Practical 1

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June 15, 2025

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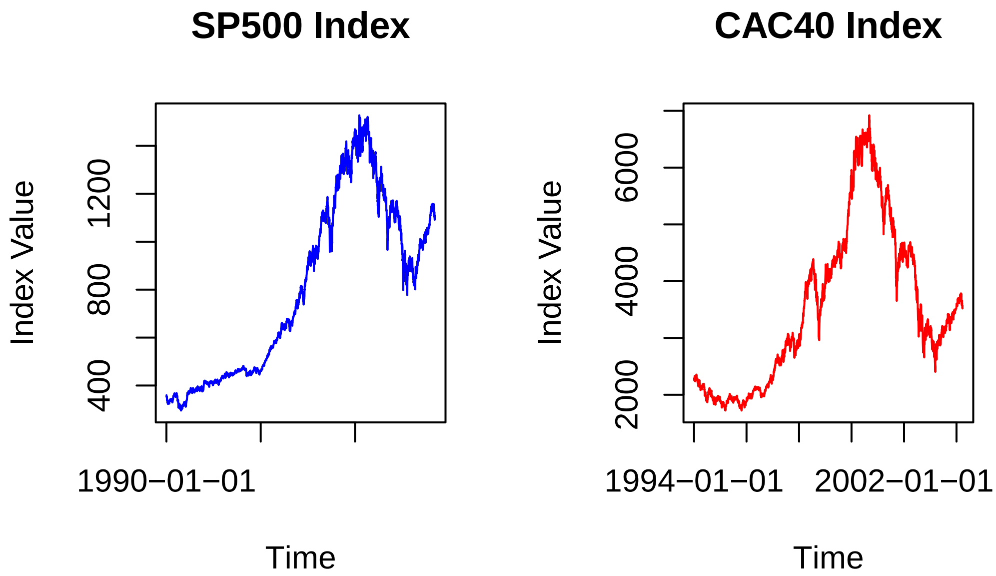
Part 1 - Financial Returns & Normality

Question a)

Read in the data. Then, assess the stationarity of the (raw) stock indices.

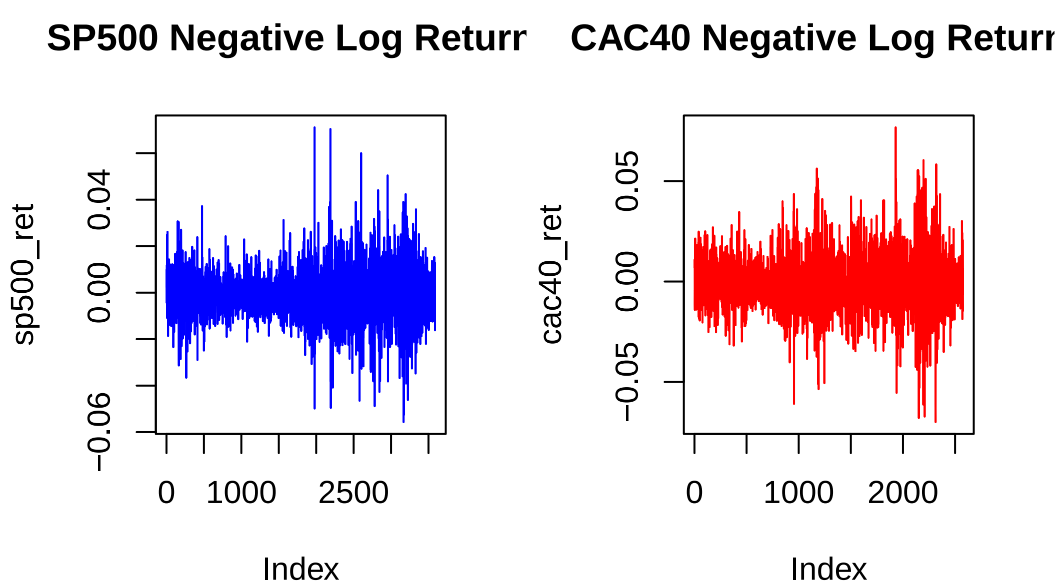
```
	Augmented Dickey-Fuller Test

data:  sp500_ts
Dickey-Fuller = -1.2128, Lag order = 15, p-value = 0.9044
alternative hypothesis: stationary

	Augmented Dickey-Fuller Test

data:  cac40_ts
Dickey-Fuller = -0.7332, Lag order = 13, p-value = 0.9676
alternative hypothesis: stationary
```



As a result, we can see that for SP500, the p-value is 0.9044 and for CAC40, the p-value is 0.9676. As they are both higher than 0.05, we can conclude that both series are not stationary.

Question b)

> Create a function to transform the daily stock indices into their daily negative log returns counterparts. Plot the latter series and assess their stationarity. To compare the series, also plot the negative log returns on a common scale to all indices.



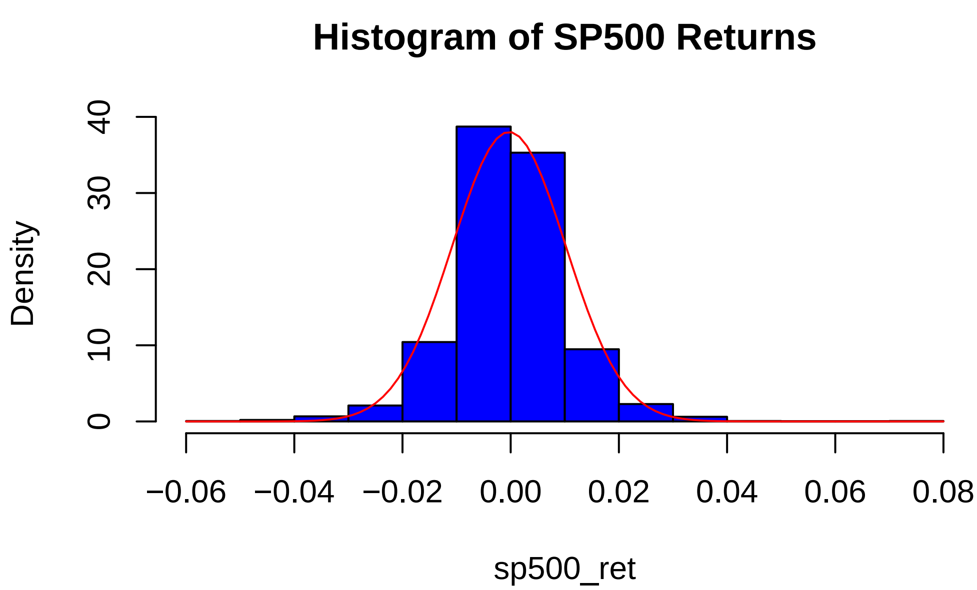

```
[1] "ADF Test for SP500:"
```

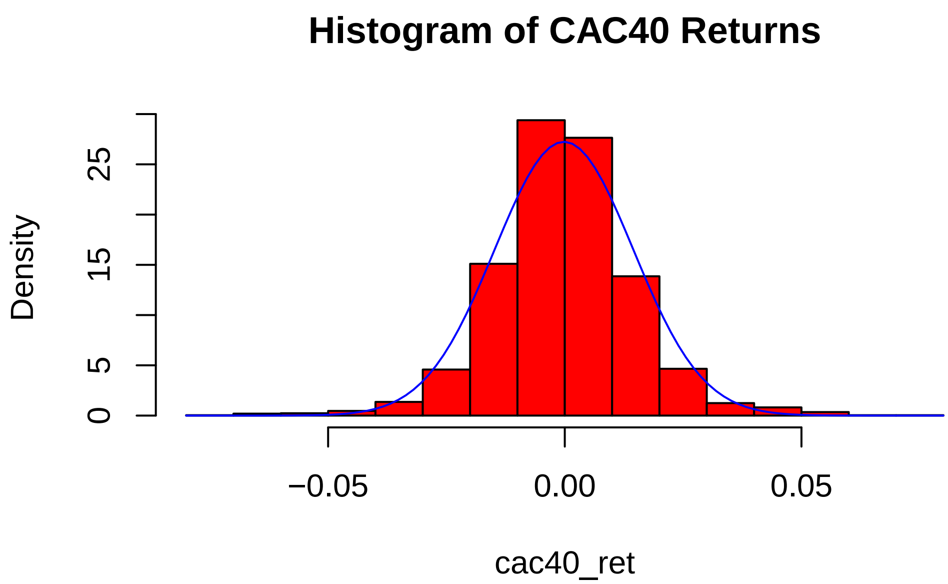
```
    Augmented Dickey-Fuller Test

data:  sp500_ret
Dickey-Fuller = -14.843, Lag order = 15, p-value = 0.01
alternative hypothesis: stationary


[1] "ADF Test for CAC40:"


    Augmented Dickey-Fuller Test

data:  cac40_ret
Dickey-Fuller = -13.34, Lag order = 13, p-value = 0.01
alternative hypothesis: stationary
```

As both plots seem to not show trends, periodic cycles and a stable variance, the series seem to be stationary. To verify this, the ADF test shows that both p-values (p-value = 0.01) are lower than 0.05, so we reject the hypothesis and thus, both series are stationary.

Question c)

> Draw histograms of the negative log returns and compare them to the Normal distribution. What do you observe?

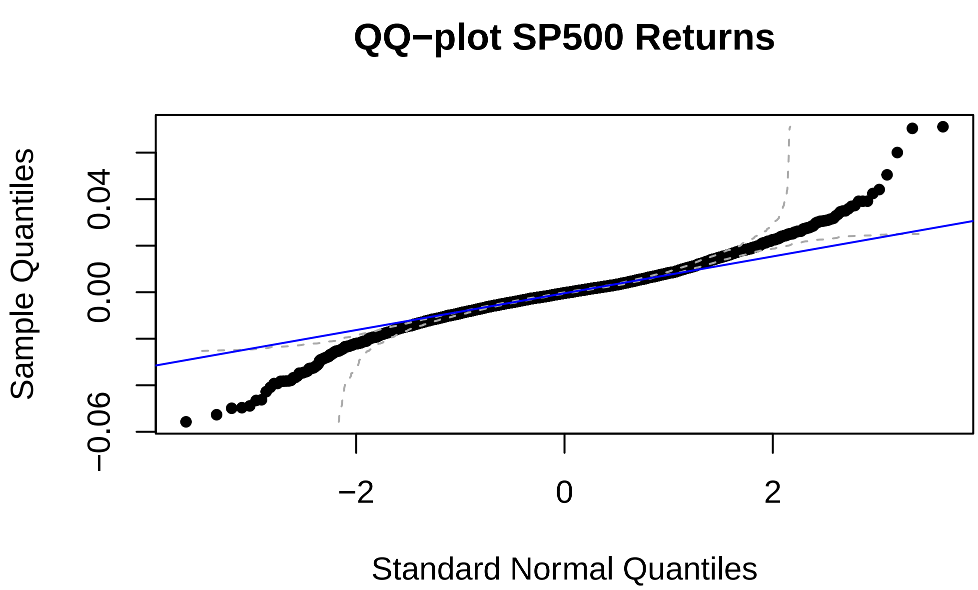


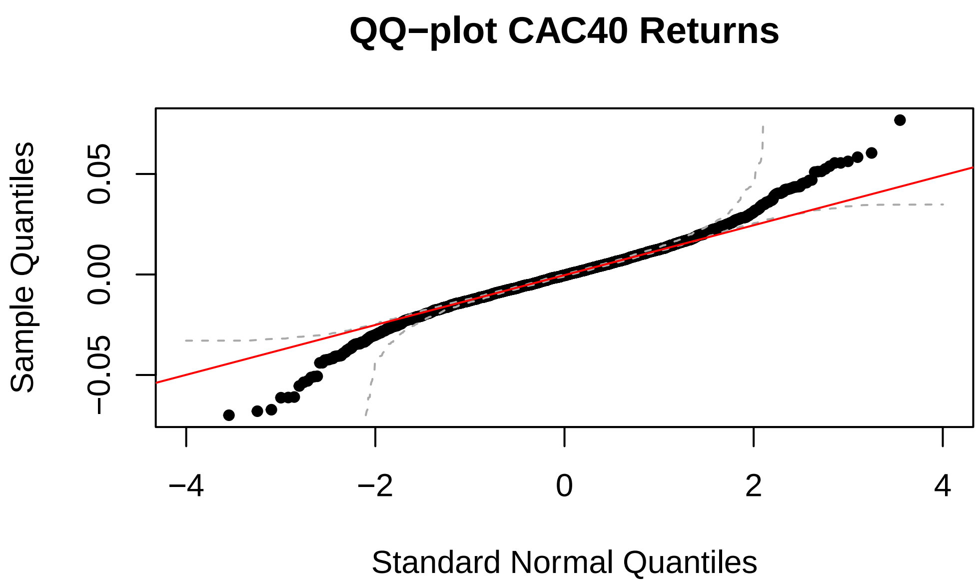


Both histograms have bell-shaped distributions, but are not perfectly aligned with the normal curve. ALso, the tails seeem to go further than what the normal disctribution curve predicts, which can indicate a higher probability of extreme returns than expected in a normal model.

Question d)

Check the normality assumption of the negative log returns using QQ-plots. What is your conclusion?





On both the lower and upper extremes, the points deviate from the line, showing heavier tails than a normal distribution. So, the negative log returns don't seem to follow normal distribution.

Question e)

> Formally test the normality assumption of the negative log returns using an Anderson-Darling testing procedure. Do you reject the Normal hypothesis?

```
    Anderson-Darling normality test

data:  sp500_ret
A = 29.24, p-value < 2.2e-16

    Anderson-Darling normality test

data:  cac40_ret
A = 10.33, p-value < 2.2e-16
```

As both p-values are lower than 0.05 (both at 2.2e-16), we reject the null hypothesis, meaning that the returns are not normally distributed.

Question f)

> Use the fitdistr() function from the MASS package in order to obtain the (maximum-likelihood estimated) parameters of distributions you could imagine for the negative log returns. Try to fit at least two different distributions on the data and, using an information criteria (such as the AIC), decide which distributional framework fits best for each of the series.

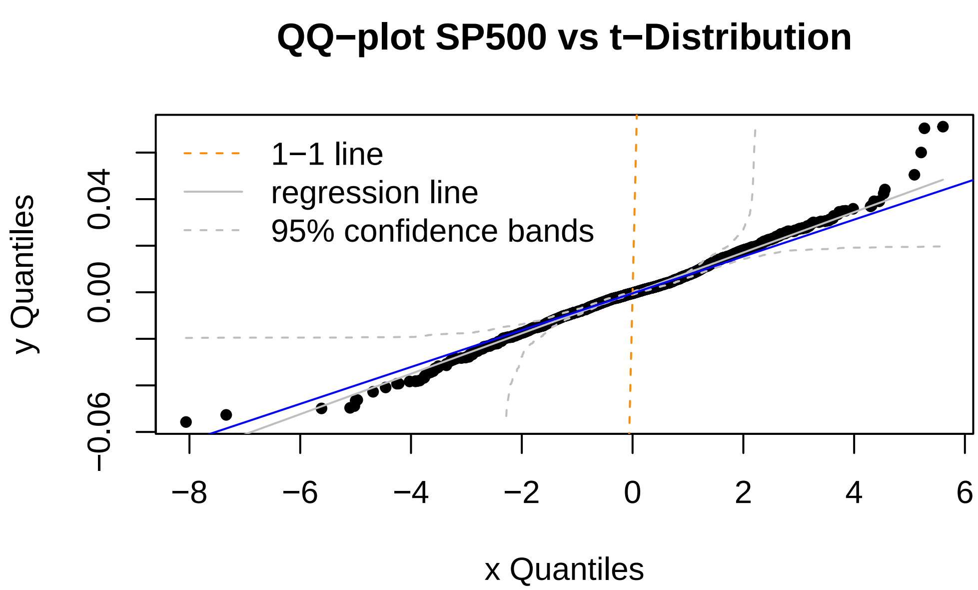
```
      mean            sd
  -0.0003139511    0.0104904994
 ( 0.0001751582) ( 0.0001238556)
```

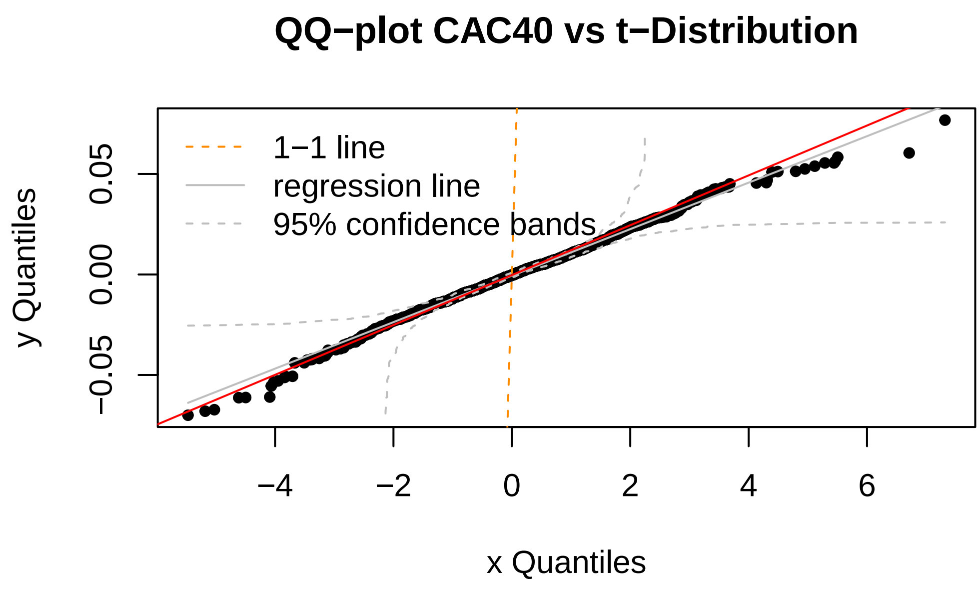
```
        m               s               df
 -0.0003823438    0.0083396816    6.5020438529
( 0.0001642079) ( 0.0011490812) ( 5.6443703001)

       mean             sd
 -0.0001723074    0.0146403328
( 0.0002884550) ( 0.0002039685)

        m               s               df
 -0.0003374078    0.0116334321    5.2499036218
( 0.0002632892) ( 0.0002745656) ( 0.5356059623)

[1] -22510.5

[1] -22898.7

[1] -14447.55

[1] -14626
```

As the Student model yields lower AIC than the normal model, it indicates a better balance between the goodness-of-fit and the complexity. It is a more appropriate choice of model.

Question g)

> If this has not been done in (f), fit a t-distribution to the negative log returns using fitdistr(). Using a QQ-plot for each of the series, decide whether the fit is better than with a Normal distribution, based on your answer in (d).



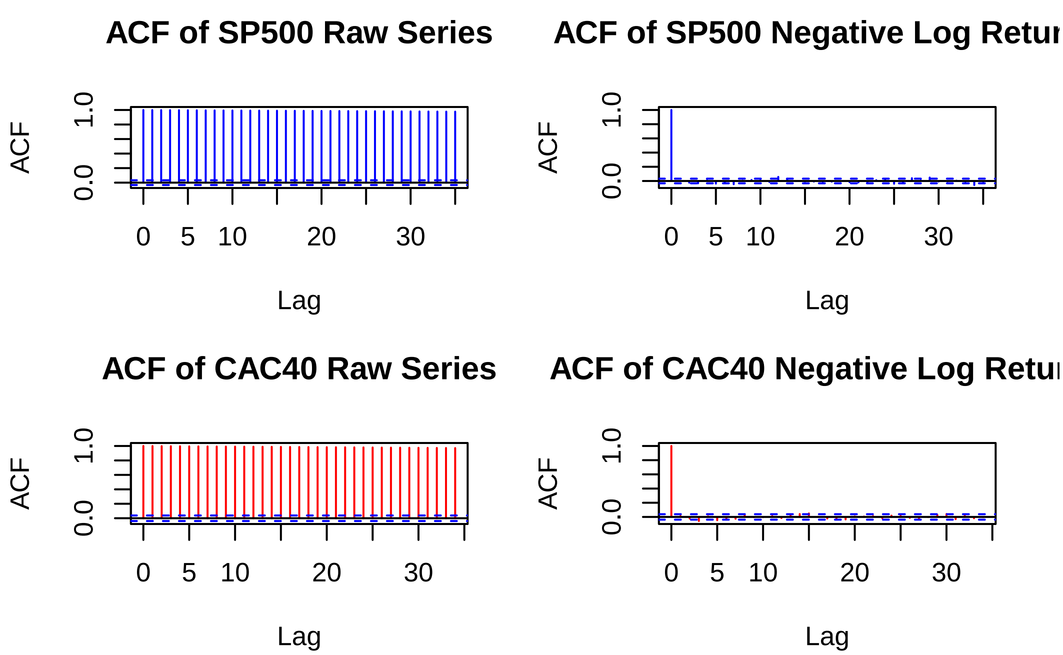


Despite some deviations from points in the extremes, the QQ-plots confirm that a Student's t-distribution provides a better fit for both the SP500 and CAC40 log returns compared to a normal distribution. The heavier tails of the t-distribution captures the extremes better. It is also consistent with question d).

Part 2 - Financial time series, volatility and the random walk hypothesis

Question a)

> Plot the ACF of all the series in Part 1 (i.e. the raw series as well as the negative log returns). What do you observe?



The raw series show high autocorrelation at all lags, implying non-stationarity. For the negative log returns, it shows little to no autocorrelation, suggesting that the daily returns are close to uncorrelated.

Question b)

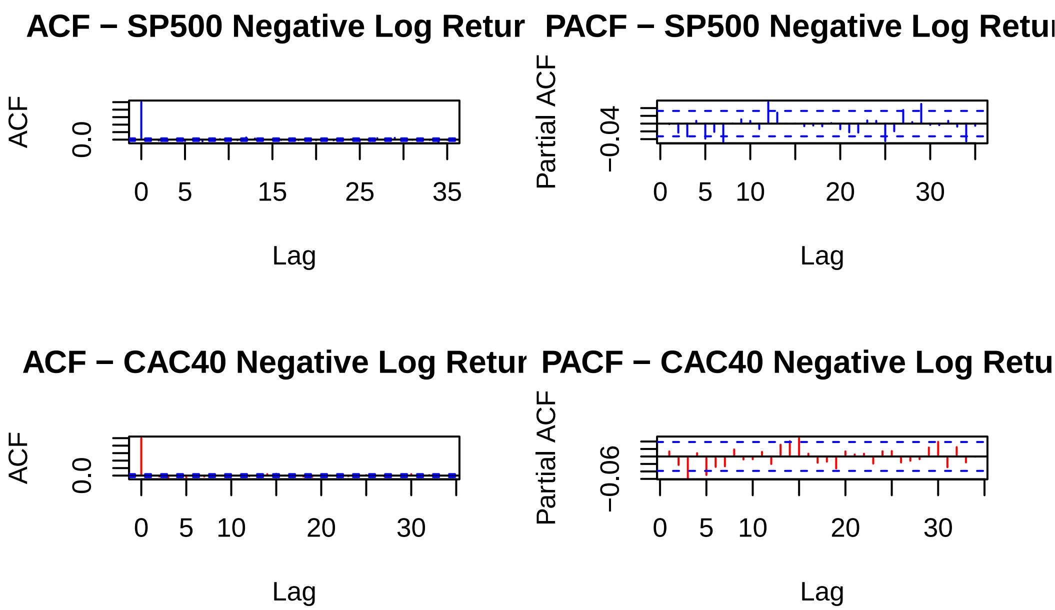
> Use a Ljung-Box procedure to formally test for (temporal) serial dependence in the series. What is your conclusion?

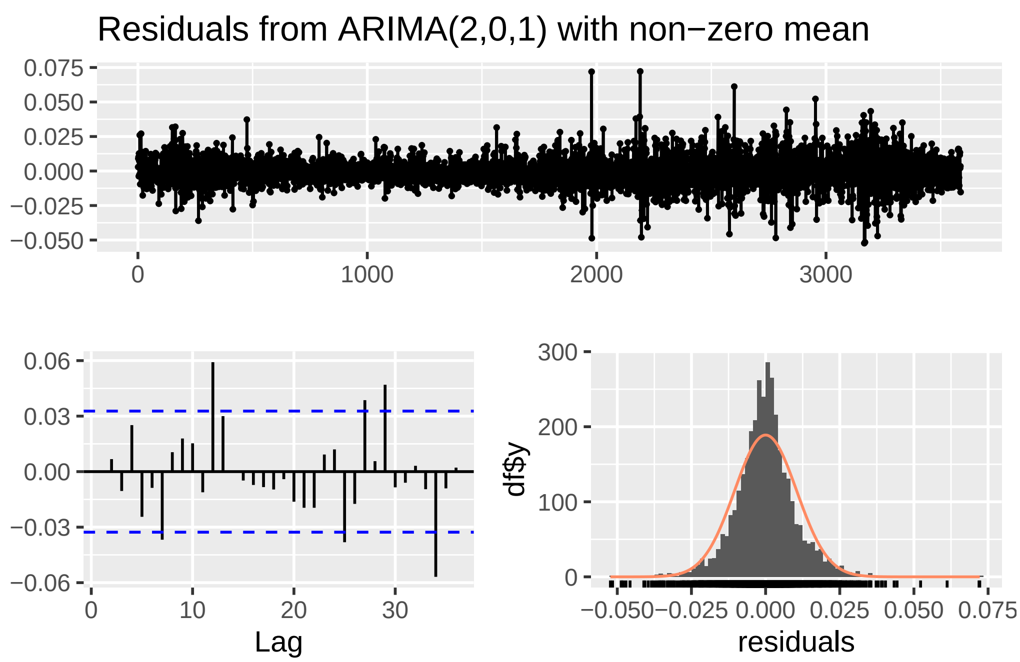
```
    Box-Ljung test

data:  sp500
X-squared = 71035, df = 20, p-value < 2.2e-16

    Box-Ljung test

data:  sp500_ret
X-squared = 38.931, df = 20, p-value = 0.0068

    Box-Ljung test

data:  cac40
X-squared = 50889, df = 20, p-value < 2.2e-16

    Box-Ljung test

data:  cac40_ret
X-squared = 41.079, df = 20, p-value = 0.003639
```

Looking at the raw series for both CAC40 and SP500, we obtain a p-value that is lower than 0.05 (both at 2.2e-16), so we reject the null hypothesis and the raw series show autocorrelation. For both negative log returns, the p-values are higher than the raw series (0.0068 for SP500, 0.003639 for CAC40), but still smaller than the p-value, so again they show autocorrelation.

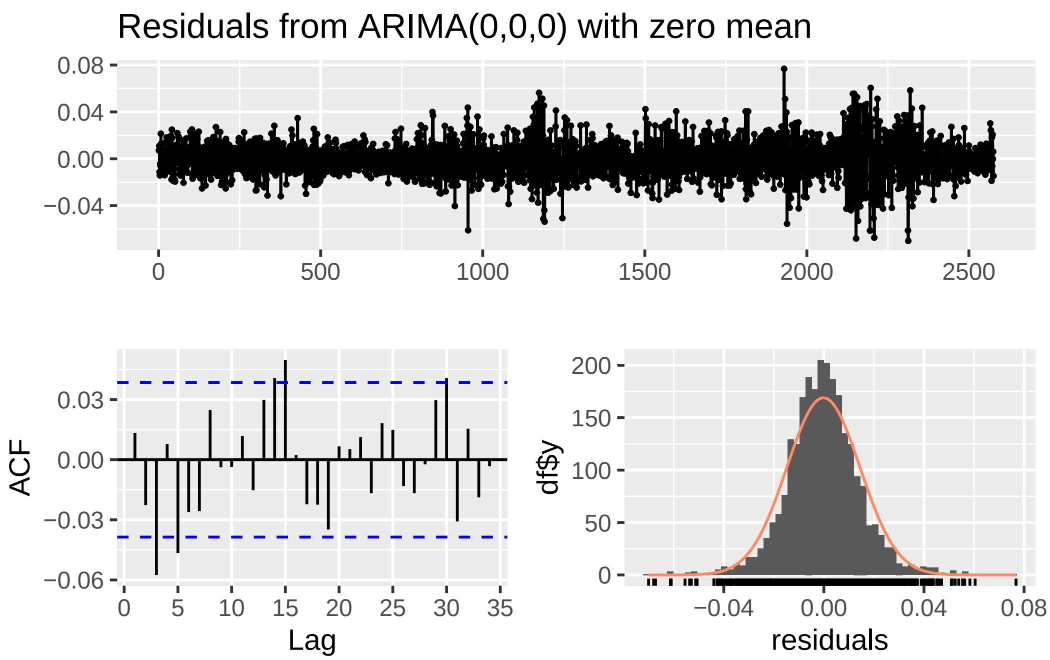
Question c)

> Propose ARIMA models for each of the negative log returns series, based on visualisation tools (e.g. ACF and PACF). Select an ARIMA model using auto.arima() (forecast package) to each of the negative log returns series. Comment on the difference. Assess the residuals of the resulting models.






```
	Ljung-Box test

data:  Residuals from ARIMA(2,0,1) with non-zero mean
Q* = 12.498, df = 7, p-value = 0.08534

Model df: 3.   Total lags used: 10
```



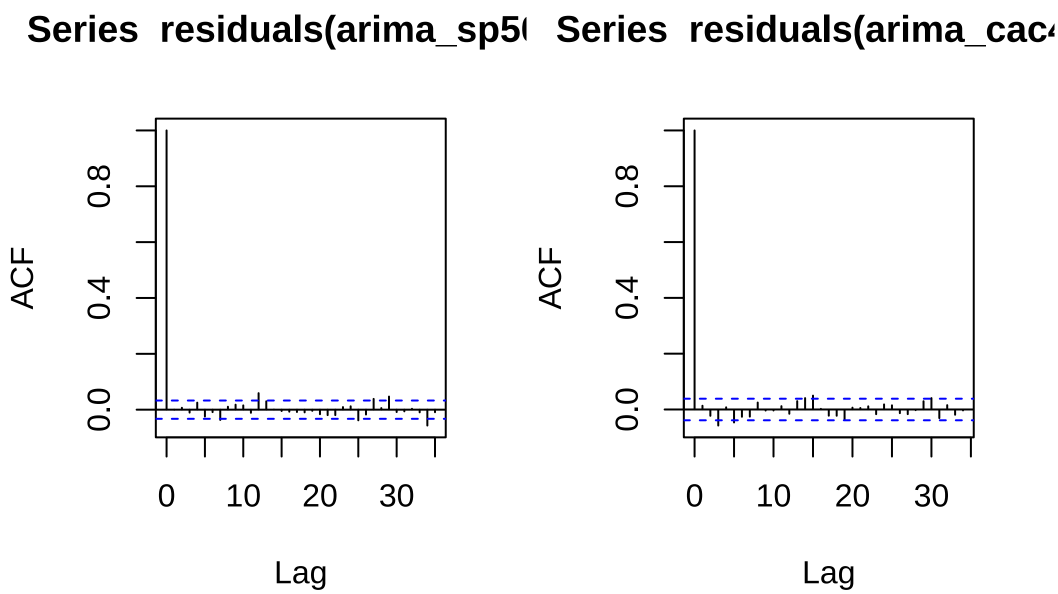

```
    Ljung-Box test

data:  Residuals from ARIMA(0,0,0) with zero mean
Q* = 21.191, df = 10, p-value = 0.0198

Model df: 0.   Total lags used: 10


Series: sp500_ret
ARIMA(2,0,1) with non-zero mean

Coefficients:
         ar1      ar2      ma1    mean
      0.7796  -0.0291  -0.7827  -3e-04
s.e.  0.1035   0.0180   0.1024   2e-04

sigma^2 = 0.0001099:  log likelihood = 11261.47
AIC=-22512.94   AICc=-22512.92   BIC=-22482.01

Training set error measures:
                       ME       RMSE        MAE  MPE MAPE      MASE
Training set 2.252009e-06 0.01047816 0.00748605 -Inf  Inf 0.6910122
                      ACF1
Training set -0.0003543668


Series: cac40_ret
ARIMA(0,0,0) with zero mean

sigma^2 = 0.0002144:  log likelihood = 7225.6
AIC=-14449.19   AICc=-14449.19   BIC=-14443.34

Training set error measures:
```

```
                          ME       RMSE        MAE MPE MAPE      MASE       ACF1
Training set -0.0001723074 0.01464135 0.01089581 100  100 0.7046982 0.01343285
```

The ACF & PACF for both SP500 & CAC40 negative log returns show almost no significant autocorrelation, indicating that the returns behave in a similar way to white noise.It is also confirmed by checking the residuals. We can also observe that the auto.arima() function selected very simple models.

Question d)

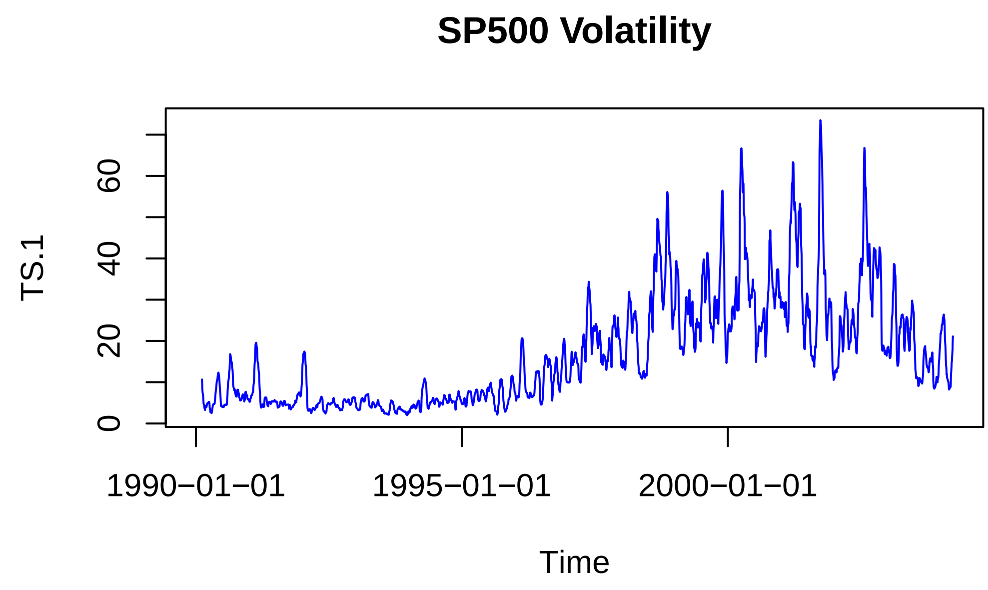
> Assess the residuals of the resulting models from (c), both their raw values and their absolute values, through visual tools (such as the ACF) and formal tests (e.g. Ljung-Box). What do you conclude about the independence assumption?

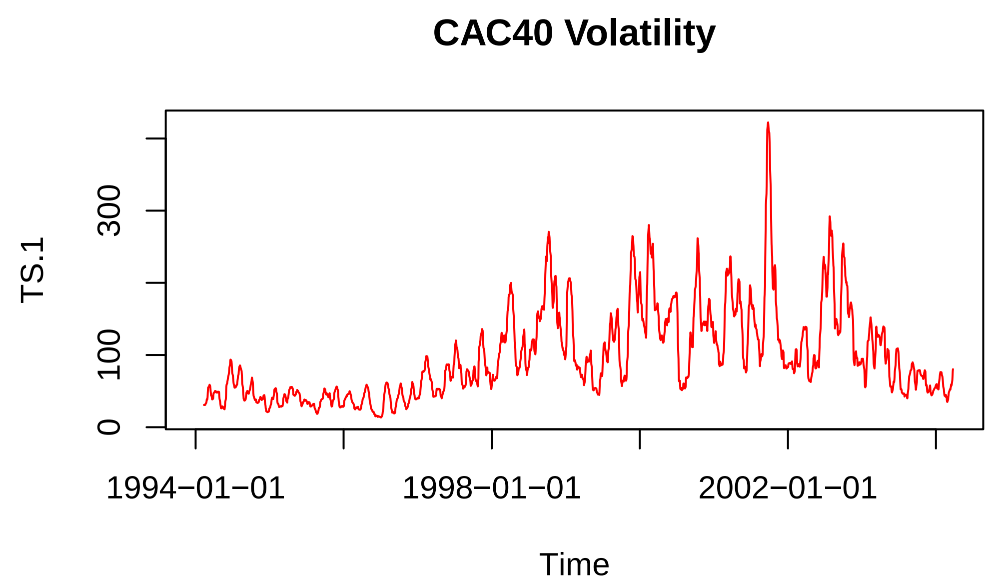


The independence assumption holds for the raw residuals but not for the volatility patterns (autocorrelation in the absolute residuals). So, further modeling could be used to fully capture the dynamics of the residuals.

Question e)

> Plot the volatility of the raw series of indices. What is your conclusion on the homoscedasticity assumption?





The data shows heteroskedasticity, as the volatility (or variance) is not constant over time.

Question f)

Fit GARCH models to the negative log returns of each series with both standardised and skewed t-distributions, with order (1, 1), using the garchFit() function from the fGarch library. Assess the quality of the fit by evaluating the residuals.

```
Title:
 GARCH Modelling

Call:
```

```
 garchFit(formula = ~garch(1, 1), data = sp500_ret, cond.dist = "std",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x11c5f9c90>
 [data = sp500_ret]

Conditional Distribution:
 std

Coefficient(s):
         mu        omega       alpha1        beta1        shape
-5.6197e-04   3.2345e-07   5.0937e-02   9.4765e-01   7.1692e+00

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -5.620e-04   1.250e-04   -4.497 6.90e-06 ***
omega   3.234e-07   1.442e-07    2.242   0.0249 *
alpha1  5.094e-02   7.720e-03    6.598 4.16e-11 ***
beta1   9.476e-01   7.660e-03  123.714  < 2e-16 ***
shape   7.169e+00   8.189e-01    8.755  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 11798.72    normalized:  3.289301

Description:
 Sun Jun 15 09:32:11 2025 by user:


Standardised Residuals Tests:
                                  Statistic     p-Value
 Jarque-Bera Test   R    Chi^2  661.3620424 0.000000000
 Shapiro-Wilk Test  R    W        0.9842328 0.000000000
 Ljung-Box Test     R    Q(10)   18.0504009 0.054119378
 Ljung-Box Test     R    Q(15)   34.5937376 0.002808328
 Ljung-Box Test     R    Q(20)   36.2946867 0.014198662
 Ljung-Box Test     R^2  Q(10)    7.7661792 0.651664153
 Ljung-Box Test     R^2  Q(15)   10.7404577 0.770766521
 Ljung-Box Test     R^2  Q(20)   15.4320193 0.751176895
 LM Arch Test       R    TR^2    10.0402112 0.612432829

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.575815 -6.567193 -6.575819 -6.572742


Title:
```

```
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = sp500_ret, cond.dist = "sstd",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x119d04438>
 [data = sp500_ret]

Conditional Distribution:
 sstd

Coefficient(s):
        mu        omega       alpha1        beta1         skew        shape
-5.1103e-04   3.3855e-07   5.1496e-02   9.4679e-01   1.0316e+00   7.3173e+00

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -5.110e-04   1.308e-04   -3.906  9.4e-05 ***
omega   3.386e-07   1.472e-07    2.299   0.0215 *
alpha1  5.150e-02   7.770e-03    6.628  3.4e-11 ***
beta1   9.468e-01   7.772e-03  121.816  < 2e-16 ***
skew    1.032e+00   2.406e-02   42.875  < 2e-16 ***
shape   7.317e+00   8.596e-01    8.512  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 11799.62    normalized:  3.28955

Description:
 Sun Jun 15 09:32:12 2025 by user:


Standardised Residuals Tests:
                                  Statistic     p-Value
 Jarque-Bera Test   R    Chi^2  659.5154771 0.000000000
 Shapiro-Wilk Test  R    W        0.9842608 0.000000000
 Ljung-Box Test     R    Q(10)   17.9330634 0.056103319
 Ljung-Box Test     R    Q(15)   34.5128308 0.002883347
 Ljung-Box Test     R    Q(20)   36.1922613 0.014599763
 Ljung-Box Test     R^2  Q(10)    7.5116696 0.676416753
 Ljung-Box Test     R^2  Q(15)   10.5335043 0.784908436
 Ljung-Box Test     R^2  Q(20)   15.1615192 0.767092017
 LM Arch Test       R    TR^2     9.8488270 0.629221076

Information Criterion Statistics:
```

```
      AIC       BIC       SIC      HQIC 
-6.575755 -6.565409 -6.575761 -6.572067 


Title:
 GARCH Modelling 

Call:
 garchFit(formula = ~garch(1, 1), data = cac40_ret, cond.dist = "std", 
    trace = FALSE) 

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x119fa62b0>
 [data = cac40_ret]

Conditional Distribution:
 std 

Coefficient(s):
         mu        omega       alpha1        beta1        shape  
-5.2677e-04   2.0876e-06   6.4925e-02   9.2750e-01   1.0000e+01  

Std. Errors:
 based on Hessian 

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)    
mu     -5.268e-04   2.343e-04   -2.248  0.02459 *  
omega   2.088e-06   7.696e-07    2.712  0.00668 ** 
alpha1  6.493e-02   9.599e-03    6.764 1.35e-11 ***
beta1   9.275e-01   1.043e-02   88.943  < 2e-16 ***
shape   1.000e+01   1.361e+00    7.350 1.99e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 7496.61    normalized:  2.910175 

Description:
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Standardised Residuals Tests:
                                 Statistic      p-Value
 Jarque-Bera Test   R    Chi^2  36.1606492 1.405448e-08
 Shapiro-Wilk Test  R    W       0.9968239 3.245060e-05
 Ljung-Box Test     R    Q(10)  12.1818800 2.730684e-01
 Ljung-Box Test     R    Q(15)  20.7654478 1.444922e-01
 Ljung-Box Test     R    Q(20)  22.3308637 3.228303e-01
 Ljung-Box Test     R^2  Q(10)  12.6755294 2.423833e-01
 Ljung-Box Test     R^2  Q(15)  13.1950834 5.872327e-01
 Ljung-Box Test     R^2  Q(20)  14.8593966 7.843956e-01
```
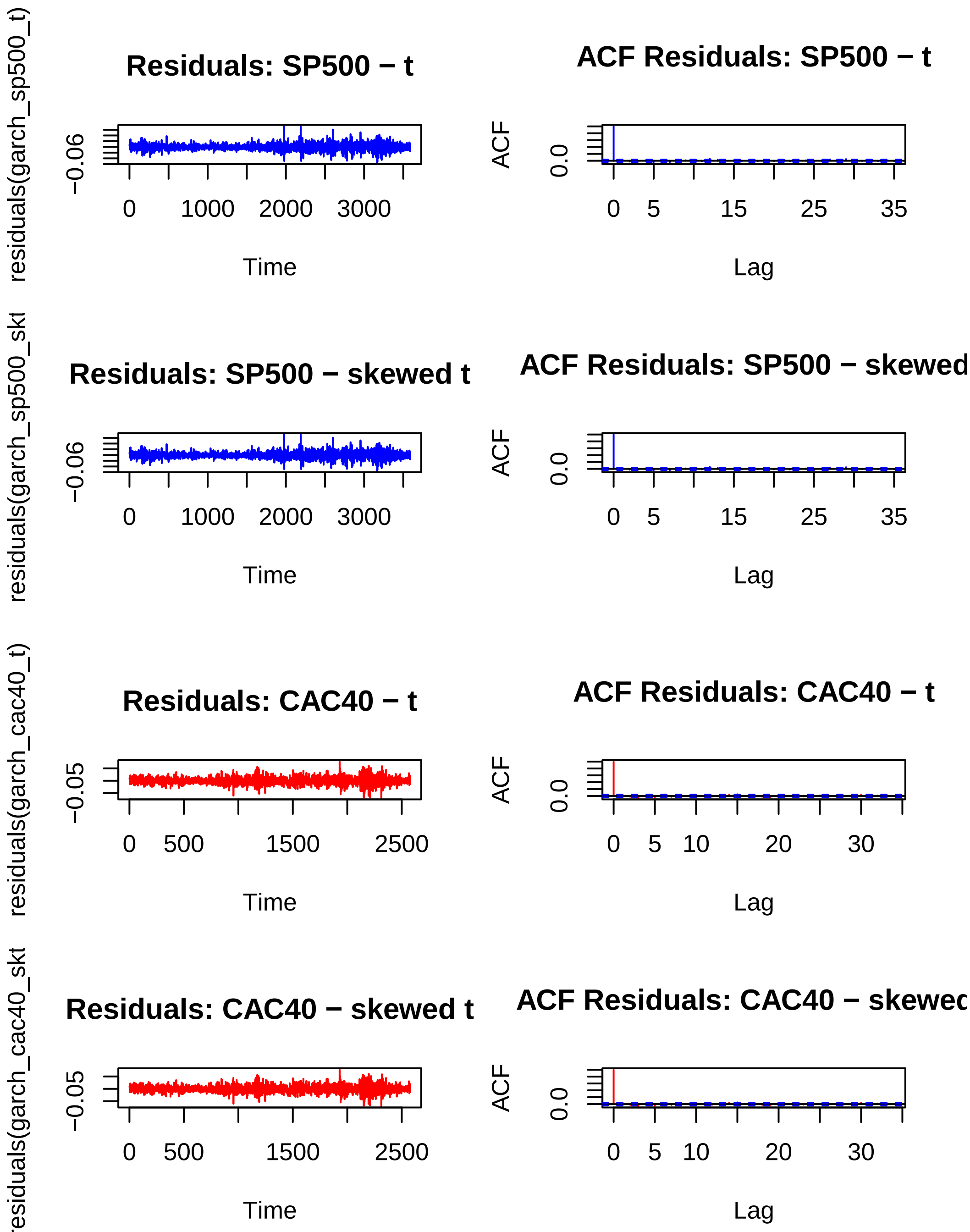

```
 LM Arch Test       R    TR^2   13.7118254 3.194879e-01

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.816467 -5.805105 -5.816475 -5.812348


Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = cac40_ret, cond.dist = "sstd",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x10d779a18>
 [data = cac40_ret]

Conditional Distribution:
 sstd

Coefficient(s):
         mu        omega       alpha1        beta1         skew        shape
-4.1284e-04   2.0402e-06   6.5401e-02   9.2730e-01   1.0840e+00   1.0000e+01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -4.128e-04   2.377e-04   -1.737  0.08245 .
omega   2.040e-06   7.576e-07    2.693  0.00708 **
alpha1  6.540e-02   9.543e-03    6.853 7.23e-12 ***
beta1   9.273e-01   1.030e-02   90.004  < 2e-16 ***
skew    1.084e+00   3.327e-02   32.581  < 2e-16 ***
shape   1.000e+01   1.342e+00    7.452 9.24e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 7500.141    normalized:  2.911545

Description:
 Sun Jun 15 09:32:12 2025 by user:


Standardised Residuals Tests:
                                 Statistic      p-Value
 Jarque-Bera Test   R    Chi^2  36.6432206 1.104144e-08
 Shapiro-Wilk Test  R    W       0.9967943 2.934125e-05
 Ljung-Box Test     R    Q(10)  12.0943005 2.787949e-01
 Ljung-Box Test     R    Q(15)  20.7596486 1.446860e-01
```

```
 Ljung-Box Test     R    Q(20)  22.3409037 3.223024e-01
 Ljung-Box Test     R^2  Q(10)  12.3893095 2.598459e-01
 Ljung-Box Test     R^2  Q(15)  12.9276394 6.078871e-01
 Ljung-Box Test     R^2  Q(20)  14.5756952 8.001500e-01
 LM Arch Test       R    TR^2   13.4251438 3.389111e-01

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.818432 -5.804797 -5.818443 -5.813490
```




```
    Box-Ljung test

data:  residuals(garch_sp500_t)
```

```
X-squared = 38.931, df = 20, p-value = 0.0068


    Box-Ljung test

data:  residuals(garch_sp500_skt)
X-squared = 38.931, df = 20, p-value = 0.0068


    Box-Ljung test

data:  residuals(garch_cac40_t)
X-squared = 41.079, df = 20, p-value = 0.003639


    Box-Ljung test

data:  residuals(garch_cac40_skt)
X-squared = 41.079, df = 20, p-value = 0.003639
```

By evaluating the residuals, we observe that for both SP500 and CAC40, the p-values are below 0.05, meaning there is still autocorrelation and residuals are not white noise. Looking at the AIC/BIC and the log-likelihood, it seems that the skewed distribution is slightly better for both models especially for CAC40. Thus, the Garch models are a good start but can be improved.

Question g)

> Residual serial correlation can be present when fitting a GARCH directly on the negative log returns. Hence, in order to circumvent this problem, it is possible to use the following two-step approach: • fit an ARMA(p,q) on the negative log returns; • fit a GARCH(1,1) on the residuals of the ARMA(p,q) fit. Proceed with the above recipe. Assess the quality of the above fit.

To fit an ARMA(p,q) on the negative log returns:

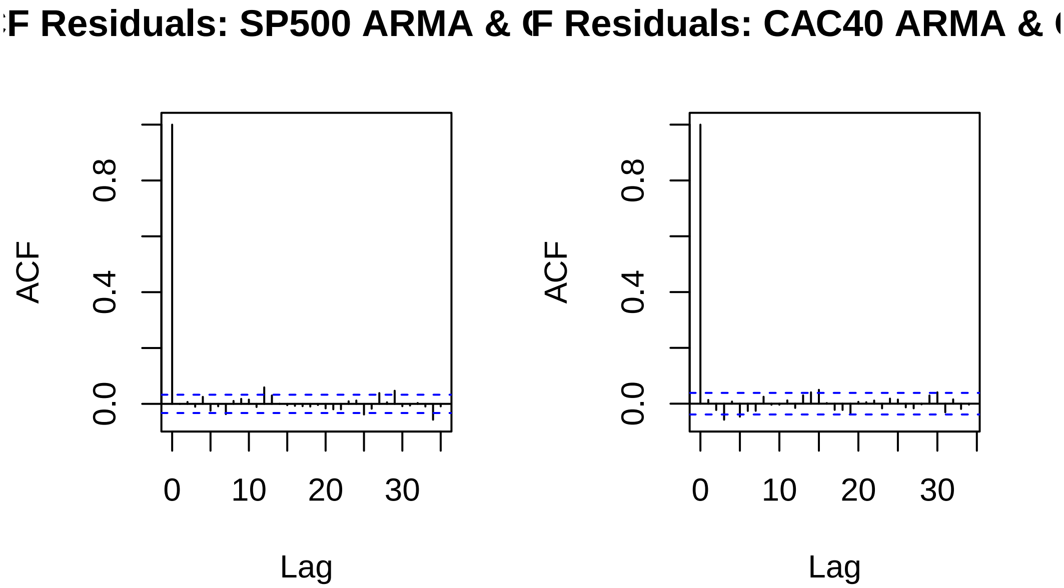
To fit GARCH(1,1) on the ARMA residuals:

To assess the fit quality:

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = res_sp500, cond.dist = "sstd",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x12b07fb58>
 [data = res_sp500]

Conditional Distribution:
 sstd

Coefficient(s):
```

```
         mu         omega        alpha1         beta1          skew         shape
-2.2520e-05   3.3032e-07   5.0301e-02   9.4808e-01   1.0551e+00   7.1935e+00

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -2.252e-05   1.325e-04   -0.170   0.8651
omega   3.303e-07   1.438e-07    2.298   0.0216 *
alpha1  5.030e-02   7.540e-03    6.671 2.53e-11 ***
beta1   9.481e-01   7.530e-03  125.907  < 2e-16 ***
skew    1.055e+00   2.446e-02   43.141  < 2e-16 ***
shape   7.193e+00   8.408e-01    8.556  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 11806.58    normalized:  3.291491

Description:
 Sun Jun 15 09:32:14 2025 by user:


Standardised Residuals Tests:
                                  Statistic    p-Value
 Jarque-Bera Test   R    Chi^2  725.5180693 0.00000000
 Shapiro-Wilk Test  R    W        0.9831582 0.00000000
 Ljung-Box Test     R    Q(10)   10.4216838 0.40430817
 Ljung-Box Test     R    Q(15)   27.7618601 0.02310814
 Ljung-Box Test     R    Q(20)   29.1478746 0.08488825
 Ljung-Box Test     R^2  Q(10)    8.0543302 0.62352994
 Ljung-Box Test     R^2  Q(15)   11.0084514 0.75199504
 Ljung-Box Test     R^2  Q(20)   15.6980861 0.73516897
 LM Arch Test       R    TR^2    10.2583123 0.59331040

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.579637 -6.569291 -6.579642 -6.575949


Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = res_cac40, cond.dist = "sstd",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x139e5c278>
 [data = res_cac40]
```

```
Conditional Distribution:
 sstd

Coefficient(s):
         mu        omega       alpha1        beta1         skew        shape
-4.1284e-04   2.0402e-06   6.5401e-02   9.2730e-01   1.0840e+00   1.0000e+01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -4.128e-04   2.377e-04   -1.737  0.08245 .
omega   2.040e-06   7.576e-07    2.693  0.00708 **
alpha1  6.540e-02   9.543e-03    6.853 7.23e-12 ***
beta1   9.273e-01   1.030e-02   90.004  < 2e-16 ***
skew    1.084e+00   3.327e-02   32.581  < 2e-16 ***
shape   1.000e+01   1.342e+00    7.452 9.24e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 7500.141    normalized:  2.911545

Description:
 Sun Jun 15 09:32:14 2025 by user:


Standardised Residuals Tests:
                                 Statistic      p-Value
 Jarque-Bera Test   R    Chi^2  36.6432206 1.104144e-08
 Shapiro-Wilk Test  R    W       0.9967943 2.934125e-05
 Ljung-Box Test     R    Q(10)  12.0943005 2.787949e-01
 Ljung-Box Test     R    Q(15)  20.7596486 1.446860e-01
 Ljung-Box Test     R    Q(20)  22.3409037 3.223024e-01
 Ljung-Box Test     R^2  Q(10)  12.3893095 2.598459e-01
 Ljung-Box Test     R^2  Q(15)  12.9276394 6.078871e-01
 Ljung-Box Test     R^2  Q(20)  14.5756952 8.001500e-01
 LM Arch Test       R    TR^2   13.4251438 3.389111e-01

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.818432 -5.804797 -5.818443 -5.813490
```




```
    Box-Ljung test

data:  residuals(garch_sp500_arma_res)
X-squared = 30.657, df = 20, p-value = 0.05989


    Box-Ljung test

data:  residuals(garch_cac40_arma_res)
X-squared = 41.079, df = 20, p-value = 0.003639
```

For SP500, we observe that the ARMA + GARCH fit improves the model quality quite clearly: higher log-likelihood, lower AIC and no autocorrelation

On the contrary, CAC40 has an identical log-likelihood and AIC as before, with a p-value still too low (so still some significant autocorrelation). It could be good to tune ARMA better or to try another GARCH variant.

Question h)

> Use the garchAuto.R script in order to fit a GARCH on the residuals of the ARMA(p,q) from (g). Assess the quality of the fit.

```
Loading required package: parallel

   Analyzing (0,0,1,1) with sged distribution done.Good model. AIC = -6.577199, forecast: 0
   Analyzing (0,1,1,1) with sged distribution done.Good model. AIC = -6.576938, forecast: 2e-04
   Analyzing (0,2,1,1) with sged distribution done.Good model. AIC = -6.576852, forecast: 2e-04
   Analyzing (0,3,1,1) with sged distribution done.Good model. AIC = -6.577534, forecast: 1e-04
   Analyzing (0,4,1,1) with sged distribution done.Good model. AIC = -6.577077, forecast: 1e-04
   Analyzing (0,5,1,1) with sged distribution done.Good model. AIC = -6.578182, forecast: -2e-04
   Analyzing (1,0,1,1) with sged distribution done.Good model. AIC = -6.576932, forecast: 2e-04
```

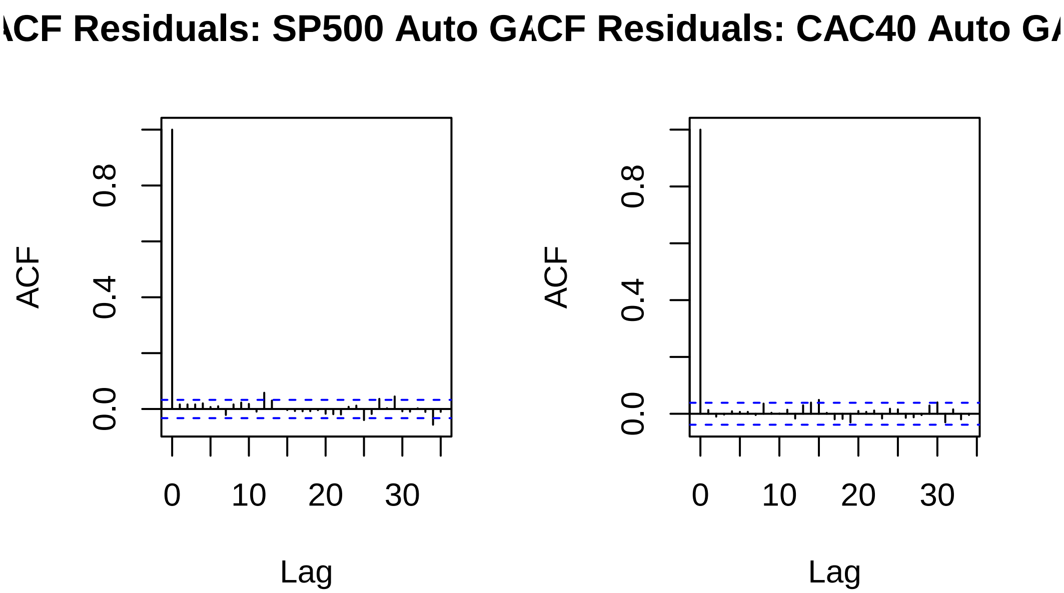
```
Analyzing (1,1,1,1) with sged distribution done.Bad model.
Analyzing (1,2,1,1) with sged distribution done.Bad model.
Analyzing (1,3,1,1) with sged distribution done.Good model. AIC = -6.577338, forecast: 2e-04
Analyzing (1,4,1,1) with sged distribution done.Good model. AIC = -6.576873, forecast: 2e-04
Analyzing (1,5,1,1) with sged distribution done.Good model. AIC = -6.578753, forecast: -1e-04
Analyzing (2,0,1,1) with sged distribution done.Good model. AIC = -6.576831, forecast: 2e-04
Analyzing (2,1,1,1) with sged distribution done.Bad model.
Analyzing (2,2,1,1) with sged distribution done.Bad model.
Analyzing (2,3,1,1) with sged distribution done.Good model. AIC = -6.578384, forecast: -4e-04
Analyzing (2,4,1,1) with sged distribution done.Bad model.
Analyzing (2,5,1,1) with sged distribution done.Bad model.
Analyzing (3,0,1,1) with sged distribution done.Good model. AIC = -6.577463, forecast: 1e-04
Analyzing (3,1,1,1) with sged distribution done.Good model. AIC = -6.577279, forecast: 2e-04
Analyzing (3,2,1,1) with sged distribution done.Good model. AIC = -6.578381, forecast: -4e-04
Analyzing (3,3,1,1) with sged distribution done.Bad model.
Analyzing (3,4,1,1) with sged distribution done.Bad model.
Analyzing (3,5,1,1) with sged distribution done.Bad model.
Analyzing (4,0,1,1) with sged distribution done.Good model. AIC = -6.577022, forecast: 2e-04
Analyzing (4,1,1,1) with sged distribution done.Good model. AIC = -6.576817, forecast: 2e-04
Analyzing (4,2,1,1) with sged distribution done.Bad model.
Analyzing (4,3,1,1) with sged distribution done.Bad model.
Analyzing (4,4,1,1) with sged distribution done.Bad model.
Analyzing (4,5,1,1) with sged distribution done.Bad model.
Analyzing (5,0,1,1) with sged distribution done.Good model. AIC = -6.578058, forecast: -2e-04
Analyzing (5,1,1,1) with sged distribution done.Good model. AIC = -6.578805, forecast: -2e-04
Analyzing (5,2,1,1) with sged distribution done.Bad model.
Analyzing (5,3,1,1) with sged distribution done.Bad model.
Analyzing (5,4,1,1) with sged distribution done.Bad model.
Analyzing (5,5,1,1) with sged distribution done.Bad model.
```

```
Analyzing (0,0,1,1) with sged distribution done.Good model. AIC = -5.820957, forecast: -4e-04
Analyzing (0,1,1,1) with sged distribution done.Good model. AIC = -5.820384, forecast: -4e-04
Analyzing (0,2,1,1) with sged distribution done.Good model. AIC = -5.820133, forecast: -5e-04
Analyzing (0,3,1,1) with sged distribution done.Good model. AIC = -5.822429, forecast: -6e-04
Analyzing (0,4,1,1) with sged distribution done.Good model. AIC = -5.82211, forecast: -7e-04
Analyzing (0,5,1,1) with sged distribution done.Good model. AIC = -5.824134, forecast: -4e-04
Analyzing (1,0,1,1) with sged distribution done.Good model. AIC = -5.820385, forecast: -4e-04
Analyzing (1,1,1,1) with sged distribution done.Bad model.
Analyzing (1,2,1,1) with sged distribution done.Good model. AIC = -5.821597, forecast: -0.0014
Analyzing (1,3,1,1) with sged distribution done.Bad model.
Analyzing (1,4,1,1) with sged distribution done.Bad model.
Analyzing (1,5,1,1) with sged distribution done.Good model. AIC = -5.824193, forecast: -8e-04
Analyzing (2,0,1,1) with sged distribution done.Good model. AIC = -5.820152, forecast: -5e-04
Analyzing (2,1,1,1) with sged distribution done.Good model. AIC = -5.821675, forecast: -0.0015
Analyzing (2,2,1,1) with sged distribution done.Bad model.
Analyzing (2,3,1,1) with sged distribution done.Good model. AIC = -5.823722, forecast: -7e-04
Analyzing (2,4,1,1) with sged distribution done.Good model. AIC = -5.823997, forecast: -0.001
Analyzing (2,5,1,1) with sged distribution done.Good model. AIC = -5.824055, forecast: -2e-04
Analyzing (3,0,1,1) with sged distribution done.Good model. AIC = -5.822217, forecast: -5e-04
Analyzing (3,1,1,1) with sged distribution done.Good model. AIC = -5.821607, forecast: 4e-04
Analyzing (3,2,1,1) with sged distribution done.Good model. AIC = -5.823868, forecast: -8e-04
Analyzing (3,3,1,1) with sged distribution done.Bad model.
```

```
   Analyzing (3,4,1,1) with sged distribution done.Bad model.
   Analyzing (3,5,1,1) with sged distribution done.Bad model.
   Analyzing (4,0,1,1) with sged distribution done.Good model. AIC = -5.821934, forecast: -5e-04
   Analyzing (4,1,1,1) with sged distribution done.Good model. AIC = -5.822103, forecast: 3e-04
   Analyzing (4,2,1,1) with sged distribution done.Bad model.
   Analyzing (4,3,1,1) with sged distribution done.Bad model.
   Analyzing (4,4,1,1) with sged distribution done.Bad model.
   Analyzing (4,5,1,1) with sged distribution done.Bad model.
   Analyzing (5,0,1,1) with sged distribution done.Good model. AIC = -5.823899, forecast: -2e-04
   Analyzing (5,1,1,1) with sged distribution done.Good model. AIC = -5.824684, forecast: -7e-04
   Analyzing (5,2,1,1) with sged distribution done.Bad model.
   Analyzing (5,3,1,1) with sged distribution done.Bad model.
   Analyzing (5,4,1,1) with sged distribution done.Bad model.
   Analyzing (5,5,1,1) with sged distribution done.Bad model.

Title:
 GARCH Modelling

Call:
 garchFit(formula = formula, data = data, cond.dist = ll$dist,
    trace = FALSE)

Mean and Variance Equation:
 data ~ arma(5, 1) + garch(1, 1)
<environment: 0x12eb37890>
 [data = data]

Conditional Distribution:
 sged

Coefficient(s):
         mu          ar1          ar2          ar3          ar4          ar5
-2.2520e-05   6.1560e-01  -5.1324e-05  -1.9771e-02   2.1914e-02  -3.3774e-02
        ma1        omega       alpha1        beta1         skew        shape
-6.3161e-01   3.8994e-07   5.2238e-02   9.4520e-01   1.0729e+00   1.3954e+00

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -2.252e-05   5.003e-05   -0.450 0.652641
ar1     6.156e-01   1.885e-01    3.265 0.001093 **
ar2    -5.132e-05   2.043e-02   -0.003 0.997996
ar3    -1.977e-02   1.974e-02   -1.001 0.316667
ar4     2.191e-02   2.052e-02    1.068 0.285454
ar5    -3.377e-02   1.918e-02   -1.761 0.078316 .
ma1    -6.316e-01   1.888e-01   -3.345 0.000823 ***
omega   3.899e-07   1.556e-07    2.506 0.012194 *
alpha1  5.224e-02   7.840e-03    6.663 2.69e-11 ***
beta1   9.452e-01   7.986e-03  118.360  < 2e-16 ***
skew    1.073e+00   2.364e-02   45.383  < 2e-16 ***
```

```
shape   1.395e+00   4.584e-02   30.442  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 11811.09    normalized:  3.292748

Description:
 Sun Jun 15 09:33:13 2025 by user:


Standardised Residuals Tests:
                                 Statistic     p-Value
 Jarque-Bera Test   R    Chi^2  766.0267687 0.000000000
 Shapiro-Wilk Test  R    W        0.9822839 0.000000000
 Ljung-Box Test     R    Q(10)   14.5209369 0.150528097
 Ljung-Box Test     R    Q(15)   31.5502756 0.007408813
 Ljung-Box Test     R    Q(20)   32.8242579 0.035269235
 Ljung-Box Test     R^2  Q(10)    7.8181492 0.646594554
 Ljung-Box Test     R^2  Q(15)   11.0400638 0.749749175
 Ljung-Box Test     R^2  Q(20)   15.6369602 0.738875811
 LM Arch Test       R    TR^2    10.1047177 0.606774224

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.578805 -6.558113 -6.578827 -6.571430


Title:
 GARCH Modelling

Call:
 garchFit(formula = formula, data = data, cond.dist = ll$dist,
    trace = FALSE)

Mean and Variance Equation:
 data ~ arma(5, 1) + garch(1, 1)
<environment: 0x11d2f0a20>
 [data = data]

Conditional Distribution:
 sged

Coefficient(s):
         mu          ar1          ar2          ar3          ar4          ar5
-1.8718e-04   6.2888e-01  -1.3514e-02  -4.6209e-02   3.3830e-02  -5.3382e-02
        ma1        omega       alpha1        beta1         skew        shape
-6.3026e-01   2.0199e-06   6.5264e-02   9.2512e-01   1.1129e+00   1.7928e+00

Std. Errors:
 based on Hessian

Error Analysis:
```

```
         Estimate  Std. Error  t value Pr(>|t|)    
mu     -1.872e-04   1.063e-04   -1.761  0.07831 .  
ar1     6.289e-01   1.329e-01    4.731 2.23e-06 ***
ar2    -1.351e-02   2.371e-02   -0.570  0.56869    
ar3    -4.621e-02   2.397e-02   -1.928  0.05390 .  
ar4     3.383e-02   2.492e-02    1.357  0.17469    
ar5    -5.338e-02   2.107e-02   -2.534  0.01127 *  
ma1    -6.303e-01   1.317e-01   -4.784 1.72e-06 ***
omega   2.020e-06   7.125e-07    2.835  0.00458 ** 
alpha1  6.526e-02   8.804e-03    7.413 1.23e-13 ***
beta1   9.251e-01   9.948e-03   92.996  < 2e-16 ***
skew    1.113e+00   3.255e-02   34.186  < 2e-16 ***
shape   1.793e+00   7.472e-02   23.993  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 7514.193    normalized:  2.917 

Description:
 Sun Jun 15 09:33:58 2025 by user:  


Standardised Residuals Tests:
                                 Statistic      p-Value
 Jarque-Bera Test   R    Chi^2  42.7453156 5.223633e-10
 Shapiro-Wilk Test  R    W       0.9961469 3.556087e-06
 Ljung-Box Test     R    Q(10)   8.3033881 5.992288e-01
 Ljung-Box Test     R    Q(15)  17.6818769 2.797593e-01
 Ljung-Box Test     R    Q(20)  19.6727478 4.785610e-01
 Ljung-Box Test     R^2  Q(10)  13.5760142 1.932238e-01
 Ljung-Box Test     R^2  Q(15)  14.0134143 5.245120e-01
 Ljung-Box Test     R^2  Q(20)  16.4207731 6.902011e-01
 LM Arch Test       R    TR^2   14.2477321 2.851692e-01

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC 
-5.824684 -5.797414 -5.824727 -5.814799 
```




```
    Box-Ljung test

data:  residuals(best_garch_sp500)
X-squared = 28.262, df = 20, p-value = 0.1033

    Box-Ljung test

data:  residuals(best_garch_cac40)
X-squared = 22.765, df = 20, p-value = 0.3005
```

As a result, both SP500 and CAC40 now have the best fit compared to the previous fits we have tried. Residuals are now white noise for both indices (so no more autocorrelation) and volatility are well-captured.