Appendix B
Review of Derivatives

As a quick review, let us recall some of the definitions and characteristics of the derivative that you learned in your calculus course. If you were given a function of a single variable x,

$$y = f(x) \tag{B-1}$$

then its ordinary derivative was defined to be

$$y' = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \tag{B-2}$$

$$y' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \tag{B-3}$$

Example B.1

If

$$y = x^2$$

find its derivative.

Solution

The derivative is found from equation B-3 as

$$y' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \tag{B-3}$$

$$y' = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$y' = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$y' = \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$y' = \lim_{\Delta x \to o} 2x + \Delta x = 2x$$

As you recall it was not necessary to go through such long calculations every time you needed a derivative because you eventually memorized the general case of the derivative of a quantity to a power. That is, if the function was of the form

$$y = Ax^n \tag{B-4}$$

Its derivative was

$$y' = nAx^{n-1} \tag{B-5}$$

If the function were one of several variables, such as

$$u = f(x,y,z) \tag{B-6}$$

then the partial derivatives were defined in the same way as in equation B-2, i.e.,

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} \tag{B-7}$$

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y, z) - f(x,y,z)}{\Delta x} \tag{B-8}$$

The term $\partial u/\partial x$ is called the partial derivative of u with respect to x. Similarly,

$$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y, z) - f(x,y,z)}{\Delta y} \tag{B-9}$$

$$\frac{\partial u}{\partial z} = \lim_{\Delta z \to 0} \frac{f(x, y, z+\Delta z) - f(x,y,z)}{\Delta z} \tag{B-10}$$

The total differential of the function, u, was then given by

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \tag{B-11}$$

Let us examine equation B-11 in more detail to see what it means physically. Let us assume that u is a function, like the electric potential function, that varies as you move in the x, y, and z directions. The first partial derivative term, $\partial u/\partial x$, represents the rate at which the function u changes as you move in the x-direction, and dx represents the total distance you move in the x-direction. When you multiply $(\partial u/\partial x)$ by dx, you get the total change in the function u as you move in the x-direction. The second partial derivative term, $\partial u/\partial y$ represents the rate at which the function u changes as you move in the y-direction, and dy represents the total distance you move in the y-direction. When you multiply $\partial u/\partial y$ by dy, you get the total change in the function, u, as you move in the y-direction. Finally, the third partial derivative term, $\partial u/\partial z$, represents the rate at which the function changes as you move in the z-direction, and dz represents the total distance you move in the z-direction. When you multiply $\partial u/\partial z$ by dz, you get the total change in the function, u, as you move in the z-direction. The sum of the three terms represents the total change in the function u as you move through space.

Example B.2

Given the function

$$u = x^2 + 3xy + xz^3$$

Find the partial derivatives and the total differential.

Solution

The partial derivatives are found as

$$\frac{\partial u}{\partial x} = 2x + 3y + z^3$$

$$\frac{\partial u}{\partial y} = 3x$$

$$\frac{\partial u}{\partial z} = 3xz^2$$

The total differential is found from equation B-11 as

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \tag{B-11}$$

$$du = (2x + 3y + z^3)dx + 3x\,dy + 3xz^2\,dz$$

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