DNA 3218T

A METHOD FOR ESTIMATING THE X-RAY PRODUCED ELECTROMAGNETIC PULSE OBSERVED IN THE SOURCE REGION OF A HIGH-ALTITUDE BURST

TN 181

D.F. Higgins
C.L. Longmire
A.A. O'Dell
Mission Research Corporation
735 State Street
Santa Barbara, California 93102

~~29 November 1973~~ February 1973

Topical Report

CONTRACT No. DASA 01-71-C-0105

THIS WORK SPONSORED BY THE DEFENSE NUCLEAR AGENCY
UNDER SUBTASK EA094-05.

Prepared for
Director
DEFENSE NUCLEAR AGENCY
Washington, D. C. 20305

TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

This report describes an approximate method of calculating the electromagnetic pulse (EMP) generated by the X-ray output of a nuclear weapon.

Most of the calculations of EMP generation made in the past have used the gamma rays rather than the X rays. This choice was appropriate for observers located below about 30-km altitude, regardless of the burst location. For low-altitude bursts the X rays are normally absorbed in the air within a few tens of meters from the burst, whereas the more energetic gamma rays proceed outwards to hundreds of meters and even kilometers from the burst. The high electrical conductivity induced in the air in the gamma-ray EMP source region effectively screens more distant observers from electrical effects produced by the X rays near the burst. The same effect occurs for high-altitude bursts, as long as the observer is below about 30-km altitude. Although the X rays may produce an EMP in the altitude range 50 to 100 km where they are normally absorbed, this EMP is attenuated in the gamma-ray absorption region between 20 and 40 km altitude, and is replaced by the gamma-ray EMP for lower observers.

On the other hand, if both the burst and the observer are at altitudes higher than about 50 km, the X-ray induced EMP may be dominant, for two reasons. First, the gamma rays interact only weakly with the thin atmosphere at these altitudes, whereas the X rays are strongly absorbed between 50- and 100-km altitude. Second, there are so many more X rays than gamma rays (by a factor 10^4 to 10^5).

We do not, however, expect the X-ray induced EMP, in its domain, to be larger than the gamma-ray induced EMP seen by low-altitude observers, because of the phenomenon of saturation. The peak electric field $\vec{E}$ tends to be determined, approximately, by a balance between the photon-induced source current and the neutralizing

secondary electron current driven by $\vec{E}$. The photoelectrons from the X rays produce a smaller ratio of net current to secondary ionization than do the higher energy Compton recoil electrons from the gamma rays, and so should lead to a substantially smaller saturated $\vec{E}$ at balance. The main possibility of finding a large EMP in the X-ray domain would therefore appear to come at early times for a fast-rising X-ray flux, before the secondary electrons have had time to build up. Consideration of the rate of build up of secondary ionization has led us to expect that the most significant part of the EMP will occur at times of the order of 10^{-8} second, or less, after the beginning of the X-ray flux.

This short expected time scale has influenced the choice of approximations made in the following sections of this report for the purpose of simplifying the calculations. For example, in such short times, the geomagnetic deflection of the photoelectrons is small, so that the net transverse current is small compared with the net radial current. As a result, we expect that the dominant part of the EMP will be simply a radial electric field. Since a radial $\vec{E}$ field does not propagate (as do transverse fields), its determination requires solution of time-dependent equations at only one point in space, rather than partial differential equations in space and time. This fact greatly simplifies the calculation.

This approximation, that the radial $\vec{E}$ is the only significant field, is made in this report. By use of several other simplifying approximations, we derive a set of ordinary differential equations in time for $\vec{E}$, for the photoelectron and secondary electron currents, and for two other required parameters. In another report we give calculated results for some typical cases, together with a discussion of the results and of the validity of the approximations in hindsight.[8]

Of course this non-propagating EMP would be seen only by observers in the X-ray absorption region.

2. DEVELOPMENT OF THE EQUATIONS

2.1 Maxwell's Equations

In general, the time variations of the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are given by the two Maxwell equations

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \tag{1}$$

$$\frac{1}{c}\frac{\partial \vec{E}}{\partial t} = -4\pi\vec{J} + \nabla \times \vec{B} \,. \tag{2}$$

The above equations are written in cgs Gaussian units; thus charge and electric fields are in esu and currents and magnetic fields are in emu. The current density $\vec{J}$ is assumed to include both the primary source current J_p and the secondary current J_s.

In our case the source current J_p is approximately radial (from the burst point) and depends only very slowly on the (spherical coordinate) angles. Therefore, from Equation 2, $\vec{E}$ is also radial and depends only very slowly on angles. Since the curl of such an $\vec{E}$ is small, Equation 1 indicates that $\vec{B}$ will be small, and we shall neglect $\vec{B}$ altogether. Thus Equation 2 becomes

$$\frac{1}{c}\frac{\partial E}{\partial t} = -4\pi[J_p + J_s] \tag{3}$$

where E is the radial component of $\vec{E}$. To solve this equation for the electric field as a function of time at a given point we need to know the time histories of both J_p and J_s, the primary and secondary current densities.

2.2 Primary Current Density

The X-ray pulse produced by a nuclear burst interacts with the nearby air atoms to create "free" Compton and photoelectrons.

These electrons will be ejected with some radial velocity, v_r. The current due to a single electron of velocity v_r is given by

$$i_e = - \frac{e}{c} v_r \text{ (abamp)} . \tag{4}$$

To calculate the total current density, J_p, we multiply i_e by the number of electrons per cm^3 having radial velocity v_r and integrate over all velocities. In other words, what is needed is the velocity distribution function $f(v_r, t)$ describing the number of electrons per cm^3 per unit (radial) velocity as a function of time. Then the total current density, J_{total}, is given by

$$J_{total} = - \frac{e}{c} \int f(v_r, t) v_r \, dv_r . \tag{5}$$

Note that in the most general case, the velocity distribution function will include both primary and secondary electrons so that J_{total} is the sum of J_p and J_S. For the present, however, let us focus our attention on the primary current alone.

The problem with this formulation for J_{total} is that, in general, the function $f(v_r, t)$ is difficult to evaluate. The initial value of f can be determined from an assumed X-ray flux and the Compton and photo effect differential cross sections. However, the distribution function changes from this initial value due to several factors. The electric field built up by the current tends to slow down the electrons. Ionization along the electron's path implies an effective drag force, also slowing the electron. Electron scattering will, to first order change only the electron's direction and not the magnitude of its velocity; but the radial component, which determines the net current, will be changed. (One might also consider electron-ion recombination, but the rate of this process at high altitudes is very slow and can be ignored in these calculations.)

One can find this distribution as a function of time by solving an appropriate form of the Boltzmann equation (see Appendix 1).

However, since solving the Boltzmann equation is a rather tedious and complicated task, we have chosen alternatively to make several simplifying assumptions to facilitate a first estimate of the X-ray EMP fields.

We can avoid solving the full Boltzmann equation by making use of the fact that the X-ray produced electrons "live" a long time compared with the duration of the main part of the EMP, which we expect to be of the order of 10^{-8} second. We shall show below that the slowing-down time of the photoelectrons is longer than this by a factor of 10 or more. Thus the photoelectrons may be assumed to have approximately constant energy over the period of interest, and this result allows a simplified treatment of the primary current, as will be seen below.

Consider a beam of monoenergetic X rays generated from a weapon burst very high in the atmosphere as indicated in Figure 1. The energy deposited per cm^3 at point P in the atmosphere can be written as

$$N = \phi \kappa_a \rho e^{-\int_0^R \kappa_T \rho(R)\, dR} , \qquad (6)$$

where ϕ is the unattenuated (by air) X-ray intensity, κ_a is the mass absorption coefficient (cm^2/gm) of air, ρ is the air density (gm/cm^3), and κ_T is the mass attenuation coefficient. The integral quantity

$$L \equiv \int_0^R \kappa_T \rho(R)\, dR ,$$

is simply the number of X-ray mean-free paths between the burst point and P. If we assume an exponential atmosphere, then

$$\rho(h) = \rho_0 e^{-h/h_0} , \qquad (7)$$

where h_0 is the scale height (~ 7 km). Using Equation 7 in the expression for L, we find

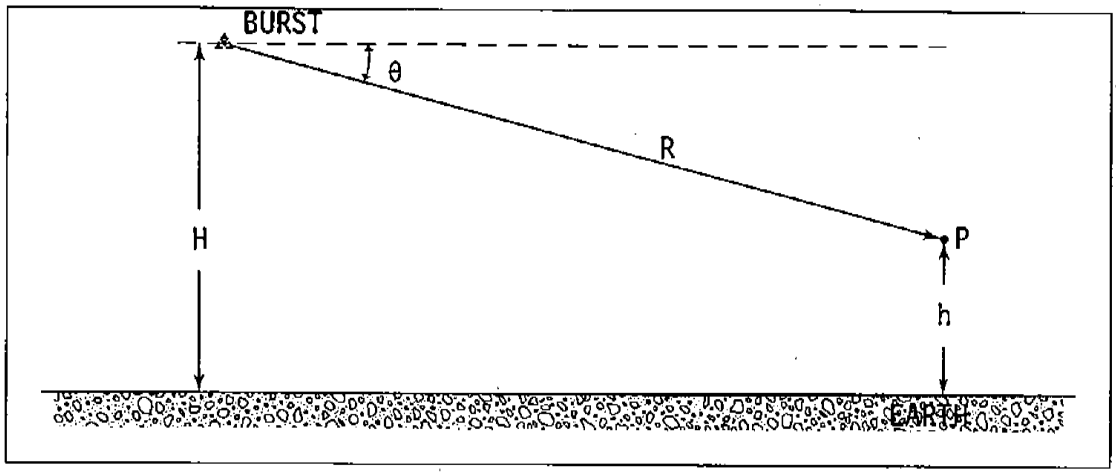

Figure 1. Assumed geometry of problem.

$$L = \int_0^R \kappa_T \rho(R)\, dR = \int_h^H \kappa_T \frac{\rho(h)\, dh}{\sin\theta}$$

$$= \int_h^H \frac{\kappa_T \rho_0}{\sin\theta} e^{-h/h_0}\, dh$$

$$= \frac{\kappa_T \rho_0 h_0}{\sin\theta}\left[e^{-h/h_0} - e^{-H/h_0} \right] ,$$

which for $H >> h_0$ can be written approximately as

$$L = \frac{\kappa_T h_0}{\sin\theta} \rho_0 e^{-h/h_0} = \frac{\kappa_T h_0 \rho}{\sin\theta} . \qquad (8)$$

Incorporating Equation 8 into Equation 6, we have

$$N = \phi \kappa_a \rho e^{-\frac{\kappa_T h_0 \rho}{\sin\theta}} .$$

We can determine the altitude h at which maximum energy deposition occurs by requiring $dN/d\rho = 0$. Thus we find that the density ρ_m which maximizes N is

$$\rho_m = \frac{\sin\theta}{\kappa_T h_0} \quad . \tag{9}$$

By using Equation 7 we can then write the equivalent altitude h_m as

$$h_m = h_0 \ell n\left(\frac{\kappa_T h_0 \rho_0}{\sin\theta}\right) \quad . \tag{10}$$

As a specific example, we have taken the dip angle θ to be 30°, the scale height to be 7.0 km, and ρ_0 to be 1.43×10^{-3} gm/cm^3. We now wish to estimate the lifetime (before stopping) of photoelectrons produced by the X rays. For a given X-ray energy u we first calculate ρ_m from Equation 9 and h_m from Equation 10. Assuming the photoelectron is born at altitude h_m with energy u, the electron range r (cm) can then be computed at the local density ρ_m. The lifetime of the photoelectron will be of the order of

$$\Delta t \approx \frac{r}{v}$$

where v is the initial velocity of the ejected photoelectron. Table 1 lists the results of such calculations for selected X-ray energies.

Table 1. Electron lifetime at altitude of maximum absorption.

u keV	v/c	K(u) cm^2/gm	ρ_m gm/cm^3	h_m km	r(range) cm	$\Delta t = r/v$ sec
5	.14	32.4	2.3×10^{-8}	78	3.0×10^{3}	7.1×10^{-7}
10	.20	4.6	1.6×10^{-7}	65	1.5×10^{3}	2.5×10^{-7}
15	.24	1.6	4.7×10^{-7}	57	1.1×10^{3}	1.5×10^{-7}
20	.28	.79	9.4×10^{-7}	51	9.0×10^{2}	1.1×10^{-7}
30	.34	.37	2.0×10^{-6}	45	8.8×10^{2}	8.6×10^{-8}

We see from the table that the lifetime is indeed long compared with the expected duration of the pulse. This means that the density N(t) of active primary electrons is simply the time integral of their source, and that their average energy is roughly independent of time and equal to their average energy at birth. The primary current density is

$$J_p = -\frac{e}{c} N(t)\bar{v}_r(t) \ . \tag{11}$$

Here $\bar{v}_r$ is the mean radial velocity of the primary electrons at a given time; $\bar{v}_r$ varies with time due to the action of electric field and slowing down forces and to the continual injection of newly created electrons.

Let us make a small step δt in time and consider the change δJ_p in the primary current density. First, J_p will change because new electrons have been injected, and this change will be

$$\delta J_1 = -\frac{e}{c} \delta N \langle v_r \rangle \ , \tag{12}$$

where $\langle v_r \rangle$ denotes the mean radial velocity of the primary electrons at birth. Second, J_p will change because of a change in the velocity of the electrons previously injected, and this change will be

$$\delta J_2 = -\frac{e}{c} N \bar{a}_r \delta t \tag{13}$$

where $\bar{a}_r$ is the average acceleration of the electrons as given by Newton's law. Combining the two changes in J_p and dividing by δt, we find the differential equation

$$\frac{dJ_p}{dt} = -\frac{e}{c} S_0 \langle v_r \rangle - \frac{e}{c} N_p \bar{a}_r \ , \tag{14}$$

where S_0 is the rate of production of new photoelectrons and $\bar{a}_r$ is given by Newton's law as

$$\bar{a}_r = -\frac{eE}{m} - \frac{C_D}{m} \bar{v}_r \ . \tag{15}$$

Here C_D is defined as a drag coefficient which characterizes the average energy loss of the electrons as they pass through the air. The fact that the electrons are not slowed down very rapidly (i.e., they have a relatively long lifetime, Δt - see Table 1) means that C_D is almost constant. In fact, we evaluate C_D by averaging over the electron distribution at birth. It is the fact that the electron distribution does not change very much during times of interest that makes this a reasonable approximation.

Thus, one can rewrite Equation 14 as

$$\frac{dJ_P}{dt} = - e\bar{\beta}S_0(t) + \frac{e^2N(t)}{mc} E - \frac{C_D}{m} J_P \ , \tag{16}$$

where

$$\bar{\beta} = \frac{\langle v_r \rangle}{c} \ , \tag{17}$$

and

$$N(t) = \int_0^t S_0(t')dt' \ . \tag{18}$$

In these calculations, we will consider only photoelectrons since at photon energies in the X-ray range it is easily shown that Compton electrons have a negligible effect on the source current.[1,9]

Thus, Equation 16 gives us differential equation describing the time history of one of the current terms on the right hand side of Equation 3. Before going on to write down equations for the other term in Equation 3, let us further consider the right-hand side of Equation 16.

Let us first consider the source rate, $S_0(t)$. One can write

$$S_0(t) = G(t)\int_0^1 P(\beta)d\beta \ , \tag{19}$$

where

$$G(t) = \frac{ab}{b - a}\left(e^{-at} - e^{-bt}\right), \tag{20}$$

is a normalized time function used to describe the time history of the X-ray pulse produced by the nuclear weapon being considered and $P(\beta)$ is the velocity spectrum at birth of the photoelectrons summed over all angles. We have implicitly assumed that the weapon X-ray emission spectrum is time-independent. The velocity is written in terms of the dimensionless variable $\beta = v/c$. Now,

$$P(\beta)d\beta = F_\chi(u)\kappa_p(u)\rho\, du \tag{21}$$

where

$F_\chi(u)$ = the X-ray fluence of the weapon (photons/cm^2 keV)

$\kappa_p(u)$ = the photo effect absorption coefficient (cm^2/gm)

ρ = the local air density (gm/cm^3)

u = the photon energy (keV) .

In Equation 21 we have ignored the small effect of the electron's binding energy (a few hundred ev compared to tens of keV for the photon energy). The relation between u (units of keV) and β is then

$$du = mc^2\beta d\beta = 511\beta d\beta \text{ [keV]} \tag{22}$$

so that

$$P(\beta) = 511\beta\rho F_\chi(u)\kappa_p(u)\left[\frac{\text{electrons}}{\text{cm}^3}\right]. \tag{23}$$

In the region of interest, the photo effect mass absorption coefficient can be approximated by the equation[10]

$$\kappa_p(u) = 4.4\left(\frac{10}{u}\right)^{2.87}. \tag{24}$$

The uncollided X-ray fluence at the calculational point is given by the expression

$$F_X(u) = 2.08 \times 10^{17} \frac{Y_X}{R^2} \frac{\underline{X}(u)}{u} e^{-L(u)} \tag{25}$$

where

R = the slant range from the burst point (km)

Y_X = X-ray yield of the weapon (kt)

$\underline{X}(u)$ = the normalized energy spectrum of the weapon ($\int_0^\infty \underline{X}(u)du = 1$)

and

L(u) = the number of mean free paths between the burst and point of calculation.

We have not included the effects of scattered X rays. However, those X rays which are scattered into the calculational point at rather large angles will be sufficiently time delayed to be unimportant. Small angle (Rayleigh) scattering does not significantly alter the X-ray energy or direction and thus can be neglected. We can simply delete the coherent scattering cross section from $\kappa_T(u)$. Thus, one can write

$$L(u) = \kappa_T(u) \int_0^R \frac{\rho(h)}{\sin\theta} dr \tag{26}$$

where

ρ = the air density

θ = the dip angle measured from the burst point

and

$\kappa_T(u)$ = the total mass attenuation coefficient including Compton and photoelectric processes.

For X rays, $\kappa_T(u)$ can be described by the fit[10]

$$\kappa_T(u) = 4.4\left(\frac{10}{u}\right)^{2.87} + \frac{0.2}{1 + 3.52 \times 10^{-3} u} \left[\frac{cm^2}{gm}\right] . \tag{27}$$

Assuming the exponential atmosphere

$$\rho(h) = 1.43 \times 10^{-3} e^{-h/7.0} \left[\frac{gm}{cm^3}\right], \tag{28}$$

then one can write

$$L(u) = \frac{\kappa_T(u)}{\sin\theta} \left[M(h) - M(h_b)\right] \tag{29}$$

where

$$M(h) = 1.0 \times 10^{3} e^{-h/7.0} \left[\frac{gm}{cm^2}\right] \tag{30}$$

h_b = height of burst [km]

h = height of point of calculation [km]

and

θ = the dip angle from burst to point of observation.

Thus, given the point of burst, the point of observation, and the characteristics of the weapon (i.e., X-ray yield, X-ray time history, and energy spectrum) the above equations can be used to calculate the velocity distribution $P(\beta)$ from Equation 23 and thereby calculate the source term $S_0(t)$ by numerical integration (Equation 19).

Now let us consider how one goes about calculating the $\overline{\beta}$ term in Equation 16. To calculate $\overline{\beta}$ one needs to know the average radial velocity of the photoelectrons at birth. The radial component of velocity of an electron of velocity v is just (see Figure 2)

$$v_r = v\mu \tag{31}$$

where

$$\mu \equiv \cos\theta \ . \tag{32}$$

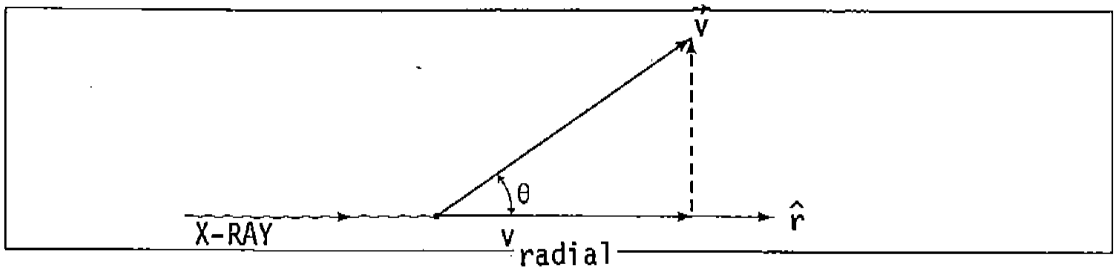

Figure 2. Electron production geometry.

The velocity distribution $P(\beta)$ was defined as the number of electrons of velocity β per unit volume summed over all angles so that the angular distribution was just integrated out. To include the angular distribution we define a differential velocity distribution

$$P_{\mu}(\beta, \mu) = P(\beta)\Phi(\beta, \mu) \tag{33}$$

where the angular distribution $\Phi(\beta, \mu)$ is normalized so that

$$\int_{-1}^{1} \Phi(\beta, \mu)d\mu = 1 \ . \tag{34}$$

For the photo effect (Reference 1)

$$\Phi(\beta, \mu) \propto \frac{\sin^2\theta}{(1 - \beta\cos\theta)^4} = \frac{1 - \mu^2}{(1 - \beta\mu)^4} \ . \tag{35}$$

Using the normalization requirement of Equation 34,

$$\Phi(\beta, \mu) = \frac{3(1 - \beta^2)^2(1 - \mu^2)}{4(1 - \beta\mu)^4} \ . \tag{36}$$

Thus, one can write $\overline{\beta}$ as

$$\overline{\beta} = \frac{\int_0^1 \int_{-1}^1 P_{\mu}(\beta, \mu)\mu\beta d\mu d\beta}{\int_0^1 \int_{-1}^1 P_{\mu}(\beta, \mu)d\mu d\beta}$$

which becomes

$$\overline{\beta} = \frac{\left[\int_0^1 \beta P(\beta)\,d\beta\right]\left[\int_{-1}^1 \mu\Phi(\beta, \mu)\,d\mu\right]}{\int_0^1 P(\beta)\,d\beta} . \qquad (37)$$

The angular integral can be analytically evaluated as

$$I_1(\beta) = \int_{-1}^{1} \mu\Phi(\beta, \mu)\,d\mu$$

$$= \left[\frac{1}{\beta^4} \ln\left(\frac{1 + \beta}{1 - \beta}\right) - \frac{2(3 - 5\beta^2)}{3\beta^3(1 - \beta^2)^2}\right] \frac{3(1 - \beta^2)^2}{4} . \qquad (38)$$

The integrals over β must be carried out numerically since P(β) depends on the energy spectrum of the specific weapon being considered.

Now consider the second term on the right hand side of Equation 16. N(t) is simply the total number of electrons produced (since losses due to recombination are assumed to be zero). Thus

$$N(t) = \int_0^t S_0(t')\,dt'$$

$$= \left[\int_0^t G(t')\,dt'\right]\left[\int_0^1 P(\beta)\,d\beta\right]$$

$$= \left[1 + \frac{a}{b - a} e^{-bt} - \frac{b}{b - a} e^{-at}\right]\left[\int_0^1 P(\beta)\,d\beta\right] . \qquad (39)$$

Note that this ignores transport effects where different numbers of electrons leave the vicinity of their birth than enter it. Ignoring such effects is a valid approximation since v_r/c is small compared to 1. (See Reference 8).

Now let us return to the drag term of Equation 16. As noted, we expect the drag coefficient, C_D, to vary only slightly as a function of time over times of interest. The fact that the electrons live a long time indicates that the whole drag term is probably fairly small compared to ther terms in Equation 16.

To evaluate the drag coefficient we first need to find the effective drag force. This can be found from the relation between extreme range and energy. The range curve is fit fairly well by the expression

$$R = \frac{2.4 \times 10^{-4}}{\rho} \left(\frac{w}{10}\right)^{1.82} \text{[cm]} \tag{40}$$

where

w = the electron energy in keV

and

ρ = the air density in gm/cm^3.

From Equation 40

$$dw = \left[\frac{A\rho}{w^{0.82}}\right] dR \tag{41}$$

where

$$A = 1.52 \times 10^5 \ .$$

The term inside the brackets is just the effective drag force acting on the electron. [The term in brackets must be multiplied by 1.6×10^{-9} to get the force in dynes.] Thus we can write the vector drag force, $\vec{F}_D$, as

$$\vec{F}_D = - 1.6 \times 10^{-9} \frac{A\rho}{w^{0.82}} \frac{\vec{v}}{v} \ . \tag{42}$$

The drag force is directed opposite to the direction of motion of the electron. The effect of the drag force on the radial component of the velocity is then described by

$$\left(\frac{dv_r}{dt}\right)_D = - \frac{1.6 \times 10^{-9}}{m} \frac{A\rho}{w^{0.82}} \frac{1}{v} v_r \ . \tag{43}$$

To define the drag coefficient we will average the above equation over the initial velocity distribution with the result that

$$\left(\frac{d\bar{v}_r}{dt}\right)_D = - \frac{1}{m} C_D \bar{v}_r \tag{44}$$

where

$$C_D = \overline{\left[\frac{1.6 \times 10^{-9} A\rho}{w^{0.82} v}\right]} = \frac{\frac{1.6 \times 10^{-9} A\rho}{\left(\frac{511}{2}\right)^{0.82} c} \int_0^1 \frac{P(\beta)d\beta}{\beta^{2.64}}}{\int_0^1 P(\beta)d\beta} \ , \tag{45}$$

where we have written the kinetic energy, w, in terms of the dimensionless velocity variable, $\beta = v/c$, and averaged over the distribution $P(\beta)$ given in Equation 23. This reduces to

$$C_D = 8.6 \times 10^{-17} \rho \frac{\int_0^1 \beta^{-2.64} P(\beta)d\beta}{\int_0^1 P(\beta)d\beta} \ , \tag{46}$$

where C_D has the units of dynes/cm/sec for ρ in gm/cm^3. The integral for C_D in Equation 46 is evaluated numerically. It is readily seen that C_D is a function of velocity and that the averaging over the initial velocity distribution is only valid if this velocity is not changed very much by the drag force.

This completes our discussion of the differential equation for the primary current density.

2.3 Secondary Current Density

The other term in Maxwell's equation that must be evaluated in order to calculate the electric field is the secondary current density, J_S. The secondary current term appears because the photoelectrons ionize the air as they pass through, creating numerous low-energy (few ev range) secondary electrons. These low-energy electrons will be acted on by the existing electric field to move with some average drift velocity, v_d, along the electric field lines. As a result one can write

$$J_S = -\frac{e}{c} N_e v_d , \tag{47}$$

where

N_e = the number per unit volume of low energy electrons.

Both N_e and v_D are functions of time which must be determined. To do this we will use an electron swarm theory treatment to deal with low energy electrons. Swarm theory assigns a mean temperature to the electrons and relates their drift velocity to the electric field and various collision frequencies. We find that the behavior of the swarm can be characterized by three equations.

The first of these equations is written as

$$\frac{dN_e}{dt} = S_1 + C_1 N_e . \tag{48}$$

This equation simply states that the rate of change of number density is equal to the electron source rates (i.e., number conservation). As written here, S_1 is the rate at which low-energy electrons are produced by the photoelectrons and $C_1 N_e$ is the rate at which the low-energy electrons produce further electrons by cascading. [C_1 is just the Townsend coefficient times the drift velocity and is thus a function of the energy of the swarm.]

The second swarm equation is

$$\frac{1}{N_e}\frac{d(N_e U)}{dt} = -\frac{2}{3} e v_d E - \nu_w [U - U_0] + S_2 \ , \qquad (49)$$

where

U = characteristic electron energy = kT_e where k is the Boltzmann constant and T_e is the effective electron temperature

ν_w = energy transfer collision frequency

U_0 = ambient air temperature (kT_{air})

and

S_2 = net rate that energy is added to the swarm.

This equation is just an expression for energy conservation. The 2/3 factor in the first term on the right-hand side of the equation just converts from kinetic energy $3kT_e/2$ to characteristic energy units (kT_e).

The third equation used to describe the electron swarm is written as

$$\frac{1}{N_e}\frac{d[N_e v_d]}{dt} = -\frac{e}{m} E - \nu_m v_d \ , \qquad (50)$$

where ν_m is the momentum transfer collision frequency. This equation is just Newton's law relating rate of change of momentum to the applied force.

Both Equation 49 and 50 can be rewritten with all the N_e terms on the right hand side. Thus

$$\frac{dU}{dt} = -\frac{2}{3} e v_d E - \nu_w [U - U_0] + S_2 - \frac{U}{N_e}\frac{dN_e}{dt} \qquad (51)$$

and

$$\frac{dv_d}{dt} = -\frac{e}{m} E - \nu_m v_d - \frac{v_d}{N_e} \frac{dN_e}{dt} . \qquad (52)$$

Therefore, we have written down three coupled differential equations (48, 51, and 52) for the parameters N_e, U, and v_d. [Equation 48 is coupled to the other two because C_1 is a function of U.] Since these swarm parameters depend on the electric field we have a set of five coupled differential equations (3, 16, 48, 50, and 51) to be solved simultaneously.

Now let us discuss the various source terms in the swarm equations. To begin with consider S_1, the rate at which low-energy electrons are produced due to ionization by the photoelectrons. We can calculate this rate from the range versus energy relationship

$$R = \frac{2.4 \times 10^{-4}}{\rho} \left(\frac{w}{10}\right)^{1.82} . \qquad (40)$$

The rate of energy loss of the photoelectron is then

$$\frac{dw}{dt} = \frac{1.52 \times 10^5 \rho}{w^{0.82}} \frac{dR}{dt}$$

$$= \frac{1.52 \times 10^5 \rho c}{(511/2)^{0.82}} \beta^{0.64} , \qquad (53)$$

where we have used the fact that

$$\frac{dR}{dt} = v = c\beta . \qquad (54)$$

Equation 53 is the rate of energy loss of a photoelectron of velocity β. The total rate of energy loss, I, is just the sum over the velocity distribution $P(\beta)$: i.e.,

$$I = 4.84 \times 10^{13} \rho \int_0^1 \beta^{0.64} P(\beta) d\beta . \qquad (55)$$

Now, it is well known that a high energy electron produces about one ionization electron for every 34 ev of energy loss when averaged over the entire range. However, some of these ionization electrons are produced from further ionization of the high energy secondaries rather than directly from the primaries. At early times, before the secondaries can produce further ionization, about one electron is produced for every 80 ev lost by the photoelectron (see Reference 2). Therefore, let us define w_I as the amount of primary energy lost in the production of an ionization electron, where $34 \leq w_I \leq 80$ ev. Then the rate of production of swarm electrons due to photoelectron ionization is just

$$S_1 = \frac{I}{w_I} = \frac{4.84 \times 10^{13}}{w_I} \rho \int_0^1 \beta^{0.64} P(\beta) d\beta \tag{56}$$

where S_1 has units of [electrons/cm^{3}sec], w_I is thus a parameter that can be varied to see its effect on functions being calculated.

The second source term to consider is S_2, the rate at which energy is added to the swarm. One can approximate S_2 by the equation

$$S_2 = \frac{2}{3}\left(\frac{\bar{w}}{N_e} S_1 - w_c C_1\right) \tag{57}$$

where $\bar{w}$ is the average energy of the ionization electrons produced by the photoelectrons and w_c is the average energy lost whenever cascading produces electrons ($w_c \simeq 14$ ev the ionization potential of air). The 2/3 factor converts the kinetic energy to temperature units. Note that S_2 has the units of ev/sec.

The value of $\bar{w}$ is not known with any high degree of accuracy and is thus left as a parameter that can be varied. However, if one uses a $w_I \simeq 80$ ev, the value of $\bar{w}$ can be estimated at about 50 ev. If the primary loses about 80 ev to create a secondary, about 14 ev will be used up for ionization and another 16 ev may typically end up as various inelastic excitations, leaving the secondary with about 50 ev

kinetic energy. (Note that these energies are just estimates.)

Another way of estimating $\bar{w}$ is by use of the approximate energy distribution of the secondaries (as discussed in Reference 2) where

$$dn = \frac{2\pi}{w_0} \frac{1}{(w_0^2 + w^2)} \, dw \tag{58}$$

where

dn = differential number density of secondary electrons

w = secondary kinetic energy (ev)

and

$w_0 \simeq 8$ ev.

Then

$$\bar{w} = \int_0^{W/2} w \, dn \tag{59}$$

$$= \frac{w_0}{\pi} \ln \left(1 + \frac{W^2}{4w_0^2}\right)$$

where W is the energy of the most energetic electron being considered. For W = 100 keV it is found that $\bar{w}$ = 44.5 ev. Thus the value of $\bar{w}$ to be used in Equation 57 is somewhere around 40 to 50 ev for w_I values of about 80 ev.

Now let us discuss some of the other terms beginning with C_1. C_1 is the rate at which swarm electrons are produced by cascading, i.e., the rate at which energetic swarm electrons create further electrons by ionization. For average swarm energies less than the ionization potential, cascading occurs due to the few electrons in the tail of the distribution that have high enough energy to strip electrons from neighboring atoms. Values of C_1 in this low-energy regime come from measurements of the Townsend coefficient and the drift velocity. When the average electron energy exceeds the ionization potential one can

just use the total ionization cross sections to find the cascading rate.

Figure 3 is a plot of the cascading rate as a function of swarm electron temperature, U. The curve shown is a plot of the fit

$$\frac{C_1}{N} = \frac{3.3 \times 10^{-14} U^{8.7}}{(1 + 5.87 U^{5.5})(1 + 1.29 \times 10^{-3} U^{3.2})} + 3.26 \times 10^{-7} e^{-44.08/U} \qquad (60)$$

where N is the number density of air molecules. The data points shown up to 3 ev are from measurements made by Phelps (Reference 3) while those data points above 20 ev are calculated from total ionization cross sections (Reference 4). No data could be found in the 3 to 12 ev range and the accuracy of the fit might be questioned in this region. The above function is simply a smooth fit connecting the low and high-energy data. The fit is not accurate for U greater than about 67 ev.

The last two input parameters of interest are the energy transfer and momentum transfer collision frequencies. These collision frequencies are based on experimental data and are functions of the swarm internal energy U. The experimental data can be fitted fairly well by the expressions

$$\frac{\nu_w}{N_0} = 1.0 \times 10^{-11} + \frac{A_1 U^{3.22}}{(1 + A_2 U^{9.15})^{.307}} + \frac{B_1 U^{5.22}}{(1 + B_2 U^{4.74})^{.965}} \qquad (61)$$

where

$A_1 = 2.43 \times 10^{-8}$

$A_2 = 2.59 \times 10^{7}$

$B_1 = 4.13 \times 10^{-10}$

$B_2 = .072$

N_0 = number density of air molecules

U = internal energy (temperature) in ev

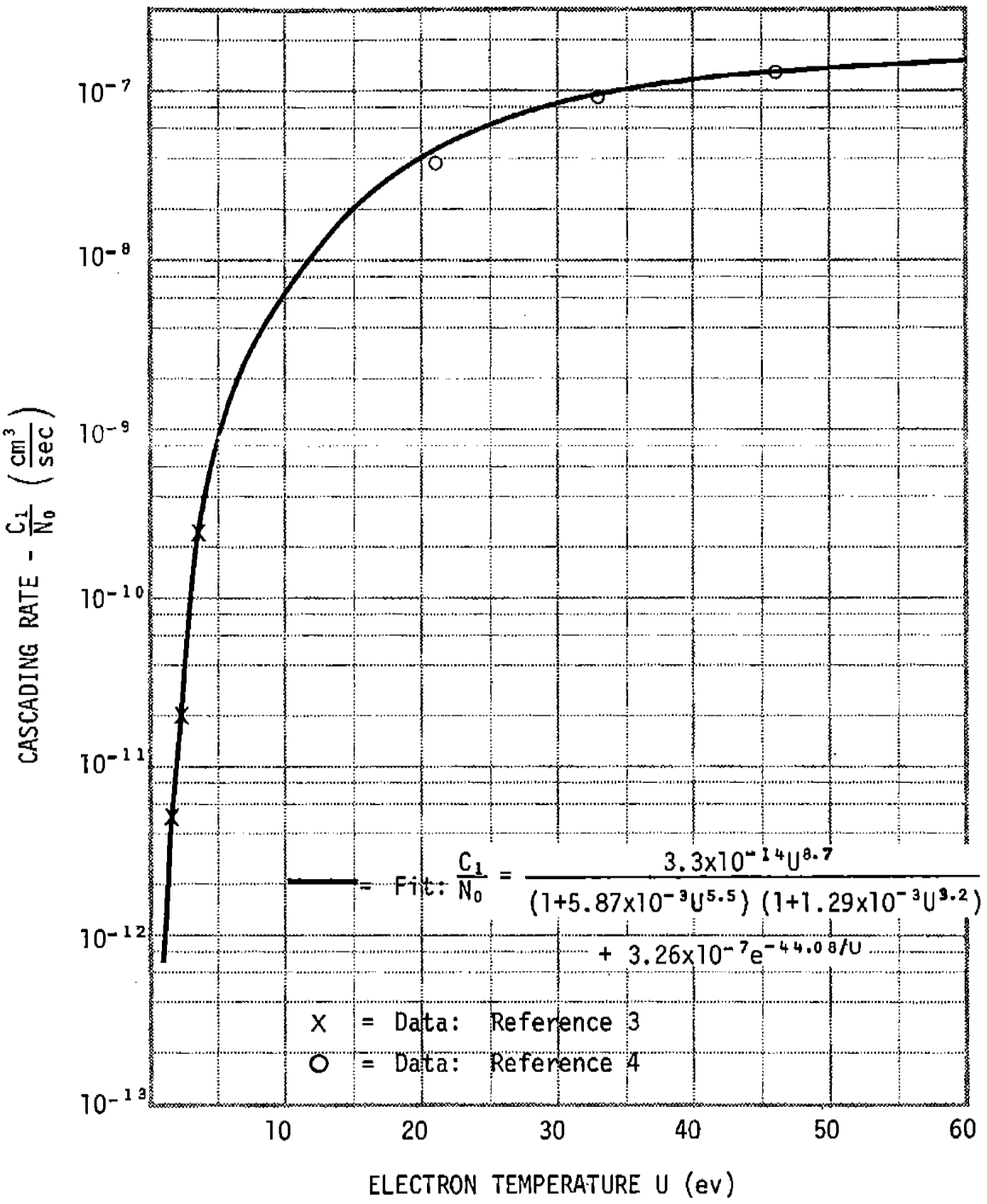

Figure 3. Cascading rate, C_1, as a function of temperature U.

and

$$\frac{\nu_m}{N_0} = \frac{1.25 \times 10^{-7} U^{.935}}{(1 + 1.26\, U^{1.67})^{.373}} \,. \tag{62}$$

The fits for both ν_w and ν_m are based on measurements by Phelps (References 3 and 5) for U less than 3 ev. For U greater than 3 ev the fits are based on cross-section data (References 3 and 6). No data was found for U greater than about 67 ev. ν_w and ν_m are plotted as a function of U in Figures 4 and 5.

Thus we now have expressions for the source terms S_1 and S_2 and the energy dependent parameters C_1, ν_w, and ν_m. The differential equations for N_e, U, and v_d can now be written as difference equations and solved numerically.

2.4 Differential Equations to be Solved

In the previous sections we have developed five first-order coupled differential equations that must be solved in order to evaluate the X-ray produced EMP fields seen at high altitudes. We rewrite the equations here as a group.

$$\frac{dE}{dt} = - 4\pi c \left[J_p - \frac{e}{c} N_e v_d \right], \tag{3}$$

$$\frac{dJ_p}{dt} = - e\bar{\beta} S_0(t) + \frac{e^2 N(t)}{mc} E - \frac{C_D}{m} J_p \,, \tag{16}$$

$$\frac{dN_e}{dt} = S_1(t) + C_1(U)\, N_e \,, \tag{48}$$

$$\frac{dU}{dt} = - \frac{2}{3} \frac{e}{1.6 \times 10^{-12}} v_d E - \nu_w(U)[U - U_0] + S_2(t) - \frac{U}{N_e} \frac{dN_e}{dt} \,, \tag{51}$$

and

$$\frac{dv_d}{dt} = - \frac{e}{m} E - \nu_m(U) v_d - \frac{v_d}{N_e} \frac{dN_e}{dt} \,, \tag{52}$$

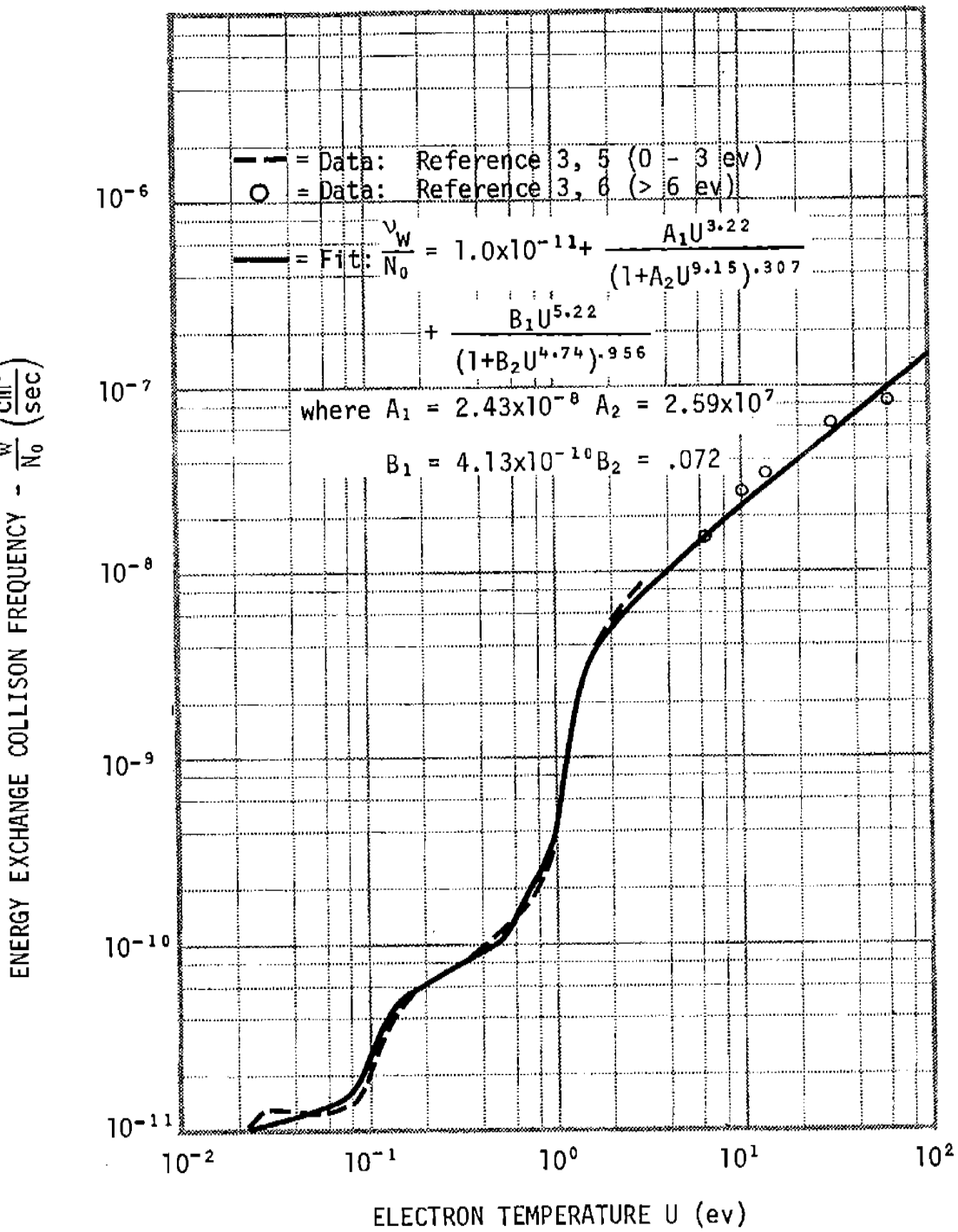

Figure 4. Energy exchange collision frequency, ν_w, as a function of U.

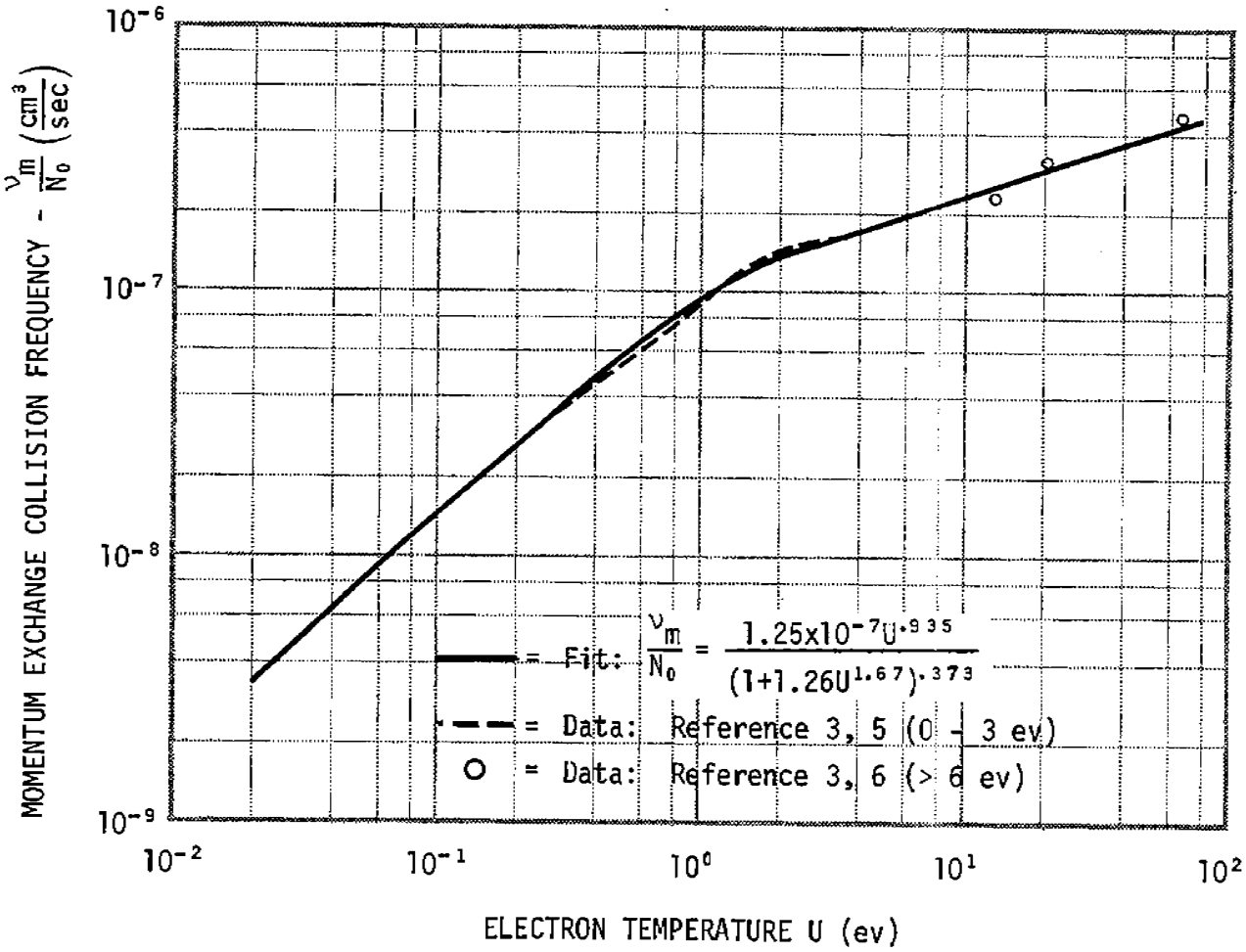

Figure 5. Momentum exchange collision frequency, ν_m, as a function of U.

where

e = the magnitude of the electron's charge

m = the electron's mass

c = the velocity of light

and all units are in the cgs Gaussian system except U which has the energy dimension of electron volts.

3. SUMMARY

In this report we have discussed a means of calculating the X-ray produced EMP fields seen by an observer inside the source region of a high-altitude burst. The electric field is assumed to be primarily radial; thus, the time rate of change of this radial field is directly proportional to the total current density. The current density is then broken into two parts, the primary current and the secondary current, which are calculated separately. The primary current is produced by the relatively high-energy photoelectrons created by the X-ray flux of the weapon. An average radial velocity and number density are calculated for these electrons. The time rate of change of the primary current is then expressed in terms of changes in these quantities. The secondary current is due to low-energy ionization electrons drifting in the electric field. A swarm theory treatment is used to deal with these electrons. This treatment involves writing down three differential equations relating the average electron energy, number density, and drift velocity. The secondary current is then just proportional to the product of the electron density and drift velocity. As a result of all this we obtain five coupled differential equations which must be solved to find the electric field.

REFERENCES

1. Heitler, W., "The Quantum Theory of Radiation," Oxford University Press, Third Edition, 1954, p 206.

2. Longmire, C. L., and H. J. Longley, Improvements in the Treatment of Compton Current and Air Conductivity in EMP Problems," MRC-N-2, (DNA 3192T), Mission Research Corporation, Santa Barbara, California, October 1971.

3. Phelps, A. V., and W. H. Kasner, Studies and Experimental Work on Atomic Collision Processes Occurring in Atmospheric Gases, AFWL-TR-66-34, Air Force Weapons Laboratory, Albuquerque, New Mexico, May 1966.

4. DASA Reaction Rate Handbook, Chapter 9, July 1967.

5. Phelps, A. V., Westinghouse Research Laboratories, private communication.

6. Frost, L. S., and A. V. Phelps, "Rotational Excitation and Momentum Transfer Cross Sections for Electrons in H_2 and N_2 from Transport Coefficients," Physical Review, 127, September 1963.

7. Longmire, C. L., "Elementary Plasma Physics," Interscience, pp. 15-21, 1963.

8. O'Dell, A. A., C. L. Longmire, D. F. Higgins, unpublished MRC report.

9. Fermi, E., "Nuclear Physics", University of Chicago Press, 1950.

10. McMaster, W. H., et al., Compilation of X-ray Cross Sections, Lawrence Livermore Laboratories, UCRL-50174, May 1969.

APPENDIX I

BOLTZMANN EQUATION APPROACH TO FINDING THE ELECTRON DISTRIBUTION FUNCTION

The X-ray produced electrons can be described by the phase space distribution function, $f(\vec{r}, \vec{v}, t)$. Continuity requires that

$$\frac{\partial f}{\partial t} + \nabla_6 \cdot J_6 = \text{Sources} - \text{Sinks} , \tag{I-1}$$

where

$$J_6 = f v_6 . \tag{I-2}$$

J_6 is essentially just a six-dimensional current vector. The components of v_6 and ∇_6 can be written as (Reference 7)

$$v_6 = (v_x, v_y, v_z, a_x, a_y, a_z), \tag{I-3}$$

$$\nabla_6 = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial v_x}, \frac{\partial}{\partial v_y}, \frac{\partial}{\partial v_z} \right) . \tag{I-4}$$

Now

$$\nabla_6 \cdot J_6 = \nabla_6 \cdot (f v_6) = v_6 \cdot (\nabla_6 f) + f(\nabla_6 \cdot v_6) \tag{I-5}$$

and

$$\nabla_6 \cdot v_6 = \nabla_v \cdot \vec{a} , \tag{I-6}$$

where

∇_v = the gradient in velocity space, i.e., $\nabla_v = \left(\frac{\partial}{\partial v_x}, \frac{\partial}{\partial v_y}, \frac{\partial}{\partial v_z} \right)$

and

$\vec{a}$ = the acceleration vector (a_x, a_y, a_z).

Note that the quantity $\nabla_v \cdot \vec{a}$ is zero if the acceleration is not a function of velocity. However, if we include an effective drag force which depends on velocity, the term cannot be dropped.

Putting (I-1) and (I-5) together, one obtains

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{a} \cdot \nabla_v f + f\nabla_v \cdot \vec{a} = \text{Sources} - \text{Sinks}. \qquad \text{(I-7)}$$

Now, let us assume that the distribution function $f(\vec{r}, \vec{v}, t)$ is a function of the velocity vector and the retarded time only; i.e., $f = f(\vec{v}, T)$ where $T = t - r/c$. (c is the velocity of light.) Further, note that the acceleration, $\vec{a}$, is just the net force divided by the electron's mass, m. Then one can use Equation I-7 to write down the modified Boltzmann equation

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \frac{e}{m}\vec{E} \cdot \nabla_v f + \frac{1}{m}\nabla_v \cdot (\vec{F}_D f) = S_0 + S_c \qquad \text{(I-8)}$$

where

$\vec{E}$ = electric field (in radial direction)

$\vec{F}_D$ = mean drag force due to ionization (in $-\vec{v}$ direction)

$-e$ = electron charge

S_0 = source - sinks (if any)

S_c = scattering term with mean energy loss subtracted (mean energy loss included in $\vec{F}_D$).

If f is a function of $\vec{v}$ and T only, then

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f = (1 - \frac{v_r}{c})\frac{\partial f}{\partial T}, \qquad \text{(I-9)}$$

where

v_r = radial component of $\vec{v}$.

Since $\vec{E}$ is radial,

$$\vec{E} \cdot \nabla_v = E \frac{\partial}{\partial v_r} , \tag{I-10}$$

and since $\vec{F}_D$ is in direction of $-\vec{v}$

$$\nabla_v \cdot (\vec{F}_D f) = \frac{1}{v^2} \frac{\partial}{\partial v}\left(v^2 F_D f\right). \tag{I-11}$$

If we use the velocity variables

$$v \equiv |\vec{v}|, \tag{I-12}$$

$$\mu \equiv \frac{v_r}{v} \equiv \cos\theta , \tag{I-13}$$

where

θ = the angle the velocity vector makes with respect to the radial direction

then

$$\frac{\partial}{\partial v_r} = \mu \frac{\partial}{\partial v} + \frac{1 - \mu^2}{v} \frac{\partial}{\partial \mu} . \tag{I-14}$$

Thus the Boltzmann equation becomes

$$\left(1 - \frac{v\mu}{c}\right) \frac{\partial f}{\partial T} - \frac{eE}{m}\left(\mu \frac{\partial f}{\partial v} + \frac{1 - \mu^2}{v} \frac{\partial f}{\partial \mu}\right) + \frac{1}{mv^2} \frac{\partial}{\partial v} (v^2 F_D f)$$

$$= S_0 + S_c , \tag{I-15}$$

where the differential number of electrons per cubic centimeter, dn, is given by

$$dn = 2\pi f v^2 dv d\mu . \tag{I-16}$$

The radial current density, J_{total}, is then given by

$$J_{total} = - \frac{2\pi e}{c} \iint \mu v f(\mu, v, t) v^2 dv d\mu . \tag{I-17}$$

If we include in our source term both the high-energy electrons (Compton and photo-effect sources) and the low energy conduction

electrons (from ionization) then J_{total} can be substituted directly in Maxwell's equation

$$\frac{1}{c}\frac{\partial E}{\partial t} = -\,4\pi J_{total} \,. \tag{I-18}$$

Writing a source term for the low-energy electrons is rather difficult. Thus another possibility is to treat only the high-energy source electrons with the Boltzmann equation and use swarm theory to describe the low-energy conduction electrons. In this case, only Compton and photo-effect sources would be included in Equation I-15.

As an example of a source term, let us consider the photo-effect electrons. Let the photoelectric source density of electrons of velocity v equal P(v) electrons/(cm^3-sec-cm/sec) summed over all angles

$$P(v) = \frac{mv}{1.6 \times 10^{-9}}\,\rho\,\kappa_p\,F_x \,, \tag{I-19}$$

where

ρ = the air density

κ_p = the absorption coefficient

F_x = the incident X-ray flux.

Let $\Phi_p(\mu)$ be the normalized angular distribution of the photoelectrons at birth; i.e., $\int_{-1}^{1}\Phi(\mu)d\mu = 1$. Then the photoelectric source function, S_{0p}, is just

$$S_{0p} = \frac{1}{2\pi v^2}\,P(v)\Phi_p(\mu) \,. \tag{I-20}$$

Similarly, one can work out a Compton electron source function and an ionization electron source function.

The other term in Equation I-15 which must be considered is the scattering term, S_c. The average energy loss of the electron as it moves through the air is included in the drag force, $\vec{F}_D$. The term S_c, then, describes the variations from this average. Now, to first order, scattering will change only the direction of the electron's velocity vector and not its magnitude. Because the scattering angle per collision is predominately very small, the scattering can be treated by the diffusion approximation, with

$$S_c = \alpha \nabla_\theta^2 f \ , \tag{I-21}$$

where ∇_θ^2 is the angular part of the Laplacian operator in velocity space, and the diffusion coefficient α is one fourth of the mean square scattering angle per unit time. (See Reference 2 for a further discussion of this treatment of scattering.) Equation I-21 can also be written

$$\begin{aligned} S_c &= \alpha \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial f}{\partial\theta} \right) \\ &= - \alpha \frac{\partial}{\partial\mu} \left((1 - \mu^2) \frac{\partial f}{\partial\mu} \right) . \end{aligned} \tag{I-22}$$

Combining all these terms the Boltzmann equation becomes a rather complicated partial differential equation describing the distribution function, $f(v, \mu, t)$. A solution of this equation would be required for a more complete treatment of the X-ray EMP problem than was discussed in the main body of this report.

The primary difference between the use of the Boltzmann equation and the average velocity approach used in the main section of this report is based on the treatment of the effective drag force used to describe energy loss due to ionization along the electron's path. In the Boltzmann equation, the drag force, F_D, can in general be a function of both velocity and time and the assumption that the photoelectrons live a relatively long time is not necessary.