

Analysis of NuFIT Best-fit points

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1 Notation

$$\begin{aligned}\Delta M &:= M_2 - M_1 > 0 & \delta M &:= \Delta M / M_1 \\ \Delta M^2 &:= M_2^2 - M_1^2 & \delta M^2 &:= \Delta M^2 / M_1^2 \\ \Delta m &:= m_{\text{heavier}} - m_{\text{lighter}} & \rho_m &:= \Delta m / \sum_i m_i & m_{\text{tot}} &:= \sum_i m_i\end{aligned}$$

One should note that $\rho_m \sim 0.7$ for NH and < 0.03 for IH. Also,

$$m_{\text{heavier}} = m_3 \quad (m_2), \quad m_{\text{lighter}} = m_2 \quad (m_1) \quad \text{for NH (IH)}.$$

Yukawa matrix follows the notation of 1611 paper.

$$\gamma_I = \frac{(yy^\dagger)_{II}}{8\pi}, \quad \Gamma_I = M_I \gamma_I.$$

We use the Casas–Ibarra parameterization with $z = w + iy$ and $\zeta = \pm 1$:

$$y = i\langle\phi_0\rangle^{-1} \sqrt{M_{\text{diag}}} R \sqrt{m_{\text{diag}}} U^\dagger. \quad (1.1)$$

With $v = \sqrt{\langle\phi_0\rangle} \approx 246 \text{ GeV}$,

$$y_{Ii} y_{Jj}^* = \frac{2}{v^2} \sum_{ab} \sqrt{M_I} R_{Ia} \sqrt{m_a} (U^\dagger)_{ai} U_{jb} \sqrt{m_b} (R^\dagger)_{bJ} \sqrt{M_J}, \quad (1.2)$$

$$(yy^\dagger)_{IJ} = \frac{2}{v^2} \sum_a \sqrt{M_I} R_{Ia} m_a (R^\dagger)_{aJ} \sqrt{M_J}, \quad (1.3)$$

$$(y^\dagger y)_{ij} = \frac{2}{v^2} \sum_{abI} U_{ia} \sqrt{m_a} (R^\dagger)_{aI} M_I R_{Ib} \sqrt{m_b} (U^\dagger)_{bj}. \quad (1.4)$$

Regardless of the hierarchy, these Yukawa combinations are rewritten by

$$(yy^\dagger)_{IJ} = \frac{m_{\text{tot}} \sqrt{M_I M_J}}{v^2} W_{IJ}; \quad (1.5)$$

$$W_{11} = \cosh 2y - \rho_m \cos 2w, \quad W_{22} = \cosh 2y + \rho_m \cos 2w, \quad (1.6)$$

$$W_{12} = W_{21}^* = i \sinh 2y + \rho_m \sin 2w. \quad (1.7)$$

2 Neutrino-option condition

The neutrino-option condition is given by, matching at $Q_0 = M_1 e^{-3/4}$,

$$\mu_{\text{EFT}}^2(Q_0) = \frac{M_1^2}{16\pi^2} \text{Tr}(yy^\dagger) =: \frac{M_1^2}{16\pi^2(v^2/2)} T \quad (2.1)$$

where $v = \sqrt{2}\langle\phi_0\rangle \approx 246 \text{ GeV}$ and T is evaluated as

$$T := \frac{v^2}{2} \text{Tr}(yy^\dagger) = \frac{M_1 m_{\text{tot}}}{2} [\delta M \rho_m \cos 2w + (2 + \delta M) \cosh 2y], \quad (2.2)$$

which has a lower bound, while its upper bound is controlled by $y = \text{Im } z$:

$$\left(\frac{T}{M_1}\right)_{\min} = m_{\text{tot}} + m_{\text{lighter}} \delta M; \quad \frac{T}{M_1} \approx m_{\text{tot}} \cosh 2y. \quad (2.3)$$

As the neutrino option requires

$$\frac{T}{M_1} = \mu_{\text{EFT}}^2(Q_0) \left(\frac{M_1^3}{16\pi^2\langle\phi_0\rangle^2}\right)^{-1} =: f(M_1), \quad (2.4)$$

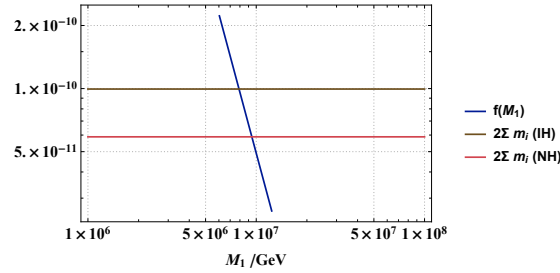
an upper bound on M_1 is obtained by

$$M_1 < 9.4 \text{ (7.9)} \times 10^6 \text{ GeV} \quad \text{for NH (IH)}, \quad (2.5)$$

while M_1 has no lower bound since we can always choose y such that

$$\cosh 2y \simeq \frac{f(M_1)}{m_{\text{tot}}} \quad (2.6)$$

to satisfy the neutrino option condition as long as δM is tiny.



3 Leptogenesis

The resulting lepton asymmetry is approximately given by

$$\delta\eta_l \simeq \sum_{I\alpha} \frac{\epsilon_{I\alpha}}{K_\alpha^{\text{eff}} \min(z_c, z_\alpha)} = \sum_\alpha \frac{\sum_I \epsilon_{I\alpha}}{K_\alpha^{\text{eff}} \min(z_c, z_\alpha)}. \quad (3.1)$$

3.1 Numerator

The parameter $\epsilon_{I\alpha}$ is given by two functions,

$$F_{I\alpha} := \frac{\text{Im} [y_{I\alpha} y_{J\alpha}^* (yy^\dagger)_{IJ}]}{(yy^\dagger)_{II} (yy^\dagger)_{JJ}} \bigg|_{J=3-I}, \quad F'_{I\alpha} := \frac{\text{Im} [y_{I\alpha} y_{J\alpha}^* (yy^\dagger)_{JI}]}{(yy^\dagger)_{II} (yy^\dagger)_{JJ}} \bigg|_{J=3-I}. \quad (3.2)$$

I emphasize that **these functions are independent of $M_{1,2}$** ; and the asymmetry parameter is given by

$$\epsilon_{I\alpha} = F_{I\alpha} f_{IJ}^{\text{vertex}} + \left(F_{I\alpha} + \frac{M_I}{M_J} F'_{I\alpha} \right) (f_{IJ}^{\text{osc}} + f_{IJ}^{\text{mix}}). \quad (3.3)$$

So the dependence of $\epsilon_{I\alpha}$ on M_I is encapsulated into the functions f and M_I/M_J :

$$f_{IJ}^{\text{vertex}} := \frac{\Gamma_J}{M_I} \left[1 - \left(1 + \frac{M_J^2}{M_I^2} \right) \ln \left(1 + \frac{M_I^2}{M_J^2} \right) \right], \quad (3.4)$$

$$f_{IJ}^{\text{mix}} := \frac{(M_I^2 - M_J^2) M_I \Gamma_J}{(M_I^2 - M_J^2)^2 + M_I^2 \Gamma_J^2}, \quad (3.5)$$

$$f_{IJ}^{\text{osc}} := \frac{(M_I^2 - M_J^2) M_I \Gamma_J}{(M_I^2 - M_J^2)^2 + M_I^2 \Gamma_J^2 \mu_{IJ} \rho_{\text{osc}}}, \quad (3.6)$$

where

$$\mu_{IJ} = \frac{M_J}{M_I} + \frac{\Gamma_I}{\Gamma_J}, \quad \rho_{\text{osc}} = \frac{\det [\text{Re}(yy^\dagger)]}{(yy^\dagger)_{11} (yy^\dagger)_{22}} = \frac{\cosh^2 2y - \rho_m^2}{\cosh^2 2y - \rho_m^2 \cos^2 2w}. \quad (3.7)$$

Note that $\mu_{IJ} = \mathcal{O}(1)$ and $0 < \rho^{\text{osc}} < 1$.

The resonant leptogenesis is thus governed by the ratio

$$R_{IJ} := \frac{M_I \Gamma_J}{M_I^2 - M_J^2} = \frac{W_{JJ} m_{\text{tot}} M_J}{8\pi v^2} \frac{M_I M_J}{M_I^2 - M_J^2}; \quad f_{IJ}^{\text{osc}} = \frac{R_{IJ}}{1 + R_{IJ}^2 \mu_{IJ} \rho_{\text{osc}}}. \quad (3.8)$$

In the parameter region of our interest, $R_{IJ} \ll 1$ and

$$f_{IJ}^{\text{mix}} \sim f_{IJ}^{\text{osc}} \sim R_{IJ} \gg f_{IJ}^{\text{vertex}} \simeq \frac{M_I^2 - M_J^2}{M_I^2} R_{IJ}. \quad (3.9)$$

We then define

$$F_\alpha^\pm := (F_{2\alpha} + F'_{2\alpha}) \pm (F_{1\alpha} + F'_{1\alpha}) \quad (3.10)$$

and evaluate them, which yields $F_\alpha^+ = 0$ and

$$F_\alpha^- = \frac{4 \text{Re}(yy^\dagger)_{12} \text{Im}(y_{1\alpha}^* y_{2\alpha})}{(yy^\dagger)_{11} (yy^\dagger)_{22}} \quad (3.11)$$

$$= \frac{2\rho_m \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w} 2v^2 m_{\text{tot}} \sqrt{M_1 M_2} \text{Im}(y_{1\alpha}^* y_{2\alpha}) \quad (3.12)$$

$$= \frac{2\rho_m \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w} \left[G_\alpha^{(1)} \zeta \cosh 2y - G_\alpha^{(2)} \sinh 2y \right], \quad (3.13)$$

where

$$G_\alpha^{(1)}(\text{NH}) = \frac{4\sqrt{m_2 m_3}}{m_2 + m_3} \text{Im}(U_{\alpha 2} U_{\alpha 3}^*) \quad (3.14)$$

$$G_\alpha^{(2)}(\text{NH}) = (1 + \rho_m)|U_{\alpha 3}|^2 + (1 - \rho_m)|U_{\alpha 2}|^2 \quad (3.15)$$

$$G_\alpha^{(1)}(\text{IH}) = \frac{4\sqrt{m_1 m_2}}{m_1 + m_2} \text{Im}(U_{\alpha 1} U_{\alpha 2}^*) \quad (3.16)$$

$$G_\alpha^{(2)}(\text{IH}) = (1 + \rho_m)|U_{\alpha 2}|^2 + (1 - \rho_m)|U_{\alpha 1}|^2 \quad (3.17)$$

Therefore,

$$\sum_I \epsilon_{I\alpha}^{\text{vertex}} \approx \frac{F_\alpha^-}{2} (2R_{21} - 2R_{12}) \quad (3.18)$$

$$= \frac{m_{\text{tot}}}{8\pi v^2} F_\alpha^- [W_{11}M_1 + W_{22}M_2] \frac{M_1 M_2}{M_2^2 - M_1^2} \quad (3.19)$$

$$\simeq \frac{\rho_m m_{\text{tot}}}{2\pi v^2} \frac{M_1^3}{M_2^2 - M_1^2} \frac{\cosh 2y \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w} \left[G_\alpha^{(1)} \zeta \cosh 2y - G_\alpha^{(2)} \sinh 2y \right]. \quad (3.20)$$

3.1.1 Inverted Hierarchy

For IH case, this analytic expression is further simplified since $\rho_m \ll 1$:

$$\sum_I \epsilon_{I\alpha}^{\text{vertex}} \approx \frac{m_{\text{tot}} \rho_m}{2\pi v^2} \frac{M_1^3}{M_2^2 - M_1^2} \left[G_\alpha^{(1)} \zeta - G_\alpha^{(2)} \tanh 2y \right] \sin 2w. \quad (3.21)$$

where 4 from mixing and 1 from oscillation, and the parameters at the best-fit point are

$$G_\alpha^{(1)}(\text{IH}) = \{-0.90s_\sigma, 0.14c_\sigma + 0.39s_\sigma, 0.52s_\sigma - 0.14c_\sigma\}, \quad (3.22)$$

$$G_\alpha^{(2)}(\text{IH}) = \{0.98, 0.43, 0.60\}. \quad (3.23)$$

3.1.2 Normal Hierarchy

At the best-fit point of NH case, $\rho_m^2 = 0.501$ and the parameters are given by

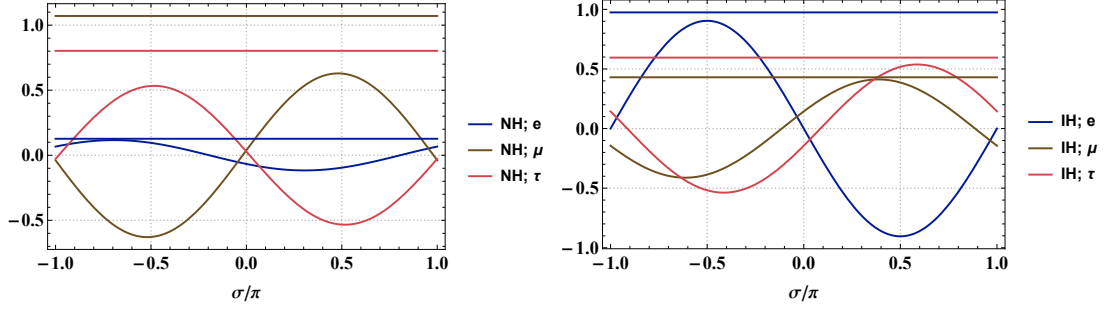
$$G_\alpha^{(1)}(\text{NH}) = \{-0.067c_\sigma - 0.095s_\sigma, 0.039c_\sigma + 0.63s_\sigma, 0.028c_\sigma - 0.53s_\sigma\}, \quad (3.24)$$

$$G_\alpha^{(2)}(\text{NH}) = \{0.13, 1.07, 0.80\}. \quad (3.25)$$

3.2 Denominator

Now we are to evaluate the denominator,

$$D_\alpha := K_\alpha^{\text{eff}} \min(z_c, z_\alpha), \quad (3.26)$$



where

$$z_\alpha = 1.25 \log 25 K_\alpha^{\text{eff}}, \quad z_c = \frac{M_1}{149 \text{ GeV}}, \quad (3.27)$$

and

$$K_\alpha^{\text{eff}} = \kappa_\alpha \sum_I \frac{\Gamma_I}{H_N} \frac{|y_{I\alpha}|^2}{(yy^\dagger)_{II}} = \kappa_\alpha \sum_I \frac{M_I |y_{I\alpha}|^2}{8\pi H_N}. \quad (3.28)$$

We further evaluate it as

$$K_\alpha^{\text{eff}} \approx \frac{M_1}{8\pi H_N} (y^\dagger y)_{\alpha\alpha} \approx \frac{M_1^2}{4\pi v^2 H_N} \left(U_{\text{PMNS}} \sqrt{m} R^\dagger R \sqrt{m} U_{\text{PMNS}}^\dagger \right)_{\alpha\alpha}, \quad (3.29)$$

where we proceed as

$$K'_\alpha := \frac{2}{m_{\text{tot}}} \left(U_{\text{PMNS}} \sqrt{m} R^\dagger R \sqrt{m} U_{\text{PMNS}}^\dagger \right)_{\alpha\alpha} \quad (3.30)$$

$$= G_\alpha^{(2)} \cosh 2y - G_\alpha^{(1)} \zeta \sinh 2y, \quad (3.31)$$

$$K_\alpha^{\text{eff}} \approx \frac{M_1^2}{4\pi v^2 H_N} \frac{m_{\text{tot}}}{2} K'_\alpha = \frac{m_{\text{tot}} M_{\text{pl}}}{8\pi v^2 \cdot 1.66 \sqrt{g_*}} K'_\alpha. \quad (3.32)$$

As we see above, K'_α are $\mathcal{O}(1)$ -parameters; thus

$$z_\alpha = 1.25 \log \frac{25 m_{\text{tot}} M_{\text{pl}}}{8\pi v^2 \cdot 1.66 \sqrt{g_*}} K'_\alpha \approx 10 \quad (3.33)$$

for both hierarchies, which is smaller than z_c in the region of our interest. Therefore, we evaluate the denominator as

$$D_\alpha \approx \frac{1}{10} \cdot \frac{m_{\text{tot}} M_{\text{pl}}}{8\pi v^2 \cdot 1.66 \sqrt{g_*}} \left(G_\alpha^{(2)} \cosh 2y - G_\alpha^{(1)} \zeta \sinh 2y \right). \quad (3.34)$$

3.3 Total asymmetry

Accordingly, as long as $R_{IJ} \ll 1$, i.e.,

$$\delta M \gg \frac{M_1 m_{\text{tot}}}{16\pi v^2}, \quad (3.35)$$

we approximate the total lepton asymmetry as

$$\delta\eta_l \approx \sum_{\alpha} \frac{\frac{\rho_m m_{\text{tot}}}{2\pi v^2} \frac{M_1^3}{M_2^2 - M_1^2} \frac{\cosh 2y \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w} \left[G_{\alpha}^{(1)} \zeta \cosh 2y - G_{\alpha}^{(2)} \sinh 2y \right]}{\frac{1}{10} \cdot \frac{m_{\text{tot}} M_{\text{pl}}}{8\pi v^2 \cdot 1.66 \sqrt{g_*}} \left(G_{\alpha}^{(2)} \cosh 2y - G_{\alpha}^{(1)} \zeta \sinh 2y \right)} \quad (3.36)$$

$$= -\frac{10 \cdot 1.66 \sqrt{g_*} \rho_m}{M_{\text{pl}}} \frac{4M_1^3}{M_2^2 - M_1^2} \frac{\cosh 2y \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w} \sum_{\alpha} \frac{G_{\alpha}^{(1)} \zeta - G_{\alpha}^{(2)} \tanh 2y}{G_{\alpha}^{(1)} \zeta \tanh 2y - G_{\alpha}^{(2)}} \quad (3.37)$$

$$= -C \frac{M_1^3}{M_2^2 - M_1^2} K(z) \sum_{\alpha} G_{\alpha}(z, \zeta), \quad (3.38)$$

where

$$C := \frac{4 \cdot 10 \cdot 1.66 \sqrt{g_*} \rho_m}{M_{\text{pl}}} \approx \frac{4.0 \times 10^{-17} \quad (4.2 \times 10^{-19})}{\text{GeV}} \quad \text{for NH (IH)}, \quad (3.39)$$

$$K(z) := \frac{\cosh 2y \sin 2w}{\cosh^2 2y - \rho_m^2 \cos^2 2w}, \quad G_{\alpha}(z, \zeta) := \frac{G_{\alpha}^{(1)} \zeta - G_{\alpha}^{(2)} \tanh 2y}{G_{\alpha}^{(1)} \zeta \tanh 2y - G_{\alpha}^{(2)}}. \quad (3.40)$$

Here, $G_{\alpha}(z, \zeta)$ depends on U_{PMNS} and m_i , while $K(z)$ has no dependence on U_{PMNS} .

4 (a few) Discussion

We are interested in the lower bound on M_1 . The neutrino-option condition may compensate smaller M_1 by having larger $|y|$ as

$$\cosh 2y \simeq \frac{f(M_1)}{m_{\text{tot}}}, \quad (4.1)$$

but for larger $|y|$ the leptogenesis gets worse:

$$K(z) \approx \frac{\sin 2w}{\cosh 2y}, \quad G_{\alpha}(z, \zeta) \approx \text{sign } y; \quad (4.2)$$

thus, even with $|\sin 2w| = 1$ with normal hierarchy,

$$|\delta\eta_l| \approx C \frac{M_1^3}{M_2^2 - M_1^2} \frac{1}{\cosh 2y} \approx \frac{8\pi v^2 \cdot C m_{\text{tot}}}{f(M_1)^2} R_{IJ}. \quad (4.3)$$

Restricting with numerical evaluation, this yields

$$|\delta\eta_l| \approx 10^{-8} \left(\frac{M_1}{4.3 \text{ (8.5)} \times 10^5 \text{ GeV}} \right)^6 R_{IJ} \quad (4.4)$$

Therefore, leptogenesis provides a lower bound at, at least, 430 TeV (850 TeV) for NH (IH).