CVEL

ELECTROMAGNETIC MODELING

THE CLEMSON UNIVERSITY VEHICULAR ELECTRONICS LABORATORY

The Finite Difference Time Domain Method

The Finite Difference Time Domain (FDTD) method, as first proposed by Yee [1], is a direct solution of Maxwell's time dependent curl equations. It uses simple central-difference approximations to evaluate the space and time derivatives. A basic element of the FDTD space lattice is illustrated in Figure 2. An electric-field grid is offset from a magnetic-field grid in both space and time. A first-order central-difference approximation can be expressed as,

$$\frac{1}{\Delta l}\left[E_{z1}(t)+E_{y2}(t)-E_{z3}(t)-E_{y4}(t)\right]=-\frac{\mu_0}{2\Delta t}\left[H_{x0}(t+\Delta t)-H_{x0}(t-\Delta t)\right] \qquad (1)$$

where Δl is the length of one side of the cubical cell in Figure 2. $Hxo(t+\Delta t)$ is the only unknown in this equation, since all other quantities were found in a previous time step. In this way, the electric field values at time t are used to find the magnetic field values at time $t+\Delta t$. A similar central-difference approximation of Equation (1) can then be applied to find the electric field values at time $t+2\Delta t$ from the magnetic field values at time $t+\Delta t$. By alternately calculating the electric and magnetic fields at each time step, fields are propagated throughout the grid.

Time stepping is continued until a steady state solution or the desired response is obtained. The required computer storage and running time is proportional to the electrical size of the volume being modeled and the grid resolution.

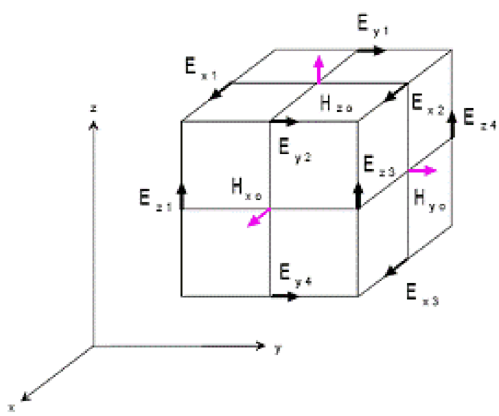

Figure 2: Basic Element of the FDTD Space Lattice.

For an open region problem, an absorbing boundary condition (ABC) is used to truncate the computational domain. One technique, which is obtained by factoring the wave equation to permit only outgoing waves, is differential based ABCs, such as those proposed by Engquist [2], Lindman [3], Mur [4], Liao [5], Keys [6], and Higdon [7]. Another is material based ABCs that are constructed so that the fields are dampened as they propagate into an absorbing medium. Rappaport [8] proposed an ABC employing pyramid-shaped absorber material. In 1994, Berenger [9] introduced the perfectly matched layer (PML) absorbing boundary condition. This ABC outperforms any that had been proposed previously and is widely used today. Andrew [10] compared the accuracy of the Berenger perfectly matched layer and the Lindman higher-order ABCs for the FDTD method. Accuracy studies of ABCs have also been conducted for dispersive media [11, 12]. In 2003, Diaz [13] introduced a new radiation boundary condition for FDTD based on self-teleportation of fields.

Because the basic elements are cubes, curved surfaces on a scatterer must be staircased. For many configurations this does not present a problem. However for configurations with sharp, acute edges, this approximation may lead to significant errors [14, 15], and an adequately staircased approximation may require a very small grid size. Surface-conforming FDTD techniques with non-rectangular elements have been introduced to combat this problem [16-23].

Since all of the elements in an FDTD analysis must generally be the same size, the size of the elements is determined by the smallest structural details that need to be modeled. If an object under consideration contains small-scale geometries, such as a narrow slot or a very thin wire, an excessively fine grid would have to be used to accurately model the associated fields. To overcome these shortcomings, sub-cellular structures [24-33] have been introduced. Sub-cellular structures are essentially special FDTD cells whose boundary conditions have been altered to model small structures contained within the cells.

One major advantage of the FDTD method is the ability to obtain wideband results using a transient excitation in one simulation. Frequency domain results can be obtained by applying a discrete Fourier transform to the time domain results. Since many materials have frequency dependent properties, it is necessary to take special precautions to model these materials correctly with a time-domain technique. In 1990, Luebbers [34] used a recursive convolution scheme to model a Debye media; this was the first frequency dependent FDTD formation. Kashiwa and co-workers [35-37] published the first papers utilizing the auxiliary differential equation (ADE) method to model Debye media, Lorentz media, and media obeying the Cole-Cole Circular Arc law. In 1992, Sullivan [38] proposed a dispersive formulation based on Z transforms. Petropoulos [39] provided a comparison of the stability and phase error among frequency dispersive FDTD methods.

Lossy surfaces can be modeled in FDTD codes by utilizing a surface impedance boundary condition (SIBC) [40-42]. A thin material sheet model has also been developed for the FDTD method [43-45].

A primary advantage of FDTD methods is their great flexibility. Arbitrary signal waveforms can be modeled as they propagate through complex configurations of conductors, dielectrics, and lossy non-linear non-isotropic materials. Another advantage of FDTD techniques is that they are readily implemented on massively parallel computers, particularly vector processors and SIMD (single-instruction-multiple-data) machines.

Time stepping techniques like FDTD are subject to dispersion errors when the time step is too large for the given problem size. Many researchers have studied the numerical dispersion error inherent in the FDTD method [46-54], but it is easily controlled by using appropriately small time steps.

Table 6 lists various strengths and weakness of FDTD modeling techniques. Table 7 lists various CEM modeling codes that employ an FDTD solver.

Table 6: Strengths and Weaknesses of the Finite Difference Time Domain Method

FDTD Modeling Strengths	***FDTD Modeling Weaknesses***
- **Excels at modeling inhomogeneous or complex materials** - **Excels at modeling very large problems** - **Runs efficiently on highly parallel computers**	- Absorbing boundary required for modeling unbounded problems, but PML boundaries work very well. - Difficult to model thin wires - Uniform cells must be small enough to model necessary detail, but still fill the entire volume. - High Q structures are not modeled efficiently

Table 7: CEM Modeling Codes that use the Finite Difference Time Domain Method

Software Title	***Description***	***Source***
ApsimFDTD	3D full-wave	Applied Simulation Tech.
CST M. Studio - TS	3D full-wave	CST
EMA3D	3D full-wave	Electromagnetic Appl.
EMPIRE XCcel	3D full-wave	Empire
EZ-FDTD	3D full-wave	EMS-Plus
Fidelity	3D full-wave	Zeland Software
GEMS	3D full-wave	2COMU
LC	3D full-wave	Cray Research
PAM-CEM	3D full-wave	ESI Group
SEMCAD X	3D full-wave	Schmid & Partner Eng.
Toy	3D full-wave	The CEMTACH Group
XFDTD	3D full-wave	Remcom

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