unit vectors $\mathbf{i}$ and $\mathbf{j}$ to describe the direction of the force vector. Coulomb's law can be written in a more general form as follows. Let q_1 be the primary charge that we are interested in. We will introduce a unit vector $\mathbf{r}_o$ that points everywhere radially away from the charge q_1 as shown in figure 2.5(a).

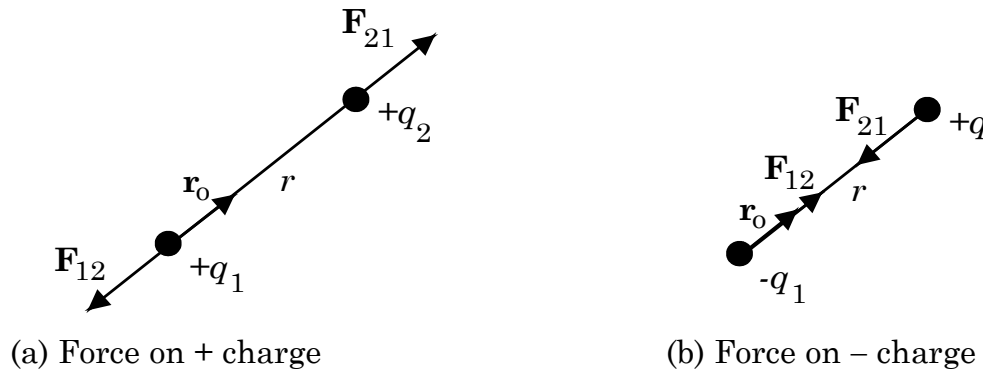(a) Force on + charge   (b) Force on – charge

Figure 2.5 Coulomb's law in vector form.

If a charge $+ q_2$ is brought into the vicinity of charge q_1, it will experience the force

$$\mathbf{F}_{21} = k\frac{q_2 q_1}{r^2}\mathbf{r}_o$$

(2.5)

where $\mathbf{F}_{21}$ is the force on charge q_2 caused by charge q_1. If q_1 and q_2 are of like sign then the force on the secondary charge q_2 is in the same direction as the unit vector $\mathbf{r}_o$, and the force is one of repulsion as expected and is shown in figure 2.5(a). If the primary charge is negative, that is, $-q_1$, then the charges are of opposite sign and the force on charge q_2 is in the opposite direction of the unit vector $\mathbf{r}_o$, and the force is one of attraction as seen in figure 2.5(b). By Newton's third law, the force on charge q_1 is equal and opposite to the force on charge q_2 as expected. That is,

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

2.5 Forces Caused by a Continuous Distribution of Charge

As we have seen in the last section, when there are multiple discrete charges in a region, then the force on any one of those charges is found by the vector sum of the forces associated with the other charges. That is, the resultant force on any one charge was given by equation 2.4 as

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + ...$$

A shorthand notation for equation 2.4 can be written as

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i$$

(2.6)

where, again, Σ, the Greek letter sigma, means "the sum of" and the sum goes from i = 1 to i = N.

Besides the forces caused by a discrete distribution of charge, the force on a single charge q_o caused by a continuous distribution of charge can be handled in a similar way. *The uniform distribution of charge can be broken up into a large number of infinitesimal elements of charge, dq, and each element of charge will produce an element of force d$\mathbf{F}$ on the discrete charge q_o.* Coulomb's Law can then be written as

$$d\mathbf{F} = k\frac{q_o dq}{r^2}\mathbf{r}_o$$

(2.7)

where r is the distance from the element of charge dq to the single discrete charge q_o. $\mathbf{r}_o$ is a unit vector that points from the element of charge dq, and points toward the discrete charge q_o. *The total force $\mathbf{F}$ on the discrete charge q_o, caused by the*

*forces from the entire distribution of all the dq's is again a sum, but since the elements of charge dq, are infinitesimal the sum becomes the integral of all the elements of force d**F**.* That is, the force is found as

$$\mathbf{F} = \int d\mathbf{F} = \int k\frac{q_o dq}{r^2}\mathbf{r}_o \tag{2.8}$$

Remember that q_o is a constant in this integration.

　　As an example of the force caused by a continuous distribution, let us find the force acting on a charge q_o located at the origin of a semicircular ring of charge as shown in figure 2.6. The ring has a radius r and carries a continuous distribution of charge q. A small element of this charge dq is shown in the right quadrant of the diagram. This element of charge dq exerts the force $d\mathbf{F}$ on the discrete charge q_o, given by equation 2.7, and is shown in the diagram. The total force $\mathbf{F}$ is found from equation 2.8, which is a vector integration.
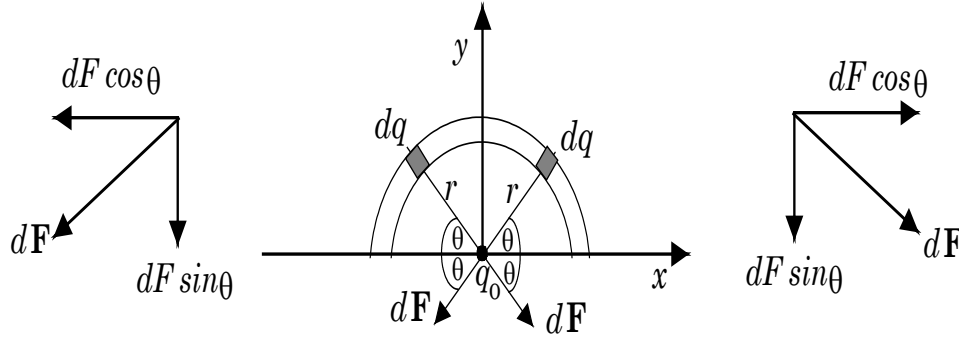

Figure 2.6 Force on a point charge caused by a continuous distribution of charge.

　　The vector integration can be simplified into a scalar integration by noting that the element of charge dq has a mirror image of charge of the same magnitude on the other half of the semicircular ring as shown in the second quadrant of the diagram. Each charge produces an element of force $d\mathbf{F}$. This element of force can be broken up into two components, one, $dF\cos\theta$, which lies along the x-axis, while the second component, $dF\sin\theta$, lies along the negative y-axis. When we add up (integrate) the effect of all the charges dq we see that *half the components $dF\cos\theta$, are in the positive x-direction, while the other half are in the negative x-direction. Thus, the sum of all the x-components of the forces will be zero, and we need only consider the y-components. Both y-components, dF sinθ, are in the negative y-direction,* and the total force can be written as the scalar integration

$$F = \int 2dF\sin\theta = \int 2k\frac{q_o dq}{r^2}\sin\theta \tag{2.9}$$

Notice that, in general, k and q_o, are constants and, for this particular problem, r, the radius of the ring is also a constant. Hence, they can all be taken outside the integral sign. The integral is now given as

$$F = \frac{2kq_o}{r^2} \int \sin\theta \, dq \tag{2.10}$$

Notice that the integration is over dq. Rather than integrating over a charge, it is easier to integrate over a geometrical figure. We define a **linear charge density** λ as the amount of charge per unit length, that is,

$$\lambda = \frac{q}{s} \tag{2.11}$$

where q is the total charge on the semicircular ring and s is the total length of the semicircular ring and also represents the arc of the semicircular ring. Solving equation 2.11 for q gives

$$q = \lambda s$$

and upon differentiating for the element of charge, dq, we get

$$dq = \lambda \, ds \tag{2.12}$$

Replacing equation 2.12 into equation 2.10 we get

$$F = \frac{2kq_o}{r^2} \int \sin\theta \, \lambda ds \tag{2.13}$$

Now the $\sin\theta$ and ds are not independent and the relation between them must be stated before the integration can begin. The angle θ is related to the radius r and the arc s of a circle by

$$s = r\theta$$

Differentiating this equation we get

$$ds = r \, d\theta \tag{2.14}$$

where ds is an element of an arc of the semicircular ring, r is the radius of the ring, and $d\theta$ is the small angle subtended by the arc ds. Replacing equation 2.14 into equation 2.13 gives

$$F = \frac{2kq_o}{r^2} \int \sin\theta \, \lambda r d\theta$$

Assuming λ is a constant, it can be taken out of the integral to yield

$$F = \frac{2kq_o\lambda}{r} \int_0^{\pi/2} \sin\theta \, d\theta \tag{2.15}$$

Notice that the integration is now over the angle θ, and the integration is from $\theta = 0$ to $\theta = \pi/2$. Integrating equation 2.15 gives

$$F = \frac{2kq_o\lambda}{r}(-\cos\theta)\,|_0^{\pi/2}$$
$$F = -\frac{2kq_o\lambda}{r}\left(\cos\frac{\pi}{2} - \cos 0\right)$$
$$F = -\frac{2kq_o\lambda}{r}(0 - 1)$$
$$F = \frac{2kq_o\lambda}{r} \tag{2.16}$$

Equation 2.16 gives the force acting on the discrete charge q_o in terms of the linear charge density λ. It can be expressed in terms of the total charge q on the semicircular ring by writing equation 2.11 as

$$\lambda = \frac{q}{s} = \frac{q}{\pi r} \tag{2.17}$$

Notice that the arc s of the semicircular ring is equal to half the circumference of a circle, namely, πr. Substituting equation 2.17 into equation 2.16 we get

$$F = \frac{2kq_o}{r}\frac{q}{\pi r}$$
$$F = \frac{2kq_oq}{\pi r^2} \tag{2.18}$$

Equation 2.18 gives the force on a discrete charge q_o by a continuous distribution of charge q along the surface of a semicircular ring of radius r.

The force on a discrete charge q_o caused by any other continuous distribution of charge can be found in the same way.

Summary of Important Concepts

Electrostatics - the study of electric charges at rest under the action of electric forces.

The Fundamental Principle of Electrostatics - like electric charges repel each other while unlike electric charges attract each other.

Quarks - elementary particles of matter. There are six quarks. They are: up, down, strange, charm, bottom, and top. The proton and neutron are made of quarks, but the electron is not.

Conductors - materials that permit the free flow of electric charge through them.

Insulators or Dielectrics - materials that do not permit the free flow of electric charge through them.

Coulomb's law - The force between point charges q_1 and q_2 is directly proportional to the product of their charges and inversely proportional to the square of the distance separating them. The direction of the force lies along the line separating the charges.

Linear charge density λ is defined as the amount of charge per unit length.

Summary of Important Equations

Coulomb's law
$$F = \frac{kq_1q_2}{r^2}$$
(2.1)

Force caused by multiple discrete charges $\quad \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + ...$ (2.4)

Coulomb's law in vector form
$$\mathbf{F}_{21} = k\frac{q_2q_1}{r^2}\mathbf{r}_o$$
(2.5)

Coulomb's law in differential form
$$d\mathbf{F} = k\frac{q_odq}{r^2}\mathbf{r}_o$$
(2.7)

Forced caused by a continuous charge distribution $\quad \mathbf{F} = \int d\mathbf{F} = \int k\frac{q_odq}{r^2}\mathbf{r}_o$ (2.8)

Linear charge density
$$\lambda = \frac{q}{s}$$
(2.11)

Linear element of charge $\quad\quad dq = \lambda\ ds$ (2.12)

Problems For Chapter 2

Section 2.3 Coulomb's Law.
1. A point charge of 4.00 μC is placed 25.0 cm from another point charge of −5.00 μC. Calculate the magnitude and direction of the force on each charge.
2. If the force of repulsion between two protons is equal to the weight of the proton, how far apart are the protons?
3. What equal positive charges would have to be placed on the earth and the moon to neutralize the gravitational force between them?
4. What is the velocity of an electron in the hydrogen atom if the centripetal force is supplied by the coulomb force between the electron and proton? The radius of the electron orbit is 5.29×10^{-11} m.

5. Two identical metal spheres are placed 15.0 cm apart. A charge of 6.00 μC is placed on one sphere while a charge of –2.00 μC is placed upon the other. What is the force on each sphere? If the two spheres are brought together and touched and then separated to their original separation, what will be the force on each sphere?

6. Two identical metal spheres, carrying different opposite charges, *attract* each other with a force of 5.00×10^{-6} N when they are 5.00 cm apart. The spheres are then touched together and then removed to the original separation where now a force of *repulsion* of 1.00×10^{-6} N is observed. What is the charge on each sphere after touching and before touching?

Section 2.4 Multiple Charges.

7. Three charges $q_1 = 2.00$ μC, $q_2 = 5.00$ μC, and $q_3 = 8.00$ μC are placed along the x-axis at 0.00, 45.0 cm, and 72.4 cm respectively. Find the force on each charge.

8. Two charges $q_1 = 4.00$ μC and $q_2 = 8.00$ μC are separated by the distance r = 50.0 cm as shown. Where should a third charge be placed on the line between them such that the resultant force on it will be zero? Does it matter if the third charge is positive or negative?
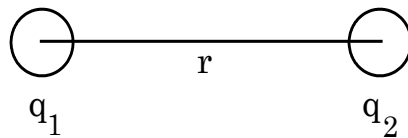
Diagram for Problem 8.

9. Repeat problem 8 but now let charge q_1 be negative, and find any position on the line, either to the left of q_1, between q_1 and q_2, or to the right of q_2, where a third charge can be placed that experiences a zero resultant force.

10. Three charges of 2.00 μC, –4.00 μC, and 6.00 μC are placed at the vertices of an equilateral triangle of length 10.0 cm on a side. Find the resultant force on each charge.

11. If $q_1 = 5.00$ μC $= q_2 = q_3 = q_4$ are located on the corners of a square of length 20.0 cm, find the resultant force on q_3.
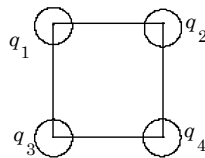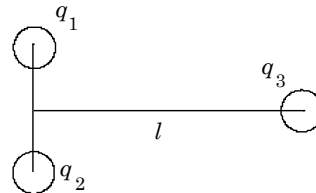

Diagram for problem 11.   Diagram for problem 13.

12. Charges of 2.54 μC, –7.86 μC, 5.34 μC, and –3.78 μC are placed on the corners of a square of side 23.5 cm. Find the resultant force on the first charge.

13. Find the force on charge $q_3 = 2.00$ μC, if $q_1 = + 5.00$ μC and $q_2 = - 5.00$ μC. The distance separating charges q_1 and q_2 is 5.00 cm, and $l = 1.00$ m.

14. Find the resultant force on charge q_3 in the diagram if $q_1 = 2.00$ µC, $q_2 = -7.00$ µC, $q_3 = 5.00$ µC, $r_{12} = 0.750$ m, $r_{23} = 0.600$ m.
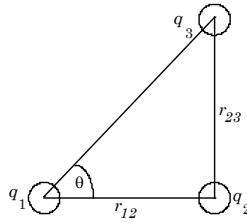

Diagram for problem 14.

Section 2.5 Force Caused by a Continuous Distribution of Charge.

15. A thin nonconducting rod of length l carries a uniform charge per unit length λ. The rod lies on the $+ x$-axis with one end at the point x_o and the other end at $x = x_o + l$. A point charge q_o lies on the x-axis at the origin. Find the force acting on charge q_o.

16. This is similar to problem 15 except the point charge q_o is located on the y-axis at the point y_o. (a) Find the x-component of the force on the point charge caused by the line of charge. (b) Find the y-component of the force on the point charge caused by the line of charge.

Additional Problems.

17. Find the force on charge $q_5 = 5.00$ µC, located at the center of a square 25.0 cm on a side if $q_1 = q_2 = 3.00$ µC and $q_3 = q_4 = 6.00$ µC.
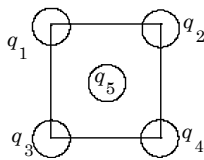
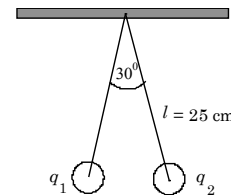

Diagram for problem 17.

Diagram for problem 19.

18. Two small, equally charged, spheres of mass 0.500 gm are suspended from the same point by a silk fiber 50.0 cm long. The repulsion between them keeps them 15.0 cm apart. What is the charge on each sphere?

19. Two pith balls of 10.0 gm mass are hung from ends of a string 25.0 cm long, as shown. When the balls are charged with equal amounts of charge, the threads separate to an angle of 30.0^0. What is the charge on each ball?

20. Two 10.0 gm pith balls are hung from the ends of two 25.0 cm long strings as shown. When an equal and opposite charge is placed on each ball, their separation is reduced from 10.0 cm to 8.00 cm. Find the tension in each string and the charge on each ball.
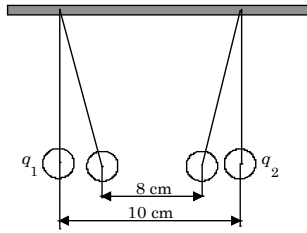
Diagram for problem 20.

21. A charge of 15.0 µC is on a metallic sphere 10.0 cm radius. It is then touched to a sphere of 5.00 cm radius, until the surface charge density is the same on both spheres. What is the charge on each sphere after they are separated? (The surface charge density σ is the charge per unit area and is given by $\sigma = q/A$.)

22. Two small spheres carrying charges $q_1 = 7.00$ µC and $q_2 = 5.00$ µC are separated by 20.0 cm. If q_2 were free to move what would its initial acceleration be? Sphere 2 has the mass $m_2 = 15.0$ gm.

23. Where should a fourth charge, $q_4 = 3.00$ µC, be placed to give a net force of zero on charge q_3? Charge $q_1 = 2.00$ µC, $q_2 = 4.00$ µC, and $q_3 = 2.00$ µC.
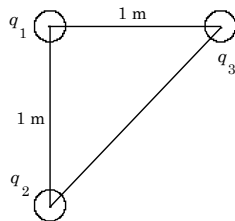

Diagram for problem 23.
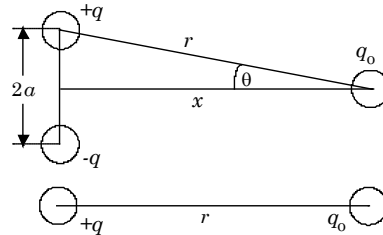

Diagram for problem 25.

24. Charge $q_1 = 3.00$ µC, is located at the coordinates (0,2) and charge $q_2 = 6.00$ µC, is located at the coordinates (1,0) of a Cartesian coordinate system. Find the coordinates of a third charge that will experience a zero net force.

25. The configuration of a positive charge q separated by a distance $2a$ from a negative charge $-q$ is called an electric dipole. Show that the force exerted by an electric dipole on a point charge q_o, located as shown in the diagram varies as $1/r^3$ while the force between a point charge q and the point charge q_o varies as $1/r^2$. Which force is the weaker?

26. A plastic rod 50.0 cm long, has a charge $+ q_1 = 2.00$ µC at each end. The rod is then hung from a string and placed so that each charge is only 5.00 cm from negative charges $q_2 = - 10.0$ µC as shown in the diagram. Find the torque acting on the string.

27. A charge of 5.00 µC is uniformly distributed over a copper ring 2.00 cm in radius. What force will this ring exert on a point charge of 8.00 µC that is placed 3.00 m away from the ring. Indicate what assumptions you make to solve this problem.
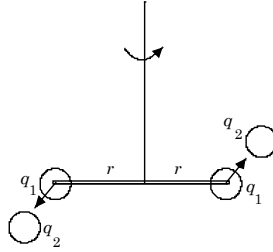
Diagram for problem 26.

28. A charge of 2.50 μC is placed at the center of a hollow sphere of charge of 8.00 μC. What is the resultant force on the charge placed at the center of the sphere? Indicate what assumptions you make to solve this problem.

29. A thin nonconducting rod of length l carries a nonuniform charge density $\lambda = Ax^2$. The rod lies on the x-axis with one end at the origin and the other end at $x = l$. A point charge q_0 lies on the x-axis at the point x_0. Find the force acting on charge q_0.

30. Find the force acting on a point charge q_0 located at the origin of a semicircular ring of charge as shown in figure 2-6. The ring has a radius r and carries a nonuniform continuous distribution of charge density $\lambda = A \sin\theta$.

To go to another chapter in the text, return to the table of contents by clicking on this sentence.