$$
\begin{aligned}
\hat{\Gamma}^i &= \tilde{\Gamma}^i + 2\tilde{\gamma}^{ij} Z_j \\
\Rightarrow Z_i &= \frac{1}{2}\tilde{\gamma}_{ij}(\hat{\Gamma}^j - \tilde{\Gamma}^j).
\end{aligned} \tag{1}
$$

Where $\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\tilde{\Gamma}^i{}_{jk} = \tilde{\gamma}^{ij}\tilde{\gamma}^{kl}\partial_l\tilde{\gamma}_{jk}$.

All terms in equations of CCZ4 formulism depend on Z_i are Z_i, $D_i Z_j$ and $D^i Z_i$, so we will be focusing on deriving expression of $D_i Z_j$. And we are working on the conformal metric $\gamma_{ij} = e^{4\phi}\tilde{\gamma}_{ij} = \chi^{-2}\tilde{\gamma}_{ij}$.

$$
\begin{aligned}
D_i Z_j &= \tilde{D}_i Z_j - 2(\partial_i\phi Z_j + \partial_j\phi Z_i - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_k\phi Z_l) \\
&= \tilde{D}_i Z_j + \frac{1}{\chi}(\partial_i\chi Z_j + \partial_j\chi Z_i - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_k\chi Z_l)
\end{aligned} \tag{2}
$$

Where

$$
\begin{aligned}
\tilde{D}_i Z_j &= \frac{1}{2}\tilde{\gamma}_{jk}(\tilde{D}_i\hat{\Gamma}^k - \tilde{D}_i\tilde{\Gamma}^k) \\
&= \frac{1}{2}\tilde{\gamma}_{jk}(\partial_i\hat{\Gamma}^k + \tilde{\Gamma}^k_{il}\hat{\Gamma}^l) - \frac{1}{2}\tilde{\gamma}_{jk}(\tilde{\gamma}^{kl}\tilde{\gamma}^{mn}\tilde{D}_i(\partial_m\tilde{\gamma}_{ln})) \\
&= \frac{1}{2}\tilde{\gamma}_{jk}(\partial_i\hat{\Gamma}^k + \tilde{\Gamma}^k_{il}\hat{\Gamma}^l) - \frac{1}{2}\tilde{\gamma}^{mn}(\partial_i\partial_m\tilde{\gamma}_{jn} - \tilde{\Gamma}^p{}_{im}\partial_p\tilde{\gamma}_{jn} - \tilde{\Gamma}^p{}_{ij}\partial_m\tilde{\gamma}_{pn} - \tilde{\Gamma}^p{}_{in}\partial_m\tilde{\gamma}_{jp})
\end{aligned} \tag{3}
$$

And we can also derive $D_i Z^i$ and $(D_i Z_j)^{TF}$ from the value of $D_i Z_j$.