

Sphere::intersect

Find distance t to intersection \vec{w} of a straight line with a spherical surface.

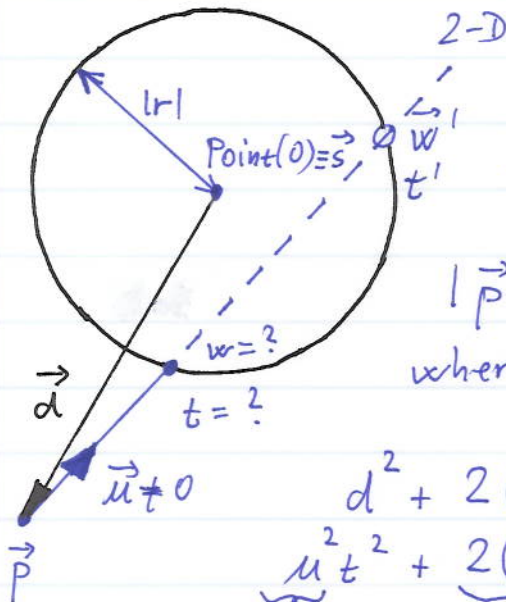
$$\text{Sphere: } |\vec{w} - \vec{s}|^2 = r^2 \quad (1)$$

$$\text{Line: } \vec{w} = \vec{p} + \vec{u}t \quad (2)$$

Substitute (2) into (1) and solve for t :

$$|\vec{p} + \vec{u}t - \vec{s}|^2 = |\vec{d} + \vec{u}t|^2 = r^2$$

where $\vec{d} \equiv \vec{p} - \vec{s}$



$$d^2 + 2(\vec{d} \cdot \vec{u})t + u^2 t^2 = r^2$$

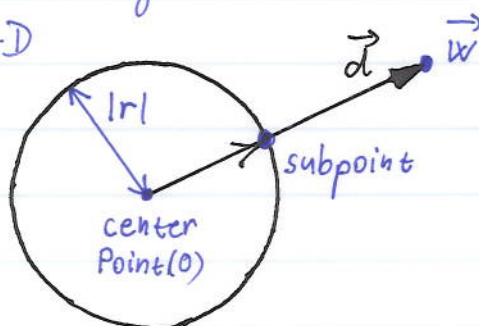
$$\underbrace{u^2 t^2}_{a \neq 0} + \underbrace{2(\vec{d} \cdot \vec{u})t}_b + \underbrace{(d^2 - r^2)}_c = 0 \quad (3)$$

If there are two positive roots t , return the smaller one;
otherwise return the larger root, or signal no solution.
If a valid t is available, use it in (2) to get \vec{w} .

Sphere::subpoint, Sphere::distance

Send the line from center through \vec{w} ; its intersection with sphere is the subpoint (if $\vec{w} = \text{center}$, return no solution for the subpoint).
The returned distance between \vec{w} and the subpoint carries a sign according to the table.

2-D



$\frac{\vec{w} \cdot \vec{n}}{r}$	inside	outside
inward	positive	negative
outward	negative	positive