Time Domain Modeling of Electromagnetic Field Coupling to Transmission lines

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Abs'. act: Transient behaviour of a finite length horizontal wire above a perfectly conducting ground, illuminated by the electromagnetic pulse, is analyzed in the time domain. Mathematical model is based on the thin wire approximation and the corresponding time domain Hallen integral equation. Integral equation is solved by the space-time finite element procedure providing accurate and stable results.

The numerical results for the current distribution along the unloaded and loaded wires are presented in the paper.

INTRODUCTION

The direct time domain analysis of electromagnetic field coupling to the transmmision lines can be performed using a thin wire scatterer or transmission line model [1]. The first approach, based on scattering theory, is a more rigorous one. The basic restrictions are concerned with the problem of the long computational time required for the calculations pertaining to the long lines. The transmission line (TL) approximation is usually considered as a compromise between a quasi-static approximation and the scatterer model. It is a satisfactory approximation for very long lines. However, the effects at the line ends cannot be taken into account utilizing this approach. So, if one deals with the lines of the finite length the scattering theory has to be used.

Since the scope of this paper is related to the modeling of finite length wires, the scattering theory is applied. Up to now, the time domain Pocklington integral equation was used by several authors, [2], [3], [4], for direct time domain modeling of radiation and scattering from thin wire structures. However, the usage of the moment method (combined with several time marching schemes) for handling the time domain Pocklington equation has some serious drawbacks due to the stability problems of numerical results [5]. This problem can be avoided by using the Hallen integral equation as in [6] and [7], instead of commonly used Pocklington one. The Hallen integral equation approach developed for the time domain analysis of the thin-wire radiation and scattering in free space was recently extended by the authors to the problem of horizontal unloaded wire over a perfect ground [8]. This paper offers a formulation for the resistively loaded wire over a perfect ground. The results for current distribution are obtained by an efficient space-time numerical procedure developed in [9] and promoted in [7].

FORMULATION OF THE PROBLEM

A straight overhead line of finite length L and radius a, located horizontally over a perfectly conducting ground is shown in Fig. 1.

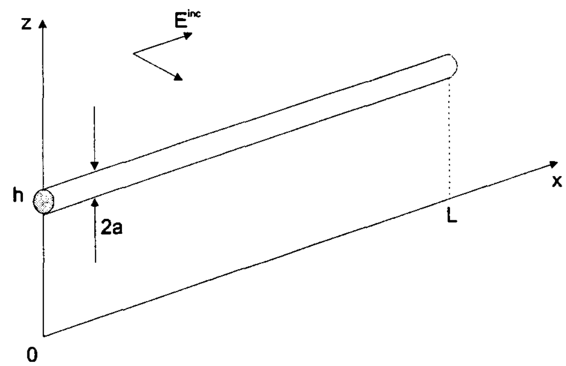

Figure 1. Geometry of the overhead line

The wire can be unloaded or it can contain distributed or concentrated resistive load. It is excited by the electromagnetic pulse (EMP).

It is well known that the relation for the tangential component of the electric field along the loaded wire can be written in the form:

$$E_x^{inc} + E_x^{sct} = IR_L \tag{1}$$

where E_x^{inc} is the tangential incident field, E_x^{sct} is the scattered field, $I(x')$ is the equivalent axial current distribution along the antenna and R_L is the resistive load per unit length of the wire.

Starting from Maxwell equation and following the condition (1) the integral equation is obtained for the configuration of Fig. 1 [9]:

$$-\varepsilon \frac{\partial E_x^{inc}}{\partial t} = \left[\frac{\partial^2}{\partial x^2} - \frac{1}{c}\frac{\partial^2}{\partial t^2}\right] \left\{\int_0^L \frac{I(x', t - R/c)}{4\pi R} dx' - \int_0^L \frac{I(x', t - R^*/c)}{4\pi R^*} dx'\right\} - R_L(x) I(x,t) \tag{2}$$

where c is the velocity of light, $R=[(x-x')^2 + a^2]^{1/2}$ is the distance from the source point (the equivalent current in the wire axis) to the observation point (on the wire surface), and $R^*=[(x-x')^2+4h^2]^{1/2}$ is the distance from the source point (the equivalent current in the wire axis) to the observation point (on the image wire surface).

Applying the superposition principle and integrating the equation (2) the corresponding Hallen integral equation can be writen in the form:

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$$
\int_0^L \frac{I(x',t-R/c)}{4\pi R}dx' - \int_0^L \frac{I(x',t-R^*/c)}{4\pi R^*}dx' = F_0(t-\frac{x}{c}) + F_L(t-\frac{L-x}{c}) + \frac{1}{2Z_0}\int_0^L E_x^{inc}(x',t-\frac{|x-x'|}{c})dx' - \int_0^L R_L(x')I(x',t-\frac{|x-x'|}{c})dx' \tag{3}
$$

where Z_0 is the wave impedance of a free space, $F_0(t)$ and $F_L(t)$ are the unknown functions representing multiple reflections of the current at the line ends. These functions can be expressed by means of the unknown current distribution invoking the Hallen equation for x=0 and x=L, as documented in [6] and [7]:

$$
F_0(t) = \sum_{n=0}^{\infty} K_0\left(t-\frac{2nL}{c}\right) - \sum_{n=0}^{\infty} K_L\left(t-\frac{(2n+1)L}{c}\right) \tag{4}
$$

$$
F_L(t) = \sum_{n=0}^{\infty} K_L\left(t-\frac{2nL}{c}\right) - \sum_{n=0}^{\infty} K_0\left(t-\frac{(2n+1)L}{c}\right) \tag{5}
$$

The auxilliary functions K_0 and K_L are then defined as follows:

$$
K_0(t) = \int_0^L \frac{I(x',t-R_0/c)}{4\pi R_0}dx' - \int_0^L \frac{I(x',t-R_0^*/c)}{4\pi R_0^*}dx' - \frac{1}{2Z_0}\int_0^L E_x^{inc}(x',t-\frac{x'}{c})dx' + \int_0^L R_L(x')I(x',t-\frac{x'}{c})dx' \tag{6}
$$

$$
K_L(t) = \int_0^L \frac{I(x',t-R_L/c)}{4\pi R_L}dx' - \int_0^L \frac{I(x',t-R_L^*/c)}{4\pi R_L^*}dx' - \frac{1}{2Z_0}\int_0^L E_x^{inc}(x',t-\frac{L-x'}{c})dx' + \int_0^L R_L(x')I(x',t-\frac{L-x'}{c})dx' \tag{7}
$$

Numerical modeling of the Hallen equation is performed by an efficient space-time finite element/marching-on-in-time procedure developed for solving the time domain integral equations, [7].

NUMERICAL MODELING

The numerical procedure developed for the Hallen equation pertaining to the unloaded antennas [8] adjusted for the loaded antenna case. If the finite element mesh (space discretization) is sufficiently dense, the space and time dicretization (which are essentially coupled) can be performed separately. In this work, the Galerkin-Bubnov variant of the finite element integral equation method (FEIEM) combined with marching-on-in-time algorithm is used for solving equation (3). The local approximation for current on a finite element, according to the usual space discretization procedure, is given in the form:

$$
I(x't') = \{f\}^T\{I\} \tag{8}
$$

where {f} is a vector containing shape functions, and {I} is the time-dependent solution vector. In the next step, using the weighted residual approach and the finite element procedure, one obtains the local system of equations for the i-th source and the j-th observation finite element:

$$
\begin{aligned}
&\int_{\Delta l_j}\int_{\Delta l_i} \{f\}_j\{f\}_i^T \frac{1}{4\pi R}dx'dx\{I\}\Big|_{t-\frac{R}{c}} \\
&- \int_{\Delta l_j}\int_{\Delta l_i} \{f\}_j\{f\}_i^T \frac{1}{4\pi R^*}dx'dx = \{I\}\Big|_{t-\frac{R^*}{c}} \\
&= \int_{\Delta l_j} F_0(t-\frac{x}{c})\{f\}_j dx + \int_{\Delta l_j} F_L(t-\frac{L-x}{c})\{f\}_j dx + \\
&+ \frac{1}{2Z_0}\int_{\Delta l_j}\int_{\Delta l_i} E_x^{inc}(x',t-\frac{|x-x'|}{c})\{f\}_j dx'dx \\
&- \frac{1}{2Z_0}\int_{\Delta l_j}\int_{\Delta l_i} R_L(x')\{f\}_j\{f\}_i^T dx'dx\{I\}\Big|_{t-\frac{|x-x'|}{c}}
\end{aligned} \tag{9}
$$

which is, for convenience, written in the matrix form:

$$
\begin{aligned}
&[A]\{I\}\Big|_{t-\frac{R}{c}} - [A_i^*]\{I\}\Big|_{t-\frac{R^*}{c}} = [B]\{E\}\Big|_{t-\frac{|x-x'|}{c}} - [R]\{I\}\Big|_{t-\frac{|x-x'|}{c}} \\
&+ [C]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2nL}{c}-\frac{R_0}{c}} - [C_i^*]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2nL}{c}-\frac{R_0^*}{c}} \\
&- [B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2nL}{c}-\frac{x'}{c}} + [R]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2nL}{c}-\frac{x'}{c}} \\
&- [D]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2n+1}{c}L-\frac{R_L}{c}} + [D_i^*]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2n+1}{c}L-\frac{R_L^*}{c}} \\
&+ [B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2n+1}{c}L-\frac{L-x'}{c}} - [R]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{x}{c}-\frac{2n+1}{c}L-\frac{L-x'}{c}} \\
&+ [D]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2nL}{c}-\frac{R_L}{c}} - [D_i^*]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2nL}{c}-\frac{R_L^*}{c}} \\
&- [B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2nL}{c}-\frac{L-x'}{c}} + [R]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2nL}{c}-\frac{L-x'}{c}} \\
&- [C]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2n+1}{c}L-\frac{R_0}{c}} + [C_i^*]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2n+1}{c}L-\frac{R_0^*}{c}} \\
&+ [B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2n+1}{c}L-\frac{x'}{c}} - [R]\left\{\sum_{n=0}^{\infty} I^n\right\}\Bigg|_{t-\frac{L-x}{c}-\frac{2n+1}{c}L-\frac{x'}{c}}
\end{aligned} \tag{10}
$$

where:

$$
[A] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{1}{4\pi R}\{f\}_j\{f\}_i^T dx'dx, \qquad [A_i^*] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{1}{4\pi R^*}\{f\}_j\{f\}_i^T dx'dx \tag{11}
$$

$$
[B] = \frac{1}{2Z_0}\int_{\Delta l_j}\int_{\Delta l_i} \{f\}_j\{f\}_i^T dx'dx \tag{12}
$$

$$
[R] = \frac{1}{2Z_0}\int_{\Delta l_j}\int_{\Delta l_i} R_L(x')\{f\}_j\{f\}_i^T dx'dx \tag{13}
$$

$$[C]=\int_{\Delta l_i}\int_{\Delta l_i}\frac{1}{4\pi R_0}\{f\}_j\{f\}_i^T dx'dx,$$
$$[C_i^*]=\int_{\Delta l_i}\int_{\Delta l_i}\frac{1}{4\pi R_0^*}\{f\}_j\{f\}_i^T dx'dx \tag{14}$$

$$[D]=\int_{\Delta l_i}\int_{\Delta l_i}\frac{1}{4\pi R_L}\{f\}_j\{f\}_i^T dx'dx$$
$$[D_i^*]=\int_{\Delta l_i}\int_{\Delta l_i}\frac{1}{4\pi R_L^*}\{f\}_j\{f\}_i^T dx'dx \tag{15}$$

R_0 and R_L are distances from the source elements to the wire ends, while R_0^* and R_L^* are corresponding distances related to the image wire. Vector {E} denotes the excitation function.

When the space discretization procedure is performed, the weighted residual approach is also used for the time discretisation procedure. The solution in time on the i-th finite element is given by:

$$I_i(t')=\sum_{k=1}^{N_t} I_i^k T^k(t') \tag{16}$$

where I_i^k are the unknown coefficients, T^k are the time domain shape functions and N_t is the total number of time samples. Taking the Dirac impulses as test functions, the recurrence formula for the space-time dependent current can be written as follows:

$$I_{j|_{t_n}}=\frac{-\sum_{i=1}^{N_e}\left(\bar{A}_{ji}I_i\big|_{t_n-\frac{R}{c}}-\bar{A}^*_{ji_i}I_i\big|_{t_n-\frac{R^*}{c}}\right)+g^*_{j_i}\Big|_{\text{all retarded times}}}{A_{jj}+A^*_{jj_i}} \tag{17}$$

where A_{ji} are the global matrix terms, g_{j1}^* is the whole right-hand side of the expression (21) containing the excitation and the currents at previous instants, and the overbar line denotes that the self term is omitted.

It is worth noting that the time domain procedure discussed so far is stable for an arbitrary time interval and does not require the implementation of certain smoothing procedures to average the solution in the later time instants, contrary to most of the known techniques [5].

NUMERICAL RESULTS

Fig. 2 shows the results of the induced current at the center of the unloaded line of length L=130m and radius a=0.1m. The line is assumed to be insulated in free space and illuminated by a Gaussian pulse given in the form: $E(t)=\exp(g^2(t-t_0)^2)$ with $g=9.1*10^7 s^{-1}$ and t_0=0.3334ns. The results obtained in this work by FEIEM seem to be in a good agreement with the results available in [4].

Fig. 3 shows the transient response at the center of the wire with L=10m and $a=6.738*10^{-2}$m. The wire is loaded at its center with concentrated resistive loading of 1Ω. It is insulated in free space and excited by EMP of the type $E(t)=E_0(\exp(-at)-\exp(-bt))$. The results calculated by FEIEM are compared with the results published in [10]. Although, the results are obtained by different techniques a good agreement is evident.

Finally, Fig. 4 shows the induced current along the unloaded line with L=20m and $a=7.15\ 10^{-2}$ m, horizontally located over a perfectly conducting ground at height h=5m, for several time instants. The line is excited with the same EMP as in the previous case.

It is worth mentioning that in all calculations the time increment was carefully chosen to satisfy the inequality $\Delta t \le \Delta x/c$, since this condition actually ensures that the distance of the space nodes should at least correspond to the distance needed for the electromagnetic field to propagate between two sample points in time.

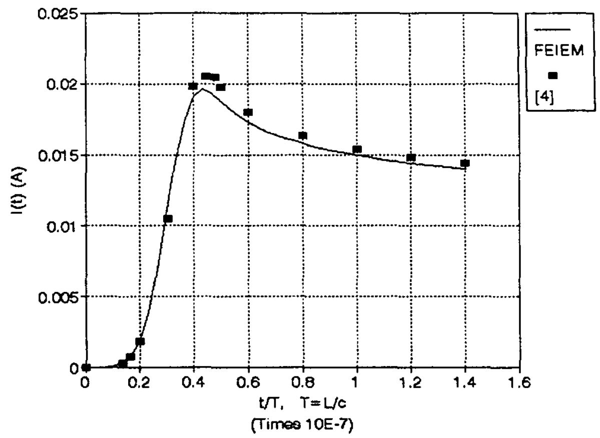

Figure 2. Comparison of results for the current at the center of unloaded line; L=130m, a=0.1m, insulated in free space and excited by Gaussian pulse $E(t)=\exp(-g^2(t-t_0)^2)$, $g=9.1*10^7 s^{-1}$, $t_0=3.334*10^{-8}$s

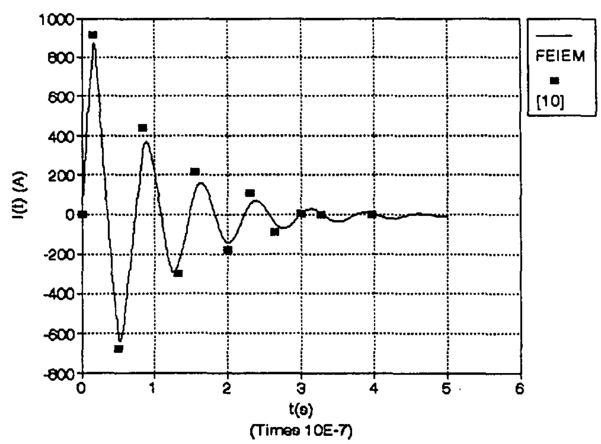

Figure 3. Comparison of results obtained for the current at the center of the line; L=10m, $a=6.738*10^{-2}$m, insulated in free space, loaded at its center with concentrated resistive load R=1Ω, and excited by EMP $E(t)=E_0(e^{-at}-e^{-bt})$, E_0=52.5kV/m, $a=4*10^{6} s^{-1}$, $b=4.76*10^{8} s^{-1}$

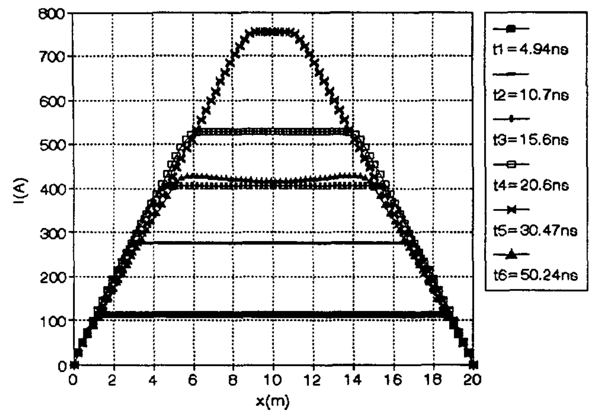

Figure 4. Time history for spatial current distribution along the unloaded line above perfect ground; L=20m, a=7.15 10^{-2} m, h=5m, excited by the EMP $E(t)=E_0(e^{-at}-e^{-bt})$, E_0=52.5kV/m, a=$4*10^{6}$ s^{-1}, b=$4.76*10^{8}$$s^{-1}$

CONCLUSION

The transient response of a transmission line to an EMP excitation is obtained directly in the time domain. The mathematical model is based on a scattering theory and the space-time Hallen integral equation related to the case of the loaded wire located over a perfect ground and excited by the electromagnetic pulse. The corresponding integral equation is solved via finite element/marching-on-in-time procedure. The proposed formulation and the computational technique could be readily applied to any configuration of multiconductor transmission lines, where the coupling between the lines must be taken into account.

REFERENCES

[1] M. Ianoz: Electromagnetic Field Coupling to Lines Cables and Networks, A review of Problems and Solutions, Proc. Int. Conf. on Electromagnetics in Advanced Applications, Turin, Sept. 12-15, 1995, pp. 75-80.

[2] S. Tkatchenko, F. Rachidi, M. Ianoz: Electromagnetic Field Coupling to a Line of Finite length: Theory and Fast Iterative Solutions in Frequency and Time Domains, IEEE Trans. EMC Vol.37, No. 4, Nov. 1995, pp 509-518.

[3] F. Arreghini, M. Ianoz, F. Rachidi: Frequency and Time-Domain Approaches in EMP Coupling. A Comparison Between Different Methods of Calculation, Proc. 2nd Int. Conf. on Electromagnetics in Aerospace Applications, Turin, Sept. 17-20, 1991, pp. 209-214.

[4] J.A. Landt, E.K. Miller: Transient Response of the Infinite Cylindrical Antenna and Scatterer, IEEE Trans. AP-24, No. 3., March 1976, pp 246-251.

[5] S.M. Rao, T.K. Sarkar, S.A. Dianat: A Novel technique to the Solution of Transient Electromagnetic Scattering from Thin Wires, IEEE trans. AP-34, No 5, May 1986, pp 630-634.

[6] A.G. Tijhuis, Z.Q. Peng, A.R. Bretones: Transient Excitation of a Straight Thin-Wire Segment: A New Look at an Old Problem, IEEE trans. AP-40, No 10, October 1992, pp 1132-1146.

[7] D. Poljak, V.Roje: Finite Element technique for Solving Time-Domain Hallen Integral Equation, Proc. IEE 10th Internat. Conf. of Antennas and Propag., Edinburgh, April 1997, pp 1.225-1.228.

[8] D. Poljak, V.Roje: Time Domain Analysis of a Thin Wire Parallel to a perfectly Conducting Ground Plane, Proc. Int. Conf. on Applied Electromagnetics and Communications, Dubrovnik, Oct. 15-17, 1997, pp. 30-33.

[9] D.Poljak: Transient response of wire antennas in the presence of an imperfectly conducting half-space, Ph.D. thesis, University of Split, 1996.

[10] E.K. Miller, J.A. Landt: Direct Time-Domain Techniques for Transient Radiation and Scattering from Wires, Proc. IEEE, Vol. 168, No.11, Nov. 1980, pp. 1396-1423.