Massive Neutrinos and the Higgs Mass Window

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If neutrino masses are produced by a see-saw mechanism the Standard Model prediction for the Higgs mass window (defined by upper (perturbativity) and lower (stability) bounds) can be substantially affected. Actually the Higgs mass window can close completely, which settles an upper bound on the Majorana mass for the right-handed neutrinos, M, ranging from 10^{13} GeV for three generations of quasi-degenerate massive neutrinos with $m_{\nu} \simeq 2$ eV, to 5×10^{14} GeV for just one relevant generation with $m_{\nu} \simeq 0.1$ eV. A slightly weaker upper bound on M, coming from the requirement that the neutrino Yukawa couplings do not develop a Landau pole, is also discussed.

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Observations of the flux of atmospheric neutrinos by SuperKamiokande [1] provide strong evidence for neutrino oscillations, which in turn imply that (at least two species of) neutrinos must be massive. Additional support to this hypothesis is given by the need of neutrino oscillations to explain the solar neutrino flux deficit and the possible essential role of the neutrinos in the large scale structure of the universe [2]. Much work has been devoted in the last months in order to guess and to explain the structure and the origin of the neutrino mass matrices capable to account for the different observations [3].

In this letter, we would like to point out the consequences that massive neutrinos have for the Standard Model (SM), in particular for its still unprobed sector, namely the Higgs sector. As it is known, the Higgs mass in the SM is bounded from above from the requirement of validity of perturbation theory, and from below by the requirement of stability of the Higgs potential [4,5]. More precisely, if we demand both requirements until the Planck scale $(M_{P\ell})$, then the allowed window for the Higgs mass is

$$137 \text{ GeV} \lesssim M_H \lesssim 175 \text{ GeV}$$
 . (1)

We will show that if neutrino masses are produced by a see-saw mechanism, the previous prediction for the Higgs mass can be substantially modified.

The simplest extension of the SM Lagrangian, capable to account for neutrino masses, reads

$$\mathcal{L}_{\nu} = -\bar{\nu}_{R} \mathcal{M}_{D} \nu_{L} - \frac{1}{2} \bar{\nu}_{R} \mathcal{M}_{M} \bar{\nu}_{R}^{T} + \text{ h.c.} , \qquad (2)$$

where \mathcal{M}_D is the Dirac mass matrix $(\mathcal{M}_D = \frac{1}{\sqrt{2}} \langle \phi \rangle \mathbf{Y}_{\nu}$, where ϕ is the neutral component of the Higgs field and \mathbf{Y}_{ν} is the matrix of the neutrino Yukawa couplings) and \mathcal{M}_M is the Majorana mass matrix for the right-handed neutrinos.

In order to discuss the impact of the previous extension of the SM on the Higgs sector, it is convenient to start

with the case where there is a hierarchy of left-handed neutrino masses, $m_{\nu_1}^2 \ll m_{\nu_2}^2 \ll m_{\nu_3}^2$, presumably inherited by a similar hierarchy in the Dirac-Yukawa couplings. In this case, there is only one relevant generation of neutrinos for our purposes (the most massive one), so \mathcal{M}_D and \mathcal{M}_M become the single parameters m_D and M. As we will see the extension of the results to the general case is straightforward. Now, the two associated neutrino eigenvalues arising from (2) are

$$m_{\nu_{1,2}} = \frac{1}{2} \left(M \mp \sqrt{M^2 + 4m_D^2} \right)$$
 (3)

For $M \gg \langle \phi \rangle$, we get $m_{\nu_1} \simeq m_D^2/M$, $m_{\nu_2} \simeq M$; ν_1, ν_2 correspond essentially to the left- and right-handed neutrinos respectively. For energy scales below M we can integrate out the ν_2 neutrino, so \mathcal{L}_{ν} effectively becomes

$$\mathcal{L}_{\nu}^{\text{eff}} = -\frac{1}{2}\nu_L^T m_{\nu_1} \nu_L + \text{h.c.} \equiv -\frac{1}{4}\kappa \nu_L^T \langle \phi \rangle^2 \nu_L + \text{h.c.} \quad (4)$$

where $v=\langle\phi\rangle$ at the physical vacuum (i.e. v=246 GeV) and for convenience we have introduced the effective coupling κ ($\kappa v^2=2\,m_D^2/M$ at the M-scale). For scales below M, κ is a running parameter, whose beta function is essentially given by [6]

$$\beta_{\kappa} = \frac{1}{16\pi^2} \left(-3g_2^2 + 2\lambda + 6Y_t^2 \right) \kappa , \qquad (5)$$

where g_2, λ, Y_t are the SU(2) gauge coupling, the Higgs quartic coupling and the top Yukawa coupling respectively. Thus, for a given physical mass of ν_1 (to be identified with the low-energy value of $\kappa v^2/2$) and for a given value of the Majorana mass, M, the Dirac mass m_D (and thus Y_{ν}) is unambiguously fixed.

The one-loop effective potential $V(\phi)$ has the form

$$V = V_{SM} + \Delta V_{\nu} \tag{6}$$

where V_{SM} is the usual one-loop SM potential, consisting of the tree level part $V_{tree} = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^2 + \Omega$, plus

the ordinary radiative corrections dominated by the top Yukawa coupling, Y_t , and

$$\Delta V_{\nu} = -\frac{1}{32\pi^2} \left[m_{\nu_1}^4 \log \frac{m_{\nu_1}^2}{\mu^2} + \theta_{\nu} m_{\nu_2}^4 \log \frac{m_{\nu_2}^2}{\mu^2} \right] , \quad (7)$$

where $m_{\nu_{1,2}}$ are given by (3) (note that they are functions of ϕ), μ is the renormalization scale* and $\theta_{\nu} \equiv \theta(\mu - M)$ [7,8] is a step θ -function accounting for the threshold at the M-scale.

Above M, Y_{ν} runs with the scale, while the various SM parameters get contributions from Y_{ν} to their beta-coefficients. The relevant ones are [9]

$$\beta_{Y_{\nu}} = \frac{1}{16\pi^{2}} \theta_{\nu} \left[3Y_{t}^{2} - \left(\frac{3}{4}g_{1}^{2} + \frac{9}{4}g_{2}^{2} \right) + \frac{5}{2}Y_{\nu}^{2} \right] Y_{\nu}$$

$$\beta_{\lambda}^{(\nu)} = \frac{1}{16\pi^{2}} \left[4\theta_{\nu} (\lambda Y_{\nu}^{2} - Y_{\nu}^{4}) \right], \ \gamma_{\phi}^{(\nu)} = \frac{1}{16\pi^{2}} \left[\theta_{\nu} Y_{\nu}^{2} \right]$$

$$\beta_{\Omega}^{(\nu)} = -\frac{1}{16\pi^{2}} \left[\theta_{\nu} M^{4} \right], \quad \beta_{m^{2}}^{(\nu)} = \frac{1}{16\pi^{2}} \left[4\theta_{\nu} M^{2} Y_{\nu}^{2} \right]$$

$$\beta_{Y_{t}}^{(\nu)} = \frac{1}{16\pi^{2}} \left[\theta_{\nu} Y_{\nu}^{2} Y_{t} \right], \tag{8}$$

where $\beta_i^{(\nu)}$ is the contribution of Y_{ν} to the β_i coefficient. The matching of the complete and the effective theory at M requires to introduce threshold corrections. In particular, the matching of V requires to introduce a threshold contribution below M: $\Delta_{th}V = -\frac{2}{64\pi^2} \left[m_{\nu_2}^4 \log \frac{m_{\nu_2}^2}{M^2} \right]$, whose expansion gives the threshold corrections to the m^2 and λ parameters, namely $\Delta_{th}m^2 = \frac{1}{16\pi^2}Y_{\nu}^2M^2$, $\Delta_{th}\lambda = -\frac{5}{16\pi^2}Y_{\nu}^4$.

Now we are prepared to compute the modification of the perturbativity and stability bounds on the SM Higgs mass. The perturbativity bound arises from the requirement that λ does not enter the non-perturbative regime below a certain scale Λ . If the SM is to be valid until $M_{P\ell}$ or some high scale M_X , Λ should be identified with those scales. Since for $\mu < M$ the running of λ is as in the SM, the perturbativity bound for $\Lambda < M$ remains the same. However, for $\mu > M$ the contribution of the neutrinos to the λ running (which for large λ is dominated by the positive λY_{ν}^2 term in eq. (8)) speeds up the increasing of λ , thus leading to a more stringent upper bound on M_H .

On the other hand, the stability bound on the Higgs mass arises from demanding that the potential does not develop an instability for large values of ϕ . The instability arises from the fact that λ can be driven to negative values at large enough scales. Notice that to evaluate the potential for large values of ϕ one has to plug a renormalization scale $\mu \sim \phi$ in order to avoid large logarithms. Hence, if the $\lambda \phi^4$ term, which is dominant for large ϕ , becomes negative, so does the potential. If we demand this not to happen below a scale Λ , i.e. we require $\lambda(\Lambda) \geq 0$, this translates into a lower bound on M_H . Again, for $\Lambda < M$ the values of λ are as in the SM, and thus the corresponding lower bound on M_H . However, for $\Lambda > M$ the values of λ get modified by the neutrino contribution to β_{λ} . For small values of λ this contribution is dominated by the negative Y_{ν}^4 term in (8). Hence, λ is driven more rapidly to negative values and the lower bound becomes more stringent too.

For a more careful analysis of the stability bound, one has to consider not just the tree-level potential (which is in fact dominated by the $\lambda\phi^4$ term), but also the radiative corrections. A practical way to include them [5] is to take $\mu=\alpha\phi$ ($\alpha\simeq 1$ is always a correct choice) and then to extract the $\sim\phi^4$ contributions from ΔV^{1-loop} , neglecting (for $\phi\gg M$) the M/ϕ factors. These ϕ^4 contributions can be incorporated into an effective quartic parameter:

$$\lambda_{eff}(\mu < M) = \lambda - \frac{1}{16\pi^2} \left[6Y_t^4 \log \frac{Y_t^2}{2} \right]$$

$$\lambda_{eff}(\mu > M) = \lambda - \frac{1}{16\pi^2} \left[6Y_t^4 \log \frac{Y_t^2}{2} + 2Y_{\nu}^4 \log \frac{Y_{\nu}^2}{2} \right] \quad (9)$$

Notice that for $\mu < M$ the ν_1 contribution (the only neutrino which is present) is negligible, as $m_{\nu_1} \ll m_t$. However, for $\mu > M$ both neutrinos contribute in the same (non-negligible) amount. The stability bound reads, in this more accurate form, as $\lambda_{eff}(\Lambda) \geq 0$.

To summarize, the presence of the neutrinos strengthens both the upper and the lower bound on the SM Higgs mass, thus narrowing the Higgs window. The quantitative effect depends just on the value of the "left-handed" neutrino mass, m_{ν_1} , and on the value of the Majorana mass, M (the Yukawa coupling, Y_{ν} , can be extracted from those two in the way explained above).

Before presenting numerical results, let us extend the analysis to the case where all the neutrinos have similar masses, and therefore all of them are equally relevant. Then, κ , \mathcal{M}_M , \mathcal{M}_D and \mathbf{Y}_{ν} are flavor matrices. The corresponding effective mass matrix for the left-handed neutrinos (i.e. the analogue to $m_{\nu_1} \simeq m_D^2/M$ in the single neutrino case) is given by

$$\mathcal{M}_{\nu} = \mathcal{M}_{D}^{T} \mathcal{M}_{M}^{-1} \mathcal{M}_{D} \tag{10}$$

The beta functions given in eqs. (8) get slightly modified, depending now on the whole matrix \mathbf{Y}_{ν} (the extension is quite trivial and can be found in [9]). Consequently, the precise results will depend in principle on the textures of

^{*}More precisely, $\mu^2=e^{3/2}\bar{\mu}^2$, where $\bar{\mu}$ is the usual $\overline{\rm MS}$ renormalization scale.

[†]Note that the threshold correction for m^2 is very sizeable, exhibiting an explicit implementation of the SM gauge hierarchy problem. Working within the SM framework, it is necessary to fine-tune the m^2 at high energy in order to reproduce the usual low-energy physics. However, this conceptual shortcoming does not play any role in our analysis and the corresponding results, since the $m^2\phi^2$ contribution to the effective potential is always negligible compared to the $\lambda\phi^4$ one.

the \mathcal{M}_D (or \mathbf{Y}_{ν}) and \mathcal{M}_M matrices. However, as we will see, in practice the form of the textures is not important in our analysis.

One can always choose a basis of ν_{L_i} , ν_{R_i} in which m_D , and thus \mathbf{Y}_{ν} , is diagonal (in this basis the matrix of Yukawa couplings of the charged leptons is in general non-diagonal, but these couplings are negligible in our analysis). Then, the relevant formulas, eqs. (7)-(9) become trivially extended to the 3-generation case [9]. If the physical neutrinos are quasi-degenerated (which, according to the observations must occur for $m_{\nu_i} \gtrsim 0.1 eV$), it is hard to believe that this fact can naturally come from a diagonal \mathcal{M}_D with non-degenerated entries, compensated by a non-trivial structure of the \mathcal{M}_M matrix, see eq. (10). One can arrange things in this way, but it is extremely artificial. (In this case, the largest Yukawa coupling would dominate all the equations, and we simply would be back to the case of a single relevant neutrino). Therefore, one expects that both \mathcal{M}_D and \mathcal{M}_M have essentially degenerated eigenvalues, m_D and M, so that at the end of the day all the neutrinos have masses $m_{\nu_i} \simeq m_D^2/M$.

On the other hand, it has been claimed in the literature [10] that if neutrino masses are to play an essential cosmological role, beside explaining the atmospheric and solar neutrino fluxes, the texture of the effective left-handed neutrino mass must be of the bimaximal mixing type

$$\mathcal{M}_{\nu} = m_{\nu} \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix}$$
 (11)

where m_{ν} is the common neutrino mass. This texture produces three degenerate left-handed neutrinos with maximal mixing angles θ_{12} and θ_{23} . This is the texture we have used for our numerical analysis, although, as explained above, other textures will produce similar results. Notice that for our analysis we need not just the form of \mathcal{M}_{ν} , but also the form of the matrix of the Yukawa couplings, \mathbf{Y}_{ν} , and thus the form of \mathcal{M}_{M} (see eq. (10)). Our choice has been to take \mathbf{Y}_{ν} proportional to the identity, i.e $\mathbf{Y}_{\nu} = Y_{\nu}$ diag(1, 1, 1). As discussed above, this is a perfectly reasonable choice in this case. Then the form of the right-handed Majorana matrix is forced to be

$$\mathcal{M}_M^{-1} = \frac{1}{Mm_{\nu}} \mathcal{M}_{\nu} . \tag{12}$$

It is straightforward to check that with these ansatzs, the mass-squared eigenvalues of the three "left-handed" and the three "right-handed" neutrinos, say $(m_{\nu_{1,2}}^{(i)})^2$, with i=1,2,3, are as in eq. (3). Thus, we see that in this particularly well-motivated case the texture plays no significant role indeed.

Let us now present the results. Fig. 1 shows the evolution of the Higgs window as a function of the scale Λ until which the theory is valid. In order to illustrate the effect

of the neutrinos we have chosen a typical case, namely $m_{\nu}=0.1~{\rm eV}$ (for one or the three generations of neutrinos) and $M\simeq 3\times 10^{14}~{\rm GeV}$. The pure SM window is also shown to facilitate the comparison. We note that the strengthening of the stability (lower) bound is the main responsible of the substantial narrowing of the allowed window.

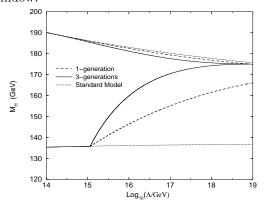


FIG. 1. The Higgs window as a function of the scale Λ for $m_{\nu}=0.1~{\rm eV}$ and $M=2.8\times 10^{14}~{\rm GeV}.$

Fig. 2 shows the variation of the Higgs window with the Majorana mass, M, for $\Lambda = M_{P\ell}$ (continuous lines) and $\Lambda = 10^{16}$ GeV (dashed lines) for just one relevant generation of massive neutrinos with $m_{\nu} = 0.1$ eV (the other generations may be massive but hierarchically smaller).

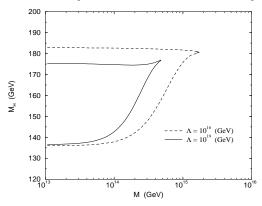


FIG. 2. The Higgs window as a function of the Majorana mass, M for $\Lambda = M_{P\ell}$ (continuous lines) and $\Lambda = 10^{16}$ GeV (dashed lines) for the case of just one massive neutrino with $m_{\nu} = 0.1$ eV.

This case corresponds to the most conservative scenario concerning the effect of neutrinos on the Higgs window. The values of the bounds for "low" Majorana masses $(M \lesssim 10^{13} \text{ GeV})$ coincide with those of the pure SM without neutrinos. Fig. 3 is analogous to Fig. 2, but for three generations of neutrinos, with degenerate masses $m_{\nu}=2 \text{ eV}$. This case corresponds to the scenario where the effect of the neutrinos in the Higgs sector is maximized.

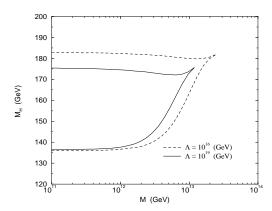


FIG. 3. The same as Fig.1, but with three generations of massive neutrinos with $m_{\nu}=2$ eV.

It is apparent from Figs. 2 and 3 that above a certain value of the Majorana mass the Higgs window closes up, disappearing. In principle, this could seem paradoxical, since one expects that for some fine-tuned initial value of λ (and thus of M_H) the evolution of $\lambda(\mu)$ for large μ will be just between the two bounds, say $0 \le \lambda(\mu) \le 4\pi$. So one would expect, for large values of M, an extremely narrow window, but a window after all. The fact that actually forbids that window is that for large values of M the neutrino Yukawa couplings themselves develop a Landau pole below Λ . Alternatively, we can set Y_{ν} at the Landau pole at $M_{P\ell}$ (i.e. $Y_{\nu}(M_{P\ell}) \gg 1$) and evaluate the corresponding low energy value of m_{ν} , through the renormalization group equations (RGE) of Y_{ν} and κ , for a certain value of the Majorana mass M.

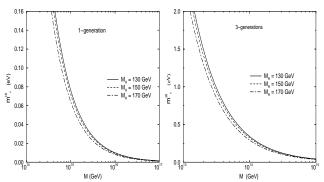


FIG. 4. Upper bound on the neutrino mass, m_{ν}^{IR} , vs. the Majorana mass M for one and three generations of massive neutrinos (left and right panels respectively).

This "infrared fixed point" value, say m_{ν}^{IR} , represents an upper bound for the neutrino mass. The dependence of m_{ν}^{IR} on M is illustrated in Fig. 4 for one and three generations of massive neutrinos. Due to the dependence of the κ RGE on λ , the value of m_{ν}^{IR} presents a slight dependence on the value of M_H , as is shown in the figure.

In conclusion, we have shown that if neutrino masses are produced by a see-saw mechanism, the SM prediction for the Higgs mass window (defined by upper (perturbativity) and lower (stability) bounds) is substantially affected in an amount that depends on the value of the

Majorana mass for the right-handed neutrinos, M. Actually, for values of M above a certain value, the Higgs window closes, setting an upper bound on M. This varies from 10^{13} GeV for three generations of massive neutrinos with $m_{\nu} \simeq 2$ eV to 5×10^{14} GeV for just one relevant generation with $m_{\nu} \simeq 0.1$ eV. We have also discussed a second (slightly weaker) upper bound on M, coming from the requirement that the neutrino Yukawa couplings do not develop a Landau pole. The whole analysis and results are practically independent of the details of the model (i.e. the particular structure of the neutrino mass matrices).

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