Chapter 5: Nuclear Physics

"Some 15 years ago the radiation of uranium was discovered by Henri Becquerel and two years later the study of this phenomenon was extended to other substances, first by me, and then by Pierre Curie and myself. This study rapidly led us to the discovery of new elements, the radiation of which, while being analogous with that of uranium, was far more intense. All the elements emitting such radiation I have termed radioactive, and the new property of matter revealed in this emission has thus received the name radioactivity."   Marie Curie, 1911

5.1 Introduction

In 1896, Henri Becquerel (1852-1908) found that an ore containing uranium emits an invisible radiation that can penetrate paper and expose a photographic plate. After Becquerel's discovery, Marie (1867-1934) and Pierre (1859-1906) Curie discovered two new radioactive elements that they called polonium and radium. The Curies performed many experiments on these new elements and found that their radioactivity was unaffected by any physical or chemical process. As seen in chapter 4, chemical effects are caused by the interaction with atomic electrons. The reason for the lack of chemical changes affecting the radioactivity implied that radioactivity has nothing to do with the orbital electrons. Hence, the radioactivity must come from within the nucleus.

Rutherford investigated this invisible radiation from the atomic nucleus by letting it move in a magnetic field that is perpendicular to the paper, as shown in figure 5.1. Some of the particles were bent upward, some downward, while others
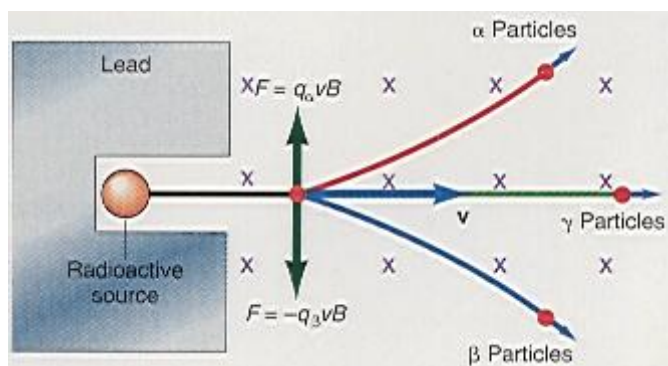

Figure 5.1 Radioactive particles.

went straight through the magnetic field without being bent at all. The particles that were bent upward were called alpha particles, α; those bent downward, beta particles, β; and those that were not deviated, gamma particles, γ.

We saw in general physics that the magnitude of the force that acts on a particle of charge q moving at a velocity **v** in a magnetic field **B** is given by

$$F = qvB \sin \theta$$

(Recall that the direction of the magnetic force is found by the right-hand rule. Place your right hand in the direction of the velocity vector **v.** On rotating your hand toward the magnetic field **B,** your thumb points in the direction of the force **F** acting on the particle.) If the charge of the α particle is positive, then the force acts upward in figure 5.1, and the α particle should be deflected upward. Because the α particle is observed to move upward, its charge must indeed be positive. Its magnitude was found to be twice that of the electronic charge. (Later the α particle was found to be the nucleus of the helium atom.)

Because the β particle is deflected downward in the magnetic field, it must have a negative charge. (The β particles were found to be high energy electrons.) The fact that the γ particle was not deflected in the magnetic field indicated that the γ particle contained no electric charge. The γ particles have since been found to be very energetic photons.

The energies of the α, β, and γ particles are of the order of 0.1 MeV up to 10 MeV, whereas energies of the orbital electrons are of the order of electron volts. Also, the α particles were found to be barely able to penetrate a piece of paper, whereas β particles could penetrate a few millimeters of aluminum, and the γ rays could penetrate several centimeters of lead. Hence, these high energies were further evidence to support the idea that these energetic particles must be coming from the nucleus itself.

5.2 Nuclear Structure

After quantum mechanics successfully explained the properties of the atom, the next questions asked were, What is the nature and structure of the nucleus? How are the protons and neutrons arranged in the nucleus? Why doesn't the nucleus blow itself apart by the repulsive force of the protons? If β particles that come out of a nucleus are electrons, are there electrons in the nucleus? We will discuss these questions shortly.

As seen in chapter 4, the nucleus is composed of protons and neutrons. These protons and neutrons are collectively called *nucleons. The number of protons in the nucleus is given by the atomic number Z, whereas the mass number A is equal to the number of protons plus neutrons in the nucleus. The number of neutrons in a nucleus is given by the neutron number N, which is just the difference between the mass number and the atomic number, that is,*

$$N = A - Z \tag{5.1}$$

A nucleus is represented symbolically in the form

$$^{A}_{Z}X \tag{5.2}$$

with the mass number A displayed as a superscript and the atomic number Z displayed as a subscript and where X is the nucleus of the chemical element that is given by the atomic number Z. As an example, the notation

$$^{12}_{6}C$$

represents the nucleus of the carbon atom that has an atomic number of 6 indicating that it has 6 protons, while the 12 is the mass number indicating that there are 12 nucleons in the nucleus. The number of neutrons, given by equation 5.1, is

$$N = A - Z$$
$$= 12 - 6$$
$$= 6$$

Every chemical element is found to have isotopes. *An **isotope** of a chemical element has the same number of protons as the element but a different number of neutrons than the element.* Hence, an isotope of a chemical element has the same atomic number Z but a different mass number A and a different neutron number N. Since the chemical properties of an element are determined by the number of orbiting electrons, an isotope also has the same number of electrons and hence reacts chemically in the same way as the parent element. Its only observable difference chemically is its different atomic mass, which comes from the excess or deficiency of neutrons in the nucleus.

An example of an isotope is the carbon isotope

$$^{14}_{6}C$$

which has the same 6 protons as the parent element but now has 14 nucleons, indicating that there are now $14 - 6 = 8$ neutrons. The simplest element, hydrogen, has two isotopes, so there are three types of hydrogen:

$^{1}_{1}H$ — Normal hydrogen contains 1 proton and 0 neutrons

$^{2}_{1}H$ — Deuterium contains 1 proton and 1 neutron

$^{3}_{1}H$ — Tritium contains 1 proton and 2 neutrons

Most elements have two or more stable isotopes. Hence, any chemical sample usually contains isotopes. The **atomic mass** of an element is really an average of the masses of the different isotopes. The abundance of isotopes of a particular element is usually quite small. For example, deuterium has an abundance of only 0.015%. Hence, the actual atomic mass is very close to the mass number A. There

are a few exceptions to this, however, one being the chemical element chlorine. As seen from the table of the elements, the atomic mass of chlorine is 35.5, rounded to three significant figures. Contained in that chlorine sample is $^{35}_{17}Cl$ and $^{37}_{17}Cl$ The abundance of $^{35}_{17}Cl$ is 75.5%, whereas the abundance of $^{37}_{17}Cl$ is 24.5%. The atomic mass of chlorine is the average of these two forms of chlorine, weighted by the amount of each present in a sample. Thus, the atomic mass of chlorine is

$$\text{Atomic mass} = 35(0.755) + 37(0.245) = 35.5$$

In general, the atomic mass of any element is

$$\text{Atomic mass} = A_1(\% \text{ Abundance}) + A_2(\% \text{ Abundance}) + A_3(\% \text{ Abundance}) +$$

where A_1, A_2, and A_3, is the mass number of a particular isotope.

In chapters 3 and 4, the masses of the proton and neutron are given as

$$m_p = 1.6726 \times 10^{-27} \text{ kg} = 1.00726 \text{ u} = 938.256 \text{ MeV}$$
$$m_n = 1.6749 \times 10^{-27} \text{ kg} = 1.00865 \text{ u} = 939.550 \text{ MeV}$$

The protons in a nucleus are charged positively, thus Coulomb's law mandates a force of repulsion between these protons and the nucleus should blow itself apart. Because the nucleus does not blow itself apart, we conclude that there must be another force within the nucleus holding these protons together. *This nuclear force is called the strong nuclear force or the strong interaction. The strong force acts not only on protons but also on neutrons and is thus the force that binds the nucleus together.* The strong force has a very short range. That is, it acts within a distance of approximately 10^{-14} m, the order of the size of the nucleus. Outside the nucleus, there is no trace whatsoever of this force. The strong nuclear force is the strongest force known.

If we plot the number of neutrons in a nucleus N against the number of protons in that same nucleus Z for several nuclei, we obtain a graph similar to the one in figure 5.2. For light nuclei the number of neutrons is approximately equal
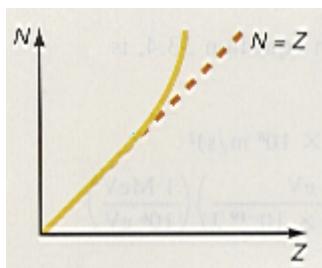

Figure 5.2 Plot of N vs. Z for atomic nuclei.

to the number of protons, as seen by the line labeled $N = Z$. As the atomic number Z increases there are more neutrons in the nucleus than there are protons. Recall

that the electrostatic repulsive force acts only between the protons, while the strong nuclear force of attraction acts between the protons and the neutrons. Hence, the additional neutrons increase the attractive force without increasing the repulsive electric force and, thereby, add to the stability of the nucleus. Whenever the nuclear force of attraction is greater than the electrostatic force of repulsion, the nucleus is stable.   *Whenever the nuclear force is less than the electrostatic force, the nucleus breaks up or decays, and emits radioactive particles*. The chemical elements with atomic numbers Z greater than 83 have unstable nuclei and decay.

The internal structure of the nucleus is determined in much the same way as in Rutherford scattering. The nucleus is bombarded by high energy electrons (several hundred mega electron volts), that penetrate the nucleus and react electrically with the protons within the nucleus. The results of such scattering experiments seem to indicate that the protons and neutrons are distributed rather evenly throughout the nucleus, and the nucleus itself is generally spherical or ellipsoidal in shape.

It is usually assumed that the whole is always equal to the sum of its parts. This is not so in the nucleus. *The results of experiments on the masses of different nuclei shows that the mass of the nucleus is always less than the total mass of all the protons and neutrons making up the nucleus.* In the nucleus, the missing mass is called the **mass defect,** Δm, given by

$$\Delta m = Zm_{\text{p}} + (A - Z)m_{\text{n}} - m_{\text{nucleus}} \qquad (5.3)$$

Because Z is the total number of protons, and $m_{\mathbf{p}}$ is the mass of a proton, $Zm_{\mathbf{p}}$ is the total mass of all the protons. As shown in equation 5.1, $A - Z$ is the total number of neutrons, and since $m_{\mathbf{n}}$ is the mass of a single neutron, $(A - Z)m_{\mathbf{n}}$ is the total mass of all the neutrons. The term m_{nucleus} is the experimentally measured mass of the entire nucleus. Hence, equation 5.3 represents the difference in mass between the sum of the masses of its constituents and the mass of the nucleus itself.

The missing mass is converted to energy in the formation of the nucleus. This energy is found from Einstein's mass-energy relation,

$$E = (\Delta m)c^2 \qquad (5.4)$$

and is called the **binding energy** (BE) of the nucleus. From equations 5.3 and 5.4, the binding energy of a nucleus is

$$\text{BE} = (\Delta m)c^2 = Zm_p c^2 + (A - Z)m_n c^2 - m_{nucleus}c^2 \qquad (5.5)$$

Example 5.1

The mass defect and the binding energy of the deuteron. Find the mass defect and the binding energy of the deuteron nucleus. The experimental mass of the deuteron is 3.3435×10^{-27} kg.

Solution

The mass defect for the deuteron is found from equation 5.3, with

$$\Delta m = m_{\mathbf{p}} + m_{\mathbf{n}} - m_{\mathrm{D}}$$
$$= 1.6726 \times 10^{-27} \text{ kg} + 1.6749 \times 10^{-27} \text{ kg} - 3.3435 \times 10^{-27} \text{ kg}$$
$$= 4.00 \times 10^{-30} \text{ kg}$$

The binding energy of the deuteron, found from equation 5.4, is

$$\mathrm{BE} = (\Delta m)c^2$$
$$= (4.00 \times 10^{-30} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2$$
$$= (3.5950 \times 10^{-13} \text{ J})\left(\frac{1 \text{ eV}}{1.60218 \times 10^{-19} \text{ J}}\right)\left(\frac{1 \text{ MeV}}{10^6 \text{eV}}\right)$$
$$= 2.24 \text{ MeV}$$

Therefore, the bound constituents have less energy than when they are free. That is, the binding energy comes from the mass that is lost in the process of formation. Conversely, an amount of energy equal to the binding energy is the amount of energy that must be supplied to a nucleus if the nucleus is to be broken up into protons and neutrons. Thus, the binding energy of a nucleus is similar to the ionization energy of an electron in the atom.

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5.3 Radioactive Decay Law

The spontaneous emission of radiation from the nucleus of an atom is called **radioactivity.** Radioactivity is the result of the decay or disintegration of unstable nuclei. Radioactivity occurs naturally from all the chemical elements with atomic numbers greater than 83, and can occur naturally from some of the isotopes of the chemical elements below atomic number 83. Some can also occur artificially from nearly all of the chemical elements.

The rate of radioactive emission is measured by the *radioactive decay law.* The number of nuclei dN that disintegrate during a particular time interval dt is directly proportional to the number of nuclei N present. That is

$$\frac{dN}{dt} \propto -N$$

To make an equality of this, we introduce the constant of proportionality λ, called the *decay constant* or *disintegration constant,* and we obtain

$$\frac{dN}{dt} = -\lambda N \tag{5.6}$$

The minus sign in equation 5.6 is necessary because the final number of nuclei N_f is always less than the initial number of nuclei N_i; hence $dN = N_f - N_i$ is always a negative quantity because there is always less radioactive nuclei with time. The decay constant λ is a function of the particular isotope of the chemical element. A large value of λ indicates a large decay rate, whereas a small value of λ indicates a small decay rate. The quantity $-dN/dt$ in equation 5.6 is the rate at which the nuclei decay with time and it is also called the **activity** and designated by the symbol A. Hence,

$$A = -\frac{dN}{dt} = \lambda N \tag{5.7}$$

We must be careful in what follows not to confuse the symbol A for activity with the same symbol A for mass number. It should always be clear in the particular context used.

The SI unit of activity is the becquerel where 1 becquerel (Bq) is equal to one decay per second. That is,

$$1 \text{ Bq} = 1 \text{ decay/s}$$

An older unit of activity, the curie, abbreviated Ci, is equivalent to

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

Smaller units of activity are the millicurie (10^{-3} curie = mCi) and the microcurie (10^{-6} curie = μCi).

The total number of nuclei present at any instant of time is found by integrating equation 5.6, from t = 0 to the time t as

$$\frac{dN}{dt} = -\lambda N$$
$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{0}^{t} \lambda dt$$

Notice that when $t = 0$, the lower limit on the right-hand side of the equation, the initial number of nuclei present is N_0, the lower limit on the left-hand side of the equation, and when $t = t$, the upper limit on the right-hand side of the equation, the number of nuclei present is N, the upper limit on the left-hand side of the equation. Upon integrating we get

$$\ln N \mid_{N_0}^{N} = -\lambda t \mid_{0}^{t}$$
$$\ln N - \ln N_0 = -\lambda t$$
$$\ln \frac{N}{N_0} = -\lambda t$$

Since $e^{\ln x} = x$, we now take e to both sides of the equation to get

$$\frac{N}{N_0} = e^{-\lambda t}$$

and solving for N we obtain

$$N = N_0 e^{-\lambda t} \qquad (5.8)$$

Equation 5.8 is the radioactive decay law and it gives the total number of nuclei N present at any instant of time t. N₀ is the number of nuclei present at the time t = 0, which is the time that the observations of the nuclei is started. A plot of the radioactive decay law, equation 5.8, is shown in figure 5.3. The curve represents the number of radioactive nuclei still present at any time *t*. A very interesting quantity is found by looking for the time it takes for half of the original nuclei to decay,
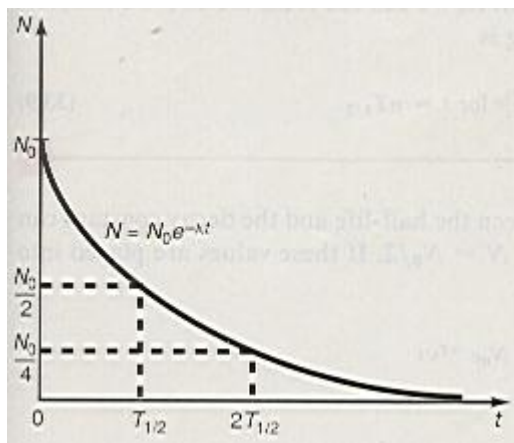

Figure 5.3 The radioactive decay law.

so that only half of the original nuclei are still present. Half the original nuclei is $N_0/2$ and is shown in the figure. A horizontal line for the value of $N_0/2$ is drawn until it intersects the curve $N = N_0 e^{-\lambda t}$. A vertical line is dropped from this point to the *t*-axis. The value of the time read on the *t*-axis is the time it takes for half the original nuclei to decay. Hence, this time read from the *t*-axis is called the half-life of the radioactive nuclei and is denoted by $T_{1/2}$. *The **half-life** of a radioactive substance is thus the time it takes for half the original radioactive nuclei to decay.*

Example 5.2

The number of radioactive nuclei for several half-lives. One mole of a radioactive substance starts to decay. How many radioactive nuclei will be left after $t =$ (a) $T_{1/2}$, (b) $2T_{1/2}$, (c) $3 T_{1/2}$, (d) $4 T_{1/2}$, and (e) $nT_{1/2}$ half-lives?

Solution

Since one mole of any substance contains 6.022×10^{23} atoms/mole (Avogadro's number), $N_0 = 6.022 \times 10^{23}$ nuclei.

a. At the end of one half-life, there will be

$$N_0 = \frac{6.022 \times 10^{23} \text{ nuclei}}{2} = 3.011 \times 10^{23} \text{ nuclei}$$

b. At the end of another half-life, $t = 2T_{1/2}$, half of those present at the time $t = T_{1/2}$ will be lost, or

$$N = \frac{1}{2}\frac{N_0}{2} = \frac{N_0}{4} = \frac{6.022 \times 10^{23} \text{ nuclei}}{4}$$
$$= 1.506 \times 10^{23} \text{ nuclei}$$

c. At $t = 3T_{1/2}$, the number of radioactive nuclei remaining is

$$N = \frac{1}{2}\frac{N_0}{4} = \frac{N_0}{8} = 0.753 \times 10^{23} \text{ nuclei}$$

d. At $t = 4T_{1/2}$, the number of radioactive nuclei remaining is

$$N = \frac{1}{2}\frac{N_0}{8} = \frac{N_0}{16} = 0.376 \times 10^{23} \text{ nuclei}$$

e. At a period of time equal to n half-lives, we can see from the above examples that the number of nuclei remaining is

$$N = \frac{N_0}{2^n} \qquad \text{for } t = nT_{1/2} \tag{5.9}$$

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An important relationship between the half-life and the decay constant can be found by noting that when $t = T_{1/2}$, $N = N_0/2$. If these values are placed into the decay law in equation 5.8, we get

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

or

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking the natural logarithm of both sides of this equation, we get

$$\ln \frac{1}{2} = \ln e^{-\lambda T_{1/2}} \tag{5.10}$$

But the natural logarithm ln is the inverse of the exponential function e, and when applied successively, as in the right-hand side of equation 5.10, they cancel each other leaving only the function. Hence, equation 5.10 becomes

$$\ln \frac{1}{2} = -\lambda T_{1/2}$$

Taking the natural logarithm ln of ½ on the electronic calculator gives −0.693. Thus,

$$-0.693 = -\lambda T_{1/2}$$

Solving for the decay constant, we get

$$\lambda = \frac{0.693}{T_{1/2}} \tag{5.11}$$

Thus, if we know the half-life $T_{1/2}$ of a radioactive nuclide, we can find its decay constant λ from equation 5.11. Conversely, if we know λ, then we can find the half-life from equation 5.11.

Example 5.3

Finding the decay constant and the activity for $^{90}_{38}Sr$ The half-life of strontium-90, $^{90}_{38}Sr$, is 28.8 yr. Find (a) its decay constant and (b) its activity for 1 g of the material.

Solution

a. The decay constant, found from equation 5.11, is

$$\lambda = \frac{0.693}{T_{1/2}}$$
$$= \left(\frac{0.693}{28.8 \text{ yr}}\right)\left(\frac{1 \text{ yr}}{365 \text{ days}}\right)\left(\frac{1 \text{ day}}{24 \text{ hr}}\right)\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right)$$
$$= 7.63 \times 10^{-10} \text{ /s}$$

b. Before the activity can be determined, the number of nuclei present must be known. The atomic mass of $^{90}_{38}Sr$ is 89.907746. Thus, 1 mole of it has a mass of approximately 89.91 g. The mass of 1 mole contains Avogadro's number or 6.022×10^{23} molecules. We find the number of molecules in 1 g of the material from the ratio

$$\frac{N_0}{N_A} = \frac{1 \text{ g}}{89.91 \text{ g}}$$

or the number of nuclei in 1 g of strontium-90 is

$$N_0 = \frac{1 \text{ g}}{89.91 \text{ g}} (N_A)$$

$$= \frac{1}{89.91} (6.022 \times 10^{23}) = 6.70 \times 10^{21} \text{ nuclei}$$

We can now find the activity from equation 5.7 as

$$A_0 = \lambda N_0$$
$$= (7.63 \times 10^{-10} \text{ 1/s})(6.70 \times 10^{21} \text{ nuclei})$$
$$= 5.11 \times 10^{12} \text{ nuclear disintegrations/s}$$

To go to this Interactive Example click on this sentence.

Note that the activity (that is, the number of disintegrations per second) is not a constant because it depends on N, which is decreasing with time by equation 5.8. In fact, if equation 5.8 is substituted into equation 5.7 for the activity, we get

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

Letting

$$\lambda N_0 = A_0$$

the rate at which the nuclei are decaying at the time $t = 0$, we obtain for the activity

$$A = A_0 e^{-\lambda t} \tag{5.12}$$

Recalling that the activity is the number of disintegrations per second, we see that the rate of decay is not a constant but decreases exponentially. A plot of the activity as a function of time is shown in figure 5.4(a). Notice the similarity of this diagram with figure 5.3. For the time t, equal to a half-life, the activity is

$$A = A_0 e^{-\lambda T_{1/2}}$$

Substituting λ from equation 5.11, gives

$$A = A_0 e^{-[0.693/T_{1/2}]T_{1/2}} = A_0 e^{-0.693}$$

Using the electronic calculator, we obtain $e^{-0.693} = 0.500 = \frac{1}{2}$. Hence,

$$A = \frac{A_0}{2} \qquad \text{for } t = T_{1/2} \tag{5.13}$$

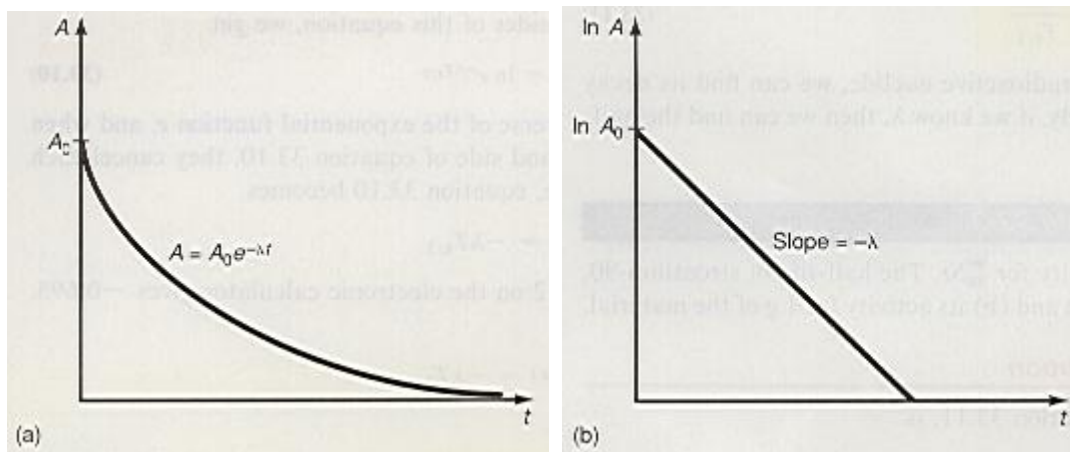That is, the rate of decay is cut in half for a time period of one half-life.

Figure 5.4 Radioactive activity.

Example 5.4

The number of radioactive nuclei and their rate of decay. Find the number of radioactive nuclei and their rate of decay for $t = T_{1/2}$ in the 1.00-g sample in example 5.3.

Solution

The number of nuclei left at the end of one half-life, found from equation 5.9, is

$$N = \frac{N_0}{2}$$
$$= \frac{6.70 \times 10^{21}}{2} \text{ nuclei}$$
$$= 3.35 \times 10^{21} \text{ nuclei}$$

While the rate of decay at the end of one half-life is found from equation 5.13 as

$$A = \frac{A_0}{2}$$
$$= \frac{5.11 \times 10^{12}}{2} \text{ decays/s}$$
$$= 2.55 \times 10^{12} \text{ decays/s}$$

Thus, at the end of 28 years, the number of strontium-90 radioactive nuclei have been cut in half and the rate at which they decay is also cut in half. That is, *there are less radioactive nuclei present at the end of the half-life, but the rate at which they decay also decreases.*

To go to this Interactive Example click on this sentence.

Example 5.5

The decay constant and activity for $^{91}_{38}\text{Sr}$ The half-life of $^{91}_{38}\text{Sr}$ is 9.70 hr. Find (a) its decay constant and (b) its activity for 1.00 g of the material.

Solution

a. This problem is very similar to example 5.3 except that this isotope of strontium has a very short half-life. The decay constant, found from equation 5.11, is

$$\lambda = \frac{0.693}{T_{1/2}}$$
$$= \left(\frac{0.693}{9.70 \text{ hr}}\right)\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right)$$
$$= 1.99 \times 10^{-5} \text{ /s}$$

b. The number of nuclei in a 1-g sample is found as before but the atomic mass of $^{91}_{38}\text{Sr}$ is 90.90. Hence,

$$N_0 = \frac{1 \text{ g}}{90.90 \text{ g}}(N_A)$$
$$= \frac{1}{90.90}(6.022 \times 10^{23} \text{ nuclei})$$
$$= 6.63 \times 10^{21} \text{ nuclei}$$

The activity, again found from equation 5.7, is

$$A_0 = \lambda N_0$$
$$= (1.99 \times 10^{-5} \text{ /s})(6.63 \times 10^{21} \text{ nuclei})$$
$$= 1.32 \times 10^{17} \text{ disintegrations/s}$$

Comparing this example to example 5.3, we see *that for a smaller half-life, we get a larger decay constant λ, and hence a greater activity, or decays per second.*

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The decay constant λ can be found experimentally using the following technique. First, let us return to equation 5.12 and take the natural logarithm of both sides of the equation, that is,
$$\ln A = \ln(A_0 e^{-\lambda t})$$

Now from the rules of manipulating logarithms, the logarithms of a product is equal to the sum of the logarithms of each term. Therefore,

$$\ln A = \ln A_0 + \ln e^{-\lambda t}$$

But as mentioned before, the natural log and the exponential are inverses of each other, and hence

$$\ln e^{-\lambda t} = -\lambda t$$

Thus,

$$\ln A = \ln A_0 - \lambda t$$

Rearranging, this becomes

$$\ln A = -\lambda t + \ln A_0 \qquad\qquad (5.14)$$

If we now go into the laboratory and count the number of disintegrations per unit time A, at different times t, we can plot the $\ln A$ on the y-axis versus t on the x-axis, and obtain the straight line shown in figure 5.4(b). The slope of the line is $-\lambda$, and thus, λ can be determined experimentally. Once we know λ, we determine the half-life from equation 5.11.

We should also note that sometimes it is convenient to use the mean life or average life T_{avg} of a sample. The mean or average life is defined as the average lifetime of all the particles in a given sample of the material. It turns out to be just the reciprocal of the decay constant, that is,

$$T_{avg} = \frac{1}{\lambda} \qquad\qquad (5.15)$$

5.4 Forms of Radioactivity

Up to now, only the number of decaying nuclei has been discussed, without specifying the details of the disintegrations. Nuclei can decay by:

1. Alpha decay, α
2. Beta decay, β^-
3. Beta decay, β^+, positron emission
4. Electron capture
5. Gamma decay, γ

Now let us discuss each of these in more detail.

Alpha Decay
When a nucleus has too many protons compared to the number of neutrons, the electrostatic force of repulsion starts to dominate the nuclear force of attraction. When this occurs, the nucleus is unstable and emits an α particle in radioactive decay. The nucleus thus loses two protons and two neutrons. Hence, its atomic

number Z, which represents the number of protons in the nucleus, decreases by 2, while its mass number A, which is equal to the number of protons and neutrons in the nucleus, decreases by 4. Before the decay, the nucleus is called the *"parent" nucleus;* after the decay the nucleus is referred to as the *"daughter" nucleus.* Hence, *we represent an* **alpha decay** *symbolically as*

$$\prescript{A}{Z}{X} \rightarrow \prescript{A-4}{Z-2}{X} + \prescript{4}{2}{He} \tag{5.16}$$

where $\prescript{A}{Z}{X}$ is the parent nucleus, which decays into the daughter nucleus $\prescript{A-4}{Z-2}{X}$, and $\prescript{4}{2}{He}$ is the α particle, which is the helium nucleus. *Notice that the atomic number Z has decreased by two units. This means that in alpha decay, one chemical element of atomic number Z has been transmuted into a new chemical element of atomic number Z– 2.* The dream of the ancient alchemists was to transmute the chemical elements, in particular to turn the baser metals into gold. This result was never attained because they were working with chemical reactions, which as has been seen, depends on the electronic structure of the atom and not its nucleus.

An example of a naturally occurring alpha decay can be found in uranium-238, which decays by α particle emission with a half-life of 4.51×10^9 yr. We find its daughter nucleus by using equation 5.16. Hence,

$$\prescript{238}{92}{U} \rightarrow \prescript{234}{90}{X} + \prescript{4}{2}{He} \tag{5.17}$$

Notice that the atomic number Z has dropped from 92 to 90. Consulting the table of the elements, we see that the chemical element with $Z = 90$ is thorium. Hence, uranium has been transmuted to thorium by the emission of an α particle. Equation 5.17 is now written as

$$\prescript{238}{92}{U} \rightarrow \prescript{234}{90}{Th} + \prescript{4}{2}{He} \tag{5.18}$$

Also note from the periodic table that the mass number A for thorium is 232, whereas in equation 5.18, the mass number is 234. This means that an isotope of thorium has been formed. (As a matter of fact $\prescript{234}{90}{Th}$ is an unstable isotope and it also decays, only this time by beta emission. We will say more about this later.)

Beta Decay, β^-

In beta decay, an electron is observed to leave the nucleus. However, as seen in chapter 3, an electron cannot be contained in a nucleus because of the Heisenberg uncertainty principle. Hence, the electron must be created within the nucleus at the moment of its emission. In fact, it has been found that a neutron within the nucleus decays into a proton and an electron, plus another particle called an *antineutrino.* The antineutrino is designated by the Greek letter nu, ν, with a bar over the ν, that is $\bar{\nu}$ The antineutrino is the antiparticle of the neutrino ν. The neutron decay is written as

$$\prescript{1}{0}{n} \rightarrow \prescript{1}{1}{p} + \prescript{0}{-1}{e} + \bar{\nu} \tag{5.19}$$

The notation $_{-1}^{0}$ is used to designate the electron or β^- particle. It has a mass number A of 0 because it has no nucleons, and an atomic number of -1 to signify that it is a negative particle. The proton is written as $_{1}^{1}p$ because it has a mass number and atomic number of one. Hence, in beta decay the nucleus loses a neutron but gains a proton, while the β^- particle, the electron, and the antineutrino are emitted from the nucleus. Thus, the atomic number Z increases by 1 in the decay because the nucleus gained a proton, but the mass number A stays the same because even though 1 neutron is lost, we have gained 1 proton.

It is perhaps appropriate to mention an interesting historical point here. The original assumption about neutron decay shown in equation 5.19 did not contain the antineutrino particle. The original decay seemed to violate the principle of conservation of energy. However, Wolfgang Pauli proposed the existence of a particle to account for the missing energy. Since the particle had to be neutral because of the law of conservation of electrical charge, the new particle was called a "neutrino" by the Italian-American physicist Enrico Fermi (1902-1954), for the "little" neutral particle. The antineutrino is the antiparticle of the neutrino, it is a particle of the same mass (zero rest mass) but has a spin component opposite to that of the neutrino. The neutrino was found experimentally in 1956. It is such an elusive particle that some move right through the earth without ever hitting anything.

A **beta decay,** β^- can be written symbolically as

$$_{Z}^{A}X \rightarrow\ _{Z+1}^{A}X +\ _{-1}^{0}e + \bar{\nu} \tag{5.20}$$

Note that in beta decay, Z increases to $Z + 1$. *Hence, a chemical element of atomic number Z is transmuted into another chemical element of atomic number $Z + 1$.*

As an example, the isotope $_{90}^{234}Th$ is unstable and decays by beta emission with a half-life of 24 days. Its decay can be represented with the use of equation 5.20 as

$$_{90}^{234}Th \rightarrow\ _{91}^{234}X +\ _{-1}^{0}e + \bar{\nu}$$
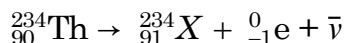
Looking up the periodic table of the elements, we see that the chemical element corresponding to the atomic number 91 is protactinium (Pa). Hence, the element thorium has been transmuted to the element protactinium. Also note from the periodic table that the mass number A for protactinium should be 231. Since we have a mass number of 234, this is an isotope of protactinium. (One that is also unstable and decays again.) The beta decay of thorium is now written as
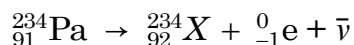
$$_{90}^{234}Th \rightarrow\ _{91}^{234}Pa +\ _{-1}^{0}e + \bar{\nu}$$
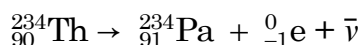
Example 5.6

Beta decay, β^-. The element $^{234}_{91}\text{Pa}$ is unstable and decays by beta emission with a half-life of 6.66 hr. Find the nuclear reaction and the daughter nuclei.

Solution

Because $^{234}_{91}\text{Pa}$ decays by beta emission, it follows the form of equation 5.20. Hence,

$$^{234}_{91}\text{Pa} \rightarrow {}^{234}_{92}X + {}^{0}_{-1}\text{e} + \bar{\nu}$$

But from the table of elements, $Z = 92$ is the atomic number of uranium. Hence, the daughter nuclei is $^{234}_{92}\text{U}$, and the entire reaction is written as

$$^{234}_{91}\text{Pa} \rightarrow {}^{234}_{92}\text{U} + {}^{0}_{-1}\text{e} + \bar{\nu}$$
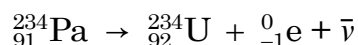
### *Beta Decay,* β^+ **Positron Emission**
In this type of decay, a positron is emitted from the nucleus. A *positron* is the antiparticle of the electron. It has all the characteristics of the electron except it carries a positive charge. Because there are no positrons in the nucleus, a positron must be created immediately before emission. Positron emission is the result of the decay of a proton into a neutron, a positron, and a neutrino ν, and is written symbolically as

$$^{1}_{1}\text{p} \rightarrow {}^{1}_{0}\text{n} + {}^{0}_{+1}\text{e} + \nu \tag{5.21}$$

The positron $^{0}_{+1}\text{e}$ is emitted with the neutrino ν. The neutron stays behind in the nucleus. *Hence, in a* **beta decay,** β^+ *the atomic number Z decreases by one because of the loss of the proton. The mass number A stays the same because even though a proton is lost, a neutron is created to keep the same number of nucleons. Hence, a β^+ decay can be written symbolically a*s

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}X + {}^{0}_{+1}\text{e} + \nu \tag{5.22}$$
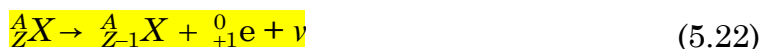
As an example, the isotope of aluminum $^{26}_{13}\text{Al}$ is unstable and decays by β^+ emission with a half-life of 7.40×10^5 yr. The reaction is written with the help of equation 5.22 as

$$^{26}_{13}\text{Al} \rightarrow {}^{26}_{12}X + {}^{0}_{+1}\text{e} + \nu$$
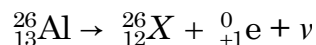
Looking at the periodic table of the elements, we find that the atomic number 12 corresponds to the chemical element magnesium Mg. Hence,

$$^{26}_{13}\text{Al} \rightarrow {}^{26}_{12}\text{Mg} + {}^{0}_{+1}\text{e} + \nu$$

Because the mass number A of magnesium is 24, we see that this transmutation created an isotope of magnesium.

It is important to note that the decay of the proton, equation 5.21, can only occur within the nucleus. A free proton cannot decay into a neutron because the mass of the proton is less than the mass of the neutron.

Electron Capture

Occasionally an orbital electron gets too close to the nucleus and gets absorbed by the nucleus. Since the electron cannot remain as an electron within the nucleus, it combines with a proton and in the process creates a neutron and a neutrino. We represent this as

$$_{-1}^{0}e + _{1}^{1}p \rightarrow _{0}^{1}n + v \qquad (5.23)$$

The net result of this process decreases the number of protons in the nucleus by one hence changing Z to $Z - 1$, while keeping the number of nucleons A constant. Hence, this decay can be written as

$$_{-1}^{0}e + _{Z}^{A}X \rightarrow _{Z-1}^{A}X + v \qquad (5.24)$$

When the electron that is close to the nucleus is captured by the nucleus, it leaves a vacancy in the electron orbit. An electron from a higher energy orbit falls into this vacancy. The difference in the energy of the electron in the higher orbit from the energy in the lower orbit is emitted as a photon in the X-ray portion of the spectrum.

As an example of electron capture, we consider the isotope of mercury $_{80}^{197}Hg$ that decays by electron capture with a half-life of 65 hr. This decay can be represented, with the help of equation 5.24, as

$$_{-1}^{0}e + _{80}^{197}Hg \rightarrow _{79}^{197}X + v$$
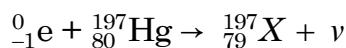
Consulting the table of elements, we find that the atomic number $Z = 79$ represents the chemical element gold, Au. Hence,

$$_{-1}^{0}e + _{80}^{197}Hg \rightarrow _{79}^{197}Au + v$$
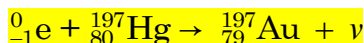
Thus, the dreams of the ancient alchemists have been fulfilled. An isotope of mercury has been transmuted into the element gold. Also note that mass number A of gold is 197. Hence, the transmutation has given the stable element gold.

Gamma Decay

A nucleus undergoing a decay is sometimes left in an excited state. Just as an electron in an excited state of an atom emits a photon and drops down to the ground state, a proton or neutron can be in an excited state in the nucleus. When the

nucleon drops back to its ground state, it also emits a photon. Because the energy given off is so large, the frequency of the photon is in the gamma ray portion of the electromagnetic spectrum. Hence, the excited nucleus returns to its ground state and a gamma ray is emitted. Thus *gamma decay is represented symbolically as*

$$\,_{Z}^{A}X^{*} \rightarrow \,_{Z}^{A}X + \gamma \tag{5.25}$$

Where the * on the nucleus indicates an excited state. *In this type of decay, neither the atomic number Z nor the mass number A changes. Hence, gamma decay does not transmute any of the chemical elements.*

5.5 Radioactive Series

As indicated earlier, elements with atomic numbers Z greater than 83 are unstable and decay naturally. Most of these unstable elements have very short lifetimes and decay rather quickly. Hence, they are not easily found in nature. The exceptions to this are the elements thorium-232, uranium-238, and the uranium isotope 235. The element $\,_{90}^{232}\text{Th}$ has a half-life of 1.39×10^{10} yr, $\,_{92}^{238}\text{U}$ has a half-life of 4.50×10^{9} yr, and $\,_{92}^{235}\text{U}$ has a half-life of 7.10×10^{8} yr. Moreover, these elements decay into a series of daughters, granddaughters, great granddaughters, and so on.

As an example, the series decay $\,_{90}^{232}\text{Th}$ is shown in figure 5.5, which is a plot of the neutron number N versus the atomic number Z. Because $\,_{90}^{232}\text{Th}$ has a Z value of 90 and an N value of 232 – 90 = 142, $\,_{90}^{232}\text{Th}$ is plotted with the coordinates $N = 142$ and $Z = 90$. First $\,_{90}^{232}\text{Th}$ decays by alpha emission with a half-life of 1.39×10^{10} yr. As seen in section 5.4, equation 5.16, an alpha decay changes the atomic number Z to $Z - 2$, and decreases the mass number A by 4 to $A - 4$. Thus, $\,_{90}^{232}\text{Th}$ decays as

$$\,_{90}^{232}\text{Th} \rightarrow \,_{88}^{228}X + \,_{2}^{4}\text{He}$$
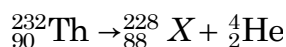
But atomic number 88 corresponds to the chemical element radium (Ra). Hence,

$$\,_{90}^{232}\text{Th} \rightarrow \,_{88}^{228}\text{Ra} + \,_{2}^{4}\text{He}$$
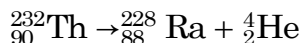
The neutron number N for $\,_{88}^{228}\text{Ra}$ is 228 – 88 = 140. Thus, $\,_{88}^{228}\text{Ra}$ is found in the diagram with coordinates, $N = 140$ and $Z = 88$. The original neutron number is given by

$$N_0 = A - Z$$

But in alpha emission, A goes to $A - 4$ and Z goes to $Z - 2$, equation 5.16. Hence, the new neutron number is given by
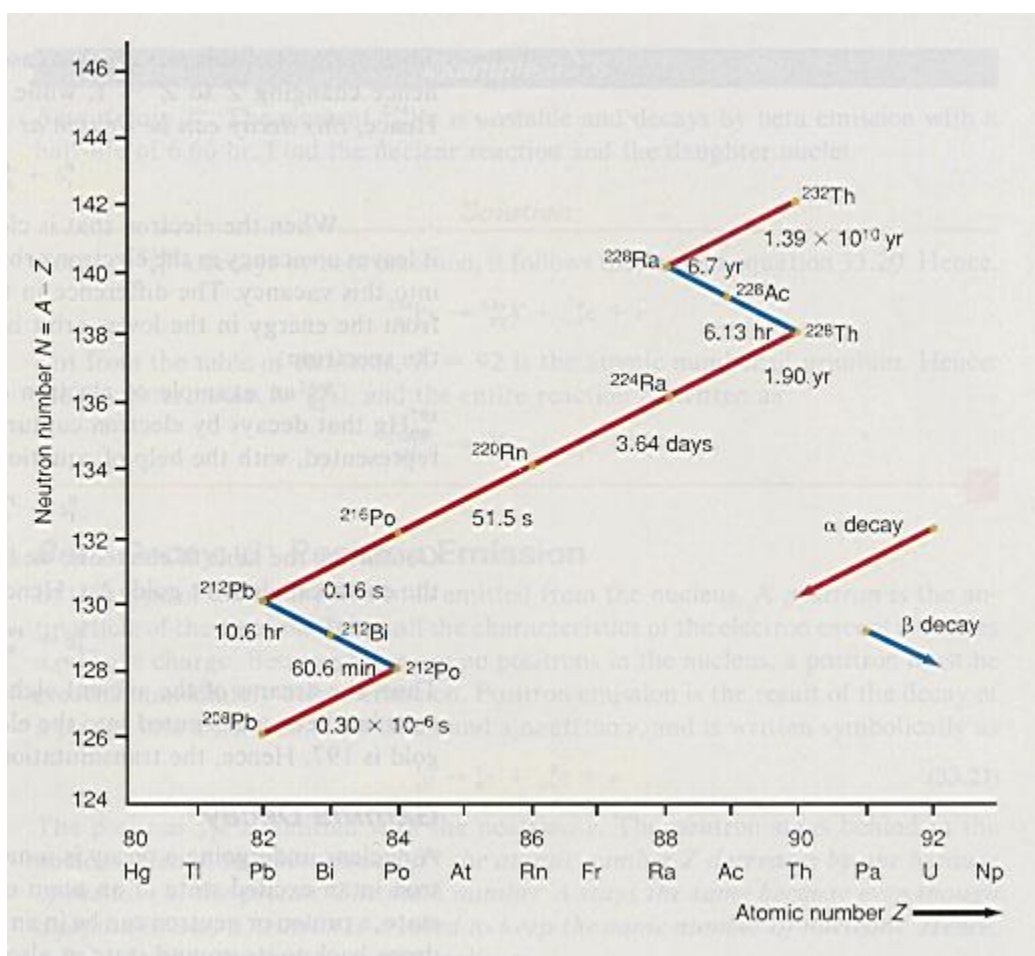
$$N_1 = (A - 4) - (Z - 2)$$

Figure 5.5 Thorium $^{232}_{90}$Th decay series.

$$= A - Z - 2$$
$$N_1 = N_0 - 2 \qquad \text{(alpha decay)} \qquad (5.26)$$

Thus, for all alpha emissions, the neutron number decreases by 2. Hence, in the diagram, for every alpha emission the element has both N and Z decreased by 2.

Radium-228 is also unstable and decays by beta emission with a half-life of 6.7 yr. As shown in equation 5.20, the value of the atomic number Z increases to $Z + 1$, while the mass number A remains the same. The neutron number for beta emission becomes

$$N_1 = A - (Z + 1) = A - Z - 1$$
$$N_1 = N_0 - 1 \qquad \text{(beta}^- \text{ decay)} \qquad (5.27)$$

Thus, $^{228}_{88}$Ra becomes actinium, $^{228}_{89}$Ac, with coordinates $N = 139$ and $Z = 89$.

Therefore, in the series diagram, alpha emission appears as a line sloping down toward the left, with both N and Z decreasing by 2 units. Beta emission, on the other hand, appears as a line sloping downward to the right with N decreasing by 1

and Z increasing by 1. The entire decay of the family is shown in figure 5.5: thorium-232 decays by α emission to radium-228, which then decays by β⁻ emission to actinium-228, which then decays by β⁻ to thorium-228, which then decays by α emission to radium-224, which then decays by α emission to radon-220, which then decays by α emission to polonium-216, which then decays by α emission to lead-212, which then decays by β⁻ to bismuth-212, which then decays by β⁻ to polonium-212, which finally decays by α emission to the stable lead-208. The half-life for each decay is shown in the diagram.

The radioactive chain is called a *series*. The decay series for uranium- 238 is shown in figure 5.6. It starts with $^{238}_{92}$U and ends in the stable isotope of lead-206. Figure 5.7 shows the decay series for uranium-235. As we can see, the series ends with the stable chemical element lead-207. Figure 5.8 shows the neptunium series that ends in the stable chemical element bismuth-209. Neptunium is called a *transuranic element* because it lies beyond uranium in the periodic table. Uranium with an atomic number $Z = 92$ is the highest chemical element found in nature. Elements with Z greater than 92 have been made by man. Many different isotopes of these new elements can also be created.

As an example of the creation of a transuranic element, bombarding $^{238}_{92}$U with neutrons creates neptunium by the reaction

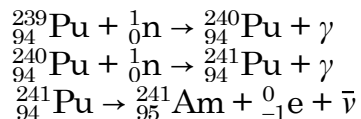$$^{238}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{239}_{93}\text{Np} + ^{0}_{-1}\text{e} + \bar{\nu} \tag{5.28}$$
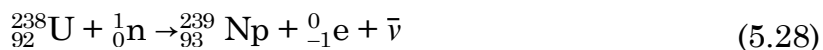
That is, $^{238}_{92}$U absorbs the neutron and then goes through a beta decay by emitting an electron. The atomic number is increased by one, from $Z = 92$ to $Z = 93$, thus creating an isotope of a new chemical element, which is called neptunium.

Neptunium-239 is itself unstable and decays by beta emission creating still another chemical element called plutonium, according to the reaction

$$^{239}_{93}\text{Np} \rightarrow ^{239}_{94}\text{Pu} + ^{0}_{-1}\text{e} + \bar{\nu} \tag{5.29}$$

The next transuranic element to be created was americium, which was created by the series of processes given by

$$^{239}_{94}\text{Pu} + ^{1}_{0}\text{n} \rightarrow ^{240}_{94}\text{Pu} + \gamma$$
$$^{240}_{94}\text{Pu} + ^{1}_{0}\text{n} \rightarrow ^{241}_{94}\text{Pu} + \gamma$$
$$^{241}_{94}\text{Pu} \rightarrow ^{241}_{95}\text{Am} + ^{0}_{-1}\text{e} + \bar{\nu}$$

That is, plutonium is bombarded with neutrons until the isotope $^{241}_{94}$Pu is created, which then beta decays producing the isotope of the new chemical element americium. Bombarding elements with various other particles and elements, created still more elements. As examples of the creation of some other new elements, we have
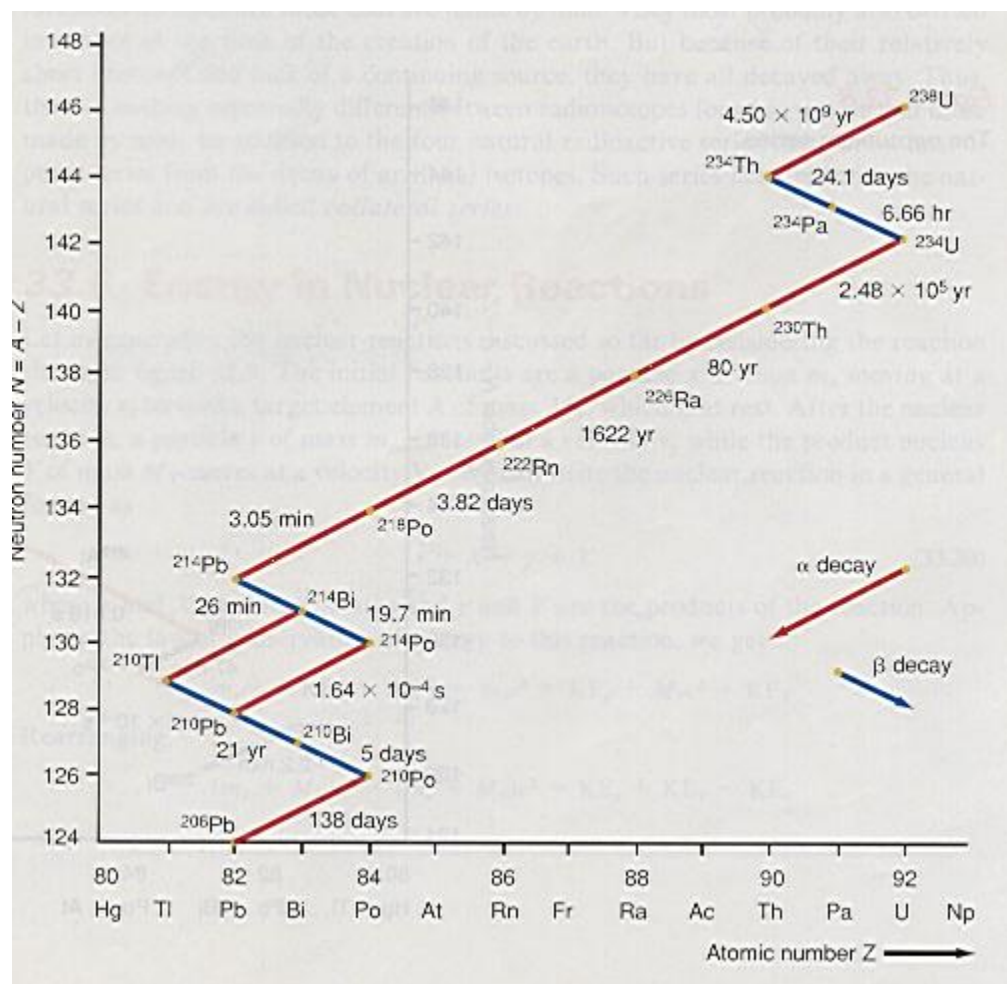
Figure 5.6 Uranium $^{238}_{92}$U decay series.

(curium)	$^{239}_{94}\text{Pu} + ^4_2\text{He} \rightarrow ^{242}_{96}\text{Cm} + ^1_0\text{n}$
(berkelium)	$^{241}_{94}\text{Am} + ^4_2\text{He} \rightarrow ^{243}_{97}\text{Bk} + ^0_{-1}\text{e} + \bar{\nu} + 2\,^1_0\text{n}$
(californium)	$^{242}_{96}\text{Cm} + ^4_2\text{He} \rightarrow ^{245}_{98}\text{Cf} + ^1_0\text{n}$
(nobelium)	$^{246}_{96}\text{Cm} + ^{12}_6\text{C} \rightarrow ^{254}_{102}\text{No} + 4\,^1_0\text{n}$
(lawrencium)	$^{252}_{98}\text{Cf} + ^{10}_5\text{B} \rightarrow ^{257}_{103}\text{Lr} + 5\,^1_0\text{n}$

The neptunium decay series was later found to actually start with plutonium, $^{241}_{94}\text{Pu}$, which decays by beta emission to americium, $^{241}_{95}\text{Am}$, which then decays by alpha emission to neptunium, $^{237}_{93}\text{Np}$.

Because of the very long lifetimes of the parent element of these series, most of the members of the series are found naturally. An equilibrium condition is established within the series with as many isotopes decaying as are being formed. Artificial isotopes are those that are made by man. They most probably also existed
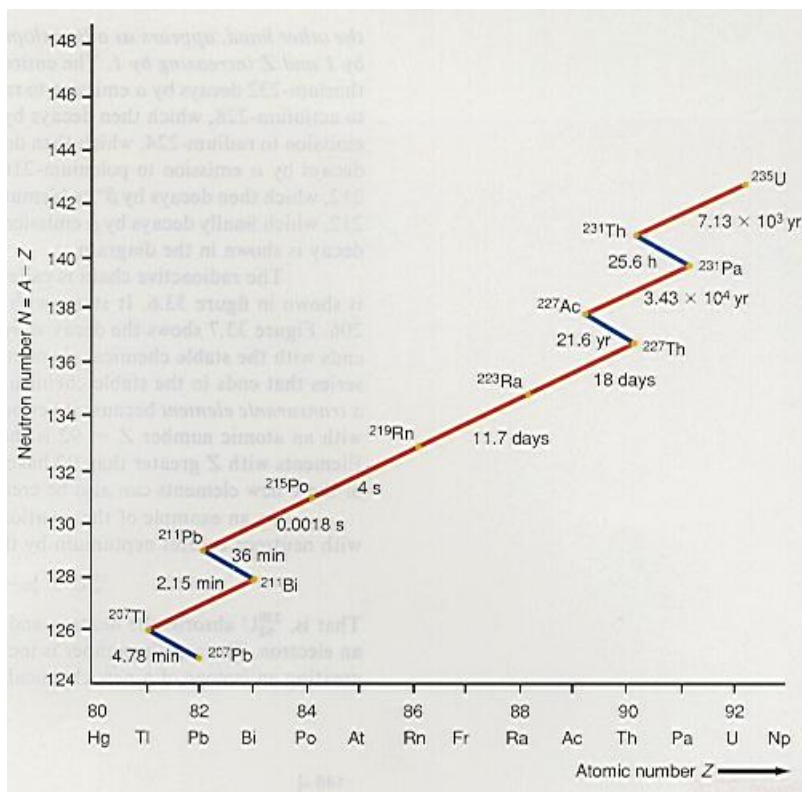
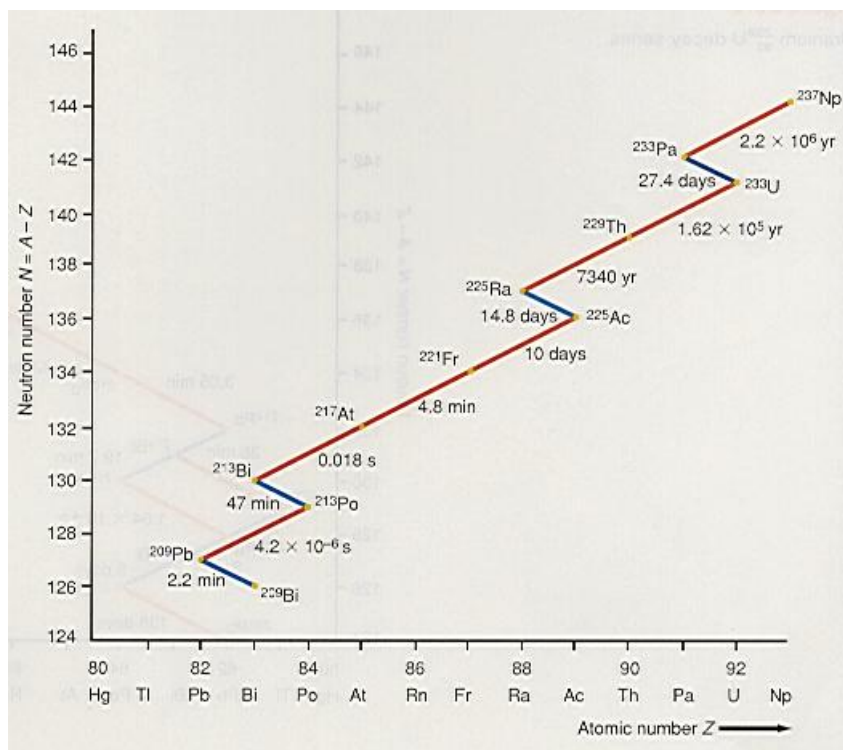**Figure 5.7** Uranium $^{235}_{92}\text{U}$ decay series.

Figure 5.8 The neptunium series.

in nature at the time of the creation of the earth. But because of their relatively short lifetimes and lack of a continuing source, they have all decayed away. Thus, there is nothing essentially different between radioisotopes found in nature and those made by man. In addition to the four natural radioactive series there are a host of other series from the decay of artificial isotopes. Such series are similar to the natural series and are called *collateral series*.

5.6 Energy in Nuclear Reactions

Let us generalize the nuclear reactions discussed so far by considering the reaction shown in figure 5.9. The initial reactants are a particle x of mass m_x moving at a velocity $\mathbf{v}_x$ toward a target element X of mass M_X, which is at rest. After the nuclear reaction, a particle y of mass m_y leaves with a velocity $\mathbf{v}_y$ while the product nucleus
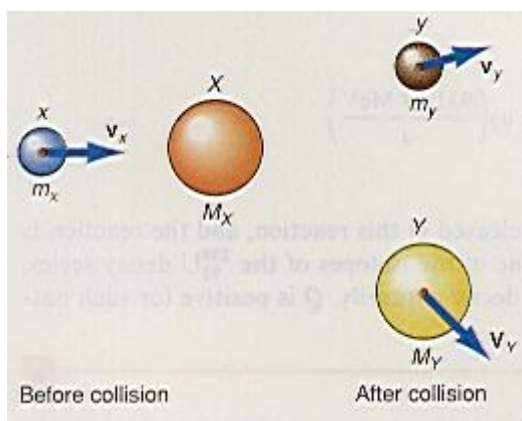

Figure 5.9 Nuclear reaction as a collision.

Y of mass M_Y moves at a velocity $\mathbf{V}_Y$.   We can write the nuclear reaction in a general format as

$$x + X = y + Y \tag{5.30}$$

where x and X are the reactants and y and Y are the products of the reaction. Applying the law of conservation of energy to this reaction, we get

$$m_x c^2 + KE_x + M_X c^2 = m_y c^2 + KE_y + M_Y c^2 + KE_Y$$

Rearranging,

$$(m_x + M_X)c^2 - (m_y + M_Y)c^2 = KE_y + KE_Y - KE_x$$

The **Q value of a nuclear reaction** *is now defined as the energy available in a reaction caused by the difference in mass between the reactants and the products.* Thus,

$$Q = (m_x + M_X)c^2 - (m_y + M_Y)c^2 \tag{5.31}$$

or

$$Q = [(\text{Input mass}) - (\text{Output mass})]c^2 \qquad (5.32)$$

or

$$Q = E_{in} - E_{out} \qquad (5.33)$$

That is, the Q value is the difference between the energy put into a nuclear reaction E_{in} and the energy that comes out E_{out}.

If $m_x + M_X$ is greater than $m_y + M_Y$, then Q is greater than zero ($Q > 0$). That is, the input mass energy is greater than the output mass energy. Thus, mass is lost in the nuclear reaction and an amount of energy Q is released in the process. *A nuclear reaction in which energy is released is called an* **exoergic reaction** *(sometimes called an exothermic reaction).*

Example 5.7

Energy released in a nuclear reaction. How much energy is released or absorbed in the following reaction?

$$^{219}_{86}\text{Rn} \rightarrow\,^{215}_{84}\text{Po} +\,^{4}_{2}\text{He}$$
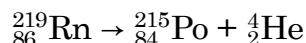
Solution

The mass of radon-219 is 219.009523 unified mass units (u) while the mass of polonium, $^{215}_{84}\text{Po}$, is 214.999469 u. The mass of the α particle is 4.002603 u. The total output mass is

$$\begin{array}{r} 214.999469 \text{ u} \\ +4.002603 \text{ u} \\ \hline 219.002072 \text{ u} \end{array}$$

Hence, the difference in mass between the input mass and the output mass is

$$219.009523 \text{ u} -\ 219.002072 \text{ u} = 0.007451 \text{ u}$$

Converting this to an energy

$$Q = \frac{(0.007451 \text{ u})(931.49 \text{ MeV})}{\text{u}}$$

$$= 6.94 \text{ MeV}$$

Since Q is greater than zero, energy is released in this reaction, and the reaction is exoergic. We might note that $^{219}_{86}\text{Rn}$ is one of the isotopes of the $^{235}_{92}\text{U}$ decay series. Because all of the isotopes of this chain decay naturally, Q is positive for such natural decays.

To go to this Interactive Example click on this sentence.

If in a nuclear reaction, $m_x + M_X$ is less than $m_y + M_Y$, then the Q value is negative ($Q < 0$). In such a reaction, mass is created if an amount of energy Q is added to the system. This energy is usually added by way of the kinetic energy of the reacting particle and nuclei. *A nuclear reaction in which energy is added to the system is called an* **endoergic reaction** *(sometimes called an endothermic reaction).*

A nuclear reaction proceeds naturally in the direction of minimum energy. Thus, in the decay of a natural radioactive nuclide, the nucleus emits a particle in order to reach a lower equilibrium energy state. The excess energy is given off in the process. Endoergic reactions, on the other hand, do not occur naturally in the physical world because the energy of the reactants is less than the required energy for the products to be created. Thus, endoergic reactions cannot take place unless energy is added to the system. The energy is added by accelerating the particle to very high speeds in an accelerator. When the particle hits the target, this additional kinetic energy is the energy necessary to make the reaction proceed. It is sometimes necessary to have additional kinetic energy to overcome the Coulomb barrier.

Example 5.8

Find the Q value of a nuclear reaction. The first artificial transmutation of an element was performed by Rutherford in 1919 when he bombarded nitrogen with alpha particles according to the reaction

$$^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{p}$$
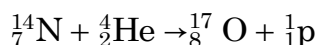
Find the Q value associated with this reaction.

Solution

The Q value, found from equation 5.31, is

$$Q = (m_x + M_X)c^2 - (m_y + M_Y)c^2$$

where

$$m_x = m(^{4}_{2}\text{H}) = 4.002603 \text{ u}$$
$$\underline{M_X = m\,(^{14}_{7}\text{N}) = 14.003242 \text{ u}}$$
$$(m_x + M_X) = 18.005845 \text{ u}$$

and

$$m_y = m_p = 1.007825 \text{ u}$$
$$\underline{M_Y = m(^{17}_{8}\text{O}) = 16.999133 \text{ u}}$$
$$(m_y + M_Y) = 18.006958 \text{ u}$$

Hence,

$$Q = 18.005845 \text{ u} - 18.006958 \text{ u}$$

$$= - \ (0.001113 \ \text{u})\frac{(931.49 \ \text{MeV})}{1 \ \text{u}}$$

$$= -1.04 \ \text{MeV}$$

Therefore, Q is negative, and this much energy must be supplied to start the reaction. The initial α particle used by Rutherford had energies of about 5.5 MeV, well above the amount of energy needed.

To go to this Interactive Example click on this sentence.

We should note that the amount of energy necessary to break up the nucleus, its Q value, is the same as the binding energy of the nucleus, BE, discussed in section 5.2. We can now write a nuclear reaction in the form

$$x + X \rightarrow y + Y + Q \tag{5.34}$$

When $Q > 0$, Q is the amount of energy released in a reaction. When $Q < 0$, Q is the amount of energy that must be added to the system in order for the reaction to proceed.
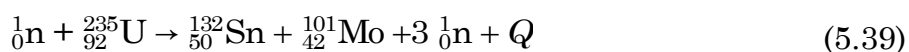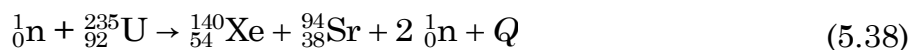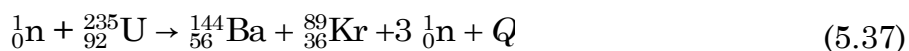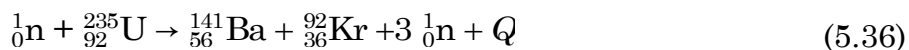

5.7 Nuclear Fission

In 1934, Enrico Fermi, beginning at the bottom of the periodic table, fired neutrons at each chemical element in order to create isotopes of the elements. He systematically worked his way up the periodic table until he came to the last known element, at that time, uranium. He assumed that bombarding uranium with neutrons would make it unstable. He then felt that if the unstable uranium nucleus went through a beta decay, the atomic number would increase from 92 to 93 and he would have created a new element. (He was the first to coin the word *transuranic.*) However after the bombardment of uranium, he could not figure out what the products of the reaction were.

From 1935 through 1938, the experiments were repeated in Germany by Otto Hahn and Lise Meitner. The German chemist, Ida Noddack, analyzed the products of the reaction and said that *it appeared as if the uranium atom had been split into two lighter elements.* Lise Meitner and her nephew, Otto Frisch, considered these results and concluded that indeed the atom had been split into two lighter elements. The splitting of an atom resembled the splitting of one living cell into two cells of equal size. This biological process is called fission. Otto Frisch then used this biological term, fission, to describe the splitting of an atom. Hence, **nuclear fission** *is the process of splitting a heavy atom into two lighter atoms.* The isotope of uranium that undergoes fission is $^{235}_{92}\text{U}$. The process can be described in general as

$$\ce{^1_0 n} + \ce{^{235}_{92} U} \rightarrow y + Y + \ce{^1_0 n} + Q \tag{5.35}$$

The fission process does not always produce the same fragments, however. It was found that the product or fragment nuclei, y and $Y,$ varied between the elements Z = 36 to Z = 60. Some typical fission reactions are:

$$\ce{^1_0 n} + \ce{^{235}_{92} U} \rightarrow \ce{^{141}_{56} Ba} + \ce{^{92}_{36} Kr} + 3\,\ce{^1_0 n} + Q \tag{5.36}$$

$$\ce{^1_0 n} + \ce{^{235}_{92} U} \rightarrow \ce{^{144}_{56} Ba} + \ce{^{89}_{36} Kr} + 3\,\ce{^1_0 n} + Q \tag{5.37}$$

$$\ce{^1_0 n} + \ce{^{235}_{92} U} \rightarrow \ce{^{140}_{54} Xe} + \ce{^{94}_{38} Sr} + 2\,\ce{^1_0 n} + Q \tag{5.38}$$

$$\ce{^1_0 n} + \ce{^{235}_{92} U} \rightarrow \ce{^{132}_{50} Sn} + \ce{^{101}_{42} Mo} + 3\,\ce{^1_0 n} + Q \tag{5.39}$$

In all cases, the masses of the product nuclei are less than the masses of the reactants, indicating that the Q value is greater than zero. The reaction is, therefore, exoergic and energy is given off in the process.

Example 5.9

The Q value of a nuclear fission reaction. Find the Q value associated with the nuclear fission process given by equation 5.36.

Solution

The mass of the reactants are
$$m_n = 1.008665 \text{ u}$$
$$m(\ce{^{235}_{92} U}) = 235.043933 \text{ u}$$
$$m_n + m(\ce{^{235}_{92} U}) = 236.052598 \text{ u}$$

The mass of the products are
$$3m_n = 3.025995 \text{ u}$$
$$m(\ce{^{141}_{56} Ba}) = 140.913740 \text{ u}$$
$$m(\ce{^{92}_{36} Kr}) = 91.925765 \text{ u}$$
$$m_{Ba} + m_{Kr} = 235.865500 \text{ u}$$

The mass lost in the process is

$$\Delta m = 236.052598 \text{ u} - 235.865500 \text{ u}$$
$$= +\,0.187098 \text{ u}$$

The Q value is obtained by multiplying Δm by the conversion factor 931.49 MeV = 1 u.

$$Q = (0.187098 \text{ u})\frac{(931.49 \text{ MeV})}{\text{u}}$$

$$= 174 \text{ MeV}$$

Hence, the splitting of only one nucleus of $_{92}^{235}\text{U}$ gives off an enormous quantity of energy. The actual energy in the fission process turns out to be even greater than this because the fragments themselves are radioactive and give off an additional 15 to 20 MeV of energy as they decay. Hence, in the entire fission process of $_{92}^{235}\text{U}$, some 200 MeV of energy are given off per nucleus.

To go to this Interactive Example click on this sentence.

Example 5.10

The energy of fission of uranium. If 1 kg of $_{92}^{235}\text{U}$ were to go through the fission process, how much energy would be released?

Solution

Because the mass of any quantity is equal to the mass of one atom times the total number of atoms, that is,

$$m = m_{\text{atom}}N$$

the number of atoms is

$$N = \frac{m}{m_{\text{atom}}}$$

$$= \left(\frac{1 \text{ kg}}{235.04 \text{ u}}\right)\left(\frac{1 \text{ u}}{1.66 \times 10^{-27}\text{kg}}\right)$$

$$= 2.56 \times 10^{24} \text{ atoms}$$

But the number of nuclei is exactly equal to the number of atoms, hence, there are 2.56×10^{24} uranium nuclei in 1 kg of uranium-235. Assuming a total energy release of 200 MeV per nucleus, the total energy released is

$$E = \frac{(200 \text{ MeV})}{\text{nucleus}}(2.56 \times 10^{24} \text{ nuclei})$$

$$= 5.12 \times 10^{26} \text{ MeV} = 8.19 \times 10^{13} \text{ J}$$

which is an absolutely immense amount of energy. It is comparable to the amount of energy released from the explosion of 20,000 tons of TNT.

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A theoretical model of nuclear fission, developed by Niels Bohr and John A. Wheeler, and called the *liquid-drop model,* is sketched in figure 5.10. When the
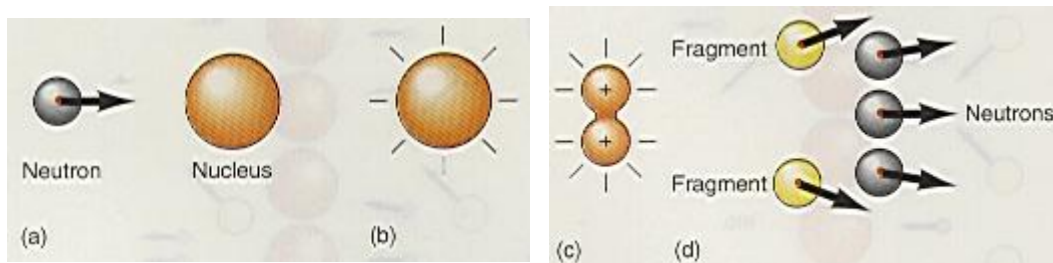

Figure 5.10 The liquid drop model of nuclear fission.

bombarding neutron is captured by the uranium nucleus, the nucleus becomes unstable, vibrates, and becomes deformed as in figure 5.10(c). In the deformed state, the nuclear force is not as great as usual because the nucleus is spread so far apart. The Coulomb force of repulsion is, however, just as strong as always and acts to split the drop (nucleus) into fragments, figure 5.10(d). Thus, the uranium nucleus is split into fragment nuclei accompanied by extra neutrons and a large amount of energy.

All in all, there are about 90 different daughter nuclei formed in the fission process. The initial neutrons that are used to bombard the uranium are called *slow neutrons* because they have very small kinetic energies and, hence, low velocities and, therefore, move slowly. The slow neutrons have a large probability of capture by the uranium-235 nucleus because they are moving so slowly. There are about two or three neutrons released per each fission.

A historical anecdote relating to nuclear fission might be interesting to mention here. In 1906, at McGill University in Montreal, Canada, Lord Rutherford said: "If it were ever found possible to control at will, the rate of disintegration of the radioactive elements, an enormous amount of energy could be obtained from a small quantity of matter."[1]   With age, Rutherford was to change that vision to, "The energy produced by breaking down the atom is a very poor kind of thing. Anyone who expects a source of power from the transformation of these atoms is talking moonshine." That statement was a challenge to Leo Szilard (1898-1964), a Hungarian physicist working for Rutherford. Szilard thought, "What if you found an element in which nuclei throw off energy? What if you could make it happen at will? What if this element's atoms threw off two new neutrons to strike two more nuclei. Two twos are four, four fours are sixteen — in a flash, the number would be astronomical. Moonshine? All you need do is to find the right element!"[2]

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[1] *Uranium, The Deadly Element,* by Lennard Bikel, p. 43.
[2] Ibid, p. 72.

A by-product of fission is that it produces the same particles that initiated the fission in the first place, namely neutrons. If more neutrons are produced than started the reaction, the result is a multiplication. If the excess product neutrons can initiate more fission, more neutrons are produced to produce more fission, and so on and on. The result is a chain reaction, as shown in figure 5.11. The
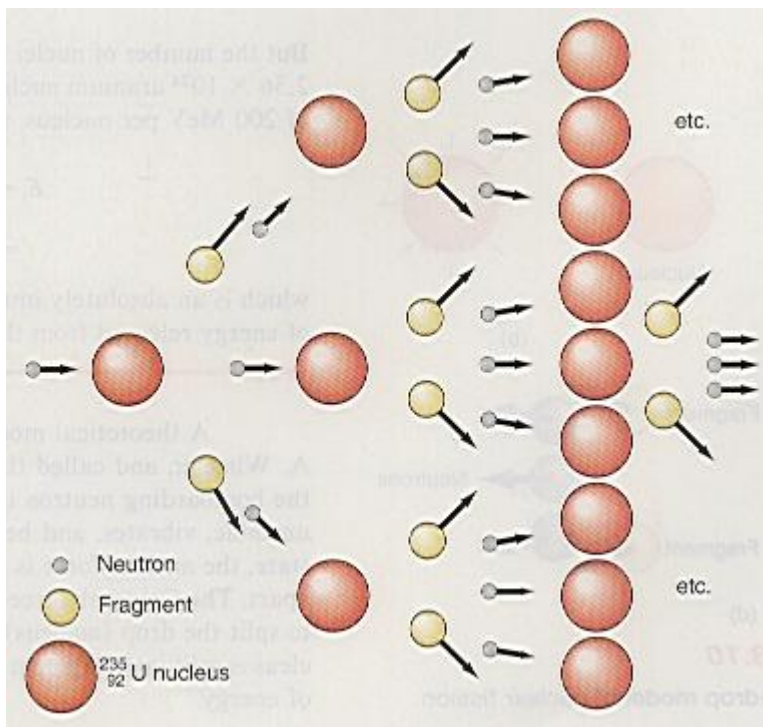

Figure 5.11 The chain reaction.

multiplication of neutrons is given by a multiplication factor, k. If $k < 1$, the reaction gives less neutrons than initiated the reaction, and the chain dies out. If $k > 1$ the reaction gives too many neutrons and the reaction escalates and runs wild. If $k = 1$, just the right number of neutrons are produced to keep the process going at a constant rate.

Natural uranium contains 99.3% of $^{238}_{92}U$ and only 0.7% of $^{235}_{92}U$, and cannot chain react. To get a chain reaction, the percentage of $^{235}_{92}U$ must be increased. Weapons grade uranium contains about 50% of $^{235}_{92}U$, whereas *nuclear reactor grade uranium contains only about 3.6% of* $^{235}_{92}U$, which is much too small to produce a nuclear explosion.

To finish our little story, Leo Szilard filed an application in the London Patents Office on June 28, 1934. It was the world's first registration of a nuclear process chain reaction by neutron bombardment. Szilard was afraid his chain reaction idea would fall into Nazi hands, so he assigned his patent to the British Admiralty. In general, the heavier the nucleus that is split, the greater the energy given off. Szilard made the mistake of proposing to split the lighter elements instead of the heavier ones.

The Atomic Bomb

The possibility of a chain reaction in uranium with its extremely large energy release, led some of the nuclear scientists to conceive of making a bomb — an atomic bomb. The Second World War was raging in Europe and the scientists were afraid that Hitler might develop such a bomb. Such a bomb in his hands, it was felt, would mean the end of the civilized world. For our own protection, it was imperative that we should develop such a bomb as quickly as possible. Leo Szilard and Edward Teller (later to become the Father of the hydrogen bomb), both Hungarian physicists who were refugees from Hitler's Europe, approached Albert Einstein and had him draft a letter to President Roosevelt on the possibility of making an atomic bomb. The letter was given to Dr. Alexander Sachs who personally delivered it to President Roosevelt on October 11, 1939. Ironically, the final decision to go ahead with the development of the A-bomb was made on December 6, 1941, under the name of the Manhattan Project.

In order to make an atomic bomb, enough uranium-235 had to be assembled to make the chain reaction. The amount of mass of uranium-235 needed to start the chain reaction was called the *critical mass.* The uranium-235 had to consist of two pieces, both below the critical mass. When one piece, in the form of a bullet, was fired into the second piece, the critical mass was obtained and the chain reaction would lead to a violent explosion. This was the type of bomb called the "Thin Man" that was detonated at Hiroshima on August 5, 1945. The difficulty with a uranium bomb was that it was relatively difficult to separate $^{235}_{92}U$ from $^{238}_{92}U$.

As already seen in equation 5.28, bombarding $^{238}_{92}U$ with neutrons produces the element neptunium, $^{239}_{93}Np$, which decays into plutonium, $^{239}_{94}Pu$, equation 5.29. It turns out that plutonium-239 is even more fissionable than uranium-235, so a much smaller mass of it is necessary for its critical mass. By making a nuclear reactor, which we will discuss in a moment, a very large, relatively cheap supply of plutonium was made available. So the Manhattan Project proceeded to make another type of atomic bomb — a plutonium bomb. The plutonium bomb was made in the form of a sphere, with pieces of plutonium, each below the critical mass, at the edge of the sphere, as shown in figure 5.12. For ignition, a series of chemical explosions fired the plutonium pieces all toward the center of the sphere at the same time. When all these pieces of plutonium came together they constituted the critical mass of plutonium, and the chain reaction was initiated and the bomb exploded.

The first test of an atomic device, was a test of the plutonium bomb on July 16, 1945, at a site called "Trinity" in the New Mexico desert. The first plutonium bomb, called "Fat Boy" was dropped on Nagasaki on August 9, 1945.
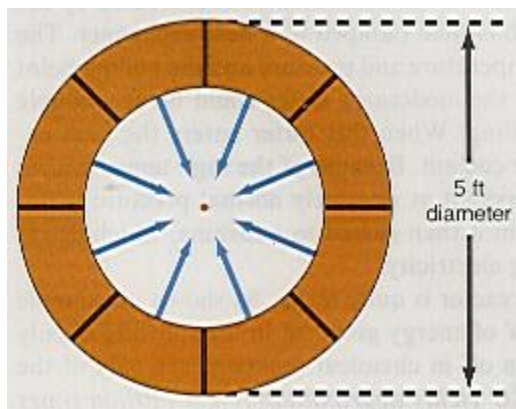
Figure 5.12 Triggering the plutonium bomb.

Fission Nuclear Reactors

The first nuclear reactor was built by Enrico Fermi on the squash court under the west stands of Stagg Field at the University of Chicago. It was started in October of 1942 and began operating on December 2, 1942. This was the first controlled use of nuclear fission.

A typical nuclear reactor is sketched in figure 5.13. The reactor itself contains uranium, $^{238}_{92}U$, enriched with 3.6% of $^{235}_{92}U$ Neutrons are given off by a reaction such as equation 5.36. The neutrons given off have a rather high kinetic energy and are called *fast neutrons* because of the high speed associated with the large kinetic energy. These fast neutrons are moving too fast to initiate more fission reactions and must be slowed down. One such way is to enclose the entire reactor in a water bath under high pressure. Such a reactor is called a pressurized water reactor (PWR). The neutrons now collide with the water molecules and are slowed down so that they can be used in the fission process. The water is called the *moderator,* because it moderates or slows down the neutrons. The slow neutrons now proceed to split more $^{235}_{92}U$ nuclei until a chain reaction is obtained. The chain reaction is not allowed to run wild as in an atomic bomb but is controlled by a series of rods, usually made of cadmium, that are inserted into the reactor. Cadmium is an element that is capable of absorbing a large number of neutrons without becoming unstable or radioactive. Hence, when the cadmium control rods are inserted into the reactor they absorb neutrons to cut down on the number of neutrons that are available for the fission process. In this way, the fission reaction is controlled. The water moderator also acts as a coolant. The tremendous heat generated by the fission process heats up the water, which is then pumped to a heat exchanger. The hot moderator water is at a very high temperature and pressure, and the boiling point of water increases with pressure. Thus, the moderator water could be at a couple of hundred degrees Celsius without boiling. When this water enters the heat exchanger, it heats up the secondary water coolant. Because of the high temperatures of the primary coolant, the secondary coolant at relatively normal
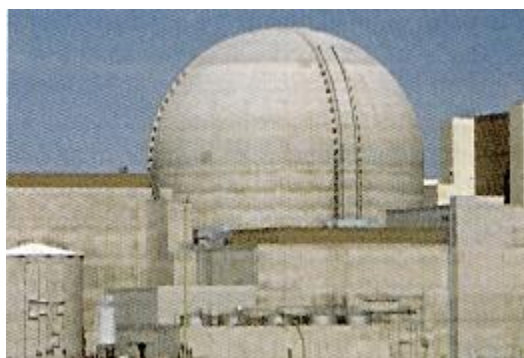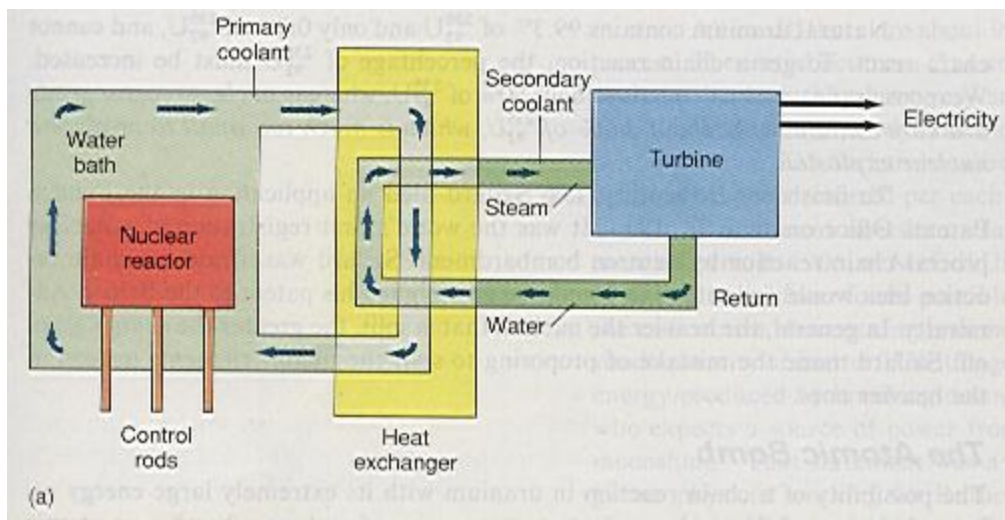
Figure 5.13 A typical nuclear reactor.

pressure is immediately converted to steam. This steam is then passed to a turbine, which drives an electric generator, thereby producing electricity.

The energy that comes from a reactor is quite large. As shown in example 5.10, there are approximately 200 MeV of energy given off in the splitting of only one $^{235}_{92}U$ nucleus. Typical energies given off in chemical reactions are only of the order of 3 or 4 eV. Hence, *fission of $^{235}_{92}U$* yields approximately 2.5 million times as much energy as found in the combination of the same mass of carbon (such as in coal or gasoline).

The one drawback to a fission reactor is the nuclear waste material. As shown in equations 5.36 through 5.39, fission fragments such as $^{141}_{56}Ba$, $^{92}_{36}Kr$, $^{144}_{56}Ba$, $^{89}_{36}Kr$, $^{140}_{54}Xe$, $^{94}_{38}Sr$, $^{132}_{50}Sn$, and $^{101}_{42}Mo$, are some of the possible products of the reaction. These isotopes are unstable and decay into other radioactive nuclei. Eventually all these dangerous radioactive waste nuclei must be discarded. Some of them have relatively long half-lives and will, therefore, be around for a long time. They cannot be dumped into oceans or left in any place where they will contaminate the environment, such as through the soil or the air. They must not be allowed to get into the drinking water. The best place so far found to store these wastes is in the bottom of old salt mines, which are very dry and are
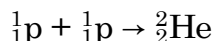
thousands of feet below the surface of the earth. Here they can sit and decay without polluting the environment.

One unfounded fear of many people is that a nuclear reactor may explode like an atomic bomb and kill all the people in its neighborhood. A nuclear reactor does not contain enough $^{235}_{92}U$ to explode as an atomic bomb. What is more, the cadmium control rod's normal position is in the reactor. They must be pulled out to get and keep the reactor in operation. Any failure of any mechanism of the reactor causes the control rods to fall back into the reactor, thereby, stopping the chain reaction and shutting down the reactor.

Another type of a fission nuclear reactor is the *breeder reactor*. A breeder reactor uses uranium $^{238}_{92}U$, or thorium $^{232}_{90}Th$, as the nuclear fuel and uses fast high-energy neutrons instead of the slow ones used in the PWR. The fast neutrons react with the $^{238}_{92}U$, according to equation 5.28, and form neptunium, $^{239}_{93}Np$ The neptunium, $^{239}_{93}Np$, decays according to equation 5.29, and produces plutonium, $^{239}_{94}Pu$ The plutonium is highly fissionable and it too can supply energy in the reactor. The net result of forming plutonium in the reactor is to create more fissionable material than is used. Hence, the name breeder reactor; it "breeds" nuclear fuel. Of course, the breeder reactor can also generate electricity while it is creating more fuel. Breeder reactors are used to create plutonium for nuclear weapons.

5.8 Nuclear Fusion

It has been long observed that the sun emits tremendous quantities of energy for an enormous quantity of time. There was much speculation as to the source of this energy. In 1938, Hans Bethe (1906-) suggested that the fusion of hydrogen nuclei into helium nuclei was responsible for the tremendous energy released. ***Nuclear fusion** is a process in which lighter nuclei are joined together to produce a heavier nucleus and a good deal of energy*. Bethe proposed that the energy was released in the sun in what he called the proton-proton cycle. The first part of the cycle consists of two protons combining to form an unstable isotope of helium.

The enormous energy generated by stars like our Sun is a result of nuclear fusion.

$$^1_1p + ^1_1p \rightarrow ^2_2He$$

But one of these combined protons in the nucleus of the unstable isotope immediately decays by equation 5.21 as

$$^1_1p \rightarrow ^1_0n + ^0_{+1}e + v$$
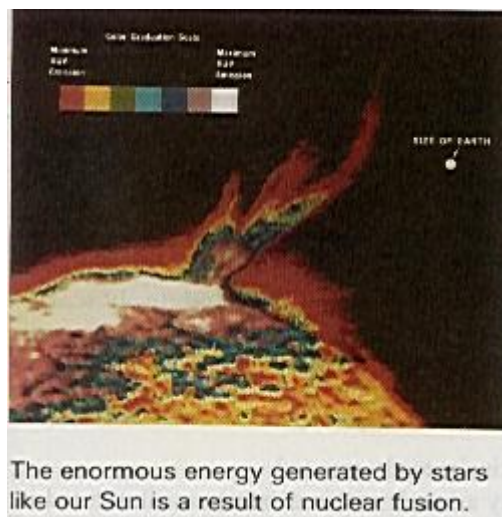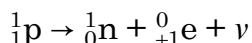
The neutron now combines with the first proton to form the deuteron, and we can write the entire reaction as

$$_1^1\text{p} + _1^1\text{p} \rightarrow _1^2\text{H} + _{+1}^0\text{e} + \nu \qquad (5.40)$$

Note here that the decay of a proton or a neutron in the nucleus is caused by the weak nuclear force, which we will describe in more detail in chapter 6. The deuteron formed in equation 5.40 now combines with another proton to form the isotope of helium, 3 2 He, according to the reaction

$$_1^2\text{H} + _1^1\text{p} \rightarrow _2^3\text{He} + \gamma \qquad (5.41)$$

The process represented by equations 5.40 and 5.41 must occur twice to form two $_2^3\text{He}$ nuclei, which then react according to the equation

$$_2^3\text{He} + _2^3\text{He} \rightarrow _2^4\text{He} + 2\,_1^1\text{p} \qquad (5.42)$$

Thus, the stable element helium has been formed from the fusion of the nuclei of the hydrogen atom. We can write the entire proton-proton cycle in the shorthand version as

$$4\,_1^1\text{p} \rightarrow _2^4\text{He} + 2\,_{+1}^0\text{e} + 2\gamma + 2\nu + Q \qquad (5.43)$$
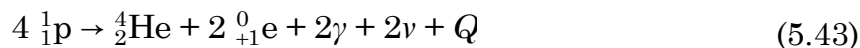
The net Q value, or energy released in the process, is about 26 MeV.

The Hydrogen Bomb

In July of 1942, Robert Oppenheimer (1904-1967), reporting on the work of Edward Teller, Enrico Fermi, and Hans Bethe, noted that the extremely high temperature of an atomic bomb could be used to trigger a fusion reaction in deuterium, thus producing a fusion bomb or a hydrogen bomb. The reaction between deuterium and tritium, both isotopes of hydrogen, is given by

$$_1^2\text{H} + _1^3\text{H} \rightarrow _2^4\text{He} + _0^1\text{n} + 17.6\,\text{MeV}$$
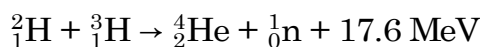
Deuterium is relatively abundant in ocean water but tritium is relatively scarce. However, tritium can be generated in a nuclear reactor by surrounding the core with lithium. The neutron from the reactor causes the reaction

$$_3^7\text{Li} + _0^1\text{n} \rightarrow _2^4\text{He} + _1^3\text{H} + _0^1\text{n}$$
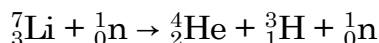
Thus, all the tritium desired can be relatively easily created.

The hydrogen bomb is effectively a bomb within a bomb, as illustrated in figure 5.14. A conventional atomic bomb made of plutonium is ignited. The tremendous heat given off by the A-bomb supplies the high temperature to start the

fusion process of the deuterium-tritium mixture. The size of an A-bomb is limited by the critical mass of plutonium. We cannot assemble an amount of plutonium
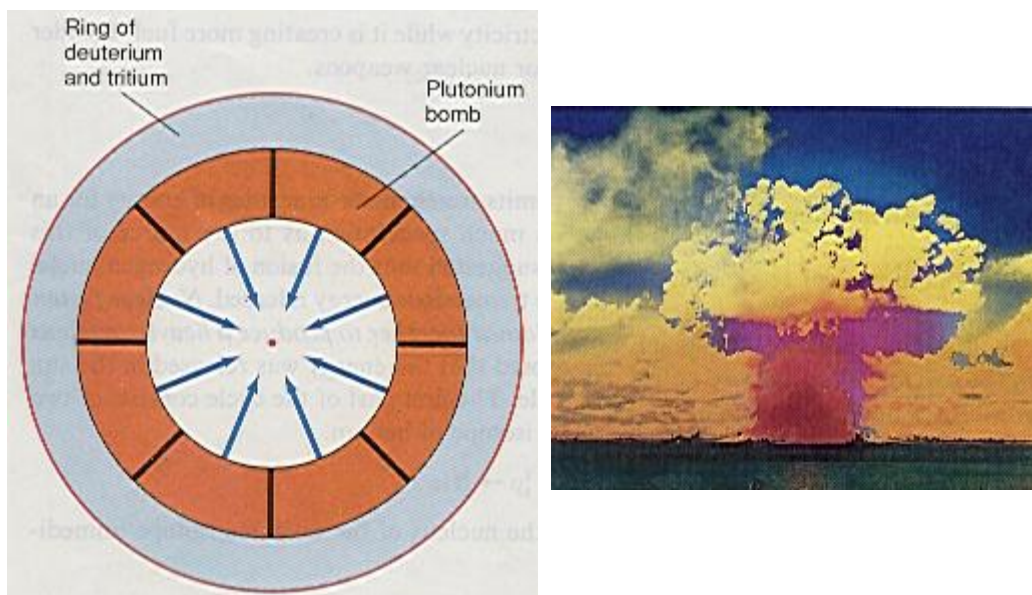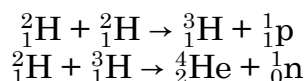

Figure 5.14 A hydrogen bomb. The energy released in an explosion resulting from the nuclear fusion of even small amounts of material is enormous.

greater than the critical mass without it exploding. We can, however, assemble as much deuterium and tritium as we please. It will never go off unless supplied with the extremely high temperature necessary for fusion.

The first H-bomb was detonated on October 31, 1952. It completely eliminated the island of Eniwetok in the Marshall Islands. The Soviet Union quickly followed suit by exploding their H-Bomb on August 12, 1953. The Soviets used lithium in place of tritium in their fusion reaction, because it is cheaper and more easily available.

The Fusion Reactor
One of the difficulties of a fission reactor is the radioactive fragments or waste that is a by-product of the reaction. *The fusion process has no by-product that is radioactive.* That is, the only result of fusion, is helium, which is an inert gas and is not radioactive. The proton-proton cycle in the sun is too slow to take place in a reactor. Hence, the fusion cycle in a fusion reactor is given by

$$_{1}^{2}\text{H} + _{1}^{2}\text{H} \rightarrow _{1}^{3}\text{H} + _{1}^{1}\text{p}$$
$$_{1}^{2}\text{H} + _{1}^{3}\text{H} \rightarrow _{2}^{4}\text{He} + _{0}^{1}\text{n}$$

The difficulty in the design of a fusion reactor has to do with the extremely high temperatures associated with the fusion process, that is, millions of kelvins. (Remember the surface temperature of the sun is about 6000 K, and the core,

thousands of times higher.) At these high temperatures, all materials that could be made to contain the reaction would melt. The task of building a fusion reactor is not, however, impossible, just difficult. At the high temperatures of fusion, electrons and nuclei are completely separated from each other in what is called a *plasma,* an ionized fluid. Because of the electric charges of the fluid, the fusion reaction can be contained within magnetic fields. Experimental fusion reactors have been built on a very limited scale using magnetic confinement with some slight success. A great deal of work still has to be done to perfect the fusion reactor. This work must be done, because the fusion reactor promises to be a source of enormous energy produced by a very cheap fuel, effectively water, with no radioactive contaminants as a by-product.
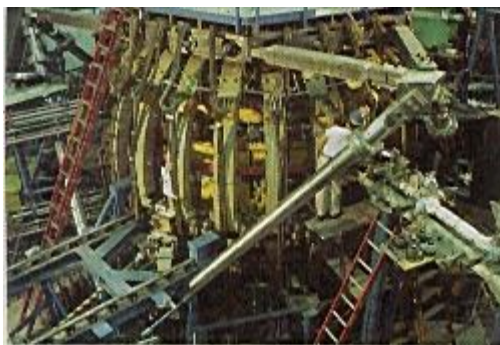

A tokamak apparatus, designed to generate the magnetic field necessary for containing a fusion reaction.
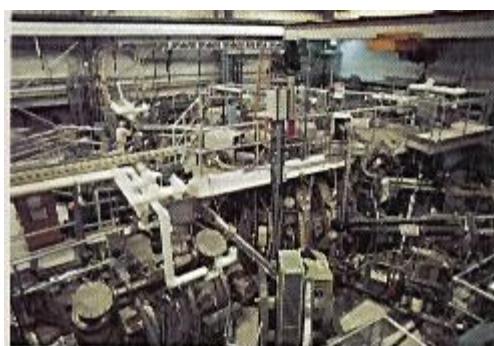
Thermonuclear fusion reactor.

5.9 Nucleosynthesis

It is a fact of life that we all take for granted the things that are around us. On this planet earth, the materials we see are made out of molecules and atoms. We saw in the discussion of the periodic table of the elements in chapter 4 how each element differs from each other by the number of electrons, protons, and neutrons contained within each atom. *But what is the origin of all these elements? How were they originally formed? The elements found on the earth and throughout the universe were originally synthesized by the process of fusion within the stars, a process called* **nucleosynthesis.** The proton-proton cycle formed helium from hydrogen. As a continuation of that cycle, ^3_2He can fuse with ^4_2He to produce beryllium according to the equation

$$^3_2\text{He} + {}^4_2\text{He} \rightarrow {}^7_4\text{Be} + \gamma$$
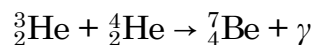
Beryllium can now capture an electron to form lithium according to the relation

$$^7_4\text{Be} + {}^0_{-1}\text{e} \rightarrow {}^7_3\text{Li} + \nu$$
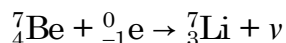
As most of the hydrogen is used, the helium nuclei can now fuse to form the following nuclei
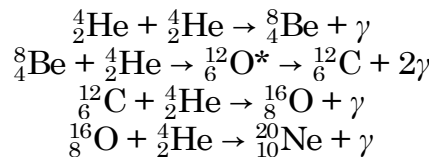
$$\begin{aligned}
{}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} &\rightarrow {}^{8}_{4}\text{Be} + \gamma \\
{}^{8}_{4}\text{Be} + {}^{4}_{2}\text{He} &\rightarrow {}^{12}_{6}\text{O*} \rightarrow {}^{12}_{6}\text{C} + 2\gamma \\
{}^{12}_{6}\text{C} + {}^{4}_{2}\text{He} &\rightarrow {}^{16}_{8}\text{O} + \gamma \\
{}^{16}_{8}\text{O} + {}^{4}_{2}\text{He} &\rightarrow {}^{20}_{10}\text{Ne} + \gamma
\end{aligned}$$

If the star continues burning, additional elements are formed, such as

$$\begin{aligned}
{}^{12}_{6}\text{C} + {}^{1}_{1}\text{p} &\rightarrow {}^{13}_{7}\text{N} + \gamma \\
{}^{13}_{7}\text{N} &\rightarrow {}^{13}_{6}\text{C} + {}^{0}_{+1}\text{e} + \nu \\
{}^{13}_{6}\text{C} + {}^{1}_{1}\text{p} &\rightarrow {}^{14}_{7}\text{N} + \gamma \\
{}^{14}_{7}\text{N} + {}^{1}_{1}\text{p} &\rightarrow {}^{15}_{8}\text{O} + \gamma \\
{}^{15}_{8}\text{O} &\rightarrow {}^{15}_{7}\text{N} + {}^{0}_{+1}\text{e} + \nu \\
{}^{15}_{7}\text{N} + {}^{1}_{1}\text{p} &\rightarrow {}^{16}_{8}\text{O} + \gamma \\
{}^{16}_{8}\text{O} + {}^{1}_{1}\text{p} &\rightarrow {}^{17}_{9}\text{F} + \gamma
\end{aligned}$$
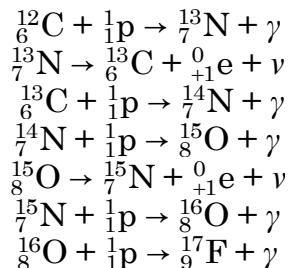
As the nuclear fusion process continues, all the chemical elements and their isotopes up to about an atomic number of 56 or so, are created within the stars. If the star is large enough it eventually explodes as a supernova, spewing its contents into interstellar space. It is believed that the high temperatures in the explosion cause the formation of the higher chemical elements above iron. Some of the dust from these clouds is gradually pulled together by gravity to form still new stars. *Hence, stars are factories for the creation of the chemical elements.* If some of these fragments of the supernova are caught up in the gravitational field of another star, they could, with the correct initial velocity, go into orbit around the new star. The captured fragments of the star would slowly condense and become a planet with a complete set of elements as are now found on the planet earth.

Have you ever wondered … ?
An Essay on the Application of Physics
Radioactive Dating

Have you ever wondered how scientists are able to determine the age of very old objects? The technique used to determine their age is called **radioactive dating** and it is based upon the amount of unstable isotopes still contained in them. Perhaps the most famous of these techniques is *carbon dating.* Cosmic rays, which are high-energy protons and neutrons from outer space, impinge on the earth's upper atmosphere, and cause nuclear reactions with the nitrogen present there. The result of these nuclear reactions is to create an unstable isotope of carbon, namely, ${}^{14}_{6}\text{C}$, which has a half-life of 5770 yr. It is assumed that the total amount of this isotope remains constant with time because of an equilibrium between the amount being formed at any time and the amount decaying at any time. This

isotope of carbon combines chemically with the oxygen, O_2, in the atmosphere to form carbon dioxide, CO_2. Most of the carbon dioxide in the atmosphere is, of course, formed from ordinary carbon, $_6^{12}C$ Because the chemical properties depend

Figure 5H.1 A fossil of a seed fern. How can you tell how old it is?

on the orbital electrons and not the nucleus, $_6^{14}C$ reacts chemically the same as $_6^{12}C$ Hence, we cannot determine chemically whether the carbon dioxide is made from carbon $_6^{12}C$ or carbon $_6^{14}C$.

The green plants in the environment convert water, H_2O, and carbon dioxide, CO_2, into carbohydrates by the process of photosynthesis. Hence, the radioactive isotope $_6^{14}C$ becomes a part of every living plant. Animals and humans eat these plants while also exhaling carbon dioxide. Thus plants, animals, and humans are found to contain the radioactive isotope $_6^{14}C$ The ratio of the carbon isotope $_6^{14}C$ to ordinary carbon $_6^{12}C$ is a constant in the atmosphere and all living things. The ratio is of course quite small, approximately 1.3×10^{-12}. That is, the amount of carbon $_6^{14}C$ is equal to 0.0000000000013 times the amount of ordinary carbon. Whenever any living thing dies, the radioactive isotope $_6^{14}C$ is no longer replenished and decreases by beta decay according to the reaction

$$_6^{14}C \rightarrow {}_7^{14}N + {}_{-1}^{0}e + \bar{\nu} \tag{5H.1}$$

Thus, the ratio of $_6^{14}C/_6^{12}C$ is no longer a constant, but starts to decay with time. Thus, by knowing the present ratio of $_6^{14}C/_6^{12}C$, the age of the particular object can be determined. In practice, the amount of $_6^{14}C$ nuclei is relatively difficult to measure, whereas its activity, the number of disintegrations per unit time, is not. Using equation 5.12 for the activity of a radioactive nucleus, we get

$$\frac{A}{A_0} = e^{-\lambda t}$$

Taking the natural logarithms of both sides of the equation, we get

$$\ln\left(\frac{A}{A_0}\right) = -\lambda t$$

Solving for the time t, we get

$$t = \frac{-\ln(A/A_0)}{\lambda}$$

(5H.2)

Thus, if the activity A_0 of a present living thing is known, and the activity A of the object we wish to date is measured, we can solve equation 5H.2 for its age.

Example 5H.1

Carbon dating. A piece of wood believed to be from an ancient Egyptian tomb is tested in the laboratory for its carbon-14 activity. It is found that the old wood has an activity of 10.0 disintegrations/min, whereas a new piece of wood has an activity of 15.0 disintegrations/min. Find the age of the wood.

Solution

First, we find the decay constant of $^{14}_{6}\text{C}$ from equation 5.11 as

$$\lambda = \frac{0.693}{T_{1/2}}$$
$$= \left(\frac{0.693}{5770 \text{ yr}}\right)\left(\frac{1 \text{ yr}}{3.1535 \times 10^7 \text{ s}}\right)$$
$$= 3.81 \times 10^{-12} \text{ /s}$$

We find the age of the wood from equation 5H.2 with $A = 10.0$ disintegrations/min and $A_0 = 15$ disintegrations/min. Thus,

$$t = \frac{-\ln(A/A_0)}{\lambda}$$
$$t = \frac{-\ln(10.0/15.0)}{3.81 \times 10^{-12}/\text{s}}$$
$$= (1.06 \times 10^{11} \text{ s})\left(\frac{1 \text{ yr}}{3.1535 \times 10^7 \text{ s}}\right)$$
$$= 3370 \text{ yr}$$

Hence, the wood must be 3370 yr old.

To go to this Interactive Example click on this sentence.

Similar dating techniques are used in geology to determine the age of rocks. As an example, the uranium atom $^{238}_{92}\text{U}$ decays through a series of steps and ends up as the stable isotope of lead, $^{206}_{82}\text{Pb}$ The ratio of the abundance of $^{238}_{92}\text{U}$ to $^{206}_{82}\text{Pb}$ can be used to determine the age of a rock.

The Language of Physics

Atomic number Z
The number of protons or electrons in an atom (p.).

Mass number A
The number of protons plus neutrons in the nucleus (p.).

Neutron number N
The number of neutrons in the nucleus. It is equal to the difference between the mass number and the atomic number (p.).

Isotope
An isotope of a chemical element has the same number of protons as the element but a different number of neutrons. An isotope reacts chemically in the same way as the parent element. Its observable difference is its different atomic mass, which comes from the excess or deficiency of neutrons in the nucleus (p.).

Atomic mass
The mass of a chemical element that is listed in the periodic table of the elements. That atomic mass is an average of the masses of its different isotopes (p.).

Strong nuclear force
The force that binds protons and neutrons together in the nucleus. Whenever the nuclear force is less than the electrostatic force, the nucleus breaks up or decays, and emits radioactive particles (p.).

Mass defect
The difference in mass between the sum of the masses of the constituents of a nucleus and the mass of the nucleus (p.).

Binding energy
The energy that binds the nucleus together. It is the mass defect expressed as an energy (p.).

Radioactivity
The spontaneous disintegration of the nuclei of an atom with the emission of α, β, or γ particles (p.).

Activity
The rate at which nuclei decay with time (p.).

Half-life
The time it takes for half the original radioactive nuclei to decay (p.).

Alpha decay
A disintegration of an atomic nucleus whereby an α particle is emitted. The original element of atomic number Z is transmuted into a new chemical element of atomic number $Z - 2$ (p.).

Beta decay, β^-
A nuclear decay whereby a neutron within the nucleus decays into a proton, an electron, and an antineutrino. The proton stays in the nucleus, but the electron and antineutrino are emitted. Thus, the atomic number Z increases by 1, but the mass number A stays the same. Hence, a chemical element Z is transmuted into the element $Z + 1$ (p.).

Beta decay, β^+
A nuclear decay whereby a proton within the nucleus decays into a neutron, a positron, and a neutrino. The positron and neutrino are emitted but the neutron stays behind in the nucleus. The atomic number Z of the element decreases by one because of the loss of the proton. Hence, an element of atomic number Z is converted into the element $Z - 1$ (p.).

Q value of a nuclear reaction
The energy available in a reaction caused by the difference in mass between the reactants and the products (p.).

Exoergic reaction
A nuclear reaction in which energy is released. It is sometimes called an exothermic reaction (p.).

Endoergic reaction
A nuclear reaction in which energy must be added to the system to make the reaction proceed. It is sometimes called an endothermic reaction (p.).

Nuclear fission
The process of splitting a heavy atom into two lighter atoms (p.).

Nuclear fusion
The process in which lighter nuclei are joined together to produce a heavier nucleus with a large amount of energy released (p.).

Nucleosynthesis
The formation of the nuclei of all the chemical elements by the process of fusion within the stars (p.).

Radioactive dating
A technique in which the age of very old objects can be determined by the amount of unstable isotopes still contained in them (p.).

Summary of Important Equations

Neutron number	$N = A - Z$	(5.1)
Representation of a nucleus	$^A_Z X$	(5.2)
Mass defect	$\Delta m = Z m_\mathbf{p} + (A - Z) m_\mathbf{n} - m_{\text{nucleus}} c^2$	(5.3)
Binding energy	$BE = (\Delta m) c^2 = Z m_p c^2 + (A - Z) m_n c^2 - m_{nucleus} c^2$	(5.5)
Rate of nuclear decay	$\dfrac{dN}{dt} = -\lambda N$	(5.6)
Activity	$A = -\dfrac{dN}{dt} = \lambda N$	(5.7)
Radioactive decay law	$N = N_0 e^{-\lambda t}$	(5.8)
Decay constant	$\lambda = \dfrac{0.693}{T_{1/2}}$	(5.11)
Alpha decay	$^A_Z X \rightarrow {}^{A-4}_{Z-2} X + {}^4_2 \text{He}$	(5.16)
Neutron decay	$^1_0 \text{n} \rightarrow {}^1_1 \text{p} + {}^0_{-1}\text{e} + \bar{\nu}$	(5.19)
Beta− decay	$^A_Z X \rightarrow {}^A_{Z+1} X + {}^0_{-1}\text{e} + \bar{\nu}$	(5.20)
Proton decay	$^1_1 \text{p} \rightarrow {}^1_0 \text{n} + {}^0_{+1}\text{e} + \nu$	(5.21)
Beta+ decay	$^A_Z X \rightarrow {}^A_{Z-1} X + {}^0_{+1}\text{e} + \nu$	(5.22)
Electron capture	$^0_{-1}\text{e} + {}^1_1 \text{p} \rightarrow {}^1_0 \text{n} + \nu$	(5.23)
Electron capture	$^0_{-1}\text{e} + {}^A_Z X \rightarrow {}^A_{Z-1} X + \nu$	(5.24)
Gamma decay	$^A_Z X^* \rightarrow {}^A_Z X + \gamma$	(5.25)

Q value of a nuclear reaction

$$Q = (m_x + M_X) c^2 - (m_y + M_Y) c^2 \tag{5.31}$$

$$Q = [(\text{Input mass}) - (\text{Output mass})] c^2 \tag{5.32}$$

$$Q = E_{in} - E_{out} \tag{5.33}$$

General form of equation for nuclear reaction $\qquad x + X = y + Y + Q \qquad$ (5.34)

Nuclear fission of $^{235}_{92}\text{U}$ $\qquad {}^1_0\text{n} + {}^{235}_{92}\text{U} \rightarrow y + Y + {}^1_0\text{n} + Q \qquad$ (5.35)

Proton-proton cycle of nuclear fusion

$$^1_1\text{p} + {}^1_1\text{p} \rightarrow {}^2_1\text{H} + {}^0_{+1}\text{e} + \nu \tag{5.40}$$

$$^2_1\text{H} + {}^1_1\text{p} \rightarrow {}^3_2\text{He} + \gamma \tag{5.41}$$

$$^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2\,{}^1_1\text{p} \tag{5.42}$$

Radioactive age	$t = \dfrac{-\ln(A/A_0)}{\lambda}$	(5H.2)

Questions for Chapter 5

1. What are isotopes? What do they have in common and what are their differences?

2. What is the difference between fast neutrons and slow neutrons, and how do they have an effect on nuclear reactions?

3. What do we mean by the term critical mass?

4. Discuss the advantages and disadvantages of nuclear power compared to the use of fossil-fuel-generated power.

*5. What is a radioactive tracer and how is it used in medicine?

6. Explain the difference between nuclear fission and nuclear fusion.

7. Should an atomic bomb really be called a nuclear bomb?

8. How is the half-life of a radioactive substance related to its activity?

*9. Was the Chernobyl Nuclear Reactor explosion in the Soviet Union a nuclear explosion? Does the fact that the reactor was a breeder reactor, rather than a commercial electricity generator, have anything to do with the severity of the disaster?

Problems for Chapter 5

Section 5.2 Nuclear Structure

1. Find the atomic number, the mass number, and the neutron number for (a) $^{58}_{29}Cu$, (b) $^{24}_{11}Na$, (c) $^{210}_{84}Po$, (d) $^{45}_{20}Ca$, and (e) $^{206}_{82}Pb$.

2. Determine the number of protons and neutrons in one atom of (a) $^{87}_{37}Rb$, (b) $^{40}_{19}K$, (c) $^{137}_{55}Cs$, (d) $^{60}_{27}Co$, and (e) $^{131}_{53}I$.

3. Find the number of protons in 1 g of $^{40}_{19}K$.

4. $^{63}_{29}Cu$ has an atomic mass of 62.929595 u and an abundance of 69.09%, whereas $^{65}_{29}Cu$ has an atomic mass of 64.927786 u and an abundance of 30.91%. Find the atomic mass of the element copper.

5. $^{107}_{47}Ag$ has an atomic mass of 106.905095 u and an abundance of 51.83%, whereas $^{109}_{47}Ag$ has an atomic mass of 108.904754 u and an abundance of 48.17%. Find the atomic mass of the element silver.

6. Find the mass defect and the binding energy for the helium nucleus if the atomic mass of the helium nucleus is 4.0026 u.

7. Find the mass defect and the binding energy for tritium if the atomic mass of tritium is 3.016049 u.

8. How much energy would be released if six hydrogen atoms and six neutrons were combined to form $^{12}_{6}C$?

Section 5.3 Radioactive Decay Law

9. $^{63}_{28}Ni$ has a half-life of 92 yr. Find its decay constant.

10. $^{235}_{92}$U has a half-life of 7.038×10^8 yr. Find its decay constant.

11. An unknown sample has a decay constant of 2.83×10^{-6} 1/s. Find the half-life of the sample.

12. The decay constant of $^{14}_{6}$C is $\lambda = 3.86 \times 10^{-12}$ s^{-1}. If there are 7.35×10^{90} atoms of carbon fourteen at $t = 0$, how many of them will decay in a time of $t = 2.00 \times 10^{12}$ s?

13. A sample contains 0.200 moles of $^{65}_{30}$Zn If $^{65}_{30}$Zn has a decay constant of 3.27×10^{-8} /s, find the number of $^{65}_{30}$Zn nuclei present at the end of 1 day.

14. One gram of $^{87}_{36}$Kr has a half-life of 78.0 min. How many of these nuclei are still present at the end of 15.0 min?

15. $^{60}_{27}$Co has a half-life of 5.27 yr. How long will it take for 90.0% of the original sample to disintegrate?

16. $^{90}_{38}$Sr has a half-life of 28.8 yr. How long will it take for it to decay to 10.0% of its original value?

17. A dose of 1.85×10^6 Bq of radioactive iodine, $^{131}_{53}$I, is used in the treatment of a disorder of the thyroid gland. If its half-life is 8 days, find the activity after (a) 8 days, (b) 16 days, and (c) 32 days.

18. In a given sample of radioactive material, the number of original nuclei drops from 6.00×10^{50} to 1.50×10^{50} in 4.50 s. Find (a) the half-life and (b) the mean lifetime (τ_{avg}) of the material.

Section 33.4 Forms of Radioactivity

19. $^{220}_{86}$Rn decays by alpha emission. What isotope is formed?

20. $^{230}_{90}$Th decays by alpha emission. What isotope is formed?

21. If ^{233}U decays twice by alpha emission, what is the resulting isotope?

22. $^{214}_{84}$Po decays by β^- decay. What isotope is formed?

23. $^{210}_{82}$Pb decays by β^- decay. What isotope is formed?

24. $^{33}_{17}$Cl decays by β^+ decay. What isotope is formed?

25. $^{49}_{24}$Cr decays by β^+ decay. What isotope is formed?

26. $^{41}_{20}$Ca decays by electron capture. What isotope is formed?
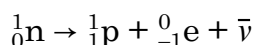
27. $^{52}_{25}$Mn decays by electron capture. What isotope is formed?

Section 5.6 Energy in Nuclear Reactions

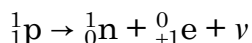28. How much energy is released or absorbed in the following reaction?

$$^{216}_{84}\text{Po} \rightarrow {}^{212}_{82}\text{Pb} + {}^{4}_{2}\text{He}$$
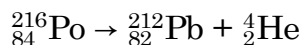
The atomic mass of $^{216}_{84}$Po is 216.0019 u, $^{4}_{2}$He is 4.002603 u, and $^{212}_{82}$Pb is 211.9919 u.

29. Determine the energy associated with the reactions

$$^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} + \bar{\nu}$$

$$_1^1\text{p} \rightarrow {}_0^1\text{n} + {}_{+1}^0\text{e} + v$$

30. Find the Q value associated with the reaction

$$_1^1\text{H} + {}_7^{14}\text{N} \rightarrow {}_8^{15}\text{O} + v$$
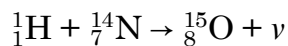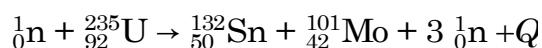
The atomic mass of $_7^{14}\text{N}$ is 14.003074 and $_8^{15}\text{O}$ is 15.003072 u.
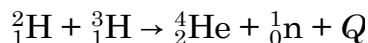   31. Find the Q value associated with the reaction

$$_6^{14}\text{C} \rightarrow {}_7^{14}\text{N} + {}_{-1}^0\text{e} + \bar{v} + Q$$

32. Find the Q value associated with the nuclear fission reaction

$$_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{50}^{132}\text{Sn} + {}_{42}^{101}\text{Mo} + 3\,{}_0^1\text{n} + Q$$
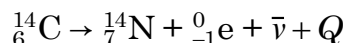
The atomic mass of $_{92}^{235}\text{U}$ is 235.043933 u, $_{50}^{132}\text{Sn}$ is 49.917756 u, and $_{42}^{101}\text{Mo}$ is 41.910346 u.
    33. Find the Q value of the fusion reaction

$$_1^2\text{H} + {}_1^3\text{H} \rightarrow {}_2^4\text{He} + {}_0^1\text{n} + Q$$

Additional Problems

    *34. A 5.00-g sample of $_{27}^{60}\text{Co}$ has a half-life of 5.27 yr. Find (a) the decay constant, (b) the activity of the material when $t = 0$, (c) the activity when $t = 1.00$ yr, and (d) the number of nuclei present after 1.00 yr.
    *35. A 5.00-g sample of $_{90}^{230}\text{Th}$ has a half-life of 80.0 yr, and a 5.00-g sample of $_{86}^{222}\text{Rn}$ has a half-life of 3.82 days. For each sample find (a) the decay constant, (b) the activity of the material when $t = 0$, (c) the activity when $t = 100$ days, (d) the number of nuclei present after 100 days. (e) Comparing the activities and the number of radioactive nuclei remaining at 100 days for the two samples, what can you conclude?
    36. If ^{231}Pa decays first by beta decay, and then by alpha emission, what is the resulting isotope?
    37. A bone from an animal is found in a very old cave. It is tested in the laboratory and it is found that it has a carbon-14 activity of 13.0 disintegrations per minute. A similar bone from a new animal is tested and found to have an activity of 25.0 disintegrations per minute. What is the age of the bone?
    38. A wooden statue is observed to have a carbon fourteen activity of 7.0 disintegrations per minute. How old is the statue? (New wood was found to have an activity of 15.0 disintegrations/min.)

Interactive Tutorials

    39. *Radioactive decay.* A mass of 8.55 g of the isotope $_{38}^{90}\text{Sr}$ has a half-life $T_{1/2} = 28.8$ yr. Find (a) the decay constant λ, (b) the number of nuclei N_0 present at the

start, (c) the activity A_0 at the start, (d) the number of nuclei N present for $t = T_{1/2}$, (e) the rate of decay of the nuclei at $t = T_{1/2}$, (f) the number of nuclei present for any time t, and (g) the activity at any time t.

To go to this Interactive Tutorial click on this sentence.

To go to another chapter, return to the table of contents by clicking on this sentence.