Solutions to Problems : Chapter 28
Problems appeared on the end of chapter 28 of the Textbook

(Problem 3, 7, 12, 14, 40, 46, 48, 50)

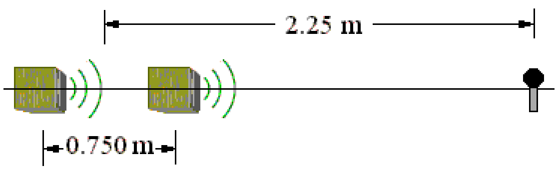
3. **Picture the Problem**: Two coherent waves interfere at an observation point 161 meters from one source and 295 meters from the other source.

   Strategy: The longest wavelength that will give constructive interference has a length equal to the path length difference.

   Solution: Calculate the path length difference: $m\lambda = \Delta\ell = 295\text{ m} - 161\text{ m} = \boxed{134\text{ m}}$

   Insight: The longest wavelength that will give destructive interference is 134 m.

7. **Picture the Problem**: The figure shows two speakers located 0.750 meters apart and a microphone located 2.25 m from the midpoint of the speakers along the line connecting the speakers. The speakers produce the same frequency sound in phase.



   Strategy: For a maximum signal to be picked up by the microphone, the difference in distance between the two speakers must be an integer number of wavelengths. Set the distance equal to one wavelength for the lowest frequency and two wavelengths for the second lowest frequency. Then divide the speed of sound by the wavelengths to calculate the frequencies.

   Solution: 1. Calculate the lowest wavelength: $\lambda = d = 0.750\text{ m}$

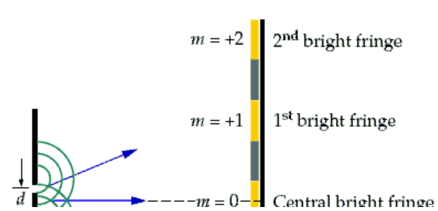
   2. Divide the speed by the frequency: $f = \dfrac{v}{\lambda} = \dfrac{343\text{ m/s}}{0.750\text{ m}} = \boxed{457\text{ Hz}}$

   3. Calculate the next lowest wavelength: $2\lambda = d = 0.750\text{ m} \Rightarrow \lambda = \tfrac{1}{2}(0.750\text{ m}) = 0.375\text{ m}$

   4. Divide the speed by the frequency: $f = \dfrac{v}{\lambda} = \dfrac{343\text{ m/s}}{0.375\text{ m}} = \boxed{915\text{ Hz}}$

   Insight: The distance to the microphone does not change the condition of constructive or destructive interference because it is parallel to the line that separates the speakers.

12. **Picture the Problem**: The figure shows a double slit that produces the first bright fringes at angles of ±35° when laser light of wavelength 670 nm illuminates the slits.



   Strategy: Solve equation 28-1 for the slit separation with $m = 1$ for the first bright fringe.

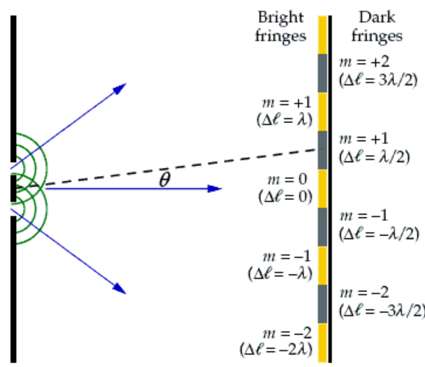
Solution: Find d for $m = 1$:

$$\sin\theta = m\frac{\lambda}{d}$$

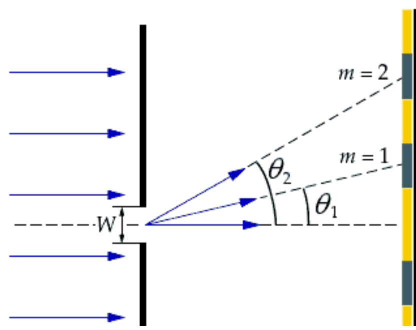
$$d = \frac{m\lambda}{\sin\theta} = \frac{1(670\text{ nm})}{\sin 35^\circ} = \boxed{1.2\ \mu\text{m}}$$

Insight: The same slit spacing would have been found using the $m = -1$ fringe at $\theta = -35°$.

14. **Picture the Problem**: The first dark fringe in Young's two-slit experiment occurs at $\theta = 0.29°$ as shown in the figure.



40. **Picture the Problem**: Light that has a wavelength of 626 nm passes through a 7.64-μm slit to a screen that is 1.85 m away. The center of the first bright fringe is a distance y above the central maximum.



Strategy: The first bright fringe above the central maximum is halfway between the first and second minima. Solve equation 28-12 for the angle $\theta_{1.5}$ that corresponds to $m = 1.5$. Then use equation 28-3 to calculate the linear distance y on the screen from the central maximum to the first bright fringe.

Solution: 1. Calculate the angle to the first bright fringe:

$$\theta_{1.5} = \sin^{-1}\frac{1.5\lambda}{W} = \sin^{-1}\frac{1.5\left(626\times10^{-9}\text{ m}\right)}{7.64\times10^{-6}\text{ m}} = 7.06°$$

2. Calculate the distance y:

$$y = L\tan\theta_{1.5} = (1.85\text{ m})\tan(7.06°) = \boxed{22.9\text{ cm}}$$

Insight: The distance to the first maximum can also be calculated by taking the average of the distances to the first and second minima.

46. Picture the Problem: Two point sources are 5.0 cm apart and are viewed through a 12-μm diameter pin hole.

Strategy: The red light has a longer wavelength and will diffract more as it passes through the pinhole. We therefore expect that the maximum distance at which the two point sources can be resolved will be smaller for the red light than it is for the blue light, because the minimum angular separation θ_{min} required to resolve the point sources with red light will be larger than it is for blue light. Use the Rayleigh criterion for resolving two objects (equation 28-15) together with equation 28-3 and the small angle approximation to determine the maximum viewing distance for each wavelength.

Solution: 1. (a) Use equation 28-15 to find θ_{min}:

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

2. Calculate L using equation 28-3:

$$y = L\tan\theta_{\min} \approx L\,\theta_{\min}$$

$$L \approx \frac{y}{\theta_{\min}} = \frac{y\,D}{1.22\,\lambda} = \frac{(5.0\text{ cm})(12\ \mu\text{m})}{1.22(0.69\ \mu\text{m})} = \boxed{71\text{ cm}}$$

3. (b) Repeat step 2 for $\lambda = 420$ nm:

$$L \approx \frac{y}{\theta_{\min}} = \frac{y\,D}{1.22\,\lambda} = \frac{(5.0\text{ cm})(12\ \mu\text{m})}{1.22(0.42\ \mu\text{m})} = \boxed{1.2\text{ m}}$$

Insight: Violet light, with the shorter wavelength, allows the observer to resolve objects that are more distant.

48. **Picture the Problem**: A telescope collects 550-nm light from two stars that have an angular separation of 2.5 arc seconds.

Strategy: Use the Rayleigh criterion (equation 28-15) to solve for the minimum aperture of the telescope. Note that
1 arc sec is 1/3600$^{\text{th}}$ of a degree.

Solution: Solve equation 28-15 for D:

$$\sin\theta_{\min} = 1.22\frac{\lambda}{D} \quad\Rightarrow\quad D = \frac{1.22\lambda}{\sin\theta_{\min}} = \frac{1.22\left(550\times10^{-9}\text{ m}\right)}{\sin\left(2.5/3600\right)^\circ} = \boxed{5.5\text{ cm}}$$

Insight: These stars could theoretically be resolved with a telescope that has an aperture greater than 2 inches.

50. **Picture the Problem**: The Hubble telescope collects light with a wavelength of 550 nm through a 2.4-m aperture and orbits at an altitude of 613 km.

Strategy: Use the Rayleigh criterion (equation 28-15) together with equation 28-3 to solve for the minimum separation distance of two objects that can be resolved by the Hubble Space Telescope camera. The small angle approximation is appropriate in this situation.

Solution: Combine equations 28-15 and 28-3 together with the small angle approximation to find y:

$$y = L\tan\theta_{\min} \approx L\theta_{\min} = L\left(1.22\frac{\lambda}{D}\right)$$

$$= \left(613\times10^{3}\text{ m}\right)\left(1.22\frac{550\times10^{-9}\text{ m}}{2.4\text{ m}}\right) = \boxed{17\text{ cm}}$$

Insight: Contrary to popular legend, this resolution does not allow the satellite to resolve the writing on a newspaper from space.