Chapter 3: Electric Fields

3.1 The Electric Field

In chapter 2, Coulomb's law of electrostatics was discussed and it was shown that, if a charge q_2 is brought into the neighborhood of another charge q_1, a force is exerted upon q_2. The magnitude of that force is given by Coulomb's law. However, it should be asked what is the mechanism that transmits the force from q_1 to q_2? Coulomb's law states only that there is a force; it says nothing about the mechanism by which the force is transmitted, and it assumes that the force is transmitted instantaneously. Such a force is called an "action at a distance" since it does not explain how the force travels through that distance. Even the ancient Greek Philosophers would not have accepted such an idea, it seems too much like magic.

To overcome this shortcoming of Coulomb's law, Michael Faraday (1791-1867) introduced the concept of an electric field. He stated that *it is an intrinsic property of nature that an electric field exists in the space around an electric charge. □This electric field is considered to be a force field that exerts a force on charges placed in the field.* As an example, around the charge q_1 there exists an electric field. When the charge q_2 is brought into the neighborhood of q_1, the electric field of q_1 interacts with q_2, thereby exerting a force on q_2. The electric field becomes the mechanism for transmitting the force from q_1 to q_2, thereby eliminating the "action at a distance" principle.

Because the electric field is considered to be a force field, the existence of an electric field and its strength is determined by the effect it produces on a positive point charge q_0 placed in the region where the existence of the field is suspected. If the point charge, called a test charge, experiences an electrical force acting upon it, then it is said that the test charge is in an electric field. *The electric field is measured in terms of a quantity called the electric field intensity. The magnitude of the electric field intensity is defined as the ratio of the force **F** acting on the small test charge, q_0, to the small test charge itself. The direction of the electric field is in the direction of the force on the positive test charge.* This can be written as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \tag{3.1}$$

i.e., the force acting per unit charge. The SI unit of electric field intensity is a newton per coulomb, abbreviated as N/C. It should also be noted at this point that the small positive test charge q_0 must be small enough so that it will not appreciably distort the electric field that you are trying to measure. (In the measurement of any physical quantity the instruments of measurement should be designed to interfere as little as possible with the quantity being measured.) To emphasize this point equation 3.1 is sometimes written in the form

$$\mathbf{E} = \lim_{q_o \to 0} \frac{\mathbf{F}}{q_0}$$

3.2 The Electric Field of A Point Charge

The electric field of a positive point charge q can be determined by following the definition in equation 3.1. A positive point charge q is shown in figure 3.1. A very small positive test charge q_o is placed in various positions around the positive
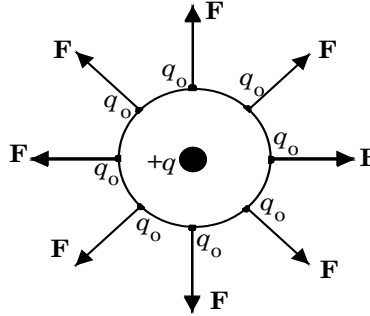

Figure 3.1 The force exerted by a positive point charge.

charge q. Because like charges repel each other, the positive test charge q_o will experience a force of repulsion from the positive point charge q. Thus, the force acting on the test charge, and hence the direction of the electric field, is always directed radially away from the point charge q. The magnitude of the electric field intensity of a point charge is found from equation 3.1, with the force found from Coulomb's law, equation 2-1.

$$F = k\frac{qq_o}{r^2} \tag{2-1}$$

That is,

$$E = \frac{F}{q_o} = \frac{k\dfrac{qq_o}{r^2}}{q_o}$$
$$E = \frac{kq}{r^2} \tag{3.2}$$

Equation 3.2 is the equation for the magnitude of the electric field intensity due to a point charge. The direction of the electric field has already been shown to be radially away from the positive point charge. If *a unit vector $\mathbf{r}_o$ is drawn pointing everywhere radially away from the point charge* then equation 3.2 can be written in the vector form

$$\mathbf{E} = k\frac{q}{r^2}\mathbf{r}_o \tag{3.3}$$

As an example, the electric field $\mathbf{E}$ at an arbitrary point P, figure 3.2(a), points radially outward if q is positive, and radially inward if q is negative, figure 3.2(b).
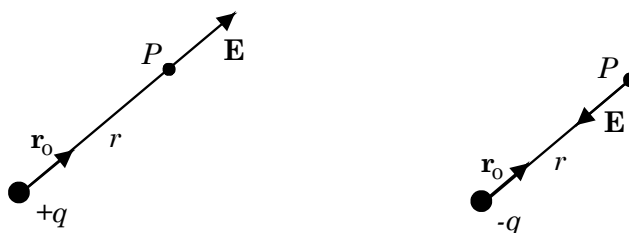
(a) Positive Charge   (b) Negative Charge

Figure 3.2 The direction of the unit vector $\mathbf{r}_0$ and the electric field vector $\mathbf{E}$.

To draw a picture of the total electric field of a positive point charge is slightly difficult because the electric field is a vector, and as you recall from the study of vectors, a vector is a quantity that has both magnitude and direction. Since the magnitude of the electric field of the point charge varies with the distance r from the charge, a picture of the total electric field would have to be a series of discrete electric vectors each one pointing radially away from the positive point charge but the length of each vector would vary depending upon how far you are away from the point charge. This is shown in figure 3.3(a).

To simplify the picture of the electric field, a series of continuous lines are drawn from the positive point charge to indicate the total electric field, as in figure 3.3(b). These continuous lines were called by Michael Faraday, lines of force because they are in the direction of the force that acts upon a positive point charge
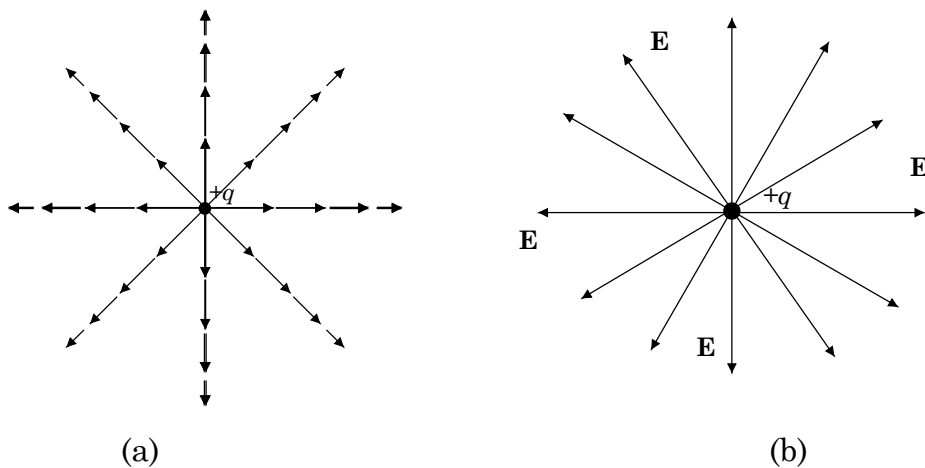
(a)   (b)

Figure 3.3 Electric field of a positive point charge.

placed in the field. They are everywhere tangent to the direction of the electric field. It must be understood however, that the magnitude of the electric field varies along the lines shown. The greater the distance from the point charge the smaller the magnitude of the electric field. With these qualifications in mind, we will say that the electric field of a positive point charge is shown in figure 3.3(b). (We will use

this same technique to depict the electric fields in the rest of this book.) Note that the electric field always emanates from a positive charge. *The electric field intensity of a point charge is directly proportional to the charge that creates it, and inversely proportional to the square of the distance from the point charge to the position where the field is being evaluated.*

The electric field intensity of a negative point charge is found in the same way. But because unlike charges attract each other, the force between the negatively charged point source and the small positive test charge is one of attraction. Hence, the force is everywhere radially inward toward the negatively charged point source and the electric field is also. Therefore, the electric field of a negative point charge is as shown in figure 3.4. The magnitude of the electric field intensity of a negative point charge is also given by equation 3.2.
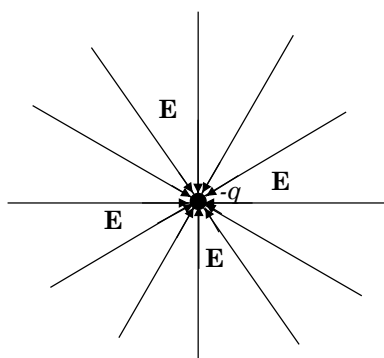

Figure 3.4 Electric field of a negative point charge.

Example 3.1

Find the magnitude of the electric field intensity at a distance of 0.500 m from a 3.00 µC charge.

Solution

The electric field intensity is found from equation 3.2 as

$$E = \frac{kq}{r^2}$$

$$= \frac{\left(9.00 \times 10^9 \text{ N m}^2/\text{C}^2\right)\left(3.00 \times 10^{-6} \text{ C}\right)}{\left(5.00 \times 10^{-1} \text{ m}\right)^2}$$

$$E = 1.08 \times 10^5 \text{ N/C}$$

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If the electric field intensity is known in a region, then the force on any charge q placed in that field is determined from equation 3.1 as

$$F = qE \tag{3.4}$$

Example 3.2

Force on a charge in an external electric field. A point charge, $q = 5.64$ µC, is placed in an electric field of 2.55×10^3 N/C. Find the magnitude of the force acting on the charge.

Solution

The magnitude of the force acting on the point charge is found from equation 3.4 as

$$F = qE$$
$$F = (5.64 \times 10^{-6} \text{ C})(2.55 \times 10^3 \text{ N/C})$$
$$F = 1.44 \times 10^{-2} \text{ N}$$

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3.3 Superposition of Electric Fields for Multiple Discrete Charges

When more than one charge is present, as in figure 3.5, the force on an arbitrary charge q is seen to be the vector sum of the forces produced by each charge, i.e.,

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots \tag{3.5}$$

But if charge q_1 produces a field $\mathbf{E}_1$, then the force on charge q produced by $\mathbf{E}_1$, is found from equation 3.4 as

$$\mathbf{F}_1 = q\mathbf{E}_1 \tag{3.6}$$

Similarly, the force on charge q produced by $\mathbf{E}_2$ is

$$\mathbf{F}_2 = q\mathbf{E}_2 \tag{3.7}$$

and finally

$$\mathbf{F}_3 = q\mathbf{E}_3 \tag{3.8}$$

Substituting equations 3.6, 3.7 and 3.8 into 3.5 gives

$$\mathbf{F} = q\mathbf{E}_1 + q\mathbf{E}_2 + q\mathbf{E}_3 + \ldots$$
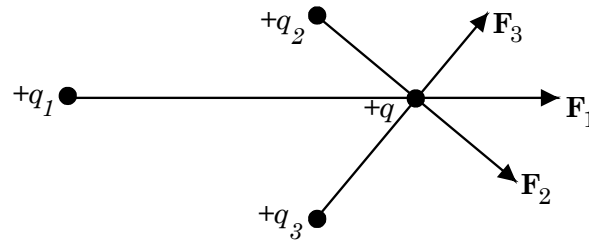
Figure 3.5 Multiple charges.

Dividing each term by q gives

$$\mathbf{F}/q = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots \tag{3.9}$$

But $\mathbf{F}/q$ is the resultant force per unit charge acting on charge q and is thus the total resultant electric field intensity $\mathbf{E}$. Therefore equation 3.9 becomes

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3 + \dots \tag{3.10}$$

Equation 3.10 is the mathematical statement of the principle of the superposition of electric fields: *When more than one charge contributes to the electric field, the resultant electric field is the vector sum of the electric fields produced by the various charges.*

Example 3.3

The electric field of two positive charges. If two equal positive charges, $q_1 = q_2 = 2.00$ μC are situated as shown, in figure 3.6, find the resultant electric field intensity at point A. The distance $r_1 = 0.819$ m, $r_2 = 0.574$ m, and $l = 1.00$ m.
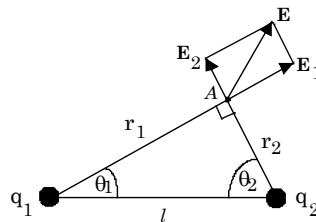

Figure 3.6 The resultant electric field for two charges.

Solution

The magnitude of the electric field intensity produced by q_1 is found from equation 3.2 as

$$E_1 = \frac{kq_1}{r_1^2}$$

$$= \frac{\left(9.00 \times 10^9 \text{ N m}^2/\text{C}^2\right)\left(2.00 \times 10^{-6} \text{ C}\right)}{\left(0.819 \text{ m}\right)^2}$$
$$E_1 = 2.68 \times 10^4 \text{ N/C}$$

The magnitude of the electric field intensity produced by q_2 is

$$E_2 = \frac{kq_2}{r_2^2}$$
$$= \frac{\left(9.00 \times 10^9 \text{ N m}^2/\text{C}^2\right)\left(2.00 \times 10^{-6} \text{ C}\right)}{\left(0.574 \text{ m}\right)^2}$$
$$E_2 = 5.46 \times 10^4 \text{ N/C}$$

The resultant electric field is found from equation 3.10 as the vector addition

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

where

$$\mathbf{E}_1 = \mathbf{i}E_{1x} + \mathbf{j}E_{1y}$$

and

$$\mathbf{E}_2 = -\mathbf{i}E_{2x} + \mathbf{j}E_{2y}$$

as can be seen in figure 3.7. Hence the resultant vector is

$$\mathbf{E} = \mathbf{i}E_{1x} + \mathbf{j}E_{1y} - \mathbf{i}E_{2x} + \mathbf{j}E_{2y}$$
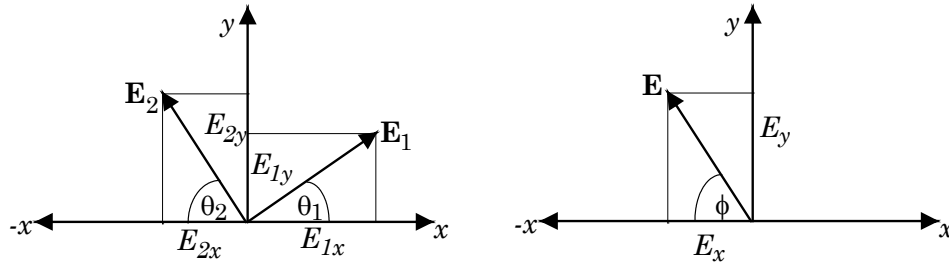$$\mathbf{E} = \mathbf{i}(E_{1x} - E_{2x}) + \mathbf{j}(E_{1y} + E_{2y})$$

Figure 3.7 The components of the electric field vectors.

Therefore,

$$E_x = E_{1x} - E_{2x}$$

and

$$E_y = E_{1y} + E_{2y}$$

The angles θ_1 and θ_2 are found from figure 3.6 as follows

$$\theta_1 = \tan^{-1}\left(\frac{r_2}{r_1}\right) = \tan^{-1}\left(\frac{0.574 \text{ m}}{0.819 \text{ m}}\right) = 35.0^0$$
$$\theta_2 = 90.0^0 - \theta_1 = 90.0^0 - 35.0^0 = 55.0^0$$

The *x*-components of the electric field intensities are

$$E_{1x} = E_1 \cos\theta_1$$
$$E_{1x} = (2.68 \times 10^4 \text{ N/C})\cos 35.0^0$$
$$E_{1x} = 2.20 \times 10^4 \text{ N/C}$$

and

$$E_{2x} = E_2 \cos\theta_2$$
$$E_{2x} = (5.46 \times 10^4 \text{ N/C})\cos 55.0^0$$
$$E_{2x} = 3.13 \times 10^4 \text{ N/C}$$

Hence the *x*-component of the resultant field is

$$E_x = E_{1x} - E_{2x}$$
$$E_x = 2.20 \times 10^4 \text{ N/C} - 3.13 \times 10^4 \text{ N/C}$$
$$E_x = -0.930 \times 10^4 \text{ N/C}$$

The *y*-components of the electric field intensities are

$$E_{1y} = E_1 \sin\theta_1$$
$$E_{1y} = (2.68 \times 10^4 \text{ N/C})\sin 35.0^0$$
$$E_{1y} = 1.54 \times 10^4 \text{ N/C}$$

and

$$E_{2y} = E_2 \sin\theta_2$$
$$E_{2y} = (5.46 \times 10^4 \text{ N/C})\sin 55.0^0$$
$$E_{2y} = 4.47 \times 10^4 \text{ N/C}$$

Therefore the *y*-component of the resultant field is

$$E_y = E_{1y} + E_{2y}$$
$$E_y = 1.54 \times 10^4 \text{ N/C} + 4.47 \times 10^4 \text{ N/C}$$
$$E_y = 6.01 \times 10^4 \text{ N/C}$$

The resultant electric field vector is given by equation 3.11 as

$$\mathbf{E} = \mathbf{i}E_x + \mathbf{j}E_y \tag{3.11}$$
$$\mathbf{E} = -(0.930 \times 10^4 \text{ N/C})\mathbf{i} + (6.01 \times 10^4 \text{ N/C})\mathbf{j}$$

The magnitude of the resultant electric field intensity at point *A* is found as

$$\mathbf{E} = \mathbf{i}E_x + \mathbf{j}E_y \tag{3.12}$$
$$\mathbf{E} = -(0.930 \times 10^4 \text{ N/C})\mathbf{i} + (6.01 \times 10^4 \text{ N/C})\mathbf{j}$$

$$E = 6.08 \times 10^4 \text{ N/C}$$

The direction of the electric field vector is found from

$$\tan \phi = \frac{E_y}{E_x}$$
$$\phi = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\left(\frac{6.01 \times 10^4 \text{ N/C}}{-0.930 \times 10^4 \text{ N/C}}\right) = \tan^{-1}(-6.46)$$
$$\phi = -81.2^0$$

Because E_x is negative, the angle ϕ lies in the second quadrant. The angle that the vector **E** makes with the positive x-axis is $\phi + 180^0 = 98.8^0$.

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Thus, by the principle of superposition, the electric field can be determined at any point for any number of charges. However, we have only found the field at one point. If it is desired to see a picture of the entire electric field, as already shown for the point charge, **E** must be evaluated vectorially at an extremely large number of points. As can be seen from this example this would be a rather lengthy job. However, the entire electric field can be readily solved by the use of a computer. The total electric field caused by two equal positive charges, q_1 and q_2, is shown in figure 3.8.
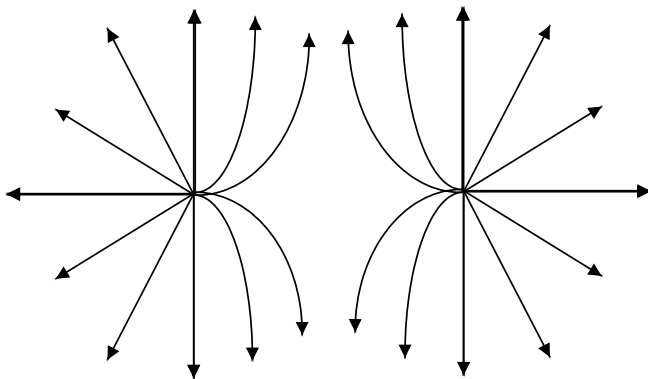

Figure 3.8 Electric field of two equal positively charged particles.

Equation 3.10 gives the electric field for any number of discrete charges. To find the electric field of a continuous distribution of charge, it is necessary to use the calculus and the sum in equation 3.10 becomes an integration. We will treat continuous charge distributions in section 3.6.

3.4 The Electric Field along the Perpendicular Bisector of an Electric Dipole

The configuration of two closely spaced, equal but opposite point charges is a very important one and is given the special name of an electric dipole. The electric field intensity at a point P along the perpendicular bisector of an electric dipole can be found with the help of figure 3.9.
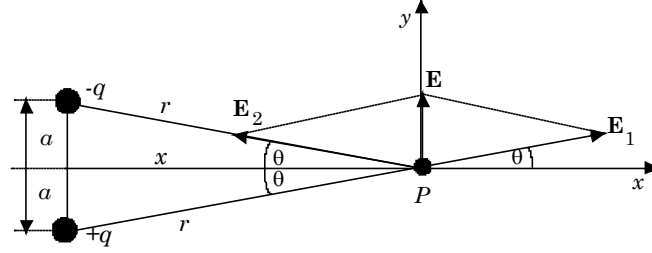

Figure 3.9 The electric dipole.

The resultant electric field intensity is found by the superposition principle as

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

The magnitudes of $\mathbf{E}_1$ and $\mathbf{E}_2$ are found as

$$E_1 = k\frac{q}{r^2} = E_2 = k\frac{q}{r^2}$$

As can be seen in figure 3.9, the electric fields are written in terms of their components as

$$\mathbf{E}_1 = \mathbf{i}E_{1x} + \mathbf{j}E_{1y}$$

and

$$\mathbf{E}_2 = -\mathbf{i}E_{2x} + \mathbf{j}E_{2y}$$

Thus the resultant electric field in terms of its components is

$$\mathbf{E} = \mathbf{i}E_{1x} + \mathbf{j}E_{1y} - \mathbf{i}E_{2x} + \mathbf{j}E_{2y}$$
$$\mathbf{E} = \mathbf{i}(E_{1x} - E_{2x}) + \mathbf{j}(E_{1y} + E_{2y})$$

Because $E_1 = E_2$, the x-component of the resultant field at the point P becomes

$$E_x = E_{1x} - E_{2x}$$
$$E_x = E_1 \cos\theta - E_2 \cos\theta$$
$$E_x = E_1 \cos\theta - E_1 \cos\theta$$
$$E_x = 0$$

Also because $E_1 = E_2$, the y-component of the resultant field at the point P is

$$E_y = E_{1y} + E_{2y}$$
$$E_y = E_1 \sin\theta + E_2 \sin\theta$$
$$E_y = E_1 \sin\theta + E_1 \sin\theta$$
$$E_y = 2E_1 \sin\theta$$

The resultant electric field becomes

$$\mathbf{E} = \mathbf{i}E_x + \mathbf{j}E_y$$
$$\mathbf{E} = \mathbf{i}(0) + \mathbf{j}(2E_1 \sin\theta)$$

Therefore the resultant electric field is

$$\mathbf{E} = 2E_1 \sin\theta \, \mathbf{j}$$

But $E_1 = kq/r^2$, hence

$$\mathbf{E} = \frac{2kq}{r^2} \sin\theta \, \mathbf{j} \qquad (3.13)$$

But from figure 3.9 it is seen that

$$\sin\theta = a/r$$

Therefore,

$$\mathbf{E} = \frac{2kq}{r^2} \frac{a}{r} \, \mathbf{j}$$
$$\mathbf{E} = \frac{k2aq}{r^3} \, \mathbf{j}$$

However, from the diagram it is seen that

$$r = \sqrt{a^2 + x^2}$$

Thus,

$$\mathbf{E} = \frac{k2aq}{\left(\sqrt{a^2 + x^2}\right)^3} \, \mathbf{j} = \frac{k2aq}{(a^2 + x^2)^{3/2}} \, \mathbf{j} \qquad (3.14)$$

The quantity $(2aq)$ which is the product of the charge q times the distance separating the charges, $2a$, is called the electric dipole moment and is designated by the letter p. That is,

$$p = 2aq \qquad (3.15)$$

The SI unit for the electric dipole is a coulomb meter, abbreviated C m. In terms of the electric dipole moment p, the electric field of a dipole along its perpendicular bisector is given by

$$\mathbf{E} = \frac{kp}{(a^2 + x^2)^{3/2}} \, \mathbf{j} \qquad (3.16)$$

In practice x is usually very much greater than a, that is $x \gg a$, so as a first approximation we can let

$$(a^2 + x^2)^{3/2} \approx (0 + x^2)^{3/2} = x^3$$

Therefore, the magnitude of the electric field intensity along the perpendicular bisector, a distance x from the dipole, is

$$\mathbf{E} = \frac{kp}{x^3}\,\mathbf{j}$$

(3.17)

The direction of the electric field along the perpendicular bisector of the electric dipole is in the $+\mathbf{j}$ direction as can be seen from figure 3.9 and is parallel to the axis of the dipole. Note that the electric field of the dipole varies as $1/x^3$, while the electric field of a point charge varies as $1/x^2$. Thus the electric field of a dipole decreases faster with distance than the electric field of a point charge.

We have found the electric field at only one point in space. If it is desired to see a picture of the entire electric field of the dipole, $\mathbf{E}$ must be evaluated vectorially at an extremely large number of points. As can be seen from this example this would be a rather lengthy job. However, the entire electric field can be readily solved by the use of a computer. The total electric field of an electric dipole is shown in figure 3.10. More detail on the electric dipole will be found in sections 3.5 and 5.7.
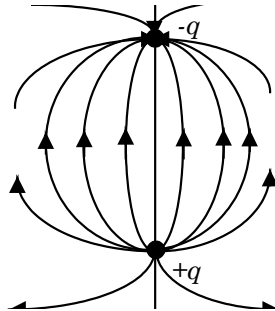

Figure 3.10 The electric field of a dipole.

Example 3.4

The electric dipole. An electric dipole consists of two charges, $q_1 = 2.00$ µC and $q_2 = -2.00$ µC separated by a distance of 5.00 mm. Find (a) the electric dipole moment and (b) the resultant electric field intensity at a point on the perpendicular bisector 1.50 m from the center of the dipole.

Solution

(a) the electric dipole moment is found from equation 3.15 as

$$p = 2aq$$
$$p = (5.00 \times 10^{-3}\,\text{m})(2.00 \times 10^{-6}\,\text{C})$$
$$p = 1.00 \times 10^{-8}\,\text{m C}$$

(b) The magnitude of the electric field intensity of the dipole is found from equation 3.16 as

$$\mathbf{E} = \frac{kp}{(a^2 + x^2)^{3/2}} \mathbf{j}$$

$$\mathbf{E} = \frac{(9.00 \times 10^9 \text{ N m}^2/\text{C}^2)(1.00 \times 10^{-8} \text{ m C})}{\left[(2.50 \times 10^{-3} \text{ m})^2 + (1.50 \text{ m})^2\right]^{3/2}} \mathbf{j}$$

$$\mathbf{E} = (26.7 \text{ N/C}) \mathbf{j}$$

Notice that the same answer would have been obtained if the quantity "a" was set equal to zero, since 1.50 m $>> 5.00 \times 10^{-3}$ m, which is consistent with the usual assumption that $x >> a$.

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3.5 The Torque on a Dipole in an External Electric Field

As shown in the last section, the electric dipole moment was defined by equation 3.15 as

$$p = 2aq$$

The electric dipole moment can be written as a vector by defining a unit vector $\mathbf{r}_o$ *that points from the negative charge to the positive charge.* The electric dipole moment can then be written as

$$\boldsymbol{p} = 2aq\boldsymbol{r}_o \tag{3.18}$$

The electric dipole moment thus points from the negative charge to the positive charge as seen in figure 3.11(a).
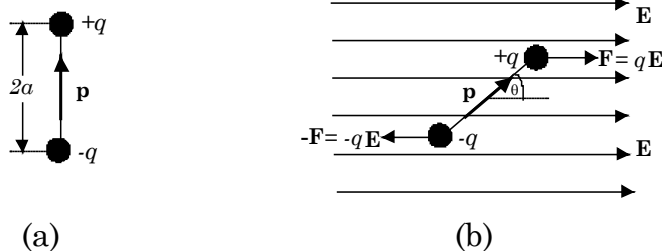

(a)   (b)

Figure 3.11 Torque on a dipole in an external field.

Let us now place this electric dipole into the external electric field shown in figure 3.11(b). The charge $+q$ experiences the force

$$\mathbf{F} = q\mathbf{E} \tag{3.19}$$

to the right as shown, while the charge $-q$ experiences the force

$$-\mathbf{F} = -q\mathbf{E} \tag{3.20}$$

to the left as shown. Hence the net force on the dipole is zero. That is, the dipole will not accelerate in the x or y-direction. However, because the forces do not act through the same point, they will produce a torque on the dipole. The torque $\boldsymbol{\tau}$ acting on the dipole is given by

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} + (-\mathbf{r}) \times (-\mathbf{F}) = 2\mathbf{r} \times \mathbf{F} \tag{3.21}$$

where $\mathbf{r}$ is the vector distance from the center of the dipole to the charge $+q$ and $-\mathbf{r}$ is the vector distance from the center of the dipole to the charge $-q$. The distance $2r$ is thus equal to $2a$ the separation between the charges and hence $2\mathbf{r}$ can be written as

$$2\mathbf{r} = 2a\mathbf{r_o} \tag{3.22}$$

where $\mathbf{r_o}$ is a unit vector pointing from the negative charge to the positive charge. Replacing equations 3.19 and 3.22 into 3.21 gives

$$\boldsymbol{\tau} = 2\mathbf{r} \times \mathbf{F} = (2a\mathbf{r_o}) \times (q\mathbf{E})$$
$$\boldsymbol{\tau} = (2aq\mathbf{r_o}) \times (\mathbf{E}) \tag{3.23}$$

But $(2aq\mathbf{r_o})$ is equal to the electric dipole moment $\mathbf{p}$ defined in equation 3.18. Replacing equation 3.18 into equation 3.23 gives for the torque acting on an electric dipole in an external field

$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E} \tag{3.24}$$

The magnitude of the torque is given by the definition of the cross product as

$$\tau = pE \sin\theta \tag{3.25}$$

When p is perpendicular to the external electric field E, θ is equal to 90^0, and the $\sin 90^0 = 1$. Therefore the torque acting on the dipole will be at its maximum value, and will act to rotate the dipole clockwise in figure 3.11b. As the dipole rotates, the angle θ will decrease until the electric dipole moment becomes parallel to the external electric field, and the angle θ will be equal to zero. At this point the torque acting on the dipole becomes zero, because the $\sin\theta$ term in equation 3.25, will be zero. *Hence, whenever an electric dipole is placed in an external electric field, a torque will act on the dipole causing it to rotate in the external field until it becomes aligned with the external field.*

Example 3.5

Torque acting on an electric dipole. The electric dipole of example 3.4 is placed in a uniform electric field of $\mathbf{E}$ = (500 N/C) $\mathbf{i}$, at an angle of 35.0^0 with the *x*-axis. Find the torque acting on the dipole.

Solution

The torque acting on the dipole is found from equation 3.25 as

$$\tau = pE \sin\theta$$
$$\tau = (1.00 \times 10^{-8} \text{ m C})(500 \text{ N/C})\sin 35.0^0$$
$$\tau = 2.87 \times 10^{-6} \text{ m N}$$

To go to this Interactive Example click on this sentence.

3.6 Electric Fields of Continuous Charge Distributions

As we saw in section 3.3, when there are multiple discrete charges in a region, then the electric field produced by those charges at any point is found by the vector sum of the electric fields associated with each of the charges. That is, the resultant electric field intensity for a group of point charges was given by equation 3.10 as

$$E = E_1 + E_2 + E_3 + ... \tag{3.10}$$

Equation 3.10 can be written in the shorthand notation as

$$\mathbf{E} = \sum_{n-1}^{N} \mathbf{E}_i \tag{3.26}$$

where, again, Σ means "the sum of" and the sum goes from $i = 1$ to $i = N$, the total number of charges present.

If the charge distribution is a continuous one, the field it sets up at any point P can be computed by dividing the continuous distribution of charge into a large number of infinitesimal elements of charge, dq. Each element of charge dq acts like a point charge and will produce an element of electric field intensity, $d\mathbf{E}$, at the point P, given by

$$d\mathbf{E} = k\frac{dq}{r^2}\mathbf{r}_o \tag{3.27}$$

where r is the distance from the element of charge dq to the field point P. $\mathbf{r}_o$ is a unit vector that points radially away from the element of charge dq and will point toward the field point P. The total electric field intensity $\mathbf{E}$ at the point P caused by

the electric field from the entire distribution of all the dq's is again a sum, but since the elements of charge dq are infinitesimal, the sum becomes the integral of all the elements of electric field $d\mathbf{E}$. That is, *the total electric field of a continuous distribution of charge is found as*

$$\mathbf{E} = \int d\mathbf{E} = \int k\frac{dq}{r^2}\mathbf{r}_o$$

(3.28)

We will now look at some specific examples of the electric fields caused by continuous charge distributions.

3.7 The Electric Field on Axis of a Charged Rod

Let us find the electric field at the point P, the origin of our coordinate system in figure 3.12, for a rod of charge that lies along the x-axis. The charge q is distributed uniformly over the rod. We divide the rod up into small elements of charge dq as shown. Each of these elements of charge will produce an element of electric field $d\mathbf{E}$. The element of charge dq located at the position x will produce the element of electric field $d\mathbf{E}$ given by
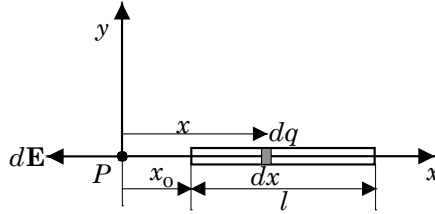

Figure 3.12 Electric field on axis for a rod of charge.

$$d\mathbf{E} = k\frac{dq}{x^2}(-\mathbf{i})$$

(3.29)

The $-\mathbf{i}$ indicates that the element of electric field points in the negative x-direction. The total electric field at the point P is the sum or integral of each of these $d\mathbf{E}$'s and is given by equation 3.28 as

$$\mathbf{E} = \int d\mathbf{E} = \int k\frac{dq}{r^2}\mathbf{r}_o$$

(3.28)

$$\mathbf{E} = \int d\mathbf{E} = \int k\frac{dq}{x^2}(-\mathbf{i})$$

(3.30)

The *linear charge density λ is defined as the charge per unit length* and can be written for the rod as

$$\lambda = q/x$$

(3.31)

The total charge on the rod can now be written as

$$q = \lambda x$$

(3.32)

and its differential by

$$dq = \lambda dx \qquad (3.33)$$

Substituting equation 3.33 back into equation 3.30 gives

$$\mathbf{E} = -\int k\frac{dq}{x^2}\mathbf{i} = -\int k\frac{\lambda dx}{x^2}\mathbf{i}$$

The integration is over x and as can be seen from the figure, the limits of integration will go from x_o to $x_o + l$, where l is the length of the rod. That is,

$$\mathbf{E} = -\int_{x_o}^{x_o+l} k\lambda\frac{dx}{x^2}\mathbf{i}$$

But k is a constant and λ, the charge per unit length, is also a constant because the charge is distributed uniformly over the rod and can be taken outside of the integral. Therefore,

$$\mathbf{E} = -k\lambda \int_{x_o}^{x_o+l} \frac{dx}{x^2}\mathbf{i} = -k\lambda \int_{x_o}^{x_o+l} x^{-2}dx\, \mathbf{i}$$
$$\mathbf{E} = -k\lambda[-x^{-1}]_{x_o}^{x_o+l}\, \mathbf{i}$$
$$\mathbf{E} = -k\lambda\left[\frac{-1}{x_o+l} - \frac{-1}{x_o}\right]\mathbf{i}$$
$$\mathbf{E} = -k\lambda\left[\frac{1}{x_o} - \frac{1}{x_o+l}\right]\mathbf{i}$$
$$\mathbf{E} = -k\lambda\left[\frac{x_o+l-x_o}{(x_o)(x_o+l)}\right]\mathbf{i}$$

Hence, the electric field at the point P is found to be

$$\mathbf{E} = -\frac{k\lambda l}{(x_o)(x_o+l)}\, \mathbf{i} \qquad (3.34)$$

If we prefer we can write the electric field in terms of the total charge q on the rod instead of the linear charge density l, since $\lambda = q/l$ from equation 3.31. That is, the total electric field at the point P caused by the rod of charge is given by

$$\mathbf{E} = -\frac{kq}{(x_o)(x_o+l)}\, \mathbf{i} \qquad (3.35)$$

Example 3.6

Electric field on axis for a rod of charge and the force associated with it. A charge q_o = 5.50 μC is placed at the origin and a rod of uniform charge density of 200 μC/m is located on the x axis at x_o = 10.0 cm. The rod has a length of 12.7 cm. Find the force acting on the charge q_o.

Solution

The total charge on the rod is found from equation 3.32 as

$$q = \lambda x$$
$$q = (200 \times 10^{-6} \text{ C/m})(0.127 \text{ m})$$
$$q = 2.54 \times 10^{-5} \text{ C}$$

The electric field at the point P is found from equation 3.35 as

$$\mathbf{E} = -\frac{kq}{(x_o)(x_o + l)} \mathbf{i}$$
$$\mathbf{E} = -\frac{(9.00 \times 10^9 \text{ N m}^2/\text{C}^2)(2.54 \times 10^{-5} \text{ C })}{(0.10 \text{ m})(0.10 \text{ m} + 0.127 \text{ m})} \mathbf{i}$$
$$\mathbf{E} = -(1.01 \times 10^7 \text{ N/C})\mathbf{i}$$

The force acting on the charge q_o is found from equation 3.4 as

$$\mathbf{F} = q_o\mathbf{E}$$
$$\mathbf{F} = (5.50 \times 10^{-6} \text{ C})(-1.01 \times 10^7 \text{ N/C}) \mathbf{i}$$
$$\mathbf{F} = -(55.5 \text{ N})\mathbf{i}$$

To go to this Interactive Example click on this sentence.

3.8 The Electric Field on Axis for a Ring of Charge

Let us determine the electric field at the point P, a distance x from the center of a ring of charge of radius "a" as shown in figure 3.13. We will assume that the charge
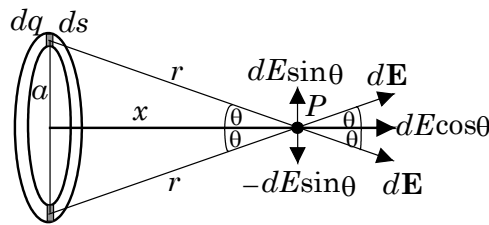

Figure 3.13 The electric field of a ring of charge.

is distributed uniformly along the ring, and the ring contains a total charge q. The charge per unit length of the ring, λ, is defined as

$$\lambda = q/s \tag{3.36}$$

where s is the entire length or arc of the ring (circumference). Let us now consider a small element ds at the top of the ring that contains a small element of charge dq. The total charge contained in this element dq is found from equation 3.36 as

$$q = \lambda s \qquad (3.37)$$

Hence,

$$dq = \lambda \, ds \qquad (3.38)$$

This element of charge dq can be considered as a point charge and it sets up a differential electric field $d\mathbf{E}$ whose magnitude is given by

$$dE = k\frac{dq}{r^2} \qquad (3.39)$$

and is shown in figure 3.13. The total electric field at the point P is obtained by adding up, integrating, all the small element $d\mathbf{E}$'s caused by all the dq's. That is,

$$\mathbf{E} = \int d\mathbf{E} \qquad (3.40)$$

As can be seen in the diagram, the differential of the electric field has two components, $dE\cos\theta$ and $- dE\sin\theta$. To simplify the integration we make use of the symmetry of the problem by noting that the element dq at the top of the ring will have the component $- dE\sin\theta$, while the element dq of charge at the bottom of the ring will have a component $+ dE\sin\theta$. Hence, by symmetry the term $dE\sin\theta$ will always have another term that is equal and opposite, and hence the sum of all the $dE\sin\theta$'s will add to zero. Hence only the $dE\cos\theta$'s will contribute to the electric field, and thus the electric field will point along the axis of the ring in the $\mathbf{i}$ direction. Therefore, the total electric field can now be written as

$$\mathbf{E} = \int \boldsymbol{dE}\cos\theta \, \mathbf{i} \qquad (3.41)$$

Replacing equation 3.39 into equation 3.41 we get

$$\mathbf{E} = \int \boldsymbol{dE}\cos\theta \, \mathbf{i} = \int k\frac{dq}{r^2}\cos\theta \, \mathbf{i} \qquad (3.42)$$

From the diagram we see that

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{a^2 + x^2}} \qquad (3.43)$$

Replacing equations 3.38 and 3.43 into equation 3.42 we get

$$\mathbf{E} = \int k\frac{dq}{r^2}\frac{x}{\sqrt{a^2 + x^2}} \, \mathbf{i} = \int kx\frac{dq}{r^3}\mathbf{i} = \int k\frac{x}{(a^2 + x^2)^{3/2}}\lambda ds \, \mathbf{i}$$

But k, x, λ, and a are constants and can be taken outside of the integral. Thus,

$$\mathbf{E} = \frac{kx\lambda}{(a^2 + x^2)^{3/2}} \int ds \, \mathbf{i}$$

(3.44)

The integration is over ds which is an element of arc of the ring. But the sum of all the ds's of the ring is just the circumference of the ring itself, that is,

$$\int ds = 2\pi a$$

(3.45)

Replacing equation 3.45 into equation 3.44 gives

$$\mathbf{E} = \frac{kx\lambda}{(a^2 + x^2)^{3/2}}(2\pi a) \, \mathbf{i}$$

(3.46)

But the charge per unit length, $\lambda = q/s = q/(2\pi a)$ from equation 3.36, hence substituting this back into equation 3.46 we get

$$\mathbf{E} = \frac{kx}{(a^2 + x^2)^{3/2}} \frac{q}{(2\pi a)}(2\pi a) \, \mathbf{i}$$

(3.47)

Therefore the electric field at the point x due to a ring of charge of radius "a" carrying a total charge q is given by

$$\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}} \, \mathbf{i}$$

(3.48)

Example 3.7

Electric field of a ring of charge. Find the electric field on the axis of a ring of charge at a very large distance from the ring of charge. That is, assume $x \gg a$.

Solution

If $x \gg a$ then as an approximation

$$(a^2 + x^2)^{3/2} \approx x^3$$

Replacing this assumption into equation 3.48 yields

$$\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}} \, \mathbf{i} = \frac{kqx}{x^3} \, \mathbf{i}$$

$$\mathbf{E} = \frac{kq}{x^2} \, \mathbf{i}$$

But this is the electric field of a point charge. Thus at very large distances from the ring of charge, the ring of charge looks like a point charge as would be expected.

Example 3.8

The electric field at the center of a ring of charge. Find the electric field at the center of a ring of charge.

Solution

At the center of the ring $x = 0$. Therefore the electric field at the center of a ring of charge is found from equation 3.48 with x set equal to zero. That is,

$$\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}}\,\mathbf{i} = \frac{kq(0)}{(a^2 + 0)^{3/2}}\,\mathbf{i}$$
$$\mathbf{E} = 0$$

Example 3.9

Maximum value of the electric field of a ring of charge. Find the point on the x-axis where the electric field of a ring of charge takes on its maximum value.

Solution

As you recall from your calculus course the maximum value of a function is found by taking the first derivative of the function with respect to x and setting that derivative equal to zero. Hence, the maximum value of the electric field is found by taking the first derivative of equation 3.48 and setting it equal to zero. Therefore,

$$\frac{dE}{dx} = \frac{d}{dx}\left[\frac{kqx}{(a^2 + x^2)^{3/2}}\right] = \frac{d}{dx}\left[kqx(a^2 + x^2)^{-3/2}\right] = 0$$
$$\frac{dE}{dx} = kq\left[x(-3/2)(a^2 + x^2)^{-5/2}(2x) + (a^2 + x^2)^{-3/2}\right] = 0$$
$$2x^2(-3/2)(a^2 + x^2)^{-5/2} + (a^2 + x^2)^{-3/2} = 0$$
$$x^2(-3)(a^2 + x^2)^{-5/2} = -(a^2 + x^2)^{-3/2}$$
$$3x^2(a^2 + x^2)^{-2/2} = 1$$
$$3x^2 = (a^2 + x^2)$$
$$3x^2 - x^2 = a^2$$
$$2x^2 = a^2$$
$$x = \frac{\pm a}{\sqrt{2}}$$

Thus, the maximum value of the electric field on the x-axis occurs at $x = \pm a/\sqrt{2}$, or approximately at 7/10 of the radius of the ring in front of or behind the ring.

3.9 The Electric Field on Axis for a Disk of Charge

Let us find the electric field on axis at the point P in figure 3.14(a) for a uniform disk of charge. Since a disk can be generated by adding up many rings of different radii, *the electric field of a disk of charge can be generated by adding up (integrating) the electric field of many rings of charge.* Thus the electric field of a disk of charge will be given by

(a)   (b)
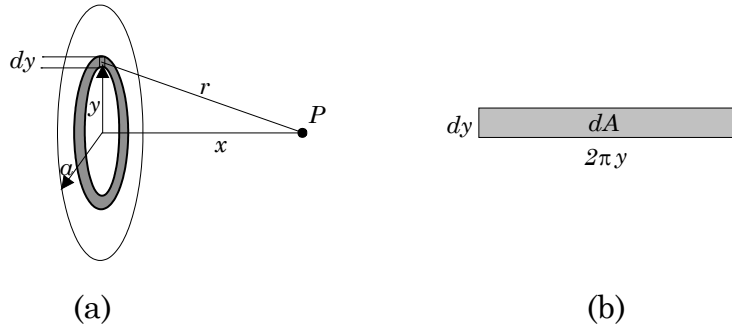
Figure 3.14 The electric field of a disk of charge.

$$\boldsymbol{E}_{\text{disk}} = \int d\boldsymbol{E}_{\text{ring}} \tag{3.49}$$

We found in the last section that the electric field on axis at the distance x from the center of a ring of charge of radius "a" was given by

$$\mathbf{E} = \frac{kqx}{(a^2+x^2)^{3/2}}\,\mathbf{i} \tag{3.48}$$

In the present problem the ring will be of radius y and we will add up all the rings from a radius of 0 to the radius "a", the radius of the disk. Hence, equation 3.48 will be written as

$$\mathbf{E}_{\text{ring}} = \frac{kqx}{(y^2+x^2)^{3/2}}\,\mathbf{i} \tag{3.50}$$

We now consider the charge on this ring to be a small element dq of the total charge that will be found on the disk. This element of charge dq will then produce an element of electric field $d\mathbf{E}$, that lies along the axis of the disk in the $\mathbf{i}$ direction. That is,

$$d\mathbf{E}_{\text{ring}} = \frac{kx}{(y^2+x^2)^{3/2}}dq\,\mathbf{i} \tag{3.51}$$

Replacing equation 3.51 back into equation 3.49 for the electric field of the disk we get

$$\boldsymbol{E}_{\text{disk}} = \int d\boldsymbol{E}_{\text{ring}} = \int \frac{kx}{(y^2 + x^2)^{3/2}} dq \, \mathbf{i}$$

(3.52)

When dealing with a rod of charge or a ring of charge which has the charge distributed along a line, we introduced the concept of the *linear charge density λ as the charge per unit length.* When dealing with a disk, the electric charge is distributed across a surface. Therefore, *we now define a surface charge density σ as the charge per unit area* and it is given by

$$\sigma = \frac{q}{A}$$

(3.53)

In terms of the surface charge density, the charge on the disk is given by

$$q = \sigma A$$

(3.54)

Its differential dq, is the amount of charge on the ring, i.e.,

$$dq = \sigma dA$$

(3.55)

where dA is the area of the ring. To determine the area of the ring, let us take the ring in figure 3.14a and unfold it as shown in figure 3.14b. The length of the ring is the circumference of the inner circle of the ring, $2\pi y$, and its width is the differential thickness of the ring, dy. The area of the ring dA is then given by the product of its length and width as

$$dA = (2\pi y) \, dy$$

(3.56)

Replacing equation 3.56 back into 3.55 gives for the element of charge of the ring

$$dq = \sigma(2\pi y) \, dy$$

(3.57)

Replacing equation 3.57 back into equation 3.52 we get for the electric field of the disk

$$\boldsymbol{E}_{\text{disk}} = \int \frac{kx}{(y^2 + x^2)^{3/2}} \sigma 2\pi y dy \, \mathbf{i}$$

(3.58)

Taking the constants outside of the integral we get

$$\boldsymbol{E}_{\text{disk}} = kx2\pi\sigma \int \frac{ydy}{(y^2 + x^2)^{3/2}} \, \mathbf{i}$$
$$\boldsymbol{E}_{\text{disk}} = \left(\frac{1}{4\pi\varepsilon_o}\right) x2\pi\sigma \int \frac{ydy}{(y^2 + x^2)^{3/2}} \, \mathbf{i}$$

$$\mathbf{E}_{\text{disk}} = \left(\frac{\sigma x}{2\varepsilon_o}\right) \int_0^a \frac{y\,dy}{(y^2 + x^2)^{3/2}}\, \mathbf{i}$$

(3.60)

Note that we have introduced the limits of integration 0 to a, that is, we add up all the rings from a radius of 0 to the radius "a", the radius of the disk. To determine the electric field it is necessary to solve the integral

$$I = \int \frac{y\,dy}{(y^2 + x^2)^{3/2}}$$

Let us make the substitution

$$u = (y^2 + x^2)$$

(3.61)

Hence, its differential is

$$du = 2y\,dy$$

Also

$$(y^2 + x^2)^{3/2} = u^{3/2}$$

and

$$y\,dy = du/2$$

Replacing these substitutions into our integral we obtain

$$I = \int \frac{y\,dy}{(y^2 + x^2)^{3/2}} = \int \frac{\frac{du}{2}}{u^{3/2}}$$
$$I = \int \frac{1}{2} u^{-3/2}\,du$$
$$I = \frac{1}{2}\frac{u^{\left(-\frac{3}{2}\right)+1}}{(-3/2 + 1)} = -u^{-1/2} = \frac{-1}{u^{1/2}}$$

Hence

$$I = \int_0^a \frac{y\,dy}{(y^2 + x^2)^{3/2}} = -\frac{1}{(y^2 + x^2)^{1/2}}\Big|_0^a$$
$$I = \int_0^a \frac{y\,dy}{(y^2 + x^2)^{3/2}} = -\frac{1}{(a^2 + x^2)^{1/2}} - \frac{-1}{(0 + x^2)^{1/2}}$$
$$I = \int_0^a \frac{y\,dy}{(y^2 + x^2)^{3/2}} = \frac{1}{x} - \frac{1}{(a^2 + x^2)^{1/2}}$$

(3.62)

Replacing equation 3.62 back into equation 3.60 we obtain

$$\mathbf{E}_{\text{disk}} = \left(\frac{\sigma x}{2\varepsilon_o}\right) \int_0^a \frac{y\,dy}{(y^2 + x^2)^{3/2}}\,\mathbf{i} = \left(\frac{\sigma x}{2\varepsilon_o}\right)\left[\frac{1}{x} - \frac{1}{(a^2 + x^2)^{1/2}}\right]\mathbf{i}$$
$$\mathbf{E}_{\text{disk}} = \left(\frac{\sigma}{2\varepsilon_o}\right)\left[1 - \frac{x}{(a^2 + x^2)^{1/2}}\right]\mathbf{i}$$

(3.63)

Equation 3.63 gives the electric field at the position x on the axis of a disk of charge of radius "a", carrying a uniform surface charge density σ. The direction of the

electric field of the disk of charge is in the **i**-direction just as the electric field of the ring of charge.

Example 3.10

The electric field of a disk of charge. (a) Find the electric field at the point $x = 15.0$ cm in front of a disk of 10.0 cm radius carrying a uniform surface charge density of 200 µC/m². (b) Find the force acting on an electron placed at this point.

Solution

(a) The electric field of the disk of charge is found from equation 3.63 as

$$\mathbf{E}_{disk} = \left(\frac{\sigma}{2\varepsilon_o}\right)\left[1 - \frac{x}{(a^2 + x^2)^{1/2}}\right]\mathbf{i} \tag{3.63}$$

$$\mathbf{E}_{disk} = \left(\frac{200 \times 10^{-6} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}\right)\left[1 - \frac{0.150 \text{ m}}{((0.100 \text{ m})^2 + (0.150)^2)^{1/2}}\right]\mathbf{i}$$

$$\mathbf{E}_{disk} = (1.13 \times 10^7 \text{ N/C})[1 - 0.832]\,\mathbf{i}$$

$$\mathbf{E}_{disk} = (1.90 \times 10^6 \text{ N/C})\mathbf{i}$$

(b) The force on an electron placed at this position is found as

$$\mathbf{F} = q\mathbf{E}$$

$$\mathbf{F} = (-1.60 \times 10^{-19} \text{ C})(1.90 \times 10^6 \text{ N/C})\mathbf{i}$$

$$\mathbf{F} = -(3.04 \times 10^{-13} \text{ N})\mathbf{i}$$

To go to this Interactive Example click on this sentence.

3.10 The Electric Field due to a Finite Line of Charge

Let us determine the electric field at a distance y from the line of charge shown in figure 3.15. The charge is distributed uniformly along the line with a linear charge
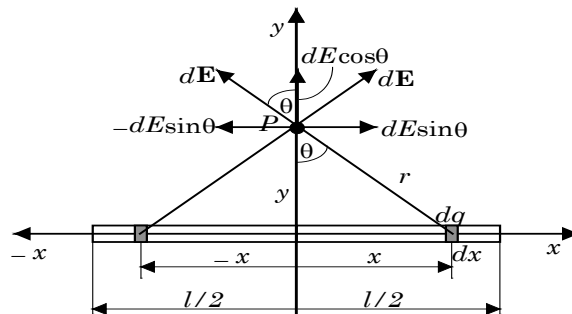

Figure 3.15 The electric field of a finite line of charge.

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density λ. Consider an element of charge dq located at the position x on the right hand side of the line as shown. It acts like a point charge creating an element of electric field $d\mathbf{E}$ at the point P. The total electric field at the point P is equal to the sum or integral of all the $d\mathbf{E}$'s associated with all the dq's constituting the line of charge. That is,

$$\mathbf{E} = \int d\mathbf{E} \qquad\qquad (3.64)$$

But as can be seen in figure 3.15, d$\mathbf{E}$ has the two components $dE \cos\theta$ and $-dE \sin\theta$. Notice that the element dq at the position $-x$ on the left hand side of the line will have the components $dE \cos\theta$ and $+dE \sin\theta$. Because of the symmetry, all the $-dE$ $\sin\theta$ terms from the right hand side of the line will be canceled out by all the $+dE$ $\sin\theta$ terms from the left hand side of the line during the sum. Hence only the dE $\cos\theta$ terms, which are all in the $+$ $\mathbf{j}$ direction, will contribute to the electric field at point P and equation 3.64 can be rewritten as

$$\mathbf{E} = \int dE \cos\theta \, \mathbf{j} \qquad\qquad (3.64)$$

Since each dq acts like a point charge, the magnitude of the element of electric field is given by

$$dE = k\frac{dq}{r^2} \qquad\qquad (3.65)$$

Replacing equation 3.65 back into equation 3.64 we get

$$\boldsymbol{E} = \int k\frac{dq}{r^2} \cos\theta \, \mathbf{j} \qquad\qquad (3.66)$$

Because the charge is distributed uniformly along the line, the linear charge density λ is given by

$$\lambda = q/x \qquad\qquad (3.67)$$

and the total charge on the line by

$$q = \lambda x \qquad\qquad (3.68)$$

The element of charge dq contained within the element of line dx is given by the differential as

$$dq = \lambda \, dx \qquad\qquad (3.69)$$

Replacing equation 3.69 back into equation 3.66 gives

$$\boldsymbol{E} = \int k\frac{\lambda dx}{r^2} \cos\theta \, \mathbf{j} \qquad\qquad (3.70)$$

Before any integration can be started it is necessary to notice that x, r, and θ, are not independent but, as can be seen in the diagram, are related to each other as

.

$$r^2 = x^2 + y^2 \tag{3.71}$$

and

$$\cos\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}} \tag{3.72}$$

Replacing equations 3.71 and 3.72 back into 3.70 we obtain

$$\boldsymbol{E} = \int k\frac{\lambda dx}{r^2}\cos\theta\, \mathbf{j} = \int k\frac{\lambda dx}{x^2+y^2}\frac{y}{\sqrt{x^2+y^2}}\, \mathbf{j}$$

$$\boldsymbol{E} = \int k\frac{\lambda y\, dx}{(x^2+y^2)^{3/2}}\, \mathbf{j}$$

Since the center of the line of charge of total length l is situated at the origin, the limits on the integration will go from $-l/2$ to $+l/2$. Hence,

$$\boldsymbol{E} = \int_{-l/2}^{l/2} k\frac{\lambda y\, dx}{(x^2+y^2)^{3/2}}\, \mathbf{j}$$

Because k, λ, and y are constants in this problem they can be taken outside of the integral and we obtain for the electric field

$$\boldsymbol{E} = k\lambda y \int_{-l/2}^{l/2}\frac{dx}{(x^2+y^2)^{3/2}}\, \mathbf{j} \tag{3.73}$$

We must now evaluate the integral

$$I = \int_{-l/2}^{l/2}\frac{dx}{(x^2+y^2)^{3/2}} \tag{3.74}$$

Up to this point in our study we have been evaluating the integrals directly. However it is not necessary to continually reinvent the wheel. Hence, from this point on we will take advantage of a table of integrals to evaluate most of the integrals in this text. Hence using a standard table of integrals as found in the appendix, equation 3.74 becomes

$$I = \int_{-l/2}^{l/2}\frac{dx}{(x^2+y^2)^{3/2}} = \frac{x}{y^2(x^2+y^2)^{1/2}}\Big|_{-l/2}^{l/2} \tag{3.75}$$

$$I = \frac{l/2}{y^2((l/2)^2+y^2)^{1/2}} - \frac{(-l/2)}{y^2((-l/2)^2+y^2)^{1/2}}$$

$$I = \frac{l}{y^2\sqrt{(l/2)^2+y^2}} \tag{3.76}$$

.

Replacing the value of the integral, equation 3.76 back into equation 3.73 for the electric field we obtain

$$E = k\lambda y \int_{-l/2}^{l/2} \frac{dx}{(x^2 + y^2)^{3/2}} \, \mathbf{j}$$

(3.73)

$$E = k\lambda y \frac{l}{y^2 \sqrt{(l/2)^2 + y^2}} \, \mathbf{j}$$

$$E = \frac{k\lambda l}{y\sqrt{(l/2)^2 + y^2}} \, \mathbf{j}$$

(3.77)

substituting for the linear charge density λ from equation 3.67 (*λ = q/l*) *we obtain the electric field at a distance y from a line of charge of length l as*

$$E = \frac{kq}{y\sqrt{(l/2)^2 + y^2}} \, \mathbf{j}$$

(3.78)

Example 3.11

The equation for the electric field of an infinite line of charge. Find the electric field at a distance *y* from an *infinite* line of charge.

Solution

The electric field for an infinite line of charge can be obtained from equation 3.77 for a finite line of charge by letting the length *l* approach infinity. First let us divide the numerator and denominator of 3.77 by *l*. That is,

$$E = \frac{k\lambda l}{y\sqrt{(l/2)^2 + y^2}} \, \mathbf{j}$$

(3.77)

$$E = \frac{k\lambda l/l}{y\sqrt{(l/2l)^2 + (y/l)^2}} \, \mathbf{j}$$

$$E = \frac{k\lambda}{y\sqrt{(1/2)^2 + (y/l)^2}} \, \mathbf{j}$$

(3.78)

Taking the limit as the length *l* approaches infinity we obtain the electric field of an infinite line of charge as

$$E = \lim_{l\to\infty} \frac{k\lambda}{y\sqrt{(1/2)^2 + (y/l)^2}} \, \mathbf{j}$$

But the term (*y/l*) approaches zero as *l* approaches infinity. Hence,

$$E = \frac{2k\lambda}{y} \, \mathbf{j}$$

(3.79)

.

Equation 3.79 gives the electric field at a distance y from an infinite line of charge carrying a uniform linear charge density λ.

Example 3.12

The electric field of a finite line of charge. Find the electric field at a distance of 15.0 cm from a line of charge 35.0 cm long carrying a linear charge density of 25.0 µC/m.

Solution

The electric field is found from equation 3.77 as

$$E = \frac{k\lambda l}{y\sqrt{(l/2)^2 + y^2}}\, \mathbf{j}$$

$$E = \frac{(9.00\times 10^9 \text{ N m}^2/\text{C}^2)(25.0\times 10^{-6} \text{ C/m})(0.350 \text{ m})}{(0.150 \text{ m})\sqrt{((0.350 \text{ m})/2)^2 + (0.150 \text{ m})^2}}\, \mathbf{j}$$

$$\mathbf{E} = (2.28 \times 10^6 \text{ N/C})\, \mathbf{j}$$

(3.77)

To go to this Interactive Example click on this sentence.

Example 3.13

The electric field of an infinite line of charge. Find the electric field at a distance of 15.0 cm from an infinite line of charge carrying a linear charge density of 25.0 µC/m.

Solution

The electric field is found from equation 3.79 as

$$E = \frac{2k\lambda}{y}\, \mathbf{j}$$

$$E = \frac{2(9.00\times 10^9 \text{ N m}^2/\text{C}^2)(25.0\times 10^{-6} \text{ C/m})}{(0.150 \text{ m})}\, \mathbf{j}$$

$$\mathbf{E} = (3.00 \times 10^6 \text{ N/C})\, \mathbf{j}$$

(3.79)

To go to this Interactive Example click on this sentence.

Notice that the value of the electric field at the same distance from a finite line of charge carrying the same linear charge density as the infinite line of charge is

different than the electric field for an infinite line of charge. There are times when we approximate the linear charge as being infinite for convenience. But remember it is only an approximation. If you are very close to the line of charge, the approximation is usually good. If you are far from the line of charge, the approximation is not so good.

3.11 The Electric Field due to an Infinite Plane Sheet of Charge

Let us determine the electric field a distance z from an infinite sheet of charge carrying a uniform surface charge density σ. *Since a sheet can be generated by adding a large number of lines, the electric field of a sheet of charge can be found by adding, or integrating, the electric field of a large number of lines of charge.* Figure 3.16 shows a portion of an infinite sheet of charge. A portion of an infinite line of charge is located in the *x-y* plane at the position *y*, as shown. The line of
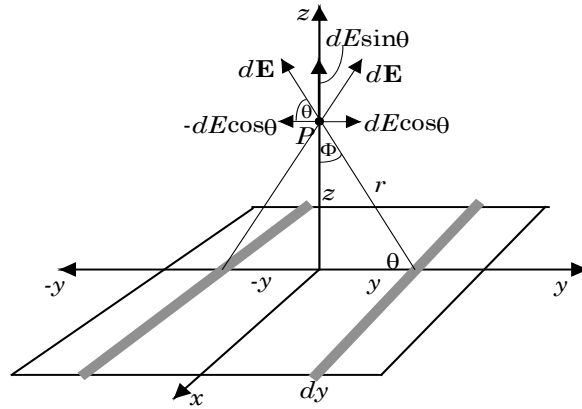
Figure 3.16 The electric field of an infinite sheet of charge.

charge is considered as an element of the total charge on the sheet and it contributes an element of electric field $d\mathbf{E}$ at the point P in the diagram. The total electric field at P will be the sum or integral of all the $d\mathbf{E}$'s from all the lines of charge. That is

$$\boldsymbol{E}_{sheet} = \int d\boldsymbol{E}_{line} \qquad (3.80)$$

As we saw in the last section the electric field of an infinite line of charge was given by equation 3.79 as

$$E = \frac{2k\lambda}{y} \, \mathbf{j} \qquad (3.79)$$

y was the distance from the line of charge and in this case corresponds to the distance r in figure 3.16. The unit vector $\mathbf{j}$ will be replaced by a unit vector $\mathbf{r}_0$ that points radially away from the line of charge. Hence in terms of the present problem the electric field of an infinite line of charge can be written as

.

$$E_{line} = \frac{2k\lambda}{r} \, \mathbf{r}_o \tag{3.81}$$

The linear charge density $\lambda = q/l$, the charge per unit length. Hence,

$$E_{line} = \frac{2kq}{rl} \, \mathbf{r}_o \tag{3.82}$$

and the infinitesimal amount of electric field caused by the element of the line charge is

$$d\mathbf{E}_{line} = \frac{2kdq}{rl} \, \mathbf{r}_o \tag{3.83}$$

We thus think of the line as containing an element of charge dq which is a very small portion of the entire charge q distributed over the sheet. The entire charge q can be written in terms of the surface charge density σ as

$$q = \sigma A$$

and its differential as

$$dq = \sigma dA \tag{3.84}$$

The area of the line can be written as

$$dA = l \, dy \tag{3.85}$$

Combining equations 3.84 and 3.85 gives for the element of charge dq

$$dq = \sigma l \, dy \tag{3.86}$$

Replacing equation 3.86 into 3.83 gives

$$d\mathbf{E}_{line} = \frac{2k\sigma l \, dy}{rl} \, \mathbf{r}_o$$

Thus the line of charge produces an element of electric field at the point P given by

$$d\mathbf{E}_{line} = \frac{2k\sigma \, dy}{r} \, \mathbf{r}_o \tag{3.87}$$

As can be seen in the diagram, the element of the electric field of the line on the right hand side of the sheet has the two components, $dE \sin\theta$ and $-dE \cos\theta$, while the element of the electric field of the line on the left hand side of the sheet has the two components, $dE \sin\theta$ and $+dE \cos\theta$. Hence in summing up all the dE's the components $+dE \cos\theta$ and $-dE \cos\theta$ will add to zero. Therefore the total electric field at P will come only from the $dE \sin\theta$ components and, as can be seen in figure 3.16, will point in the **k** direction. Thus

$$\mathbf{E}_{sheet} = \int d\mathbf{E}_{line} = \int dE_{line} \sin\theta \, \mathbf{k} \tag{3.88}$$

$$E_{sheet} = \int \frac{2k\sigma \, dy}{r} \, \sin\theta \, \mathbf{k} \qquad (3.89)$$

But, as can be seen in the diagram, r, y, and θ are not independent and before we can integrate equation 3.89 we must express them in terms of their y-dependence as

$$r = \sqrt{y^2 + z^2}$$
$$\sin\theta = \frac{z}{r} = \frac{z}{\sqrt{y^2 + z^2}}$$

Therefore,

$$E_{sheet} = \int \frac{2k\sigma \, dy}{r} \, \sin\theta \, \mathbf{k} = \int \frac{2k\sigma \, dy}{\sqrt{y^2 + z^2}} \frac{z}{\sqrt{y^2 + z^2}} \, \mathbf{k} \qquad (3.90)$$

$$E_{sheet} = \int \frac{2k\sigma z \, dy}{y^2 + z^2} \, \mathbf{k} \qquad (3.91)$$

Because k, z and σ are constants they can be taken outside of the integral. Also, since we are determining the electric field of an infinite sheet of charge, y will vary from $-\infty$ to $+\infty$ and hence the limits of integration will go from $-\infty$ to $+\infty$, thus

$$E_{sheet} = 2k\sigma z \int_{-\infty}^{\infty} \frac{dy}{y^2 + z^2} \, \mathbf{k} \qquad (3.92)$$

We must now evaluate the integral

$$I = \int_{-\infty}^{\infty} \frac{dy}{y^2 + z^2} \qquad (3.93)$$

From the table of integrals in the appendix we obtain

$$I = \int_{-\infty}^{\infty} \frac{dy}{y^2 + z^2} = \frac{1}{z} \tan^{-1} \frac{y}{z} \Big|_{-\infty}^{\infty} \qquad (3.94)$$

Now the inverse tangent of y/z is the angle whose tangent is y/z. Hence, $\tan^{-1}(y/z)$ is the complementary angle of θ which we will designate as the angle ϕ and is shown in figure 3.16. When y approaches infinity the angle ϕ approaches $\pi/2$ and when y approaches minus infinity the angle ϕ approaches $-\pi/2$. Therefore

$$I = \frac{1}{z} \tan^{-1} \frac{y}{z} \Big|_{-\infty}^{\infty} = \frac{1}{z} \left[\tan^{-1} \frac{\infty}{z} - \tan^{-1} \frac{-\infty}{z} \right] \qquad (3.95)$$

$$I = \frac{1}{z} \left[\frac{\pi}{2} - \frac{(-\pi)}{2} \right] = \frac{\pi}{z} \qquad (3.96)$$

Replacing equation 3.96 back into equation 3.92 we get

$$E_{sheet} = 2k\sigma z \frac{\pi}{z} \, \mathbf{k} = \frac{2\sigma\pi}{4\pi\varepsilon_o} \, \mathbf{k}$$

Finally, *the electric field of an infinite sheet of charge carrying a surface charge density σ is given by*

$$E_{sheet} = \frac{\sigma}{2\varepsilon_o}\, \mathbf{k}$$

(3.97)

The direction of the electric field is perpendicular to the sheet of charge.

Example 3.14

The electric field of an infinite sheet of charge. (a) Find the electric field of an infinite sheet of charge carrying a surface charge density of 200 µC/m². (b) Find the force on an electron placed 10.0 cm in front of the sheet of charge.

Solution

(a) The electric field of an infinite sheet of charge is given by equation 3.97 as

$$E_{sheet} = \frac{\sigma}{2\varepsilon_o}\, \mathbf{k}$$

(3.97)

$$E_{sheet} = \frac{200 \times 10^{-6}\text{C/m}^2}{2(8.85 \times 10^{-12}\text{C}^2/\text{N m}^2)}\, \mathbf{k}$$

$$\mathbf{E}_{sheet} = (1.13 \times 10^7 \text{ N/C})\, \mathbf{k}$$

(b) The force on an electron placed 10.0 cm in front of the sheet of charge is found as

$$\mathbf{F} = q\mathbf{E}$$
$$\mathbf{F} = (-1.60 \times 10^{-9}\text{ C})(1.13 \times 10^7 \text{ N/C})\, \mathbf{k}$$
$$\mathbf{F} = -(1.81 \times 10^{-12}\text{ N})\, \mathbf{k}$$

To go to this Interactive Example click on this sentence.

Example 3.15

The electric field between two parallel sheets of charge. An infinite sheet of positive surface charge density $+\sigma$ is placed parallel to an infinite sheet of negative surface charge density $-\sigma$. Find the electric field between the two parallel sheets of charge.

Solution

The electric field between the two sheets is found by the superposition principle as

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$$

But since the electric field emanates from a positive charge distribution and into a negative charge distribution, the two electric fields between the sheets are both in

.
the same direction. Hence they are added directly as

$$E = \frac{\sigma}{2\varepsilon_o}\, \mathbf{k} \; + \; \frac{\sigma}{2\varepsilon_o}\, \mathbf{k}$$

Hence *the electric field between two equal but opposite parallel infinite charge distributions is given by*

$$E = \frac{\sigma}{\varepsilon_o}\, \mathbf{k}$$

(3.98)

Summary of Important Concepts

Electric field
It is an intrinsic property of nature that an electric field exists in the space around an electric charge. The electric field is considered to be a force field that exerts a force on charges placed in the field (p.).

Electric field intensity
The electric field is measured in terms of the electric field intensity. The magnitude of the electric field intensity is defined as the ratio of the force acting on a small test charge to the magnitude of the small test charge. The direction of the electric field is in the direction of the force on the positive test charge (p.).

Electric field intensity of a point charge
The electric field of a point charge is directly proportional to the charge that creates it, and inversely proportional to the square of the distance from the point charge to the position where the field is being evaluated (p.).

Superposition of electric fields
When more than one charge contributes to the electric field, the resultant electric field is the vector sum of the electric fields produced by each of the various charges (p.).

Summary of Important Equations

Definition of the electric field intensity $\quad\quad\quad\quad \mathbf{E} = \dfrac{\mathbf{F}}{q_0}$ (3.1)

Electric field intensity of a point charge $\quad\quad\quad\quad E = \dfrac{kq}{r^2}$ (3.2)

$$\mathbf{E} = k\frac{q}{r^2}\mathbf{r}_o$$ (3.3)

Force on a charge q in an electric field $\quad\quad\quad \mathbf{F} = q\mathbf{E}$ (3.4)

Superposition principle

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots \qquad (3.5)$$
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots \qquad (3.10)$$
$$\mathbf{E} = \sum_{n-1}^{N} \mathbf{E}_i \qquad (3.26)$$

Electric dipole moment

$$p = 2aq \qquad (3.15)$$
$$\boldsymbol{p} = 2aq\boldsymbol{r}_o \qquad (3.18)$$

Electric field of a dipole along its perpendicular bisector

$$\mathbf{E} = \frac{kp}{(a^2 + x^2)^{3/2}}\,\mathbf{j} \qquad (3.16)$$
$$\mathbf{E} = \frac{kp}{x^3}\,\mathbf{j} \qquad (3.17)$$

Torque on a dipole in an external electric field

$$\tau = \boldsymbol{p} \times \boldsymbol{E} \qquad (3.24)$$
$$\tau = pE\,\sin\theta \qquad (3.25)$$

An element of an electric field
$$\mathbf{E} = k\frac{q}{r^2}\mathbf{r}_o \qquad (3.27)$$

Electric field of a continuous distribution of charge
$$\mathbf{E} = \int d\mathbf{E} = \int k\frac{dq}{r^2}\mathbf{r}_o \qquad (3.28)$$

Electric field caused by a rod of charge
$$\mathbf{E} = -\frac{kq}{(x_o)(x_o + l)}\,\mathbf{i} \qquad (3.35)$$

Electric field due to a ring of charge
$$\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}}\,\mathbf{i} \qquad (3.48)$$

Electric field of a disk of charge
$$\boldsymbol{E}_{disk} = \int d\boldsymbol{E}_{ring} \qquad (3.49)$$
$$\mathbf{E}_{disk} = \left(\frac{\sigma}{2\varepsilon_o}\right)\left[1 - \frac{x}{(a^2 + x^2)^{1/2}}\right]\mathbf{i} \qquad (3.63)$$

The surface charge density
$$\sigma = \frac{q}{A} \qquad (3.53)$$

The differential of the charge
$$dq = \sigma dA \qquad (3.55)$$

Questions for Chapter 3

1. Describe as many different types of fields as you can.

*2. Because you cannot really see an electric field, is anything gained by using the concept of a field rather than an "action at a distance" concept?

*3. Is there any experimental evidence that can substantiate the existence of an electric field rather than the concept of an "action at a distance"?

4. Is the force of gravity also an "action at a distance" ?Should a gravitational field be introduced to explain gravity? What is the equivalent gravitational "charge"?

*5. If there are positive and negative electrical charges, could there be positive and negative masses? If there were, what would their characteristics be?

*6. Michael Faraday introduced the concept of lines of force to explain electrical interactions. What is a line of force and how is it like an electric field line? Is there any difference?

Problems for Chapter 3

Section 3.2 The Electric Field of a Point Charge.

1. Find the electric field 2.00 m from a point charge of 3.00 pC.

2. A point charge, q = 3.75 μC, is placed in an electric field of 250 N/C. Find the force on the charge.

Section 3.3 Superposition of Electric Fields For Multiple Discrete Charges.

3. Find the electric field at point A in the diagram if (a) q_1 = 2.00 μC, and q_2 = 3.00 μC. and (b) q_1 = 2.00 μC and q_2 = − 3.00 μC.
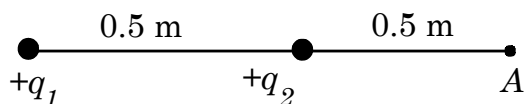
Diagram for problem 3.

4. A point charge of +2.00 μC is 30.0 cm from a charge of +3.00 μC. Where is the electric field between the charges equal to zero?

5. Find the electric field at the apex of the triangle shown in the diagram if q_1 = 2.00 μC and q_2 = 3.00 μC. What force would act on a 6.00 μC charge placed at this point?
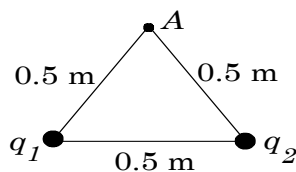
Diagram for Problem 5 and 6.

6. Find the electric field at point A in the diagram if q_1 = 2.00 μC and q_2 = −3.00 μC.

.

7. Charges of 2.00 μC, 4.00 μC, −6.00 μC and 8.00 μC are placed at the corner of a square of 50.0 cm length. Find the electric field at the center of the square.

Section 3.4 Electric Field of an Electric Dipole.

8. Find the electric dipole moment of a charge of 4.50 μC separated by 5.00 cm from a charge of − 4.50 μC.

9. If a charge of 2.00 μC is separated by 4.00 cm from a charge of −2.00 μC find the electric field at a distance of 5.00 m, perpendicular to the axis of the dipole.

Additional Problems.

10. Find the electric field at point A in the diagram if charge q_1 = 2.63 μC and q_2 = − 2.63 μC, d = 10.0 cm, r_1 = 50.0 cm, θ_1 = 25.0⁰. Hint: first find r_2 by the law of cosines, then with r_2 known, use the law of cosines again to find the angle θ_2.
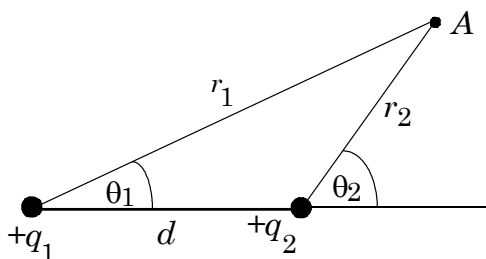
Diagram for problem 10.

Additional Problems

11. A point charge of 3.00 pC is located at the origin of a coordinate system. (a) What is the electric field at x = 50.0 cm? (b) What force would act on a 2.00 μC charge placed at x = 50.0 cm?

12. A point charge of 2.00 μC is 30.0 cm from a charge of 3.00 μC. Find the electric field half way between the charges.

13. Starting with equation 3-63 for the electric field on the x-axis for a uniform disk of charge, find the electric field on the x-axis for an infinite sheet of charge in the y-z plane. (Hint: An infinite sheet of charge can be generated from a finite disk of charge by letting the radius of the disk of charge approach infinity.)

14. An electron is placed between two charged parallel plates. What must the value of the electric field be in order that the electron be in equilibrium between the electric force and the gravitational force?

15. Find the equation for the electric field at the point P in figure 3-12 for a rod of charge that has a nonuniform linear charge density λ given by $\lambda = Ax^2$.

16. Find the equation for the electric field at the point P in figure 3-13 for a ring of charge that has a *nonuniform* linear charge density λ given by $\lambda = A \sin\phi$, where ϕ is the angle between the y-axis and the location of the element of charge dq, and A is a constant.
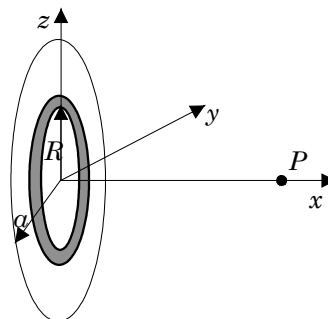
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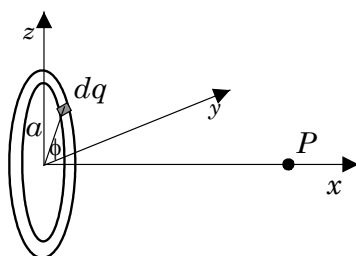
Diagram for problem 16.

Diagram for problem 17.

17. Starting with the equation for the electric field of a uniform ring of charge, find the electric field on the *x*-axis for a *nonuniform* disk of charge. The surface charge density on the rings vary linearly with the radius of the ring, i.e., $\sigma = Cy$ where y is the radius of each ring and C is a constant.

18. Find the equation for the electric field between two disks of charge. The first one carries the charge density $+ \sigma$ while the second carries the charge density $- \sigma$.

19. A wire of length l, lying along the *x*-axis, carries a charge of λ per unit length. Find the electric field at the arbitrary point P a distance R from the wire. Show that this reduces to the solution of the problem in section 3.10 when the point P is along the perpendicular bisector of the wire.
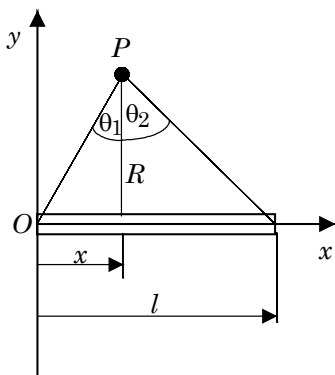
Diagram for problem 19.
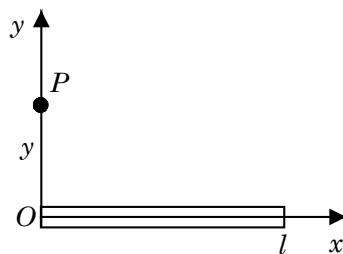
Diagram for problem 20.

20. A uniform rod of charge lies along the *x*-axis and carries a charge of λ per unit length. One end of the rod is located at the origin of the coordinate system, while the other end is at a distance l along the *x*-axis. Find the electric field at the point P a distance y along the *y*-axis.

21. One line of charge lies on the *x*-axis while a second line of charge lies on the *y*-axis as shown. Find the electric field at the point P in the diagram.
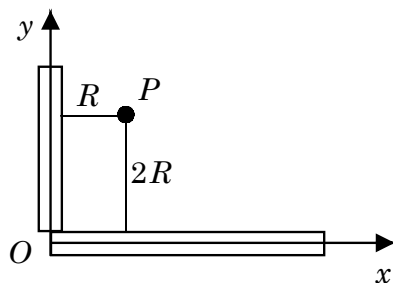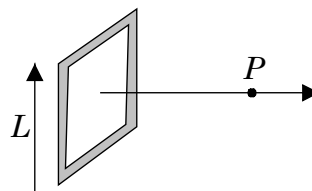
Diagram for problem 21.   Diagram for problem 22.

22. A wire carrying a charge density λ is bent into the form of a square of length L. Find the electric field at the point P perpendicular to the plane of the square, on a line passing through the center of the square.

23. Show that the rectangular components of the electric field of a dipole for an arbitrary point P in the x-y plane are given by

$$E_x = \frac{k3pxy}{(x^2 + y^2)^{5/2}}$$
$$E_y = \frac{kp(2y^2 - x^2)}{(x^2 + y^2)^{5/2}}$$
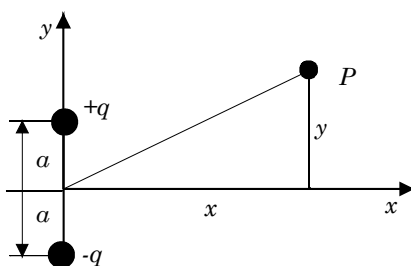
Diagram for problem 23.

24. The differential equation for the lines of force of an electric field in rectangular coordinates is given by

$$\frac{dx}{E_x} = \frac{dy}{E_y} = \frac{dz}{E_z}$$

Using the rectangular components of the electric field of an electric dipole, find the equation for the lines of force of an electric dipole.

25. The differential equation for the lines of force of an electric field in polar coordinates is given by

$$\frac{dr}{E_r} = \frac{rd\theta}{E_\theta}$$

Using the polar components of the electric field of an electric dipole, find the equation for the lines of force of an electric dipole in polar coordinates

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26. A thin rod carrying a charge q spread uniformly along its length is bent into a semicircle of radius r. Find the electric field at the center of the Semicircle.
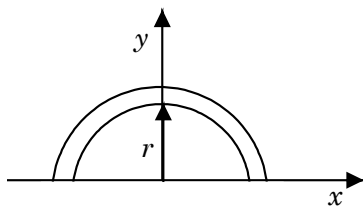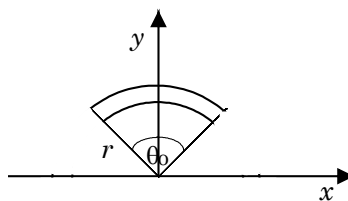

Diagram for problem 26.   Diagram for problem 27.

27. A thin rod carrying a charge q spread uniformly along its length is bent into an arc of a circle of radius r. The arc subtends an angle θ_o, as shown in the diagram. Find the electric field at the center of the circle.

28. A hemispherical insulating cup of inner radius R carries a total charge q over its surface. Find the electric field at the center of the cup.
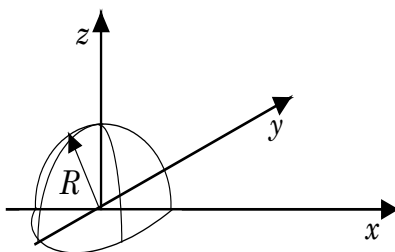
Diagram for problem 28.

29. Find the point on the x-axis where the electric field of a disk of charge takes on its maximum value.

30. A sphere of 10.0 cm radius has a charge of 8.00 μC distributed uniformly over its surface. Find the surface charge density σ.

31. A sphere of 10.0 cm radius has a charge of 8.00 μC distributed uniformly throughout its volume. Find the volume charge density ρ.

32. A sphere has a surface charge density σ given by $\sigma = \sigma_o r$. Find the total charge contained on the surface of the sphere.

33. A sphere has a volume charge density ρ given by $\rho = \rho_o/r^2$. Find the total charge contained within the sphere.

34. A 10.0 g pith ball, carrying a charge of 1.52 μC hangs from a 25.0 cm thread attached to a vertical wall of charge carrying a surface charge density $\sigma = 3.57$ μC/m². Find the angle that the thread makes with the vertical.

35. Find the electric field at the point P halfway between an infinite sheet of charge carrying a surface charge density $\sigma = 84.5$ μC/m² and a point charge carrying a charge $q = 5.25$ μC as shown in the diagram. The distance d between the sheet of charge and the point charge is 7.55 cm.
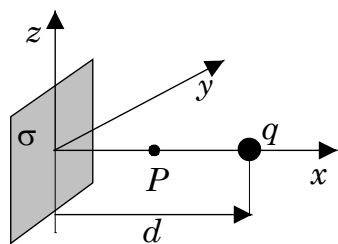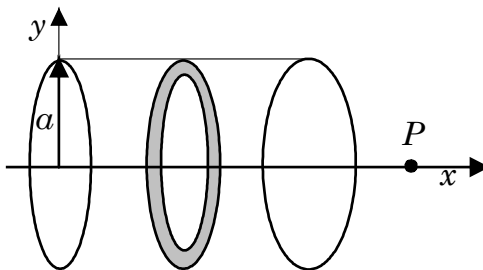
Diagram for problem 35.

Diagram for problem 36.

36. Find the electric field **E** on the *x*-axis at the point *P* of the cylindrical shell of radius a shown in the diagram. Hint: the cylindrical shell is made up of a sum of rings of charges.

37. Find the electric field **E** on the *x*-axis at the point *P* of the solid cylinder of radius a shown in the diagram. Hint: the solid cylinder is made up of a sum of disks of charges.
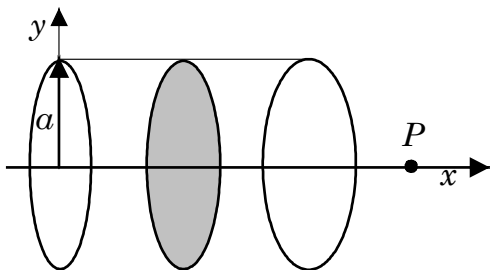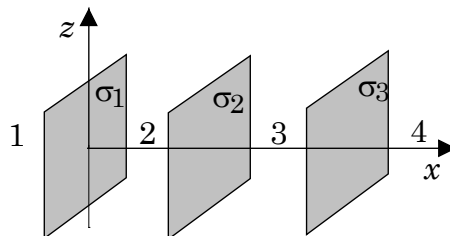

Diagram for problem 37.

Diagram for problem 38.

38. Three infinite sheets of charge are arranged as shown. Sheet 1 carries a surface charge density $\sigma_1 = 1$ µC/m², sheet 2 carries a surface charge density $\sigma_2 = 2$ µC/m², while sheet 3 carries a surface charge density $\sigma_3 = 3$ µC/m². Find the electric field **E** in regions 1, 2, 3, and 4.

39. Show that the solution for the electric field on axis of a ring of charge, equation 3-48, reduces to the electric field of a point charge if *x* is very much greater than a, the radius of the ring. Hint: use the binomial theorem.

40. Two charged rods, one carrying a positive charge density λ and one carrying a negative charge density $-\lambda$ are bent into the semicircle shown. Find the equation for the electric field **E** at the point *P*.
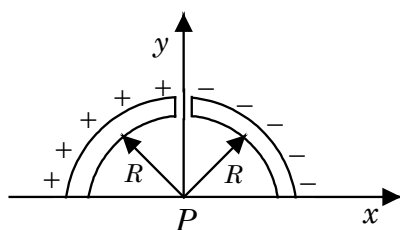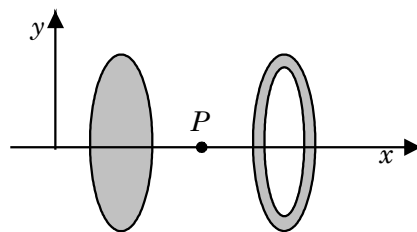

Diagram for problem 40.

Diagram for problem 41.

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41. Find the electric field at the point P midway between a disk of charge carrying a surface charge density $\sigma = 250$ $\mu C/m^2$ and a ring of charge carrying a total charge $q = 5.60$ μC. The radius of the disk is $a_{disk} = 0.150$ m, and the radius of the ring is $a_{ring} = 0.150$ m.

42. Find the electric field $\mathbf{E}$ at the origin of the coordinate system shown for the two rods of charge. The linear charge density $\lambda_1 = 20.0$ $\mu C/m$ while $\lambda_2 = 35.0$ $\mu C/m$. The distance $a = 0.150$ m while the distance $b = 0.250$ m.
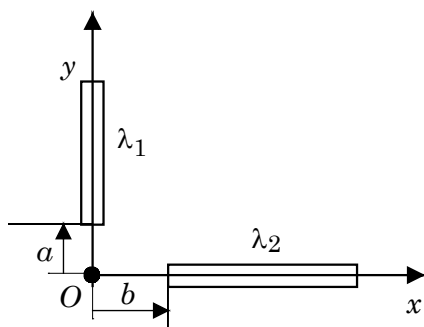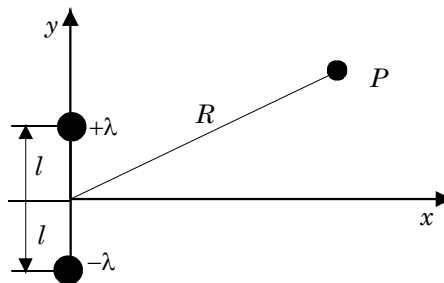

Diagram for problem 42.

Diagram for problem 43.

43. Two essentially infinite lines of charge in the y-z plane are carrying linear charge densities of $+ \lambda$ and $- \lambda$. Find the equation for the electric field at the arbitrary point P. What would the electric dipole moment be?

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