A COMPUTER CODE FOR

HIGH ALTITUDE EMP

THESIS

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Terry C. Chapman
Capt USAF

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The code can be used to calculate the peak value of the electric field at a target anywhere on or above ground level,

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20. resulting from a nuclear burst above 60 km altitude with a gamma yield up to 60 tons. Either the direct or the ground reflected wave can be calculated. With special care, bursts up to 1 kt of gamma yield can be used.

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A COMPUTER CODE FOR HIGH ALTITUDE EMP

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
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by

Terry C. Chapman, B.S.
Capt USAF
Graduate Nuclear Engineering

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Preface

It is my pleasure to recognize several people who contributed in various ways to make this work possible.

I want to thank my advisor, Maj Carl T. Case. His patience and helpful suggestions were important factors to the successful conclusion of this work. In addition, I want to point out that the theoretical portion of this work is based largely on a series of lectures he gave while teaching the Electromagnetic Waves (EE 6.30) course during the summer quarter of 1973. His unusual ability to present difficult topics in a way that is easily understood by the student was the largest and most important contribution to the success of this work.

I want to thank Dr. Charles J. Bridgman for his suggestions and helpful comments made during the early stages of the work.

I want to thank Capt Frank N. Fredrickson and Lt John R. Lillis for taking the time to discuss various problems, ideas, and solutions with me.

Last, but not least, I want to gratefully acknowledge the large contribution of my wife, Karen. She offered moral support, punched computer cards, typed drafts, and in many other ways contributed to the successful conclusion of this work.

Terry C. Chapman

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Abstract

A relatively inexpensive computer code is developed to calculate the peak value of the electric field contained in an electromagnetic pulse generated by the gamma rays from a high altitude nuclear burst. The code is based on the Karzas and Latter theory for the production of Compton electrons and their interaction with the earth's magnetic field.

The code can be used to calculate the peak value of the electric field at a target anywhere on or above ground level, resulting from a nuclear burst above 60 km altitude with a gamma yield up to 60 tons. Either the direct or the ground reflected wave can be calculated. With special care, bursts up to 1 kt of gamma yield can be used.

A COMPUTER CODE FOR HIGH ALTITUDE EMP

I. Introduction

The effects of a nuclear environment on aerospace systems is an important factor in systems analysis. During the past few years several students have worked with Professor Bridgman at the Air Force Institute of Technology (AFIT) on a computer code to determine survivability of a system with known nuclear vulnerabilities from a variable nuclear threat. The AFIT survivability code capabilities include blast, thermal, x-ray, gamma ray, and neutron effects. The high altitude EMP code presented in this report is intended to be used in conjuction with the AFIT survivability code.

The EMP (electromagnetic pulse) from a nuclear weapon is usually considered to be a radiating electromagnetic wave of short duration containing many frequencies. However, the nuclear generated EMP was not studied seriously until a considerable time after the first nuclear explosion. At present there is a significant amount of work being done to model EMP generation and effects. For example the Air Force Weapons Laboratory (AFWL) and several civilian companies under contract to the USAF are working in the field.

There are several different types of EMP with distinctions made between the mechanisms which generate them. Kinsley (Ref 1) presents a comprehensive discussion of the various types of EMP. For example, a nuclear burst on the ground produces an EMP with different characteristics than those from a high altitude burst. Also, nuclear burst products interacting directly with a system can produce an EMP within the system or even within the circuits of the system. This report considers only the EMP generated by high altitude burst gamma rays interacting with the atmosphere.

The high altitude EMP code developed in this report is based on the theory of Karzas and Latter (Ref 2). Briefly, the theory develops a model for the interaction of Compton electrons with the geomagnetic field. The Compton electrons are produced by prompt gamma radiation from the burst in a reasonably well defined region in the atmosphere. Several simplifications are made before arriving at the final equations.

Since several of the simplifications and assumptions used are implicit in the presentation of the theory, it is appropriate to list them here. Only one group of monoenergetic unscattered gamma rays are considered to produce Compton electrons. Each gamma which interacts is assumed to produce one and only one Compton electron initially traveling precisely in the radial direction. No angular distribution of Compton electrons is allowed. All Compton electrons are assumed to have the same energy. Curvature of the Earth's magnetic field is ignored. The electromagnetic fields are not self-consistent, that is, only the geomagnetic field is considered to affect the motion of the Compton electrons. Cascading of secondary electrons and recombination of ions is ignored. The low frequency portion of the pulse is not considered. The Earth is assumed to be flat and the finite conductivity of ground is not considered. The burst is assumed to be far from the absorption region. Only gamma ray effects are considered.

Although the final model is somewhat restricted by these assumptions and simplifications, the end result is a relatively inexpensive computer code which gives a peak value of the electric field at any target point on or above the ground, which is an upper bound on the actual peak value.

Section II of this report develops the theory and derives the equations used in the code. Section III describes the calculational procedures used in the code. Section IV presents a sample of typical results and a study of input parameter variation. Section V is a discussion of the code's limitations and uses, with recommendations for possible improvements. Appendix A is a code user's guide. Appendix B is the detailed flow charts for the entire code. And finally Appendix C is a listing of the complete code.

II. Theory

Overview

The EMP problem is a problem in classic electromagnetic theory. A solution of Maxwell's equations is a solution of the problem. In this case it is necessary to model the current and charge densities generated by the gamma rays in the absorption region to obtain the sources and conductivity needed to solve Maxwell's equations.

Expressions for the current sources and conductivity are obtained in four steps. The transport of the gammas from the burst to the absorption region is used to obtain the number density of reacting gammas. This result is used with the models for the current and charge densities to obtain preliminary expressions. Then after considering the relativistic motion of the Compton electrons, the preliminary expressions are transformed to spherical coordinates.

After presenting Maxwell's equations in a convenient form, they are transformed to spherical coordinates and retarded time. A high frequency approximation is then made to arrive at the final equations.

Electron Current and Density

Gamma Transport. Consider the geometry shown in Fig. 1. The nuclear burst occurs at the origin at time, $t = 0$. The gamma rays move to point r' in time t' and at that point and time interact to create Compton electrons. It is assumed

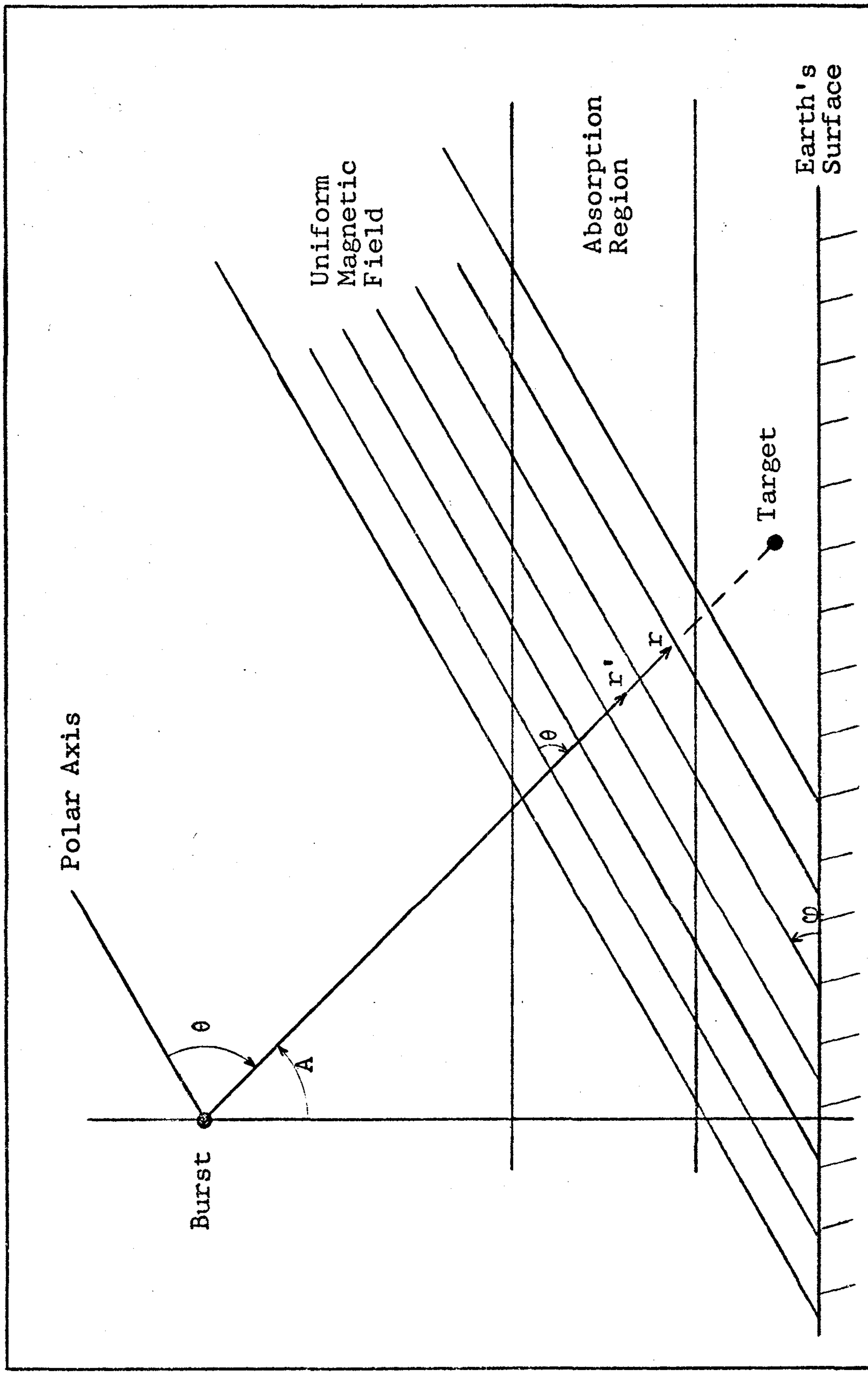


Fig. 1. Geometry of the Burst

that each gamma creates one and only one Compton electron traveling in the radial direction with the maximum Compton recoil energy.

The gamma ray emission rate can be taken as

$$\frac{dN(t)}{dt} = \frac{Y}{E} f(t) \tag{1}$$

where

$N(t)$ = number of gamma rays emitted

Y = gamma ray yield of burst

E = mean energy of the gamma rays

$f(t)$ = time dependence of the yield

and

$$\int_{-\infty}^{\infty} f(t)dt = 1 \tag{2}$$

The number density of gammas, $g(r)$, which interact at a point, r, can be taken as

$$g(r) = \frac{Y}{E} \frac{\exp\left\{-\int_0^r \frac{dr'}{\lambda(r')}\right\}}{4\pi r^2 \lambda(r)} \tag{3}$$

where

λ = mean free path for production of Compton electrons.

<u>Electron Currents and Densities</u>. The rate of production of primary (Compton) electron density, n_{pri}, is

$$\frac{dn_{pri}}{dt} = g(r)f(t - r/c) \tag{4}$$

Following the Karzas-Latter approach (Ref 2) it is assumed that the electrons maintain their initial speed, V_0, throughout their range, R, and then abruptly stop. Also, it is assumed that the secondary electrons are made at a uniform rate during the lifetime, R/V_0, of the Compton electrons. Therefore, the rate of production of secondary electron density, n_{sec}, is

$$\frac{dn_{sec}}{dt} = \left\{ \frac{E_{pri}/33ev}{R/V_0} \right\} n_{pri} = \frac{qV_0}{R} n_{pri} \tag{5}$$

where

E_{pri} = the initial energy of the Compton electrons

R = the range of the Compton electrons in air

q = $E_{pri}/33ev$

$33ev$ = average ionization energy per molecule for air

V_0 = the speed of the Compton electrons

R/V_0 = the lifetime of the Compton electrons

Now consider the current resulting from the Compton electrons. The differential current is the charge times the velocity times the differential density of electrons. Hence

$$d\vec{J}^c = -e\vec{V}(t-t')g(r')f(t' - r'/c)dt' \tag{6}$$

where

$\vec{V}(t-t')$ = velocity of the Compton electrons at time t which were created at time t'.

Putting (6) into integral form gives

$$\vec{J}^c = -e \int_{t-R/V_0}^{t} g(r')f(t' - r'/c)\vec{V}(t-t')dt' \qquad (7)$$

Now let

$$\tau' = t - t' \qquad (8a)$$

$$\tau = t - (r/c) \qquad (8b)$$

$$X(\tau') = X(\tau) = r - r' \qquad (8c)$$

Also note that

$$(r-r') \ll r \text{ or } r' \qquad (9)$$

for distant explosions (see Fig. 1). So,

$$g(r) \simeq g(r') \qquad (10)$$

Using Eqs (8), (9), and (10) in Eq (7) gives

$$\vec{J}^c = -eg(r) \int_0^{R/V_0} \vec{V}(\tau')f\left(\tau - \tau' + \frac{X(\tau')}{c}\right)d\tau' \qquad (11)$$

Using similar arguments,

$$n_{pri} = g(r) \int_0^{R/V_0} f\left(\tau - \tau' + \frac{X(\tau')}{c}\right) d\tau' \qquad (12)$$

And putting Eq (12) into Eq (5) yields

$$n_{sec} = \frac{qV_0}{R} \int_{-\infty}^{\tau} n_{pri}(\tau')d\tau'$$

$$= g(r) \frac{qV_0}{R} \int_{-\infty}^{\tau} \left[\int_0^{R/V_0} f\left(\tau' - \tau'' + \frac{X(\tau'')}{c}\right) d\tau'' \right] d\tau' \quad (13)$$

<u>Relativistic Electron Motion</u>. Equations (11), (12), and (13) contain $r(\tau)$ and $X(\tau)$ which are not yet defined in an easily obtained form. The equation of motion for a Compton electron is

$$\frac{d}{dt}(m\gamma\vec{V}) = -e\vec{V}X\vec{B}_0 \quad (14)$$

where

m = electron rest mass

$\gamma = [1-(V/c)^2]^{-1/2}$

$\vec{B}_0$ = earth's magnetic field = $B_0\hat{U}_z$

Again it is assumed that V_0 is constant throughout the electron's lifetime.

With $\omega = eB_0/m\gamma$ Eq (14) becomes

$$\frac{d}{d\tau}\vec{V}(\tau) = -\vec{V}(\tau)X\hat{U}_z\omega \quad (15)$$

Breaking Eq (15) into its rectangular components

$$\frac{dV_x}{d\tau} = -\omega V_y \quad (16a)$$

$$\frac{dV_y}{d\tau} = \omega V_x \tag{16b}$$

$$\frac{dV_z}{d\tau} = 0 \tag{16c}$$

A solution for this set of equations is

$$V_x = V_\perp \cos \omega\tau \tag{17a}$$

$$V_y = V_\perp \sin \omega\tau \tag{17b}$$

$$V_z = V_\parallel \tag{17c}$$

where $V_\perp$ is the initial velocity component perpendicular to $\vec{B}_0$ and $V_\parallel$ is the initial velocity component parallel to $\vec{B}_0$ and both are constants with respect to τ.

<u>Transformation to Spherical Coordinates</u>. It is convenient to transform the above solution to a spherical coordinate system with its origin at the burst point. The transformation from rectangular to spherical coordinates is

$$V_r = V_x \sin\theta \cos\phi + V_y \sin\theta \sin\phi + V_z \cos\theta \tag{18a}$$

$$V_\theta = V_x \cos\theta \cos\phi + V_y \cos\theta \sin\phi - V_z \cos\theta \tag{18b}$$

$$V_\phi = -V_x \sin\phi + V_y \cos\phi \tag{18c}$$

Without loss of generality the coordinates can be chosen such that $\vec{V}$ lies in the X-Y plane, hence $\phi = 0$, and the transformation becomes

$$V_r = V_x \sin\theta + V_z \cos\theta \tag{19a}$$

$$V_\theta = V_x \cos\theta - V_z \sin\theta \tag{19b}$$

$$V_\phi = V_y \tag{19c}$$

Note that

$$V_\perp = V_0 \sin\theta \tag{20a}$$

$$V_\parallel = V_0 \cos\theta \tag{20b}$$

Putting Eqs (17) and (20) into Eq (19) gives

$$V_r = V_0[\sin^2\theta \cos\omega\tau + \cos^2\theta] \tag{21a}$$

$$V_\theta = V_0[\cos\theta \sin\theta \cos\omega\tau - \sin\theta \cos\theta] \tag{21b}$$

$$V_\phi = V_0[\sin\theta \sin\omega\tau] \tag{21c}$$

Now $X(\tau)$ can be written as

$$X(\tau) = \int_0^\tau V_r d\tau = V_0[\sin^2\theta \frac{\sin\omega r}{\omega} + \tau \cos^2\theta] \tag{22}$$

Equations (21) and (22) substituted into Eq (11) give

$$J_r^c = -eg(r)V_0 \int_0^{R/V_0} f(T)[\cos^2\theta + \sin^2\theta \cos\omega\tau']d\tau' \tag{23}$$

$$J_\theta^c = -eg(r)V_0 \int_0^{R/V_0} f(T)[\sin\theta \cos\theta (\cos\omega\tau'-1)]d\tau' \tag{24}$$

$$J_\phi^c = -eg(r)V_0 \int_0^{R/V_0} f(T)[\sin\theta \sin\omega\tau']d\tau' \tag{25}$$

where

$$T = \tau - (1 - \beta \cos^2 \theta)\tau' + \beta \sin^2 \theta \frac{\sin \omega\tau'}{\omega} \qquad (26a)$$

with

$$\beta = V_0/c \qquad (26b)$$

Equations (23), (24), (25), and (26) provide the Compton currents within the absorption region in a form which can be used in the final field equations. In addition to the Compton currents, an expression for the conductivity within the absorption region is needed.

Equations (21) and (22) substituted into Eq (13) give

$$n_{sec}(\tau) = \frac{qV_0}{R} g(r) \int_{-\infty}^{\tau} [\int_0^{R/V_0} f(T')d\tau'']d\tau' \qquad (27)$$

where

$$T' = \tau' - (1 - \beta \cos^2 \theta)\tau'' + \beta \sin^2 \theta \frac{\sin \omega\tau''}{\omega} \qquad (28)$$

Consider the equation of motion for secondary electrons. Neglecting the $\vec{V}X\vec{B}_0$ term, which is small compared to the other terms (Ref 2) it is

$$m \frac{d\vec{V}}{dt} = -e\vec{E} - m\vec{V}\nu_c \qquad (29)$$

where

ν_c = electron collision frequency.

These secondary electrons have velocities in the thermal region and are assumed to reach their maximum velocity instantly. In this case, Eq (29) becomes

$$\vec{V} = \frac{-e}{m\nu_c}\vec{E} \tag{30}$$

The current source from the secondary electrons is

$$\vec{J}^{sec} = -e\vec{V}(\tau)n_{sec}(\tau) = \frac{n_{sec}(\tau)}{m\nu_c}e^2\vec{E} \tag{31}$$

or in terms of conductivity

$$\vec{J}^{sec} = \sigma(\tau)\vec{E} \tag{32}$$

where

$$\sigma(\tau) = \frac{n_{sec}(\tau)}{m\nu_c}e^2 \tag{33}$$

Equations (32) and (33) provide the needed expressions for the conductivity.

Electromagnetic Fields from High Altitude Currents

Maxwell's Equations. Now that the Compton currents and the conductivity due to secondary electrons have been obtained, consider the field equations.

Maxwell's equations are

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \tag{34a}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \tag{34b}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{q_v}{\varepsilon_0} \tag{34c}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \tag{34d}$$

where

$\vec{J}$ = total current density

q_v = total charge density

Continuity of charge requires

$$\frac{\partial q_v}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0 \tag{35}$$

It is convenient to combine the above equations into equations containing only $\vec{E}$ in one and only $\vec{B}$ in the other. Doing so gives

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0\frac{\partial\vec{J}}{\partial t} + \frac{1}{\varepsilon_0}\vec{\nabla}q_v \tag{36}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} = -\mu_0\vec{\nabla}\times\vec{J} \tag{37}$$

<u>Transformation to Spherical Coordinates and Retarded Time</u>. Equations (36) and (37) will now be transformed to spherical coordinates and retarded time. Consider the transformation

$$\tau = t - r/c \tag{38a}$$

$$r' = r \tag{38b}$$

$$\theta' = \theta \tag{38c}$$

$$\phi' = \phi \tag{38d}$$

This is a spherical coordinate system where time is measured at each radial point in terms of the arrival of the bomb gamma rays at that point.

Using Eq (38) it is easily shown that

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} \tag{39}$$

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r'} - \frac{1}{c}\frac{\partial}{\partial \tau} \tag{40}$$

$$\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta'} \tag{41}$$

$$\frac{\partial}{\partial \phi} = \frac{\partial}{\partial \phi'} \tag{42}$$

Thus the operator

$$\frac{\partial}{\partial t}$$

transforms to

$$\frac{\partial}{\partial \tau}$$

and the operator

$$\vec{\nabla}$$

transforms to

$$\vec{\nabla} - \hat{U}_r \frac{1}{c} \frac{\partial}{\partial \tau}$$

Similarly, the operator

$$\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$

transforms to

$$\nabla^2 - \frac{2}{c} \frac{\partial}{\partial \tau} \frac{1}{r} \frac{\partial}{\partial r}\, r$$

Equation (36) now becomes

$$\left[\nabla^2 - \frac{2}{c} \frac{1}{r} \frac{\partial}{\partial \tau} \frac{\partial}{\partial r}\, r\right] \vec{E} = \mu_0 \frac{\partial \vec{J}}{\partial \tau} + \frac{1}{\varepsilon_0} \vec{\nabla} q_v - \hat{U}_r \frac{1}{c} \frac{\partial q_v}{\partial \tau} \qquad (43)$$

and Eq (35) becomes

$$\frac{\partial q_v}{\partial \tau} = -\vec{\nabla}\cdot\vec{J} + \hat{U}_r \frac{1}{c} \frac{\partial}{\partial \tau} \cdot \vec{J} = -\vec{\nabla}\cdot\vec{J} + \frac{1}{c} \frac{\partial J_r}{\partial \tau} \qquad (44)$$

Using Eq (44) in Eq (43) and rearranging gives

$$-\nabla^2 \vec{E} + \hat{U}_r \frac{1}{c\varepsilon_0} \vec{\nabla}\cdot\vec{J} + \frac{1}{\varepsilon_0} \vec{\nabla} q_v$$

$$+ \frac{\partial}{\partial \tau} \left[\frac{2}{c} \frac{1}{r} \frac{\partial}{\partial \tau} (r\vec{E}) + \mu_0 (\vec{J} - \hat{U}_r J_r)\right] = 0 \qquad (45)$$

Similarly, Eq (37) becomes

$$-\nabla^2\vec{B} + \mu_0\vec{\nabla}\times\vec{J} + \frac{\partial}{\partial\tau}\left[\frac{2}{rc}\frac{\partial}{\partial r}(r\vec{B})\right]$$

$$+\frac{\partial}{\partial\tau}\left[\frac{\mu_0}{c}(\hat{U}_\phi J_\phi - \hat{U}_\theta J_\theta)\right] = 0 \qquad (46)$$

<u>High Frequency Approximation</u>. Again, following the Karzas-Latter model, note that the variation of currents with distance is slow compared to variations with time and that the fields resulting from the transverse currents are rapidly varying in character, as are the currents themselves. Therefore, only the $\partial/\partial\tau$ terms are kept in the transverse field equations. Since the radial components do not propagate outside of the absorption region, they are not considered further in this report.

The transverse equations become

$$\frac{\partial}{\partial\tau}\left[\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) + \mu_0 J_\theta\right] = 0 \qquad (47)$$

$$\frac{\partial}{\partial\tau}\left[\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) + \mu_0 J_\phi\right] = 0 \qquad (48)$$

$$\frac{\partial}{\partial\tau}\left[\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{\mu_0}{c}J_\phi\right] = 0 \qquad (49)$$

$$\frac{\partial}{\partial\tau}\left[\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi) + \frac{\mu_0}{c}J_\theta\right] = 0 \qquad (50)$$

These equations are called the Karzas-Latter high frequency approximation for the EMP fields, and they are useful in the range $0 < \tau < 100$ shakes.

Integration with respect to time yields

$$\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) + \mu_0 J_\theta = 0 \qquad (51)$$

$$\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) + \mu_0 J_\phi = 0$$

$$\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{\mu_0}{c} J_\phi = 0 \qquad (53)$$

$$\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi) + \frac{\mu_0}{c} J_\theta = 0 \qquad (54)$$

Recall that the total current density is

$$\vec{J} = \vec{J}^{pri} + \vec{J}^{sec} = \vec{J}^{c} + \sigma(\tau)\vec{E} \qquad (55)$$

so Eqs (51) and (52) become

$$\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) + \mu_0 J_\theta^c + \mu_0\sigma(\tau)E_\theta = 0 \qquad (56)$$

$$\frac{2}{c}\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) + \mu_0 J_\phi^c + \mu_0\sigma(\tau)E_\phi = 0 \qquad (57)$$

With the aid of a computer, it is now possible to obtain numerical solutions for the above equations which will yield a slightly high estimate of the peak value of the EMP pulse resulting from a high altitude burst.

Below the absorption region the Compton currents and the conductivity are zero. In this case, Eqs (56) and (57) have the following solutions:

$$E_\theta = C_1/r \tag{58}$$

$$E_\phi = C_2/r \tag{59}$$

where C_1 and C_2 are determined by the values of E_θ, E_ϕ, and r at the bottom of the absorption region.

III. Code Description

General Approach

Equations (56), (57), (58), and (59) were chosen as the simplest ones to solve numerically. Of course, Eqs (24), (25), (27), and (33) are used to obtain the Compton currents and conductivity needed to solve Eqs (56) and (57).

The B - field equations are not solved since

$$E = cB \tag{60}$$

can be used to obtain B once E is found. This relationship is based on the assumption that the EMP pulse is a spherical wave propagating in free space, below the absorption region.

The function used for the time dependence of the weapon yield is the one recommended by Pomranning (Ref 3).

$$f(\tau) = (1/N) \frac{(\alpha+\beta) \exp (\tau-\tau_0)}{\beta + \alpha \exp [(\alpha+\beta)(\tau-\tau_0)]} \tag{61}$$

where N is chosen such that

$$\int_0^\infty f(\tau) d\tau = 1 \tag{62}$$

and $\alpha > \beta$.

This function rises like $e^{\alpha\tau}$ for small τ, falls like $e^{-\beta\tau}$ for large τ, and has a single maximum at τ_0.

Figure 2 presents a flow chart which is descriptive of the approach taken solving the equations. The top of the absorption region is assumed to be at 50 km altitude and the bottom of the absorption region is assumed to be

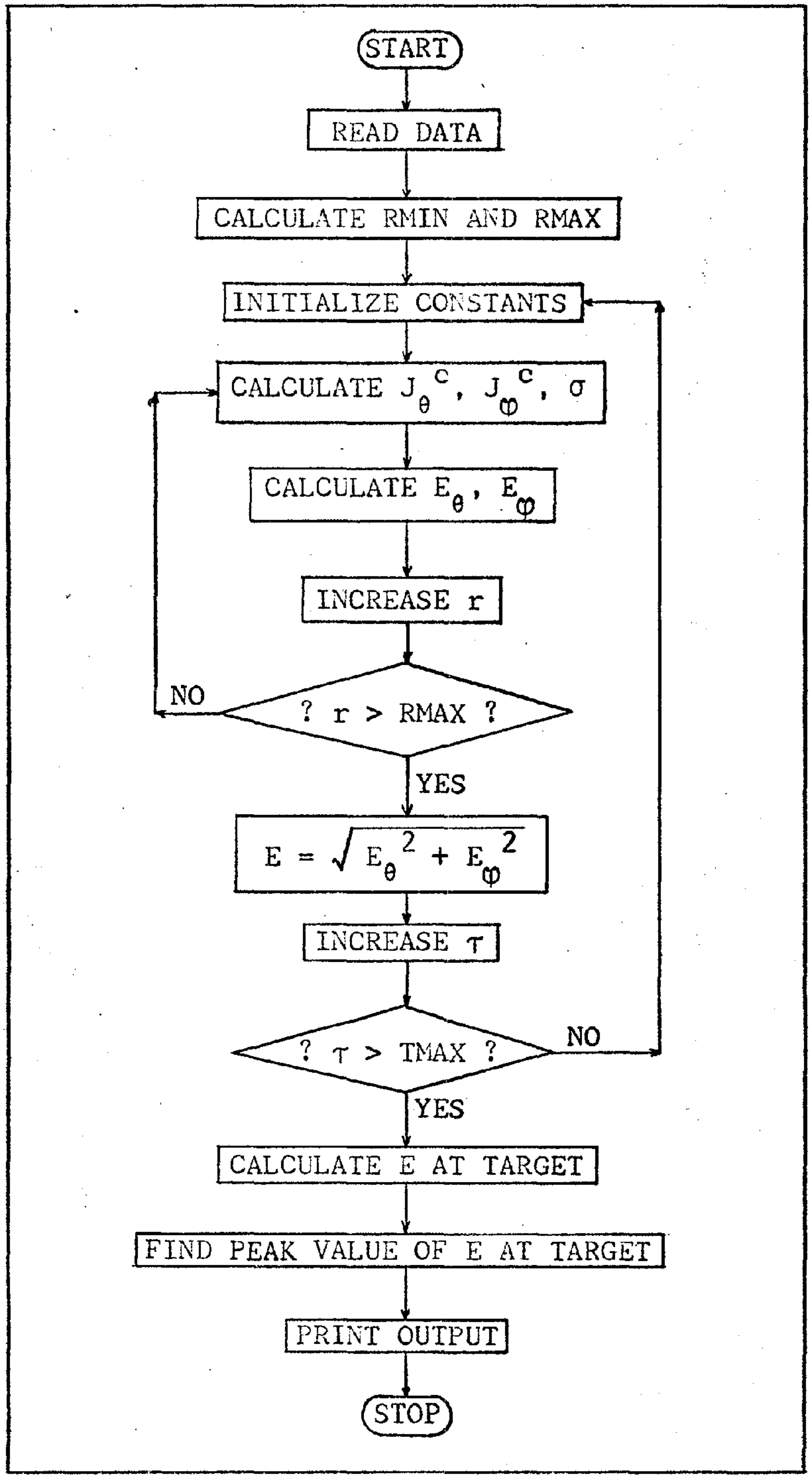


Fig. 2. Descriptive Flow Chart

at 20 km altitude. Calculations by Latter and LeLevier (Ref 4) indicate that 20 km to 50 km is the altitude where most of the prompt gamma ray energy is deposited.

Figure 3 depicts the target geometry. The value for RMIN is determined by the intersection of the line of sight with the 50 km altitude. The value for RMAX is determined by the intersection of the line of sight with the 20 km altitude. If the target is in the absorption region the target altitude determines RMAX for the direct wave calculation. These two values of r are the limits on the mesh in the r direction. The line of sight is divided into the desired number of steps along r for the integration on r in the absorption region.

The retarded time direction of the mesh is divided into 0.1 shake steps up to 10 shakes and then 1.0 shake steps on up to 100 shakes. Calculation can be stopped at any desired TMAX from 10 to 100 shakes, which is the upper limit of the usefulness of the high frequency approximation.

If the ground reflected wave is to be calculated, the mirror image of the target, below ground, is used to find the line of sight from the burst to the target. (Refer to Fig. 3.)

At r = RMIN all of the fields are assumed to be zero. For each τ, equations (57) and (58) are integrated over r from RMIN to RMAX and the value of E at the bottom of the absorption region is stored. At each step in r, equations (24), (25), and (27) are numerically integrated. Then

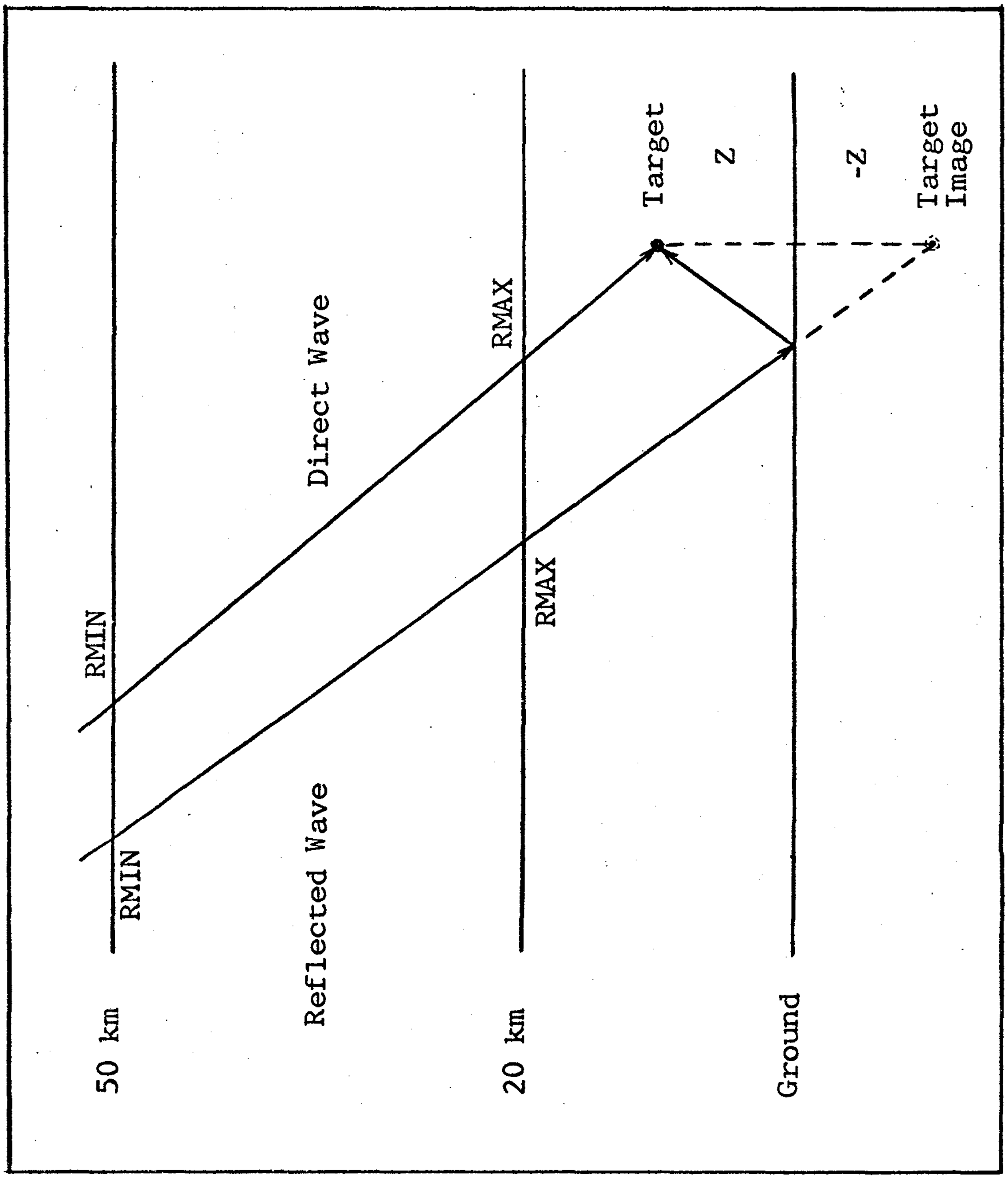


Fig. 3. Target Geometry

equations (58) and (59) are combined into

$$E = \frac{(RMAX)(E_{RMAX})}{r_{target}} \tag{63}$$

to find E at the target.

The E array is then searched to find the peak value before printing out the results.

Inputs

The code uses a right handed Cartesian coordinate system with the ground in the X-Y plane, the $\vec{B}_0$ vector in the Y-Z plane, and $\hat{j}$ pointing towards the equator. For example, in the northern hemisphere, $\hat{i}$ is magnetic west, $\hat{j}$ is magnetic south, and $\hat{k}$ is altitude. The origin of the coordinate system is always at ground zero, directly below the burst. Note that this coordinate system is not the same as the Cartesian systems used earlier.

Refering to the above coordinate system the target coordinates, (X,Y,Z), are read in using units of meters. If the reflected wave is to be calculated the altitude is read in as a negative number, (X,Y,-Z).

The height of the burst is read in using units of kilometers. The gamma yield of the burst is read in using units of kilotons.

The magnitude of the Earth's magnetic field is read in using units of webers per square meter. The dip angle (ϕ in Fig. 1) of the magnetic field is read in using units of degrees.

NDELR, the desired number of steps to be used in the integration over r in the absorption region, is read in as any integer in the closed interval [50, 500].

TMAX, the retarded time where calculations are to be stopped, is read in, using units of shakes, as any integer in the closed interval [10, 100].

Preliminary Calculations

Before starting the numerical integrations, the code performs several preliminary calculations. The input data is converted to MKS units. The reflected wave is used whenever Z is greater than 49 km or less than 0. The target coordinates are transformed to a spherical coordinate system with the burst at the origin and the polar axis parallel to $\vec{B}_0$. The line of sight intersections with the absorption region are determined. And finally, the constant angles required by the code, θ and A, (see Fig. 1) are calculated.

Calculation of Compton Currents and Conductivity

The two Compton currents, J_θ^c and J_ϕ^c are calculated at each r, τ mesh point by numerically integrating equations (24) and (25). The step size used is 0.1 times the Compton lifetime, R/V_0. The integration itself is done using the 4^{th} order Runge-Kutta method (Ref 5). It should be noted that both the mean free path for Compton interaction and the Compton lifetime are exponentially scaled from sea level values using a 7 km scale height. However, the Compton lifetime is not allowed to be greater than 100 shakes, since this is the maximum time of interest.

Monoenergetic gammas of energy 1.5 Mev are assumed. The most energetic Compton electrons resulting from 1.5 Mev gammas have a speed of 2.88 $(10)^8$ m/sec. Therefore $V_0 = 2.88\ (10)^8$ m/sec.

Since the integration on τ'' in equation (27) is also over the Compton lifetime, this integration is carried out simultaneously with the Compton current integrations. Again, the 4^{th} order Runge-Kutta method is used. It is broken into two parts, one for $-\infty < \tau' < 0$ and the other for $0 < \tau' < \tau$. In this case, $-\infty$ is defined to be the time when the first gamma ray reached the top of the absorption region, since no secondaries can be produced before that time.

The integration on τ' in equations (27) is also broken into two parts, one for $-\infty < \tau' < 0$ and the other for $0 < \tau' < \tau$. In the first case, integration is started at $\tau' = 0$ and proceeds to $\tau' = -(r\text{-RMIN})/V_0$ in steps of $\Delta\tau' = -\Delta r/V_0$. In the second case, integration is started at $\tau' = 0$ and proceeds to $\tau' = \tau$ in steps of $\Delta\tau' = \Delta\tau$. In both cases, simple step integration is used. That is

$$\int f(\tau')d\tau' = \sum_{\text{all } i} (\Delta\tau_i')[f(\tau_i')] \tag{64}$$

The integration over τ' is carried out parallel to the integration of (56) and (57) over r (using space as a pseudo retarded time) and simultaneously with the increase in τ as the space integrations are repeated for each new τ.

This rather involved approach to solving equation (27) is necessary to save running time. A direct approach, with separate integrations, would at least triple or quadruple the total running time required for execution of the code.

Integration of the Field Equations

For each τ, equations (56) and (57) are integrated from r = RMIN to r = RMAX in steps of Δr = (RMAX-RMIN)/NDELR using the 4^{th} order Runge-Kutta method. Then the magnitude of E is found from the two components and the result is stored in the E array. τ is increased by $\Delta\tau$ and the whole process is repeated until τ reaches TMAX.

On completion of the iterations, each member of the E array is multiplied by RMAX/r_{target} (equation 62). Then the E array is searched to find the peak value.

Outputs

There are several output options available in the code. The basic output it:

1. Gamma yield and altitude of burst.
2. Target coordinates from ground zero.
3. Distance from burst to target.
4. A message indicating whether the direct or the reflected wave is being calculated.

5. The time period covered by the calculation.

6. The time when the peak value occured.

7. The peak value of E at the target.

8. The τ and E arrays.

In addition, a linear and a log-log plot of $E(\tau)$ can be obtained. Also, a listing of the values of E at the bottom of the absorption region for each τ can be obtained. Either or both of these two options can be added to the basic output.

IV. Results and Input Parameter Variation

The output from a typical run is shown in Fig. 4. The $E(\tau)$ calculated during the run is shown in Fig. 5. The input data for this run was:

X	= 0 meters	(65a)
Y	= 0 meters	(65b)
Z	= 0 meters	(65c)
HOB	= 100 km	(65d)
Y_γ	= .001 kt	(65e)
B_0	= $2(10)^{-5}$ wb/m^2	(65f)
Dip Angle	= 20°	(65g)
NDELR	= 50	(65h)
TMAX	= 20 shakes	(65i)

The CDC 6600 Computer required 191 sec and 33000_8 words of central memory to execute this run.

The peak value of E, 6400 V/m, obtained in this run compares favorably with Karzas-Latter's order of magnitude estimate of 10^4 V/m (Ref 2) from similar input data.

In order to gain a better knowledge of the operating capabilities of the code, the effect of varying input parameters one at a time was studied. The basic set of parameters used was:

```
THE BURST WITH GAMMA YIELD CF 1.000E-03 KILOTCNS
IS AT AN ALTITUCE OF 1.000E+02 KILOMETERS.

THE TARGET IS AT CCORCINATES      0.              0.              0.
WHICH IS 1.000E+05 METERS FRCM THE BURST



DIRECT WAVE IS BEING CALCULATED




ITERATION TERMINATED AFTER 20.0 SHAKES



PEAK OCCURRED AT  2.1 SHAKES



* * * * * * * * * * * * * * * * * * * * * * * * * *
* PEAK EFIELD AT TARGET IS  6.448E+03 VOLTS/METER *
* * * * * * * * * * * * * * * * * * * * * * * * * *
```

Fig. 4. Output from a Typical Run

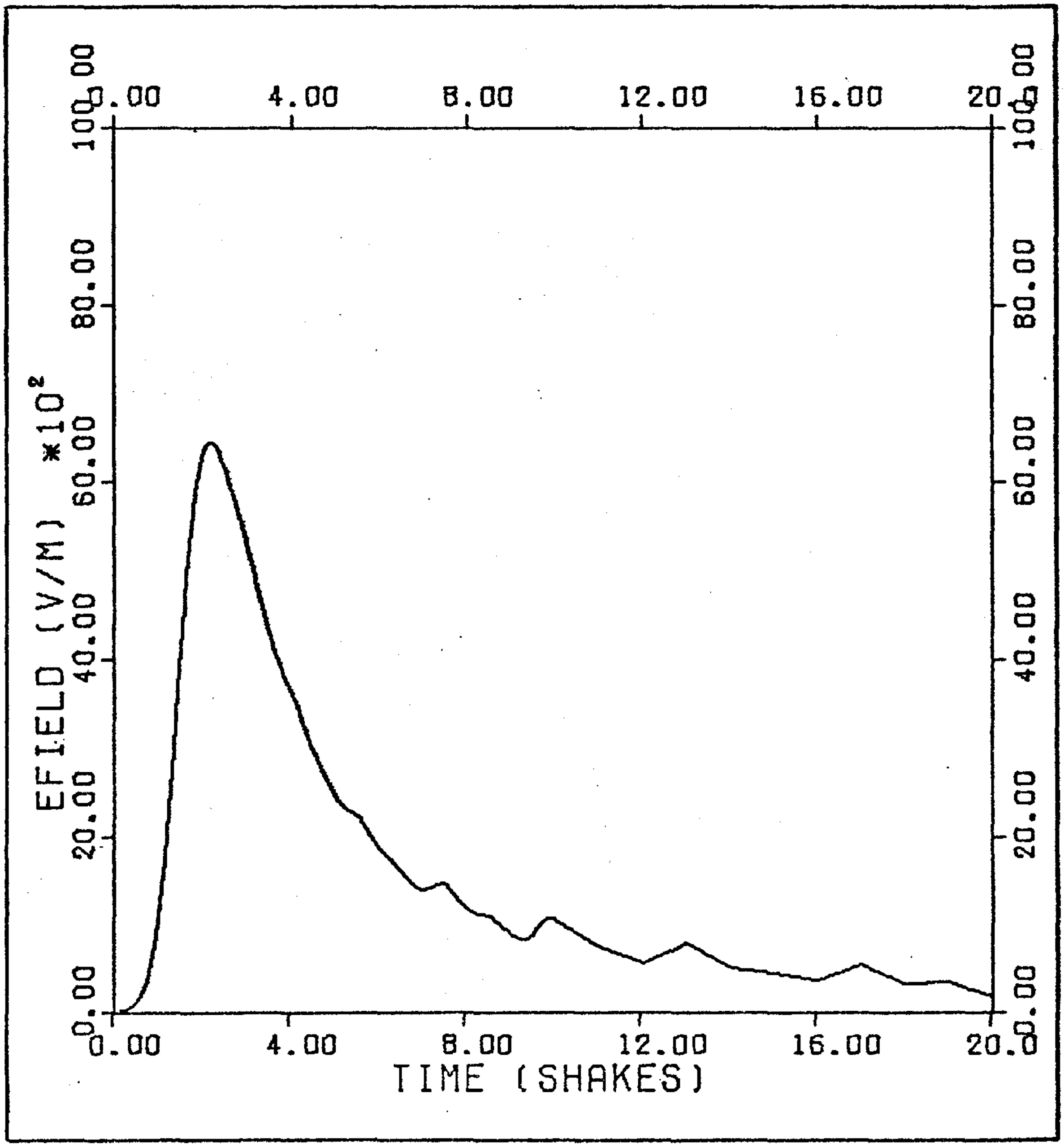


Fig. 5. Plot of E(τ) at target from a typical run

$$X = 0 \text{ meters} \tag{66a}$$

$$Y = 0 \text{ meters} \tag{66b}$$

$$Z = 0 \text{ meters} \tag{66c}$$

$$HOB = 100 \text{ km} \tag{66d}$$

$$Y_\gamma = .001 \text{ kt} \tag{66e}$$

Each of the above parameters was systematically varied while holding the others constant. The other inputs were held constant at the values shown in equations (65).

The results of the variation in X are shown in Fig. 6. Since the X axis is perpendicular to the magnetic field the symmetry about X = 0 is expected. The decrease in peak value of E for increasing distance from ground zero is due to the increasing distance from the burst.

The results of the variation in Y are shown in Fig. 7. Here the peak values of E depend on the angle between $\vec{r}$ and $\vec{B}_0$, θ. When $\theta = 180°$ ($A = -70°$ and $Y = -275$ km) the peak E drops to zero. The maximum peak E is skewed toward $A = 20°$ ($\theta = 90°$ and $Y = 36$ km). The maximum is not exactly at $A = 20°$ because of the increased distance from the burst. These characteristics are expected since an electron moving perpendicular to the magnetic field would feel the strongest acceleration from it while an electron moving parallel to the magnetic field would feel no acceleration at all.

The results of variation in Z are shown in Fig. 8. In this case, both the direct and the reflected waves were calculated at each point below the top of the absorption

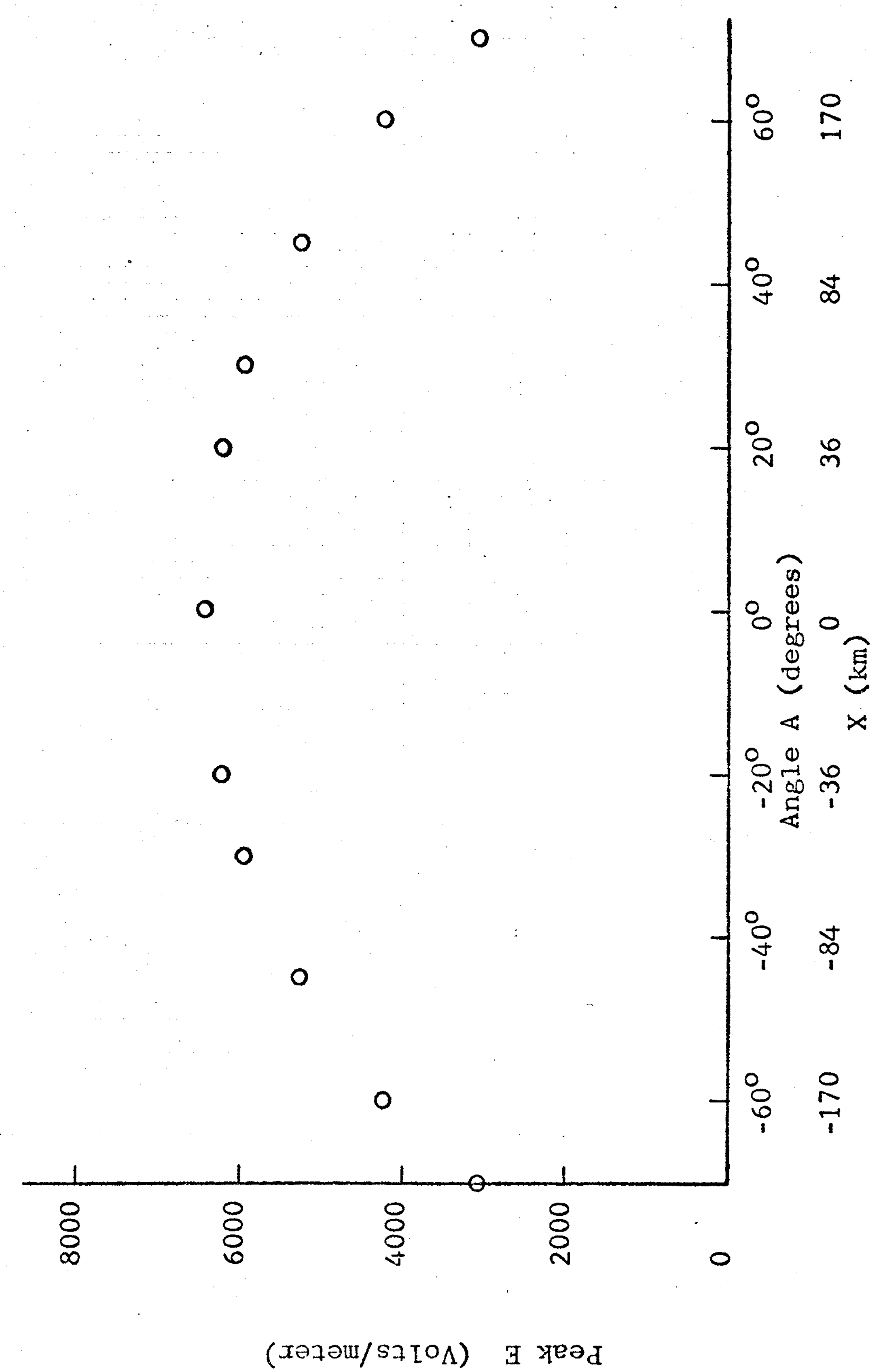


Fig. 6. Variation in the X Direction (Y=0, Z=0, HOB=100km, Y_{γ}=0.001kt)

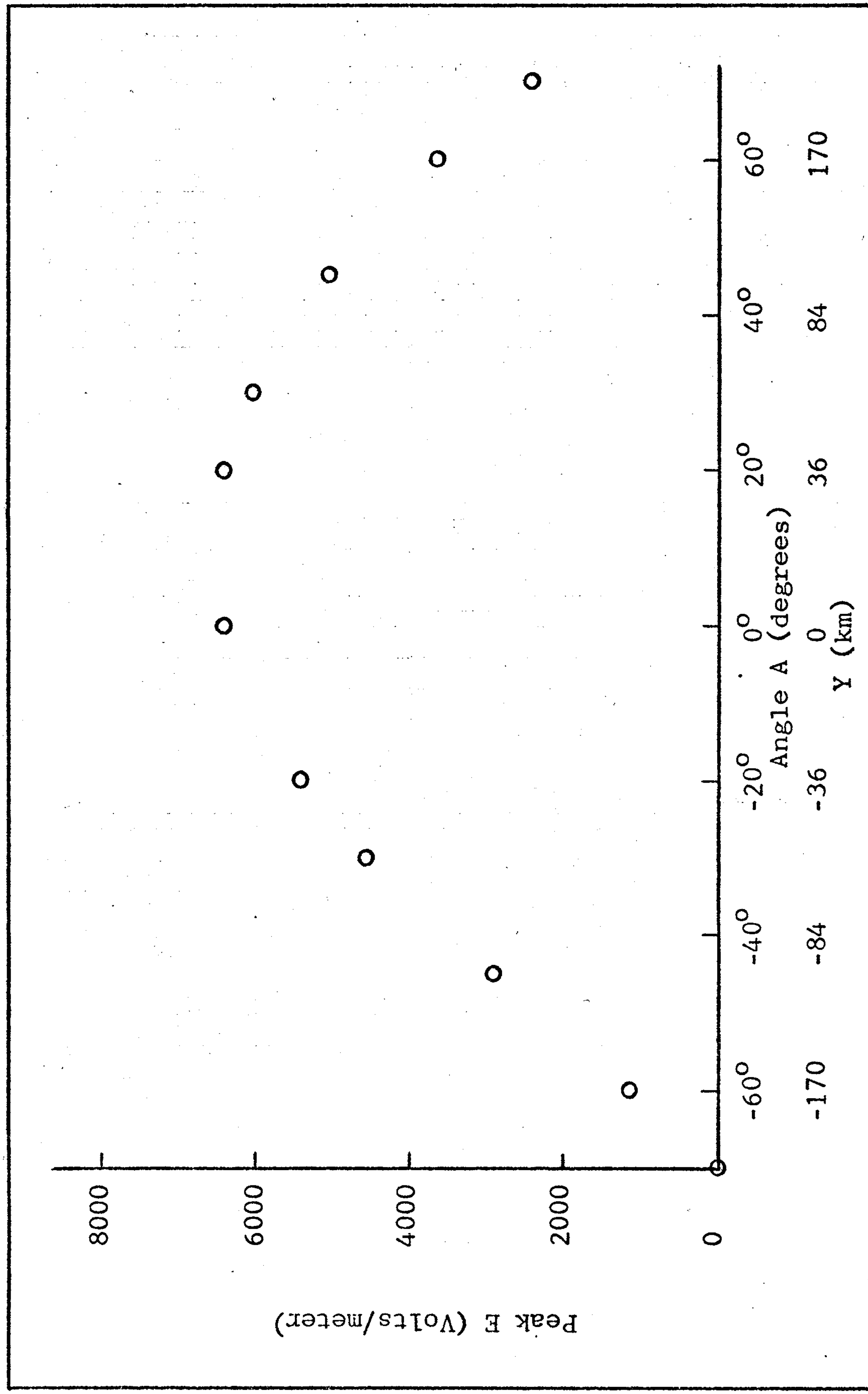


Fig. 7. Variation in Y direction (X=0, Z=0, HOB=100km, Y_γ=.001kt)

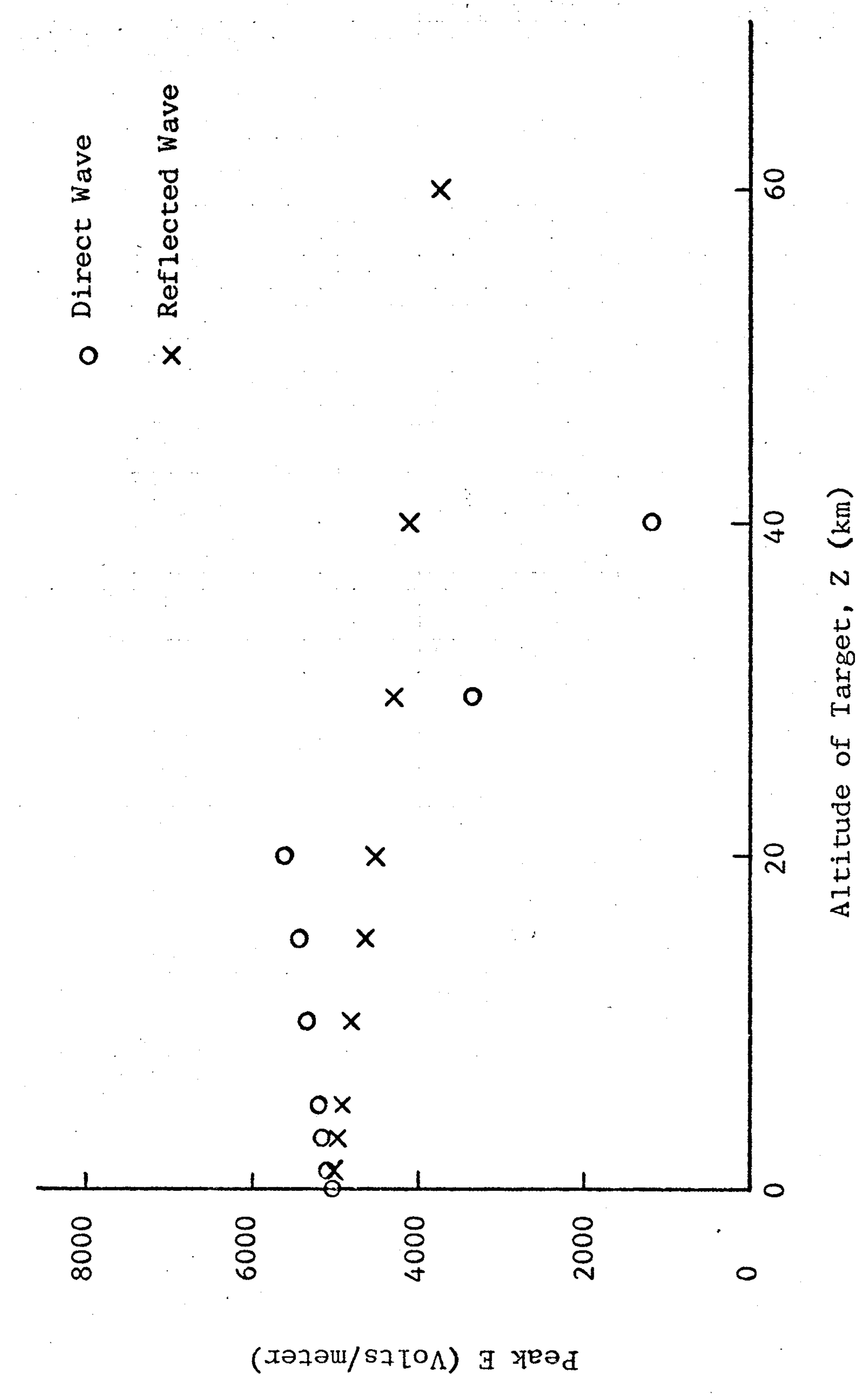


Fig. 8. Variation in Z Direction (X=0, Y=+100km, Y_γ=.001kt, HOB=100km)

region. Note that Y = 100 km for these runs. As expected, the direct wave falls off rapidly as the target altitude passes through the absorption region, since less of the absorption region contributes to the wave with each increase in altitude. The crossover point where the reflected wave becomes the largest occured at 25 km in this case. Above ground zero the crossover point was 29.4 km. The altitude of the crossover point is both yield and geometry dependent. It is necessary for the user to calculate both waves whenever there is any doubt which one is the largest.

The reflected wave calculation assumes 100% reflection from the ground and no attenuation in the absorption region or the ionosphere. These assumptions are reasonable if it is recalled that only the high frequency component is being considered and that it requires at least

$$\frac{40 \text{ km}}{3(10)^8 \text{m/sec}} = 133 \ \mu \text{ sec} \tag{67}$$

for the wave to leave the absorption region, reach the earth, be reflected, and return to the absorption region. This length of time is enough for a significant number of the free electrons to recombine and reduce the effective conductivity of the absorption region.

The results of variation in HOB are shown in Fig. 9. For all values of HOB attempted below 60 km the code went unstable. Infinite values for E were obtained which resulted in abnormal termination of the calculations. This is

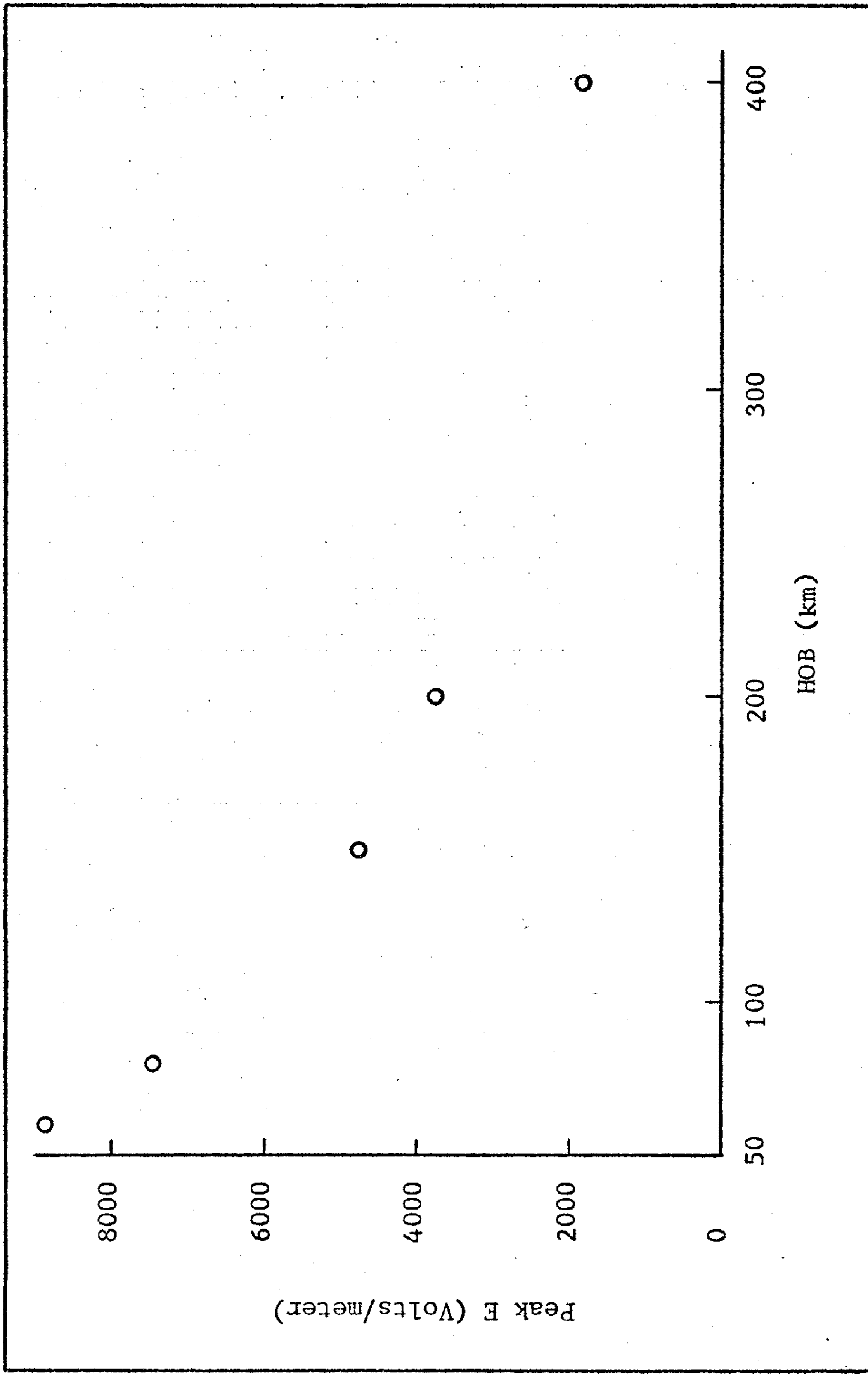


Fig. 9. Variation in Height of Burst (X=0, Y=0, Z=0, Y_γ=0.001kt)

expected since the burst is assumed to be distant from the absorption region (equations 9 and 10).

The results of variation in gamma yield are shown in Fig. 10. For all gamma yields attempted above 60 tons the code went unstable, giving infinite values for E. However, the instability always occured at times later than the natural peak value of E. For example, with 80 tons of gamma yield, the natural peak occured at 1 shake and the instability occured at 10 shakes. By using the natural peak value and ignoring the instability, reasonable values for peak E were obtained up to 1 kt of gamma yield.

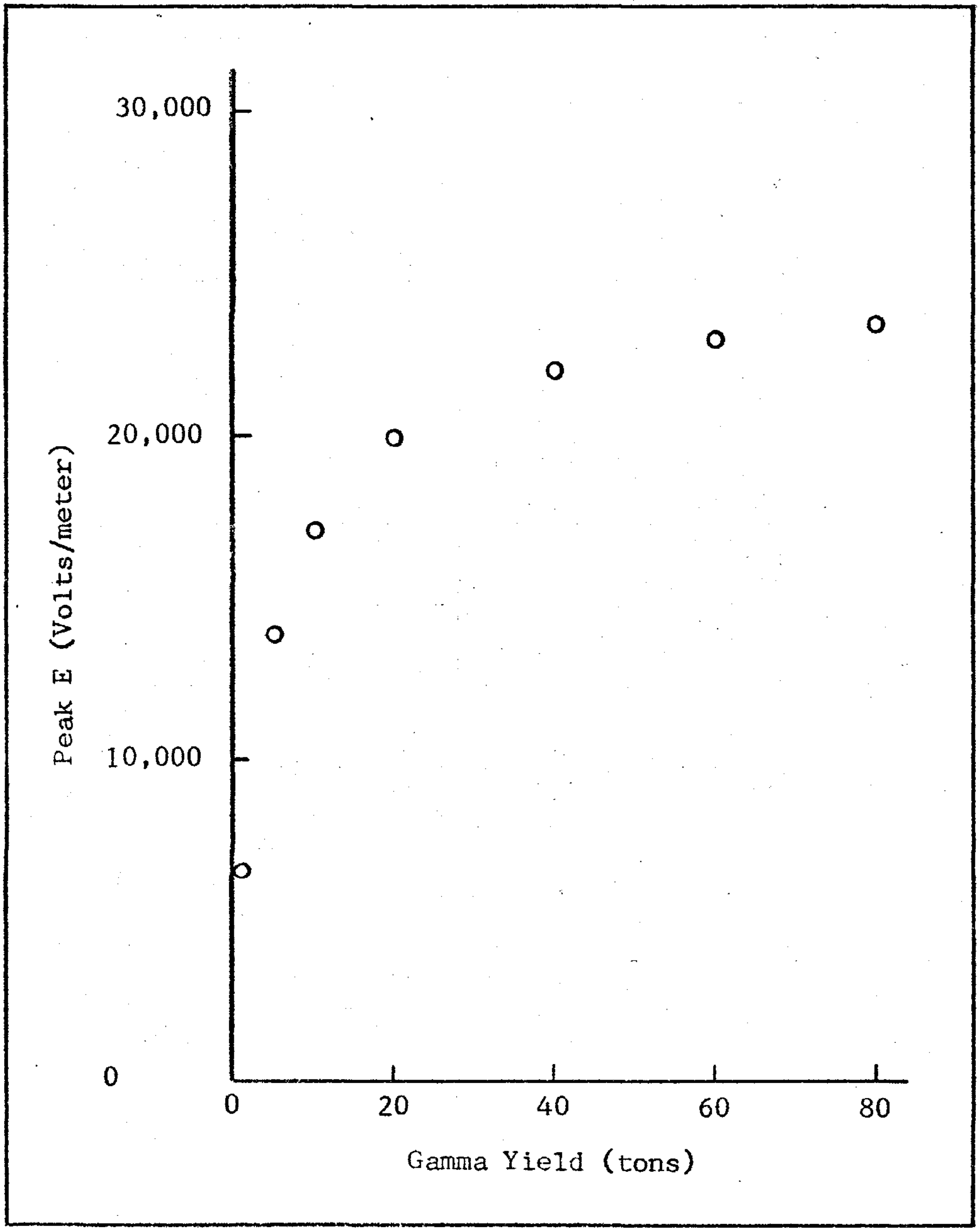


Fig. 10. Variation in Gamma Yield (X=0, Y=0, Z=0, HOB=100km)

V. Discussion and Recommendations

Limitations

Most of the limitations of the code are inherent in the model upon which it is based. Approximations such as a flat earth, a uniform magnetic field, and constant speed Compton electrons can be improved only by changing the model. In addition, the effect of the self generated electromagnetic fields on the motion of the Compton electrons is ignored, as is recombination of both primary and secondary electrons. The possibility of a single gamma ray interacting to produce more than one Compton electron is not allowed. In the absorption region the contribution of the non-propagating radial component of the electric field is neglected. Also, the model is not easily adapted to multi-group gamma transport, or to multiple burst calculations.

The code calculates only the effect of the gamma rays. The user must keep in mind that X-ray generated EMP becomes important for bursts above 100 km.

The code does not account for the increase in altitude of the absorption region for slant angles (angle A in Fig. 1) greater than 60° which is indicated by Latter and LeLevier (Ref 4).

Since 97% of the running time of the code is used for numerical iteration it is not practical to adapt the code to run more than one target at a time. Two targets would merely double the running time, so it is simpler to just make two runs. Typical requirements are 200 seconds running time with 33000_8 words of central memory on the CDC 6600 computer using NDELR = 50 and TMAX = 20 shakes.

Uses

The code can be used to calculate the peak value of the E field at a target, anywhere on or above ground level, resulting from a nuclear burst above 60 km altitude with a gamma yield up to 60 tons. Either the direct or the ground reflected wave can be calculated. With special care, bursts up to 1 kt of gamma yield can be used.

Recommendations

In the interest of accuracy, the targets should be located such that the slant angle, A, is between -60° and +60°.

By accepting a much longer running time the accuracy and hopefully, the stability of the code could be improved by using a smaller step size in the integration of the Compton current equations. Reducing the step size from one tenth of the Compton lifetime to one shake would require approximately ten times as much running time as the code presently requires. This possibility should be investigated further to determine the optimum step size for obtaining the best relationship between accuracy and running time.

Another possibility for increasing the accuracy and stability of the code is to reduce the step size in r. The present code has the capability of dividing the absorption region into 500 steps in r along the line of sight. Of course, the running time required for 500 steps is ten times that required for 50 steps. A modification of the code to allow more than 500 steps would increase the amount of computer core required as well as increasing the running time. This provides another area for investigation to determine the best trade off point between accuracy and running cost.

These two possibilities could be investigated with minor modifications to the present code. However, the computer time required would be considerable.

In addition, there are numerous possibilities for improvements in the model itself. Some of the more important ones are;

Using multigroup gamma transport.

Using multigroup Compton electrons.

Allowing angular distribution of Compton electrons.

Using self consistent electromagnetic fields.

Including the low frequency components.

Each of these would require major modifications to the present code.

Bibliography

1. Kinsley, O.V. Introduction to the Electromagnetic Pulse, Wright-Patterson AFB: Air Force Institute of Technology, March 1971. (GNE/PH/71-4).

2. Karzas, W. J. and R. Latter. "Detection of the Electromagnetic Radiation from Nuclear Explosions in Space", Physical Review, Vol 137, No. 5B. pages 1369-1378, March 8, 1965. (Also published as EMP Theoretical Note 40).

3. Pomranning, G. C. "Early Time Air Fireball Model for a Near-Surface Burst", DNA 3029T, March 1973.

4. Latter, R. and R. E. LeLevier. "Detection of Ionization Effects from Nuclear Explosions in Space", Journal of Geophysical Research, Vol. 68, No. 6, March 15, 1963.

5. Wylie, C. R. Jr. Advanced Engineering Mathematics, New York: McGraw-Hill Book Co. 1966. (Third Edition).

6. Lecture Notes, Electromagnetic Waves, EE 6.30, Air Force Institute of Technology, Wright-Patterson AFB, Summer, 1973. (Course taught by Maj. Carl T. Case.)

Appendix A

EMP Code User's Guide

EMP Code User's Guide

The code is run the same as any other Fortran Extended program, but due to the running time it should be converted to binary form before execution. The plotting subroutine requires an on-line plotter and both linear and log plotting library subroutines.

The input data is read in the following order:

Data card #1, using FORMAT (7F10.0, 2I5), contains;

X,Y,Z	The target coordinates in meters
HOB	The height of the burst in kilometers ($60 \text{ km} \leq \text{HOB}$)
GAMYLD	The gamma yield in kilotons ($\text{GAMYLD} \leq 1 \text{ kt}$)
BFIELD	The Earth's magnetic field in wb/m^2
BANGLE	The magnetic field dip angle in degrees
NDELR	The number of steps in r taken through the absorption region ($50 \leq \text{NDELR} \leq 500$)
OUT	The output control parameter

Data card #2, using FORMAT (I3), contains;

ITER	The time period covered by the iterations in shakes ($10 \leq \text{ITER} \leq 100$) (ITER = TMAX)

Data card #3, using FORMAT (4F10.0), contains;

A	Pomranning constant α in inverse shakes
B	Pomranning constant β in inverse shakes
RN	Pomranning constant N in shakes
TO	Pomranning constant τ_0 in shakes

Default values are provided for BANGLE, BFIELD, and NDELR. They are 40°, 0.00002 wb/m^2, and 50 respectively. If these default values are desired, zero must be punched in their respective card fields.

The ground reflected wave at the target is obtained by reading in the target altitude, Z, as a negative number. For any target within the absorption region, both the direct and the ground reflected wave should be calculated to determine which one is the strongest.

For values of GAMYLD between 0.06 kilotons and 1.0 kilotons the code will most likely go unstable. This instability occurs after the real peak has been calculated, but the peak value printed out may not be the real peak. Since execution is terminated when the field becomes greater than 1E15 V/m, the array search can result in a false peak value. In this case, the arry itself (or the plot) can be used to determine the real peak value.

Increasing NDELR makes the step size in r through the absorption region smaller and the calculation becomes more accurate. However, total running time varies directly with changes in NDELR. For example, using NDELR = 100 instead of NDELR = 50 will approximately double the running time required for NDELR = 50.

There are four output options provided. Option 0 prints out the informative messages, the calculated peak value at the target, the E array, and the τ array. Option 1 adds a linear plot of the first 20 shakes and a log-log plot of 100 shakes of E as a function of τ at the target. Option 2 includes both Option 0 and Option 1 and adds a printout of E and σ as a function of τ at the bottom of the absorption region. Option 3 deletes the plots from Option 2. The last two options are primarily for debugging since a partial printout is made for each completed iteration even if execution is terminated before the iterations are completed. The first two options are best for production runs.

The only requirements on the Pomranning constants are N must be chosen such that equations (61) and (62) are satisfied, all of them must be positive, and $\alpha > \beta$.

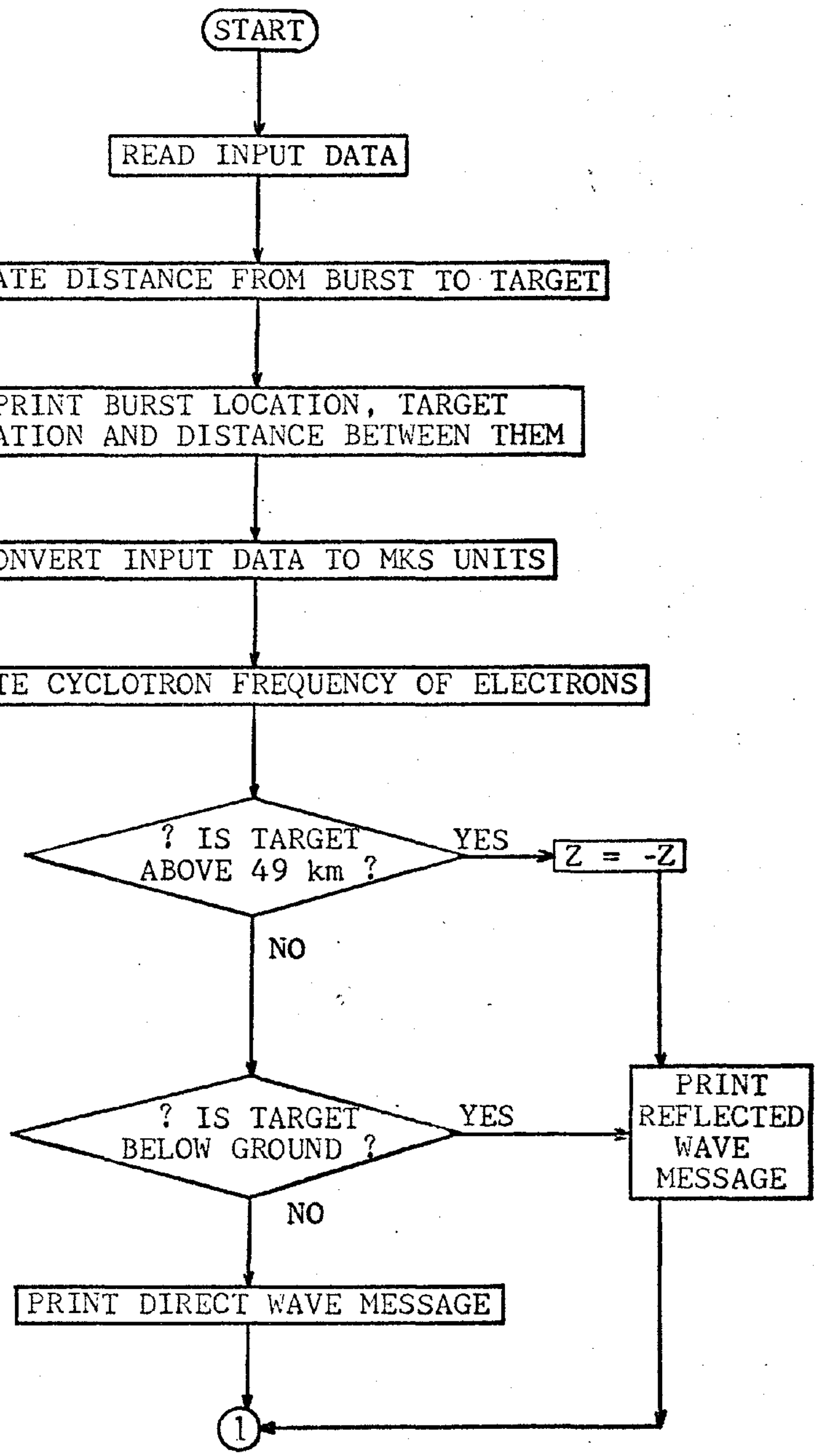
Increasing ITER also increases the running time. For ITER = 10 shakes, running time is approximately 180 seconds on the CDC 6600 computer. For ITER = 100 shakes, running time is approximately 340 seconds. A good compromise, which gives nice looking plots, is ITER = 20 shakes with a running time of approximately 200 seconds.

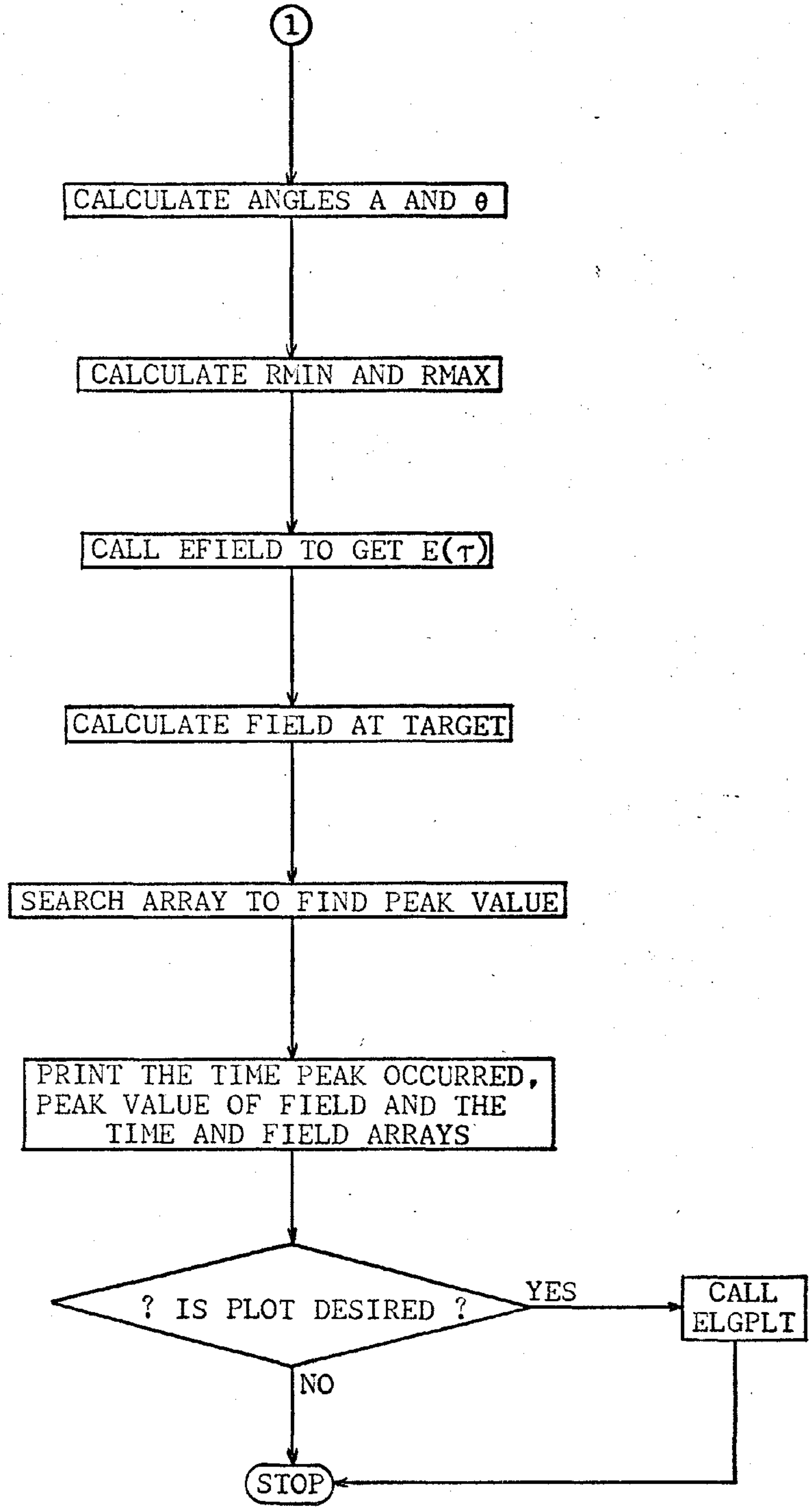
In binary form, the code requires 33000_8 words of core on the CDC 6600 computer.

Appendix B

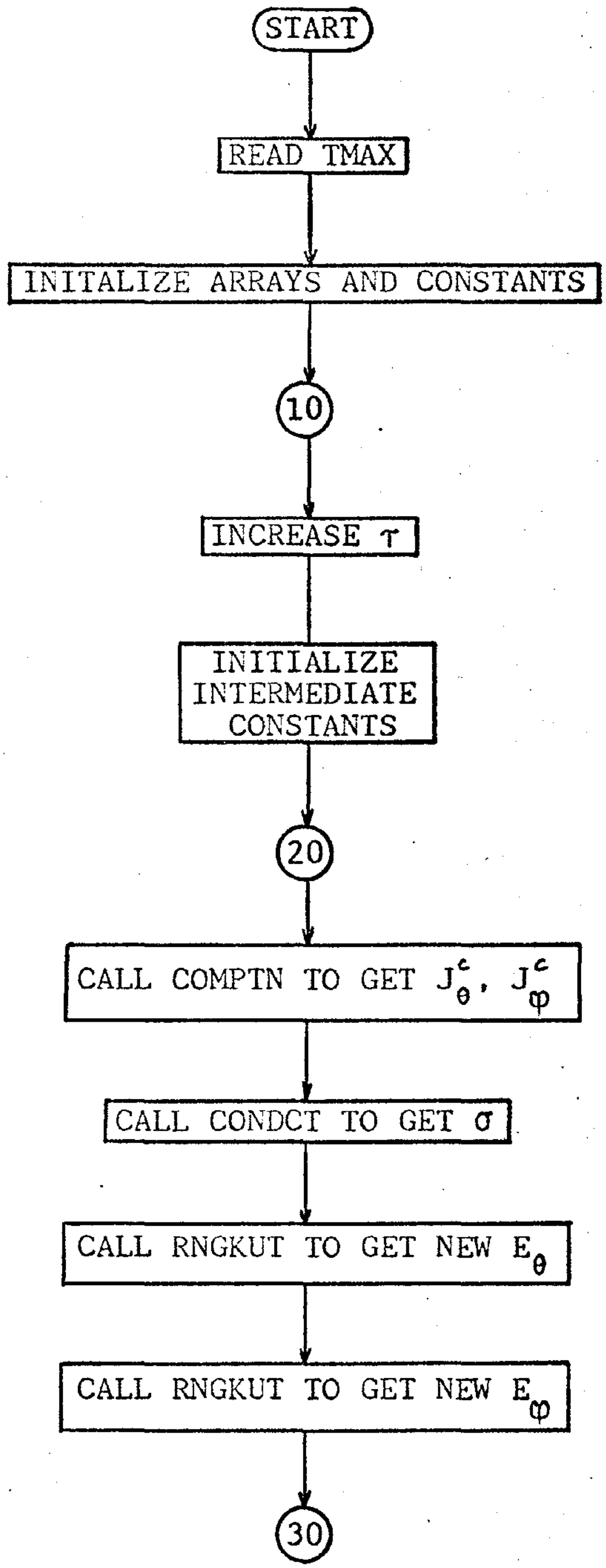
EMP Code Flow Charts

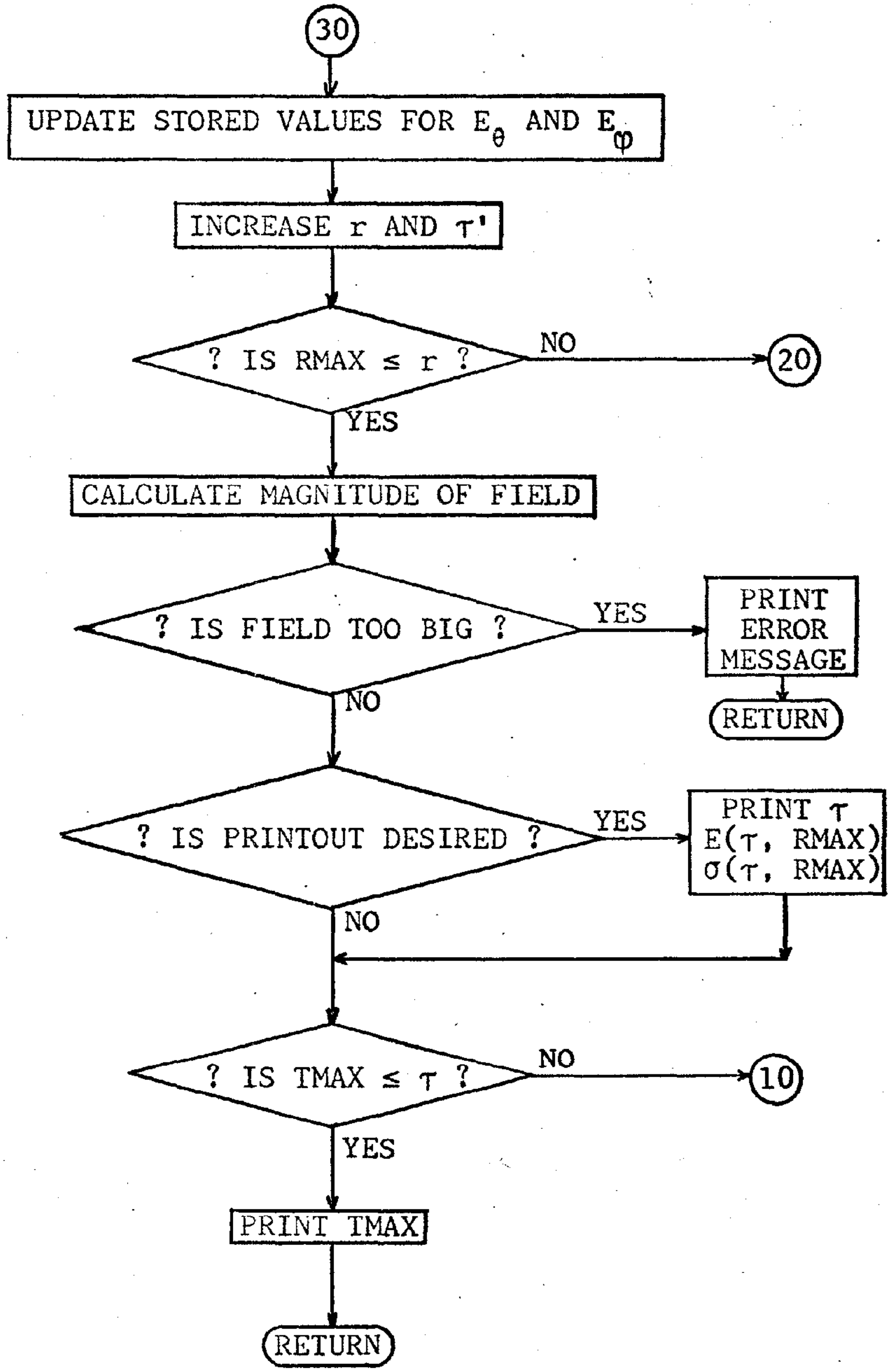
PROGRAM CONTRL



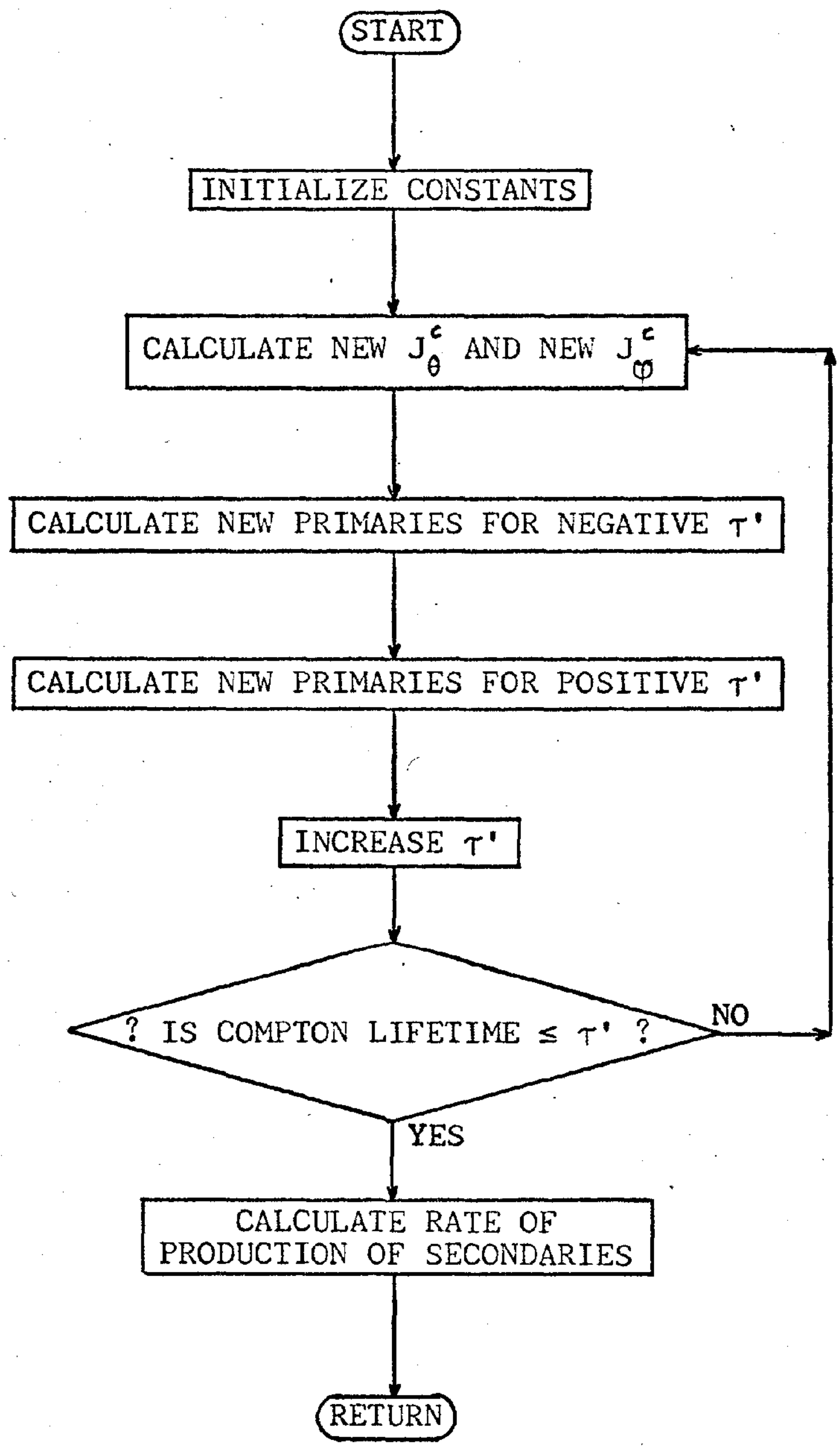

1
CALCULATE ANGLES A AND θ
CALCULATE RMIN AND RMAX
CALL EFIELD TO GET E(τ)
CALCULATE FIELD AT TARGET
SEARCH ARRAY TO FIND PEAK VALUE
PRINT THE TIME PEAK OCCURRED, PEAK VALUE OF FIELD AND THE TIME AND FIELD ARRAYS
? IS PLOT DESIRED ?
YES
CALL ELGPLT
NO
STOP

SUBROUTINE EFIELD

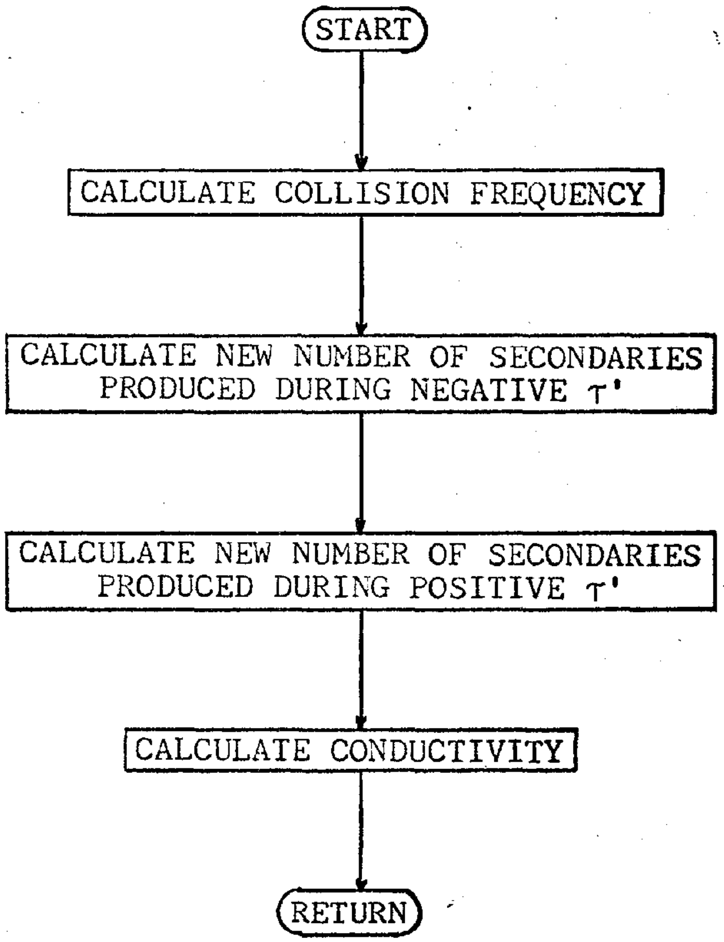



30
UPDATE STORED VALUES FOR E_θ AND E_φ
INCREASE r AND τ'
? IS RMAX ≤ r ?
NO
20
YES
CALCULATE MAGNITUDE OF FIELD
? IS FIELD TOO BIG ?
YES
PRINT ERROR MESSAGE
RETURN
NO
? IS PRINTOUT DESIRED ?
YES
PRINT τ $E(\tau, RMAX)$ $\sigma(\tau, RMAX)$
NO
? IS TMAX ≤ τ ?
NO
10
YES
PRINT TMAX
RETURN

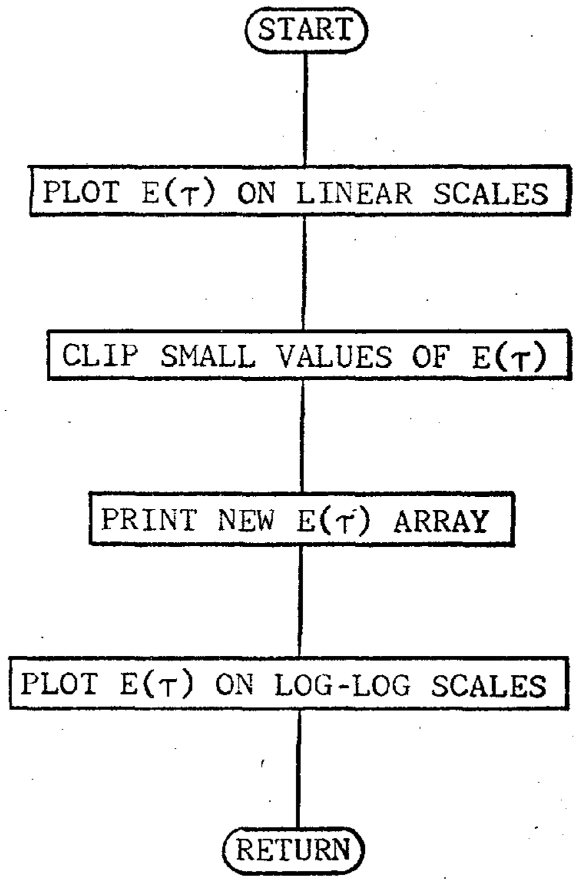
SUBROUTINE COMPTN



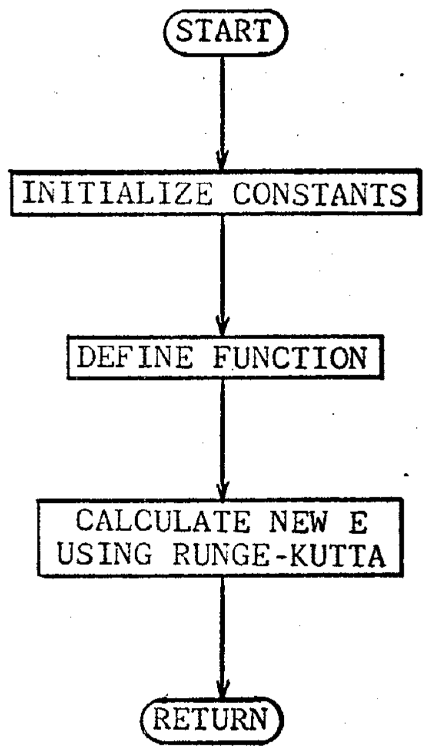
SUBROUTINE CONDCT



SUBROUTINE ELGPLT



SUBROUTINE RNGKUT



Appendix C

EMP Code Listing

```
      PROGRAM CONTRL (INPUT,OUTPUT,PLOT)
C
C        THIS PROGRAM CONTROLS THE SUBROUTINES
C
      COMMON OUT
      INTEGER OUT
      DIMENSION E(192),TIME(192),STORE2(500)
C
C        X,Y,Z IS THE TARGET LOCATION IN METERS
C             FOR THE NORTHERN HEMISPHERE
C             X IS MAGNETIC WEST
C             Y IS MAGNETIC SOUTH
C             Z IS ALTITUDE
C
C        HOB IS HEIGHT OF BURST IN KILOMETERS >  50KM
C
C        GAMYLD IS GAMMA YIELD OF BURST IN KILOTONS
C
C        BFIELD IS THE MAGNITUDE OF EARTHS MAGNETIC FIELD IN THE
C        ABSORPTION REGION BELOW THE BURST IN WEBERS/SQUARE METER
C
C        BANGLE IS THE DIP ANGLE OF THE MAGNETIC FIELD IN DEGREES
C
C        NDELR IS THE NUMBER OF STEPS BETWEEN RMIN AND RMAX
C        50<=NDELR<=500
C
C        OUT IS THE OUTPUT CONTROL PARAMETER
C        OUT=0 ==> PRINT PEAK VALUE AND ARRAYS
C        OUT=1 ==> PRINT PEAK VALUE AND MAKE PLOT
C        OUT=2 ==> PRINT EVERYTHING AND MAKE PLOT
C        OUT=3 ==> PRINT EVERYTHING
C
      READ 1001,X,Y,Z,HOB,GAMYLD,BFIELD,BANGLE,NDELR,OUT
      R=SQRT(X*X+Y*Y+(HOB*1000.-Z)**2)
      PRINT 2006,GAMYLD,HOB,X,Y,Z,R
C
```

```
C         SET UP DEFAULT VALUES
C
      IF(BANGLE.EQ.0.) BANGLE=40.
      IF(BFIELD.EQ.0.) BFIELD=0.00002
      IF(NDELR.EQ.0) NDELR=50
C
C         CONVERT DATA TC MKS UNITS
C
      HOB=HOB*1000.
      GAMYLD=2.61625E25*GAMYLD
      BANGLE=0.017453295*BANGLE
      OMEGA=1.6E-19*BFIELD/(3.505*9.11E-31)
C
C         PRINT TYPE OF CALCULATICN
C
      REFLCT=49000.
      IF(Z.GT.REFLCT)PRINT 2007
      IF(Z.LT.0.0) PRINT 2008
      IF(Z.LE.REFLCT.AND.Z.GE.0.0) PRINT 2009
C
C         REFLECTED WAVE CALCULATICN ASSUMES 100% REFLECTION
C         AND USES MIRROR IMAGE CF TARGET BELOW GROUND
C         SET Z = -Z IF REFLECTED WAVE IS TO BE USED
C
      IF(Z.GT.REFLCT) Z=-Z
      IF(Z.GT.HOB-1000.) PRINT 2007
      IF(Z.GT.HOB-1000.) Z=-Z
C
C         DETERMINE ANGLES
C
      R=SQRT(X*X+Y*Y+(HOB-Z)**2)
      A=ACOS((HOB-Z)/R)
      THETA=ACOS(COS(BANGLE)*Y/R+SIN(BANGLE)*(Z-HOB)/R)
C
C         DETERMINE RMIN AND RMAX
C
```

```
      ZRMIN=5.E4
      IF(HOB.LT.ZRMIN) ZRMIN=HOB
      TA=(ZRMIN-HOB)/(Z-HOB)
      XRMIN=TA*X
      YRMIN=TA*Y
      RMIN=SQRT(XRMIN**2+YRMIN**2+(ZRMIN-HOB)**2)
      ZRMAX=Z
      IF(Z.LT.2.E4) ZRMAX=2.E4
      TB=(ZRMAX-HOB)/(Z-HOB)
      XRMAX=TB*X
      YRMAX=TB*Y
      RMAX=SQRT(XRMAX**2+YRMAX**2+(ZRMAX-HOB)**2)
C
C         CALCULATE EFIELD AT BOTTOM OF ABSORPTION REGION
C
      CALL EFIELD(E,TIME,RMIN,RMAX,NDELR,HOB,A,THETA,OMEGA,GAMYLD,
     1STORE2)
C
C         CALCULATE EFIELD AT TARGET
C
      IF(R.LE.RMAX) GO TO 3
      DO 1 I=1,190
 1    E(I)=E(I)*RMAX/R
C
C         FIND PEAK VALUE OF EFIELD
C
 3    BIG=0.0
      DO 2 I=1,190
      IF(E(I).GT.BIG) 4,2
 4    BIG=E(I) $ IT=I
 2    CONTINUE
C
C         PRINT OUTPUT
C
      PRINT 2010,TIME(IT)
      PRINT 2005,BIG
```

```
      PRINT 2001                                                         CNTL1090
      PRINT 2002,(TIME(I),I=1,190)                                       CNTL1100
      PRINT 2003                                                         CNTL1110
      PRINT 2004,(E(I),I=1,190)                                          CNTL1120
C                                                                        CNTL1130
C         IF DESIRED, MAKE PLCT                                          CNTL1140
C                                                                        CNTL1150
      IF(OUT.LE.0.CR.OUT.GE.3) STCF                                      CNTL1160
      CALL ELGPLT (E,TIME,BIG)                                           CNTL1170
      STOP                                                               CNTL1180
 1001 FCRMAT(7F10.0,2I5)                                                 CNTL1190
 2010 FCRMAT(//5X,"PEAK OCCURREC AT",F5.1," SHAKES"//)                   CNTL1200
 2009 FCRMAT(5X,"DIRECT WAVE IS EEING CALCULATED"/////)                  CNTL1210
 2008 FCRMAT(5X,"REFLECTED WAVE IS BEING CALCULATED"/////)               CNTL1220
 2007 FCRMAT(5X,"TARGET IS ABCVE AESCRPTION REGICN SO REFLECTED WAVE IS CNTL1230
     1BEING CALCULATED"/////)                                            CNTL1240
 2006 FCRMAT("1    THE BURST WITH GAMMA YIELC CF",1PE10.3," KILOTCNS"    CNTL1250
     1    /5X,"IS AT AN ALTITUCE CF",1PE10.3," KILOMETERS."              CNTL1260
     2    //5X,"THE TARGET IS AT CCORDINATES",3(5X,1PE10.3)              CNTL1270
     3     /5X,"WHICH IS",1PE10.3," METERS FRCM THE BURST"/////)         CNTL1280
 2005 FCRMAT(//5X,"* * * * * * * * * * * * * * * * * * * * * * * * * * *" CNTL1290
     1         /5X,"* PEAK EFIELC AT TARGET IS ",1PE10.3," VOLTS/METER *"CNTL1300
     2         /5X,"* * * * * * * * * * * * * * * * * * * * * * * * * *")CNTL1310
 2004 FCRMAT(19(10(3X,1PE10.3)/))                                        CNTL1320
 2003 FCRMAT(///5X,"EFIELC VALUES AT TARGET (IN V/M) ARE"//)             CNTL1330
 2002 FCRMAT(19(10(4X,F5.1,4X)/))                                        CNTL1340
 2001 FCRMAT("1  TIMES USED (IN SHAKES) ARE"//)                          CNTL1350
      END                                                                CNTL1360
```

```
      SUBROUTINE EFIELD(E,TIME,RMIN,RMAX,NDELR,HOB,A,THETA,OMEGA,GAMYLD,
     1STORE2)
C
C
C         CALCULATE THE EFIELD IN THE ABSORPTION REGION
C
      DIMENSION E(190),TIME(190),STORE2(NDELR)
      REAL JTHETA,JPHI
      INTEGER OUT
      COMMON OUT,AP,BP,RNP,TOP
C
C         ITER IS TIME OF ITERATION IN SHAKES 10<=ITER<=100
C         READ ITER AND CHANGE IT TO NUMBER OF TIME STEPS
C
      READ 100,ITER
      ITER=100+(ITER-10)
C
C         INITIALIZE ARRAYS AND CONSTANTS
C
      READ 101,AP,BP,RNP,TOP
  101 FORMAT(4F10.0)
      DO 61 J=1,100
      E(J)=0.0 $ TIME(J)=0.1*J
   61 CONTINUE
      DO 71 J=101,190
      E(J)=0.0 $ TIME(J)=10.+(J-100.)
   71 CONTINUE
      DO 51 L=1,NDELR $ STORE2(L)=0.
   51 CONTINUE
      ETHE=0. $ DELRN=NDELR $ T=0. $ DT=1. $ EPHI=0.
      DELTAR=(RMAX-RMIN)/DELRN $ R=RMIN+DELTAR $ RNP=1.E-3*RNP
C
C         START INTEGRATIONS
C         OUTSIDE LOOP IS FOR CALCULATION IN RETARDED TIME
C         INSIDE LOOP IS FOR INTEGRATION IN R AT EACH TIME STEP
C
```

```
      DO 21 I=1,190 $ T=T+(1.E-9)*DT $ TIME(I)=T*(1.E8)
      IT=I
      IF(I.GT.ITER) GO TO 42
      TP=-DELTAR/2.88E8$ DTP=TP $ SIGMA=0.$ STORE1=0.
      DO 31 K=1,NDELR
      CALL COMPTN(JTHETA,JPHI,T,R,A,THETA,OMEGA,HOB,GAMYLD,TP,PRI,PRI2)
      CALL CONDCT(SIGMA,PRI,DTP,DT,HOB,R,A,STORE1,STORE2,K,NDELR,PRI2)
      CALL RNGKUT(ETHENW,ETHE,R,DELTAR,SIGMA,JTHETA)
      CALL RNGKUT(EPHINW,EPHI,R,DELTAR,SIGMA,JPHI)
      ETHE=ETHENW $ EPHI=EPHINW $ R=R+DELTAR
      TP=TP+DTP
   31 CONTINUE
C
C         FIND MAGNITUDE OF EFIELD
C
      E(I)=SQRT(ETHE**2+EPHI**2)
C
C         CHECK FOR DIVERGENCE OF SOLUTION
C
      IF(E(I).GT.1.E15) GO TO 52
      IF (I.EQ.100) DT=10.
      R=RMIN+DELTAR
C
C         IF DESIRED, PRINT OUTPUT
C
      IF (OUT-1.LE.0) GO TO 21
      PRINT 5,I,TIME(I),E(I),SIGMA
   21 CONTINUE
C
C         PRINT MESSAGE AFTER TERMINATION OF TIME LOOP
C
   42 PRINT 201,TIME(ITER)
      RETURN
C
C         PRINT MESSAGE AFTER ABNORMAL TERMINATION OF TIME LOOP
C
```

```
52    PRINT 301
      PRINT 201,TIME(IT)
      IF(IT.LT.10)RETURN
C
C         SET LAST 5 VALUES OF EFIELD TO 0.0 TO AVOID INCORRECT PEAK
C
      E(IT)=E(IT-1)=E(IT-2)=E(IT-3)=E(IT-4)=E(IT-5)=0.0
      RETURN
100   FORMAT (I3)
5     FORMAT("   I =",I4,"      TIME =",F6.1," SHAKES     E(T,RMAX) =",
     11PE10.3," VOLTS/METER      SIGMA =",1PE10.3," MHO/METER")
201   FORMAT(//5X,"ITERATION TERMINATED AFTER",F5.1," SHAKES"//)
301   FORMAT(//15X,"******"/15X,"***** SOLUTION HAS GONE UNSTABLE"
     1/15X,"******"//)
      END


      SUBROUTINE CONDCT(SIGMA,PHI,CTP,DT,HOB,R,A,STORE1,STORE2,K,NDELR,
     1PRI2)
C
C         CALCULATES SIGMA AFTER FINDING
C         NSECONDARY FROM NPRIMARY
C
C         STORE1 CONTAINS INTEGRAL FOR NEGATIVE TAU
C         STORE2 CONTAINS INTEGRAL FOR POSITIVE TAU
C
      DIMENSION STORE2(NDELR)
      COLISN=4.E12*EXP((R*COS(A)-HOB)/7000.)
      STORE1=STORE1-PRI*CTP
      STORE2(K)=STORE2(K)+PRI2*DT*(1.0E-9)
      SEC=STORE2(K)-STORE1
      SIGMA=(1.6E-19**2)*SEC/(COLISN*9.11E-31)
      RETURN $ END
```

```
      SUBROUTINE COMPTN(JTHETA,JPHI,T,R,A,THETA,OMEGA,HOB,GAMYLD,TF,PRI,
     1PRI2)
C
C         CALCULATE THE TWO COMPONENTS OF THE
C         COMPTON CURRENT AT GIVEN T AND R
C         CALCULATE NUMBER OF PRIMARY ELECTRONS
C
C         JTHETA IS THETA COMPONENT OF COMPTON CURRENT
C         JPHI IS PHI COMPONENT OF COMPTON CURRENT
C         TMAX IS COMPTON LIFETIME
C         PATH IS ALTITUDE SCALED COMPTON MEAN FREE PATH
C         TP IS NEGATIVE RETARDED TIME
C         T IS POSITIVE RETARDED TIME
C         PRI IS NUMBER OF PRIMARY ELECTRONS GENERATED DURING TP
C         PRI2 IS NUMBER OF PRIMARY ELECTRONS GENERATED DURING T
C         TPRIME IS VARIABLE OF INTEGRATION
C
C         INITIALIZE CONSTANTS
C
      REAL JTHETA,JPHI
      JTHETA=0. $ JPHI=0. $ TPRIME=(5.E-9) $ TMAX=CLIFE(R,A,HOB)$PRI=0.
      DT=TMAX/10.$ PATH=309.*EXP((HOB-R*COS(A))/7000.)$TPRIME=DT$W=DT/2.
      PRI2=0.
C
C         RUNGE-KUTTA INTEGRATION OF COMPTON CURRENT
C
      DO 31 K=1,10
      RK1=DT*CMTHET(HOB,R,A,THETA,OMEGA,PATH,T,TPRIME,GAMYLD)
      RK2=DT*CMTHET(HOB,R,A,THETA,OMEGA,PATH,T,TPRIME+W,GAMYLD)
      RK3=RK2
      RK4=DT*CMTHET(HOB,R,A,THETA,OMEGA,PATH,T,TPRIME+DT,GAMYLD)
      JTHETA=JTHETA+(RK1+2.*(RK2+RK3)+RK4)/6.
      RK5=DT*CMPHI (HOB,R,A,THETA,OMEGA,PATH,T,TPRIME,GAMYLD)
      RK6=DT*CMPHI (HOB,R,A,THETA,OMEGA,PATH,T,TPRIME+W,GAMYLD)
      RK7=RK6
      RK8=DT*CMPHI (HOB,R,A,THETA,OMEGA,PATH,T,TPRIME+DT,GAMYLD)
```

```
      JPHI=JPHI+(RK5+2.*(RK6+RK7)+RK8)/6.                               CMTN 370
C                                                                       CMTN 380
C         RUNGE-KUTTA INTEGRATION OF PRIMARIES                          CMTN 390
C                                                                       CMTN 400
      RKP1=DT*RKCMTN(R,THETA,OMEGA,TP,TPRIME)                           CMTN 410
      RKP2=DT*RKCMTN(R,THETA,OMEGA,TP,TPRIME+W)                         CMTN 420
      RKP3=RKP2                                                         CMTN 430
      RKP4=DT*RKCMTN(R,THETA,OMEGA,TP,TPRIME+DT)                        CMTN 440
      PRI=PRI+(RKP1+2.*(RKP2+RKP3)+RKP4)/6.                             CMTN 450
      RP1=DT*RKCMTN(R,THETA,OMEGA,T,TPRIME)                             CMTN 460
      RP2=DT*RKCMTN(R,THETA,OMEGA,T,TPRIME+W)                           CMTN 470
      RP3=RP2                                                           CMTN 480
      RP4=DT*RKCMTN(R,THETA,OMEGA,T,TPRIME+DT)                          CMTN 490
      PRI2=PRI2+(RP1+2.*(RP2+RP3)+RP4)/6.                               CMTN 500
      TPRIME=TPRIME+DT                                                  CMTN 510
   31 CONTINUE                                                          CMTN 520
C                                                                       CMTN 530
C         MULTIPLY PRIMARIES BY Q*G(R)/TMAX                             CMTN 540
C         TO OBTAIN RATE OF PRODUCTION OF SECONDARIES                   CMTN 545
C                                                                       CMTN 550
      PRI=PRI*5.000E4*GOFR(R,A,HOB,PATH,GAMYLD)/TMAX                    CMTN 560
      PRI2=PRI2*5.0E4*GOFR(R,A,HOB,PATH,GAMYLD)/TMAX                    CMTN 570
      RETURN $ END                                                      CMTN 580
```

```
      FUNCTION CMPHI(HOB,R,A,THETA,OMEGA,PATH,T,TPRIME,GAMYLD)
C
C        CALCULATES F(T,R) FOR RUNGE-KUTTA INTEGRATION
C        OF PHI COMPONENT OF COMPTON CURRENT
C
      SOLVE=TOFT(T,TPRIME,THETA,OMEGA)
      SOLVE=FOFT(SOLVE)
      SOLVE=SOLVE*(-4.608E-11)*GCFR(R,A,HOB,PATH,GAMYLD)
      CMPHI=SOLVE*SIN(THETA)*SIN(OMEGA*TPRIME)
      RETURN $ END

      FUNCTION CMTHET(HOB,R,A,THETA,OMEGA,PATH,T,TPRIME,GAMYLD)
C
C        CALCULATES F(T,R) FOR RUNGE-KUTTA INTEGRATION
C        OF THETA COMPONENT OF COMPTON CURRENT
C
      SOLVE=TOFT(T,TPRIME,THETA,OMEGA)
      SOLVE=FOFT(SOLVE)
      SOLVE=SOLVE*(-4.608E-11)*GCFR(R,A,HOB,PATH,GAMYLD)
      CMTHET=SOLVE*SIN(THETA)*COS(THETA)*(COS(OMEGA*TPRIME)-1.)
      RETURN $ END
```

```
      SUBROUTINE RNGKUT (E1,E,R,H,SIGMA,COMPTJ)                          RNKT1010
C                                                                        RNKT1020
C         E(I+1) IS CALCULATED FROM E(I)                                 RNKT1030
C         USING THE RUNGE-KUTTA METHOD                                   RNKT1040
C                                                                        RNKT1050
      DATA (C=3.0E8),(RMUO=12.56637E-7)                                  RNKT1060
      EFUN(R,E)=-(1./R+C*RMUO*SIGMA/2.)*E-COMPTJ*C*RMUO/2.               RNKT1070
      RK1=H*EFUN(R,E)                                                    RNKT1080
      RK2=H*EFUN(R+H/2.,E+RK1/2.)                                        RNKT1090
      RK3=H*EFUN(R+H/2.,E+RK2/2.)                                        RNKT1100
      RK4=H*EFUN(R+H,E+RK3)                                              RNKT1110
      E1=E+(RK1+2.*(RK2+RK3)+RK4)/6.                                     RNKT1120
      RETURN $ END                                                       RNKT1130
```

```
      FUNCTION RKCMTN(R,THETA,OMEGA,TP,TPRIME)                           RKCM1010
C                                                                        RKCM1020
C         CALCULATES F(T) FOR RUNGE-KUTTA                                RKCM1030
C         INTEGRATION OF PRIMARY ELECTRONS                               RKCM1040
C                                                                        RKCM1050
      SOLVE=TOFT(TP,TPRIME,THETA,OMEGA)                                  RKCM1060
      RKCMTN=FOFT(SOLVE)                                                 RKCM1070
      RETURN $ END                                                       RKCM1080
```

```
      FLNCTICN GOFR (R,A,HOP,FATH,GAMYLD)                              GCFR1010
C                                                                      GOFR1020
C         SOLVES VIRGIN TRANSFCRT AND USES REACTION RATE TO            GCFR1030
C         CALCULATE THE NUMBER CENSITY OF RACIAL ELECTRONS             GCFR1040
C                                                                      GOFR1050
      SCLVE=(.0226275/COS(A))*(-1.+EXP(R*COS(A)/7000.))*EXP(-HOB/7C00.) GCFR1060
      DENOM=12.56637*R*R*PATH*1.5                                      GOFR1070
      GCFR=EXP(-SOLVE)*GAMYLD/DENCM                                    GCFR1080
      RETURN $ END                                                     GOFR1090


      FUNCTICN CLIFE (R,A,HOB)                                         CLIF1010
C                                                                      CLIF1020
C         CALCULATES COMPTON LIFETIME AT RADIUS = R                    CLIF1030
C         MAX ACCEPTABLE LIFETIME = 100 SHAKES FCR                     CLIF1040
C         THE KARZAS-LATTER HIGH FREQUENCY APPRCX                      CLIF1050
C                                                                      CLIF1060
      CLIFE=1.041667E-8*EXP((HCE-R*COS(A))/7000.)                      CLIF1070
      IF(CLIFE.GT.1.E-6) CLIFE=(1.E-6)                                 CLIF1080
      RETURN $ END                                                     CLIF1090
```

```
      FUNCTION TOFT (T,TPRIME,THETA,OMEGA)                              TOFT1010
C                                                                       TOFT1020
C        T(T) IS TIME TRANSFORMED TO KARZAS-LATTER FORM                 TOFT1030
C                                                                       TOFT1040
      B=0.358                                                           TOFT1050
      FIRST=T-(1.-B*(COS(THETA)**2))*TPRIME                             TOFT1060
      SECOND=B*(SIN(THETA)**2)*SIN(OMEGA*TPRIME)/OMEGA                  TOFT1070
      TOFT=FIRST+SECOND                                                 TOFT1080
      RETURN $ END                                                      TOFT1090
```

```
      FUNCTION FOFT(T)                                                  FOFT1010
C                                                                       FOFT1020
C         F(T) IS THE POMRANNING MODEL FOR TIME DEPENDENCE              FOFT1030
C         OF NUCLEAR WEAPON YIELD IN RETARDED TIME                      FOFT1040
C                                                                       FOFT1050
      INTEGER OUT                                                       FOFT1060
      COMMON OUT,A,B,RN,TO                                              FOFT1070
      TSHAKE=1.E8*T                                                     FOFT1080
      DENOM=(B+A*EXP((A+B)*(TSHAKE-TO)))*RN                             FOFT1090
      FOFT=(A+B)*EXP(A*(TSHAKE-TO))/DENOM                               FOFT1100
      RETURN $ END                                                      FOFT1110
```

```
      SUBROUTINE ELGPLT(EPLOT,TIME,BIG)
C
C        THE FIRST 20 SHAKES OF E(T) IS PLOTTED ON LINEAR SCALES
C
      DIMENSION EPLOT(192),TIME(192) ,E(112),T(112)
      MAG = 5 $ SMALL = 0.0
      CHECK = 0.0
      DO 6 I=1,110
      E(I)=EPLOT(T) $ T(I)=TIME(I)
 6    CONTINUE
      CALL PLOT (0.0,-8.0,-3)
      CALL PLOT (2.0,2.0,-3)
      CALL SCALE (T   ,5.,110,1)
      CALL SCALE (E    ,5.,110,1)
      CALL LINE(T   ,E    ,110,1,0,0)
      CALL   AXIS(0.,0.,13HTIME (SHAKES),-13,5.,0.,T   (111),T   (112))
      CALL   AXIS(0.,0.,12HEFIELD (V/M),12,5.,90.,E    (111),E    (112))
      CALL   AXIS(0.,5.,2H  ,2,5.,0.,T   (111),T   (112))
      CALL   AXIS(5.,0.,2H  ,-2,5.,90.,E    (111),E    (112))
      CALL PLOT(10.0,2.0,-3)
C
C        SMALL VALUES OF E(T) ARE CLIPPED OFF
C        AND E(T) IS PLOTTED ON LOG-LOG SCALES
C
      DO 3 I=1,190
      IF((EPLOT(I)/BIG).LT.(10.**(-MAG)))2,3
 2    EPLOT(I)=BIG*(10.**(-MAG)) $ CHECK=1.
 3    CONTINUE
      IF(CHECK.EQ.0.) GO TO 11
      PRINT 2005
      PRINT 2006,(EPLOT(I),I=1,190)
      SMALL=BIG*(10.**(-MAG))
      PRINT 2007,SMALL,BIG
 11   CONTINUE
      CALL PLOT (0.0,-8.0,-3)
      CALL PLOT (2.0,2.0,-3)
```

```
      CALL LGSCAL (TIME,5.0,190)
      CALL LGSCAL (EPLOT,5.0,190)
      CALL LGLINE (TIME,EPLOT,190,0,0,0)
      CALL LGAXIS(0.,0.,13HTIME (SHAKES),-13,5.,0.,TIME(191),TIME(192))
      CALL LGAXIS(0.,0.,12HEFIELD (V/M),12,5.,90.,EPLOT(191),EPLOT(192))
      CALL LGAXIS(0.,5.,2H  ,2,5.,0.,TIME(191),TIME(192))
      CALL LGAXIS(5.,0.,2H  ,-2,5.,90.,EPLOT(191),EPLOT(192))
      CALL PLOTE
      RETURN
 2005 FORMAT("1    * * * WARNING! PLOT OF E(T) HAS BEEN CLIPPED * * *"/)
 2006 FORMAT(/5X,"CLIPPED E(T) IS"//19(10(3X,1PE10.3)/))
 2007 FORMAT(//"   MINIMUM E(T) IS",1PE10.3/"   MAXIMUM E(T) IS",1PE10.3
     1)
      END
```

Vita

Terry C. Chapman was born on 21 August 1943 in Vancouver, Washington. He graduated from high school in Manitou Springs, Colorado in 1961. He attended the University of Colorado in Boulder where he was elected to Tau Beta Pi, Engineering Honor Society and Sigma Pi Sigma, Physics Honor Society. He received the degree Bachelor of Science, Engineering Physics and a commission in the U. S. Air Force from the University of Colorado in 1969. After attending communications training, he was assigned to Kelly AFB, Texas as a communications operations officer in 1970. He entered the Air Force Institute of Technology in 1972.

Permanent Address: P.O. Box 1144
Manitou Springs, Colorado 80829

This thesis was typed by Ladonna Stitzel.