

Detector :: do-Ray

Element of solid angle from pmh-2015-0210, page 1.

$$\begin{aligned}\Delta\Omega &= \int_{\text{wedge}} d\Omega = \int_{\varphi_{\min}}^{\varphi_{\max}} d\varphi \int_{\vartheta_{\min}}^{\vartheta_{\max}} \sin\vartheta d\vartheta = \Delta\varphi (\cos\vartheta_{\min} - \cos\vartheta_{\max}) \\ &= 2\Delta\varphi \sin\frac{\vartheta_{\max} + \vartheta_{\min}}{2} \sin\frac{\vartheta_{\max} - \vartheta_{\min}}{2}\end{aligned}$$

Central Ray: $\vartheta=0$, $\vartheta_{\min}=0$, $\vartheta_{\max}=\frac{\Delta\vartheta}{2}$
 $\varphi_{\min}=0$, $\varphi_{\max}=2\pi$

$$\Delta\Omega = 2 \times 2\pi \times \sin\frac{\frac{\Delta\vartheta}{2}}{2} \times \sin\frac{\frac{\Delta\vartheta}{2}}{2} = \underline{\underline{4\pi \sin^2\frac{\Delta\vartheta}{4}}}$$

if $\Delta\vartheta \rightarrow 0$, then $\Delta\Omega \approx 4\pi \left(\frac{\Delta\vartheta}{4}\right)^2 = \pi \left(\frac{\Delta\vartheta}{2}\right)^2 \checkmark$

All other Rays: $\vartheta \neq 0$, $\vartheta_{\min} = \vartheta - \frac{\Delta\vartheta}{2}$, $\vartheta_{\max} = \vartheta + \frac{\Delta\vartheta}{2}$

$$\underline{\underline{\Delta\Omega = 2\Delta\varphi \sin\vartheta \sin\frac{\Delta\vartheta}{2}}}$$