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REMOTE SENSING FOR EARTH OBSERVATION AND SURVEILLANCE: HOMEWORK ASSIGNEMENT 2

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Chapter 1

Problem 1: performance and design of a spaceborn SAR

The BIOMASS Mission is to be launched by the European Space Agency in 2022. It will be implemented as a spaceborne SAR operating at P-Band. BIOMASS primary goal is to map forest Above Ground Biomass (AGB) worldwide to within an accuracy of 20%. AGB is estimated from SAR intensity based on semi-empirical laws. As a reference, you can assume the following law:

$$AGB = A\sigma_{hv}^{p} \tag{1.1}$$

where AGB is expressed in tons per hectare, σ_{hv} is the backscatter coefficient of vegetation in HV polarization, and $A = 2.4 * 10^6$ and $p = 2.6$ are two empirically derived constants. Students are required to consider the following points:

1. Derive the relative accuracy to be required on σ_{hv} to ensure an accuracy of 20% on AGB.

2. Determine the size of the averaging window to meet the requirement derived at point 1).

3. Determine a), b), c) as necessary to ensure a Signal to Noise Ratio of at least $10dB$ at HV polarization across the whole range of AGB values:

   (a) Pulse Repetition Interval (PRI)

   (b) Duration of the transmitted pulses

   (c) Transmitted power for each transmitted pulse

4. Discuss whether your response at points 1) and 2) needs to be modified when accounting for thermal noise.

5. Discuss the choice of the focusing processor (is 1D azimuth matched filtering enough?).

6. One of BIOMASS secondary goals is to map terrain topography using SAR Interferometry. By virtue of the penetration capabilities of P-Band waves, one can assume that the signal in HH polarization is mostly contributed by trunk-ground double bounce scattering from several tree trunks within any resolution cell. Accordingly, the phase center can be (at least approximately) assumed to be directly associated with terrain topography.

(a) Evaluate the amount of phase noise in HH interferograms for a height of height of ambiguity $z_{amb} = 80m$

(b) Determine the size of the averaging window required to estimate topography to within an accuracy of 5 m

Reference parameters:

- Azimuth resolution: 10 m (processing a full synthetic aperture)
- Carrier frequency: $f_0 = 435MHz$
- Transmitted bandwidth: 6 MHz (maximum allowed by ITU regulations for frequency allocation)
- Orbital height: 666 Km
- Reference incidence angle (off-nadir): $\theta = 28^o$
- Reference noise temperature: T = 290 K (as usual)
- Reference range of AGB: (250-500) tons per hectare
- Reference value for signal backscatter in HH polarization: $\sigma_{hh} = -8dB$ (assuming the signal is dominated by trunkground double bounce scattering).

1.1 Derive the relative accuracy to be required on σ_{hv} to ensure an accuracy of 20% on AGB

The relative accuracy for a variable x is defined as $\frac{\sigma_x}{x}$, where σ_x is the standard deviation of the estimation error. To derive the relative accuracy to be required on σ_{hv} we have to start from the AGB law 1.1. Deriving the law we obtain:

$$dAGB = A * p * \sigma_{hv}^{(p-1)} * d\sigma_{hv} \tag{1.2}$$

Then we obtain the accuracies dividing 1.2 by 1.1:

$$\frac{dAGB}{AGB} = \frac{A * p * \sigma_{hv}^{(p-1)} * d\sigma_{hv}}{A\sigma_{hv}^{p}} \tag{1.3}$$

Now we can set the accuracy on AGB equal to 20% to obtain the required accuracy on σ_{hv}:

$$accuracy_{AGB} = p * accuracy_{\sigma_{hv}} = 0.2 \tag{1.4}$$

$$accuracy_{\sigma_{hv}} = \frac{0.2}{p} \tag{1.5}$$

So we obtain that to ensure an accuracy of 20% on AGB we need to have an accuracy on σ_{hv} of at least 0.0769, that is 7.69%.

1.2 Determine the size of the averaging window to meet the requirement derived at point 1)

At point 1 we obtained the required accuracy on σ_{hv}; from it we can derive the equivalent number of independent pixels in the estimation window necessary to obtain that accuracy:

$$accuracy_{\sigma_{hv}} = \frac{1}{\sqrt{L_{eq}}} \tag{1.6}$$

$$L_{eq} = \left(\frac{1}{accuracy_{\sigma_{hv}}}\right)^2 \tag{1.7}$$

We obtain $L_{eq} = 169$. Now we can calculate the real number of pixels in the estimation window L:

$$L = L_{eq}\frac{sampling}{resolution} \tag{1.8}$$

As a first approximation we can consider we have exactly the equivalent number of pixels, and so $L = L_{eq} = 169$. To obtain the area of the averaging window, first we have to calculate the area of each pixel, that is the product of range and azimuth resolution. We already have the azimuth resolution $\rho_x = 10m$, and we can calculate the range resolution:

$$\rho_r = \frac{c}{2B} \tag{1.9}$$

where c is the speed of light, and B is the given bandwidth. We obtain $\rho_r = 25m$. Now we can calculate the pixel (resolution cell) area:

$$A_{pixel} = \rho_x * \rho_r \tag{1.10}$$

So we have $A_{pixel} = 250m^2$. As we have 169 pixels, we can consider we have a 13x13 grid of pixels. Now we can calculate the area of the averaging window:

$$A_{averagingwindow} = 13 * \rho_x * 13 * \rho_r \tag{1.11}$$

We obtain $A_{averagingwindow} = 42250m^2$. In reality we will always have to account for sampling, so we will have a real number of pixels higher than the equivalent one. So we can have a 14x14, 15x15 or even bigger averaging window, that will obviously have a bigger area with respect to the 13x13 averaging window obtained with $L = L_{eq}$.

1.3 Determine a), b), c) as necessary to ensure a Signal to Noise Ratio of at least $10dB$ at HV polarization across the whole range of AGB values:

The first step to solve this point is to make hypotesis about important antenna values we need to obtain the desired parameters:

- antenna diameter $d = 12m$,
- ambiguous range $R_{amb} = 50000m$,
- duty cycle $\delta = 0.1$,

- atmospheric losses $L_{atm,dB} = 3dB$,
- noise figure $F_{dB} = 3dB$.

We also need the forest Radar Cross Section across the whole range of AGB values. To simplify the calculations we calculate RCS only for the minimum and the maximum values. We need the backscatter coefficient in the two conditions and antenna illumination area:

$$\sigma_{hv,min} = \left(\frac{AGB_{min}}{A}\right)^{\left(\frac{1}{p}\right)} \tag{1.12}$$

$$\sigma_{hv,MAX} = \left(\frac{AGB_{MAX}}{A}\right)^{\left(\frac{1}{p}\right)} \tag{1.13}$$

$$\lambda = \frac{c}{f_0} \tag{1.14}$$

$$angle_{ill} = \frac{\lambda}{2\rho_x} \tag{1.15}$$

$$r = \frac{H}{cos(\theta)} \tag{1.16}$$

$$Area_{ill} = r * \rho_r * angle_{ill} \tag{1.17}$$

$$RCS_{min} = \sigma_{hv,min} * Area_{ill} \tag{1.18}$$

$$RCS_{MAX} = \sigma_{hv,MAX} * Area_{ill} \tag{1.19}$$

We obtain $RCS_{min} = 19118m^2$ and $RCS_{MAX} = 24959m^2$

1.3.1 (a) Pulse Repetition Interval PRI

We can easily calculate the Pulse Repetition Interval, and then we can also obtain the Pulse Repetiotion Frequency:

$$PRI = \frac{2}{c} * R_{amb} \tag{1.20}$$

$$PRF = \frac{1}{PRI} \tag{1.21}$$

We obtain $PRI = 3.333 * 10^{-4}s$ and $PRF = 3000Hz$

1.3.2 (b) Duration of the transmitted pulse

We can calculate the duration of the transmitted pulse, also known as observation time, with the following equation:

$$T_{obs} = PRI * \delta \tag{1.22}$$

obtaining $T_{obs} = 3.333 * 10^{-5}s$

1.3.3 (c) Transmitted power for each transmitted pulse

To obtain the transmitted power for each transmitted pulse we need to use the antenna equation. To use it we need to calculate antenna area A and gain G:

$$A = \pi * \frac{d^2}{4} \tag{1.23}$$

$$G = \frac{4\pi}{\lambda^2} * A \tag{1.24}$$

We have to transform all the quantities froom dB to linear scale:

$$F = 10^{(F_{dB}/10)} \tag{1.25}$$

$$SNR = 10^{(SNR_{dB}/10)} \tag{1.26}$$

$$L_{atm} = 10^{(L_{atm,dB}/10)} \tag{1.27}$$

We also have to compute the noise spectral density N_0, that is the product between noise figure, reference temperature and Boltzmann constant $K = 1.38 * 10^{-23}(m^2kg)/(s^2K)$

$$N_0 = K * T * F \tag{1.28}$$

Now we can manipulate the radar equation to obtain the transmitted power for each transmitted pulse with a given SNR:

$$P_r = P_t * \frac{1}{(4 * pi * R^2)^2} * G * A * \frac{RCS}{L} \tag{1.29}$$

$$E_r = P_r * T_{obs} = P_t * \frac{T_obs}{(4 * pi * R^2)^2} * G * A * \frac{RCS}{L} \tag{1.30}$$

$$SNR = E_r/N_0 = P_t * \frac{Toss}{(4 * pi * R^2)^2} * G * A * \frac{RCS}{L * N_0} \tag{1.31}$$

$$P_t = \frac{SNR}{\frac{T_{obs}}{(4*\pi*r^2)^2} * G * A * \frac{RCS}{L_{atm}*N_0}} \tag{1.32}$$

For RCS_{min} we obtain $P_t = 37.817W$, for RCS_{MAX} we obtain $P_t = 28.967W$. To have a minimum SNR of 10 dB across the whole range of AGB values we have to use the power obtained with RCS_{min}. If we use that power we obtain $SNR_{dB} = 10dB$ at the minimum AGB value, and an higher SNR for the other values in the given range. For example with $P_t = 37.817W$, for RCS_{MAX}, inverting the equation 1.32, we obtain $SNR = 13.055$, that is $SNR_{dB} = 11.158dB$

1.4 Discuss whether your response at points 1) and 2) needs to be modified when accounting for thermal noise

The thermal noise is generated by the radiation emitted by any body that isn't at a temperature of 0 K. For this reason the antenna, the circuit and all the system always emit thermal noise which affects the transmitted signal. The thermal noise affects all the frequencies following the Planck's law. With some semplifications we can neglect the descending part of Planck's law and we obtain the Rayleigh-Jeans model of the noise power density:

$$dS = \frac{KTB}{\lambda^2} d\Omega \tag{1.33}$$

For a single polarization we can also obtain the noise power:

$$P_n = KTB \tag{1.34}$$

This is true for microwaves, small bandwidth, and if the antenna is enclosed by the emitting body. The thermal noise introduced by a radar system can be modeled with the noise figure F. In Earth Observation Radars

$$P_n = KTB * F \tag{1.35}$$

The thermal noise modeled in this way is a random process with these characteristics: $S_n(f)$ =constant within B, $E[|n(t)|^2] = KTBF$, $E[n(t)] = 0$. The thermal noise affects the power which must be transmitted for each transmitted pulse if we want to obtain a given minimum signal to noise ratio; If we have more thermal noise, we must increase the transmitted power to keep the same SNR. While if we keep the transmitted power constant, the thermal noise effect is to reduce the SNR. In calculating the transmitted power for each pulse in the previous question we already took into account the thermal noise, as it's always present in every radar. We set noise figure $F = 3dB$, but we obtained a power which is still feasible for a spaceborn radar to obtain a SNR of at least 10 dB. If we neglected the thermal noise, we would obtain a lower necessary power. With an higher thermal noise, if the required power increased too much, we would have to filter out the noise. The best filter, that is the one which maximizes SNR with a given transmitted power, is the matched filter. If matched filtering isn't enough, we have to modify the system to fulfil the SNR requirements without exceeding in the necessary power. A way to do it is increasing T_{obs} (pulse compression strategy), that is the duration of the transmitted pulse, as with matched filtering

$$SNR = \frac{p_r T_{obs}}{N_0} \tag{1.36}$$

But this would result in a worse range resolution, as

$$\rho_r = \frac{c}{2} * T_{obs} \tag{1.37}$$

In this way the area of a single pixel in the averaging window would increase, and so also the averaging window area calculated in the point 2 would increase. While if we act in this way to counteract the effects of the thermal noise, the response at point one isn't affected.

1.5 Discuss the choice of the focusing processor (is 1D azimuth matched filtering enough?)

After the acquisition of the data by the SAR, we have to use a focusing processor to focus the data so that we can use them. One of the simplest focusing algorothm is 1D azimuth matched filtering. It requires a small amount of time and it has a low computational cost, but it works well only under strict assumptions: plane waves, neglectable range migration, small bandwidth, small array length or synthetic aperture. If one of those assumptions isn't valid, the focused data loses resolution or is misplaced. Very often, and in our case too, in spaceborn SAR those assumptions aren't valid because of satellite orbital motion uncertainties, big synthetic aperture, impossible plane waves approximation, non negligible range migration, high bandwidth; so we can't use the 1D azimuth matched filtering focusing processor. A focusing method which is valid in SAR conditions is the time domain back projection (TDBP). It is a powerful algorithm that is used whenever the assumptions that led to the formulation of focusing by DFT do not hold. This is the case of: large bandwidth signals, large array length or synthetic aperture, array formed by antennas at random positions. If d is a vector that represents our data, and p is a vector of parameters to be estimated based on the knowledge of d, we have to assume that we know the physical mechanisms that relate the parameters p to the data d to use the TDBP. A possible option to estimate model parameters is by maximization of the cross-correlation between the data and the model. Also to use this focusing algorithm data must be previously matched filtered. In the end TDBP is the result of three operations: interpolation, phase rotation and sum over all antennas or synthetic aperture samples. More details about the two focusing algorithms can be found in the chapter 2.

1.6 One of BIOMASS secondary goals is to map terrain topography using SAR Interferometry. By virtue of the penetration capabilities of P-Band waves, one can assume that the signal in HH polarization is mostly contributed by trunk-ground double bounce scattering from several tree trunks within any resolution cell. Accordingly, the phase center can be (at least approximately) assumed to be directly associated with terrain topography

1.6.1 (a) Evaluate the amount of phase noise in HH interferograms for a height of height of ambiguity $z_{amb} = 80m$

The altitude of ambiguity z_{amb} is defined as the altitude difference that generates an interferometric phase change of 2π after interferogram flattening. We know that

$$z_{amb} = \frac{2\pi}{k_z} \tag{1.38}$$

So we can find $k_z = 0.0785$. Then, from the equation

$$\sigma_{hh} = \frac{\sigma_{\Delta\phi}}{k_z} \tag{1.39}$$

we can find the interferometric phase dispersion $\sigma_{\Delta\phi} = 0.0124$, that is $-19.05dB$. We know that

$$\Delta\phi = k_z z \tag{1.40}$$

So if $z = 0$ we have $\Delta\phi = 0$; while if $z = z_{amb}$ we obtain $\Delta\phi = 2\pi$, which is also equal to zero if we don't perform a phase unwrapping, which improves the topography estimation if we go beyond z_{amb} increasing the horizontal baseline.

1.6.2 (b) Determine the size of the averaging window required to estimate topography to within an accuracy of 5 m

As the phase center can be assumed to be directly associated with terrain topography, we can estimate topography in this way:

$$z_{estimated} = z_{reference} + \frac{\Delta\phi}{k_z} \tag{1.41}$$

From $\sigma_{\Delta\phi}$ we can calculate the SNR from the simple relation

$$\sigma_{\Delta\phi} = \frac{1}{SNR} \tag{1.42}$$

obtaining $SNR = 80.336$. Then, as we know the accuracy ΔR, we can also calculate the interferometric coherence γ:

$$\gamma = \frac{SNR}{1 + SNR} * e^{i*\frac{4\pi}{\lambda}*\Delta R} \tag{1.43}$$

whose norm is $|\gamma| = 0.9877$. From $\sigma_{\Delta\phi}$ and the interferometric coherence γ we can compute the number of pixels of the averaging window inverting the equation

$$\sigma^2_{\Delta\phi} = \frac{1 - |\gamma|^2}{2L|\gamma|^2} \tag{1.44}$$

We obtain $L = 81$, that is an averaging window of 9x9 pixels.

Chapter 2

Problem 2: some SAR processing

The p-code Generate raw SAR data was created to generate raw SAR data (before pulse compression) based on an input image provided by the user. To run the code, type:

- SAR data = Generate raw SAR data('my image.jpg')

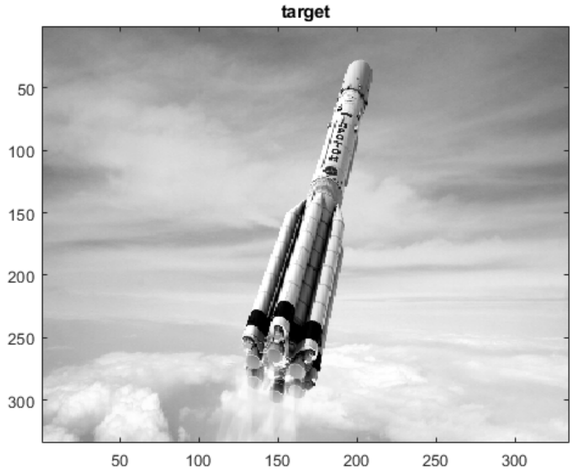
The input image can be whatever digital image on your computer. The output SAR data is a structure containing raw SAR data, a replica of the transmitted chirp waveform, plus various parameters used for data generation and useful for subsequent processing. A description of each variable is provided as well. Students are required to:

1. Focus the raw data to produce a focused SAR image.
2. Comment on the choice of the focusing algorithm and its performance in terms of image quality and computational burden.
3. Evaluate spatial resolution and check if it is consistent with the resulting focused image.

2.1 Focus the raw data to produce a focused SAR image

The starting point for the solution of this problem is to generate raw SAR data starting from an image. The following figure shows the original image and the target image generated by the p-code Generate raw SAR data.p:

(a) original image



(b) target image: raw SAR data

Figure 2.1: original and target images

The p code gives as output the following data:

- range resolution $\rho_r = 3m$
- azimuth resolution $dx_a = 2.5m$
- raw data matrix [fast time, sensor position] D,
- fast times vector [s] t_{ax},
- sensor positions along orbit vector [m] x_a,
- transmitted pulse g,
- range positions of the targets vector [m] r_{ax},
- azimuth positions of the targets vector [m] x_{ax},
- carrier frequency $f_0 = 5 * 10^9 Hz$,
- Synthetic Aperture length $A_s = 7005m$
- bandwidth $B = 5 * 10^7 Hz$.

From the data obtained from the p-code we can derive parameters which will be useful for the focusing algorithms:

- wave propagation velocity $c = 3 * 10^8 m/s$,
- wavelength $\lambda = \frac{c}{f_0} = 0.06m$,
- range resolution $\rho_r = \frac{c}{2B} = 3m$,
- minimum range $r_{min} = t_{ax}(1) * \frac{c}{2} = 699850m$,
- maximum range $r_{MAX} = t_{ax}(end) * \frac{c}{2} = 700650m$,

- maximum synthetic aperture $L_x = \frac{\lambda}{A_s} * r_{MAX} = 6.0013m$
- spatial sampling of the synthetic aperture $dx_{ant} = \frac{L_x}{2} * 0.5 = 1.5003m$
- time sampling $dt = \frac{1}{2B} = 1 * 10^{-8}s$
- number of x_a vector components N_x
- number of r_{ax} vector components N_r

Before using the focusing algorithms we have to filter the raw data to eliminate the noise. We use matched filtering (range compression) because it's the filter which maximizes the Signal to Noise Ratio:

$$h_{matched}(t) = g^*(-t) \tag{2.1}$$

Then we obtain the range compressed data by convoluting the raw data matrix D with the filter $h_{matched}$ for each azimuth position of the target ($i = 1, ..., N_x$), and then multiplying by dt for dimensional consistency:

$$D_{rc}(:, i) = conv(D(:, i), h_{matched}) * dt \tag{2.2}$$

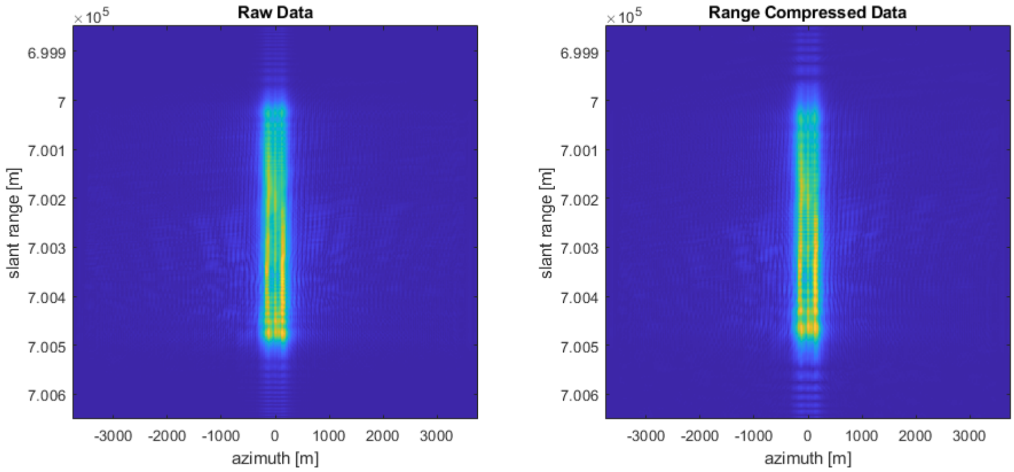
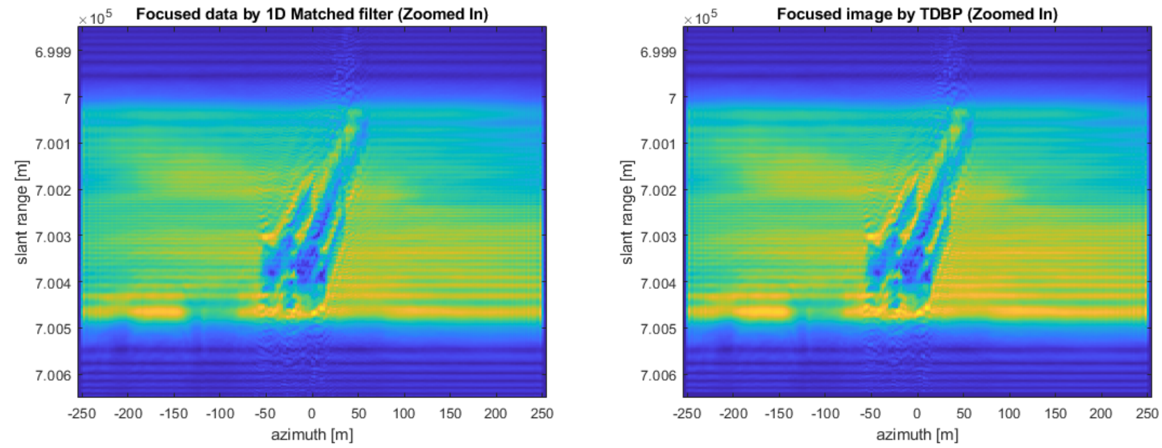


Figure 2.2: raw data and range compressed data

To focus the raw data we use two focusing algorithms: 1D matched filtering and Time Domain Back Projection (TDBP):



(a) (b)

Figure 2.3: focused images with 1D matched filtering and Time Domain Back Projection

2.2 Comment on the choice of the focusing algorithm and its performance in terms of image quality and computational burden

The first focusing algorithm we use is one of the simplest ones: the 1D matched filtering. It requires a small amount of time and it has a low computational cost, but it works well only under strict assumptions: plane waves, neglectable range migration, small bandwidth, small array length or synthetic aperture. For each range position of the target ($k = 1, ..., N_r$) we compute

$$R = \sqrt{x_a^2 + r_{ax}(k)^2} \tag{2.3}$$

to obtain the filter

$$h_{1Dmf} = e^{i*\frac{4\pi}{\lambda}*R} \tag{2.4}$$

Then we obtain the focused data convoluting the matched filtered data with the filter h_{1Dmf}, and then multiplying by dt for dimensional consistency:

$$D_{1Dmf}(k,:) = conv(D_{rc}(k,:), h_{1Dmf}) * dt \tag{2.5}$$

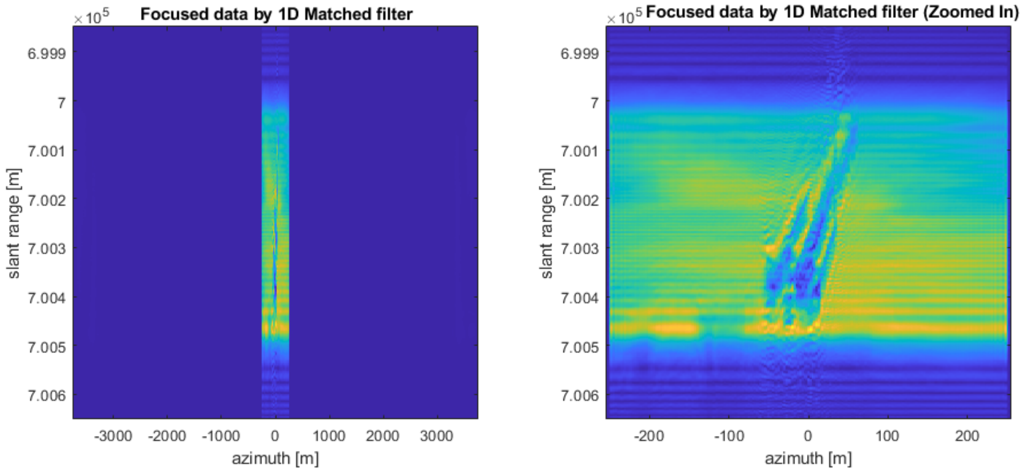


Figure 2.4: focused data with 1D matched filtering

As in this case we are under all those assumptions which allows the 1D matched filtering focusing algorithm to work well, we obtain an image with an acceptable quality with a very low computational burden: we obtained it in only 1.045 seconds.

The second focusing algorithm we use is more complex, expensive and slow, but it works also where the 1D matched filtering loses precision and resolution due to the lack of the requirements which allows it to work well: it is the Time Domain Back Projection (TDBP). The first step is to obtain the rectangular grid of coordinates slant range R and azimuth X starting from the vectors r_{ax} and x_a (we can do it with the MATLAB command $[R, X] = ndgrid(r_{ax}, x_a)$. Then for each azimuth position of the target $(n = 1, ..., N_x)$ we compute

$$R_{n_{rx}} = \sqrt{(X - x_a)^2 + R^2} \tag{2.6}$$

to obtain the filter

$$h_{TDBP} = e^{i * \frac{4\pi}{\lambda} * R_{n_{rx}}} \tag{2.7}$$

Then we obtain the focused data by multiplying the values obtained interpolating the matched filtered values, function of r_{ax}, at the points $R_{n_{rx}}$, by the filter h_{TDBP}:

$$D_{TDBP}(:, n) = interp1(r_{ax}, Drc(:, n), R_{n_{rx}}) * h_{TDBP} \tag{2.8}$$

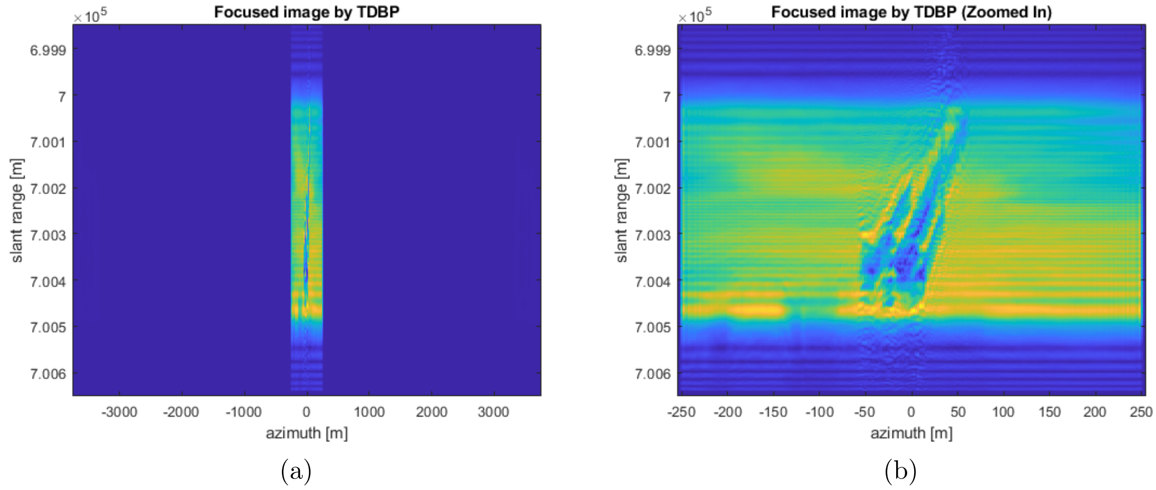


Figure 2.5: focused data with Time Domain Back Projection

As we already said we are working under those assumptions which make the 1D matched filtering a very good focusing algorithm, so the image quality doesn't considerably increase using the TDBP. But this method is much heavier from the computational burden point of view; even if we consider only the samples within a synthetic aperture to speed up the computation, it took 11 minutes and 41 seconds to focus the data with TDBP.

2.3 Evaluate spatial resolution and check if it is consistent with the resulting focused image

In slant range all pixels are distributed equidistantly, so for each of them we achieve the same resolution. Moreover in SAR the along-track (in the direction of flight) resolution is constant. As we can see in the following images, we have the same resolution for both the focusing algorithms, in both range and azimuth.

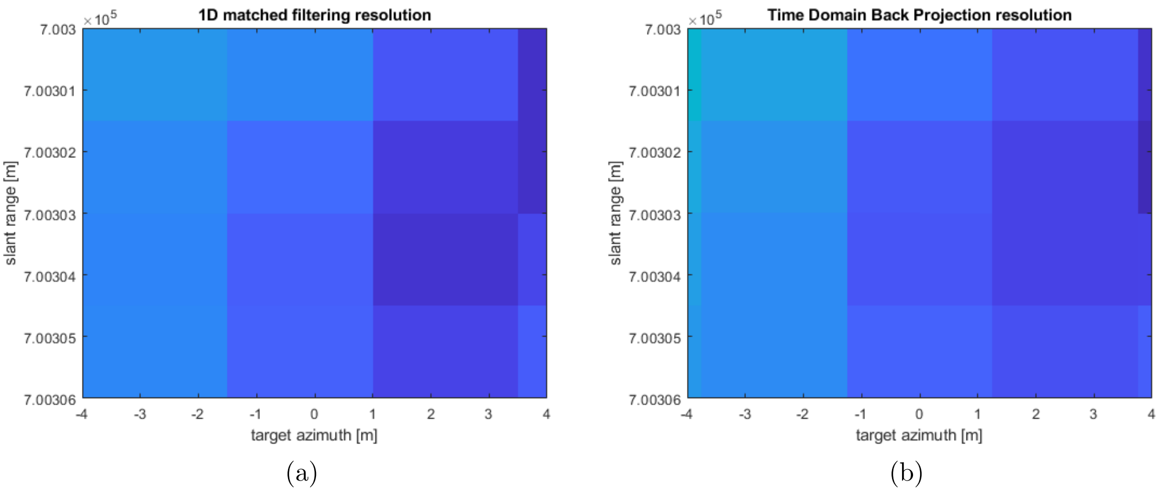


Figure 2.6: resolution cells with the two focusing algorithms

We can see that as we obtained from the beginning, we have an azimuth resolution of 2.5 m. While the range resolution is exactly half of the predicted one of 3 m.