Analysis of NuFIT Best-fit points

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1. Notation

Masses We introduce the following notations on the two heavier neutrino masses[*1] M_I and the three lighter neutrino masses m_i, one of which is zero, as

$$\begin{aligned} \Delta M &:= M_2 - M_1 > 0 & \delta M &:= \Delta M/M_1 \\ \Delta M^2 &:= M_2^2 - M_1^2 & \delta M^2 &:= \Delta M^2/M_1^2 \\ \Delta m &:= m_{\text{heavier}} - m_{\text{lighter}} & \rho_m &:= \Delta m/m_{\text{tot}}, & m_{\text{tot}} &:= \sum_i m_i, \end{aligned}$$

$$m_{\text{heavier}} = m_3\ (m_2), \qquad m_{\text{lighter}} = m_2\ (m_1) \qquad \text{for NH (IH)}.$$

Numerically, for BFP of NH (IH), $\rho_m \simeq \sqrt{0.5}$ (0.0075) and $m_{\text{tot}} \simeq 5.9\ (9.9) \times 10^{-11}\,\text{GeV}$.

Yukawa Yukawa matrix follows the notation of 1611 paper, and the CIP is given by

$$y = \mathrm{i}(v/\sqrt{2})^{-1}\sqrt{M_{\text{diag}}}R\sqrt{m_{\text{diag}}}U^\dagger \tag{1.1}$$

with $v = \sqrt{\langle\phi_0\rangle} \approx 246\,\text{GeV}$,

$$R = \begin{pmatrix} 0 & +\mathrm{c}_z & \zeta \mathrm{s}_z \\ 0 & -\mathrm{s}_z & \zeta \mathrm{c}_z \end{pmatrix} \quad \text{for NH,} \qquad R = \begin{pmatrix} +\mathrm{c}_z & \zeta \mathrm{s}_z & 0 \\ -\mathrm{s}_z & \zeta \mathrm{c}_z & 0 \end{pmatrix} \quad \text{for IH.} \tag{1.2}$$

Here, $z \in \mathbb{C}$ and $\zeta = \pm 1$:

$$z = w + \mathrm{i}x; \qquad w \in \mathbb{R}, \quad x \in \mathbb{R}, \tag{1.3}$$

and $U = U_{\text{PMNS}}$ follows 1611 paper, i.e., PDG with a phase matrix $\mathrm{diag}(1, \mathrm{e}^{\mathrm{i}\sigma}, 1)$.

$$\gamma_I = \frac{(yy^\dagger)_{II}}{8\pi}, \qquad \Gamma_I = M_I \gamma_I.$$

Yukawa products We define

$$\begin{aligned} W_1 &= W_{11} = \cosh 2x - \rho_m \cos 2w, & W_{12} &= W_{21}^* = \mathrm{i}\sinh 2x + \rho_m \sin 2w, \\ W_2 &= W_{22} = \cosh 2x + \rho_m \cos 2w, & \mu_I &= \frac{m_{\text{tot}} M_I}{8\pi v^2} \approx 3.9\ (6.5) \times 10^{-17} M_I/\text{GeV}, \end{aligned}$$

for NH (IH), which leads us to

$$(yy^\dagger)_{IJ} = \frac{m_{\text{tot}}\sqrt{M_I M_J}}{v^2} W_{IJ}, \qquad \Gamma_I = \frac{(yy^\dagger)_{II}}{8\pi} M_I = \mu_I W_I M_I. \tag{1.4}$$

[*1] We can safely neglect the differences between M_I and the (diagonalized) Majorana masses.

Effective neutrino masses Remembering that m_i is given by

$$m_i = \left[U^* \left(-\frac{v^2}{2} y^{\mathrm{T}} M^{-1} y\right) U\right]_{ii} = U^*_{i\alpha} \left(-\frac{v^2}{2} \sum_I \frac{y_{I\alpha} y_{I\beta}}{M_I}\right) U_{\beta i}, \tag{1.5}$$

we define

$$\widetilde{m_I} := \frac{v^2}{2} \sum_\alpha \frac{|y_{I\alpha}|^2}{M_I} = \frac{v^2}{2} \frac{(yy^\dagger)_{II}}{M_I}, \qquad m_* := 1.66\sqrt{g_*}\frac{8\pi(v^2/2)}{M_{\mathrm{pl}}} \sim 1 \times 10^{-12}\,\mathrm{GeV}. \tag{1.6}$$

Note that these effective neutrino masses are related to

$$\widetilde{m_I} = \frac{m_{\mathrm{tot}}}{2} W_I. \tag{1.7}$$

2. Neutrino-option condition

The neutrino-option condition is given by, matching at $Q_0 = M_1\,\mathrm{e}^{-3/4}$,

$$\mu^2_{\mathrm{EFT}}(Q_0) = \frac{M_1^2}{16\pi^2}\,\mathrm{Tr}(yy^\dagger). \tag{2.1}$$

Defining

$$f(M_1) := \left.\frac{8\pi^2 v^2 \cdot \mu^2_{\mathrm{EFT}}(Q_0)}{M_1^3}\right|_{Q=M_1 \exp(-3/4)}, \tag{2.2}$$

we can rewrite the neutrino-option condition as

$$f(M_1) = \frac{v^2}{2} \frac{\mathrm{Tr}(yy^\dagger)}{M_1}. \tag{2.3}$$

The right-hand side is parameterized as[*2]

$$\frac{v^2}{2} \frac{\mathrm{Tr}(yy^\dagger)}{M_1} = m_{\mathrm{tot}} \left[\cosh 2x + \frac{\delta M}{2}(\cosh 2x + \rho_m \cos 2w)\right] > m_{\mathrm{tot}}. \tag{2.4}$$

Therefore, the neutrino-option condition gives a constraint

$$f(M_1) > m_{\mathrm{tot}}. \tag{2.5}$$

This is translated to an upper bound on M_1, which is

$$M_1 < 9.4\ (7.9) \times 10^6\,\mathrm{GeV} \quad \text{for NH (IH)} \tag{2.6}$$

Meanwhile, for $M_1 \ll 10^7\,\mathrm{GeV}$, we can fulfill the neutrino-option condition with

$$\cosh 2x \simeq \frac{f(M_1)}{m_{\mathrm{tot}}}. \tag{2.7}$$

For example, for $M_1 = 4\ (1)\ \times 10^6\,\mathrm{GeV}$, the condition is satisfied with $\cosh 2x \sim 10\ (1000)$.

[*2] Notice that $\cosh 2x = (\widetilde{m_1} + \widetilde{m_2})/m_{\mathrm{tot}}$, where $\widetilde{m_I}$ is an effective neutrino parameter.

3. Leptogenesis

The resulting lepton asymmetry is approximately given by

$$\delta\eta_l \simeq \sum_{I\alpha} \frac{\epsilon_{I\alpha}}{K_\alpha^{\mathrm{eff}} \min(z_c, z_\alpha)} = \sum_{\alpha} \frac{\sum_I \epsilon_{I\alpha}}{K_\alpha^{\mathrm{eff}} \min(z_c, z_\alpha)}. \tag{3.1}$$

3.1. Numerator

The parameter $\epsilon_{I\alpha}$ is given by two functions,

$$F_{I\alpha} := \left.\frac{\mathrm{Im}\left[y_{I\alpha} y^*_{J\alpha} (yy^\dagger)_{IJ}\right]}{(yy^\dagger)_{II}(yy^\dagger)_{JJ}}\right|_{J=3-I}, \qquad F'_{I\alpha} := \left.\frac{\mathrm{Im}\left[y_{I\alpha} y^*_{J\alpha} (yy^\dagger)_{JI}\right]}{(yy^\dagger)_{II}(yy^\dagger)_{JJ}}\right|_{J=3-I}. \tag{3.2}$$

I emphasize that **these functions are independent of $\boldsymbol{M_{1,2}}$;** and the asymmetry parameter is given by

$$\epsilon_{I\alpha} = F_{I\alpha} f^{\mathrm{vertex}}_{IJ} + \left(F_{I\alpha} + \frac{M_I}{M_J} F'_{I\alpha}\right)(f^{\mathrm{osc}}_{IJ} + f^{\mathrm{mix}}_{IJ}). \tag{3.3}$$

So the dependence of $\epsilon_{I\alpha}$ on M_I is encapsuled into the functions f and M_I/M_J:

$$f^{\mathrm{vertex}}_{IJ} := \frac{\Gamma_J}{M_I}\left[1 - \left(1 + \frac{M_J^2}{M_I^2}\right)\ln\left(1 + \frac{M_I^2}{M_J^2}\right)\right], \tag{3.4}$$

$$f^{\mathrm{mix}}_{IJ} := \frac{(M_I^2 - M_J^2) M_I \Gamma_J}{(M_I^2 - M_J^2)^2 + M_I^2 \Gamma_J^2}, \tag{3.5}$$

$$f^{\mathrm{osc}}_{IJ} := \frac{(M_I^2 - M_J^2) M_I \Gamma_J}{(M_I^2 - M_J^2)^2 + M_I^2 \Gamma_J^2 \mu_{IJ} \rho_{\mathrm{osc}}}, \tag{3.6}$$

where

$$\mu_{IJ} = \frac{M_J}{M_I} + \frac{\Gamma_I}{\Gamma_J}, \qquad \rho_{\mathrm{osc}} = \frac{\det\left[\mathrm{Re}(yy^\dagger)\right]}{(yy^\dagger)_{11}(yy^\dagger)_{22}} = \frac{\cosh^2 2y - \rho_m^2}{\cosh^2 2y - \rho_m^2 \cos^2 2w}. \tag{3.7}$$

Note that $\mu_{IJ} = \mathcal{O}(1)$ and $0 < \rho^{\mathrm{osc}} < 1$.

The resonant leptogenesis is thus governed by the ratio

$$R_{IJ} := \frac{M_I \Gamma_J}{M_I^2 - M_J^2} = \frac{W_{JJ}}{8\pi} \frac{m_{\mathrm{tot}} M_J}{v^2} \frac{M_I M_J}{M_I^2 - M_J^2}; \qquad f^{\mathrm{osc}}_{IJ} = \frac{R_{IJ}}{1 + R_{IJ}^2 \mu_{IJ} \rho_{\mathrm{osc}}}. \tag{3.8}$$

In the parameter region of our interest, $R_{IJ} \ll 1$ and

$$f^{\mathrm{mix}}_{IJ} \sim f^{\mathrm{osc}}_{IJ} \sim R_{IJ} \gg f^{\mathrm{vertex}}_{IJ} \simeq \frac{M_I^2 - M_J^2}{M_I^2} R_{IJ}. \tag{3.9}$$

We then define

$$F^\pm_\alpha := (F_{2\alpha} + F'_{2\alpha}) \pm (F_{1\alpha} + F'_{1\alpha}) \tag{3.10}$$

and evaluate them, which yields $F^+_\alpha = 0$ and

$$\begin{aligned}
F^-_\alpha &= \frac{4\,\mathrm{Re}(yy^\dagger)_{12}\,\mathrm{Im}(y^*_{1\alpha} y_{2\alpha})}{(yy^\dagger)_{11}(yy^\dagger)_{22}} && (3.11)\\
&= \frac{2\rho_m \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w} 2v^2 m_{\mathrm{tot}} \sqrt{M_1 M_2}\,\mathrm{Im}(y^*_{1\alpha} y_{2\alpha}) && (3.12)\\
&= \frac{2\rho_m \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w}\left[G^{(1)}_\alpha \zeta \cosh 2y - G^{(2)}_\alpha \sinh 2y\right], && (3.13)
\end{aligned}$$

where

$$G_\alpha^{(1)}(\mathrm{NH}) = \frac{4\sqrt{m_2 m_3}}{m_2+m_3}\,\mathrm{Im}\,(U_{\alpha 2}U_{\alpha 3}^*) \tag{3.14}$$

$$G_\alpha^{(2)}(\mathrm{NH}) = (1+\rho_m)|U_{\alpha 3}|^2 + (1-\rho_m)|U_{\alpha 2}|^2 \tag{3.15}$$

$$G_\alpha^{(1)}(\mathrm{IH}) = \frac{4\sqrt{m_1 m_2}}{m_1+m_2}\,\mathrm{Im}\,(U_{\alpha 1}U_{\alpha 2}^*) \tag{3.16}$$

$$G_\alpha^{(2)}(\mathrm{IH}) = (1+\rho_m)|U_{\alpha 2}|^2 + (1-\rho_m)|U_{\alpha 1}|^2 \tag{3.17}$$

Therefore,

$$\sum_I \epsilon_{I\alpha}^{\cancel{\text{vertex}}} \approx \frac{F_\alpha^-}{2}\,(2R_{21} - 2R_{12}) \tag{3.18}$$

$$= \frac{m_{\mathrm{tot}}}{8\pi v^2} F_\alpha^- \left[W_{11}M_1 + W_{22}M_2\right] \frac{M_1 M_2}{M_2^2 - M_1^2} \tag{3.19}$$

$$\simeq \frac{\rho_m m_{\mathrm{tot}}}{2\pi v^2} \frac{M_1^3}{M_2^2 - M_1^2} \frac{\cosh 2y \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w} \left[G_\alpha^{(1)} \zeta \cosh 2y - G_\alpha^{(2)} \sinh 2y\right]. \tag{3.20}$$

3.1.1. Inverted Hierarchy

For IH case, this analytic expression is further simplified since $\rho_m \ll 1$:

$$\sum_I \epsilon_{I\alpha}^{\cancel{\text{vertex}}} \approx \frac{m_{\mathrm{tot}}\rho_m}{2\pi v^2} \frac{M_1^3}{M_2^2 - M_1^2} \left[G_\alpha^{(1)}\zeta - G_\alpha^{(2)} \tanh 2y\right] \sin 2w. \tag{3.21}$$

where 4 from mixing and 1 from oscillation, and the parameters at the best-fit point are

$$G_\alpha^{(1)}(\mathrm{IH}) = \{-0.90\mathrm{s}_\sigma, 0.14\mathrm{c}_\sigma + 0.39\mathrm{s}_\sigma, 0.52\mathrm{s}_\sigma - 0.14\mathrm{c}_\sigma\}\,, \tag{3.22}$$

$$G_\alpha^{(2)}(\mathrm{IH}) = \{0.98, 0.43, 0.60\}\,. \tag{3.23}$$

3.1.2. Normal Hierarchy

At the best-fit point of NH case, $\rho_m^2 = 0.501$ and the parameters are given by

$$G_\alpha^{(1)}(\mathrm{NH}) = \{-0.067\mathrm{c}_\sigma - 0.095\mathrm{s}_\sigma, 0.039\mathrm{c}_\sigma + 0.63\mathrm{s}_\sigma, 0.028\mathrm{c}_\sigma - 0.53\mathrm{s}_\sigma\}\,, \tag{3.24}$$

$$G_\alpha^{(2)}(\mathrm{NH}) = \{0.13, 1.07, 0.80\}\,. \tag{3.25}$$

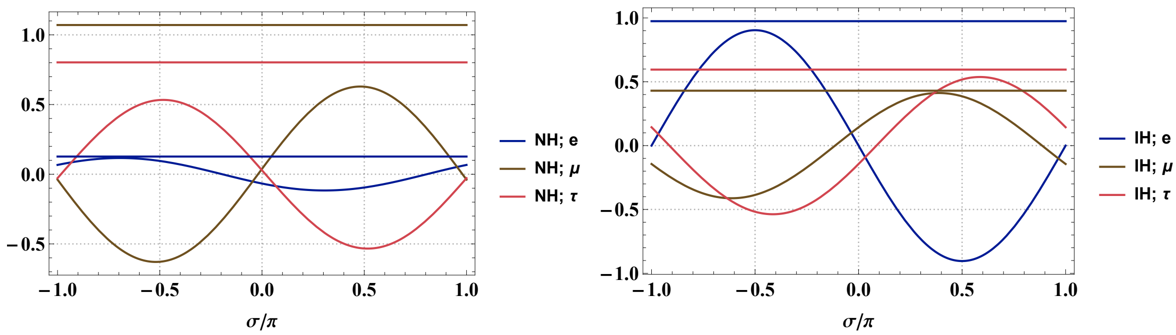
3.2. Denominator

Now we are to evaluate the denominator,

$$D_\alpha := K_\alpha^{\mathrm{eff}} \min(z_c, z_\alpha), \tag{3.26}$$

where

$$z_\alpha = 1.25 \log 25 K_\alpha^{\mathrm{eff}}, \qquad z_c = \frac{M_1}{149\,\mathrm{GeV}}, \tag{3.27}$$



and

$$K_\alpha^{\text{eff}} = \kappa_\alpha \sum_I \frac{\Gamma_I}{H_N} \frac{|y_{I\alpha}|^2}{(yy^\dagger)_{II}} = \kappa_\alpha \sum_I \frac{M_I |y_{I\alpha}|^2}{8\pi H_N}. \tag{3.28}$$

We further evaluate it as

$$K_\alpha^{\text{eff}} \approx \frac{M_1}{8\pi H_N} (y^\dagger y)_{\alpha\alpha} \approx \frac{M_1^2}{4\pi v^2 H_N} \left(U_{\text{PMNS}} \sqrt{m} R^\dagger R \sqrt{m} U_{\text{PMNS}}^\dagger \right)_{\alpha\alpha}, \tag{3.29}$$

where we proceed as

$$K'_\alpha := \frac{2}{m_{\text{tot}}} \left(U_{\text{PMNS}} \sqrt{m} R^\dagger R \sqrt{m} U_{\text{PMNS}}^\dagger \right)_{\alpha\alpha} \tag{3.30}$$

$$= G_\alpha^{(2)} \cosh 2y - G_\alpha^{(1)} \zeta \sinh 2y, \tag{3.31}$$

$$K_\alpha^{\text{eff}} \approx \frac{M_1^2}{4\pi v^2 H_N} \frac{m_{\text{tot}}}{2} K'_\alpha = \frac{m_{\text{tot}} M_{\text{pl}}}{8\pi v^2 \cdot 1.66 \sqrt{g_*}} K'_\alpha. \tag{3.32}$$

As we see above, K'_α are $\mathcal{O}(1)$-parameters; thus

$$z_\alpha = 1.25 \log \frac{25 m_{\text{tot}} M_{\text{pl}}}{8\pi v^2 \cdot 1.66 \sqrt{g_*}} K'_\alpha \approx 10 \tag{3.33}$$

for both hierarchies, which is smaller than z_c in the region of our interest. Therefore, we evaluate the denominator as

$$D_\alpha \approx \frac{1}{10} \cdot \frac{m_{\text{tot}} M_{\text{pl}}}{8\pi v^2 \cdot 1.66 \sqrt{g_*}} \left(G_\alpha^{(2)} \cosh 2y - G_\alpha^{(1)} \zeta \sinh 2y \right). \tag{3.34}$$

3.3. Total asymmetry

Accordingly, as long as $R_{IJ} \ll 1$, i.e.,

$$\delta M \gg \frac{M_1 m_{\text{tot}}}{16\pi v^2}, \tag{3.35}$$

we approximate the total lepton asymmetry as

$$\delta\eta_l \approx \sum_\alpha \frac{\frac{\rho_m m_{\rm tot}}{2\pi v^2}\frac{M_1^3}{M_2^2-M_1^2}\frac{\cosh 2y\sin 2w}{\cosh^2 2y-\rho_m^2\cos^2 2w}\left[G_\alpha^{(1)}\zeta\cosh 2y - G_\alpha^{(2)}\sinh 2y\right]}{\frac{1}{10}\cdot\frac{m_{\rm tot}M_{\rm pl}}{8\pi v^2\cdot 1.66\sqrt{g_*}}\left(G_\alpha^{(2)}\cosh 2y - G_\alpha^{(1)}\zeta\sinh 2y\right)} \tag{3.36}$$

$$= -\frac{10\cdot 1.66\sqrt{g_*}\rho_m}{M_{\rm pl}}\frac{4M_1^3}{M_2^2-M_1^2}\frac{\cosh 2y\sin 2w}{\cosh^2 2y-\rho_m^2\cos^2 2w}\sum_\alpha\frac{G_\alpha^{(1)}\zeta - G_\alpha^{(2)}\tanh 2y}{G_\alpha^{(1)}\zeta\tanh 2y - G_\alpha^{(2)}} \tag{3.37}$$

$$= -C\frac{M_1^3}{M_2^2-M_1^2}K(z)\sum_\alpha G_\alpha(z,\zeta), \tag{3.38}$$

where

$$C := \frac{4\cdot 10\cdot 1.66\sqrt{g_*}\rho_m}{M_{\rm pl}} \approx \frac{4.0\times 10^{-17}\quad (4.2\times 10^{-19})}{\rm GeV}\quad \text{for NH (IH)}, \tag{3.39}$$

$$K(z) := \frac{\cosh 2y\sin 2w}{\cosh^2 2y-\rho_m^2\cos^2 2w},\qquad G_\alpha(z,\zeta) := \frac{G_\alpha^{(1)}\zeta - G_\alpha^{(2)}\tanh 2y}{G_\alpha^{(1)}\zeta\tanh 2y - G_\alpha^{(2)}}. \tag{3.40}$$

Here, $G_a(z,\zeta)$ depends on $U_{\rm PMNS}$ and m_i, while $K(z)$ has no dependence on $U_{\rm PMNS}$.

4. (a few) Discussion

We are interested in the lower bound on M_1. The neutrino-option condition may compensate smaller M_1 by having larger $|y|$ as

$$\cosh 2y \simeq \frac{f(M_1)}{m_{\rm tot}}, \tag{4.1}$$

but for larger $|y|$ the leptogenesis gets worse:

$$K(z) \approx \frac{\sin 2w}{\cosh 2y},\qquad G_\alpha(z,\zeta) \approx \operatorname{sign} y; \tag{4.2}$$

thus, even with $|\sin 2w| = 1$ with normal hierarchy,

$$|\delta\eta_l| \approx C\frac{M_1^3}{M_2^2-M_1^2}\frac{1}{\cosh 2y} \approx \frac{8\pi v^2\cdot Cm_{\rm tot}}{f(M_1)^2}R_{IJ}. \tag{4.3}$$

Restricting with numerical evaluation, this yields

$$|\delta\eta_l| \approx 10^{-8}\left(\frac{M_1}{4.3\ (8.5)\ \times 10^5\,{\rm GeV}}\right)^6 R_{IJ} \tag{4.4}$$

Therefore, leptogenesis provides a lower bound at, at least, 430 TeV (850 TeV) for NH (IH).

A. Yukawa products

We use the Casas–Ibarra parameterization with $z = w + \mathrm{i}y$ and $\zeta = \pm 1$: With

$$y_{Ii} y^*_{Jj} = \frac{2}{v^2} \sum_{ab} \sqrt{M_I} R_{Ia} \sqrt{m_a} (U^\dagger)_{ai} U_{jb} \sqrt{m_b} (R^\dagger)_{bJ} \sqrt{M_J}, \tag{A.1}$$

$$(yy^\dagger)_{IJ} = \frac{2}{v^2} \sum_{a} \sqrt{M_I} R_{Ia} m_a (R^\dagger)_{aJ} \sqrt{M_J}, \tag{A.2}$$

$$(y^\dagger y)_{ij} = \frac{2}{v^2} \sum_{abI} U_{ia} \sqrt{m_a} (R^\dagger)_{aI} M_I R_{Ib} \sqrt{m_b} (U^\dagger)_{bj}. \tag{A.3}$$

Regardless of the hierarchy, these Yukawa combinations are rewritten by