The Cornus method of determining the elastic constants of a transparent material

Objective: Using Cornu's Method determine the elastic constants of a given transparent beam.

In physics lab III, last semester, you carried out Newton's rings experiment. Using a plano-convex lens placed on a flat glass base circular shape bright and dark fringes centred around the point of contact were obtained. Cornu's method is an extension of the Newton's rings experiment where a Perspex beam is used in place of the flat glass base. This method allows one to determine the elastic constants (Young's modulus and Poisson's ratio) of the perspex beam. Experimental set-up is shown below (Fig. 1). The beam undergoes a longitudinal bending when equal weights are hanged from its free ends. Longitudinal bending will also result in a certain amount of lateral bending, i.e., small upward bending perpendicular to the length of the beam. Due to these bending, the original circular shape of the Newton rings will change to an elliptical one. To understand the change of shape, you must recall that along any given ring the air gap should remain constant. If the lower reflecting surface is kept flat, the air-gap (between the convex surface of the lens and the flat base) is constant along a circular path, which explains the circular shape of the fringes or rings (*Question: if we replace the plano-convex lense by a flat glass slide so as to make a wedge shaped air film, what will be the shape of the resulting fringes?*). However, when the lower reflecting surface is bent the constant air-gap is traced along an elliptical path (*Question: Can you show is analytically?*) Thus, the rings change from circular to an elliptic shape (*Question: what would be the shape of interference fringes upon beam deformation if a flat glass slide is used. Assume that prior to the deformation the surface of the* *glass slide was fully in contact with beam (obviously, in this condition no fringes will be seen, the fringes will appear only after the beam is deformed)?* Let 'd_n' is the radius of the nth circular ring in the absence of bending, and d_n', d_n'' the minor and the major axis, respectively, of the same ring upon bending. Due to large downward bending along the length of the beam (longitudinal bending), d_n' will be smaller than d_n. Similarly, the small upward bending in the lateral direction causes $d_n''\ to\ be\ slighly\ larger\ than\ d_n$. Stretching a body produces an internal force called stress which prevents the body from tearing apart. The ratio of stress to strain for a given material is called Young's modulus. From the values of d_n' d_n' and d_n'' for various n under different values of attached weight W one can determine the elastic constants: Poisson's ratio and Young's Modulus of the beam as explained below.

Figure 1: Experimental set-up for the Cornu's method: (from left) Sodium lamp source for producing Newton's rings, side view and front view of the apparatus with weights. Thin air-film between a plano-convex lens and the Perspex beam produces circular rings due to interference. Upon attaching weights to the free ends of the beam and its consequent bending, the circular ring shape changes to an elliptical one.

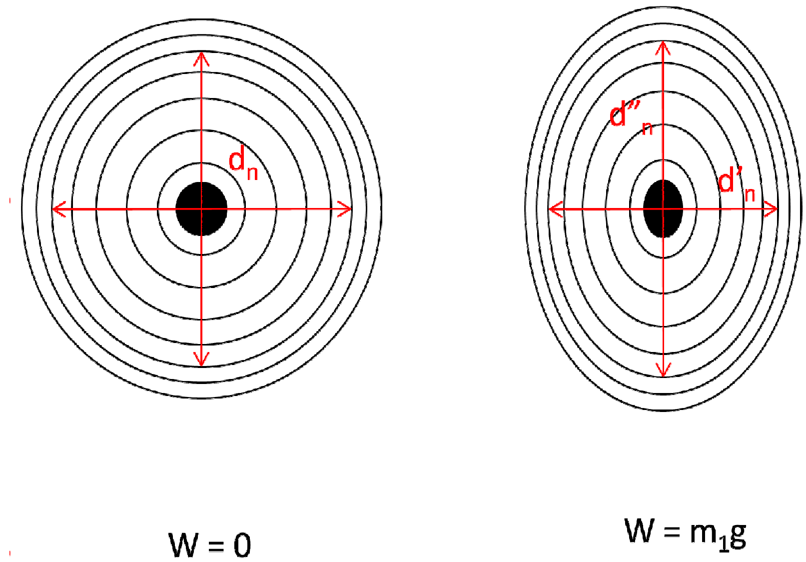

Figure2: Newton rings with and without bending shown, respectively, in the left and right panel. Note that bending is accompanied by change in shape of the rings from circular to elliptical. The diameter of the n^th dark ring *decreases* greatly along the beam length and *increases* slightly perpendicular to the beam length in accordance with change in thickness of the air film sandwiched between the beam and the convex surface of the plano-convex lens (i.e., $d'_n < d_n < d''_n$).

Here is a brief outline of the analysis:

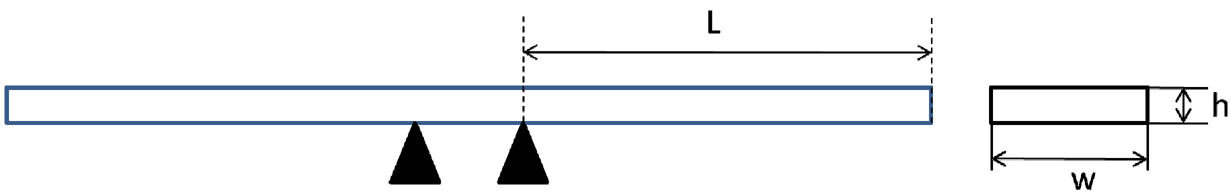

Figure 3: Side views (the unloaded condition) of the beam whose elastic constants have to be determined. The beam is supported on two triangular pegs (black triangular shape objects in the figure)

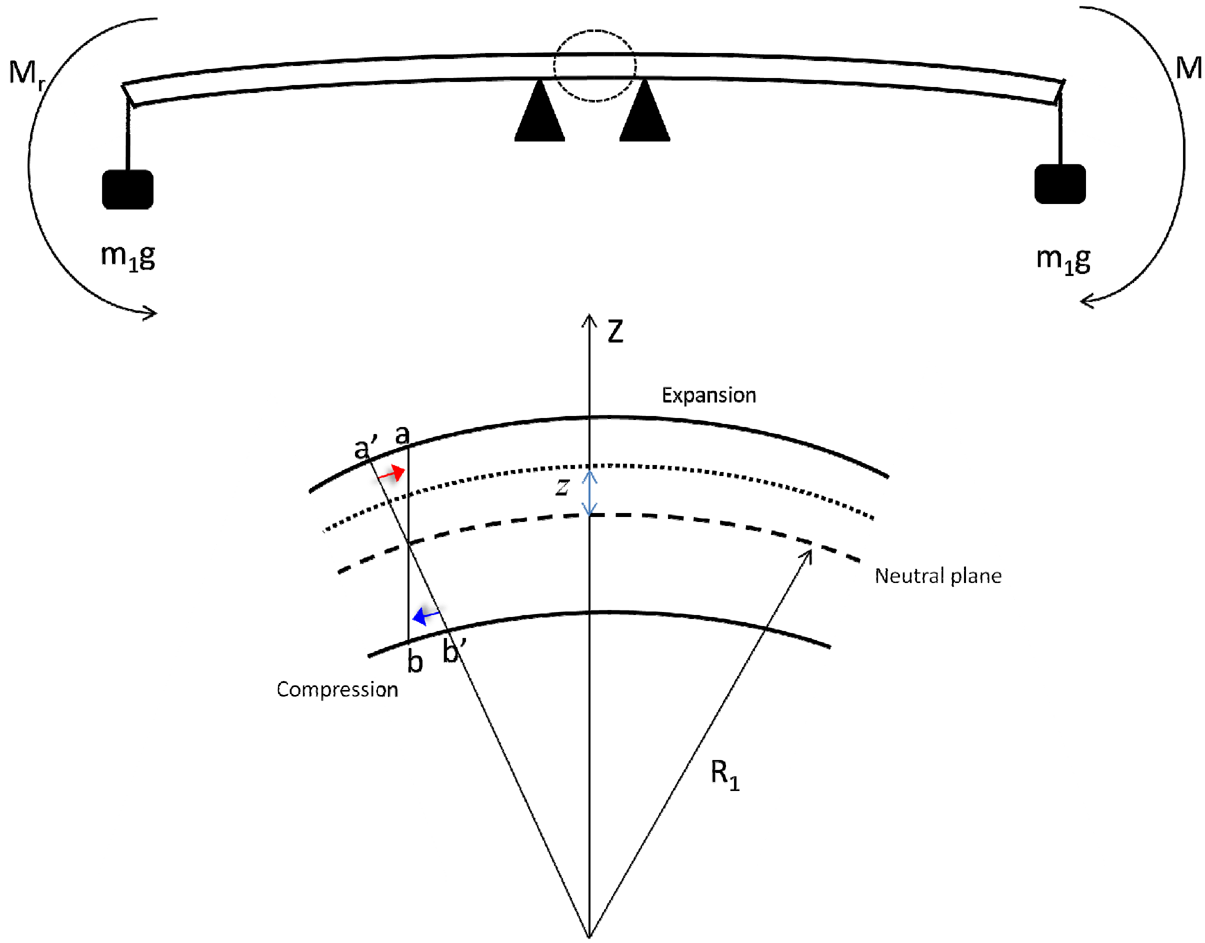

Figure 4: Bending of a beam under an applied weight W = m_1g at each end. M_r is the bending moment (= $m_1g.L$). Lower panel shows an expanded view of the region over which the Newton rings form (region enclosed in a dotted circle in the upper panel). Due to bending, the upper half (i.e., half above the neutral plane) of the beam will undergo "expansion" while the lower half (below the neutral plane) will undergo compression resulting in stress (section ab of the beam became a'b' upon bending). The *stress* generated is indicated by arrows. We can approximate a small section (region over which the Newton rings form) by the arc of a circle of radius R_1.

Calculation of Strain ε(z):

Elongation (contraction) of a plane a distance "z" above (below) the neutral plane (see fig. 4) is given by:

$$\varepsilon(z) = \frac{(R_1 + z)\theta - R_1\theta}{R_1\theta} = \frac{z}{R_1} \qquad (1)$$

Calculation of Stress σ(z):

Young's modulus (Y) = Stress/Strain = σ/ε

i.e., $\sigma(z) = Y.z/R_1$ (2)

The bending moment is therefore given by:

$$M_r = 2\int_0^{\frac{h}{2}} \sigma(z).z\,(w.dz) \quad (3)$$

$$= \frac{Y\,w\,h^3}{12\,R_1} \quad (4)$$

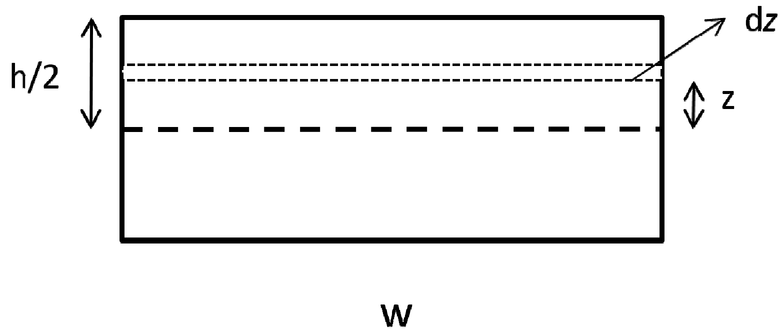

Figure 4: Cross-section of a bent beam. Broken line indicates neutral plane. Please see also Figure 6.

Under equilibrium, the internal bending moment (eq. 4) must be balanced by the moment due to weight m_1g attached to its ends (eq. 5),

$$M_r = m_1g.L \quad (5)$$

Combining (4) and (5) gives:

$$Y = \frac{12\,m_1\,g\,L\,R_1}{w\,h^3} \quad (6)$$

Thus, if we can determine R_1 from our experiment, other quantities being known, the Young's modulus of the beam can be calculated. But before that we should first calculate R_0 (the radius of curvature of the plano-convex lens used).

I. Determination of R_0 (radius of curvature of the plano-convex lens)

It is simple to Show that:

$$R_0 = \frac{d_n^2}{4n\lambda} \qquad (7)$$

Where d_n is the diameter of the n^{th} dark ring. Plot of 'd_n^2' as a function of 'n' should be a straight line with slope $4\lambda R_0$ (since the absolute value of 'n' is not known with certainty, you should not try to calculate R_0 using direct formula).

II. Determination of R_1

Using the same geometrical reasoning as is used in deriving eq. (7). It can be shown (*please try showing it*) that R_1 is given by:

$$\frac{1}{R_1} = 4\,\lambda.n\left(\frac{1}{d_n'^2} - \frac{1}{d_n^2}\right) \qquad (8)$$

By plotting (as done in part I) you can obtain R_1 from the slope of straight line, which leads to determination of Y. So, far we did not consider a small but finite lateral bending. When a beam is made to bend along its length (longitudinal bending), it also undergoes a small lateral bending (see, figure 6), resulting in a lateral strain. That is the reason why in figure 2 major axis (d_n'') of ellipse is slightly greater than the radius (d_n) of the corresponding circular ring. The measure of this tendency is called Poisson's ratio (ζ), given by:

ζ = lateral strain/longitudinal strain. With the help of eq. 1: $\zeta = R_1/R_2$. Typically R_2 is much larger than R_1. (9)

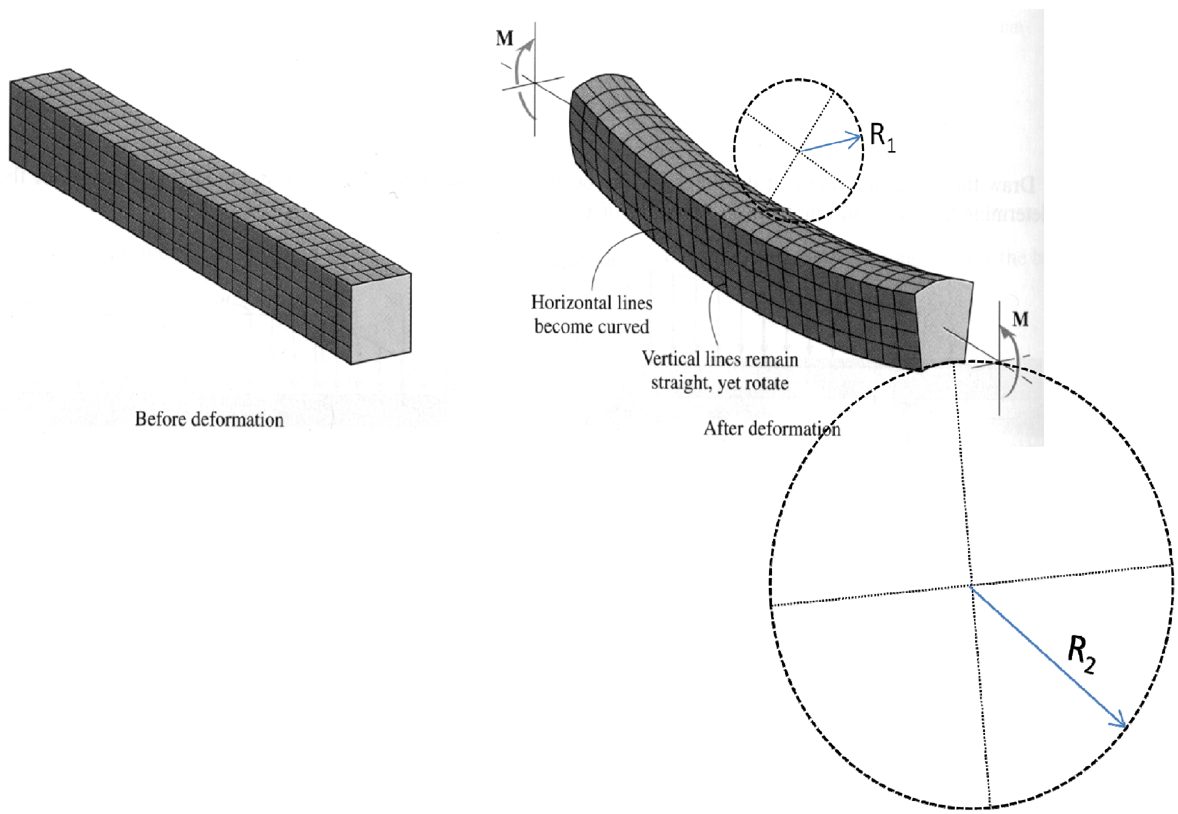

Figure. 6: Longitudinal bending (R_1) of a bean resulting in small lateral bending (R_2)

III. Determination of R_2

Show that:

$$\frac{1}{R_2} = 4\,\lambda . n \left(\frac{1}{d_n''^2} - \frac{1}{d_n^2}\right) \tag{10}$$

By plotting (as done in part I and II) you can obtain R_2 from the slope of straight line, which leads to the experimental determination of Poisson's ratio z.

Author: surjeet.singh@iiserpune.ac.in

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