Universidade do Minho
Dep. de Matemática e Aplicações

Mestrado Integrado em Engenharia Informática
Introdução aos Sistemas Dinâmicos
2019/20

edo's primeira ordem separáveis

Consulte o ficheiro 'Folha5.nb'.

Exercício 1.

(a) Soluções constantes

$$\begin{array}{rcl} \mathbb{R}^+ & \to & \mathbb{R} \\ x & \mapsto & 0 \end{array} \quad \text{e} \quad \begin{array}{rcl} \mathbb{R}^- & \to & \mathbb{R} \\ x & \mapsto & 0 \end{array}$$

Soluções não constantes

$$\begin{array}{rcl} \mathbb{R}^+ & \to & \mathbb{R} \\ x & \mapsto & c\,x^2 \end{array} \quad \text{e} \quad \begin{array}{rcl} \mathbb{R}^- & \to & \mathbb{R}\,, \\ x & \mapsto & c\,x^2 \end{array} \qquad c \in \mathbb{R}\backslash\{0\}$$

(b) Solução constante

$$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & 0 \end{array}$$

Soluções não constantes

$$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R}\,, \\ x & \mapsto & \dfrac{c}{(x^2+1)^2} \end{array} \qquad c \in \mathbb{R}\backslash\{0\}$$

(c) Soluções não constantes

$$\begin{array}{rcl} \mathbb{R}^+ & \to & \mathbb{R} \\ x & \mapsto & -x \end{array} \quad \text{e} \quad \begin{array}{rcl} \mathbb{R}^+ & \to & \mathbb{R} \\ x & \mapsto & x \end{array} \quad \text{e} \quad \begin{array}{rcl} \mathbb{R}^- & \to & \mathbb{R} \\ x & \mapsto & -x \end{array} \quad \text{e} \quad \begin{array}{rcl} \mathbb{R}^- & \to & \mathbb{R} \\ x & \mapsto & x \end{array}$$

$$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & -\sqrt{x^2+2c} \end{array} \quad \text{e} \quad \begin{array}{rcl} \mathbb{R} & \to & \mathbb{R}\,, \\ x & \mapsto & \sqrt{x^2+2c} \end{array} \qquad c \in \mathbb{R}^+$$

$$\begin{array}{rcl}]-\infty, -\sqrt{-2c}[& \to & \mathbb{R}\,, \\ x & \mapsto & -\sqrt{x^2+2c} \end{array} \qquad \begin{array}{rcl}]\sqrt{-2c}, +\infty[& \to & \mathbb{R}\,, \\ x & \mapsto & -\sqrt{x^2+2c} \end{array}$$

$$\begin{array}{rcl}]-\infty, -\sqrt{-2c}[& \to & \mathbb{R} \\ x & \mapsto & \sqrt{x^2+2c} \end{array} \quad \text{e} \quad \begin{array}{rcl}]\sqrt{-2c}, +\infty[& \to & \mathbb{R}\,, \\ x & \mapsto & \sqrt{x^2+2c} \end{array} \qquad c \in \mathbb{R}^-$$

(d) Solução constante

$$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & 0 \end{array}$$

Soluções não constantes

$$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R}\,, \\ x & \mapsto & c\,e^{x^2/2} \end{array} \qquad c \in \mathbb{R}\backslash\{0\}$$

(e) Solução constante

$$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & 0 \end{array}$$

Soluções não constantes

$$\begin{array}{rcl}]-\infty, c[& \to & \mathbb{R} \\ x & \mapsto & \dfrac{1}{x-c} \end{array} \qquad \mathrm{e} \qquad \begin{array}{rcl}]c, +\infty[& \to & \mathbb{R}\,, \\ x & \mapsto & \dfrac{1}{x-c} \end{array} \qquad c \in \mathbb{R}$$

Exercício 2.

(a) $$\begin{array}{rcl}]-2, 3[& \to & \mathbb{R} \\ t & \mapsto & \dfrac{1}{t^2-t-6} \end{array}$$

(b) $$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ t & \mapsto & \dfrac{1}{t^2-t+6} \end{array}$$

(c) $$\begin{array}{rcl} \left]\dfrac{\sqrt{15}}{2}, +\infty\right[& \to & \mathbb{R} \\ x & \mapsto & \dfrac{-1+\sqrt{4x^2-15}}{2} \end{array}$$

Exercício 3.

(a) $\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & -2\, e^{3x^2} \end{array}$

(b) $\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ t & \mapsto & e^{t^2} - 1 \end{array}$

(c) $\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & 2\, e^{\operatorname{sen}(x+1)} \end{array}$

Exercício 4.

(a) $y(x) = \dfrac{1}{c - \cos(x)}, \quad c \in \mathbb{R}$

(b) $y(x) = 0; \quad y(x) = 1; \quad \log\left|\dfrac{y-1}{y}\right| = e^x + c, \quad c \in \mathbb{R}$

(c) $y(x) = \frac{\pi}{2} + k\,\pi, \quad k \in \mathbb{Z}; \quad \operatorname{tg}(y) = 2\operatorname{sen}(2x) - 4x + c, \quad c \in \mathbb{R}$

(d) $y + \dfrac{y^2}{2} = \dfrac{x \operatorname{sen}(2x)}{2} + \dfrac{\cos(2x)}{4} + c, \quad c \in \mathbb{R}$

(e) $y(x) = \log(\operatorname{sen}(x) + c), \quad c \in \mathbb{R}$

(f) $\dfrac{3y}{2} - \dfrac{1}{4}\operatorname{sen}(2y) = x \operatorname{sen}(x) + \cos(x) + c, \quad c \in \mathbb{R}$

Exercício 5. A solução maximal que passa no ponto $P = (1, 1)$ é:

$$\begin{array}{rcl} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & 1 \end{array}$$

A solução maximal que passa no ponto $Q = (0, \frac{1}{2})$ é:

$$\begin{array}{rcl}]-\sqrt[3]{2}, +\infty[& \to & \mathbb{R} \\ x & \mapsto & 1 - \dfrac{1}{x^3+2} \end{array}$$