Universidade do Minho

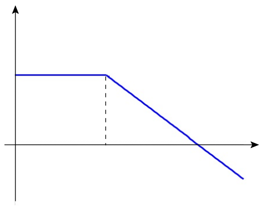
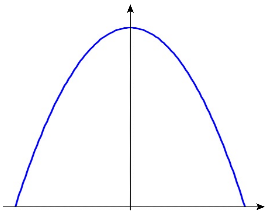
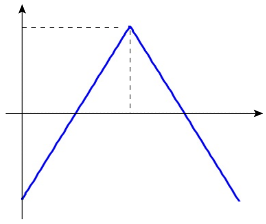
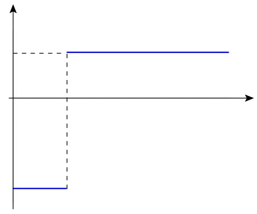
MIEInf
DMA

Cálculo

folha 7 —— 2018'19

Primitivas.

1. Considere, em cada alínea, a função $f : I \longrightarrow \mathbb{R}$, I um intervalo, representada graficamente por

(a)  (b)  (c)  (d) 

Para cada alínea esboce, caso exista, o gráfico de uma função F, primitiva de f em I, sabendo que:

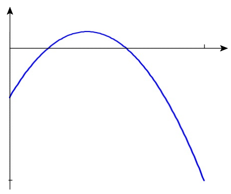
(a) $I = [0, 5]$

(b) $I = [-1, 1]$, $f(x) = 1 - x^2$ e $F(0) = 0$

(c) $I = [0, 4]$ e $F(0) = -2$

(d) $I = [0, 4]$ e $F(0) = 1$

2. Seja $f : [0, 5] \longrightarrow \mathbb{R}$ representada graficamente na figura ao lado. Considere uma função primitiva de f, $F : [0, 5] \longrightarrow \mathbb{R}$.



(a) Encontre os pontos críticos de F.

(b) Classifique os pontos críticos de F.

3. Calcule os seguintes integrais indefinidos

(a) $\displaystyle\int (3x^2 - 2x^5)\, dx$

(b) $\displaystyle\int (\sqrt{x} + 2)^2\, dx$

(c) $\displaystyle\int (2\theta + 10)^{20}\, d\theta$

(d) $\displaystyle\int x^4 (x^5 + 10)^9\, dx$

(e) $\displaystyle\int y^2 e^{y^3}\, dy$

(f) $\displaystyle\int \sqrt{2x + 1}\, dx$

(g) $\displaystyle\int \frac{2x + 1}{x^2 + x + 3}\, dx$

(h) $\displaystyle\int \frac{t}{3 - t^2}\, dt$

(i) $\displaystyle\int \frac{1}{4 - 3\,x}\, dx$

(j) $\displaystyle\int \operatorname{th} x\, dx$

(k) $\displaystyle\int \frac{1}{e^{3x}}\, dx$

(l) $\displaystyle\int \frac{-7}{\sqrt{1 - 5x}}\, dx$

(m) $\displaystyle\int \frac{\sqrt{1 + 3 \ln a}}{a}\, da$

(n) $\displaystyle\int z \operatorname{sen} z^2\, dz$

(o) $\displaystyle\int \frac{1}{x(\ln^2 x + 1)}\, dx$

(p) $\displaystyle\int \left(\frac{2}{x} - 3\right)^2 \frac{1}{x^2}\, dx$

(q) $\displaystyle\int \operatorname{sen}(\pi - 2\,x)\, dx.$

4. Usando primitivação por partes calcule:

(a) $\displaystyle\int \ln x\, dx$

(b) $\displaystyle\int x \operatorname{sen}(2x)\, dx$

(c) $\displaystyle\int \operatorname{arctg} x\, dx$

(d) $\displaystyle\int x \cos x\, dx$

(e) $\displaystyle\int \ln(1 - x)\, dx$

(f) $\displaystyle\int x \ln x\, dx$

(g) $\displaystyle\int x^2 \operatorname{sen} x\, dx$

(h) $\displaystyle\int x \operatorname{sen} x \cos x\, dx$

(i) $\displaystyle\int \ln^2 x\, dx$

(j) $\displaystyle\int e^x \cos x\, dx$

(k) $\displaystyle\int \operatorname{arcsen} x\, dx$

(l) $\displaystyle\int e^{\operatorname{sen} x} \operatorname{sen} x \cos x\, dx$

(m) $\displaystyle\int \frac{\operatorname{arcsen} \sqrt{x}}{\sqrt{x}}\, dx$

(n) $\displaystyle\int x \operatorname{arctg} x\, dx$

(o) $\displaystyle\int x^2 \ln x\, dx$

(p) $\displaystyle\int \operatorname{sen}(\ln x)\, dx$

(q) $\displaystyle\int \operatorname{ch} x \operatorname{sen}(3x)\, dx$

(r) $\displaystyle\int x^3 e^{x^2}\, dx.$

5. Encontre F, uma primitiva da função f, sabendo que $F(1) = 0$. A solução encontrada é única?

(a) $f(x) = \text{sen}\, x \cos x$

(b) $f(x) = \text{sen}(2x) \cos x$

(c) $f(x) = \text{sen}^2 x$

(d) $f(x) = \dfrac{1}{x}, x > 0$

6. Sendo $f : \mathbb{R} \longrightarrow \mathbb{R}$ definida por $f(x) = x^2 \,\text{sen}\, x$, encontre a primitiva de f cujo gráfico passa pelo ponto $(\frac{\pi}{2}, \pi)$.

7. Calcule os seguintes integrais indefinidos.

(a) $\displaystyle\int \frac{3x^2 - 4x - 1}{(x^2 - 1)(x - 2)}\, dx$

(b) $\displaystyle\int \frac{2x^2 + x + 1}{(x - 1)(x + 1)^2}\, dx$

(c) $\displaystyle\int \frac{4x^2 + x + 1}{x^3 - x}\, dx$

(d) $\displaystyle\int \frac{27}{x^4 - 3\,x^3}\, dx$

(e) $\displaystyle\int \frac{x^4 - 8}{x^3 - 2\,x^2}\, dx$

(f) $\displaystyle\int \frac{x + 3}{(x - 2)\,(x^2 - 2\,x + 5)}\, dx$

8. Calcule os seguintes integrais indefinidos usando a substituição indicada.

(a) $\displaystyle\int x\sqrt{x - 1}\, dx, \quad x = t^2 + 1$

(b) $\displaystyle\int \sqrt{1 - x^2}\, dx, \quad x = \text{sen}\, t$

(c) $\displaystyle\int \frac{e^{2x}}{1 + e^x}\, dx, \quad x = \ln t$

(d) $\displaystyle\int \sqrt{1 + x^2}\, dx, \quad x = \text{sh}\, t$

9. Calcule os seguintes integrais indefinidos

(a) $\displaystyle\int \frac{x}{x^2 - 1}\, dx$

(b) $\displaystyle\int \frac{x}{\sqrt{x^2 - 1}}\, dx$

(c) $\displaystyle\int \frac{1}{x}\ \text{sen}(\ln x)\, dx$

(d) $\displaystyle\int \frac{-3}{x\,(\ln x)^3}\, dx$

(e) $\displaystyle\int \frac{e^x}{1 + e^{2x}}\, dx$

(f) $\displaystyle\int \frac{e^x}{1 - 2e^x}\, dx$

(g) $\displaystyle\int \frac{1}{\cos^2(7x)}\, dx$

(h) $\displaystyle\int \left(\sqrt{2x - 1} - \sqrt{1 + 3x}\right)\, dx$

(i) $\displaystyle\int \frac{1}{x}\ \left(1 + \ln^2 x\right)\, dx$

(j) $\displaystyle\int \frac{2\, +\, \sqrt{\text{arctg}(2x)}}{1 + 4x^2}\, dx$

(k) $\displaystyle\int \frac{e^{\text{arctg}\, x}}{1 + x^2}\, dx$

(l) $\displaystyle\int \frac{\text{sen}\, x}{\sqrt{1 + \cos x}}\, dx$

(m) $\displaystyle\int \frac{1}{(2 + \sqrt{x})^7\, \sqrt{x}}\, dx$

(n) $\displaystyle\int \text{tg}^2\, x\, dx$

(o) $\displaystyle\int \frac{x + [\text{arcsen}(3x)]^4}{\sqrt{1 - 9\,x^2}}\, dx$

(p) $\displaystyle\int \frac{x\, e^{\sqrt{1 - x^2}}}{\sqrt{1 - x^2}}\, dx$

(q) $\displaystyle\int \frac{1}{\cos^2 x\ \text{sen}^2 x}\, dx$

(r) $\displaystyle\int \cos^2 x\ \text{sen}^2 x\, dx$

(s) $\displaystyle\int \frac{1}{1 + e^x}\, dx$

(t) $\displaystyle\int \frac{1}{x\sqrt{x^2 - 1}}\, dx$

(u) $\displaystyle\int \frac{1}{x^2\, \sqrt{4 - x^2}}\, dx.$

10. Em cada alínea, determine a única função $f : \mathbb{R} \longrightarrow \mathbb{R}$, duas vezes derivável, tal que:

(a) $f''(x) = 4x - 1, \quad x \in \mathbb{R}, \quad f(1) = 3 \quad$ e $\quad f'(2) = -2;$

(b) $f''(x) = \text{sen}\, x \cos x, \quad x \in \mathbb{R}, \quad f(0) = 0 \quad$ e $\quad f'(0) = 1.$