

RGE Validation

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1 SM parameters at $Q = m_t$

1.1 Fixing a typo

The original reference [1, v4] has a typo in threshold correction $g_Y^{(1)}$ (Eq.(95)); the second last-term should be read as $36M_W^4/(M_Z^2 - M_W^2)$. This typo is fixed in **6b18ea0**→**9e2350f**.

With the fixed version we can reproduce Table 3 from Table 2 (both in [1, v4]). For example,

```
Install["LoopTools"];
Needs["SMRGE`", "1307_3536.w1"];
table2 = <|
  "MW" -> 80.384,      "MZ" -> 91.1876,    "MH" -> 125.15,
  "MT" -> 173.34,      "MB" -> 4.2,        "MTau" -> 1.777,
  "as(MZ)" -> 0.1184,  "Gmu" -> 1/(v^2 Sqrt[2]) /. {v -> 246.21971} |>
table3[order_] := {
  #[lam], Sqrt#[ytsq], Sqrt#[g2sq], Sqrt#[g1sq]*3/5, Sqrt#[msq]
} &@ getCouplingsAtMt[table2, order]
```

will give The unfixed version gives different g_Y , while after the fix Table 3 is reproduced.

6b18ea0 ; table3[0]	0.129177	0.995614	0.652945	0.349715	125.15
6b18ea0 ; table3[1]	0.127745	0.951129	0.647545	0.358571	132.37
6b18ea0 ; table3[2]	0.126049	0.940179	0.647799	0.357464	131.554
9e2350f ; table3[0]	0.129177	0.995614	0.652945	0.349715	125.15
9e2350f ; table3[1]	0.127745	0.951129	0.647545	0.359394	132.37
9e2350f ; table3[2]	0.126049	0.940179	0.647799	0.358287	131.554

1.2 Strange results in neutrino-option paper

Neutrino-option paper [2] uses slightly different values in their Table 2:

```
table2aNeutOpt = <|
  "MW" -> 80.387,      "MZ" -> 91.1875,    "MH" -> 125.09,
  "MT" -> 173.2,       "MB" -> 4.18,        "MTau" -> 1.776,
  "as(MZ)" -> 0.1185,  "Gmu" -> 1.1663787/10^5 |>;
table2b[order_] := {
  #[lam], Sqrt#[msq], Sqrt#[g1sq], Sqrt#[g2sq], Sqrt#[g3sq],
  Sqrt#[ytsq], Sqrt#[ybsq], Sqrt#[yasq]
} &@ getCouplingsAtMt[table2aNeutOpt, order]
```

but their obtained values (Table 2b) are quite strange. First of all, they show $\hat{g}_3 = 1.22029$, but this is equal to $\sqrt{4\pi\alpha_s(M_Z)}$ and inconsistent to what they wrote,

The value of $g_3(\mu = m_t)$ is extracted from Eqn. 60 of Ref. [42] which includes higher order QCD corrections.

because this procedure should give $\hat{g}_3 = 1.16711$. SI is also sure that they did not notice the typo.

Their value of Table 2a should give the following values for Table 2b:

9e2350f ; table2b[0]	0.1291	125.09	0.451	0.653	1.16711	0.995	0.024	0.0102
9e2350f ; table2b[1]	0.1276	132.288	0.464	0.648	1.16711	0.950	0.024	0.0102
9e2350f ; table2b[2]	0.1259	131.474	0.462	0.648	1.16711	0.939	0.024	0.0102

One can reproduce most of their results, shown in their Table 2b, using [6b18ea0](#) with `WeakScaleThreshold["g3", 0,1,2] := Sqrt[4*Pi*asMZ];`

6b18ea0 -mod; table2b[0]	0.1291	125.09	0.451	0.6530	1.22029	0.995	...
6b18ea0 -mod; table2b[1]	0.1276	132.288	0.463	0.6476	1.22029	0.946	...
6b18ea0 -mod; table2b[2]	0.1258	131.431	0.461	0.6478	1.22029	0.933	...

Here, however, \hat{g}_2 is a bit different (0.5%) from their values; SI guesses they have another typo in their code.

1.3 Validation with `mr` 1.3.2

SI compares the results against `mr` 1.3.2 [3]. The code `rgecheck.cpp` given in [b543c71](#) gives, using the values in the neutrino-option paper,

<code>mr 1.3.2; order=0</code>	0.12905	125.0900	0.4514	0.6530	1.1651091	0.9948	0.0240
	0.12757	132.4737	0.4642	0.6473	1.1651091	0.9505	0.0166
	0.12588	131.2905	0.4626	0.6481	1.1651091	0.9396	0.0146

This result supports SI's value than those in the neutrino-option paper.

References

- [1] D. Buttazzo, *et al.*, *Investigating the near-criticality of the Higgs boson*, JHEP **12** (2013) 089 [arXiv:1307.3536].
- [2] I. Brivio and M. Trott, *Examining the neutrino option*, JHEP **02** (2019) 107 [arXiv:1809.03450].
- [3] B. A. Kniehl, A. F. Pikelner, and O. L. Veretin, *mr: a C++ library for the matching and running of the Standard Model parameters*, Comput. Phys. Commun. **206** (2016) 84–96 [arXiv:1601.08143]. GitHub:apik/mr.