Monte-Carlo Method for Uncertainty Quantification of SRM MEOP

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Abstract

In this report the experimental data of a Solid Rocket Motor will be analyzed for performance estimation. The lack of flexibility affecting SRMs makes the reproducibility of estimated performance an aspect to take into account for the accomplishment of any mission, ranging from small sounding rockets to the solid boosters of a launch vehicle. To this aim, a Monte-Carlo simulation will be performed in order to quantify the uncertainty in terms of MEOP for the 70-bar rocket version, including uncertainties.

Nomenclature

Nondimensional Quantities

μ	Mean value	$[-]$
σ	Standard deviation	$[-]$
n	Ballistic coefficient	$[-]$

Physical Quantities

$\dot{m}_p$	Propellant mass flow rate	[kg/s]
ρ	Density	$[\mathrm{kg/m^3}]$
a	Pre-exponential coefficient	$[\frac{\mathrm{mm}}{\mathrm{s \cdot bar}}]$
A_b	Burning Area	$[\mathrm{m^2}]$
A_t	Throat Area	$[\mathrm{m^2}]$
c^*	Characteristic velocity	[m/s]
D	Diameter	[mm]
M_{tot}	Total propellant mass	[kg]
P_c	Combustion chamber pressure	[bar]
r_b	Burning rate	[mm/s]
t_b	Burning time	[s]

Acronyms

BATES	BAllistic Test and Evaluation System
HTPB	Hydroxyl-Terminated Polybutadiene
MC	Monte Carlo
MEOP	Maximum expected operating pressure
SRM	Solid Rocket Motor

1 Introduction

The 9 propellant batches have been tested at three different chamber pressure conditions, labeled as low-, mid-, high-pressure and characterized by the following throat diameters:

Pressure Level:	**Low**	**Mid**	**High**
D_t [mm]	28.80	25.25	21.81

Each batch is nominally identical and composed of: AP (68%), Al (18%), HTPB (14%). The rocket motor used in each test is a BATES motor, characterized by the sizes reported in Fig. 1. A pressure transducer is placed inside the combustion chamber, just upstream of the grain, to measure the pressure time-history with a sampling rate of 1000 Hz. The pressure traces resulting from the test campaign are all exhibiting a trend similar to the one reported in Fig. 2.

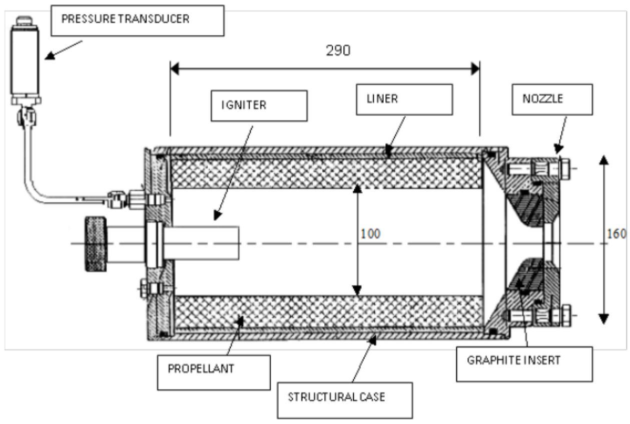

Figure 1: *BATES schematics*

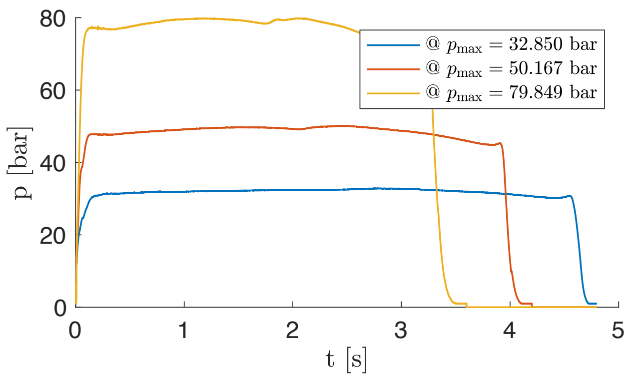

Figure 2: *Pressure traces sample*

2 Internal Ballistics Analysis

The propellant burning time depends on the burning rate r_b according to Vielle's law; thus under the assumption of steady state behavior

$$r_b = aP_c{}^n$$

and on the burning area A_b. Moreover, the A_b time-dependence leads to a non-constant combustion chamber pressure P_c according to the following equation:

$$P_c = \left(a\rho_p \frac{A_b}{A_t} c^* \right)^{\frac{1}{1-n}}$$

First, coefficients a and n are computed by fitting the data of all pressure traces; both a and n depend only on the propellant composition and not on the geometry of the motor. In order to do that, for each batch and for each pressure trace, we take the maximum pressure value, then compute the 5% and identify the action time $t_G - t_A$. Now we compute reference pressure as:

$$P_{\text{ref}} = \frac{\int_{t_A}^{t_G} P_c \, dt}{2(t_G - t_A)}$$

and set the burning time $t_E - t_B$. Finally, we compute the effective pressure

$$P_{\text{eff}} = \frac{\int_{t_B}^{t_E} P_c \, dt}{t_E - t_B}$$

and the burning rate as the ratio between the thickness and the burning time

$$r_b = \frac{w}{t_E - t_B}$$

to obtain a mean value for a and n and the respective uncertainties σ. Secondly, the characteristic velocity can be computed from the experimental data using[(1)]:

$$c^*_{exp} = \frac{\int_{t_B}^{t_E} P_c A_t \, dt}{M_{\text{tot}}}$$

(1) From propellant composition, the average density is derived: $\rho_p = 1761.933$ kg/m^3. Then, knowing the geometry of the motor - $V_{\text{tot}} = 3.553 \cdot 10^{-3}$ m^3 and the total mass results: $M_{\text{tot}} = 6.26$ kg.

3 Monte Carlo Simulation

From the previously analyzed experimental data, the mean values along with the related uncertainty can be computed for the three variables that will be taken into account, $\{a, n, c^*\}$, which yields:

Table 1: *Uncertainties computed from experimental data*

	a [mm/(s · bar^n)]	n [−]	c^* [m/s]
Mean value	1.7319	0.3811	1517.8046
Uncertainty	±0.0189657	±0.0028112	±123.3692

Using the CEA code we obtain a mean ideal c^* equal to 1579.6 m/s, that leads to a c^* efficiency of 0.9609. We may at this point assume a *Gaussian distribution* for all the variables and randomly pick 30 samples per variable, which will therefore yield a total amount of $30^3 = 27000$ MC iterations. For each iteration the burning time t_b will be computed by tracking the grain regression until burnout. At each time instant the following algorithm is thus performed:

1. $t = \bar{t} \quad \Rightarrow \quad A_b(\bar{t}) = 2\pi r_i (2x_{\text{corner}}) + 2\pi \left(\frac{D_e^2}{4} - r_i^2 \right)$

2. re-compute: $\quad P_c \underset{\substack{\downarrow \\ \text{(quasi-steady)}}}{=} \left(a \rho_p \frac{A_b}{A_t} c^* \right)^{\frac{1}{1-n}}$

3. re-compute: $\quad r_b = a \cdot {P_c}^n$

4. regression progress: $\quad \Delta x \equiv \Delta y = r_b \cdot \Delta t$

The algorithm will then result in a triple-loop, through which the samples are permuted. The convergence of the MC method can be checked by analyzing the trend of MEOP μ and σ, considering all the outputted values.

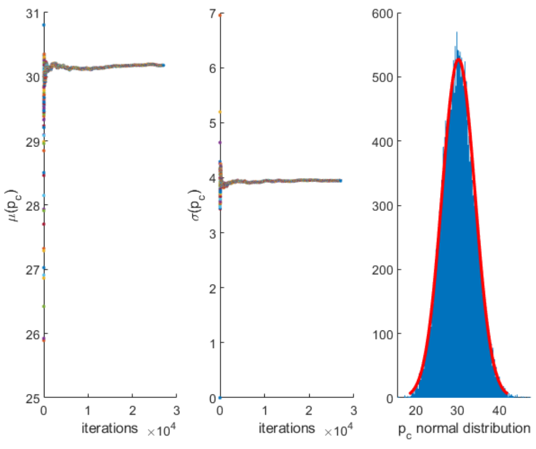

Figure 3: *MC iterations (low-pressure)*

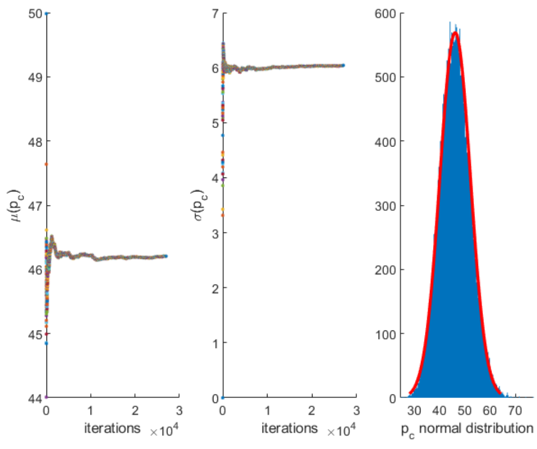

Figure 4: *MC iterations (mid-pressure)*

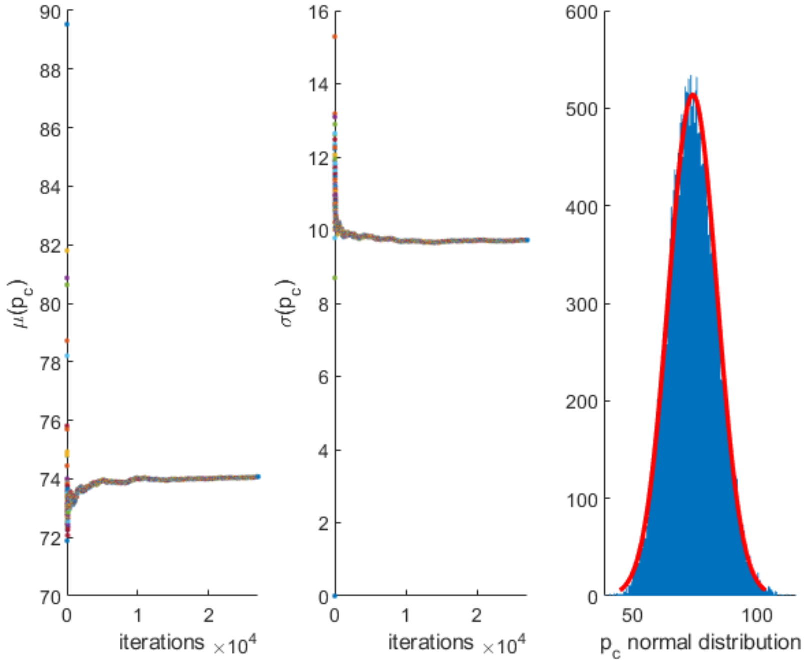

Figure 5: *MC iterations (high-pressure)*

4 Conclusions

The uncertainty propagation by means of the MC method yields the following estimation:

Table 2: *MEOP uncertainty*

Pressure Level:	**Low**	**Mid**	**High**
$\overline{P}_c$ [bar]	30.1984	46.1009	73.9827
σ_{P_c} [bar]	3.9259	5.9708	9.8497

The resulting uncertainty over the value of P_c, due to the propagation of the former, turns out to be around 13% at all pressure levels. The 70-bar rocket version P_c distribution is almost perfectly normal: it has a skewness of 0.1368 ($0 \rightarrow$ perfect normal distribution, $-2 < s < +2$ acceptable range) and a kurtosis of 3.0277 ($3 \rightarrow$ perfect normal distribution). Visual Normality Test is assessed with QQ Plot and Normal Probability Plot:

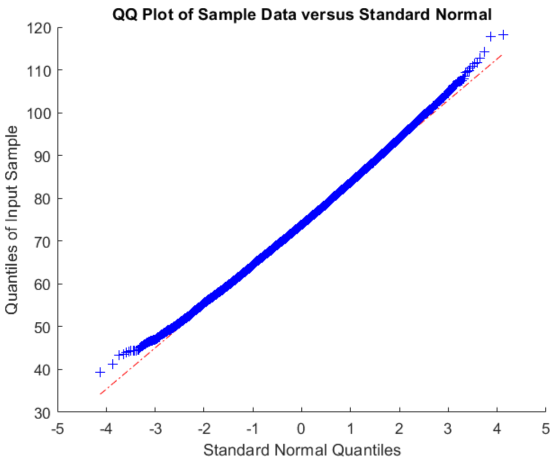

Figure 6: *Quantile-quantile plot (high-pressure)*

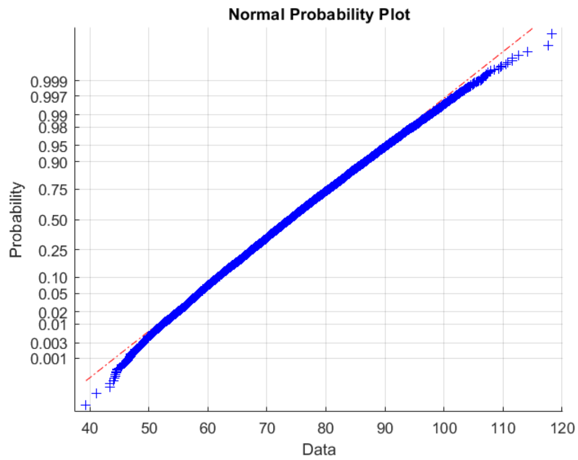

Figure 7: *Normal probability plot (high-pressure)*