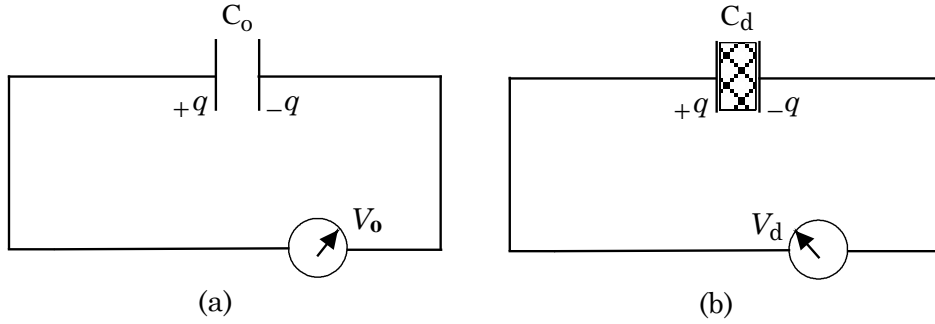


# Chapter 7: Dielectrics

## 7.1 Introduction to Dielectrics

A *dielectric* is an insulator or nonconductor of electricity. Its effects are defined in terms of the ratio of the capacitance of a capacitor with a dielectric between the plates to the capacitance of a capacitor without a dielectric between the plates. As an example consider the parallel plate capacitor  $C_0$  with air between the plates, shown in figure 7.1(a). A battery is momentarily connected to the plates of the capacitor,



**Figure 7.1** Placing a dielectric between the plates of a capacitor.

thereby leaving a charge  $q$  on the plates and establishing a potential difference  $V_0$  between them. The battery is then removed. An electrometer, which is essentially a very sensitive voltmeter, is connected across the plates and reads the potential difference  $V_0$ . The relationship between the charge, the potential, and the capacitance is, of course,

$$C_0 = \frac{q}{V_0} \quad (7.1)$$

A very interesting phenomenon occurs when an insulating material, called a *dielectric*, is placed between the plates of the capacitor, as shown in figure 7.1(b). The voltage, as recorded by the electrometer, drops to a lower value  $V_d$ . Since the charge on the plates remains the same (there is no place for it to go because the battery was disconnected), the only way the potential between the plates can change is for the capacitance to change. The capacitance with the dielectric between the plates can be written as

$$C_d = \frac{q}{V_d} \quad (7.2)$$

where  $C_d$  is the capacitance of the capacitor and  $V_d$  is the potential between the plates when there is a dielectric between them. Dividing equation 7.2 by equation 7.1 gives

$$\frac{C_d}{C_0} = \frac{q/V_d}{q/V_0} = \frac{V_0}{V_d} \quad (7.3)$$

and

## Chapter 7: Dielectrics

$$\frac{C_d}{C_o} = \frac{V_o}{V_d} \quad (7.4)$$

Because it is observed experimentally that  $V_d$  is less than  $V_o$ , the ratio  $V_o/V_d$  is greater than 1 and hence  $C_d$  must be greater than  $C_o$ . Therefore, *placing a dielectric between the plates of a capacitor increases the capacitance of that capacitor*. This should not come as too great a surprise since we saw earlier that the capacitance is a function of the geometrical configuration of the capacitor. Placing an insulator between its plates is a significant change in the capacitor configuration.

The effect of the dielectric between the plates is accounted for by the introduction of a new constant, called *the dielectric constant*  $\kappa$  and it is defined as the ratio of the capacitance with a dielectric between the plates to the capacitance with a vacuum between the plates, that is,

$$\kappa = \frac{C_d}{C_o} \quad (7.5)$$

The dielectric constant depends on the material that the dielectric is made of, and some representative values are shown in table 7.1. Note that  $\kappa$  for air is very close

<b>Table 21.1</b> Dielectric Constant and Dielectric Strength		
<b>Material</b>	<b>Dielectric Constant (<math>\kappa</math>)</b>	<b>Dielectric Strength (<math>\times 10^6</math> V/m)</b>
Vacuum	1.00000	$\infty$
Air	1.00059	3
Carbon tetrachloride	2.238	
Water	80.37	3
Plexiglass	3.12	40
Paper	3.5	40
Mica	5.40	200
Amber	2.7	
Pyrex glass	4.5	30
Bakelite	4.8	24
Ethyl alcohol	26	
Wax paper	2.2	
Benzene	2.3	
Rubber	2.94	21

to the value of  $\kappa$  for a vacuum, which allows us to treat a great many of electric problems in air as if they were in vacuum.

The capacitance of any **capacitor with a dielectric** is easily found from the juxtaposition of equation 7.5 to

$$C_d = \kappa C_o \quad (7.6)$$

For example, the capacitance of a parallel plate capacitor is found from equations 6-14 and 7.6 to be

$$C_d = \kappa \epsilon_o \frac{A}{d} \quad (7.7)$$

Quite often a new constant is used, called the *permittivity of the medium* and is defined as

$$\epsilon = \kappa \epsilon_0 \quad (7.8)$$

### Example 7.1

A parallel plate capacitor with air between the plates has a capacitance of  $3.85 \mu\text{F}$ . If mica is placed between the plates of the capacitor what will the new capacitance be?

#### Solution

The value of the dielectric constant for mica is found in table 7.1 as  $\kappa = 5.40$ . The capacitance with the dielectric placed between the plates is found from equation 7.6 as

$$\begin{aligned} C_d &= \kappa C_o \\ &= 5.40 \times 3.85 \times 10^{-6} \text{ F} \\ &= 2.08 \times 10^{-5} \text{ F} \end{aligned}$$

[To go to this Interactive Example click on this sentence.](#)

### Example 7.2

Find the permittivity of the dielectric material mica.

#### Solution

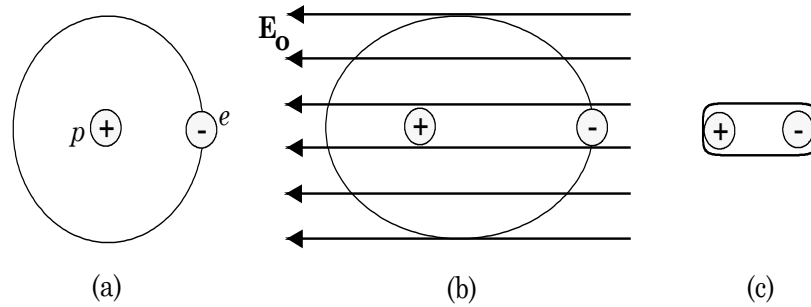
The permittivity of mica is found from equation 7.8 as

$$\begin{aligned} \epsilon &= \kappa \epsilon_0 \\ &= (5.40)(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2) \\ &= 4.78 \times 10^{-11} \text{ C}^2/\text{N m}^2 \end{aligned}$$

[To go to this Interactive Example click on this sentence.](#)

## 7.2 Atomic Description of a Dielectric

As was mentioned in chapter 2, all atoms are electrically neutral. We can see the reason for this from figure 7.2(a). The positive charge is located at the center of the atom. The electron revolves around the nucleus. Sometimes it is to the right of



**Figure 7.2** Placing an atom in a uniform electric field.

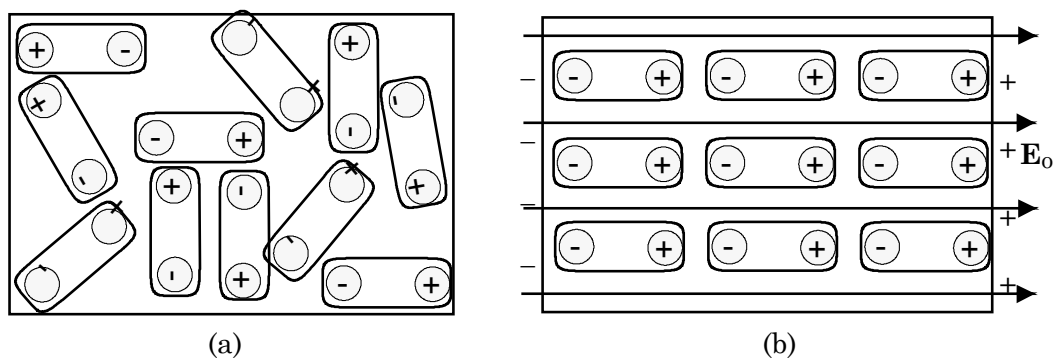
the nucleus, sometimes to the left, sometimes it is above the nucleus, sometimes below. Because of this symmetry, its *mean* position appears as if it were at the center of the nucleus. Therefore, it appears as though there is a positive charge and a negative charge at the center of the atom. These equal but opposite charges give the effect of neutralizing each other, and hence the atom does not appear to have any charges and appears neutral. The electric field of the positive charge is radially outward, whereas the average electric field of the negative charge is radially inward. These equal but opposite average electric fields have the effect of neutralizing each other, and hence the atom also does not appear to have any electric field associated with it. Because of this basic symmetry of the atom, all atoms are electrically neutral.

If the atom is now placed in a uniform external electric field  $E_0$ , we observe a change in this symmetry, as shown in figure 7.2(b). The electron of the atom finds itself in an external field  $E_0$  and experiences the force

$$\mathbf{F} = q\mathbf{E}_0 = -e\mathbf{E}_0$$

Because the electronic charge is negative, the force on the electron is opposite to the direction of the external field  $E_0$ . When the electron is to the right of the nucleus, the external electric field pulls it even further to the right, away from the nucleus. When the electron is to the left of the nucleus, the external field again pulls it toward the right, this time toward the nucleus. The overall effect of this external field is to change the orbit of the electron from a circle to an ellipse, as shown in figure 7.2(b). The mean position of the negative electron no longer coincides with the positive nucleus, but because of the new symmetry is displaced to the right of the nucleus, as shown in figure 21.8(c). The apparently neutral atom has been converted to an electric dipole by the uniform external electric field  $\mathbf{E}_0$ .

When two or more atoms combine they form a molecule. Many molecules have symmetric charge distributions and therefore also appear electrically neutral. Some molecules, however, do not have symmetric charge distributions and therefore exhibit an electric dipole moment. Such molecules are called *polar molecules*. A slab of material made from polar molecules is shown in figure 7.3(a). In general, the electric dipoles are arranged randomly. If the dielectric is placed in an external



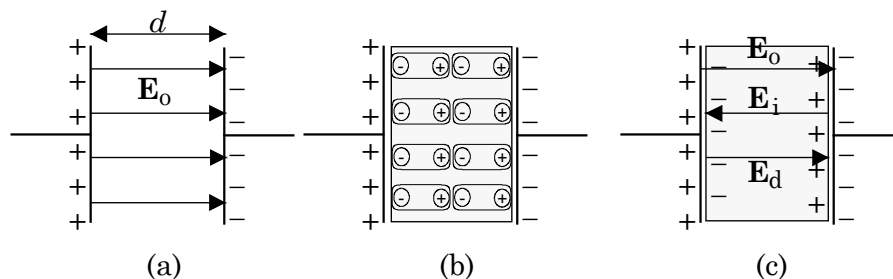
**Figure 7.3** A polar dielectric.

electric field, electric forces act on the dipoles until the dipoles become aligned with the field, as shown in figure 7.3(b).

The net effect is that a dielectric material made of dipole molecules has those dipoles aligned in an external electric field. A dielectric material not composed of polar molecules has electric dipoles induced in them by the external electric field, as in figure 7.2. These dipoles are in turn aligned by the external electric field. Therefore, all insulating materials become polarized in an external electric field. The overall effect is to have positive charges on the right of the slab in figure 7.3(b) and negative charges to the left. These charges are not free charges, but are bound to the atoms of the dielectric. They are merely the displaced charges and not a flow of charges.

### 7.3 A Parallel Plate Capacitor with A Dielectric Between the Plates

With this digression into the atomic nature of dielectrics, we can now explain the effect of a dielectric placed between the plates of a capacitor. Figure 7.4(a) shows a charged capacitor with air between the plates. A uniform electric field  $E_0$  is found



**Figure 7.4** Electric field between the plates of the capacitor.

between the plates. A slab of dielectric material is now placed between the plates, as shown in figure 7.4(b), and completely fills the space between the plates. The uni-

## Chapter 7: Dielectrics

form electric field  $E_0$  polarizes the dielectric by aligning the electric dipoles, figure 7.4(b). The net effect is to displace positive bound charges to the right of the slab and negative bound charges to the left of the slab. These induced bound charges have an electric field  $E_i$  associated with them. The induced electric field  $E_i$  emanates from the positive bound charges on the right of the dielectric and terminates on the negative bound charges on the left of the dielectric, and, as we can see from figure 7.4(c),  $E_i$  is opposite to the original electric field of the capacitor. Hence, the total electric field within the dielectric is the vector sum of the two electric fields, that is,

$$\mathbf{E}_d = \mathbf{E}_0 + \mathbf{E}_i$$

or in terms of their magnitudes

$$E_d = E_0 - E_i \quad (7.9)$$

*Therefore, placing a dielectric between the plates of a capacitor reduces the electric field between the plates of the capacitor.*

The potential difference  $V_0$  across the plates of the capacitor, when there is no dielectric between the plates, is given by

$$V_0 = E_0 d \quad (7.10)$$

While the potential difference  $V_d$  across the capacitor when there is a dielectric between the plates is

$$V_d = E_d d \quad (7.11)$$

Dividing equation 7.10 by equation 7.11 gives

$$\frac{V_0}{V_d} = \frac{E_0}{E_d} \quad (7.12)$$

But from equation 7.3 the ratio of the capacitance with a dielectric to the capacitance without a dielectric was

$$\frac{C_d}{C_0} = \frac{V_0}{V_d} \quad (7.3)$$

Combining this result with equation 7.12 gives

$$\frac{C_d}{C_0} = \frac{V_0}{V_d} = \frac{E_0}{E_d}$$

Recall that the ratio of  $C_d/C_0$  was defined in equation 7.5 as the dielectric constant,  $\kappa$ . Therefore,

$$\frac{C_d}{C_0} = \frac{V_0}{V_d} = \frac{E_0}{E_d} = \kappa \quad (7.13)$$

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The effect of the dielectric between the plates is summarized from equation 7.13 as follows:

1. The capacitance with a dielectric increases to

$$C_d = \kappa C_o \quad (7.6)$$

2. The potential difference between the plates decreases to

$$V_d = \frac{V_o}{\kappa} \quad (7.14)$$

and

3. the electric field between the plates decreases to

$$E_d = \frac{E_o}{\kappa} \quad (7.15)$$

The value of the induced electric field  $E_i$  within the dielectric can be found from equation 7.15 and 21-32 as

$$E_d = E_o - E_i = \frac{E_o}{\kappa}$$

Solving for  $E_i$  gives

$$E_i = E_o - \frac{E_o}{\kappa} = E_o \left(1 - \frac{1}{\kappa}\right)$$
$$E_i = E_o \left(\frac{\kappa - 1}{\kappa}\right) \quad (7.16)$$

The presence of a dielectric between the plates of a capacitor not only increases the capacitance of the capacitor, but it also allows a much larger voltage to be applied across the plates. When the voltage across the plates of an air capacitor exceeds  $3.00 \times 10^6$  V/m, the air between the plates is no longer an insulator, but conducts electric charge from the positive plate to the negative plate as a *spark* or *arc discharge*. The dielectric is said to break down. The capacitor is now useless as a capacitor since it now conducts electricity through it and is said to be *short circuited*. The value of the potential difference per unit plate separation when the dielectric breaks down is called the **dielectric strength** of the medium, and is given in the second column of table 7.1. If mica is placed between the plates, the breakdown voltage rises to  $200 \times 10^6$  V/m, a value that is 66 times greater than the value for air.

### Example 7.3

*A capacitor with a dielectric.* A parallel plate capacitor, having a plate area of  $25.0 \text{ cm}^2$  and a plate separation of  $2.00 \text{ mm}$ , is charged by a  $100 \text{ V}$  battery. The battery is then removed. Find: (a) the capacitance of the capacitor, (b) the charge on the plates of the capacitor. A slab of mica is then placed between the plates of the capacitor. Find, (c) the new value of the capacitance, (d) the potential difference across the ca-

pacitor with the dielectric, (e) the initial electric field between the plates, (f) the final electric field between the plates, (g) the initial energy stored in the capacitor, and (h) the final energy stored in the capacitor.

### ***Solution***

---

(a) The original capacitance of the capacitor, found from equation 6-14, is

$$\begin{aligned}
 C_o &= \epsilon_o \frac{A}{d} = \left( 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N m}^2} \right) \left( \frac{25.0 \text{ cm}^2}{0.002 \text{ m}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right)^2 \\
 C_o &= \left( 1.11 \times 10^{-11} \frac{\text{C}^2}{\text{N m}} \right) \left( \frac{\text{N m}}{\text{J}} \right) \\
 C_o &= \left( 1.11 \times 10^{-11} \frac{\text{C}}{\text{J/C}} \right) \left( \frac{\text{J/C}}{\text{V}} \right) \left( \frac{\text{F}}{\text{C/V}} \right) \\
 C_o &= 1.11 \times 10^{-11} \text{ F} = 11.1 \times 10^{-12} \text{ F} \\
 C_o &= 11.1 \text{ pF}
 \end{aligned}$$

(b) The charge on the plates of the capacitor, found from equation 6-15, is

$$\begin{aligned}
 q &= C_o V_o = (11.1 \times 10^{-12} \text{ F})(100 \text{ V}) \frac{(\text{C/V})}{(\text{F})} \\
 q &= 11.1 \times 10^{-10} \text{ C}
 \end{aligned}$$

This charge remains the same throughout the rest of the problem because the battery was disconnected.

(c) The value of the capacitance when a mica dielectric is placed between the plates is found from equation 7.6 and table 7.1 as

$$\begin{aligned}
 C_d &= \kappa C_o \\
 C_d &= (5.40)(11.1 \text{ pF}) \\
 C_d &= 59.9 \text{ pF}
 \end{aligned}$$

(d) The new potential difference between the plates is found from equation 7.14 as

$$V_d = \frac{V_o}{\kappa} = \frac{100 \text{ V}}{5.40} = 18.5 \text{ V}$$

This could also have been found by equation 7.2 as

$$V_d = \frac{q}{C_d} = \frac{11.1 \times 10^{-10} \text{ C}}{59.9 \times 10^{-12} \text{ F}} = 18.5 \text{ V}$$

(e) The initial electric field between the plates is found from equation 7.10 as



## Chapter 7: Dielectrics

$$E_o = \frac{V_o}{d} = \frac{100 \text{ V}}{0.002 \text{ m}} = 5.00 \times 10^4 \text{ V/m}$$

(f) The final electric field between the plates, when the dielectric is present, is found from equation 7.15 as

$$E_d = \frac{E_o}{\kappa} = \frac{5.00 \times 10^4 \text{ V/m}}{5.40} = 9.25 \times 10^3 \text{ V/m}$$

This could also have been found from equation 7.11 as

$$E_d = \frac{V_d}{d} = \frac{18.5 \text{ V}}{0.002 \text{ m}} = 9.25 \times 10^3 \text{ V/m}$$

(g) The initial energy stored in the capacitor is found from equation 6-50 as

$$\begin{aligned} W_o &= \frac{1}{2} C_o V_o^2 = \frac{1}{2} (11.1 \times 10^{-12} \text{ F})(100 \text{ V})^2 \\ W_o &= 5.55 \times 10^{-8} \frac{\text{C V}^2 (\text{J})}{\text{V} (\text{CV})} \\ W_o &= 5.55 \times 10^{-8} \text{ J} \end{aligned}$$

(h) The final energy stored in the capacitor when there is a dielectric between the plates is

$$W_d = \frac{1}{2} C_d V_d^2 \quad (7.17)$$

The energy can be computed directly from equation 7.17, but it is instructive to do the following. The capacitance  $C_d$  can be substituted from equation 7.6 and the potential difference can be substituted from equation 7.14 into equation 7.17 to yield

$$\begin{aligned} W_d &= \frac{1}{2} C_d V_d^2 = \frac{1}{2} (\kappa C_o) \left( \frac{V_o}{\kappa} \right)^2 \\ W_d &= \frac{1}{\kappa} \left( \frac{1}{2} C_o V_o^2 \right) \\ W_d &= \frac{W_o}{\kappa} \end{aligned} \quad (7.18)$$

*This is the energy stored in the capacitance with a dielectric between the plates.* Substituting the numbers into equation 7.18 gives

$$\begin{aligned} W_d &= \frac{W_o}{\kappa} = \frac{5.55 \times 10^{-8} \text{ J}}{5.40} \\ W_d &= 1.03 \times 10^{-8} \text{ J} \end{aligned}$$

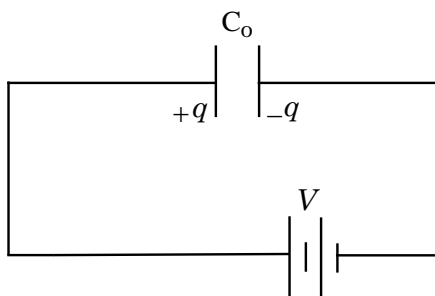
*The energy stored in the capacitor with a dielectric is less than without the dielectric.* This, at first, may seem to be a violation of the law of conservation of energy, since

there is a decrease in energy. But this decrease in energy can be accounted for by the work done in moving the dielectric slab between the plates of the capacitor.

**To go to this Interactive Example click on this sentence.**

### Example 7.4

A capacitor connected to a battery. A 6.00 pF air capacitor is connected to a 100 V battery (figure 7.5). A piece of mica is now placed between the plates of the capacitor. Find: (a) the original charge on the plates of the capacitor, (b) the new value of the capacitance with the dielectric, (c) the new charge on the plates.



**Figure 7.5** Placing a dielectric between the plates while maintaining a constant potential between the plates.

### Solution

(a) The original charge on the plates is found from

$$q_o = C_o V_o = (6.00 \times 10^{-12} \text{ F})(100 \text{ V}) \frac{(\text{C/V})}{(\text{F})}$$

$$q_o = 6.00 \times 10^{-10} \text{ C}$$

(b) The new value of the capacitance is found from

$$C_d = \kappa C_o = (5.40)(6.00 \times 10^{-12} \text{ F})$$

$$C_d = 32.4 \times 10^{-12} \text{ F}$$

$$C_d = 32.4 \text{ pF}$$

(c) In this example, the battery is not disconnected from the battery. Therefore the potential across the plates remains the same whether the capacitor has a dielectric or not. Because the capacitance of the capacitor has changed, the charge on the plates must now change and is found from

$$\begin{aligned} q_d &= C_d V_d \\ q_d &= (32.4 \times 10^{-12} \text{ F})(100 \text{ V}) \\ q_d &= 32.4 \times 10^{-10} \text{ C} \end{aligned}$$

By keeping the potential across the plates constant, more charge was drawn from the battery. A clear distinction must be made between this example and the previous one. In example 7.3 the charge remained constant because the battery was disconnected, and the potential varied by the introduction of the dielectric. In this example, the potential remained a constant, because the battery was *not disconnected*, and the charge varied with the introduction of the dielectric.

**To go to this Interactive Example click on this sentence.**

## 7.4 Dielectrics and Coulomb's Law

When Coulomb's law was treated in chapter 2, the charges were located in a vacuum or air. What happens if the charges are encased in a dielectric material? We will see that Coulomb's law must be modified to take into account the dielectric material. The electric field in a vacuum was defined as  $E = F/q_0$  in equation 3-1. Calling the electric field when no dielectric is present  $E_0$ , equation 3-1 becomes

$$E_0 = \frac{F}{q} \quad (7.19)$$

But if a dielectric medium is present the electric field within the dielectric is given by equation 7.15 as

$$E_d = \frac{E_0}{\kappa} \quad (7.15)$$

Replacing equation 7.19 into equation 7.15 gives

$$E_d = \frac{F}{\kappa q} \quad (7.20)$$

Comparing equations 7.20 with equation 7.19 prompts us to define the electric field in a dielectric as the ratio of the force acting on a particle within the dielectric to the charge, i.e.,

$$E_d = \frac{F_d}{q} \quad (7.21)$$

Comparing equations 7.20 and 7.21 allows us to define the force on a particle in a dielectric as

## Chapter 7: Dielectrics

$$F_d = \frac{F}{\kappa} \quad (7.22)$$

Using Coulomb's law, we can now find *the force between point charges in a dielectric medium as*

$$F_d = \frac{F}{\kappa} = \frac{1}{4\pi\epsilon_o\kappa} \frac{q_1q_2}{r^2}$$

Recall from equation 7.8 that the permittivity of the medium is defined as

$$\epsilon = \kappa\epsilon_0$$

*In general then, Coulomb's law should be expressed as*

$$F = \frac{1}{4\pi\epsilon} \frac{q_1q_2}{r^2} \quad (7.23)$$

As the special case for a vacuum  $\kappa = 1$ , and equation 7.23 reduces to

$$F = \frac{1}{4\pi\epsilon_o} \frac{q_1q_2}{r^2}$$

the form of Coulomb's law used in chapter 2.

### **Example 7.5**

Find the effect of the dielectric on the force between one  $\text{Na}^+$  ion, and one  $\text{Cl}^-$  ion, separated by a distance of  $1.00 \times 10^{-7}$  cm, for the salt sodium chloride, NaCl, which is common table salt when it is placed in water.

### **Solution**

Although NaCl consists of a lattice structure of  $\text{Na}^+$  and  $\text{Cl}^-$  ions, let us just consider the electric force between one  $\text{Na}^+$  ion, and one  $\text{Cl}^-$  ion separated by a distance of  $1.00 \times 10^{-7}$  cm. The force between the ions is given by Coulomb's law as

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_o} \frac{q_1q_2}{r^2} \\ &= \left(9.00 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}\right) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(1.00 \times 10^{-9} \text{ m})^2} \\ &= 2.30 \times 10^{-10} \text{ N} \end{aligned}$$

If this salt is now placed in water,  $\kappa_w = 80.0$ , the force between the ions becomes

$$F_d = \frac{1}{4\pi\epsilon_o\kappa} \frac{q_1q_2}{r^2}$$

$$\begin{aligned}
 &= \left( \frac{9.00 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}}{80.0} \right) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(1.00 \times 10^{-9} \text{ m})^2} \\
 &= 0.0288 \times 10^{-10} \text{ N}
 \end{aligned}$$

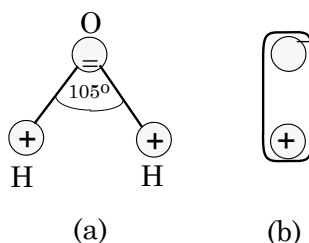
This could also be found as

$$\begin{aligned}
 F_d &= \frac{F}{\kappa} \\
 &= \frac{2.30 \times 10^{-10} \text{ N}}{80.0} \\
 &= 0.0288 \times 10^{-10} \text{ N}
 \end{aligned}$$

The force between the ions in the water solution has decreased by a factor of 1/80. This is why NaCl readily dissolves in water. The force between the ions becomes small enough for simple thermal energy to pull the ions apart. In fact, most chemicals are soluble in water because of this high value of 80 for the dielectric constant. If the NaCl salt is placed in benzene,  $\kappa = 2.3$ , the force between the  $\text{Na}^+$  and  $\text{Cl}^-$  ions is not reduced enough to allow the ions to disassociate, and hence NaCl does not dissolve in benzene.

**To go to this Interactive Example click on this sentence.**

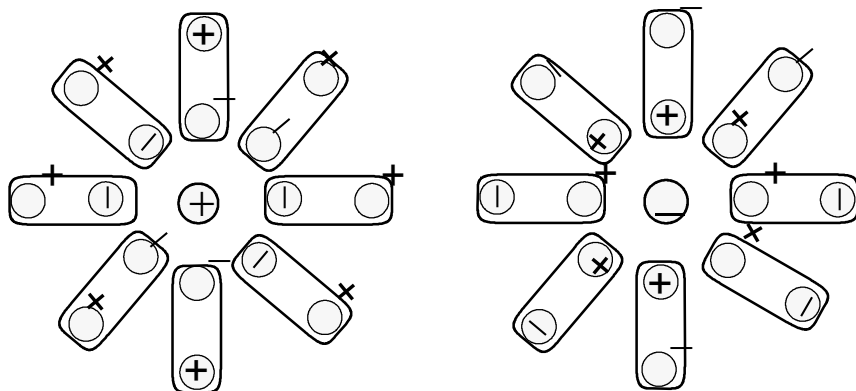
The water molecule,  $\text{H}_2\text{O}$ , is composed of two atoms of hydrogen and one atom of oxygen. The molecular configuration is not spherically symmetric, but is rather asymmetric, as shown in figure 7.6(a). The oxygen atom has picked up two electrons, one from each hydrogen atom, and is therefore charged doubly negative. Each hydrogen atom, having lost an electron to the oxygen, is positive. The mean position of the positive charge does not coincide with the location of the negative



**Figure 7.6** The water molecule.

charge but lies below it in the diagram. Hence, the water molecule behaves as an electric dipole, as seen in figure 7.6(b), and it is a polar molecule. When a material, such as the sodium chloride, is placed in water, these water dipoles surround the  $\text{Na}^+$  and  $\text{Cl}^-$  ions, as shown in figure 7.7. The negative ends of the water molecule are attracted to the positive ions, whereas the positive end of the water molecules are attracted to the negative ion. These dipoles that surround the ions, shield the

ions and have the effect of decreasing the effective charge of the ions and hence decrease the coulomb force between them. Liquids with large values of  $\kappa$  are the most effective in decreasing the force between the separated ions and are therefore the best solvents for ionic crystals.



**Figure 7.7** Water dipoles surround ions placed in water.

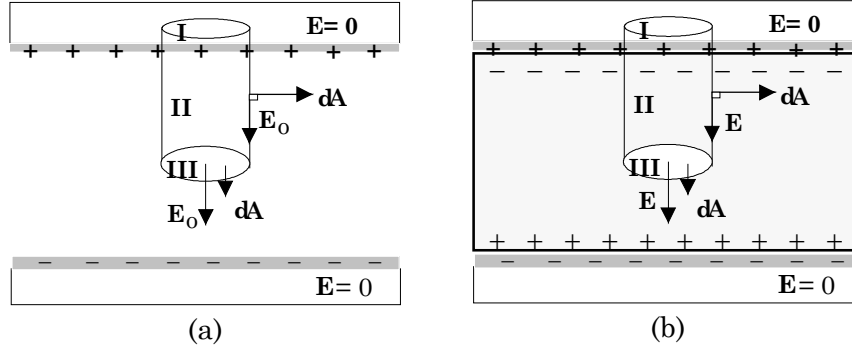
## 7.5 Dielectrics and Gauss's Law

When we studied Gauss's law in chapter 4 we found that we could apply that law to determine the electric field in a vacuum or air. But what happens if a dielectric is present? Does Gauss's law need to be modified, and if so, how should we modify it? We will first apply Gauss's law to a parallel plate capacitor with air between the plates, then we will apply it when there is a dielectric between the plates.

Let us first consider the parallel plate capacitor with air between the plates, studied in section 6.2 and reproduced here, in figure 7.8(a). The application of Gauss's law is the same as in section 6.2 and we outline it here. We start by drawing a Gaussian cylinder, as shown in the diagram. One end of the Gaussian surface lies within one of the parallel conducting plates while the other end lies in the region between the two conducting plates where we wish to find the electric field. Gauss's law is given by equation 4-14 as

$$\Phi_E = \oint \mathbf{E}_o \cdot d\mathbf{A} = \frac{q}{\epsilon_o} \quad (7.24)$$

We have used the symbol  $\mathbf{E}_o$  in equation 7.24 for the electric field when there is only air between the plates to distinguish it from when we later place a dielectric between the plates. The sum in equation 7.24 is over the entire Gaussian surface.



**Figure 7.8** Gauss's law applied to a dielectric.

As before, we break the entire surface of the cylinder into three surfaces. Surface I is the end cap on the topside of the cylinder, surface II is the main cylindrical surface, and surface III is the end cap on the bottom of the cylinder as shown in figure 7.8(a). The total flux  $\Phi$  through the Gaussian surface is the sum of the flux through each individual surface. That is,

$$\Phi = \Phi_I + \Phi_{II} + \Phi_{III}$$

where

$\Phi_I$  is the electric flux through surface I  
 $\Phi_{II}$  is the electric flux through surface II  
 $\Phi_{III}$  is the electric flux through surface III

but as we showed in section 6-2, the flux  $\Phi_I$  through surface I is zero because  $\mathbf{E} = 0$  inside the conducting plate, and the electric flux  $\Phi_{II}$  through surface II is zero because  $d\mathbf{A}$  is everywhere perpendicular to the electric field  $\mathbf{E}_0$ , and therefore  $\mathbf{E}_0 \cdot d\mathbf{A}$  is also equal to zero. Surface III is the end cap on the bottom of the cylinder and as can be seen from figure 7.8(a),  $\mathbf{E}_0$  points downward and since the area vector  $d\mathbf{A}$  is perpendicular to the surface it also points downward. Hence  $\mathbf{E}_0$  and  $d\mathbf{A}$  are parallel to each other and the angle  $\theta$  between  $\mathbf{E}_0$  and  $d\mathbf{A}$  is zero. Hence, the flux through surface III is

$$\Phi_{III} = \int_{III} \mathbf{E}_0 \cdot d\mathbf{A} = \int_{III} E_0 dA \cos \theta = \int_{III} E_0 dA \cos 0^\circ = \int_{III} E_0 dA \quad (7.25)$$

The total flux through the Gaussian surface is equal to the sum of the fluxes through the individual surfaces. Hence, Gauss's law becomes

$$\Phi_E = 0 + 0 + \int_{III} E_0 dA = \frac{q}{\epsilon_0} \quad (7.26)$$

But  $E_0$  is constant in every term of the sum in integral III and can be factored out of the integral giving

$$\Phi_E = E_0 \int dA = \frac{q}{\epsilon_0}$$

## Chapter 7: Dielectrics

But  $\oint dA = A$  the area of the end cap, hence

$$E_o A = \frac{q}{\epsilon_o}$$

$A$  is the magnitude of the area of Gaussian surface III and  $q$  is the charge enclosed within Gaussian surface III. Solving for the electric field  $E_o$  between the conducting plates gives

$$E_o = \frac{q}{\epsilon_o A} \quad (7.27)$$

Equation 7.27, as it now stands, describes the electric field in terms of the charge enclosed by Gaussian surface III and the area of Gaussian surface III. *If the surface charge density on the plates is uniform, the surface charge density enclosed by Gaussian surface III is the same as the surface charge density of the entire plate. Hence the ratio of  $q/A$  for the Gaussian surface is the same as the ratio  $q/A$  for the entire plate.* Thus  $q$  in equation 7.27 can also be interpreted as the total charge  $q$  on the plates, and  $A$  can be interpreted as the total area of the conducting plate. Therefore, equation 7.27 is rewritten as

$$E_o = \frac{q}{\epsilon_o A} \quad (7.28)$$

Equation 7.28 gives us the electric field  $E_o$  between the plates of a parallel plate capacitor when there is only air between the plates.

We now place a dielectric between the parallel plates as shown in figure 7.8(b). We call the electric field  $\mathbf{E}$  now, because there is a dielectric between the plates, to distinguish it from the value of  $\mathbf{E}_o$  when there was only air between the plates. Gauss's law is again applied to the problem to give

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{(q - q')}{\epsilon_o} \quad (7.29)$$

Notice now how Gauss's law has changed. There are now two charge terms in the right-hand side of the equation. One,  $q$ , is the electric charge on the conducting plate of the capacitor as before. The other,  $-q'$ , is the induced negative surface bound charge that we saw in figure 7.4 when a dielectric is placed between the plates. Hence, the total charge contained within the Gaussian surface is now  $q - q'$ .

The rest of the analysis is similar to before. The sum in equation 7.29 is over the entire Gaussian surface. As before, we break the entire surface of the cylinder into three surfaces. Surface I is the end cap on the topside of the cylinder, surface II is the main cylindrical surface, and surface III is the end cap on the bottom of the cylinder as shown in figure 7.8(b). The total flux  $\Phi$  through the Gaussian surface is the sum of the flux through each individual surface. That is,

$$\Phi = \Phi_I + \Phi_{II} + \Phi_{III}$$



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but as we just showed, the flux  $\Phi_I$  through surface I is zero because  $\mathbf{E} = 0$  inside the conducting plate, and the electric flux  $\Phi_{II}$  through surface II is zero because  $d\mathbf{A}$  is everywhere perpendicular to the electric field  $\mathbf{E}$ , and therefore  $\mathbf{E} \cdot d\mathbf{A}$  is also equal to zero. Surface III is the end cap on the bottom of the cylinder and as can be seen from figure 7.8(b),  $\mathbf{E}$  points downward and the area vector  $d\mathbf{A}$  also points downward. Hence  $\mathbf{E}$  and  $d\mathbf{A}$  are parallel to each other and the angle  $\theta$  between  $\mathbf{E}$  and  $d\mathbf{A}$  is zero. Hence, the flux through surface III is

$$\Phi_{III} = \int_{III} \mathbf{E} \cdot d\mathbf{A} = \int_{III} E dA \cos \theta = \int_{III} E dA \cos 0^\circ = \int_{III} E dA \quad (7.30)$$

The total flux through the Gaussian surface is equal to the sum of the fluxes through the individual surfaces. Hence, Gauss's law becomes

$$\Phi_E = 0 + 0 + \int_{III} E dA = \frac{(q - q')}{\epsilon_o} \quad (7.31)$$

But  $E$  is constant in every term of the sum in integral III and can be factored out of the integral giving

$$\Phi_E = E \int dA = \frac{(q - q')}{\epsilon_o}$$

But  $\int dA = A$  the area of the end cap, hence

$$E A = \frac{(q - q')}{\epsilon_o}$$

$A$  is the magnitude of the area of Gaussian surface III and  $q$  is the free charge enclosed within the Gaussian surface, and  $-q'$  is the bound charge contained within the Gaussian surface. Solving for the electric field  $E$  between the conducting plates gives

$$E = \frac{(q - q')}{\epsilon_o A} \quad (7.32)$$

Equation 7.32, as it now stands, describes the electric field in terms of the charge  $(q - q')$  enclosed by Gaussian surface III and the area of Gaussian surface III. *If the surface free charge density on the plates is uniform, the surface free charge density enclosed by Gaussian surface III is the same as the surface free charge density of the entire plate, and if the induced bound charge density on the dielectric is uniform, the induced bound charge density enclosed by Gaussian surface III is the same as the induced bound charge density of the entire plate. Hence the ratio of  $(q - q')/A$  for the Gaussian surface is the same as the ratio  $(q - q')/A$  for the entire plate.* Thus  $q$  in equation 7.27 can also be interpreted as the total charge  $q$  on the plates,  $q'$  as the total induced bound charge on the dielectric, and  $A$  can be interpreted as the total area of the conducting plate. Therefore, equation 7.32 is rewritten as

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$$E = \frac{(q - q')}{\epsilon_o A} \quad (7.33)$$

Equation 7.33 gives us the electric field  $E$  between the plates of a parallel plate capacitor when there is a dielectric placed between the plates.

The problem with equation 7.33 as it now stands is that we don't know the actual value of  $q'$ . We can determine it as follows. Let us rewrite equation 7.33 in the form

$$E = \frac{(q - q')}{\epsilon_o A} = \frac{q}{\epsilon_o A} - \frac{q'}{\epsilon_o A} \quad (7.34)$$

But we showed in equation 7.15 that the electric field in a dielectric is related to the electric field without the dielectric by

$$E = \frac{E_o}{\kappa}$$

And the electric field  $E_o$  was determined by equation 7.28 as

$$E_o = \frac{q}{\epsilon_o A}$$

Replacing equation 7.15 and equation 7.28 into equation 7.34 yields

$$E = \frac{E_o}{\kappa} = \frac{q}{\kappa \epsilon_o A} = \frac{q}{\epsilon_o A} - \frac{q'}{\epsilon_o A} \quad (7.35)$$

Simplifying

$$\frac{q}{\kappa} = q - q'$$

Solving for  $q'$  we get

$$q' = q \left( 1 - \frac{1}{\kappa} \right) \quad (7.36)$$

Equation 7.36 gives us the induced bound charge on the dielectric.

Using equation 7.36 allows us to rewrite Gauss's law, equation 7.29, into the form

$$\begin{aligned} \Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} &= \frac{(q - q')}{\epsilon_o} = \frac{(q - q(1 - \frac{1}{\kappa}))}{\epsilon_o} = q \frac{(1 - 1 + \frac{1}{\kappa})}{\epsilon_o} \\ \Phi_E &= \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_o \kappa} \end{aligned} \quad (7.37)$$

Multiplying both side of the equation by  $\epsilon_o \kappa$  we get

$$\Phi_E (\kappa \epsilon_o) = \oint \kappa \epsilon_o \mathbf{E} \cdot d\mathbf{A} = q \quad (7.38)$$

But recall from equation 7.8 that  $\kappa \epsilon_o = \epsilon$ , where  $\kappa$  is the dielectric constant of the medium,  $\epsilon_o$  is the permittivity of air or a vacuum, and  $\epsilon$  is the permittivity of the medium. Replacing this into equation 7.38 yields

$$\epsilon\Phi_E = \oint \epsilon \mathbf{E} \cdot d\mathbf{A} = q \quad (7.39)$$

Equation 7.39 is Gauss's law when a dielectric is present. The dielectric is taken into account by the permittivity  $\epsilon$  of the medium. Notice that the charge contained within the Gaussian surface is still only the free charge  $q$ . A new electric flux density, when dielectrics are present, is defined by

$$\Psi_E = \epsilon\Phi_E \quad (7.40)$$

We now define a new quantity called the **electric displacement vector** as

$$\mathbf{D} = \epsilon \mathbf{E} = \kappa \epsilon_0 \mathbf{E} \quad (7.41)$$

With this new definition, Gauss's law becomes

$$\oint \mathbf{D} \cdot d\mathbf{A} = q \quad (7.42)$$

Equation 7.42 is the generalization of Gauss's law. It handles the problems associated with dielectrics through equation 7.41 and reduces to the specialized cases of vacuum and air by equation 7.8 because  $\kappa = 1$  for air and the permittivity reduces to  $\epsilon = \kappa \epsilon_0 = (1)\epsilon_0 = \epsilon_0$ . We will apply equation 7.42 to finding the electric fields in dielectric media in section 7.7.

## 7.6 The Polarization Vector, the Displacement Vector and the Electric Field Vector

As we have seen, when a dielectric is placed in a uniform electric field there is a bound positive charge on one side of the dielectric and a bound negative charge on the other side. We say that the dielectric material becomes polarized, and a polarization vector can be defined. Some interesting relations between the electric field vector, the displacement vector, and the polarization vector is found by analyzing equation 7.35 further.

$$E = \frac{E_o}{\kappa} = \frac{q}{\kappa \epsilon_0 A} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \quad (7.35)$$

Let us rewrite the equation in the form (by transposing)

$$\frac{q}{\epsilon_0 A} = \frac{q}{\kappa \epsilon_0 A} + \frac{q'}{\epsilon_0 A}$$

Multiply each term by  $\epsilon_0$  we get

$$\frac{q}{A} = \epsilon_0 \left( \frac{q}{\kappa \epsilon_0 A} \right) + \frac{q'}{A} \quad (7.43)$$

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But from equation 7.35, the electric field within the dielectric is given by

$$E = \frac{q}{\kappa\epsilon_o A} \quad (7.44)$$

Replacing equation 7.44 into equation 7.43 gives

$$\frac{q}{A} = \epsilon_o E + \frac{q'}{A} \quad (7.45)$$

Let us consider the last term  $q'/A$ , and multiply the numerator and denominator by the quantity  $d$ , the separation distance between the two parallel plates, and hence the thickness of the dielectric. Therefore,

$$\frac{q'}{A} = \frac{q'd}{Ad} \quad (7.46)$$

Let us consider the meaning of each of these terms. The term  $q'd$  is the product of the induced bound charge  $q'$  and the distance  $d$  separating them. But this is exactly the meaning of an electric dipole moment  $p$ . Therefore,

$$p = q'd \quad (7.47)$$

*is the electric dipole moment of the dielectric material.* The term  $Ad$  is just the volume  $v$  taken up by the dielectric material. Equation 7.46 becomes

$$\frac{q'}{A} = \frac{q'd}{Ad} = \frac{p}{v} \quad (7.48)$$

We now define a new quantity, called the electric polarization vector  $P$  as

$$P = \frac{p}{v} \quad (7.49)$$

*which is the electric dipole moment per unit volume of the dielectric material.* Replacing equation 7.49 into equation 7.45 gives

$$\frac{q}{A} = \epsilon_o E + P \quad (7.50)$$

Finally let us consider the term  $q/A$ . The electric field between the plates when a dielectric is present is given by equation 7.44 as

$$E = \frac{q}{\kappa\epsilon_o A} \quad (7.44)$$

If we solve equation 7.44 for  $q/A$  we get

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$$\frac{q}{A} = \kappa \epsilon_0 E \quad (7.51)$$

But we defined  $\kappa \epsilon_0 E$  in equation 7.41 to be the displacement vector  $D$ . Therefore we obtain

$$\frac{q}{A} = D \quad (7.52)$$

Equation 7.52 says that the magnitude of the displacement vector is simply equal to the ratio of the free charge  $q$  on the plates to the area  $A$  of the plates. Replacing equation 7.52 into equation 7.50 yields

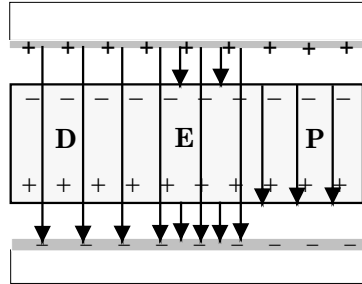
$$D = \epsilon_0 E + P \quad (7.53)$$

A generalization of equation 7.53 can be written as

$$D = \epsilon_0 E + P \quad (7.54)$$

Equation 7.53 gives the relation between the electric displacement vector  $D$ , the electric field vector  $E$ , and the polarization vector  $P$ .

The displacement vector  $D$  is associated with free charge only and originates on positive free charge and terminates on negative free charge as shown in figure 7.9. The polarization vector  $P$  is associated with bound charges only and originates



**Figure 7.9** The three electric vectors.

on negative induced bound charges and terminates on positive bound charge as for any dipole. The polarization vector lies only within the dielectric as seen in figure 7.9. The electric field vector  $E$  is associated with all charges, both free charges and bound charges and is also shown in figure 7.9.

Just as the displacement vector  $D$  can be expressed in term of the electric field vector  $E$  alone, that is

$$D = \epsilon E = \kappa \epsilon_0 E$$

The polarization vector  $P$  can also be expressed in terms of the electric field vector  $E$  alone. Recall from equation 7.48 and 7.49

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$$\frac{q'}{A} = \frac{q'd}{Ad} = \frac{p}{v} = P$$

and from equation 7.36

$$q' = q\left(1 - \frac{1}{\kappa}\right)$$

Combining these two equations we obtain

$$P = \frac{q'}{A} = \frac{q(1 - \frac{1}{\kappa})}{A} = \frac{q}{A}\left(1 - \frac{1}{\kappa}\right)$$

But  $q/A = D$ , therefore

$$P = D\left(1 - \frac{1}{\kappa}\right)$$

and since  $D = \kappa\epsilon_o E$  we get

$$\begin{aligned} P &= \kappa\epsilon_o E\left(1 - \frac{1}{\kappa}\right) = \kappa\epsilon_o E - \epsilon_o E \\ P &= \epsilon_o(\kappa - 1)E \end{aligned} \tag{7.55}$$

Equation 7.55 gives the polarization of the dielectric in terms of the electric field  $E$  and the constants  $\epsilon_o$  and  $\kappa$ . This can also be written in vector notation as

$$\mathbf{P} = \epsilon_o(\kappa - 1)\mathbf{E} \tag{7.56}$$

Notice that for a vacuum  $\kappa = 1$ , and the polarization  $P = 0$  as expected.

### Example 7.6

A potential of 50.0 V is applied to a parallel plate capacitor completely filled with a 2.50 mm piece of mica between the plates. Find the magnitude of: (a) the electric field  $E$  between the plates, (b) the electric displacement  $D$  between the plates, and (c) the polarization  $P$  between the plates.

### Solution

(a) The electric field  $E$  between the plates is found from equation 7.10 as

$$\begin{aligned} E &= \frac{V}{d} \\ E &= \frac{50.0 \text{ V}}{2.50 \times 10^{-3} \text{ m}} \\ E &= 20,000 \text{ V/m} \end{aligned}$$

(b) Using the value of  $\kappa = 5.40$  for mica from table 7.1, the electric displacement  $D$  between the plates is found from equation 7.41 as

$$\begin{aligned} D &= \kappa\epsilon_o E \\ D &= 5.40(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)(20,000 \text{ V/m}) \end{aligned}$$

$$D = 9.56 \times 10^{-7} \text{ C/m}^2$$

Notice that the units for  $D$  comes out to be  $\text{C/m}^2$

(c) the polarization  $P$  between the plates is found from equation 7.55 as

$$\begin{aligned} P &= \epsilon_0(\kappa - 1)E \\ P &= (8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)(5.40 - 1)(20,000 \text{ V/m}) \\ P &= 7.79 \times 10^{-7} \text{ C/m}^2 \end{aligned}$$

**To go to this Interactive Example click on this sentence.**

## 7.7 An Infinite Line of Charge Encased in Glass

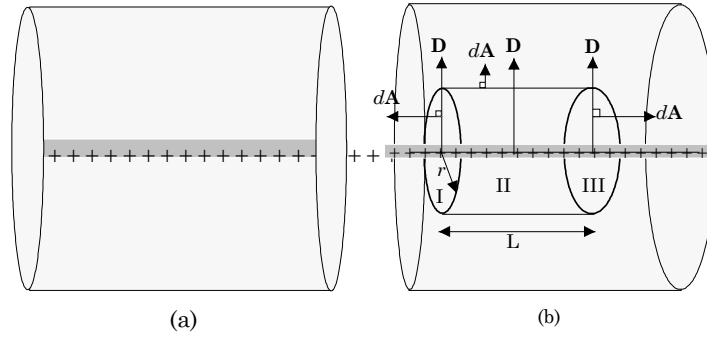
We would like to find the electric displacement  $D$  for an infinite line of charge encased in glass. The problem is similar to the problem we solved in section 4.2 for the electric field intensity  $\mathbf{E}$  at a distance  $r$  from an infinite line of charge, except the line of charge is now encased in glass. We now use Gauss's law in the form of equation 7.42 to solve for  $D$ .

A small portion of the infinite line of charge is shown in figure 7.10(a). We assume that the charge is uniformly distributed and has a linear charge density  $\lambda$ . To determine the electric displacement we must first draw a Gaussian surface. We choose as our Gaussian surface a cylindrical surface of radius  $r$  and length  $L$  around the infinite line of charge as shown in figure 7.10(b). Gauss's law for the total flux emerging from the Gaussian cylinder, equation 7.42, is applied.

$$\oint \mathbf{D} \cdot d\mathbf{A} = q$$

The integral in Gauss's law is over the entire Gaussian surface. As before, we break the entire cylindrical Gaussian surface into three surfaces. Surface I is the end cap on the left-hand side of the cylinder, surface II is the main cylindrical surface, and surface III is the end cap on the right-hand side of the cylinder as shown in figure 7.10(b). The total flux  $\Psi$  through the entire Gaussian surface is the sum of the flux through each individual surface. That is,

$$\Psi = \Psi_{\text{I}} + \Psi_{\text{II}} + \Psi_{\text{III}} \quad (7.57)$$



**Figure 7.10** The electric field of an infinite line of charge.

where

$\Psi_I$  is the flux through surface I  
 $\Psi_{II}$  is the flux through surface II  
 $\Psi_{III}$  is the flux through surface III

Hence, Gauss's law becomes

$$\oint \mathbf{D} \cdot d\mathbf{A} = \int_I \mathbf{D} \cdot d\mathbf{A} + \int_{II} \mathbf{D} \cdot d\mathbf{A} + \int_{III} \mathbf{D} \cdot d\mathbf{A} = q \quad (7.58)$$

Along cylindrical surface I,  $d\mathbf{A}$  is everywhere perpendicular to the surface and points toward the left as shown in figure 7.10(b). The electric displacement vector  $\mathbf{D}$  lies in the plane of the end cylinder cap and is everywhere perpendicular to the surface vector  $d\mathbf{A}$  of surface I and hence  $\theta = 90^\circ$ . Therefore the flux through surface I is

$$\int_I \mathbf{D} \cdot d\mathbf{A} = \int_I D dA \cos \theta = \int_I D dA \cos 90^\circ = 0$$

Surface II is the cylindrical surface itself. As can be seen in figure 7.10(b),  $\mathbf{D}$  is everywhere perpendicular to the cylindrical surface pointing outward, and the area vector  $d\mathbf{A}$  is also perpendicular to the surface and also points outward. Hence  $\mathbf{D}$  and  $d\mathbf{A}$  are parallel to each other and the angle  $\theta$  between  $\mathbf{D}$  and  $d\mathbf{A}$  is zero. Hence, the flux through surface II is

$$\int_{II} \mathbf{D} \cdot d\mathbf{A} = \int_{II} D dA \cos \theta = \int_{II} D dA \cos 0^\circ = \int_{II} D dA$$

Along cylindrical surface III,  $d\mathbf{A}$  is everywhere perpendicular to the surface and points toward the right as shown in figure 7.10(b). The electric displacement vector  $\mathbf{D}$  lies in the plane of the end cylinder cap and is everywhere perpendicular to the surface vector  $d\mathbf{A}$  of surface III and therefore  $\theta = 90^\circ$ . Hence the electric flux through surface III is

$$\int_{III} \mathbf{D} \cdot d\mathbf{A} = \int_{III} D dA \cos \theta = \int_{III} D dA \cos 90^\circ = 0$$



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Combining the flux through each portion of the cylindrical surfaces, equation 7.58 becomes

$$0 + \int_H D dA + 0 = q$$

From the symmetry of the problem, the magnitude of the electric displacement  $D$  is a constant for a fixed distance  $r$  from the line of charge, that is, there is no reason to assume that  $D$  in the  $z$ -direction is any different than  $D$  in the  $y$ -direction. Hence,  $D$  can be taken outside the integral sign to yield

$$\int_H D dA = D \int_H dA = q$$

The integral  $\int dA$  represents the sum of all the elements of area  $dA$ , and that sum is just equal to the total area of the cylindrical surface. The total area  $A$  is just the product of the length times the width of the rectangle formed by unfolding the cylindrical surface, that is,  $A = (L)(2\pi r)$ . Hence, the integral of  $dA$  is

$$\int dA = A = (L)(2\pi r)$$

Thus, Gauss's law becomes

$$D \int_H dA = D(L)(2\pi r) = q$$

Solving for the magnitude of the electric displacement  $D$  we get

$$D = \frac{q}{2\pi r L}$$

But  $q/L = \lambda$  the linear charge density. Therefore, the magnitude of the electric displacement  $D$  for an infinite line of charge is

$$D = \frac{\lambda}{2\pi r} \quad (7.59)$$

Notice that the problem is solved in the same way we solved the problem for the electric field of an infinite line of charge in chapter 4. In fact we could, if we wished, solve all those problems again in terms of the electric displacement  $D$ . However, it is not necessary to resolve all those problems again. We can use the solutions we established for  $\mathbf{E}$  and then find  $\mathbf{D}$  and  $\mathbf{P}$  by equations 7.41 and 7.55 respectively, with the appropriate value of  $\kappa$  for the dielectric.

### **Example 7.7**

An infinite line of charge encased in glass carries a linear charge density of  $2.55 \times 10^{-5}$  C/m. Find the electric displacement  $D$  at a distance of 15.5 cm from the line of charge.

**Solution**

The electric field intensity  $D$  for an infinite line of charge is found by equation 7.59 as

$$D = \frac{\lambda}{2\pi r}$$

$$D = \frac{(2.55 \times 10^{-5} \text{ C/m})}{2(\pi)(0.155 \text{ m})}$$

$$D = 2.62 \times 10^{-5} \text{ C/m}^2$$

Notice that the solution does not depend on any of the characteristics of the glass. Can this be right? Let us solve for the value of the electric field at the same position  $r$  within the dielectric. The electric field within the dielectric ( $\kappa = 4.5$ ) is found from the juxtaposition of equation 7.41 as

$$E_{\text{glass}} = \frac{D}{\kappa \epsilon_0}$$

$$E_{\text{glass}} = \frac{2.62 \times 10^{-5} \text{ C/m}^2}{(4.5)(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N m}^2})}$$

$$E_{\text{glass}} = 6.58 \times 10^5 \text{ N/C} = 6.58 \times 10^5 \text{ V/m}$$

If there were no glass dielectric present the value of the electric field would be ( $\kappa = 1$ )

$$E_{\text{air}} = \frac{D}{\kappa \epsilon_0} = \frac{D}{\epsilon_0}$$

$$E_{\text{air}} = \frac{2.62 \times 10^{-5} \text{ C/m}^2}{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N m}^2})}$$

$$E_{\text{air}} = 2.96 \times 10^6 \text{ N/C} = 2.96 \times 10^6 \text{ V/m}$$

*Thus the value of the electric field  $E$  is different when the field is measured in air versus when it is measured in the glass, yet  $D$  is the same in both cases. Hence the value of  $D$  is essentially an invariant to the material that is used. Hence the equations of electromagnetic theory are more general if written in terms of the displacement vector  $\mathbf{D}$  then in terms of the electric field vector  $\mathbf{E}$ , and in more general treatises they are.*

**To go to this Interactive Example click on this sentence.**

## 7.8 The Differential Form of Gauss's Law When a Dielectric is Present

In chapter 4 we found Gauss's law in integral form to be

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0} \quad (4-14)$$

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and in differential form

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (4-87)$$

We would like to find the equivalent equation when a dielectric is present. We already showed that the integral form of Gauss's law for a dielectric is

$$\Psi_E = \oint \mathbf{D} \cdot d\mathbf{A} = q \quad (7.42)$$

Equation 7.42 is the dielectric equivalent of equation 4-14. How do we find the counterpart to equation 4-87? Recall that in chapter 4 we also found that Gauss's divergence theorem is

$$\oint \mathbf{E} \cdot d\mathbf{A} = \int [\nabla \cdot \mathbf{E}] dV \quad (4-88)$$

Gauss's divergence theorem relates the amount of a vector quantity  $\mathbf{E}$  emerging from a surface to the divergence of that same vector quantity  $\mathbf{E}$  from out of the volume enclosed by that surface. Gauss's divergence theorem applies to any vector, and we now apply it to the displacement vector  $\mathbf{D}$  to obtain

$$\oint \mathbf{D} \cdot d\mathbf{A} = \int [\nabla \cdot \mathbf{D}] dV \quad (7.60)$$

Combining equations 7.42 and 7.60 we get

$$\oint \mathbf{D} \cdot d\mathbf{A} = q = \int [\nabla \cdot \mathbf{D}] dV \quad (7.61)$$

But the free charge  $q$  can be written in terms of the volume charge density  $\rho$  as

$$q = \int \rho dV \quad (7.62)$$

Equating equations 7.61 and 7.62 yields

$$\int [\nabla \cdot \mathbf{D}] dV = \int \rho dV$$

Since both integrations are over the same volume they can be combined as

$$\int [\nabla \cdot \mathbf{D} - \rho] dV = 0 \quad (7.63)$$

For the integral in equation 7.63 to be equal to zero for all values of  $\mathbf{D}$  and  $\rho$  the entire term in brackets must equal zero.

$$\nabla \cdot \mathbf{D} - \rho = 0$$

or

$$\nabla \cdot \mathbf{D} = \rho \quad (7.64)$$

Equation 7.64 is Gauss's law in differential form for a dielectric. It says that the divergence of  $\mathbf{D}$  is a measure of the volume free charge density  $\rho$ .

## 7.9 The Energy Stored in the Electric Field When a Dielectric is Present

Just as Gauss's law is modified for a dielectric, the energy density in the electric field is also modified. The energy density was defined in chapter 6 as the energy per unit volume, that is

$$U_E = \frac{\text{total energy}}{\text{volume}} = \frac{W}{Ad} \quad (6-55)$$

The energy stored in the electric field between the plates of a capacitor was found to be

$$W = \frac{1}{2} CV^2 \quad (6-50)$$

where the capacitance of the parallel plate capacitor is given by

$$C = \frac{\epsilon_0 A}{d} \quad (6-14)$$

When a dielectric is placed between the plates of a capacitor, the capacitance becomes

$$C_d = \kappa C_0 \quad (7.6)$$

Hence the capacitance of a parallel plate capacitor with a dielectric between the plates becomes

$$C_d = \frac{\kappa \epsilon_0 A}{d} \quad (7.65)$$

Combining equations 6-55, 6-50, and 7.65 we obtain

$$U_E = \frac{W}{Ad} = \frac{\frac{1}{2} C_d V^2}{Ad} = \frac{1}{2} \frac{\frac{\kappa \epsilon_0 A}{d} V^2}{Ad} = \frac{1}{2} \frac{\kappa \epsilon_0 A V^2}{Ad^2}$$

$$U_E = \frac{1}{2} \kappa \epsilon_0 \left( \frac{V}{d} \right)^2 \quad (7.66)$$

But  $V/d = E$  the value of the electric field. Replacing this into equation 7.66 gives

$$U_E = \frac{1}{2} \kappa \epsilon_0 E^2 \quad (7.67)$$

## Chapter 7: Dielectrics

Equation 7.67 gives the energy stored in the electric field of a capacitor when a dielectric of dielectric constant  $\kappa$  is placed between the plates. But since  $\kappa\epsilon_0 E = D$ , this can also be written in the form

$$U_E = \frac{1}{2}\kappa\epsilon_0 E^2 = \frac{1}{2}(\kappa\epsilon_0 E)E = \frac{1}{2}DE$$

or in more general form

$$U_E = \frac{1}{2}\mathbf{D} \cdot \mathbf{E} \quad (7.68)$$

Equation 7.68 gives the energy per unit volume stored in an electric field when a dielectric material is present. Although equation 7.68 was derived for a parallel plate capacitor, it is true in general for any electric field in a dielectric.

### Summary of Important Concepts

**Dielectric constant** - The dielectric constant is defined as the ratio of the capacitance with a dielectric between the plates to the capacitance with air or vacuum between the plates.

**Capacitors with a dielectric** - Placing a dielectric, an insulator, between the plates of a capacitor increases the capacitance of that capacitor; decreases the electric field and the potential difference between the plates; decreases the amount of energy that can be stored in the capacitor; and increases the dielectric strength of the capacitor.

**Dielectric strength** - The value of the potential difference per unit plate separation when the dielectric breaks down.

**Gauss's law with dielectrics** says that the electric flux  $\Psi_E$  that passes through a surface surrounding the free electric charge  $q$  is a measure of the amount of free charge  $q$  contained within the Gaussian surface.

### Summary of Important Equations

Capacitance with dielectric	$C_d = \frac{q}{V_d}$	(7.2)
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Definition of dielectric constant	$\kappa = \frac{C_d}{C_0}$	(7.5)
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Capacitance of capacitor with a dielectric	$C_d = \kappa C_0$	(7.6)
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Permittivity of the dielectric medium	$\epsilon = \kappa\epsilon_0$	(7.8)
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## Chapter 7: Dielectrics

The effect of a dielectric between the plates  $\frac{C_d}{C_o} = \frac{V_o}{V_d} = \frac{E_o}{E_d} = \kappa$  (7.13)

Potential in the dielectric  $V_d = \frac{V_o}{\kappa}$  (7.14)

Electric field within the dielectric  $E_d = \frac{E_o}{\kappa}$  (7.15)

Energy stored in dielectric capacitor  $W_d = \frac{W_o}{\kappa}$  (7.18)

Force on a particle in a dielectric medium  $F_d = \frac{F}{\kappa}$  (7.22)

Coulomb's law within a dielectric medium  $F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$  (7.23)

Bound charge on a dielectric  $q' = q(1 - \frac{1}{\kappa})$  (7.36)

Displacement vector **D**  $\mathbf{D} = \epsilon \mathbf{E} = \kappa \epsilon_o \mathbf{E}$  (7.41)

$$\mathbf{D} = \epsilon_o \mathbf{E} + \mathbf{P} \quad (7.54)$$

Polarization vector **P**  $\mathbf{P} = \epsilon_o (\kappa - 1) \mathbf{E}$  (7.56)

Displacement in a dielectric for an infinite line of charge  $D = \frac{\lambda}{2\pi r}$  (7.59)

Gauss's law in integral form for a dielectric  $\oint \mathbf{D} \cdot d\mathbf{A} = q$  (7.42)

Gauss's law in differential form for a dielectric (Maxwell's first equation)  $\nabla \cdot \mathbf{D} = \rho$  (7.64)

Energy per unit volume stored in an electric field when a dielectric material is present  $U_E = \frac{1}{2} \kappa \epsilon_o E^2$  (7.67)

$$U_E = \frac{1}{2} \mathbf{D} \cdot \mathbf{E} \quad (7.68)$$

## Questions for Chapter 7

1. If the applied potential difference across a capacitor exceeds the dielectric strength of the medium between the plates, what happens to the capacitor?
2. Can a coaxial cable have a capacitance associated with it?

## Problems for Chapter 7

1. A  $5.00\text{-}\mu\text{F}$  parallel plate capacitor has air between the plates. When an insulating material is placed between the plates, the capacitance increases to  $13.5\text{ }\mu\text{F}$ . Find the dielectric constant of the insulator.
2. If the dielectric constant of rubber is 2.94, what is its permittivity?
3. Find the capacitance of a parallel plate capacitor having an area of  $75.0\text{ cm}^2$ , a plate separation of  $3.00\text{ mm}$ , and a strip of bakelite placed between the plates.
4. A  $350\text{-}\mu\text{F}$  capacitor is constructed with bakelite between its plates. If the bakelite is removed and replaced by amber, find the new capacitance.
5. A  $0.100\text{-mm}$  piece of wax paper is to be placed between two pieces of aluminum foil to make a parallel plate capacitor of  $5\text{ }\mu\text{F}$ . What must the area of the foil be?
6. A parallel plate capacitor has plates that are  $5.00\text{ cm}$  by  $6.00\text{ cm}$  and are separated by an air gap of  $1.50\text{ mm}$ . Calculate the maximum voltage that can be applied to the capacitor before dielectric breakdown.
7. A  $1.00\text{-mm}$  piece of wax paper is placed between two pieces of aluminum foil that are  $10.0\text{ cm}$  by  $10.0\text{ cm}$ . How much energy can be stored in this capacitor if it is connected to a  $24.0\text{ V}$  battery?
8. If  $1.73 \times 10^{-3}\text{ J}$  of energy can be stored in a capacitor with air between the plates, how much energy can be stored in the capacitor if a slab of mica is placed between the plates?
9. What is the maximum charge that can be placed on the plates of an air capacitor of  $9.00\text{ }\mu\text{F}$  before breakdown if the plate separation is  $1.00\text{ mm}$ ? If a strip of mica is placed between the plates, what is the maximum charge?
10. A parallel plate capacitor has a value of  $3.00\text{ pF}$ . A  $1.00\text{-mm}$  piece of plexiglass is placed between the plates of the capacitor, which is then connected to a  $12.0\text{-V}$  battery. Find (a) the electric field  $E_0$  without the dielectric between the plates and (b) the induced electric field in the dielectric  $E_i$ .
11. A sample of  $\text{NaCl}$  is placed into a solution of ethyl alcohol. If the distance between the  $\text{Na}^+$  and the  $\text{Cl}^-$  ions is  $1.00 \times 10^{-7}\text{ cm}$ , find the force on the ions.
12. A cylindrical capacitor is made by wrapping a sheet of aluminum foil around a hollow cardboard core  $2.50\text{ cm}$  in diameter. A  $1.00\text{ mm}$  thick piece of wax paper is then wrapped around the foil and then another sheet of aluminum foil is wrapped around the wax paper. The length of the core is  $25.0\text{ cm}$ . Find the capacitance of this cylindrical capacitor.
13. Find (a) the capacitance of a parallel plate capacitor having an area of  $40.0\text{ cm}^2$  and separated by air  $2.00\text{ mm}$  thick, (b) the capacitance if  $2.00\text{ mm}$  of mica is placed between the plates, (c) if the capacitor is connected to a  $6.00\text{-V}$  battery, find the charge on the capacitor with air and with mica between the plates, (d) the electric field between the plates, and (e) the energy stored in each capacitor.
14. If a piece of mica is placed between the plates of a capacitor of  $6.00\text{ }\mu\text{F}$  and is then connected to the plates of a  $24.0\text{-V}$  battery, find the charge  $q$  and  $q'$ .

## Chapter 7: Dielectrics

15. A cumulonimbus cloud is 5.00 km long by 5.00 km wide and has its base 1.00 km above the surface of the earth. Consider the cloud and earth to be a parallel plate capacitor with air as the dielectric. (a) Find the capacitance of the cloud-earth combination. (b) Find the potential difference between the cloud and the earth when lightning occurs. (c) Calculate the charge that must be on the base of the cloud when lightning occurs.

16. One model of a red blood cell considers the cell to be a spherical capacitor. The radius of a blood cell is about  $6.00 \times 10^{-4}$  cm and the membrane wall of  $0.100 \times 10^{-4}$  cm is considered to be a dielectric material with a dielectric constant of 5.00. Find (a) the capacitance of the cell. (b) If a potential difference of 100 mV is measured across the blood cell, find the charge on a red blood cell.

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