pmh_2022-0427

$\boxed{\text{Cone::Intercept}}$ INTERSECTION OF TWO NON-PARALLEL LINES

Distance of two non-parallel lines in 3D : Vector3d

$\vec{w} = \vec{p} + \vec{u}\,t$ and $\vec{q} + \vec{v}\,s$: $\left| (\vec{p} - \vec{q}) \cdot (\widehat{\vec{u} \times \vec{v}}) \right|$ unit vector

If this distance is zero, we can find the intersection in the plane containing the two lines.

$$\underset{\text{Ray}}{p_x + u_x t} = \underset{\text{Cone}}{r_A + \Delta r \cdot s} \Rightarrow u_x t - \Delta r \cdot s = r_A - p_x$$

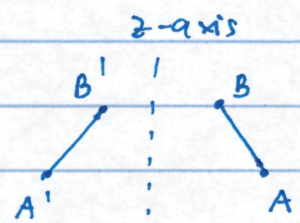
$$p_y + u_y t = z_A + \Delta z \cdot s \qquad u_y t - \Delta z \cdot s = z_A - p_y$$

$$\begin{pmatrix} u_x & -\Delta r \\ u_y & -\Delta z \end{pmatrix} \begin{pmatrix} t \\ s \end{pmatrix} = \begin{pmatrix} r_A - p_x \\ z_A - p_y \end{pmatrix}$$

$$D = u_y \Delta r - u_x \Delta z \; ; \; D_s = \begin{vmatrix} u_x & r_A - p_x \\ u_y & z_A - p_y \end{vmatrix} = u_x (z_A - p_y) - u_y (r_A - p_x)$$

$s = \frac{D_s}{D}$, where $s \in [0, 1]$ for the intersection to be contained by the Cone

intersection point $= \vec{A} + (\vec{B} - \vec{A})\,s$



Repeat for the second candidate :

where :

$A \to A'$ via $r \to -r$

$B \to B'$ $\quad z \to z$

$\vec{u}$ intersects z-axis

Transform solution back to 3D using $\varphi = \operatorname{atan2}(u_y, u_x)$.

$x = r\cos\varphi$, $y = r\sin\varphi$, $z = z$