Tarea #7

1. Calcule error absoluto y relativo p por p^*

$p=\pi$ $p^* = 22/7$

P (Real)	p^* (Aprox)
$\pi = 3.1415$	3.14
$22/7 = 3.1428$	3.14

Absoluto — Relativo

a) $|p - p^*|$ — $\left|\frac{p-p^*}{p}\right|$

$3.1415 - 3.1428$ — $3.1415 - 3.1428 / 3.1415$

$1,2644 \times 10^{-3}$ — $4,024 \times 10^{-4}$

b) $3.1415 - 3.1416$ — $\frac{3.1415 - 3.1416}{3.1415}$ — $p = \pi$ — $p^* = 3.1416$

$7,346 \times 10^{-6}$ — $2,338 \times 10^{-6}$

c) $2,7182 - 2,718$ — $(e - 2,718)/e$ — $e = 2,7182$ — $p^* = 2,718$

$2,818 \times 10^{-4}$ — $1,036 \times 10^{-4}$ — $p^* = 2,718$

d) $\sqrt{2} - 1,414$ — $\sqrt{2} - 1,414)/\sqrt{2}$ — $p = \sqrt{2}$ — $p^* = 1,414$

$2,135 \times 10^{-4}$ — $1,51 \times 10^{-4}$ — $= 1,4142$

2. a) $e^{10} - 22000$ — $(e^{10} - 22000)/e^{10}$ — $p = e^{10} = 22026.5$ — $p^* = 22000$

26.465 — $1,201 \times 10^{-3}$

b) $10^{\pi} - 1400$ — $10^{\pi} - 1400/10^{\pi}$ — $p = 10^{\pi}$ — $p^* = 1400$

$14,544$ — $1,049 \times 10^{-2}$

c) $8! - 39900$ — $(8! - 39900)/8!$ — $p = 8!$ — $p^* = 39900$

420 — $1,041 \times 10^{-2}$

d) $9! - p^*$ — $(9! - p^*)/9!$ — $p = 9!$ — $p^* = \sqrt{18\pi}(9/e)^9$

3343.1 — $9,21 \times 10^{-3}$

3. a) $p = \pi$ error relativo de 10^{-4}

$$\left|\frac{p - p^*}{p}\right| = \left|\frac{\pi - p^*}{\pi}\right| = 10^{-4} \quad \frac{\pi - p^*}{\pi} = 10^{-4} \wedge \frac{\pi - p^*}{\pi} = -10^{-4}$$

$= 3.1412784941 \quad = 3.1419068813$

b) $p = e$

$$\left|\frac{e - p^*}{e}\right| = 10^{-4} \Rightarrow \frac{e - p^*}{e} = 10^{-4} \wedge \frac{e - p^*}{e} = -10^{-4}$$

$p \in [2,71801 \; ; \; 2,7185533657]$

c) $p = \sqrt{2}$

$$\left|\frac{\sqrt{2} - p^*}{\sqrt{2}}\right| = 10^{-4} \quad \frac{\sqrt{2} - p^*}{\sqrt{2}} = 10^{-4} \wedge \frac{\sqrt{2} - p^*}{\sqrt{2}} = -10^{-4}$$

$[1,4143549884 \; ; \; 1,4140721411]$

d) $p = \sqrt[3]{7}$

$$\left|\frac{\sqrt[3]{7} - p^*}{\sqrt[3]{7}}\right| = 10^{-4} \quad \left|\frac{\sqrt[3]{7} - p^*}{\sqrt[3]{7}}\right| = 10^{-4} \wedge \left|\frac{\sqrt[3]{7} - p^*}{\sqrt[3]{7}}\right| = -10^{-4}$$

$[1,91273989 \; ; \; 1,913122476]$

4. Aritmética de redondeo

	Real (Corte)	Aproximado (Aprox)
a) $\dfrac{\frac{13}{14} - \frac{5}{7}}{2e - 5,4} =$	$13/14 = 0,92857$	$0,929$
	$5/7 = 0,71428$	$0,714$
	$2e = 5,4365$	$5,44$
$\lvert p - p^*\rvert = 0,4856$	$5,4 = 5,4$	$5,4$
$\left\lvert\frac{p - p^*}{p}\right\rvert = 0,082858$	$p = 5,8606$	$p^* = 5,375$

	Real	Aproximado
b) $-10\pi + 6e - \frac{3}{61}$	$10\pi = 31,415$	$31,4$
$\lvert p - p^*\rvert = 0,045$	$6e = 16,3096$	$16,3$
$\frac{\lvert p - p^*\rvert}{p} = 2,4693 \times 10^{-3}$	$3/61 = 0,0491803$	$0,0492$
	$p = -15,155$	$p^* = -15,2$

Real Aproximado

c) $\left(\frac{2}{9}\right)\left(\frac{9}{11}\right) = 0,18181$ $2/9 = 0,22222$ $0,222$

$|p-p^*| = 8,18\times10^{-4}$ $9/11 = 0,81818$ 0.818

$\frac{|p-p^*|}{p} = -1,045\times10^{-3}$ $p = 0,18181$ $p^* = 0,182$

d) $\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$ $\sqrt{13} = 3,60555$ $3,606$

$\sqrt{11} = 3,3166$ $3,32$

$|p-p^*| = -0,41739$ $p = 23,958$ $p^* = 24$

$\left|\frac{p-p^*}{p}\right| = 1,7421\times10^{-3}$

5. Calcule errores absolutos y relativos mediante polinomio

$$f(x) = x - \left(\frac{1}{3}\right)x^3 + \left(\frac{1}{5}\right)x^5$$

a) $4\left[\text{arc}\left(\frac{1}{2}\right) + \text{arc}\left(\frac{1}{3}\right)\right] = 3,1415$

$|\pi - p^*| = 3,98\times10^{-3}$

$\frac{|\pi - p^*|}{\pi} = 1,267\times10^{-3}$

b) $16\left[\text{arc}\left(\frac{1}{5}\right) - 4\,\text{arc}\left(\frac{1}{239}\right)\right] = 3,41621029$

$|\pi - p^*| = 2,83\times10^{-5}$

$\left|\frac{\pi - p^*}{\pi}\right| = 9,03\times10^{-6}$

6 # e se puede definir $e = \sum_{n=0}^{\infty}\left(\frac{1}{n!}\right) \Rightarrow n! = n(n-1)\ldots2\cdot1$

$n \neq 0$ y $0! = 1$

a. $\sum_{n=0}^{5}\left(\frac{1}{n!}\right) = 2,7166$

$|e - p^*| = 1,615\times10^{-3}$

$\left|\frac{e-p^*}{e}\right| = 5,94\times10^{-4}$

b. $\sum_{n=0}^{10}\left(\frac{1}{n!}\right) = 2,7182$

$|e - p^*| = 2,731\times10^{-8}$

$\left|\frac{e-p^*}{e}\right| = 1,004\times10^{-8}$

7- $x = \dfrac{x_0 y_1 - x_1 y_0}{y_1 - y_0}$ $\boxed{1}$ $x = x_0 - \dfrac{(x_1 - x_0) y_0}{y_1 - y_0}$

$= \dfrac{1,31 \cdot 5,76 - 1,93 \cdot 3,24}{5,76 - 3,24}$ $= 1,31 - \dfrac{(1,93 - 1,31)\,3,24}{5,76 - 3,24}$

$= 0,5128571429$ $= 0,5128571429$

Aunque el resultado es el mismo la optimización puede verse en la cantidad de bits cuando los datos sean muy grandes la segunda opción puede ser mejor