In[1]:=
```
(* 9904295 *)
$Assumptions = {M > 0};
MM = {{0, mD}, {mD, M}};
mD = ϕ Y / √2 ;
V = 1/2 λ H^4 - m^2 H^2 + Ω /. H → ϕ / √2
{mn1, mn2} = Eigenvalues[MM]
```

Out[4]= $-\frac{1}{2} m^2 \phi^2 + \frac{\lambda \phi^4}{8} + \Omega$

Out[5]= $\left\{\frac{1}{2}\left(M - \sqrt{M^2 + 2 Y^2 \phi^2}\right), \frac{1}{2}\left(M + \sqrt{M^2 + 2 Y^2 \phi^2}\right)\right\}$

For renormalization scale Q < M, the effective potential V(***ϕ***) regularized/renormalized by DREG/MS-bar is given by

In[6]:=
```
ΔVlow = -2/(64 π^2) (#^4 (Log[#^2/Q^2] - 3/2)) & /@ {mn1} // Total;
```

Meanwhile, if we renormalize the potential at Q > M, we obtain

In[7]:=
```
ΔVhigh = -2/(64 π^2) (#^4 (Log[#^2/Q^2] - 3/2)) & /@ {mn1, mn2} // Total;
```

The potential at ***ϕ*** << M ~ Q is respectively given by

In[8]:=
```
Series[V + ΔVlow, {ϕ, 0, 4}] // FullSimplify
Series[V + ΔVhigh, {ϕ, 0, 4}] // FullSimplify
```

Out[8]= $\Omega - \frac{m^2 \phi^2}{2} + \frac{\lambda \phi^4}{8} + O[\phi]^5$

Out[9]= $$\left(\frac{3 M^4}{64 \pi^2} + \Omega - \frac{M^4 \operatorname{Log}\left[\frac{M^2}{Q^2}\right]}{32 \pi^2}\right) + \frac{\left(-8 m^2 \pi^2 + M^2 Y^2 - M^2 Y^2 \operatorname{Log}\left[\frac{M^2}{Q^2}\right]\right) \phi^2}{16 \pi^2} - \frac{\left(Y^4 - 8 \pi^2 \lambda + Y^4 \operatorname{Log}\left[\frac{M^2}{Q^2}\right]\right) \phi^4}{64 \pi^2} + O[\phi]^5$$

In **Δ**Vlow (**Δ**Vhigh), the couplings are renormalized at Q < M (Q>M).
Since these two should be equivalent, ***λ***(Q) for Q < M and Q > M should give the same value if evaluated at Q=M.

In[10]:=
$$\text{quartic[Q\_] := If}\left[Q < M, \frac{\lambda\text{low}[Q]\,\phi^4}{8}, -\frac{\left(Y^4 - 8\pi^2\lambda\text{high}[Q] + Y^4\,\text{Log}\left[\frac{M^2}{Q^2}\right]\right)\phi^4}{64\pi^2}\right]$$

$$\text{quadratic[Q\_] := If}\left[Q < M, -\frac{\text{msqlow}[Q]\,\phi^2}{2}, \frac{\left(-8\,\text{msqhigh}[Q]\,\pi^2 + M^2 Y^2 - M^2 Y^2\,\text{Log}\left[\frac{M^2}{Q^2}\right]\right)\phi^2}{16\pi^2}\right]$$

$$\text{const[Q\_] := If}\left[Q < M, \Omega, \frac{c\,M^4 + 32\pi^2\,\Omega - M^4\,\text{Log}\left[\frac{M^2}{Q^2}\right]}{32\pi^2}\right]$$

```
{Refine[quartic[Q], Q < M] /. Q → M, Refine[quartic[Q], Q > M] /. Q → M};
sol1 = Solve[%[[1]] == %[[2]], λlow[M]] // Expand
```

Out[14]=
$$\left\{\left\{\lambda\text{low}[M] \to -\frac{Y^4}{8\pi^2} + \lambda\text{high}[M]\right\}\right\}$$

In[15]:=
```
{Refine[quadratic[Q], Q < M] /. Q → M, Refine[quadratic[Q], Q > M] /. Q → M}
sol2 = Solve[%[[1]] == %[[2]], msqlow[M]] // FullSimplify
```

Out[15]=
$$\left\{-\frac{1}{2}\phi^2\,\text{msqlow}[M], \frac{\phi^2\left(M^2 Y^2 - 8\pi^2\,\text{msqhigh}[M]\right)}{16\pi^2}\right\}$$

Out[16]=
$$\left\{\left\{\text{msqlow}[M] \to -\frac{M^2 Y^2}{8\pi^2} + \text{msqhigh}[M]\right\}\right\}$$

These solution means m^2**[**Q**]** and $\boldsymbol{\lambda}$[Q] in the effective theory ("low") at Q=M is given in terms of m^2**[**Q**]** and $\boldsymbol{\lambda}$[Q] in the full theory by

In[17]:=
```
{sol1, sol2}
```

Out[17]=
$$\left\{\left\{\left\{\lambda\text{low}[M] \to -\frac{Y^4}{8\pi^2} + \lambda\text{high}[M]\right\}\right\}, \left\{\left\{\text{msqlow}[M] \to -\frac{M^2 Y^2}{8\pi^2} + \text{msqhigh}[M]\right\}\right\}\right\}$$

Note that the original paper does the matching at Q = M Exp[-3/4];

In[18]:=
```
{Refine[quartic[Q], Q < M] /. Q → M Exp[-3/4],
  Refine[quartic[Q], Q > M] /. Q → M Exp[-3/4]};
sol1 = Solve[%[[1]] == %[[2]], λlow[_]] // Expand
{Refine[quadratic[Q], Q < M] /. Q → M Exp[-3/4],
  Refine[quadratic[Q], Q > M] /. Q → M Exp[-3/4]};
sol1 = Solve[%[[1]] == %[[2]], msqlow[_]] // Expand
```

Out[19]=
$$\left\{\left\{\lambda\text{low}[\_] \to -\frac{5\,Y^4}{16\pi^2} + \lambda\text{high}\left[\frac{M}{e^{3/4}}\right]\right\}\right\}$$

Out[21]=
$$\left\{\left\{\text{msqlow}[\_] \to \frac{M^2 Y^2}{16\pi^2} + \text{msqhigh}\left[\frac{M}{e^{3/4}}\right]\right\}\right\}$$