

Magnetism: quantities, units and relationships

If you occasionally need to design a wound component, but do not deal with the science of magnetic fields on a daily basis, then you may become confused about what the many [terms](#) used in the data sheet for the core represent, [how they are related](#) and how you can use them to [produce a practical inductor](#).

About your browser: if this character '×' does not look like a multiplication sign, or you see lots of question marks '?' or symbols like '□' or sequences like '&cannot;' then please [accept my apologies](#).

See also ...

[↑ Producing [wound components](#)] [[Air coils](#)] [[Power loss](#) in wound components] [The [force](#) produced by a magnetic field] [[Faraday's law](#)] [The [magnetic properties of materials](#)] [[Unit Systems](#)]

Index to magnetic terms & units in the SI

This set of web pages uses the system of units known as the SI (Système International). For more information on the SI, and how it compares with other systems, see [Unit Systems in Electromagnetism](#).

Quantity name	Quantity symbol	Quantity name	Quantity symbol
coercivity	H_c	core factor	$\Sigma l/A$
current	I	effective area	A_e
effective length	l_e	effective permeability	μ_e
flux linkage	λ	induced voltage	u
inductance	L	inductance factor	A_l
initial permeability	μ_i	intensity of magnetization	I
magnetic field strength	H	magnetic flux	Φ
magnetic flux density	B	magnetic mass susceptibility	χ_p
magnetic moment	m	magnetic polarization	J
magnetic susceptibility	χ	magnetization	M
magnetomotive force	F_m	permeability	μ
permeability of vacuum	μ_0	relative permeability	μ_r
reluctance	R_m	remnance	B_r

Magnetic quantities in the SI

An Example Toroid Core

As a concrete example for the calculations throughout this

page we consider the 'recommended' toroid, or *ring core*, used in this [Faculty](#). Manufacturers use toroids to derive material characteristics because there is no [gap](#), even a residual one. Such tests are done using fully wound cores rather than just the two turns here; but, providing the [permeability](#) is high, then the error will be small.

Parameter	Symbol	Value
Effective magnetic path length	l_e	$27.6 \times 10^{-3} \text{ m}$
Effective core area	A_e	$19.4 \times 10^{-6} \text{ m}^2$
Relative permeability	μ_r	2490
Inductance factor	A_l	2200 nH
saturation flux density	B_{sat}	360 mT

Data for approved toroid

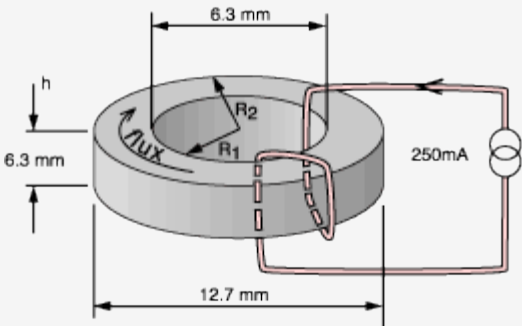


Fig. TDE: An example toroid.

Let's take a worked example to find the [inductance](#) for the winding shown with just two turns ($N=2$).

$$\Sigma l/A = l_e / A_e = 27.6 \times 10^{-3} / 19.4 \times 10^{-6} = 1420 \text{ m}^{-1}$$

$$\mu = \mu_0 \times \mu_r = 1.257 \times 10^{-6} \times 2490 = 3.13 \times 10^{-3} \text{ Hm}^{-1}$$

$$R_m = (\Sigma l/A) / \mu = 1420 / 3.13 \times 10^{-3} = 4.55 \times 10^5 \text{ A-t Wb}^{-1}$$

$$A_l = 10^9 / R_m = 10^9 / 4.55 \times 10^5 = 2200 \text{ nH per turn}^2$$

$$L = A_l \times N^2 = 2200 \times 10^{-9} \times 2^2 = 8.8 \text{ }\mu\text{H}$$

[[↑ Top](#) of page]

Core Factor

Quantity name	core factor or geometric core constant
Quantity symbol	$\Sigma l/A$
Unit name	per metre
Unit symbols	m^{-1}

Core Factor in the [SI](#)

The idea of core factor is, apart from adding to the jargon $-($, to encapsulate in one figure the contribution to core [reluctance](#) made by the size and shape of the core. It is usually quoted in the data sheet but it is calculated as -

$$\Sigma l/A = l_e / A_e \text{ m}^{-1}$$

So for our [example toroid](#) we find -

$$\Sigma l/A = 27.6 \times 10^{-3} / 19.4 \times 10^{-6} = 1420 \text{ m}^{-1}$$

Core factors are often specified in millimetres⁻¹. You should then multiply by 1000 before using them in the [formula for reluctance](#).

[[↑ Top](#) of page]

Effective Area

Quantity name	effective Area
Quantity symbol	A_e
Unit name	square metre
Unit symbols	m^2

Effective Area in the [SI](#)

The 'effective area' of a core represents the cross sectional area of one of its limbs. Usually this corresponds closely to the physical dimensions of the core but because [flux](#) may not be distributed completely evenly the manufacturer will specify a value for A_e which reflects this.

The need for the core area arises when you want to relate the [flux density](#) in the core (limited by the [material type](#)) to the total [flux](#) it carries -

$$A_e = \Phi / B$$

In the [example toroid](#) the area could be determined approximately as the product of the core height and the difference between the major and minor radii -

$$A_e = 6.3 \times ((12.7 - 6.3) / 2) = 20.2 \text{ mm}^2$$

However, because the flux concentrates where the path length is shorter it is better to use the value stated by the manufacturer - 19.4 mm². For the simple toroidal shape A_e is calculated ([Snelling](#)) as

$$A_e = h \times \ln^2(R_2/R_1) / (1/R_1 - 1/R_2) \text{ m}^2$$

This assumes square edges to the toroid; real ones are often rounded.

There is a slight twist to the question of area: the manufacturer's value for A_e will give the correct results when used to compute the core [reluctance](#) but it may not be perfect for computing the [saturation](#) flux (which depends upon the narrowest part of the core or A_{\min}). In a well designed core A_{\min} won't be very different from A_e , but keep it in mind.

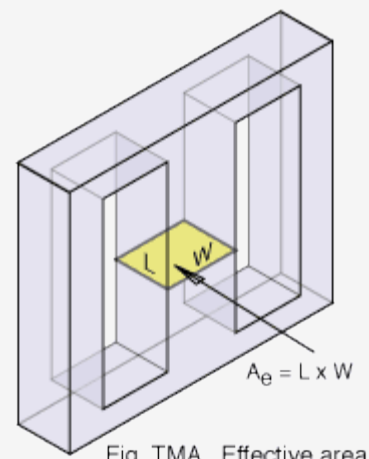


Fig. TMA Effective area

⚠ Effective area is usually quoted in millimetres squared. Many formulae in data books implicitly assume that a numerical value in mm² be used. Other books, and these notes, assume metres squared.

[↑ [Top](#) of page]

Effective Length

Quantity name	effective length
Quantity symbol	l_e
Unit name	metre
Unit symbols	m

Effective Length in the [SI](#)

The 'effective length' of a core is a measure of the distance which [flux](#) lines travel in making a complete circuit of it. Usually this corresponds closely to the physical dimensions of the core but because flux has a tendency to concentrate on the inside corners of the path the manufacturer will specify a value for l_e which reflects this.

In the [toroid example](#) the path length could be determined approximately as -

$$l_e = \pi \times (12.7 + 6.3) / 2 = 29.8 \text{ mm}$$

However, because the flux concentrates where the path length is shorter it is better to use the value stated by the manufacturer - 27.6 mm. For a simple toroidal shape l_e is calculated as

$$l_e = 2\pi \times \ln(R_2 / R_1) / (1 / R_1 - 1 / R_2)$$

Another common core type, the EE, is shown in Fig: EEE. The red line represents the shortest path which a flux line could take to go round the core. The green line is the longest. Shown in blue is a path whose length is that of the short path plus four sectors whose radius is sufficient to take the path mid-way down the limbs.

$$l_e = 2(3.8 + 1.2) + \pi((2.63 - 1.2) / 2) = 12.25 \text{ mm}$$

Equation TMB

This is all a bit approximate; but bear in mind that since manufacturing tolerances on [permeability](#) are often 25% there isn't much point in being more exact.

⚠ Effective length is usually quoted in millimetres. Many formulae in data books implicitly assume that a numerical value in mm be used. Other books, and these notes, assume metres.

[↑ [Top](#) of page]

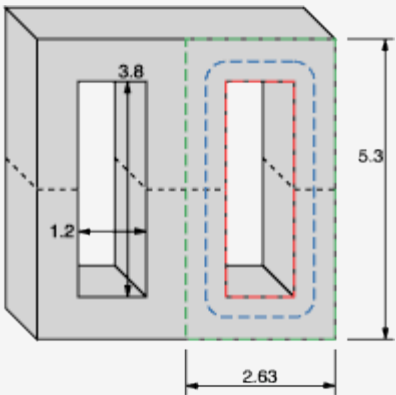


Fig. EEE: An example EE core.

Magnetomotive Force

Quantity name	magnetomotive force, alias magnetic potential
Quantity symbol	F_m , \mathcal{H} or \mathfrak{F}
Unit name	ampere
Unit symbol	A

Magnetomotive Force in the [SI](#)

Duality with the Electric World

Quantity	Unit	Formula
Magnetomotive force	amperes	$F_m = \mathcal{H} \times \mathcal{L}_e$
Electromotive force	volts	$V = E$ (Electric field strength) $\times l$ (distance)

MMF can be thought of as the magnetic [equivalent](#) of electromotive force. You can calculate it as -

$$F_m = \mathcal{I} \times \mathcal{N} \quad \text{ampere turns}$$

Equation TMM

The units of MMF are often stated as *ampere turns* (A-t) because of this. In the [example toroid core-](#)

$$F_m = 0.25 \times 2 = 0.5 \quad \text{ampere turns}$$

Equation TMC

Don't confuse magnetomotive force with [magnetic field strength \(magnetizing force\)](#). As an analogy think of the plates of a capacitor with a certain *electromotive* force (EMF) between them. How high the electric field strength is will depend on the distance between the plates. Similarly, the magnetic field strength in a transformer core depends not just on the MMF but also on the [distance](#) that the [flux](#) must travel round it.

A magnetic field represents stored energy and

$$F_m = 2 W / \Phi$$

Equation TMF

where W is the energy in joules. You can also relate MMF to the total [flux](#) going through part of a magnetic circuit whose [reluctance](#) you know.

$$F_m = \Phi \times \mathcal{R}_m$$

[Rowland's Law](#)

There is a clear analogy here with an electric circuit and Ohm's Law, $V = I \times R$. The analogy with electric potential (voltage) leads to the alternate name *magnetic potential*. There is, however, then a risk of confusion with *magnetic vector potential* - which has quite different units.

A specific MMF is required to sustain a given field strength along a known path length -

$$F_m = \mathcal{H} \times \mathcal{L}_e$$

Equation TMP

Practical coil windings are made from copper wire which has a [current carrying capacity](#) limited mainly by its cross-section. There is therefore a limit to the MMF of a coil in continuous operation of about 3.5×10^6 ampere-turns per square metre of aperture.

[[↑ Top](#) of page]

Magnetic Field Strength

Quantity name	magnetic field strength alias magnetic field intensity alias the auxiliary field alias the H-field alias magnetizing force
Quantity symbol	H
Unit name	ampere per metre
Unit symbols	A m ⁻¹

Magnetic Field Strength in the [SI](#)

Whenever current flows it is always accompanied by a magnetic field. Scientists talk of the field as being due to 'moving electric charges' - a reasonable description of electrons flowing along a wire. The strength, or intensity, of this field surrounding a straight wire is given by

$$H = I / (2 \pi r)$$

Equation TML

where r, the distance from the wire, is small in comparison with the length of the wire. The situation for short wires is described by the [Biot-Savart equation](#).

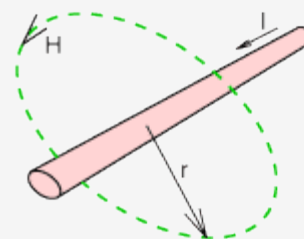


Fig. TMD The H-field near a long wire

By the way, don't confuse the speed of the charges (such as electrons) with the speed of a signal travelling down the wire they are in. Think of the signal as being the boundary between those electrons that have started to move and those that have yet to get going. The boundary might move close to the speed of light ($3 \times 10^8 \text{ m s}^{-1}$) whilst the electrons themselves drift (on average) something near to 0.1 mm s^{-1} . The electrons would be outpaced by a snail - even if it wasn't in a hurry.

You may object that magnetic fields are also produced by permanent magnets (like compass needles, door catches and fridge note holders) where no current flow is evident. It turns out that even here it is electrons moving in orbit around nuclei or spinning on their own axis which are responsible for the magnetic field.

Duality with the Electric World

Quantity	Unit	Formula
Magnetic field strength	amperes per metre	$H = E_m / l_e$
Electric field strength	volts per metre	$\epsilon = e/d$

Magnetic field strength is [analogous](#) to electric field strength. Where an electric field is set up between two plates separated by a distance, d, and having an electromotive force, e, between them the electric field is given by -

$$\epsilon = e / d \quad \text{V m}^{-1}$$

Equation TMG

Similarly, magnetic field strength is -

$$H = \frac{F_m}{l_e}$$

Equation TMH

In the [example](#) the field strength is then -

$$H = 0.5 / 27.6 \times 10^{-3} = 18.1 \text{ A m}^{-1}$$

The analogy with electric field strength is mathematical and not physical. An electric field has a clearly defined physical meaning: simply the force exerted on a 'test charge' divided by the amount of charge. Magnetic field strength cannot be measured in the same way because there is no 'magnetic monopole' equivalent to a test charge.

Do not confuse magnetic field strength with [flux density](#), B. This is closely related to field strength but depends also on the material within the field. The strict definition of H is

$$H = \frac{B}{\mu_0} - M$$

[Sommerfeld](#) Field Equation

This formula applies generally, even if the materials within the field have non-uniform [permeability](#) or a permanent [magnetic moment](#). It is rarely used in coil design because it is usually possible to simplify the calculation by assuming that within the field the permeability can be regarded as uniform. With that assumption we say instead that

$$H = \frac{B}{\mu}$$

Equation TMU

Flux also emerges from a permanent magnet even when there are no wires about to impose a field.

A field strength of about 2000 A m⁻¹ is about the limit for cores made from iron powder.

[[↑ Top](#) of page]

Magnetic Flux

Quantity name	magnetic flux
Quantity symbol	Φ
Unit name	weber
Unit symbol	Wb
Base units	kg m ² s ⁻² A ⁻¹

Magnetic Flux in the [SI](#)

We talk of magnetism in terms of lines of force or flow or *flux*. Although the Latin *fluxus*, means 'flow' the English word is older and unrelated. Flux, then, is a measure of the number of these lines - the total amount of magnetism.

You can calculate flux from the time integral of the voltage V on a winding -

$$\Phi = (1/N) \int V \cdot dt \text{ webers}$$

Equation TMX

This is one form of [Faraday's law](#). If a constant voltage is applied for a time T then this boils down to -

$$\Phi = V \times T / N \text{ Wb}$$

How much simpler can the maths get? Because of this relationship flux is sometimes specified as *volt seconds*.

Duality with the Electric World

Quantity	Unit	Formula
Magnetic flux	volt second	$\Phi = V \times T$
Electric charge	amp second (= coulomb)	$Q = I \times T$

Although as shown above flux corresponds in physical terms most closely to electric charge, you may find it easiest to envisage flux flowing round a core in the way that current flows round a circuit. When a given voltage is applied across a component with a known resistance then a specific current will flow. Similarly, application of a given [magnetomotive force](#) across a [ferromagnetic](#) component with a known [reluctance](#) results in a specific amount of magnetic flux -

$$\Phi = F_m / R_m$$

Rowland's Law

There's a clear analogy here with Ohm's Law. You can also calculate flux as

$$\Phi = I \times L / N$$

Flux can also be derived by knowing both the [magnetic flux density](#) and the area over which it applies:

$$\Phi = A_e \times B$$

Equation TMS

A magnetic field represents energy stored within the space occupied by the field. So

$$\Phi = 2W / F_m$$

Equation TMW

where W is the field energy in joules. Or, equivalently,

$$\Phi = \sqrt{(2W / R_m)}$$

Equation TMZ

[[↑ Top](#) of page]

Magnetic Flux Density

Quantity name	Magnetic flux density, alias Magnetic induction alias The B-field
Quantity symbol	B
Unit name	tesla
Unit symbol	T
Base units	kg s ⁻² A ⁻¹

Magnetic Flux Density in the [SI](#)

Duality with the Electric World

Quantity	Unit	Formula
Magnetic flux density	webers per metre ²	$B = \Phi / \text{Area}$
Electric flux density	coulombs per metre ²	$D = C / \text{Area}$

Flux density is simply the total [flux](#) divided by the [cross sectional area](#) of the part through which it flows -

$$B = \Phi / A_e \text{ teslas}$$

Thus 1 weber per square metre = 1 tesla. Flux density is related to [field strength](#) via the [permeability](#)

$$B = \mu \times H$$

Equation TMD

So for the [example core](#) -

$$B = 3.13 \times 10^{-3} \times 18.1 = 0.0567 \text{ teslas}$$

Equation TMD suggests that the 'B field' is simply an effect of which the 'H field' is the cause. Can we visualize any qualitative distinction between them? Certainly from the point of view of practical coil design there is rarely a need to go beyond equation TMD. However, the presence of [magnetized](#) materials modifies formula TMD -

$$B = \mu_0 (M + H)$$

[Sommerfeld](#) field equation

If the B field pattern around a bar magnet is compared with the H field then the lines of B form continuous loops without beginning or end whereas the lines of H may either originate or terminate at the *poles* of the magnet. A mathematical statement of this general rule is -

$$\text{div } \mathbf{B} = 0$$

[Maxwell's](#) Equation for B

You could argue that B indicates better the strength of a magnetic field than does the 'magnetic field strength' H! This is one reason why modern authors tend not to use these names and stick instead with 'B field' and 'H field'. The *definition of B* is in terms of its ability to produce a force F on a wire, length L, carrying current, I, -

$$B = F / (I \times L \times \sin\theta)$$

The Motor Equation

where θ is the angle between the wire and the field direction. So it seems that H describes the way magnetism is **generated** by moving electric charge (which is what a current is), while B is to do with the ability to be **detected** by moving charges.

In the end, both B and H are just abstractions which the maths can use to model magnetic effects. Looking for more solid explanations isn't easy.

A feel for typical magnitudes of B helps. One metre away in air from a long straight wire carrying one ampere B is exactly 200 nanoteslas. The earth's field has a value of roughly 60 microteslas (but varies from place to place). A largish permanent magnet will give 1 T, iron saturates at about 1.6 T and a super conducting electromagnet might achieve 15 T.

[[↑ Top](#) of page]

Flux Linkage

Quantity name	flux linkage
Quantity symbol	λ
Unit name	weber-turn
Unit symbol	Wb-t
Base units	$\text{kg m}^2 \text{s}^{-2} \text{A}^{-1}$

Flux Linkage in the [SI](#)

In an ideal inductor the [flux](#) generated by one of its turns would encircle all the other other turns. Real coils come close to this ideal when the cross sectional dimensions of the winding are small compared with its diameter, or if a high permeability core guides the flux right the way round. In longer [air-core coils](#) the situation is likely to be nearer to that shown in Fig.TFK: Here we see that the flux density decreases towards the ends of the coil as some flux takes a 'short cut' bypassing the outer turns. Let's assume that the current into the coil is 5 amperes and that each flux line represents 7 mWb.

The central three turns all 'link' four lines of flux: 28 mWb.
The two outer turns link just two lines of flux: 14 mWb.

We can calculate the total 'flux linkage' for the coil as:

$$\lambda = 3 \times 28 + 2 \times 14 = 112 \text{ mWb-t}$$

The usefulness of this result is that it enables us to calculate the total self [inductance](#) of the coil, L:

$$L = \lambda / I = 112 / 5 = 22.4 \text{ mH}$$

In general, where an ideal coil is assumed, you see expressions involving $N \times \Phi$ or $N \times d\Phi/dt$. For greater accuracy you substitute λ or $d\lambda/dt$.

[[↑ Top](#) of page]

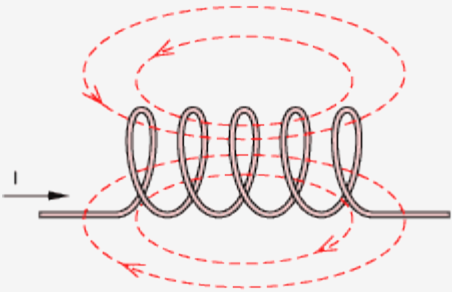


Fig. TFK: Flux Linkage

Inductance

Quantity name	Inductance
Quantity symbol	L
Unit name	henry
Unit symbol	H
Base units	$\text{kg m}^2 \text{s}^{-2} \text{A}^{-2}$

Inductance in the [SI](#)

Duality with the Electric World

Quantity	Unit	Formula
Inductance	webers per amp	$L = \Phi/I$
Capacitance	coulombs per volt	$C = Q/V$

Any length of wire has inductance. Inductance is a measure of a coil's ability to store energy in the form of a magnetic field. It is defined as the rate of change of [flux](#) with current -

$$L = N \times d\Phi / dI$$

Equation TMO

If the core material's [permeability](#) is considered constant then the relation between flux and current is linear and so:

$$L = N \times \Phi / I$$

Equation TMI

By Substitution of [Equation TMM](#) and [Rowland's Law](#) -

$$L = N^2 / R_m$$

Equation TMA

You can relate inductance directly to the energy represented by the surrounding magnetic field -

$$L = 2W / I^2$$

Equation TME

Where W is the field energy in joules.

In practice, where a high [permeability](#) core is used, inductance is usually determined from the A_l value specified by the manufacturer for the core -

$$L = 10^{-9} A_l \times N^2$$

Equation TMK

Inductance for the [toroid example](#) is then:

$$L = 2200 \times 10^{-9} \times 2^2 = 8.8 \mu H$$

If there is no [ferromagnetic](#) core so μ_r is 1.0 (the coil is '[air cored](#)') then a variety of formulae are available to estimate the inductance. The correct one to use depends upon

- Whether the coil has more than one layer of turns.
- The ratio of coil length to coil diameter.
- The shape of the cross section of a multi-layer winding.
- Whether the coil is wound on a circular, polygonal or rectangular former.
- Whether the coil is open ended, or bent round into a toroid.
- Whether the cross section of the wire is round or rectangular, tubular or solid.
- The permeability of the wire.
- The frequency of operation.
- The phase of the moon, direction of the wind etc..

Most of these variants are described in early editions of [Terman](#) or successor publications. There are too many formulae to reproduce here. You can find them all in [Grover](#).

[[↑ Top](#) of page]

Inductance Factor

Quantity name	inductance factor
Quantity symbol	A_l
Unit name	nanohenry
Unit symbol	nH
Base units	$\text{kg m}^2 \text{s}^{-2} \text{A}^{-2}$

Inductance Factor

A_l is usually called the *inductance factor*, defined

$$A_l = L \times 10^9 / N^2$$

Equation TMT

If you know the inductance factor then you can multiply by the square of the number of turns to find the [inductance](#) in nano henries. In our [example](#) core $A_l = 2200$, so the inductance is -

$$L = 2200 \times 10^{-9} \times 2^2 = 8800 \text{ nH} = 8.8 \text{ }\mu\text{H}$$

Equation TMV

The core manufacturer may directly specify an A_l value, but frequently you must derive it via the [reluctance](#), R_m . The advantage of this is that only one set of data need be provided to cover a range of cores having identical dimensions but fabricated using materials having different [permeabilities](#).

$$A_l = 10^9 / R_m$$

Equation TMY

So, for our [example toroid core](#) -

$$A_l = 10^9 / 4.55 \times 10^5 = 2200$$

Equation TSA

The inductance factor may sometimes be expressed as "millihenries per 1000 turns". This is synonymous with nanohenries per turn and takes the same numerical value.

If you have no data on the core at all then wind ten turns of wire onto it and measure the inductance (in henrys) using an inductance meter. The A_l value will be 10^7 times this reading.

A_l values are, like permeability, a non-linear function of [flux](#). The quoted values are usually measured at low (<0.1 mT) flux.

[[↑ Top](#) of page]

Reluctance

Quantity name	reluctance
Quantity symbol	R_m or \mathfrak{R}
	per henry or

Unit name	ampere-turns per weber
Unit symbols	H^{-1}
Base units	$\text{A}^2 \text{s}^2 \text{kg}^{-1} \text{m}^{-2}$

Reluctance in the [SI](#)

Reluctance is the ratio of [MMF](#) to [flux](#) -

$$R_m = E_m / \Phi$$

[Rowland's Law](#)

In a magnetic circuit this [corresponds](#) to Ohm's Law and resistance in an electric circuit. Compare

$$R_e = V / I$$

Reluctance is also proportional to the [core factor](#), $\Sigma l/A$, but inversely proportional to [permeability](#) -

$$R_m = (\Sigma l/A) / \mu$$

[Equation TMQ](#)

Again, compare

$$R_e = (\Sigma l/A) / \sigma$$

where σ is the electrical conductivity of a conductor of given length and cross-sectional area.

Take care to use the absolute rather than the relative permeability here. So for the [toroid example](#) reluctance is then:

$$R_m = 1420 / 3.13 \times 10^{-3} = 4.55 \times 10^5 \text{ A-t Wb}^{-1}$$

A magnetic field represents stored energy and

$$R_m = 2 W / \Phi^2$$

[Equation TMR](#)

where W is the energy in joules.

Although it can be a useful concept when analyzing series or parallel combinations of magnetic components reluctance is, like permeability, non-linear and must be used carefully.

[[↑ Top](#) of page]

Current

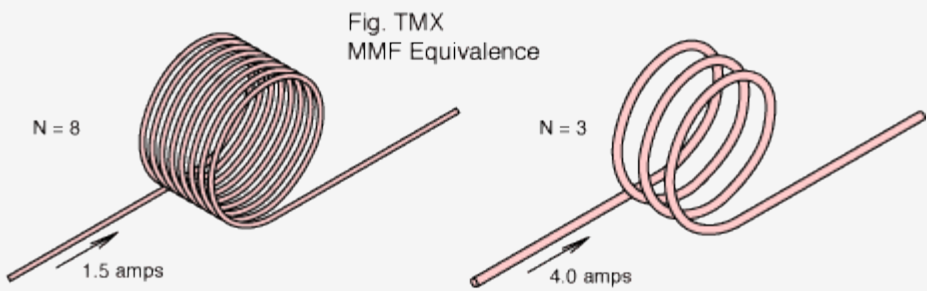
Quantity name	current
Quantity symbol	I, i
Unit name	ampere
Unit symbol	A

Current in the [SI](#)

You might be forgiven for thinking that there would be no need to spell out what current is. That's obvious surely? Your mistake is to forget how hard all writers on electromagnetism strive to obfuscate

an already difficult subject. Here's the problem.

Figure TMX shows two coils with different numbers of turns but the same [magneto-motive force](#). When considering the MMF it makes no difference whether you have twelve turns of wire carrying one amp, or three turns carrying four amps, or two turns with six amps. As far as the MMF goes it's all just 'twelve ampere-turns'. You will get just the same magnetic field in each case.



$$I = E_m / N$$

Equation TSD

Reasoning that detail about the number of turns and the number of amps doesn't matter, only the **product of the two**, some writers decide to say that the current **is** twelve amps. They write $I = 12\text{ A}$ and leave it to you to decide which scenario brought about that 'current'. This insidious practice carries over to formulae as well.

Which is fine as long as it's consistent and clear to the reader what's happening. If the current changes then, by [Faraday's Law](#) we have an induced voltage. You then have to remember that the induced voltage is **per turn** and not the the total coil voltage. Ambiguity starts to creep in.

It depends, perhaps, on whether you're more interested in physics or engineering. These pages take the latter view and distinguish current from MMF. Current here, then, is what an ordinary ammeter reads, and the number of coil [turns](#), N , is written explicitly.

The physicists get their way in the end because, although you might just speak of [reluctance](#) as 'ampere-turns per weber', [inductance](#) as 'weber-turns per ampere' is getting a little contrived - even if it does reflect the concept of [flux linkage](#) rather nicely. But [permeability](#) as 'weber-turns per ampere-metre'?

These pages are being converted to use upper case I both for direct current and for a current given as an RMS quantity; whilst the lower case i will stand for instantaneous values of time varying current.

Trivia point: why is the symbol I used for current? Allegedly, it stands for 'electric *intensity*', as opposed to 'total amount of electricity' (charge). [Maxwell](#), though, used the symbol C for current and used electric intensity to refer to the E-field: what most people today know as electric field strength. So it goes.

[[↑ Top](#) of page]

Current density

Quantity name	current density
Quantity symbol	J
Unit name	amperes per square metre

Unit symbol	A m ⁻²
--------------------	-------------------

Current density in the [SI](#)

Current density is simply the total [electric current](#) divided by the area over which it is flowing. Example: if a wire 0.7 millimetres diameter carries a current of 0.5 amperes then the current density is

$$J = 0.5 / (\pi 0.0007^2 / 4) = 1.30 \times 10^6 \text{ A m}^{-2}$$

Equation TMJ

Or 1.3 amps per millimetre². A reasonable limit for most small transformers is $3.5 \times 10^6 \text{ A m}^{-2}$.

[[↑ Top](#) of page]

The number of turns

Quantity name	Turns
Quantity symbol	N
Unit name	turn
Unit symbol	t

Turns (of wire)
See [note on units](#)

By tradition, coil calculations use the capital letter N to represent the total number of turns in the coil. Solenoid coils are sometimes described using the lower case letter n to represent the number of turns per unit length. So

$$N = n \times l_a$$

Equation TMN

Where l_a is the axial length of the coil.

Naturally, for most designs, the number of turns required is the \$64,000 question. The answer comes in a bewildering variety of forms. For the most common case, such as the [example toroid](#) core, where the manufacturer has specified A_i -

$$N = \sqrt{(10^9 \underline{L} / A_i)}$$

Equation TSB

So, if you needed 330 microhenries then

$$N = \sqrt{(10^9 \times 330 \times 10^{-6} / 2200)} = 12 \text{ turns}$$

Equation TSC

[[↑ Top](#) of page]

Relationships between magnetic quantities

Flux, field strength, permeability, reluctance it's easy to go into jargon overload. [Snelling](#) lists over 360 different symbol uses connected with ferromagnetics. There isn't even agreement about what to call some properties (I say remnance, you say remanence, he says retentivity). You will cope better if you can form a mental picture of the party that these names throw when they get together inside your transformer.

Analogy with electric quantities:

You may find it easier to obtain an intuitive grasp of the relationships between magnetic quantities by thinking in terms of 'magnetic circuits' with [flux](#) flowing round a core in a fashion analogous to current flowing round an electric circuit.

Magnetic quantity	Electric quantity
magnetomotive force	electromotive force (voltage)
magnetic field strength	electric field strength
permeability	conductivity
magnetic flux	current
magnetic flux density	current density
reluctance	resistance

Electric analogues

For example, if you have a transformer with a [gapped core](#) then imagine that the core and the gap form a series magnetic circuit with the same [flux](#) flowing through both [reluctance](#) components in an analogous fashion to a series electric circuit in which the same current flows through two resistors -

$$E_m = \Phi \times (R_{m_gap} + R_{m_core}) \text{ ampere-turns}$$

compare

$$V = I \times (R1 + R2) \text{ volts}$$

There's an entire family of formulae which take similar forms in both the electric and magnetic worlds. [Kraus](#) lists most of them.

All analogies break down when pushed too far. This one falls rather quickly if you realise that current flowing through a resistor dissipates energy while [flux](#) flowing through a [reluctance](#) does not. In fact you can ask whether flux is a real physical effect at all (in the way that electron flow is).

Sequence of operation

In transformer design you would normally like to deal in terms of the voltages on the windings. However, the key to understanding what happens in a transformer (or other wound component) is to realise that what the transformer really cares about is the **current** in the windings; and that everything follows on from that.

- The **current** in a winding produces [magneto-motive force](#) -

$$E_m = I \times N \text{ ampere-turns}$$

- The magneto-motive force produces [magnetic field](#) -

$$H = E_m / l_e \quad \text{ampere-turns per metre}$$

- The field produces [magnetic flux density](#) -

$$B = \mu \times H \quad \text{tesla}$$

- Summed over the cross-sectional area of the core this equates to a **total flux** -

$$\Phi = B \times A_e \quad \text{webers}$$

- The time-varying flux produces [induced voltage](#) (EMF) -

$$e = N \times d\Phi/dt \quad \text{volts}$$

If you can follow this five step sequence then building a mental image of a magnetic component becomes simpler. Remember, you put in a current and get back an induced voltage. In fact, if you can treat the [permeability](#) as being linear, then the constants N , l_e , μ and A_e can be lumped together into one constant for the winding which is called (surprise!) [Inductance](#), L -

$$L = \mu \times A_e \times N^2 / l_e \quad \text{henrys}$$

I give the base units for all the quantities in this equation; enabling thrill-seekers to make a [dimensional analysis](#) verifying that it is consistent. Right, so then our five step relationship between current and EMF boils down to:

$$e = L \times dI/dt \quad \text{volts}$$

You may be about to complain that you know the EMF on your winding but don't know the current in it. The answer is that the process then works in reverse - the current will build up until the induced voltage is sufficient to oppose the applied voltage. You can find out more by looking at [Faraday's law](#).

How do you take into account the presence of the secondary windings in a transformer? One way is to take the first four steps of the sequence above and apply them separately to each winding (whether primary or secondary). The arithmetic sum over all windings gives total core [flux](#). From the time rate of change of flux you then have the induced voltage in each winding (since you also know the number of turns for each). There are less tedious methods of analyzing transformer operation which you would probably do better using. But they are another story.

[[↑ Top](#) of page]

E-mail: R.Clarke@surrey.ac.uk

Last modified: 2014 July 22nd.