# Chapter 8: Magnetism

## 8.1 The Force on a Charge in a Magnetic Field - The Definition of the Magnetic Field B

Besides the existence of electric fields in nature, there are also magnetic fields. Most students have seen and played with a simple bar magnet, observing that the magnet attracts nails, paper clips, etc. Some may have even placed the bar magnet under a piece of paper and sprinkled iron fillings on the paper, observing the characteristic magnetic field of a bar magnet, figure 8.1. One end of the magnet is

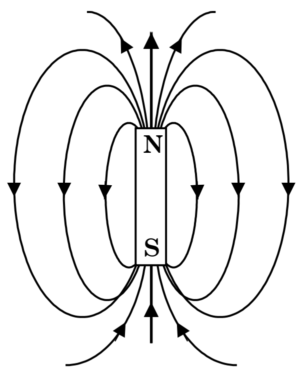


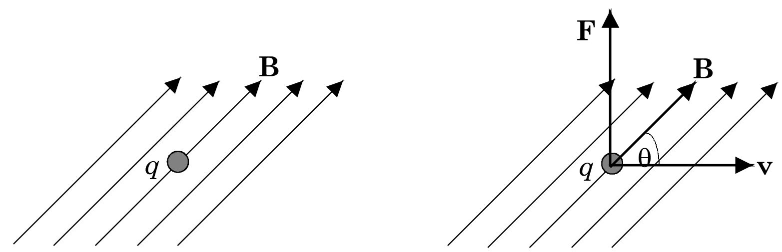

***Figure 8.1*** The magnetic field of a bar magnet.

called a north pole, while the other end is called a south pole. The magnetic field is defined to emerge from the north pole of the magnet and enter at the south pole. A compass needle, a tiny bar magnet, placed in a magnetic field aligns itself with the field. The designation of poles as north and south is arbitrary, just as electric charges are arbitrarily called positive and negative. Since the combination of a positive and negative electric charge was called an electric dipole, a bar magnet, consisting as it does of a north and a south magnetic pole, is sometimes called a magnetic dipole. The earth has a magnetic field, and what is usually called the north magnetic pole is slightly displaced from the north geographic pole of the earth. *The force between magnets is similar to the force between electric charges in that like magnetic poles repel, while unlike magnetic poles attract.* Thus, when the north pole of a compass needle points in a northerly direction on the surface of the earth, it is really being attracted toward a south pole. Hence, what is called the north magnetic pole of the earth is really a south pole. But because compass needles always point toward that pole, it is still called the north pole. The symbol **B**, will be used to designate the magnetic field. Later in this chapter it will be shown how these magnetic fields are generated, but for now let us accept the fact that magnetic fields do indeed exist.

As you may recall from chapter 3, the existence of an electric field and its strength were determined by the effect it produced on a small positive test charge, $q_o$, placed in the region where the field was assumed to exist. If the test charge experienced a force, it was said that the charge was in an electric field, and the electric field was defined to be

$$\mathbf{E} = \frac{\mathbf{F}}{q_o} \tag{3-2}$$

It is desirable to define the magnetic field in a similar way. A positive charge $q$ is placed at rest in a uniform magnetic field $\mathbf{B}$ as shown in figure 8.2(a). But to our surprise, nothing happens to the charge. No force is observed to act on the charge.




(a) No force on charge at rest (b) Force on moving charge

***Figure 8.2*** A charge in a magnetic field.

The experiment is repeated, but now the charge is fired into the magnetic field with a velocity **v**, figure 8.2(b). It is now observed that a force does indeed act on the charge. The force, however, acts at an angle of $90^0$ to the plane determined by the velocity vector **v**, and the magnetic field vector **B**. In fact, *the force acting on the moving charge is given by the cross product of* **v** *and* **B** *as*

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B} \tag{8.1}$$

The magnitude of the force is determined from the definition of the cross product, equation 1-44, and is given by

$$F = qvB\sin\theta \tag{8.2}$$

The angle θ is the angle between the velocity vector **v** and the magnetic field vector **B**. If the angle θ between **v** and **B** is $90^0$ then the magnitude of the force simplifies to

$$F = qv_{\perp}B \tag{8.3}$$

This result is now used to define the magnitude of the magnetic field B as

$$B = \frac{F}{qv_{\perp}} \tag{8.4}$$

This definition is now similar to the definition of the electric field. *The magnetic field B, called the magnetic induction or the magnetic flux density, is defined as the force per unit charge, per unit velocity, provided that v is perpendicular to B.* The

SI unit for the magnetic induction is defined from equation 8.4 to be a tesla, named after Nikola Tesla (1856-1943), where

$$1 \text{ tesla} = 1 \frac{\text{newton}}{\text{coulomb m/s}}$$

This will be abbreviated as

$$1 \text{ T} = 1 \frac{\text{N}}{\text{C m/s}}$$

The relation of the Tesla with other equivalent units for the magnetic field are

$$\text{tesla} = \frac{\text{N}}{\text{A m}} = \frac{\text{weber}}{\text{m}^2} = 10^4 \text{ gauss}$$

The gauss is a cgs unit that is still used because of its convenient size. To give you an idea of the size of the tesla, the earth's magnetic field is about 1/20000 tesla.

From the definition of the cross product it is obvious that if the charge has a velocity which is parallel to the magnetic field, then the angle θ will be zero and hence

$$F = qvB \sin 0^0 = 0 \quad (\text{for } \mathbf{v} \mid\mid \mathbf{B}) \tag{8.5}$$

Of course, if the velocity of the charge is zero then the force will also be zero. The magnetic field manifests itself to a charge only when the charge is in motion with respect to the field. The maximum force occurs when **v** is at an angle of $90^0$ to **B**, as shown in equation 8.3. It should also be stated that equation 8.1 was defined with $q$ being a positive charge. If the charge in motion is a negative particle, such as an electron, $q$ will be negative and the force on the negative particle will be in the opposite direction to the force on the positive particle.

## *Example 8.1*

*A particle in a magnetic field.* A proton is fired into a uniform magnetic field **B** of magnitude 0.500 T, at a speed of 300 m/s at an angle of $30.0^0$ to **B**. Find the force and the acceleration of the proton.

## *Solution*

The magnitude of the force acting on the proton is found from equation 8.2 as

$$F = qvB \sin\theta = (1.60 \times 10^{-19} \text{ C})(300 \text{ m/s})(0.500 \text{ T}) \text{ Sin}30.0^0$$

$$F = (1.2 \times 10^{-17} \text{ C (m/s) T})\frac{(\text{N/(C m/s)})}{(\text{ T })}$$

$$F = 1.20 \times 10^{-17} \text{ N}$$

Note how the conversion factor for a tesla was used to make the force come out in the unit of newtons. The direction of the force is perpendicular to the plane of **v** and **B** as shown in figure 8.2(b). The acceleration of the proton is found from Newton's second law as

$$a = \frac{F}{m_p} = \frac{1.20 \times 10^{-17}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = 7.19 \times 10^{9}\ \text{m/s}^2$$

As long as the particle stays within the magnetic field, at the same angle θ, the magnitude of the force and the magnitude of the acceleration is a constant. Since the magnitude of the acceleration is a constant, the kinematic equations from college physics can be used to find the position of the particle at any time.

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## 8.2 Force on a Current-Carrying Conductor in an External Magnetic Field

If a wire carrying a current $I$ is placed in an external magnetic field **B** as shown in figure 8.3, a force will be found to act on the wire. The explanation for this force can

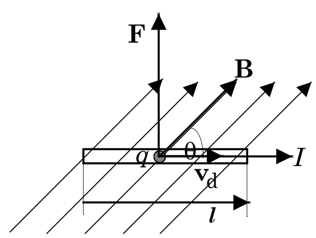



***Figure 8.3*** Force on a current-carrying wire in an external magnetic field

be found in the magnetic force acting on a charged particle in a magnetic field. If the wire is carrying a current, then there are charges in motion within the wire. These charges will be moving with a drift velocity $\mathbf{v}_d$ in the direction of the current flow. Any one of these charges $q$ will experience the force

$$\mathbf{F}_q = q\mathbf{v}_d \times \mathbf{B} \tag{8.6}$$

This force on an individual charge will cause the charge to interact with the lattice structure of the wire, exerting a force on the lattice and hence the wire itself. The drift velocity of the moving charge can be written as

$$v_d = \frac{l}{t} \tag{8.7}$$

where $\boldsymbol{l}$ is a small length of the wire in the direction of the current flow and is shown in figure 8.3, and $t$ is the time. Replacing this drift velocity in equation 8.6 gives

$$\boldsymbol{F}_q = q\left(\frac{\boldsymbol{l}}{t}\right)\times\boldsymbol{B} = \left(\frac{q}{t}\right)\boldsymbol{l}\times\boldsymbol{B} \tag{8.8}$$

The net force on the wire is the sum of the individual forces associated with each charge carrier, i.e.,

$$\boldsymbol{F} = \Sigma_q\,\boldsymbol{F}_q = \Sigma_q\left(\frac{q}{t}\right)\boldsymbol{l}\times\boldsymbol{B}$$

But $\Sigma_q\ (q/t)$ is equal to all the charges passing through a plane of the wire per unit time and is defined to be the current in the circuit, $I$. Hence *a wire carrying a current I in any external magnetic field* **B***, will experience a force given by*

$$\boldsymbol{F} = I\,\boldsymbol{l}\times\boldsymbol{B} \tag{8.9}$$

The force is again given by a cross product term, and the direction of the force is found from $\boldsymbol{l} \times \mathbf{B}$. With $\boldsymbol{l}$ in the direction of the current and **B** pointing into the page in figure 8.3, $\boldsymbol{l} \times \mathbf{B}$ is a vector that points upward. If the direction of the current flow is reversed, $\boldsymbol{l}$ would be reversed and $\boldsymbol{l} \times \mathbf{B}$ would then point downward. *The magnitude of the force is determined from equation 8.9 as*

$$F = IlB\sin\theta \tag{8.10}$$

where $\theta$ is the angle between $\boldsymbol{l}$ and **B**. Solving equation 8.10 for $B$ gives another set of units for the magnetic induction $B$, namely

$$B = \frac{F}{Il}$$

Thus,

$$1\ \text{tesla} = \frac{\text{newton}}{\text{ampere meter}}$$

This will be abbreviated as

$$1\ \text{T} = 1\ \frac{\text{N}}{\text{A m}}$$

## *Example 8.2*

*Force on a wire in a magnetic field.* A 20.0 cm wire carrying a current of 10.0 A is placed in a uniform magnetic field of 0.300 T as shown in figure 8.3. If the wire makes an angle of $40.0^0$ with the vector **B**, find the direction and magnitude of the force on the wire.

***Solution***

The direction of the force is found from equation 8.9 as

$$\mathbf{F} = I\boldsymbol{l} \times \mathbf{B}$$

In rotating the vector $\boldsymbol{l}$ toward the vector $\mathbf{B}$ in the cross product, the thumb points upward, indicating that the direction of the force is also upward. The magnitude of the force is found from equation 8.10 as

$$F = IlB \sin\theta$$
$$F = (10.0 \text{ A})(0.200 \text{ m})(0.300 \text{ T}) \sin 40.0^0$$
$$F = (0.386 \text{ A m T})\left(\frac{\text{N/(A m)}}{\text{T}}\right)$$
$$F = 0.386 \text{ N}$$

**To go to this Interactive Example click on this sentence.**

## 8.3 Force on a Semicircular Wire Carrying a Current in an External Magnetic Field

In section 8.2 we found the force on a straight piece of wire carrying a current in an external magnetic field is given by

$$\boldsymbol{F} = I\,\boldsymbol{l} \times \boldsymbol{B} \tag{8.9}$$

But what if the wire is not a straight wire, what is the force on the wire then? If the wire is not straight, we divide the wire into small little elements of length $d\boldsymbol{l}$, each in the direction of the current flow, and each length $d\boldsymbol{l}$ experiences the force $d\mathbf{F}$ given by

$$d\boldsymbol{F} = I\,d\boldsymbol{l} \times \boldsymbol{B} \tag{8.11}$$

The total force on the wire will now be just the sum, or integral, of all these $d\mathbf{F}$'s. That is,

$$\boldsymbol{F} = \int d\boldsymbol{F} = \int I\,d\boldsymbol{l} \times \boldsymbol{B} \tag{8.12}$$

As an example of the application of equation 8.12 let us find the force on a semicircular piece of wire carrying a current as shown in figure 8.4. Consider the small piece of wire $d\boldsymbol{l}$ that is shown. The magnetic field $\mathbf{B}$ points into the page and $d\boldsymbol{l} \times \mathbf{B}$ points down along the radius of the semicircle to the center of the circle. The angle between $\mathbf{B}$ and $d\boldsymbol{l}$ is $90^0$, therefore the element of force $d\mathbf{F}$ associated with

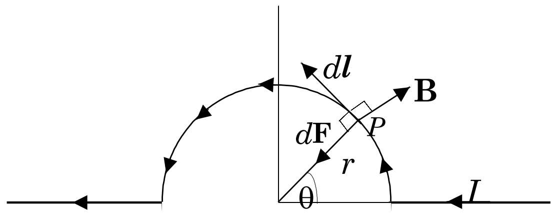



*Figure 8.4* Force on a semicircular portion of wire.

this small element $dl$ of wire is

$$dF = I\,dl\,B \sin 90^o\,(-r_o) = I\,dl\,B\,(-r_o) \tag{8.13}$$

$\mathbf{r}_o$ is a unit vector that points from the origin to the point $P$, and $d\mathbf{F}$ points in the opposite direction or $-\mathbf{r}_o$. The total force on the semicircular wire is the sum of all these $d\mathbf{F}$'s or

$$\boldsymbol{F} = \int d\boldsymbol{F} = \int_0^\pi I\,dl\,B(-\boldsymbol{r}_o) \tag{8.14}$$

But the unit vector $\mathbf{r}_o$ can be written in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ as

$$\boldsymbol{r}_o = \boldsymbol{i}\cos\theta + \boldsymbol{j}\sin\theta \tag{8.15}$$

Replacing equation 8.15 into equation 8.14 we get

$$\boldsymbol{F} = \int_0^\pi I\,dl\,B(-\boldsymbol{r}_o) = \int_0^\pi I\,B\,dl\,[-(\boldsymbol{i}\cos\theta + \boldsymbol{j}\sin\theta)]$$
$$\boldsymbol{F} = -\int_0^\pi I\,B\,dl\,\boldsymbol{i}\cos\theta - \int_0^\pi IB\,dl\,\boldsymbol{j}\sin\theta \tag{8.16}$$

But $dl$ and θ are not independent and are related by

$$dl = r\,d\theta \tag{8.17}$$

Therefore equation 8.16 becomes

$$\boldsymbol{F} = -\int_0^\pi I\,B\,rd\theta\,\boldsymbol{i}\cos\theta - \int_0^\pi IB\,rd\theta\,\boldsymbol{j}\sin\theta$$

The current $I$ in the circuit is a constant, as well as the magnetic field $B$, and the radius of the semicircle $r$, and they can each be removed from under the integral sign to yield

$$\boldsymbol{F} = -IBr\int_0^\pi \cos\theta d\theta\,\boldsymbol{i} - IBr\int_0^\pi \sin\theta d\theta\,\boldsymbol{j}$$

Performing the integrations

$$\boldsymbol{F} = -IBr\sin\theta\,|_0^\pi\,\boldsymbol{i} - IBr(-\cos\theta)\,|_0^\pi\,\boldsymbol{j}$$
$$\boldsymbol{F} = -IBr[\sin\pi - \sin 0]\,\boldsymbol{i} + IBr[\cos\pi - \cos 0]\,\boldsymbol{j}$$

$$\boldsymbol{F} = -IBr[0-0]\,\boldsymbol{i} + IBr[-1-1]\,\boldsymbol{j} = 0\,\boldsymbol{i} - 2IBr\,\boldsymbol{j}$$

and finally

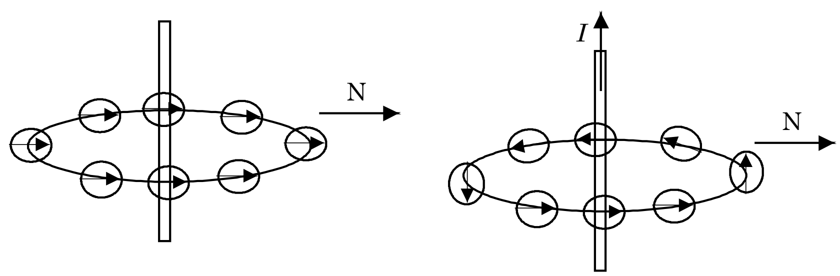
$$\boldsymbol{F} = -2rIB\,\boldsymbol{j} \tag{8.18}$$

*Equation 8.18 gives the force* **F** *that will act on the semicircular portion of the wire carrying a current I in a magnetic field B.* Notice that the force is completely in the negative **j** direction, or downward. The two components in the $x$-direction cancel each other out.

Although this problem was solved for a semicircular portion of a current-carrying wire, the procedure is essentially the same for any shape of wire. That is, $d$**F** is given by equation 8.11 and the total force will be the sum or integral of all these $d$**F**'s in equation 8.12.

## 8.4 Generation of a Magnetic Field

Although magnets had been known for hundreds of years, it was not until 1820, that Hans Christian Oersted (1777 - 1851) discovered a relation between electric current and magnetic fields. If a series of compasses are placed around a wire that is not carrying a current, all the compasses will point toward the north, the direction of the earth's magnetic field, as shown in figure 8.5(a). If, however, a current $I$ is sent through the wire, the compass needles will no longer point to the




(a) No current in wire (b) Current in wire

***Figure 8.5*** The creation of a magnetic field by an electric current

north. Instead they point in a direction which is everywhere tangential to a circle drawn around the wire, passing through each compass, as shown in figure 8.5(b). Because a compass always aligns itself in the direction of a magnetic field, the current in the wire has created a circular magnetic field directed counterclockwise around the wire. If the direction of the current is reversed, the direction of the magnetic field will also be reversed and the compasses will point in a clockwise direction. *The direction of the magnetic field around a long straight wire carrying a current, I, is easily determined by the so called "Right Hand Rule": Grasp the wire with the right hand, with the thumb in the direction of the current flow, the fingers will curl around the wire in the direction of the magnetic field.*

This observation that electric currents can create a magnetic field was responsible for linking the then two independent sciences of electricity and magnetism into the one unified science of electromagnetism. In fact, it will be shown later that all magnetic fields are caused by the flow of electric charge.

## 8.5 The Biot-Savart Law

*The Biot-Savart law relates the amount of magnetic field* $d\boldsymbol{B}$ *at the position* $\mathbf{r}$ *produced by a small element* $d\boldsymbol{l}$*, of a wire carrying a current* $I$ *and is given by*

$$d\boldsymbol{B} = \frac{\mu I}{4\pi}\frac{d\boldsymbol{l}\times\boldsymbol{r}}{r^3} \tag{8.19}$$

and is shown in figure 8.6(a). $\mu$ is a constant called the permeability of the medium.

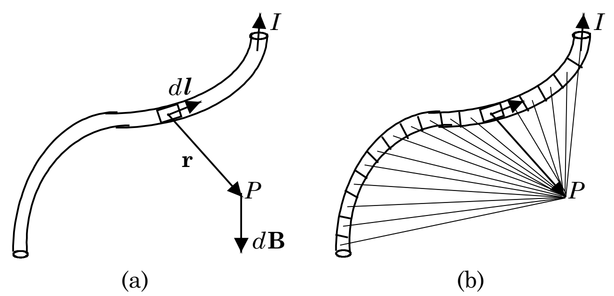



**Figure 8.6** The magnetic field produced by a current element.

In a vacuum or air, it is called the permeability of free space, and is denoted by $\mu_o$, where

$$\mu_o = 4\pi \times 10^{-7}\ \frac{\text{T m}}{\text{A}}$$

The cross product term $d\boldsymbol{l} \times \mathbf{r}$ immediately determines the direction of $d\mathbf{B}$, and is shown in figure 8.6(a). The Biot-Savart law says that the small current element $d\boldsymbol{l}$ produces a small amount of magnetic field $d\mathbf{B}$ at the point $P$. But the entire length of wire can be cut up into many $d\boldsymbol{l}$'s, and each will contribute to the total magnetic field at $P$. This is shown in figure 8.6(b). *Therefore, the total magnetic field at point P is the vector sum, and hence, integral of all the* $d\mathbf{B}$*'s associated with each current element, i.e.,*

$$\boldsymbol{B} = \int d\boldsymbol{B} \tag{8.20}$$

The computation of the magnetic field from equation 8.20 can be quite complicated for most problems because of the vector integration. However, there are some problems that can be easily solved by the Biot-Savart law, and we will solve some in the next sections.

## 8.6 The Magnetic Field at the Center of a Circular Current Loop

To determine the magnetic field at the center of a circular current loop, figure 8.7, the Biot-Savart law is used. A small element of the wire $d\boldsymbol{l}$ produces an element of

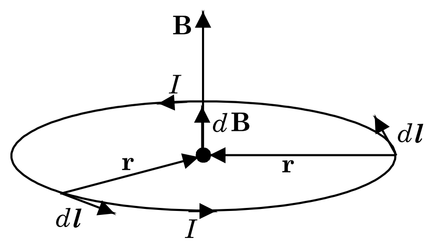



*Figure 8.7* The magnetic field at the center of a circular current loop

magnetic field $d\mathbf{B}$ at the center of the wire given by equation 8.19 as

$$d\boldsymbol{B} = \frac{\mu_o I}{4\pi}\frac{d\boldsymbol{l}\times\boldsymbol{r}}{r^3} \tag{8.19}$$

From the nature of the vector cross product, $d\boldsymbol{l} \times \mathbf{r}$, and hence $d\mathbf{B}$ points upward at the center of the circle for every current element, as seen in figure 8.7. The total magnetic field $\mathbf{B}$ which is the sum of all the $d\mathbf{B}$'s, must also point upward at the center of the loop. Therefore, the magnetic field at the center of the current loop is perpendicular to the plane formed by the loop, and points upward. Since the direction of the vector $\mathbf{B}$ is now known, equation 8.20 can be reduced to the scalar form

$$B = \int dB \tag{8.21}$$

The magnitude of $dB$ is found from equation 8.19 to be

$$dB = \frac{\mu_o I}{4\pi}\frac{dl\ r\sin\theta}{r^3} \tag{8.22}$$

Since $d\boldsymbol{l}$ is perpendicular to $\mathbf{r}$, the angle $\theta$ is equal to $90^0$, and the sine of $90^0$ is equal to 1. Therefore, $dB$ becomes

$$dB = \frac{\mu_o I dl}{4\pi r^2} \tag{8.23}$$

Replacing equation 8.23 into equation 8.21 gives the magnitude of the magnetic induction $B$ as

$$B = \int dB = \int \frac{\mu_o I dl}{4\pi r^2} \tag{8.24}$$

Because the loop is a circle of constant radius $r$ and $\mu_o I/4\pi$, is a constant, these terms will be in every term of the summation and can be factored out of the integral to yield

$$B = \frac{\mu_o I}{4\pi r^2} \int dl \tag{8.25}$$

But the summation of all the $dl$'s is simply the circumference of the wire, i.e.,

$$\int dl = 2\pi r \tag{8.26}$$

Therefore, equation 8.25 becomes

$$B = \frac{\mu_o I}{4\pi r^2}(2\pi r) \tag{8.27}$$

Canceling like terms, this becomes

$$B = \frac{\mu_o I}{2r} \tag{8.28}$$

*Equation 8.28 gives the magnetic field at the center of a circular current loop.* Notice that the magnetic field at the center of the circular current loop is directly proportional to the current $I$ - the larger the current, the larger the magnetic field; and inversely proportional to the radius $r$ of the loop - the larger the radius, the smaller the magnetic field. If there are $N$ turns of wire constituting the loop, the magnetic field at the center is

$$B = \frac{\mu_o NI}{2r} \tag{8.29}$$

The magnetic field found in this way is the magnetic field at the center of the current loop. The magnetic field all around the loop is shown in figure 8.8. Note that it looks something like the magnetic field of a bar magnet, where the top of the loop would be the north pole.

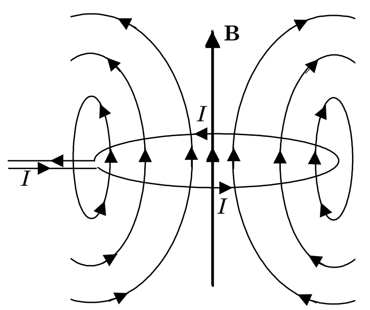



***Figure 8.8*** The magnetic field of a current loop.

## ***Example 8.3***

*The magnetic field at the center of a circular current loop.* Find the magnetic field at the center of a circular current loop of 0.500 m radius, carrying a current of 7.00 A.

### ***Solution***

The magnetic field at the center of the loop, found from equation 8.28, is

$$B = \frac{\mu_o I}{2r} = \frac{(4\pi \times 10^{-7}\text{T m/A})(7.00 \text{ A})}{2(0.500 \text{ m})}$$
$$B = 8.80 \times 10^{-6} \text{ T}$$

**To go to this Interactive Example click on this sentence.**

### *Example 8.4*

*The magnetic field at the center of a circular current loop.* Find the magnetic field at the center of a circular current loop of 10 turns, with a radius of 5.00 cm carrying a current of 10.0 A.

### *Solution*

The magnetic field is found from equation 8.29 to be

$$B = \frac{\mu_o NI}{2r} = \frac{(4\pi \times 10^{-7}\text{T m/A})(10)(10.0 \text{ A})}{2(0.050 \text{ m})}$$
$$B = 1.26 \times 10^{-3} \text{ T}$$

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## 8.7 Magnetic Field Around a Long Straight Wire

As another example of the application of the Biot-Savart law let us determined the magnetic field a distance $R$ away from a long straight wire carrying a current $I$, as shown in figure 8.9. The wire lies along the $y$-axis and is carrying a current in the positive $y$-direction as shown. A small element $\boldsymbol{dl}$ of the current carrying wire causes a small element of magnetic field $d\mathbf{B}$ at the point $P$ given by the Biot-Savart law , equation 8.19, as

$$d\boldsymbol{B} = \frac{\mu_o I}{4\pi} \frac{\boldsymbol{dl} \times \boldsymbol{r}}{r^3} \tag{8.19}$$

The total magnetic field at the point $P$ will be the sum or integral of all these $d\mathbf{B}$'s, and is given by equation 8.20.

$$\boldsymbol{B} = \int d\boldsymbol{B} \tag{8.20}$$

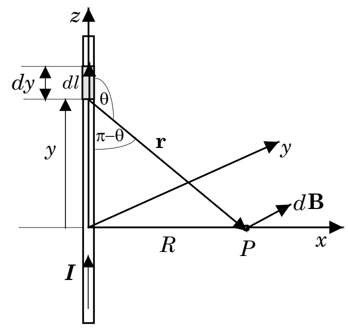


*Figure 8.9* The magnetic field around a long straight wire by use of the Biot-Savart law.

As you rotate the vector $\boldsymbol{dl}$ into $\mathbf{r}$ to determine the direction of the cross product term $\boldsymbol{dl} \times \mathbf{r}$, and hence, $d\mathbf{B}$, you notice that it points into the plane of the paper at the point $P$. Since $d\mathbf{B}$ is always into the page, $\mathbf{B}$ will also be into the page, and we only have to deal with the magnitude of $dB$ in the integration. That is $B$ is found from

$$B = \int dB = \int_{-\infty}^{+\infty} \frac{\mu_o I}{4\pi} \frac{dl \sin\theta}{r^2} \tag{8.30}$$

Notice that the long straight wire is assumed to go from $y = -\infty$ to $y = +\infty$ and these are now our limits of integration. The element of length $dl$ will now be represented by $dy$ since $y$ is the variable we are integrating over. Equation 8.30 becomes

$$B = \frac{\mu_o I}{4\pi} \int_{-\infty}^{+\infty} \frac{\sin\theta \, dy}{r^2} \tag{8.31}$$

But the variables θ, $r$, and $y$ are not independent, but are related from the geometry of figure 8.9 as

$$r = \sqrt{y^2 + R^2} \tag{8.32}$$

and from the trigonometric identity

$$\sin\theta = \sin(\pi - \theta)$$

Hence

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{y^2 + R^2}} \tag{8.33}$$

Replacing equations 8.32 and 8.33 into equation 8.31 yields

$$B = \frac{\mu_o I}{4\pi} \int_{-\infty}^{+\infty} \frac{\sin\theta \, dy}{r^2} = \frac{\mu_o I}{4\pi} \int_{-\infty}^{+\infty} \frac{R}{\sqrt{y^2 + R^2}} \frac{dy}{(y^2 + R^2)}$$

$$B = \frac{\mu_o I}{4\pi}\int_{-\infty}^{+\infty}\frac{R\,dy}{(y^2+R^2)^{\frac{3}{2}}} = \frac{\mu_o IR}{4\pi}\int_{-\infty}^{+\infty}(y^2+R^2)^{-\frac{3}{2}}\,dy \tag{8.34}$$

Because of the symmetry of the problem we can integrate from $y = 0$ to $y = +\infty$, instead of integrating from $y = -\infty$ to $y = +\infty$, by doubling the value of the integral. That is

$$\int_{-\infty}^{+\infty} = 2\int_{0}^{+\infty}$$

Hence, equation 8.34 can be written as

$$B = \frac{2\mu_o IR}{4\pi}\int_{0}^{+\infty}(y^2+R^2)^{-\frac{3}{2}}\,dy \tag{8.35}$$

But from the table of integrals we find

$$\int_{0}^{+\infty}(y^2+R^2)^{-\frac{3}{2}}\,dy = \frac{y}{R^2\sqrt{y^2+R^2}} \tag{8.36}$$

Replacing equation 8.36 into equation 8.35 gives

$$B = \frac{\mu_o IR}{2\pi}\left[\frac{y}{R^2\sqrt{y^2+R^2}}\right]_0^{\infty} \tag{8.37}$$

If we were to place the limits of integration into the present form of equation 8.37, we would get an indeterminate form. Hence we now divide both numerator and denominator by $y$ to get

$$B = \frac{\mu_o IR}{2\pi R^2}\left[\frac{1}{\sqrt{\frac{y^2}{y^2}+\frac{R^2}{y^2}}}\right]_0^{\infty} = \frac{\mu_o I}{2\pi R}\left[\frac{1}{\sqrt{1+\frac{R^2}{y^2}}}\right]_0^{\infty} = \frac{\mu_o I}{2\pi R}\left[\frac{1}{\sqrt{1+\frac{R^2}{\infty}}} - \frac{1}{\sqrt{1+\frac{R^2}{0}}}\right]$$

$$B = \frac{\mu_o I}{2\pi R}\left[\frac{1}{\sqrt{1+0}} - \frac{1}{\sqrt{1+\infty}}\right] = \frac{\mu_o I}{2\pi R}[1-0]$$

and

$$B = \frac{\mu_o I}{2\pi R} \tag{8.38}$$

*Equation 8.38 gives the value of the magnetic field B at a distance R from a long straight wire carrying a current I.*

## 8.8 Magnetic Field on Axis for a Circular Current Loop

Let us now consider a case a little more general than the one we solved in section 8.6, by finding the magnetic field **B** at a point *P* on the *z*-axis of a current loop that lies in the *x, y*-plane, figure 8.10. We consider a small element ***dl*** of the current

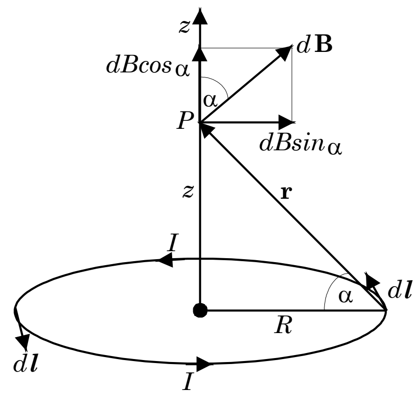


***Figure 8.10*** The magnetic field on axis for a circular current loop.

carrying wire and use the Biot-Savart law to find the element of the magnetic field $d\mathbf{B}$ associated with this element of wire ***dl***. That is,

$$d\boldsymbol{B} = \frac{\mu_o I}{4\pi}\frac{d\boldsymbol{l}\times\boldsymbol{r}}{r^3} \tag{8.19}$$

The total magnetic field at the point *P* is the sum or integral of all the $d\mathbf{B}$'s caused by all the ***dl***'s as we go around the circle, that is

$$\boldsymbol{B} = \int d\boldsymbol{B} \tag{8.20}$$

The integration in equation 8.20 is a vector integration. We simplify the problem by noting that the vector $d\mathbf{B}$ has the components $dB$ sinα and $dB$ cosα. As you can see in figure 8.10, the term $dB$ sinα points to the right in the positive *x*-direction. But diametrically opposite to the element ***dl*** considered, is another ***dl*** and there is a corresponding $d\mathbf{B}$ associated with it that will also have a component $dB$ sinα, but this component points to the left in the negative *x*-direction. For each $+dB$ sinα associated with an element ***dl*** on one side of the wire there will be a corresponding $-dB$ sinα on the opposite side and the sum of all these $dB$ sinα terms will be zero. This is equivalent to saying that the sum or integral of all these components will be zero. That is,

$$\int dB \sin a = 0$$

But notice that the terms $dB$ cosα always point in the positive *z*-direction, and it is the sum or integral of these components that will give us the magnetic field at the point *P*. Therefore, the magnetic field **B** is found from

$$B = \int dB \cos\alpha \tag{8.39}$$

Upon substituting for $dB$ we get

$$B = \int \frac{\mu_o I}{4\pi} \frac{dl \sin 90^0}{r^2} \cos\alpha$$
$$B = \frac{\mu_o I}{4\pi} \int \frac{\cos\alpha}{r^2} dl \tag{8.40}$$

But the variables $r$ and α are not independent of each other and as can be seen from the geometry of figure 8.10 the relations are

$$r = \sqrt{R^2 + z^2}$$

and

$$\cos\alpha = \frac{R}{\sqrt{R^2 + z^2}}$$

Replacing these into equations 8.40 yields

$$B = \frac{\mu_o I}{4\pi} \int \frac{\cos\alpha}{r^2} dl = \frac{\mu_o I}{4\pi} \int \frac{\frac{R}{\sqrt{R^2+z^2}}}{R^2 + z^2} dl = \frac{\mu_o IR}{4\pi} \int \frac{dl}{(R^2 + z^2)^{\frac{3}{2}}}$$

But as you can see in figure 8.10 for the point $P$, the values of $R$ and $z$ are constant and hence can be taken out of the integral sign to yield

$$B = \frac{\mu_o IR}{4\pi (R^2 + z^2)^{\frac{3}{2}}} \int dl \tag{8.41}$$

Thus the only integration is the sum of all the *dl*'s. But the sum of all the dl's is just the circumference of the circle. Therefore,

$$\int dl = 2\pi R \tag{8.42}$$

Substituting equation 8.42 into 8.41 gives

$$B = \frac{\mu_o IR}{4\pi (R^2 + z^2)^{\frac{3}{2}}} 2\pi R$$
$$B = \frac{\mu_o IR^2}{2(R^2 + z^2)^{\frac{3}{2}}} \tag{8.43}$$

*Equations 8.43 gives the magnetic field B at the point P located on the z-axis of a circular current loop of radius R carrying a current I.*

### *Example 8.5*

*A special case.* Using the solution for the magnetic field on axis for a circular current loop, find the magnetic field $B$ at the center of the circular current loop.

### *Solution*

The magnetic field at the center of the circular current loop is found from equation 8.43 by letting $z = 0$. Therefore,

$$B = \frac{\mu_o I R^2}{2(R^2 + z^2)^{\frac{3}{2}}} = \frac{\mu_o I R^2}{2(R^2 + 0)^{\frac{3}{2}}} = \frac{\mu_o I R^2}{2(R^2)^{\frac{3}{2}}} = \frac{\mu_o I R^2}{2R^3}$$

and the magnetic field $B$ at the center of the circular current loop is

$$B = \frac{\mu_o I}{2R}$$

Notice that this is the same solution we obtained in section 8.6, equation 8.28. Notice that this solution, equation 8.43, is more general than the one we derived in section 8.6.

## 8.9 Ampere's Circuital Law

Although the Biot-Savart law can be used to determine the magnetic field for different current distributions many of the derivations require vector integrations. Another simpler technique for the computation of magnetic fields, when the symmetry is appropriate, is Ampere's Circuital Law. *Ampere's Law states: along any arbitrary path encircling a total current* $I_{\text{total}}$*, the integral of the scalar product of the magnetic field* $\mathbf{B}$ *with the element of length* $d\boldsymbol{l}$ *of the path, is equal to the permeability* $\mu_o$ *times the total current* $I_{\text{total}}$ *enclosed by the path.*[1] That is,

$$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}} \tag{8.43}$$

It should be pointed out that Ampere's law is a fundamental law based on experiments and cannot be derived. Ampere's law is especially helpful in problems with symmetry and will be used in the following sections.

[1] In many problems the subscript total will be left out of the current term $I$ in the statement of Ampere's law, but it must be understood that the term $I$ in Ampere's law is always the *total current* enclosed in the path of integration.

## 8.10 The Magnetic Field Around a Long Straight Wire by Ampere's Law

To determine the magnetic field around a long straight wire by the Biot-Savart law is a little complicated, as seen in section 8.7. However, because of the symmetry of the magnetic field around a long straight wire, a simple solution to the magnetic field can be found using Ampere's law. It was pointed out in section 8.4 that the magnetic field around a long straight wire was found by experiment to be circular as shown in figure 8.11(a). In applying Ampere's law, an arbitrary path

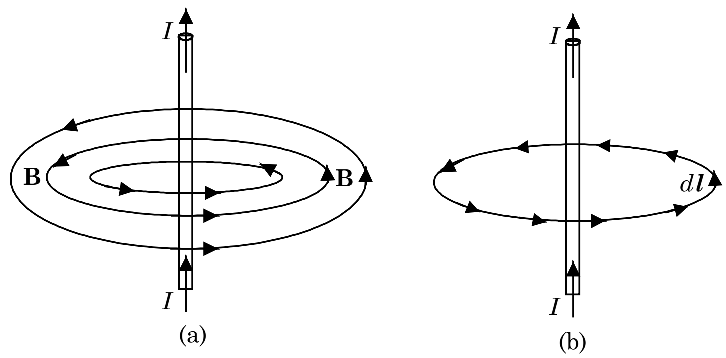


***Figure 8.11*** The magnetic field around a long straight wire.

must be drawn around the wire that contains the current $I$. The most symmetrical path that can be drawn about the wire is a concentric circle, as shown in figure 8.11(b). This circular path is divided into elements $\boldsymbol{dl}$, and then $\mathbf{B} \bullet \boldsymbol{dl}$ is computed for each element $\boldsymbol{dl}$. Because the magnetic field is circular, $\mathbf{B}$ and $\boldsymbol{dl}$ are parallel at every point along the circular path. Ampere's law becomes

$$\oint \boldsymbol{B} \bullet \boldsymbol{dl} = \mu_o I$$
$$\oint \boldsymbol{B} \bullet \boldsymbol{dl} = \oint Bdl \cos 0^0 = \oint Bdl = \mu_o I \tag{8.44}$$

But from symmetry, the value of $B$ is the same anywhere along the circular path and can be taken outside of the integral to yield

$$B \oint dl = \mu_o I \tag{8.45}$$

But the sum of all the $dl$'s, that is, $\int dl$, is the circumference of the circular path. Hence,

$$\int dl = 2\pi r \tag{8.46}$$

Therefore,

$$B(2\pi r) = \mu_o I \tag{8.47}$$

Thus, *the magnitude of the magnetic field around a long straight wire is*

$$B = \frac{\mu_o I}{2\pi r} \tag{8.48}$$

**_Example 8.6_**

*The magnetic field of a long straight wire.* A long straight wire is carrying a current of 15.0 A. Find the magnetic field 30.0 cm from the wire.

**_Solution_**

The magnetic field around the wire is found from equation 8.48 to be

$$B = \frac{\mu_o I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{T m/A})(15.0 \text{ A})}{2\pi(0.300 \text{ m})}$$
$$B = 1.00 \times 10^{-5} \text{ T}$$

**To go to this Interactive Example click on this sentence.**

## 8.11 The Magnetic Field Inside a Solenoid

A solenoid is a long coil of wire with many turns and is shown schematically in figure 8.12. Note that the magnetic field of a solenoid looks like the magnetic field of a bar magnet. The magnetic field is uniform and intense within the coils but is so small outside of the coil, that it is taken to be zero there. The magnetic field inside

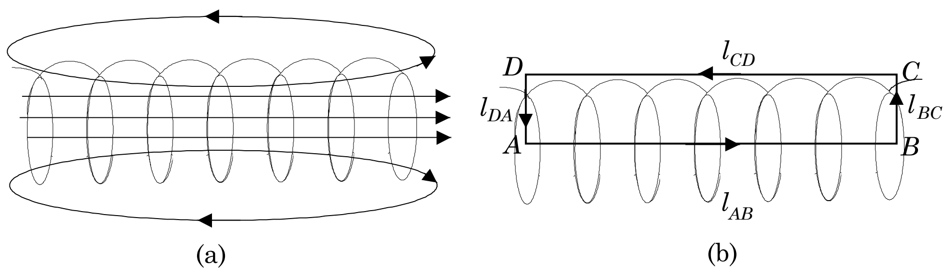


***Figure 8.12*** The magnetic field of a solenoid

the solenoid lies along the axis of the coil, as seen in figure 8.12(a). The value of **B** inside the solenoid is found from Ampere's law, by adding up the values of **B** • $dl$ along the rectangular path *ABCD* in figure 8.12(b). That is

$$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}}$$
$$\int_{AB} \boldsymbol{B} \bullet d\boldsymbol{l} + \int_{BC} \boldsymbol{B} \bullet d\boldsymbol{l} + \int_{CD} \boldsymbol{B} \bullet d\boldsymbol{l} + \int_{DA} \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}} \tag{8.49}$$

But since **B** = 0 outside the solenoid

$$\int_{CD} \boldsymbol{B} \bullet d\boldsymbol{l} = 0 \tag{8.50}$$

Because **B** is perpendicular to the paths $\boldsymbol{l}_{\text{BC}}$ and $\boldsymbol{l}_{\text{DA}}$, their scalar products must be zero, i.e.,

$$\int_{BC} \boldsymbol{B} \bullet d\boldsymbol{l} = \int_{BC} Bdl \cos 90^0 = 0 \tag{8.51}$$

and

$$\int_{DA} \boldsymbol{B} \bullet d\boldsymbol{l} = \int_{DA} Bdl \cos 90^0 = 0 \tag{8.52}$$

Now **B** is parallel to $\boldsymbol{l}_{\text{AB}}$, therefore their scalar product becomes

$$\int_{AB} \boldsymbol{B} \bullet d\boldsymbol{l} = \int_{AB} Bdl \cos 0^0 = \int_{AB} Bdl = Bl_{AB} \tag{8.53}$$

Replacing equations 8.50 through 8.53 into equation 8.49 gives

$$Bl_{AB} = \mu_o I_{\text{total}} \tag{8.54}$$

The term, $I_{\text{total}}$, represents the total amount of current contained within the path *ABCD*. Each turn of wire carries a current $I$, but there are $N$ turns of wire in the solenoid. Therefore, the total current contained within the path is

$$I_{\text{total}} = NI \tag{8.55}$$

A more convenient unit for the number of turns of wire in the coil is the number of turns per unit length, $n$. Since the coil has a length $l_{\text{AB}}$, the total number of turns $N$ can be expressed as

$$N = nl_{\text{AB}} \tag{8.56}$$

Replacing equations 8.56 and 8.55 into equation 8.54 yields

$$Bl_{AB} = \mu_o I_{\text{total}} = \mu_o NI = \mu_o nl_{AB} I \tag{8.57}$$

Simplifying, the magnetic field within the solenoid becomes

$$B = \mu_o nI \tag{8.58}$$

Notice that the magnetic field within the solenoid can be increased by increasing the current $I$ flowing in the wires, and/or by increasing the number of turns of wire per unit length, $n$.

## *Example 8.7*

*Magnetic field of a solenoid.* A solenoid 15.0 cm long is composed of 300 turns of wire. If there is a current of 5.00 A in the wire, what is the magnetic field inside the solenoid?

### *Solution*

The number of turns of wire per unit length is

$$n = \frac{N}{l} = \frac{300 \text{ turns}}{0.150 \text{ m}} = 2000 \text{ turns/m}$$

The magnetic field inside the solenoid is found from equation 8.58 as

$$B = \mu_o n I = (4\pi \times 10^{-7} \ \frac{\text{T m}}{\text{A}})\frac{(2000 \text{ turns})}{\text{m}}(5.00 \text{ A})$$

$$B = 1.26 \times 10^{-2} \text{ T}$$

**To go to this Interactive Example click on this sentence**.

## 8.12 Magnetic Field Inside a Toroid

Let us now find the magnetic field inside a toroid. A toroid is essentially a solenoid of finite length bent into the shape of a doughnut as shown in figure 8.13. To

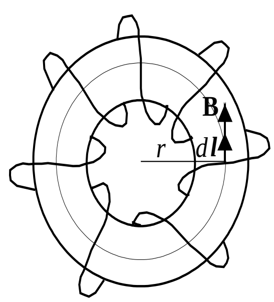


***Figure 8.13*** The magnetic field inside a toroid.

compute the magnetic field within the toroid we use Ampere's law, equation 8.43.

$$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}} \tag{8.43}$$

Since the toroid is a solenoid bent into a circular shape, and the magnetic field of a solenoid is parallel to the walls of the solenoid, the magnetic field **B** inside the toroid will take the same shape as the walls of the toroid. Hence the magnetic field inside the toroid will be circular. The path that we will use for the line integral in Ampere's law will be a circle of radius $r$ as shown in the diagram. In this way the angle θ between the magnetic field vector **B** and the element $\boldsymbol{dl}$ is $0^0$. Thus Ampere's law becomes

$$\oint \boldsymbol{B} \bullet \boldsymbol{dl} = \int B dl \cos 0^0 = \int B dl = \mu_o I_{\text{total}} \tag{8.59}$$

But from the symmetry of the problem, the magnitude of the magnetic field $B$ is a constant at a particular radius $r$ from the center of the toroid. (That is, there is no reason to assume that the magnitude of the magnetic field $B$ a distance $r$ to the right of the center of the toroid should be any different than the magnitude of the magnetic field $B$ a distance $r$ to the left of the center of the toroid.) Hence we can take $B$ outside the integral sign in equation 8.59 to obtain

$$\int B dl = B \int dl = \mu_o I_{\text{total}} \tag{8.60}$$

We now see that the integration is only over the element of path $dl$ and the sum of all these $dl$'s is just the circumference of the circle of radius $r$. Therefore Ampere's law becomes

$$B \int dl = B(2\pi r) = \mu_o I_{\text{total}} \tag{8.61}$$

Now the current term $I_{\text{total}}$ represents the total amount of current contained within the path of integration. Each turn of wire carries a current $I$, but there are $N$ turns of wire in the toroid. Therefore, the total current contained within the path of integration is

$$I_{\text{total}} = NI \tag{8.62}$$

Replacing equation 8.62 into equation 8.61 yields

$$B(2\pi r) = \mu_o I_{\text{total}} = \mu_o NI \tag{8.63}$$

Upon solving for $B$ we get

$$B = \frac{\mu_o NI}{2\pi r} \tag{8.64}$$

*Equation 8.64 gives the value of the magnetic field B inside the toroid. Notice that the value of B is not constant over the cross section of the toroid, but rather is a function of r, the distance from the center of the toroid (specifically B varies as 1/r).* For the smaller values of $r$, near the inside of the toroid, $B$ will be large and for the larger values of $r$, near the outside of the toroid, $B$ will be smaller. *Hence, the magnetic field B inside a toroid is not constant and uniform as it was in the solenoid.*

**_Example 8.8_**

*The magnetic field of a toroid.* A toroid has an inner radius of 10.0 cm and an outer radius of 20.0 cm and carries 500 turns of wire. If the current in the toroid is 5.00 A, find the minimum and maximum values of the magnetic field inside the toroid.

**_Solution_**

The minimum value of the magnetic field $B$ within the toroid occurs for the maximum value of $r$ and is found from equation 8.64 as

$$B = \frac{\mu_o NI}{2\pi r}$$
$$B = \frac{(4\pi \times 10^{-7}\text{T m/A})(500)(5.00\text{ A})}{2\pi(0.200\text{ m})}$$
$$B = 0.0025\text{ T}$$

The maximum value of the magnetic field $B$ within the toroid occurs for the minimum value of $r$ and is found from equation 8.63 as

$$B = \frac{\mu_o NI}{2\pi r}$$
$$B = \frac{(4\pi \times 10^{-7}\text{T m/A})(500)(5.00\text{ A})}{2\pi(0.100\text{ m})}$$
$$B = 0.0050\text{ T}$$

**To go to this Interactive Example click on this sentence**.

## 8.13 Torque on a Current Loop in an External Magnetic Field - The Magnetic Dipole Moment

Let us place a rectangular coil of wire in a uniform magnetic field $B$ as shown in figure 8.15(a). Notice that the magnetic field emanates from the north pole of the magnet and enters the south pole of the magnet. A current $I$ is set up in the coil in the direction indicated, by an external battery. Any segment of the coil now represents a current carrying wire in an external magnetic field and will thus experience a force on it given by equation 8.9.

$$\mathbf{F} = I\,\boldsymbol{l} \times \mathbf{B} \tag{8.9}$$

Let us divide the coil into sections of length $l_{ab}$, $l_{bc}$, $l_{cd}$, $l_{da}$ and compute the force on each segment.

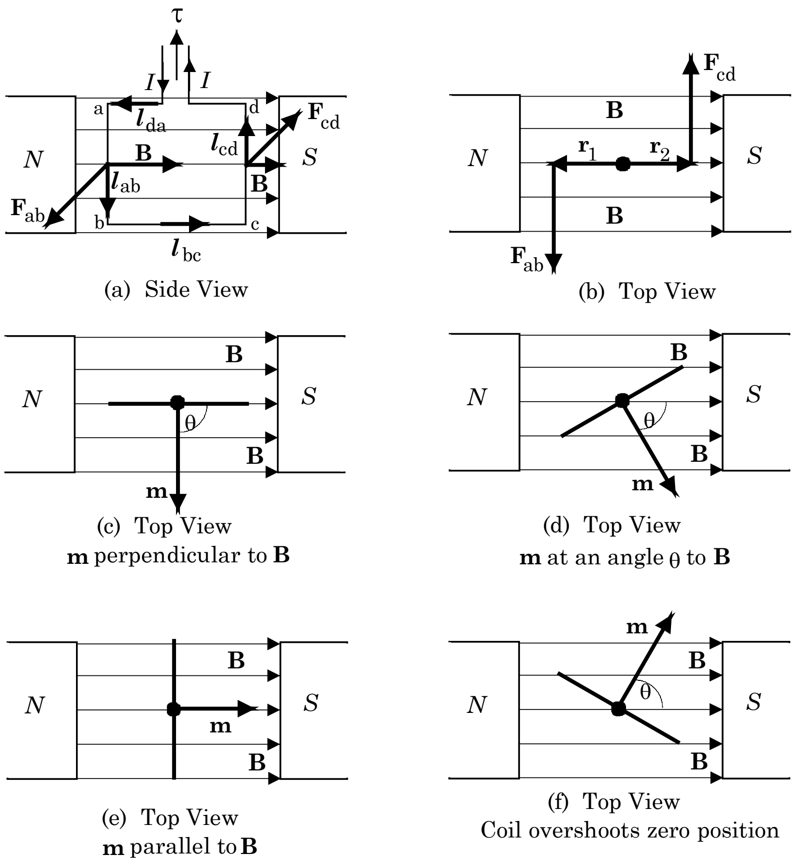


***Figure 8.15*** A coil in a magnetic field.

**Segment ab:** The force $\mathbf{F}_{ab}$ acting on segment $l_{ab}$ is given by

$$\mathbf{F}_{ab} = I\, \boldsymbol{l}_{ab} \times \mathbf{B} \tag{8.65}$$

Since $\boldsymbol{l}_{ab}$ is in the direction of the current in segment ab, it points downward in figure 8.15(a). The cross product of $\boldsymbol{l}_{ab} \times \mathbf{B}$ points outward as shown in the side view of the coil in figure 8.15(a), and from a top view in figure 8.15(b). The magnitude of the force is

$$F_{ab} = Il_{ab}B \sin 90^0 = Il_{ab}B \tag{8.66}$$

**Segment bc:** The force $\mathbf{F}_{bc}$ acting on segment $\boldsymbol{l}_{bc}$ is given by

$$\mathbf{F}_{bc} = I\boldsymbol{l}_{bc} \times \mathbf{B} \tag{8.67}$$

Since $\boldsymbol{l}_{bc}$ is in the direction of the current flow, it points to the right in figure 8.15(a), parallel to the magnetic field $\mathbf{B}$. As you recall from chapter 1, the cross product of parallel vectors is zero. That is

$$F_{bc} = Il_{bc}B \sin 0^0 = 0 \tag{8.68}$$

Thus, there is no force acting on the bottom of the wire coil in figure 8.15(a).

**Segment cd:** The force $\mathbf{F}_{cd}$ acting on segment $\boldsymbol{l}_{cd}$ is given by

$$\mathbf{F}_{cd} = I\boldsymbol{l}_{cd} \times \mathbf{B} \tag{8.69}$$

Since $\boldsymbol{l}_{cd}$ points upward and $\mathbf{B}$ points to the right in figure 8.15(a), the cross product $\boldsymbol{l}_{cd} \times \mathbf{B}$ points into the page in figure 8.15(a), and can be seen from the top view in figure 8.15(b). The magnitude of $F_{cd}$ is

$$F_{cd} = Il_{cd}B \sin 90^0 = Il_{cd}B \tag{8.70}$$

Because the magnitudes of the lengths $l_{ab}$ and $l_{cd}$ are equal, the forces are also equal. That is,

$$F_{ab} = F_{cd}$$

**Segment da:** The force $\mathbf{F}_{da}$ acting on the upper segment $\boldsymbol{l}_{da}$ is found from

$$\mathbf{F}_{da} = I\boldsymbol{l}_{da} \times \mathbf{B} \tag{8.71}$$

But $\boldsymbol{l}_{da}$ points to the left and makes an angle of $180^0$ with the magnetic field $\mathbf{B}$. Therefore,

$$F_{da} = Il_{da}B \sin 180^0 = 0 \tag{8.72}$$

because the $\sin 180^0$ is equal to zero. Hence, there is no force acting on the top wire.

The net result of the current flowing in a coil in an external magnetic field, is to produce two equal and opposite forces acting on the coil. But since the forces do not lie along the same line of action, the forces will cause a torque to act on the coil as is readily observable in figure 8.15(b). The torque acting on the coil is given by equation 1-51 as

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \tag{1-51}$$

Since both forces can produce a torque, the total torque on the coil is the sum of the two torques. That is,

$$\boldsymbol{\tau} = \mathbf{r}_1 \times \mathbf{F}_{ab} + \mathbf{r}_2 \times \mathbf{F}_{cd} \tag{8.73}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are position vectors from the axis of the coil to the point of application of the force, as can be seen in figure 8.15(b). Therefore,

$$\mathbf{r}_2 = -\mathbf{r}_1 \tag{8.74}$$

and

$$\mathbf{F}_{cd} = -\mathbf{F}_{ab} \tag{8.75}$$

as shown before. Therefore,

$$\boldsymbol{\tau} = \mathbf{r}_1 \times \mathbf{F}_{ab} + (-\mathbf{r}_1) \times (-\mathbf{F}_{ab}) \tag{8.76}$$

The total torque on the coil is therefore the clockwise torque

$$\boldsymbol{\tau} = 2\mathbf{r}_1 \times \mathbf{F}_{ab} \tag{8.77}$$

and lies along the axis of the coil, pointing upward in figure 8.15(a). As can be seen from the diagram,

$$r_1 = \frac{l_{bc}}{2} \tag{8.78}$$

while $F_{ab}$ is given by equation 8.66. Therefore the magnitude of the torque on the coil is given by

$$\tau = 2r_1F_{ab}\sin\theta = 2\frac{l_{bc}}{2}Il_{ab}B\sin\theta \tag{8.79}$$

Note that the angle θ is the angle between the radius vector **r** and the force vector **F** and the product of $l_{bc}$ and $l_{ab}$ is the area of the loop, i.e.,

$$l_{bc}l_{ab} = A \tag{8.80}$$

Equation 8.79 simplifies to

$$\tau = IAB\sin\theta \tag{8.81}$$

Although equation 8.81 has been derived for a rectangular current loop, one can prove, that it is a perfectly general result for any planar loop having an area $A$, regardless of the shape of the loop. *The torque on the coil in a magnetic field depends on the current I in the coil, the area A of the coil, the intensity of the magnetic field B, and the angle* θ *between* **r** *and* $\mathbf{F}_{ab}$. Since it was shown in section 8.6 that there is a magnetic field at the center of a current loop, and this magnetic field looks like the magnetic field of a bar magnet, *a magnetic dipole moment* ***m*** *of a current loop is defined as*

$$\mathbf{m} = IA\,\mathbf{n} \tag{8.82}$$

The magnetic dipole moment is shown in figure 8.16. **n** is a unit vector that determines the direction of **m**, and is itself determined by the direction of the

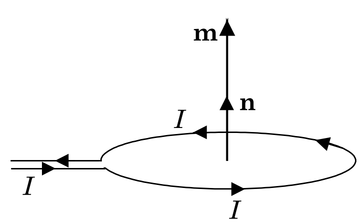


***Figure 8.16*** The magnetic dipole moment of a current loop.

current flow. If the right hand curls around the loop in the direction of the current flow, the thumb points in the direction of the unit vector **n**, and hence in the direction of the magnetic dipole moment **m**. Reversing the direction of the current flow will reverse the direction of the magnetic dipole moment. If the coil consists of $N$ loops of wire, the magnetic dipole moment will be given by

$$\mathbf{m} = NIA\,\mathbf{n} \tag{8.83}$$

Figure 8.15(c) is a top view of the coil, showing the magnetic dipole moment **m** perpendicular to the coil. The torque acting on the coil, equation 8.81, can now be written in terms of the magnetic dipole moment of the coil as

$$\tau = mB\sin\theta \tag{8.84}$$

or in the vector notation

$$\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B} \tag{8.85}$$

Note that the angle θ in equation 8.85, the angle between the magnetic dipole moment vector **m** and the magnetic field vector **B** is the same angle θ that was between the radius vector **r** and the force vector **F**. *Equation 8.85 shows that the torque acting on a coil in a magnetic field is equal to the cross product of the magnetic dipole moment **m** of the coil and the magnetic field* **B**. When **m** is perpendicular to the magnetic field **B**, θ is equal to $90^0$, and the sine of $90^0 = 1$. Therefore the torque acting on the coil will be at its maximum value, and will act to rotate the coil counterclockwise in figure 8.15(c). As the coil rotates, the angle θ will decrease, figure 8.15(d), until the magnetic dipole moment becomes parallel to the magnetic field, and the angle θ will be equal to zero, as shown in figure 8.15(e). At this point the torque acting on the coil becomes zero, because the sinθ term in equation 8.84, will be zero. Because of the inertia of the coil, however, the coil does not quite stop at this position but over shoots this position as shown in figure 8.15(f). But now the torque, given by **m** × **B**, is reversed, and the coil will rotate clockwise, until **m** is again parallel to **B** and again the torque will be zero. The coil may oscillate one or two times but because of friction it eventually will come to a stop with its magnetic dipole moment parallel to the magnetic field **B**. *In summary, the magnetic field **B** will cause a torque* **τ** *to act on the coil until the magnetic dipole moment **m** of the coil is aligned with the magnetic field **B**.*

This result should not come as too much of a surprise, for this is exactly what happens with a compass needle. The compass needle is a tiny bar magnet with a magnetic dipole moment. The earth's magnetic field acts on this dipole to align it with the earth's magnetic field. On the earth's surface, the earth's magnetic field points toward the north and the compass needle will also point toward the north.

## *Example 8.9*

*Magnetic dipole moment of a coil.* A circular coil, consisting of 10 turns of wire, 10.0 cm in diameter carries a current of 2.00 A. Find the magnetic dipole moment of the coil.

### *Solution*

The area of the coil is

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.100\text{ m})^2}{4} = 7.85 \times 10^{-3}\text{ m}^2$$

The magnitude of the magnetic dipole moment of the coil is found from equation 8.83 as

$$m = NIA$$
$$m = (10)(2.00\text{ A})(7.95 \times 10^{-3}\text{ m}^2)$$
$$m = 0.157\text{ A m}^2$$

Notice that the unit for magnetic dipole moment is an ampere meter squared.

**To go to this Interactive Example click on this sentence.**

## *Example 8.10*

*Torque on a coil in a magnetic field.* The above coil is placed in a uniform magnetic field of 0.500 T. Find the maximum torque on the coil.

### *Solution*

The torque acting on the coil is found from equation 8.84 as

$$\tau = mB\sin\theta$$

and the maximum torque occurs for $\theta = 90^0$. Therefore, the maximum torque acting on the coil is

$$\tau_{max} = mB = (0.157\text{ A m}^2)(0.500\text{ T})$$
$$\tau_{max} = (7.85 \times 10^{-2}\text{ A m}^2\text{ T})\left(\frac{\text{N/(A m)}}{\text{T}}\right)$$
$$\tau_{max} = 7.85 \times 10^{-2}\text{ m N}$$

**To go to this Interactive Example click on this sentence.**

## 8.14 Permanent Magnets and Atomic Magnets

The magnetic field of a bar magnet is shown in figure 8.17(a). The magnetic field emanates from the north pole of the magnet and enters at the south pole. The field is similar to the field of an electric dipole, shown in figure 8.17(b). The electric field emanates from the positive electric charge and terminates at the negative electric charge. Since both negative and positive electric charges can exist separately, it is reasonable to ask if magnetic poles can exist separately? The simplest test, would be to cut the bar magnet in figure 8.17(a) in half, expecting to obtain one isolated north pole and one isolated south pole. When the experiment is performed, however, two

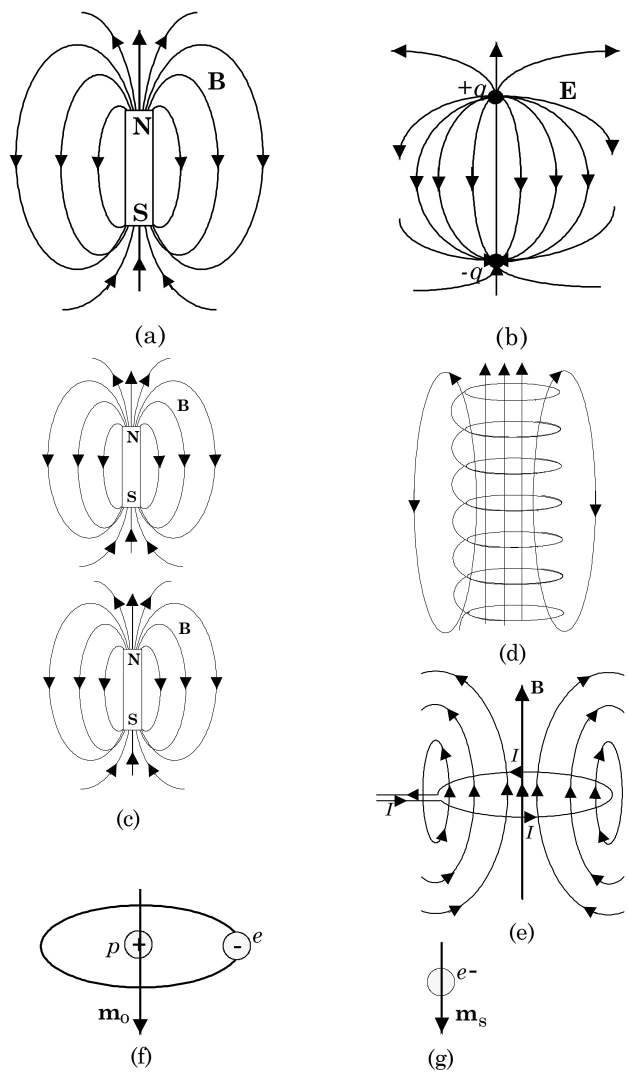


*Figure 8.17* Some Magnetic Fields

smaller bar magnets are found, each with a north pole and a south pole as seen in figure 8.17(c). *No matter how many times the bar magnet is subdivided, an isolated* *magnetic pole is never found. We always get a dipole.* Looking at the magnetic field of a solenoid, figure 8.17(d), it appears to be the same as the magnetic field of a bar magnet. So, maybe all magnetic fields are caused by electric currents. The magnetic field of a single current loop is shown in figure 8.17(e). Looking at a picture of the atom, we find the negative electron orbiting around the positive nucleus. The orbiting electron, figure 8.17(f), looks exactly like a current loop, and there is, therefore, a magnetic dipole moment of the atom, caused by the orbital electron. This orbital magnetic dipole moment is given by

$$m_o = IA \tag{8.86}$$

The atomic current is equal to the charge on the electron divided by the time $T$ for the electron to make one complete orbit, i.e.,

$$I = \frac{(-e)}{T} = -ef \tag{8.87}$$

where $f$, the reciprocal of the period, is the frequency, or the number of times the electron orbits the nucleus per second. Assuming the orbit to be a circle, it has an area

$$A = \pi r^2 \tag{8.88}$$

Hence, the orbital magnetic moment of an atom is

$$m_o = IA = -ef\pi r^2 \tag{8.89}$$

Therefore the atom itself looks like a tiny bar magnet.

Besides the orbital magnetic moment of the atom, the electron, which can be viewed as a charged sphere spinning on its axis, also has a magnetic dipole moment associated with its spin. The spin magnetic dipole moment is represented by $\mathbf{m}_s$. In general the total magnetic dipole moment of an atom is equal to the vector sum of its orbital magnetic dipole moment $\mathbf{m}_o$ and its spin magnetic dipole moment $\mathbf{m}_s$. The electrons usually fill up the shells of an atom with one spin magnetic dipole moment pointing up, and the next down. Thus, when an atom has completely closed shells, it has no magnetic dipole moment. Therefore, most chemical elements do not display magnetic behavior. In the case of iron, cobalt, and nickel, the electrons do not pair off to permit their spin magnetic dipole moments to cancel. In iron, for example, five of its electrons have parallel spins, giving it a large resultant magnetic dipole moment. Therefore each atom of iron is a tiny atomic bar magnet. When these atomic magnets are aligned in a bar of iron, we have the common bar magnet. Thus, the bar magnet is made up of atomic currents. ***This is why we can never isolate a north pole or a south pole, because they do not exist.*** *What does exist is an orbiting, rotating electric charge, that creates a magnetic dipole moment.*

## 8.15 The Potential Energy of a Magnetic Dipole in an External Magnetic Field

In section 8.13 we saw that a coil of wire of cross-sectional area $A$, with $N$ turns, carrying a current $I$, has a magnetic dipole moment $\mathbf{m}$ given by equation 8.83 as

$$\mathbf{m} = NIA\,\mathbf{n} \tag{8.83}$$

We also saw that when that coil of wire is placed in an external magnetic field $\mathbf{B}$ its magnetic dipole $\mathbf{m}$ experiences a torque given by

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \tag{8.85}$$

This torque acts to rotate the dipole until it is aligned with the external magnetic field. Because the natural position of $\mathbf{m}$ is parallel to the field, as shown in figure 8.15(e) and again here in figure 8.18(a), work must be done to rotate $\mathbf{m}$ in the

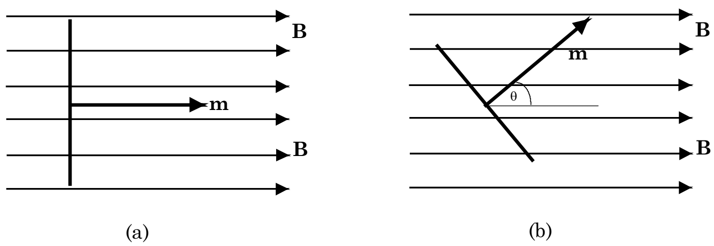


*Figure 8.18* A magnetic dipole $\mathbf{m}$ in an external magnetic field $\mathbf{B}$.

external magnetic field $\mathbf{B}$. When work was done in lifting a rock in a gravitational field, the rock then possessed potential energy. In the same way, since work must be done by an external agent to change the orientation of the dipole, the work done in rotating the dipole in the magnetic field shows up as potential energy of the dipole, figure 8.18(b). That is, the dipole now possesses an additional potential energy associated with the work done in rotating $\mathbf{m}$.

The potential energy of the dipole in an external magnetic field $\mathbf{B}$ is found by computing the work that must be done to rotate the dipole in the external magnetic field. That is,

$$\text{PE} = W = \int dW \tag{8.90}$$

Just as the element of work $dW = \mathbf{F} \bullet d\mathbf{s}$ for translational motion, the element of work for rotational motion is given by

$$dW = \boldsymbol{\tau} \bullet d\boldsymbol{\theta} \tag{8.91}$$

where $\boldsymbol{\tau}$ is the torque acting on the dipole to cause it to rotate and $d\boldsymbol{\theta}$ is the element of angle turned through. Both the torque vector $\boldsymbol{\tau}$ and the element of angle vector are perpendicular to the plane of the paper, and hence the angle between the two vectors are zero and their dot product is simply $\tau\, d\theta$. The increased potential energy becomes

$$\text{PE} = \int dW = \int \tau\, d\theta \tag{8.92}$$

The magnitude of the torque is found from equation 8.84 as

$$\tau = mB \sin\theta \tag{8.84}$$

Replacing equation 8.84 into equation 8.92 gives

$$\text{PE} = \int_{\theta_0}^{\theta} \tau d\theta = \int_{90^0}^{\theta} mB \sin\theta d\theta \tag{8.93}$$

We have made the lower limit of integration $\theta_o$ to be $90^0$, and $\theta$ the upper limit. Equation 8.93 becomes

$$\text{PE} = -mB\cos\theta \,\Big|_{90^0}^{\theta}$$
$$\text{PE} = -mB \cos\theta \tag{8.94}$$

Noticing the form of this equation, we can write it more generally as

$$\text{PE} = -\boldsymbol{m} \bullet \boldsymbol{B} \tag{8.95}$$

*Equation 8.95 gives the potential energy of a magnetic dipole* ***m*** *in an external magnetic field* ***B***.

## *Example 8.11*

*The potential energy of a magnetic dipole.* Find the potential energy of a magnetic dipole in an external magnetic field when (a) it is antiparallel to **B** (i.e., $\theta = 180^0$), (b) it is perpendicular to **B** (i.e., $\theta = 90^0$), and (c) it is aligned with **B** (i.e., $\theta = 0$).

## *Solution*

The potential energy of the dipole, found from equation 8.94, is
(a)

$$\text{PE} = -\, mB \cos 180^0$$
$$\text{PE} = +\, mB$$

(b)

$$\text{PE} = -\, mB \cos 90^0$$
$$\text{PE} = 0$$

(c)

$$\text{PE} = -\, mB \cos 0^0$$
$$\text{PE} = -\, mB$$

Thus, the magnetic dipole has its highest potential energy when it is antiparallel ($180^0$), decreases to zero when it is perpendicular ($90^0$), and decreases to its lowest potential energy, a negative value, when it is aligned with the magnetic field, $\theta = 0^0$. This is shown in figure 8.19. So, just as the rock falls from a position of high potential energy to the ground where it has its lowest potential energy, the dipole, if given a slight push to get it started, rotates from its highest potential energy (antiparallel) to its lowest potential energy (parallel).

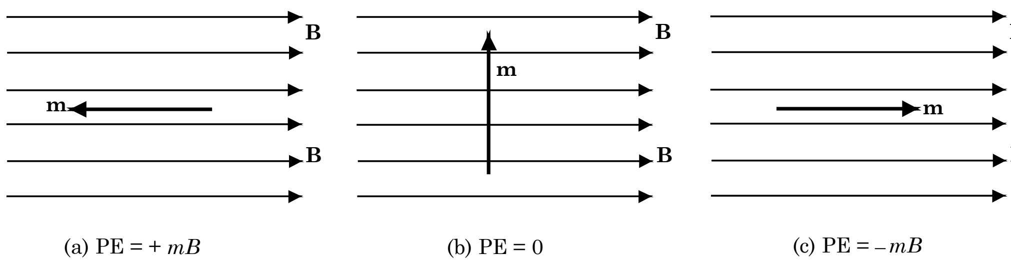


***Figure 8.19*** Potential energy of a magnetic dipole **m** in an external magnetic field **B**.

## 8.16 Magnetic Flux

Just as the electric flux was defined in chapter 4 as a quantitative measure of the number of electric field lines passing normally through a surface, the magnetic flux $\Phi_M$ can be defined as a quantitative measure of the number of magnetic field lines **B** passing normally through a particular surface area **A**. Figure 8.20(a), shows a magnetic field **B** passing through a portion of a surface of area **A**. *The magnetic flux is defined to be*

$$\Phi_M = \mathbf{B} \bullet \mathbf{A} \tag{8.96}$$

*and is a quantitative measure of the number of lines of* ***B*** *that pass normally through the surface area* ***A***. The number of lines represents the strength of the field. The vector **B**, at the point $P$ of figure 8.20(a), can be resolved into the components, $B_\perp$ the component perpendicular to the surface, and $B_{||}$ the parallel component. The perpendicular component is given by

$$B_\perp = B\cos\theta$$

while the parallel component is given by

$$B_{||} = B\sin\theta$$

The parallel component $B_{||}$ lies in the surface itself and therefore does not pass through the surface, while the perpendicular component $B_\perp$ completely passes

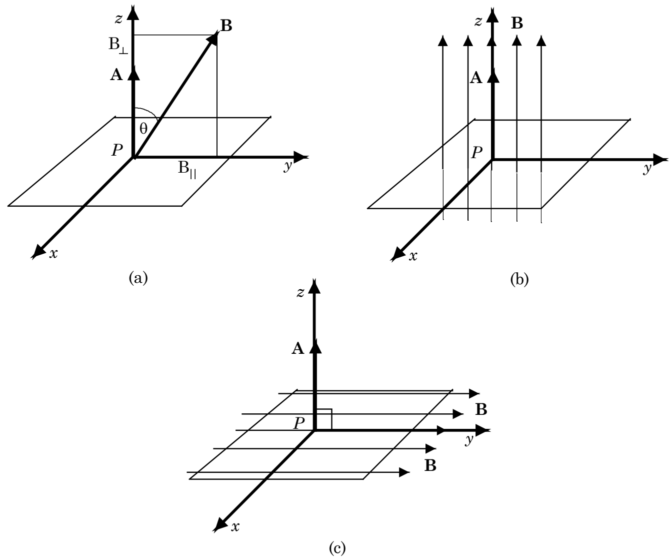


*Figure 8.20* The magnetic flux.

through the surface at the point $P$. The product of the perpendicular component $B_\perp$ and the area $A$

$$B_\perp A = (B\cos\theta)A = BA\cos\theta = \mathbf{B} \bullet \mathbf{A} = \Phi_M \tag{8.97}$$

is therefore a quantitative measure of the number of lines of **B** passing normally through the entire surface area **A**. If the angle θ in equation 8.97 is zero, then **B** is parallel to the vector **A** and all the lines of **B** pass normally through the surface area $A$, as seen in figure 8.20(b). If the angle θ in equation 8.97 is $90^0$ then **B** is perpendicular to the area vector **A**, and none of the lines of **B** pass through the surface $A$ as seen in figure 8.20(c).

One of the units for the magnetic field was shown to be 1 Tesla = Weber/m². This unit was listed in anticipation of the introduction of the magnetic flux. The unit for magnetic flux will now be defined from the formula as

$$\Phi_M = BA$$
$$\Phi_M = \frac{\text{Weber}}{\text{m}^2}\ \text{m}^2 = \text{Weber}$$

Hence, the unit for magnetic flux is the Weber. It will be abbreviated as Wb.

## *Example 8.12*

*Magnetic flux.* A magnetic field of $5.00 \times 10^{-2}$ T passes through a plane 25.0 cm by 35.0 cm at an angle of $40.0^0$ to the normal. Find the magnetic flux $\Phi_M$ passing through the plane.

***Solution***

The area of the plane is

$$A = (0.250\text{ m})(0.350\text{ m}) = 8.75 \times 10^{-2}\text{ m}^2$$

The magnetic flux is found from equation 8.97 as

$$\Phi_M = BA\cos\theta$$

$$\Phi_M = (5.00 \times 10^{-2}\text{ T})(8.75 \times 10^{-2}\text{ m}^2)\cos 40.0^0 \left(\frac{\text{Wb/m}^2}{1\text{ T}}\right)$$

$$\Phi_M = 3.35 \times 10^{-3}\text{ Wb}$$

**To go to this Interactive Example click on this sentence.**

## 8.17 Gauss's Law for Magnetism

Just as there is an electric flux $\Phi_E$ associated with an electric field **E** passing through a surface area **A,** there is also a magnetic flux $\Phi_M$ associated with a magnetic field **B** passing through a surface area **A.** The magnetic flux was defined in section 8.16, and was given by equation 8.96 as

$$\Phi_M = \mathbf{B} \bullet \mathbf{A}$$

Because of the similarity of these fluxes it is reasonable to assume that Gauss's law should also apply to magnetism. Gauss's law for electricity was found in equation 4-14 as

$$\Phi_E = \oint \boldsymbol{E} \bullet d\boldsymbol{A} = \frac{q}{\varepsilon_o} \tag{4-14}$$

Where $\Phi_E$, the electric flux was a measure of the electric charge $q$ enclosed within the Gaussian surface. It is therefore reasonable to assume that Gauss's law for magnetism should take the same form as equation 4-14. If the assumption is valid, Gauss's law for magnetism should be written as

$$\Phi_M = \oint \boldsymbol{B} \bullet d\boldsymbol{A} = \text{enclosed magnetic pole} \tag{8.98}$$

But here we run into a slight difficulty. The magnetic flux should be a measure of the amount of magnetic pole enclosed within the Gaussian surface. *But, as has been seen in section 8.14, isolated magnetic poles do not exist.* Thus the term for the enclosed magnetic pole on the right-hand side of equation 8.98 must be equal to zero. *Thus **Gauss's law for magnetism** becomes*

$$\Phi_M = \oint \boldsymbol{B} \bullet d\boldsymbol{A} = 0 \tag{8.99}$$

Just as we considered Gaussian surfaces at various positions in the field of an electric dipole in figure 4.3, we will consider Gaussian surfaces at various positions in the field of a magnetic dipole, represented by the simple bar magnet in figure 8.21.

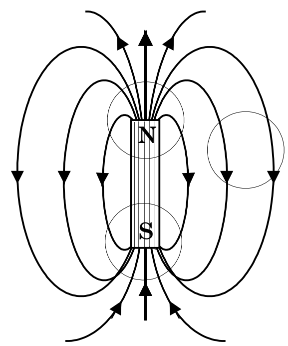


***Figure 8.21*** Gaussian surfaces and a magnetic dipole.

In a bar magnet, the magnetic field lines also go through the bar, as shown in figure 8.21. *Hence, the amount of magnetic flux entering the Gaussian surface around the north magnetic pole inside the bar magnet is equal to the amount of magnetic flux coming out of the same Gaussian surface outside of the bar magnet. Thus, the flux into the Gaussian surface is equal to the flux out of the Gaussian surface and the net flux through the Gaussian surface surrounding the north magnetic pole is zero.*

In a similar way, the magnetic field lines entering the south magnetic pole continue through the bar magnet until they emerge at the north magnetic pole. *Hence, a Gaussian surface surrounding the south magnetic pole also has a net flux of zero passing through it.* That is, the flux into the Gaussian surface is equal to the flux out because all magnetic field lines form closed loops. *The net flux through any Gaussian surface anywhere in a magnetic field is zero because there can never be any isolated magnetic poles.*

The electric field vectors in an electrostatic field always begin and end on electric charges. Because there are no isolated magnetic poles, magnetic field vectors do not begin or end on magnetic poles, but rather, are always continuous.

## 8.18 Gauss's Law for Magnetism in Differential Form

In chapter 4 we found Gauss's law in integral form for the electric field to be

$$\Phi_E = \oint \boldsymbol{E} \bullet d\boldsymbol{A} = \frac{q}{\varepsilon_o} \tag{4-14}$$

and in differential form

$$\nabla \bullet \boldsymbol{E} = \frac{\rho}{\varepsilon_o} \tag{4-87}$$

We would like to find the equivalent equation for Gauss's law in differential form for magnetism. We already showed that the integral form of Gauss's law for a magnetic field is

$$\Phi_E = \oint \boldsymbol{B} \bullet d\boldsymbol{A} = 0 \tag{8.99}$$

Equation 8.99 is the magnetic equivalent of equation 4-14. How do we find the magnetic counterpart to equation 4-87? Recall that in chapter 4 we also found that Gauss's divergence theorem was

$$\oint \boldsymbol{E} \bullet d\boldsymbol{A} = \int [\nabla \bullet \boldsymbol{E}] dV \tag{4-88}$$

Gauss's divergence theorem relates the amount of a vector quantity **E** emerging from a surface to the divergence of that same vector quantity **E** from out of the volume enclosed by that surface. Gauss's divergence theorem applies to any vector, and we now apply it to the magnetic field vector **B** to obtain

$$\oint \boldsymbol{B} \bullet d\boldsymbol{A} = \int [\nabla \bullet \boldsymbol{B}] dV \tag{8.100}$$

Combing Gauss's law, equation 8.99, and Gauss's divergence theorem, equation 8.100, we get

$$\oint \boldsymbol{B} \bullet d\boldsymbol{A} = 0 = \int [\nabla \bullet \boldsymbol{B}] dV \tag{8.101}$$

Which we can write simply as

$$\int [\nabla \bullet \boldsymbol{B}] dV = 0 \tag{8.102}$$

For the integral in equation 8.102 to be equal to zero for all values of **B** the entire term in brackets must equal zero. That is,

$$\nabla \bullet \boldsymbol{B} = 0 \tag{8.103}$$

*Equation 8.103 is Gauss's law in differential form for a magnetic field.* It says that the divergence of **B** is always equal to zero. *Equation 8.103 is another of Maxwell's equations* and we will go into more detail with it in chapter 10.

## Summary of Important Concepts

**Magnetism** - The study of magnetic forces and fields.

**Magnetic Field** - The field of force in the neighborhood of a magnetized body, or a current carrying wire. It is measured by the magnetic induction or magnetic flux density.

**Fundamental Principle of Magnetostatics** - Like magnetic poles repel each other; unlike magnetic poles attract each other.

**Magnetic Induction or Magnetic flux Density** - is equal to the force per unit charge per unit velocity that acts on a charge that is moving perpendicular to the magnetic field. It is also the force acting on a wire of unit length carrying a unit current, when placed in the magnetic field.

**Tesla** - The SI unit for the magnetic field.

**Weber** - The unit of magnetic flux.

**Right Hand Rule** - To determine the direction of the magnetic field around a wire carrying a current, grasp the wire with the right hand, with the thumb in the direction of the current, the fingers will curl around the wire in the direction of the magnetic field.

**Biot-Savart Law** - A law that relates the amount of magnetic field generated by a small element of wire carrying a current.

**Ampere's Law** - Along any arbitrary path encircling a total current, the sum of the scalar product of the magnetic induction with the element of length of the path, is equal to the permeability times the total current enclosed by the path.

**Magnetic Poles** - Since both negative and positive electric charges can exist separately, it is reasonable to ask if magnetic poles can exist separately? The simplest test, would be to cut a bar magnet in half, expecting to obtain one isolated north pole and one isolated south pole. When the experiment is performed, however, two smaller bar magnets are found, each with a north pole and a south pole. No matter how many times the bar magnet is subdivided, an isolated magnetic pole is never found. We always get a dipole. This is why we can never isolate a north pole or a south pole, because they do not exist. What does exist is an orbiting, rotating electric charge, that creates a magnetic dipole moment.

**Magnetic Flux** - The magnetic flux is a quantitative measure of the number of lines of the magnetic field **B** that pass normally through a surface area **A**.

**Gauss's law for magnetism** - The amount of magnetic flux entering a Gaussian surface is equal to the amount of magnetic flux coming out of the same Gaussian surface. The net flux through any Gaussian surface anywhere in a magnetic field is zero because there are no isolated magnetic poles, and hence, magnetic field vectors do not begin or end on magnetic poles, but rather, are always continuous.

# Summary of Important Equations

Force on a charged particle in an external magnetic field

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad (8.1)$$

$$F = qvB \sin\theta \quad (8.2)$$

Definition of the magnetic induction $B = \dfrac{F}{qv_\perp}$ (8.4)

Force on a current carrying conductor in an external magnetic field

$$\mathbf{F} = I\boldsymbol{l} \times \mathbf{B} \quad (8.23)$$

Biot-Savart Law $d\mathbf{B} = \dfrac{\mu_o I}{4\pi} \dfrac{d\boldsymbol{l} \times \mathbf{r}}{r^3}$ (8.19)

Total Magnetic field $\mathbf{B} = \int d\mathbf{B}$ (8.20)

Magnetic field at the center of a circular current loop $B = \dfrac{\mu_o I}{2r}$ (8.28)

Ampere's circuital law $\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I$ (8.50)

Magnetic field around a long straight wire $B = \dfrac{\mu_o I}{2\pi r}$ (8.48)

Magnetic field inside a solenoid $B = \mu_o n I$ (8.58)

Magnetic field inside a toroid $B = \dfrac{\mu_o NI}{2\pi r}$ (8.64)

Torque on a current loop in an external magnetic field $\tau = IAB \sin\theta$ (8.81)

Magnetic dipole moment of a current loop

$$\mathbf{m} = IA\ \mathbf{n} \quad (8.82)$$

$$\mathbf{m} = NIA\ \mathbf{n} \quad (8.83)$$

Torque on a current loop in an external magnetic field

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \quad (8.85)$$

$$\tau = mB \sin\theta \quad (8.84)$$

Potential energy of a magnetic dipole **m** in an external magnetic field **B**

$$\text{PE} = -mB \cos\theta \quad (8.94)$$

$$\text{PE} = -\boldsymbol{m} \bullet \boldsymbol{B} \quad (8.95)$$

Magnetic flux

$$\Phi_M = \mathbf{B} \bullet \mathbf{A} \quad (8.96)$$

$$\Phi_M = BA \cos\theta \quad (8.97)$$

Gauss's law for magnetism

Integral form $$\Phi_M = \oint \boldsymbol{B} \bullet d\boldsymbol{A} = 0 \tag{8.99}$$

Differential form $$\nabla \bullet \boldsymbol{B} = 0 \tag{8.103}$$

# Questions for Chapter 8

1. Should there be a law similar to Coulomb's law of electrostatics that shows the force between magnetic poles? What would be the advantages and disadvantages of such a law?

2. In the very early days of nuclear physics, nuclear radiation was described in terms of alpha, beta, and gamma particles. How did Rutherford use a magnetic field to distinguish between these particles.

3. A charge, in motion with a drift velocity $\mathbf{v}_d$ in a long straight wire, generates a magnetic field around the wire. If you were to move parallel to the wire at the same velocity, would you still observe a magnetic field? If not, where did the field go?

4. Since a moving electric charge creates a magnetic field, does moving a magnet create an electric field?

5. How can you use a magnetic field to separate isotopes of a chemical element?

6. How can you make a bar magnet?

7. What causes the earth's magnetic field? Is the field constant or does it change with time? Is it possible for the earth's poles to flip, i.e., for the north pole to become the south and vice versa?

8. If you heat a bar magnet it loses its magnetism. Why?

9. What is Magneto-hydrodynamics?

# Problems for Chapter 8

1. A proton, moving at a speed of $1.62 \times 10^3$ m/s enters a magnetic field of 0.250 T, at an angle of $43.5^0$. Find the force acting on the proton.

2. An electron moving at a speed of $3.00 \times 10^6$ m/s enters a magnetic field of 0.200 T at an angle of $35.0^0$. Find the force acting on the electron.

3. An electron, moving at a speed of $3.00 \times 10^5$ m/s, enters a magnetic field of 0.250 T, at an angle of $30.0^0$. (a) Find the force on the electron, and (b) find the acceleration of the electron.

4. What is the force on a proton moving north to south at a speed of $3.00 \times 10^5$ m/s in the earth's magnetic field, if the vertical component of the earth's magnetic field at that location is $25.0 \times 10^{-6}$ T?

5. A wire 35.0 cm long, carrying a current of 3.50 A is placed at an angle of $40.0^0$ in a uniform magnetic field of 0.002 T. Find the force on the wire.

6. What is the maximum force acting on a 20.0 cm wire, carrying a current of 5.00 A, in the magnetic field of the earth whose horizontal component is $20.0 \times 10^{-6}$ T.

7. A long straight wire carries a current of 10.0 A. Find the magnetic field 5.00 cm from the wire.

8. A power line 10.0 meters high carries a current of 200 A. Find the magnetic field of the wire at the ground.

9. A long straight wire carries a current of 10.0 A. How far from the wire will the magnetic field be (a) 1.00 T, (b) 0.100 T, (c) $1.00 \text{ x } 10^{-2}$ T, (d) $1.00 \times 10^{-3}$ T?

10. What current is necessary to generate a magnetic field of 0.100 T at a distance of 10.0 cm from a long straight wire?

11. Two long parallel wires each carry a current of 5.00 A. If the wires are 15.00 cm apart, find the magnetic field midway between them if (a) the currents are in the same direction, and (b) the currents are in the opposite direction.

12. Two parallel wires 10.0 cm apart carry currents of 10.0 A each. Find the magnetic field 5.00 cm to the left of wire 1, 5.00 cm to the right of wire 1, and 15.00 cm to the right of wire 1, if (a) the currents are in the same direction, and (b) the currents are in the opposite directions.

13. A circular loop of wire of radius 5.00 cm carries a current of 3.00 A. Find the magnetic induction at the center of the current loop.

14. A circular current loop of 10 turns carries a current of 5.00 A. If the radius of the loop is 5.00 cm, find the magnetic field at the center of the loop.

15. It is desired to neutralize the vertical component of the earth's magnetic field, $20.0 \times 10^{-6}$ T, at a particular point. A flat circular coil is mounted horizontally over this point. If the coil has 10 turns and has a radius of 10.0 cm, what current is necessary and in what direction should it flow through the coil?

16. How many loops of wire are necessary to give a magnetic field of $1.50 \times 10^{-3}$ T at the center of a circular current loop carrying a current of 10.0 A, it the radius of the loop is 5.00 cm?

17. You are asked to design a circular coil that will have a value of $3.00 \times 10^{-2}$ T at its center. Find the ratio of the current in the coil to the radius of the coil that will give this value of $B$. Pick a reasonable value for the combination of $I$ and $r$ such that the current is not too large nor the radius too small. Would it be desirable to introduce more than one loop of wire? What would be a better combination of $I$, $r$, and $N$?

18. In the Bohr model of the hydrogen atom, a negative electron orbits about a positive proton at a radius of $5.29 \times 10^{-11}$ m, at a speed of $2.19 \times 10^{6}$ m/s. (a) How long does it take for the electron to revolve around the proton? (b) From the definition of current show that the orbiting electron constitutes a current and determine its magnitude. (c) Since the orbiting electron looks like a current loop, determine the magnetic field at the location of the proton caused by the orbiting electron.

19. Everyone knows that the earth revolves around the sun. Yet, in our every day experience the sun is seen to rise in the eastern sky and set in the western sky, as though the sun revolved around the earth. The observed motion seems to depend

upon the frame of reference of the observer. When the frame of reference is placed on the earth, the sun appears to revolve around the earth. In a similar way, if the frame of reference is placed on the orbiting electron in the Bohr theory of the atom, it appears as though the proton is moving in a circular orbit about the electron. Find the value of the magnetic field at the position of the electron, caused by the proton.

20. A solenoid is 20.0 cm long and carries 500 turns of wire. If the current in the solenoid is 2.00 A, find the magnetic field inside the solenoid.

21. You are asked to design a solenoid that will give a magnetic field of 0.100 T, yet the current must not exceed 10.0 A. Find the number of turns per unit length, that the solenoid should have.

22. You are asked to design a solenoid by wrapping insulated copper wire around the hollow cardboard core of an empty roll of paper towels. The completed solenoid will then be connected to a 12.0 V battery and the maximum current that can flow in the circuit is 10.0 A. (a) What is the minimum value of the resistance of this solenoid? (b) If the wire used is #22 S & W gauge, copper wire, which has a diameter of $6.44 \times 10^{-4}$ m, what is the minimum length of this wire. (c) With the above restrictions on $I$ and $l$, how many turns of wire can you have if the diameter of the solenoid is 4.00 cm? (d) If the length of the cardboard core is 28.0 cm, and with all of the above restrictions, find the value of $B$ inside the solenoid. (e) In this design, what factors might be changed to increase the value of $B$?

23. Two parallel wires 15.0 cm apart carry currents of 10.0 A in the same direction. If the wires are 25.0 cm long, (a) find the magnitude and direction of the force on each wire. (b) If the direction of one current is reversed, find the force on each wire.

24. A coil of wire of 10.0 cm radius carries a current of 5.00 A. Find its magnetic dipole moment.

25. A coil of wire has a magnetic dipole moment of 25.0 A $m^2$. It is placed perpendicular to the horizontal magnetic field of the earth of $20.0 \times 10^{-6}$ T. What torque will act on the coil?

26. A rectangular galvanometer coil 2.00 cm by 1.50 cm has 10 turns of wire. If the current through the coil is 3.00 mA, find its magnetic dipole moment. The coil is placed in a magnetic field of 0.300 T. Find the torque on the coil when the magnetic dipole moment makes an angle of $30.0^0$ with the magnetic field.

27. A coil of 3.00 cm radius, carrying a current of 2.00 A is placed within a solenoid that carries a current of 3.00 A. If the solenoid has 5000 turns per meter, find the torque on the coil.

28. The coil in a D.C. motor is called an armature. The armature, 15.0 cm by 10.0 cm, carries a current of 3.50 A in a magnetic field of 0.250 T. Find the maximum torque exerted by the motor. If the armature rotates at 1000 revolutions per minute, what is the power rating of the motor?

29. Starting with the solution for the magnetic field on axis for a circular current loop, equation 8-43, show that very far away from the loop, that is $z >> R$, that the magnetic field looks like the magnetic field of a magnetic dipole, that is,

$$B = \frac{\mu_o}{2\pi}\frac{m}{z^3}$$

30. A rectangular coil 6.00 cm by 8.00 cm is located in a uniform magnetic field of 0.250 T. Find the flux through the coil when the plane of the coil is (a) perpendicular to **B,** (b) parallel to **B,** and (c) makes an angle of $60.0^0$ with **B.**

31. A circular coil, 6.00 cm in diameter, is placed in a uniform magnetic field of 0.300 T. Find the flux through the coil when the coil makes an angle of $53.0^0$ with **B.** What is the flux if the angle is increased to $90.0^0$?

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