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1 Graph Theory

1.1 Adjacency List

```cpp
vector<int> list[5];

void Adjacency_List(){

    // initial
    for (int i = 0; i < 5; i++)
        list[i].clear();

    int a, b;   // start & end of an edge

    while (cin >> a >> b)
        list[a].push_back(b);
        // list[b].push_back(a);
}
```

1.2 DFS

```cpp
vector<int> G[N];
bitset<N> vis;
void dfs(int s) {
    vis[s] = 1;
    for (int t : G[s]) {
        if (!vis[i])
            dfs(i);
    }
}
```

1.3 BFS

```cpp
vector<int> G[N];
bitset<N> vis;
void bfs(int s) {
    queue<int> q;
    q.push(s);
    vis[s] = 1;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int t : G[v]) {
            if (!vis[t]) {
                q.push(t);
                vis[t] = 1;
            }
        }
    }
}
```

1.4 Disjoint Set and Kruskal

```cpp
struct Edge{
    int u, v, w;
    // bool operator < (const Edge &rhs) const {
        return w < rhs.w; }
};

vector<int> parent;
vector<Edge> E;

bool cmp(Edge edge1, Edge edge2){
    return edge2.w > edge1.w;
}

int find(int x){
    if(parent[x] < 0){
        return x;
    }
    return parent[x] = find(parent[x]);
}

bool Uni(int a, int b){
    a = find(a);
    b = find(b);
    if(a == b){
        return false;
    }
    if(parent[a] > parent[b]){
        swap(a, b);
    }
    parent[a] = parent[a] + parent[b];
    parent[b] = a;
    return true;
}

void Kruskal() {

    int cost = 0;

    sort(E.begin(), E.end()); // sort by w
    // sort(E.begin(), E.end(), cmp);

    // two edge in the same tree or not
    for (auto it: E){
        it.s = Find(it.s);
        it.t = Find(it.t);
        if (Uni(it.s, it.t)){
            cost = cost + it.w;;
        }
    }
}

int main(){

    // create N space and initial -1
    parent = vector<int> (N, -1);

    for(i = 0; i < M; i++){
        cin >> u >> v >> w;
        E.push_back({u, v, w});
    }

    Kruskal();

    return 0;
}
```

1.5 Floyd-Warshall

```
for (k = 0; k < n; k++){
    for (i = 0; i < n; i++){
        for (j = 0; j < n; j++){
            w[i][j] = w[j][i] = min(w[i][j],
                max(w[i][k], w[k][j]));
```

1.6 Dijkstra

```
struct edge {
  int s, t;
  LL d;
  edge(){};
  edge(int s, int t, LL d) : s(s), t(t), d(d) {}
};

struct heap {
  LL d;
  int p; // point
  heap(){};
  heap(LL d, int p) : d(d), p(p) {}
  bool operator<(const heap &b) const { return d > b.d; }
};

int d[N], p[N];
vector<edge> edges;
vector<int> G[N];
bitset<N> vis;

void Dijkstra(int ss){

    priority_queue<heap> Q;

    for (int i = 0; i < V; i++){
        d[i] = INF;
    }

    d[ss] = 0;
    p[ss] = -1;
    vis.reset() : Q.push(heap(0, ss));
    heap x;

    while (!Q.empty()){

        x = Q.top();
        Q.pop();
        int p = x.p;

        if (vis[p])
            continue;
        vis[p] = 1;

        for (int i = 0; i < G[p].size(); i++){
            edge &e = edges[G[p][i]];
            if (d[e.t] > d[p] + e.d){
                d[e.t] = d[p] + e.d;
                p[e.t] = G[p][i];
                Q.push(heap(d[e.t], e.t));
            }
        }
    }
}
```

2 Number Theory

2.1 Modulo

- $(a+b) \bmod p = (a \bmod p + b \bmod p) \bmod p$
- $(a-b) \bmod p = (a \bmod p - b \bmod p + p) \bmod p$
- $(a*b) \bmod p = (a \bmod p \cdot b \bmod p) \bmod p$
- $(a^b) \bmod p = ((a \bmod p)^b) \bmod p$
- $((a+b) \bmod p + c) \bmod p = (a + (b+c)) \bmod p$
- $((a \cdot b) \bmod p \cdot c) \bmod p = (a \cdot (b \cdot c)) \bmod p$
- $(a+b) \bmod p = (b+a) \bmod p$
- $(a \cdot b) \bmod p = (b \cdot a) \bmod p$
- $((a+b) \bmod p \cdot c) = ((a \cdot c) \bmod p + (b \cdot c) \bmod p) \bmod p$
- $a \equiv b \pmod m \Rightarrow c \cdot m = a - b, c \in \mathbb{Z}$
  $\Rightarrow a \equiv b \pmod m \Rightarrow m \mid a - b$
- $a \equiv b \pmod c, b \equiv d \pmod c$
  則 $a \equiv d \pmod c$
- $\begin{cases} a \equiv b \pmod m \\ c \equiv d \pmod m \end{cases} \Rightarrow \begin{cases} a \pm c \equiv b \pm d \pmod m \\ a \cdot c \equiv b \cdot d \pmod m \end{cases}$

2.2 Linear Sieve

```
vector<int> p;
bitset<MAXN> is_notp;
void PrimeTable(int n){

    is_notp.reset();
    is_notp[0] = is_notp[1] = 1;

    for (int i = 2; i <= n; ++i){
        if (!is_notp[i]){
            p.push_back(i);
        }
        for (int j = 0; j < (int)p.size(); ++j){
            if (i * p[j] > n){
                break;
            }

            is_notp[i * p[j]] = 1;

            if (i % p[j] == 0){
                break;
            }
        }
    }
}
```

2.3 Prime Factorization

```
void primeFactorization(int n){
    for(int i = 0; i < (int)p.size(); i++){
        if(p[i] * p[i] > n){
            break;
        }
        if(n % p[i]){
            continue;
        }
        cout << p[i] << ' ';
        while(n % p[i] == 0){
            n /= p[i];
        }
    }
    if(n != 1){
        cout << n << ' ';
    }
    cout << '\n';
}
```

2.4 Exponentiating by Squaring

```cpp
T pow(int a, int b, int c){ // calculate a ^ b % c
    T ans = 1, tmp = a;
    for (; b; b >>= 1) {
        if (b & 1){ // b is odd
            ans = ans * tmp % c;
        }
        tmp = tmp * tmp % c;
    }
  return ans;
}
```

2.5 Euler

```cpp
int Phi(int n){
    int ans = n;
    for (int i: p) {
        if (i * i > n){
            break;
        }
        if (n % i == 0){
            ans /= i;
            ans *= i - 1;
            while (n % i == 0){
                n /= i;
            }
        }
    }
    if (n != 1) {
        ans /= n;
        ans *= n - 1;
    }
    return ans;
}
```

3 Dynamic Programming

3.1 Fibonacci

```cpp
// f(n) = f(n - 1) + f(n - 2)
// f(0) = 0, f(1) = 1
int dp[30];
int f(int n){
    if (dp[n] != -1){
        return dp[n];
    }
    return dp[n] = f(n - 1) + f(n - 2);
}

int main(){
    memset(dp, -1, sizeof(dp));
    dp[0] = 0;
    dp[1] = 1;
    cout << f(25) << '\n';
}
```

3.2 Pascal Triangle

```cpp
// init : f(i, 0) = f(i, i) = 1
// tren : f(i, j) = f(i - 1, j) + f(i - 1, j - 1)
int main(){
    int dp[30][30];
    memset(dp, 0, sizeof(dp));
    for (int i = 0; i < 30; ++i){
        dp[i][0] = dp[i][i] = 1;
    }
    for (int i = 1; i < 30; ++i){
        for (int j = 1; j < 30; ++j){
            dp[i][j] = dp[i - 1][j] + dp[i - 1][j - 1];
        }
    }
}
```

3.3 Robot

```cpp
// f(1, j) = f(i, 1) = 1
// f(i, j) = f(i - 1, j) + f(i, j - 1)
int dp[105][105];
dp[1][1] = 1;
for(int i = 1; i <= 100: ++i){
    for(int j = 1; j <= 100; ++j){
        if(i + 1 <= 100) dp[i + 1][j] += dp[i][j];
        if(j + 1 <= 100) dp[i][j + 1] += dp[i][j];
    }
}
```

3.4 Max Interval Sum

```cpp
// No Limit
int ans = A[1];
sum[1] = dp[1] = A[1];

for(int i = 2; i <= n; ++i){
    sum[i] = A[i] + sum[i - 1];
    dp[i] = min(dp[i - 1], sum[i]);
    ans = max(ans, sum[i] - dp[i - 1]);
}

// length <= L
int a[15] = {0, 6, -8, 4, -10, 7, 9, -6, 4, 5, -1};
int sum[15];

int main(){
    int L = 3, ans = 0;
    for (int i = 1; i <= 10; ++i)
    {
        sum[i] = a[i] + sum[i - 1];
    }
    deque<int> dq;
    dq.push_back(0);
    for (int i = 1; i <= 10; ++i){
        if (i - dq.front() > L){
            dq.pop_front();
        }
        ans = max(ans, sum[i] - sum[dq.front()]);
        while(!dq.empty() && sum[i] < sum[dq.back()]){
            dq.pop_back();
        }
        dq.push_back(i);
    }
    cout << ans << '\n';
}
```

3.5 Max Area

```cpp
const int N = 25;

int main(){
    int n;
    cin >> n;
    vector<int> H(n + 5), L(n + 5), R(n + 5);
    for (int i = 0; i < n; ++i){
        cin >> H[i];
    }
    stack<int> st;
    // calculate R[]
    for (int i = 0; i < n; ++i){
        while (!st.empty() && H[st.top()] > H[i]){
```

```
            R[st.top()] = i - 1;
            st.pop();
        }
        st.push(i);
    }
    while (!st.empty()){
        R[st.top()] = n - 1;
        st.pop();
    }
    // calculate L[]
    for (int i = n - 1; i >= 0; --i){
        while (!st.empty() && H[st.top()] > H[i]){
            L[st.top()] = i + 1;
            st.pop();
        }
        st.push(i);
    }
    while (!st.empty()){
        L[st.top()] = 0;
        st.pop();
    }
    int ans = 0;
    for (int i = 0; i < n; ++i){
        ans = max(ans, H[i] * (R[i] - L[i] + 1));
        cout << i << ' ' << L[i] << ' ' << R[i] <<
            '\n';
    }
    cout << ans << '\n';
}
```

3.6 LCS

```
// init：dp[i][0] = dp[0][i] = 0
// tren：dp[i][j] =
    // if a[i] = b[j]
        // dp[i - 1][j - 1] + 1
    // else
        // max(dp[i - 1][j], dp[i][j - 1])
// LIS
    // init：dp[0] = 0
    // tren：dp[i] = max{dp[j] |  j < i and A[j] <
        A[i]} + 1
// LIS → LCS (嚴格遞增)
    // A 為原序列，B = sort(A)
    // 對 A, B 做 LCS
// LCS → LIS (數字重複、有數字在 B 裡面不在 A 裡面)
    // A, B 為原本的兩序列
    // 對 A 序列作編號轉換，將轉換規則套用在 B
    // 對 B 做 LIS
int dp[a.size() + 1][b.size() + 1];
for(int i = 0; i <= a.size(); i++){
    dp[i][0]= 0;
}
for(int i = 0; i <= b.size(); i++){
    dp[0][i] = 0;
}

for(int i = 1; i <= a.size(); i++){
    for(int j = 1; j <= b.size(); j++){
        if(a[i - 1] == b[j - 1]){
            dp[i][j] = dp[i - 1][j - 1] + 1;
        }
        else{
            dp[i][j] = max(dp[i - 1][j], dp[i][j -
                1]);
        }
    }
}

return 0;
```

3.7 0-1 Bag

```
// 不放：重量和價值不變
    // to f(i, j) = f(i - 1, j)
// 放：重量 + w_i，價值 + v_i
    // to f(i, j) = f(i - 1, j - w_i) + v_i
// tren：f(i, j) = max(f(i - 1, j), f(i - 1, j - w_i)
    + v_i)
int dp[MXN + 1][MXW + 1];
memset(dp, 0, sizeof(dp));
for (int i = 1; i <= MXN; ++i){
    for (int j = 0; j < w[i]; ++j){
        dp[i][j] = dp[i - 1][j];
    }
    for (int j = w[i]; j <= MXW; ++j){
        dp[i][j] = max(dp[i - 1][j - w[i]] + v[i],
            dp[i - 1][j]);
    }
}
cout << dp[MXN][MXW] << '\n';
```

3.8 Infinite Bag

```
// f(i, j) = max(f(i - 1, j), f(i - 1, j - wi) + vi,
    f(i, j - wi) + vi)
    // coin chage
        // 最少幾枚能湊成 M 元
            //
                f(i,j)=min(f(i-1,j),f(i-1,j-ci)+1,f(i,j-ci)+1)
        // 多少種能湊成 M 元
            // f(i, j) = f(i - 1, j) + f(i, j - ci)
int dp[MXW];
memset(dp, -INF, sizeof(dp));
dp[0] = 0;
for (int i = 0; i < N; ++i){
    for (int j = w[i]; j <= MXW; ++j){
        dp[j] = max(dp[j - w[i]] + v[i], dp[j]);
    }
}
```

3.9 Tree

```
#include <bits/stdc++.h>
using namespace std;
const int MXV = 15;
vector<int> G[MXV];
int high[MXV][2];
int ans[MXV], height[MXV];

void dfs(int u){
    height[u] = 1;
    for (int v : G[u]){
        dfs(v);
        height[u] = max(height[u], height[v] + 1);
        if (high[u][0] == 0 || height[high[u][0]] <
            height[v]){
            high[u][1] = high[u][0];
            high[u][0] = v;
        }
        else if (high[u][1] == 0 ||
            height[high[u][1]] < height[v]){
            high[u][1] = v;
        }
    }
}

void dfs2(int u, int legnth){
    ans[u] = height[high[u][0]] +
        max(height[high[u][1]], legnth) + 1;
    for (int v : G[u]){
        if (v == high[u][0]){
            dfs2(v, max(height[high[u][1]], legnth) +
                1);
        }
        else{
```

```
            dfs2(v, max(height[high[u][0]], legnth) +
                1);
        }
    }
}

int main(){
    int n;
    cin >> n;
    for (int i = 1; i < n; ++i){
        int x, y;
        cin >> x >> y;
        G[x].emplace_back(y);
    }
    dfs(1);
    dfs2(1, 0);
    for (int i = 1; i <= n; ++i){
        cout << ans[i] << '\n';
    }
}
```