Capture the Problem

Sample Problems

December 2, 2025

Instructions

The capture the problem round will have a grid of 50 problems, with the grid split up into sections depending on the difficulty of the problem. Teams will be expected to quickly answer the questions. If they are the first to answer a problem, they 'capture' it and no other team will be able to solve it. This game requires both speed, accuracy and strategy.

Note

The grid that the teams will get during the contest will look much differently. There will be 5 problems in each row, and each row will have a different difficulty. The first row's problems will be worth 10 points each, and the second row's will be worth 20 and so on. We have not done that split for these problems but all of these are of medium difficulty. Furthermore, we present only 20 problems, but these problems are representative of the difficulty of the competition. The solutions are left as an exercise to the reader.

Problems

1	Let a, b, c be positive reals such that $abc = 8$ and $a+b+c = 10$. Compute the minimum value of $a^2+b^2+c^2$.	**2**	Solve for the integer n satisfying $\varphi(n) = \frac{n}{3}$ and $n < 500$.
3	A sequence is defined by $a_1 = 3$, $a_2 = 10$, and $a_{n+1} = 4a_n - 4a_{n-1}$ for $n \geq 2$. Compute a_{10}.	**4**	Let z be a complex number satisfying $\lvert z-3+4i\rvert = 5$ and $\arg(z) = \frac{\pi}{6}$. Find $\mathrm{Re}(z)$.
5	Let x be the smallest positive solution to $\frac{1}{x} + \frac{1}{x+2} = \frac{1}{6}$. Compute $6x$.	**6**	Triangle ABC has side lengths $AB = 17$, $BC = 25$, $AC = 26$. Point D on AC satisfies $\angle ABD = \angle DBC$. Compute AD.
7	Compute the number of integers $1 \leq k \leq 10^6$ such that $\gcd(k, 1000) = 20$.	**8**	Let p be the smallest prime such that $p \mid (2^{2025}-1)$ but $p \nmid (2^{675}-1)$. Compute p.
9	A function f satisfies $f(1) = 2$, $f(2) = 7$, and $f(x+y) = f(x) + f(y) + 3xy$. Compute $f(10)$.	**10**	Let $S = \sum_{k=1}^{20} \frac{1}{k(k+2)}$. Compute S as a simplified rational number.

11	Let a and b be positive reals such that $a+b=12$ and $ab=20$. Compute the minimum value of $$a^2+b^2+\frac{100}{ab}.$$	**12**	How many integers $1 \leq n \leq 2000$ satisfy $$n^2+5n \equiv 0 \pmod{12}?$$
13	Let z satisfy $\lvert z-4\rvert = 5$ and $\mathrm{Im}(z) = 3$. Compute $\lvert z\rvert$.	**14**	A sequence is defined by $a_1 = 4$, $a_2 = 9$, and $a_{n+1} = a_n + a_{n-1} + 3$. Compute a_{12}.
15	Let x be the positive solution of $$\sqrt{x+5}+\sqrt{x-3}=9.$$ Compute x.	**16**	Triangle ABC has side lengths $AB = 10$, $BC = 14$, $CA = 16$. Let m_a be the length of the median from A. Compute m_a.
17	Compute the number of integers $1 \leq k \leq 5000$ for which $\gcd(k, 360) = 12$.	**18**	Let p be the smallest prime divisor of $$3^{2024}-1.$$ Compute p.
19	Let f satisfy $f(1) = 3$, $f(2) = 11$, and $$f(x+y) = f(x) + f(y) + xy.$$ Compute $f(8)$.	**20**	Compute the value of $$\sum_{k=1}^{15} \frac{1}{(k+1)(k+3)}.$$