

RGE Validation

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1 SM parameters at $Q = m_t$

1.1 Fixing a typo

The original reference [1, v4] has a typo in threshold correction $g_Y^{(1)}$ (Eq.(95)); the second last-term should be read as $36M_W^4/(M_Z^2 - M_W^2)$. This typo is fixed in [6b18ea0](#)→[9e2350f](#).

With the fixed version we can reproduce Table 3 from Table 2 (both in [1, v4]). For example,

```
Install["LoopTools"];
Needs["SMRGE`", "1307_3536.w1"];
table2 = <|
  "MW" -> 80.384,      "MZ" -> 91.1876,    "MH" -> 125.15,
  "MT" -> 173.34,      "MB" -> 4.2,        "MTau" -> 1.777,
  "as(MZ)" -> 0.1184,  "Gmu" -> 1/(v^2 Sqrt[2]) /. {v -> 246.21971} |>
table3[order_] := {
  #[lam], Sqrt#[ytsq], Sqrt#[g2sq], Sqrt#[g1sq]*3/5, Sqrt#[msq]
} &@ getCouplingsAtMt[table2, order]
```

will give The unfixed version gives different g_Y , while after the fix Table 3 is reproduced.

6b18ea0 ; table3[0]	0.129177	0.995614	0.652945	0.349715	125.15
6b18ea0 ; table3[1]	0.127745	0.951129	0.647545	0.358571	132.37
6b18ea0 ; table3[2]	0.126049	0.940179	0.647799	0.357464	131.554
9e2350f ; table3[0]	0.129177	0.995614	0.652945	0.349715	125.15
9e2350f ; table3[1]	0.127745	0.951129	0.647545	0.359394	132.37
9e2350f ; table3[2]	0.126049	0.940179	0.647799	0.358287	131.554

1.2 Strange results in neutrino-option paper

Neutrino-option paper [2] uses slightly different values in their Table 2:

```
table2aNeutOpt = <|
  "MW" -> 80.387,      "MZ" -> 91.1875,    "MH" -> 125.09,
  "MT" -> 173.2,       "MB" -> 4.18,        "MTau" -> 1.776,
  "as(MZ)" -> 0.1185,  "Gmu" -> 1.1663787/10^5 |>;
table2b[order_] := {
  #[lam], Sqrt#[msq], Sqrt#[g1sq], Sqrt#[g2sq], Sqrt#[g3sq],
  Sqrt#[ytsq], Sqrt#[ybsq], Sqrt#[yasq]
} &@ getCouplingsAtMt[table2aNeutOpt, order]
```

but their obtained values (Table 2b) are quite strange. First of all, they show $\hat{g}_3 = 1.22029$, but this is equal to $\sqrt{4\pi\alpha_s(M_Z)}$ and inconsistent to what they wrote,

The value of $g_3(\mu = m_t)$ is extracted from Eqn. 60 of Ref. [42] which includes higher order QCD corrections.

because this procedure should give $\hat{g}_3 = 1.16711$. SI is also sure that they did not notice the typo.

Their value of Table 2a should give the following values for Table 2b:

9e2350f ; table2b[0]	0.1291	125.09	0.451	0.653	1.16711	0.995	0.024	0.0102
9e2350f ; table2b[1]	0.1276	132.288	0.464	0.648	1.16711	0.950	0.024	0.0102
9e2350f ; table2b[2]	0.1259	131.474	0.462	0.648	1.16711	0.939	0.024	0.0102

One can reproduce most of their results, shown in their Table 2b, using **6b18ea0** with `WeakScaleThreshold["g3", 0,1,2] := Sqrt[4*Pi*asMZ];`

6b18ea0 -mod; table2b[0]	0.1291	125.09	0.451	0.6530	1.22029	0.995	...
6b18ea0 -mod; table2b[1]	0.1276	132.288	0.463	0.6476	1.22029	0.946	...
6b18ea0 -mod; table2b[2]	0.1258	131.431	0.461	0.6478	1.22029	0.933	...

Here, however, \hat{g}_2 is a bit different (0.5%) from their values; SI guesses they have another typo in their code.

1.3 Validation with mr 1.3.2

SI compares the results against mr 1.3.2 [3]. The code `rgecheck.cpp` given in **current** gives, using the values in the neutrino-option paper,

mr 1.3.2; order=0	0.12905	125.0900	0.4514	0.6530	1.1651091	0.9948	0.0283
	0.12757	132.4746	0.4642	0.6473	1.1651091	0.9505	0.0199
	0.12589	131.2914	0.4626	0.6481	1.1651091	0.9396	0.0177

References

- [1] D. Buttazzo, *et al.*, *Investigating the near-criticality of the Higgs boson*, JHEP **12** (2013) 089 [arXiv:1307.3536].
- [2] I. Brivio and M. Trott, *Examining the neutrino option*, JHEP **02** (2019) 107 [arXiv:1809.03450].
- [3] B. A. Kniehl, A. F. Pikelner, and O. L. Veretin, *mr: a C++ library for the matching and running of the Standard Model parameters*, Comput. Phys. Commun. **206** (2016) 84–96 [arXiv:1601.08143]. GitHub:apik/mr.