

Solutions to Problems : Chapter 29

Problems appeared on the end of chapter 29 of the **Textbook**

(Problem 2, 16, 18, 24, 38, 42, 46, 55, 56)

2. **Picture the Problem:** An astronaut travels at a high rate of speed while observing the periodic flashes of a neon sign.

Strategy: The two events to be considered in this case are two successive flashes of the neon sign. From the point of view of the astronaut, these events occur at different places, so the astronaut observes the dilated time interval Δt and the observer on Earth measures the proper time interval $\Delta t_0 = 4.5$ s. Use equation 29-2 to calculate Δt .

Solution: Find the time interval observed by the astronaut:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{4.5 \text{ s}}{\sqrt{1 - (0.84)^2}} = \boxed{8.3 \text{ s}}$$

Insight: Time intervals between events that occur at different places are always longer than the proper time intervals.

16. **Picture the Problem:** As a spaceship travels between two planets, it measures a proper time of 1.00 s. An observer on Earth, would measure the speed of the spaceship using the dilated time.

Strategy: The two events to be considered in this case are the departure from one planet and the arrival at the other planet. These two events occur at different places in Earth's frame of reference but at the same place (just outside the spacecraft) in the spacecraft's frame of reference, so an Earth observer measures the dilated time interval Δt and the astronaut measures the proper time interval Δt_0 . Use equation 29-2 to find the dilated time and divide the distance between planets measured in Earth's frame by the dilated time to find the speed.

Solution: 1. Calculate the dilated time:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

2. Calculate $v = d/\Delta t$ in Earth's frame of reference:

$$v = \frac{d}{\Delta t} = \frac{d}{\Delta t_0} \sqrt{1 - v^2/c^2}$$

3. Square both sides of the equation and rearrange:

$$d^2 (1 - v^2/c^2) = v^2 \Delta t_0^2$$

18. **Picture the Problem:** The clocks on a plane measure the proper time of 2.0 hours for the plane to travel a given distance at 212 m/s. Ground-based clocks will measure the dilated time for this trip. Since the speed of the plane is much smaller than the speed of light, this dilated time will differ from the proper time by only a small amount.

Strategy: The two events to be considered in this case are the departure from one location and the arrival at another. These two events occur at different places in Earth's frame of reference but at the same place (just outside the plane) in the plane's frame of reference, so an Earth observer measures the dilated time interval Δt and the pilot measures the proper time interval Δt_0 . Use the binomial expansion (in Appendix A) to express the dilated time from equation 29-2 for speeds much smaller than c . Subtract the proper time to calculate the difference in times:

Solution: 1. (a) According to the ground-based clocks, the flight will take slightly more than 2.00 hours because of time dilation.

2. (b) Write equation 29-2 using the binomial expansion:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \approx \Delta t_0 \left(1 + \frac{v^2}{2c^2} \right) = \Delta t_0 + \frac{v^2}{2c^2} \Delta t_0$$

3. Subtract the proper time:

$$\Delta t - \Delta t_0 \approx \frac{v^2 \Delta t_0}{2c^2} = \frac{(212 \text{ m/s})^2 (2.00 \text{ h})}{2(3.00 \times 10^8 \text{ m/s})^2} \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = \boxed{1.80 \text{ ns}}$$

Insight: This time shift was experimentally measured by J. C. Hafele and R. E. Keating in 1971 using synchronized atomic clocks and a military jet plane (see section 29-2 of the text).

24. **Picture the Problem:** The proper length of your car is $L_{0,c} = 5.0 \text{ m}$ and the proper length of your garage is $L_{0,g} = 4.0 \text{ m}$. If your car moves at a high rate of speed relative to the garage, an observer in the garage's frame of reference measures a contracted length of the car. From inside your car you observe the garage moving at a high rate of speed and its length is contracted.

Strategy: Solve equation 29-3 for the velocity of the car, setting the contracted length of the car equal to 4.0 m, as measured by an observer in the garage's frame of reference. Then insert the velocity into equation 29-3 to find the contracted length of the garage that you measure from the point of view of the car's frame of reference.

Solution: 1. Solve equation 29-3 for the velocity: $L = L_0 \sqrt{1 - v^2/c^2} \Rightarrow v = c \sqrt{1 - L^2/L_0^2}$

2. Insert the rest and contracted lengths of the car: $v = c \sqrt{1 - (4.0 \text{ m}/5.0 \text{ m})^2} = \boxed{0.60c}$

3. (b) Calculate the contracted length of the garage: $L = L_0 \sqrt{1 - v^2/c^2} = (4.0 \text{ m}) \sqrt{1 - (0.60)^2} = \boxed{3.2 \text{ m}}$

Insight: To an observer in the garage's frame of reference, it is possible for the car to fit entirely inside the garage (before crashing into the back at $0.60c$). From your vantage point in the car, the garage is 1.8 m shorter than the car, and the car does not fit. This discrepancy is explained by the non-simultaneity of events in different reference frames; from your point of view, the front of the car exits through the back of the garage before the rear of the car enters it.

38. **Picture the Problem:** One electron (1) travels with a velocity $v_{13} = 0.84c$ relative to the laboratory (3). The second electron (2) travels in the opposite direction at a velocity $v_{21} = -0.43c$ relative to the first electron.

Strategy: Use equation 29-4 to calculate v_{23} , the velocity of the second electron relative to the laboratory.

Solution: Solve equation 29-4 for the speed of the second electron relative to the lab:

$$v_{23} = \frac{v_{21} + v_{13}}{1 + v_{21}v_{13}/c^2} = \frac{(-0.43c) + 0.84c}{1 + (-0.43)(0.84)} = \boxed{0.64c}$$

Insight: In the laboratory the second electron is traveling in the same direction as the first electron, but at a slower speed.

42. **Picture the Problem:** The relativistic momentum of an asteroid is related to the mass and speed of the asteroid.

Strategy: Solve equation 29-5 for the speed of the asteroid.

Solution: 1.

Square both sides of equation 29-5 and multiply both sides by the denominator:

$$p^2 = \frac{m^2 v^2}{1 - v^2/c^2}$$

$$p^2 (1 - v^2/c^2) = m^2 v^2$$

2. Factor out the v^2 terms: $p^2 = (m^2 + p^2/c^2)v^2$

3. Solve for the velocity:
$$v = \frac{p}{\sqrt{m^2 + p^2/c^2}} = \frac{7.74 \times 10^{20} \text{ kg} \cdot \text{m/s}}{\sqrt{(8.2 \times 10^{11} \text{ kg})^2 + (7.74 \times 10^{20} \text{ kg} \cdot \text{m/s} / 3.00 \times 10^8 \text{ m/s})^2}}$$

$$= \boxed{2.86 \times 10^8 \text{ m/s}}$$

Insight: Verify for yourself that the speed of the asteroid is $0.95c$. If the momentum had been treated classically, the speed would have been wrongly calculated as $v = 9.4 \times 10^8 \text{ m/s}$, which is greater than the speed of light.

46. **Picture the Problem:** A probe approaches a stationary asteroid at a relative speed of $0.50c$ and smashes into it. After the collision, the asteroid/probe combination travels with a speed of $0.26c$ relative to the stationary frame of reference.

Strategy: The speeds and momenta are all measured relative to the stationary frame of reference defined by the asteroid before it is struck. Calculate the initial momentum of the system using equation 29-5, and then set that momentum equal to the final momentum of the asteroid and probe. Solve the expression for the mass of the asteroid.

Solution: 1. Calculate the initial momentum of the probe:

$$p_i = \frac{m_p v_p}{\sqrt{1 - v_p^2/c^2}} = \frac{8.2 \times 10^7 \text{ kg} (0.50c)}{\sqrt{1 - 0.50^2}} = 4.734 \times 10^7 \text{ kg} \cdot c$$

2. Set the final momentum equal to the initial momentum and solve for the mass of the asteroid:

$$\begin{aligned} p_f &= p_i = \frac{(m_a + m_p) v_f}{\sqrt{1 - v_f^2/c^2}} \\ m_a &= \frac{p_i}{v_f} \sqrt{1 - v_f^2/c^2} - m_p \\ &= \frac{4.734 \times 10^7 \text{ kg} \cdot c}{0.26c} \sqrt{1 - 0.26^2} - 8.2 \times 10^7 \text{ kg} = \boxed{9.4 \times 10^7 \text{ kg}} \end{aligned}$$

Insight: The final speed is slightly greater than half the initial speed, even though the mass of the asteroid is greater than the mass of the probe. This is because the mass of the probe is effectively larger than its rest mass when it is moving at a high speed.

55. **Picture the Problem:** The relativistic kinetic energy of a rocket is a function of the mass and speed of the rocket.

Strategy: Use equation 29-9 to write the kinetic energy of the rocket in terms of its mass and speed. Solve the resulting equation for the rocket speed.

Solution: 1. Use equation 29-9 to write the kinetic energy of the rocket.

$$K = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$

2. Solve for the rocket speed: $v = c \sqrt{1 - \left(\frac{1}{1 + K/m_0 c^2} \right)^2}$

$$= c \sqrt{1 - \left(\frac{1}{1 + 2.7 \times 10^{23} \text{ J} / \left[2.7 \times 10^6 \text{ kg} (3.00 \times 10^8 \text{ m/s})^2 \right]} \right)^2} = \boxed{0.88c}$$

Insight: Note that the kinetic energy of the rocket is proportional to the mass of the rocket. However, the relationship between the kinetic energy and velocity is much more complicated.

56. **Picture the Problem:** The total energy of an object is the sum of its rest energy and its kinetic energy.

Strategy: Equation 29-7 describes the total energy of an object in terms of its mass and its speed. Set the total energy equal to a number b times its rest energy using equation 29-7, and solve for the speed.

Solution: 1. Write the energy as b times the rest energy: $E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \stackrel{\text{set}}{=} b m_0 c^2$

2. Solve for the velocity: $v = c \sqrt{1 - \frac{1}{b^2}} = c \sqrt{1 - \frac{1}{(4.5)^2}} = \boxed{0.97c}$

Insight: At what speed is the total energy double the rest energy? Answer: $0.866c$. Note that the kinetic energy depends upon the frame of reference; for every object moving at constant speed, there is a frame of reference in which the object is at rest and has zero kinetic energy, even though its kinetic energy may be large in a different frame of reference.