Dark Matter in Cosmology

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Abstract

1 Astrophysical and Cosmological methods indicating the existence of dark matter

The history of dark matter began in the 1930s, when a Dutch radio astronomer by the name of Jan Oort analyzed numbers and velocities of stars located near the Sun and reached the conclusion that the particular stars appeared to be lacking approximately 30-50% of the matter necessary in order to account for their apparent velocities. In 1933, and shortly after Oort's work, Fritz Zwicky performed similar calculations, and concluded that velocity dispersions in rich galaxy clusters require approximately 100 times more mass in order to ensure that they remain bound. Similar research including methods such as weak gravitational lensing continued more intensely during the following decades, and especially in the 1970s, when various scientific groups attempted to further constrain the existence of such an *invisible matter*, using rotation curves. The following paragraphs contain an overview of the main Astrophysical and Cosmological methods, which established the existence of dark matter in the minds of the scientific community.

1.1 Velocity Dispersion

As we mentioned above, the earliest dark matter indications appeared through calculations related to the velocity dispersion of galaxies, and, in particular, those carried out by Zwicky on the Coma Cluster of galaxies [1]. The velocities of galaxies belonging to the particular cluster were observed to differ by at least $1500 - 200\ km/s$. Assuming that the Coma Cluster has reached a stationary state, one can implement the Virial Theorem as follows:

$$2E_k = -E_{pot}$$

, where E_k and E_{pot} stand for the average kinetic and potential energy per unit mass in the system. The cluster size can be approximated with $R \sim 10^{24}\ cm$ and includes a total number of around 800 nebulae, each with a mass of $M_n \sim 10^9 M_\odot$. If we additionally assume uniform mass distribution for the contents of the cluster, its mass can be calculated by $M_{CC} \sim 800 \times M_n \sim 1.6 \times 10^{45}\ gr$. The potential energy of such a cluster is given by: $V = -\frac{3GM^2}{5R}$, resulting in an average potential energy per unit mass equal to $E_{pot} = -\frac{3GM}{5R} \sim -64 \times 10^{12}\ cm^2/s^2$. In addition, the average kinetic energy can be expressed as $E_k = \frac{1}{2}\bar{v^2} \sim 32 \times 10^{12}\ cm^2/s^2$. Using the expression of the virial theorem written above, we finally get:

$$(\bar{v^2})^{1/2} \sim 80\ km/s$$

. Such a velocity appears to be very small compared to the Doppler effects of at least 1000 km/s measured from observations of the Coma Cluster. The ultimate conclusion that can be drawn from the above is that the average density of the cluster, has to satisfy $< \rho > \sim 400\rho_{lum}$. In other words, the real density has to exceed the density of the observed luminous matter by a factor of 400, in order for the cluster to remain bound. If the density fails to comply to this restriction, the 800 nebulae of the cluster are bound to ultimately disperse and become independent of each other, thus no longer contituting a cluster of galaxies. Similar calculation have, since, been performed for many more known large structures of matter in the Universe, since Velocity Dispersion constitutes one of the basic indicators for the existence of dark matter.

1.2 Rotation Curves

The rotation curve argument is another serious indication for the existence of dark matter in the Universe from the early years of research on the topic [2], until today. Such a curve is created in a diagram of the rotational velocity versus the distance from the center of a galaxy and can be better understood by the following simplified explanation.

In the presence of *only* visible gravitational forces, the virial theorem of hydrostatic equilibrium gives:

$$Mv^2 = \frac{GM}{R}$$

or

$$\frac{v^2}{R} = \frac{GM}{R^2}$$

As a result, the *Keplerian rotation curve* follows the relation $v \propto R^{-1/2}$. However, the observations do not appear to agree with such a conclusion and instead of rotation curves which decrease with the distance, they give flat curves, indicating that the velocity has to follow $v \sim \mathit{const.}$ instead. From the virial formula above and with a constant velocity, one can also conclude that $M \propto R$ and, moreover:

$$\frac{dM}{dR} = \frac{v^2}{G}$$

But how can this conclusion be reconciled with the dark matter theory? The essence of the argument supports that the Keplerian curves do, actually, accurately represent reality, but at much larger distances than the ones we are able to observe. Therefore, a logical conclusion indicates the existence of much larger galaxies, where great quantities of dark matter extend far beyond the visible matter, in what can be appriximated as a spherically symmetric dark matter halo. The density of such a galaxy can be calculated from:

$$\rho = \frac{1}{4\pi R^2}\frac{dM}{dR} = \frac{v^2}{4\pi G R^2}$$

and, therefore, $\rho \propto R^{-2}$ beyond the visible radius.

1.3 Gravitational lensing

Another astrophysical method that has been extremely important for the establishment of the dark matter argument is that of gravitational lensing, and especially of the weak gravitational lensing. More specifically, Gravitational lensing is one of the various predictions made in Albert Einstein's theory of General relativity and refers to the deflection of light by gravitational fields, as well as to the resulting effect of that deflection on images seen by an observer. There exist various different types of gravitational lensing, depending of the strength of the effect and on the corresponding degree of image distortion of distant objects, including strong gravitational lensing, weak gravitational lensing and gravitational micro-lensing. More specifically:

- **Strong gravitational lensing**, in which case the mass of the lens is enough to produce multiple images, arcs, or even Einstein rings. Generally, the strong lensing effect requires the projected lens mass density greater than the critical density Σ_{cr}. For point-like background sources, there will be multiple images; for extended background emissions, there can be arcs or rings.

- **Weak gravitational lensing**, in which case the mass acting as a lens causes disfigurements to the background objects that are observed, but is not strong enough to produce arcs or rings.

- **Gravitational microlensing**, in which case the lens allows the observation of sources producing little or no light, due to the collection and beaming of emitted light towards the direction of the observer.

In order to better understand the concept of gravitational lensing, we will attempt to present the simple basis of the argument using Einstein's theory of General Relativity, using light originating from a point-like source. The deflection angle a can be calculated from:

$$\vec{\alpha} = \frac{4GM}{c^2 b}$$

where b is the distance from the point-like source of the beam to the observer.

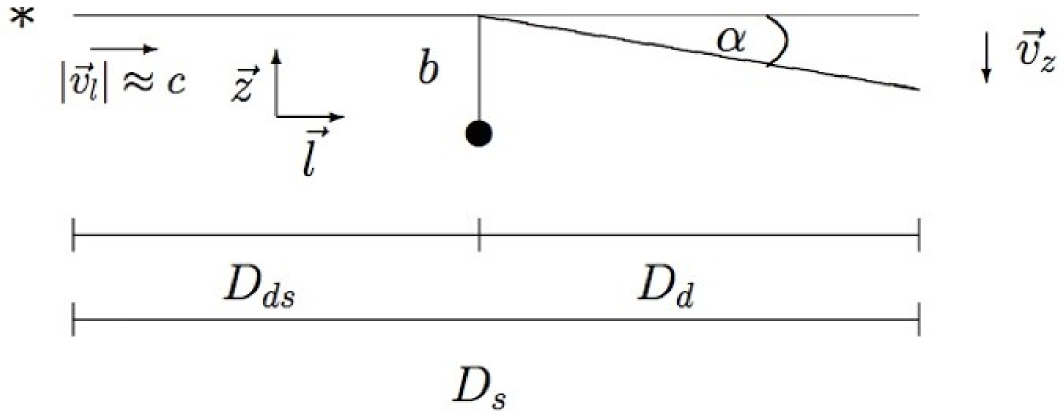

Figure 1: Deflection of a light beam originating from a point-like source

If we consider the gravitational field as an optical medium with an index of refraction $n > 1$, the particular index is such that the light beam travels slower through the medium than through vacuum, and is given by (Ehlers & Schneider 1992 [3]):

$$n = 1 - \frac{2}{c^2}\Phi = 1 + \frac{2}{c^2} \mid \Phi \mid$$

where Φ refers to the gravitational potential. Furthermore:

$$\vec{\alpha} = -\int \bigtriangledown_{\perp} n dl = \frac{2}{c^2}\int \bigtriangledown_{\perp}\Phi dl$$

which contains the integral of the potential gradient perpendicular to the light propagation direction ($\bigtriangledown_{\perp}\Phi = \frac{d\Phi}{dz}$). Therefore, for a light beam traveling at a distance $b = R_{sun}$ from the center of the sun (which is considered as a point-like source), the deflection angle α is calculated to be approximately 1.7". Further topics that are related to Gravatiational lensing include the lens equation, as well as the phenomenon that is known as the *Einstein ring*. However, such discussion goes beyond the purposes of the particular review.

1.4 Bullet clusters

The existence of two different types of matter can, also, be observed in the *Bullet Clusters*, where two clusters are undergoing high-velocity collisions, and, subsequently, emit X-rays that have been detected by the Chandra telescope. In images of such phenomena that have been captured, we can observe two different types of material. The *hot gas* is concentrated at the collision front, while some dark matter, confirmed by weak gravitational lensing techniques, is concentrated behind the collision front. We can, therefore, conclude that the dark matter detected in such phenomena, not only exists, but is also collisionless.

1.5 Baryon Acoustic Oscillations [4]

Having already mentioned various astrophysical methods of establishing the credibility of the dark matter argument, it is time to move on to some cosmological phenomena. The Baryon Acoustic Oscillations are those that give rise to the various acoustic peaks in the Cosmic Microwave Background power-spectrum. Their creation took place in the Early Universe, before the recombination of electrons and protons (for the formation of neutral hydrogen atoms) and the subsequent release of the CMB radiation. During this early age of the Universe, the various cosmological perturbations caused the excitation of sound waves in the relativistic plasma. Until the time of recombination all the various modes of different wavelength had completed a different number of oscillation periods, a fact that has been "captured" in the CMB power spectrum as the different maxima and minima. While, however, the baryon perturbations travelled outwards in the form of an acoustic wave, the dark matter perturbations grew in place. At the time of recombination, the speed of sound falls rapidly, which results in the end of the sound-wave propagation, at a moment when the baryon acoustic shell has expanded to a radius of $\sim$ 150 Mpc. It is after the time of recombination at redshift $z \sim 1000$ that dark matter perturbations, along with the baryonic perturbations contribute to the creation of all large-scale structures observed in the Universe today. If dark matter failed to exist, and its coresponding perturbations, failed to dominate over the baryonic acoustic perturbations, the resulting structures created would have to be much larger than the ones observed today. We can, therefore, confirm the existence of an additional component, apart from regular matter itself, that is essential in accounting for the cosmological structure witnessed today. Such a conclusion provides a strong ally of the dark matter argument, from a cosmological perspective.

2 ΛCDM latest measurements from the Planck telescope

The measurements made by the Planck telescope [5] are constraint using all astrophysical and cosmological methods mentioned in section 1. Some additional contraints are calculated using distance and velocity measurements of Type Ia Supernovae, which are known as "standard candles" for astrophysical observations. The total energy density of all components, considering a flat universe is expected to be $\Omega_{tot} \sim 1$. According to measurements of the telescope made in 2015, the rest of the components have the following characteristics values:

- The energy density of baryonic matter: $\Omega_b h^2 \simeq 0.02230 \pm 0.00014$
- The energy density of cold dark matter: $\Omega_c h^2 \simeq 0.1188 \pm 0.0010$
- The energy density of dark energy: $\Omega_\Lambda \simeq 0.6911 \pm 0.0062$
- Hubble's constant at the present time: $H_0 \simeq 67.74 \pm 0.46\ km/s/Mpc$
- The energy density of all matter in the universe $\Omega_m = \Omega_c + \Omega_b$: $\Omega_m \simeq 0.03089 \pm 0.0062$

The values above constitute most of the essential characteristics of the ΛCDM model, which is considered the most probable cosmological theory until today. Some of the problems of the particular model will be discussed in the following sections of the review.

3 Type of dark matter

We have already seen that there must exist dark matter, but we have yet to determine what kind of matter dark matter consists of. There exists various possibilities, of which we will discuss some together with the likelihood of dark matter to exist of these types.

3.1 Baryonic dark matter

The simplesst thing to assume is that dark matter consists of ordinary baryons, that we haven't found yet. However, this is unlikely for several reasons:

- **The amount of baryons.** There exists 2 ways in which we can measure the amount of baryons that are present in the universe. Firstly, we can measure the abundance of light elements, specifically deuterium. This abundance is closely governed by the amount of baryons that exist. Secondly, we can consider the distribution of hot and cold spots in the CMB. Both of these turn out to be in excellent accordance with one another. As we saw, the energy density for baryonic matter is $\Omega_b = 0.02230/0.49 \approx 0.05$, whereas the total energy density of all matter in the universe is approximately 0.25.

- **various candidates are unlikely**
  - **Hydrogen or helium gas** When hydrogen is frozen, it forms 'snowballs' which would evaporate. When helium is in a cool state, it should absorb light that comes from behind it, but this is scarcely witnessed. When there's hot gas, this should emit X-ray radiation, but this is also scarce.
  - **Dusts, rocks or asteroids** More complex elements may create bigger objects, such as rocks or asteroids. However, that would imply that stars should also have higher metallicity than is the case. This is thus not likely. For dust to be present, we should have more blocking of light, which is not abundant. So, also baryonic dust cannot be the constituent of dark matter.
  - **MACHOs.** MACHOs, very low luminosity stars are hard to witness, for apparent reasons. They can be found using gravitational microlensing. Although some have been found over the last decades, it does not nearly come close to explaining all the missing matter.
  - **Very massive objects.** The remnanants of very massive stars that formed early in the Galactic history might form neutron stars and massive black holes. For neutron stars to form, a star usually goes supernova, ejecting many heavy elements into the Universe, which we don't observe. Additionally, the

final mass of a neutron star is not high enough to explain all the missing matter. Very massive stars usually end their lives by collapsing into a black hole, which is unlikely to have happened enough to explain the amount of matter that is required.

4 Hot, Warm or Cold Dark Matter?

We have seen that dark matter should be non-baryonic and weakly interacting. In addition to the types of dark matter explained in section 3 we can subdivide dark matter even further into hot, warm and cold dark matter. These terms have to do with weather or not the dark matter particles where relativistic just before recombination. This difference could have a big influence on the matter structures in later stages of the universe.

Hot dark matter particles are relativistic just before recombination. Around 1980 the hot dark matter model was very popular since neutrinos seemed to be a good candidate for hot dark matter [6]. People had just discovered that neutrinos where weirdly behaving. Before this time neutrinos where thought of as massless particles. This changed since the discovery of the solar neutrino problem. Neutrinos are produced in the sun by:

$$4p \rightarrow He + 2e^- + 2\nu_e \tag{1}$$

Once we are trying to measure neutrinos on earth we only measure 1/3 of the expected value. The solution to this problem is that neutrinos oscillate, they are constantly changing flavor $(\nu_e, \nu_\mu, \nu_\tau)$. This is only possible if neutrinos have mass $(10eV < m_\nu < 100eV)$. And thus people thought neutrinos could be dark matter. However, hot dark matter has as consequence that small scale structures are damped. The density perturbations in the primordial fluid are damped out by the free-streaming, relativistic neutrinos. This has as effect that the initial structures in the early universe are of size $\lambda_\nu \simeq 40(30eV/m_\nu)MeV$ which is the typical distance a neutrino travels in the lifetime of the universe [7]. Smaller scales, like galaxies, would only form later on by fragmentation. We see that this is not the case in our universe and we conclude that λ_ν for hot dark matter is to large.

As for this reason a cold dark matter model has been studied extensively as well. In the cold dark matter model the dark matter was non-relativistic before recombination. There are many cold dark matter candidates, the most prominent once's are Weakly Interacting Massive Particles (WIMPs) and Axions.

See [6] (p372) for what is hot, warm and cold DM. Why cold most plausible?

4.1 Warm Dark Matter

Sterile Neutrinos as Dark Matter [7] warm dark matter particles have an even lower crossection that neutrinos and they are also less abundant. They have a mass of $\sim 1keV$. In this case λ_ν is much lower

4.2 Cold Dark Matter

cold dark matter: non relativistic = needed in order to have small scale structures

4.3 Cold + Hot Dark Matter?

1995 => Cold + hot [8]

5 MOND

Explaining flat rotation curves requires at least one of the following solution concepts: 1. There exists invisible matter 2. Newton's laws do not hold for galaxies [9]. Concept 1 points to dark matter, concept 2 to MOND. Note that it could conceptually be possible that the solution is found by combining both concepts.

5.1 Milgrom's law

Although Newton's laws have been widely tested in high-acceleration environments, they haven't been tested in the realm where objects undergo low acceleration, which happens at the outer edges of galaxies. This sets the way for a new gravitational force law to describe the newtonian force, which Milgrom postulated to be:

$$F_N = m\mu\left(\frac{a}{a_0}\right)a. \tag{2}$$

Here, m is the gravitational mass of the object, a is the acceleration, a_0 is a fundamental constant representing the transition from Newtonian to deep-MOND regimes and the interpolating function is said to be:

$$\mu(x) \to \begin{cases} 1 & \text{for } x \gg 1 \\ x & \text{for } x \ll 1 \end{cases} \tag{3}$$

So, in the deep-MOND regime, where $a \ll a_0$, we find that:

$$F_N = m\frac{a^2}{a_0}.$$

Now, for objects with mass m in a circular orbit, we have that:

$$\begin{aligned} \frac{GMm}{r^2} &= m\frac{(v^2/r)^2}{a_0} \\ v^4 &= \frac{GMma_0r^2}{mr^2} \\ &= GMa_0 \end{aligned}$$

Here, a_0 is experimentally found to be approximately 10^{-10} ms^{-2} [9]. This indeed gives rise to a rotation velocity that is independent of r, such that the rotation velocity is flat.

5.2 conservation of momentum

We can thus see that Milgrom's law, as written in equation 2, solves the problems that arise from the ΛCDM interpretation. However, this is merely a law, which should derive from a universal force law. However, this law does not uphold the principle of conservation of momentum. If we consider a system in which two masses, m_1 and m_2 are small enough to be in the weak acceleration limit, and in rest on the x-axis. Now, we can express the change in momentum of the system as follows:

$$\begin{aligned} \dot{p} &= \dot{p_1} + \dot{p_2} = m_1\dot{v_1} - m_2\dot{v_2} = ma_1 - ma_2 \\ &= m_1\sqrt{\frac{F_Na_0}{m_1}} - m_2\sqrt{\frac{F_Na_0}{m_2}} = \sqrt{F_Na_0}(\sqrt{m_1} - \sqrt{m_2}). \end{aligned}$$

We can immediately see that this does not equal 0 when $m_1 \neq m_2$, thus violating the principle of the conservation of momentum. Thus, equation 2 cannot be more than an approximation of a more heuristic force law. Such a law would have to be derived from a variational and action principle, leading to a modified Newtonian dynamics (MOND) theory[1].

To change the dynamics such that they will give the Milgrom's law, the classical action provides a good starting point. We describe a system in which a set of particles move in a gravitational field that arises from the matter density, $\rho = \sum_i m_i\delta(x - x_i)$ and is described by a potential, Φ_N, such that:

$$S_N = S_{\text{kin}} + S_{\text{in}} + S_{\text{grav}} = \int \frac{\rho v}{2}d^3xdt - \int \rho\Phi d^3xdt - \int \frac{|\nabla\Phi_N|^2}{8\pi G}d^3xdt$$

Now, we straightforwardly find the equation of motion to be $d^2x/dt^2 = -\nabla\Phi_N$, which can be used to find $\nabla^2\Phi_n = 4\pi G\rho$. We can now modify the gravitational action. When we do this, the equation of motion remains in tact, but we will find a different Poisson equation [9]. We find, for the gravitational action:

$$S_{\text{grav, BM}} = -\int \frac{a_0^2F(|\nabla\Phi|^2a_0^2)}{8\pi G}d^3xdt,$$

where F represents any dimensionless function. We can vary this with respect to Φ, to find:

[1] We shall discuss the classical level, and refrain from discussing the weak-field limit, which is more properly described by modified Einstein Dynamics.

$$\nabla \left[\mu \left(\frac{|\nabla \Phi|}{a_o} \right) \nabla \Phi \right] = 4\pi G \rho,$$

where $\mu(x) = F(\sqrt{x})$ and $\mu(x)$ still satisfies equation 3. Thus, when $|\nabla\Phi| \ll a_o$, we find that the potential becomes non-linear, which is contradictory to the general relativity, which predicts linear equations in the weak-field limit [10].

5.3 Gravitational waves

There are 2 ways in which MOND can alter gravitational wave (GW) physics. Firstly, MOND can violate the equivalence principle as it is an acceleration based theory. This would imply that GWs can propagate with a speed that is less than the speed of light [10]. Secondly, as we just saw that the equations are non-linear in the weak-field limit, GWs can be explained by non-linear equations.

Let's first delve into the first claim. When GWs propagate with $c_g < 1$, where the speed of light, $c = 1$, we must conclude that the arrival times of a electromagnetic signal and the gravitational wave that originate from the same source cannot be the same. However, recently a detection of a GW coming from an inspiraling neutron star binary, of which an electromagnetic counterpart was measured, was made and the time between the arrivals turned out to be 1.7 seconds [11, 12]. Additionally, when $c_g < 1$, highly energetic cosmic rays that travel with $v \to 1$ will lose energy via the Cherenkov radiation with a rate dependent on $1 - c_g$. Observing such cosmic rays on Earth allows for a lower bound on c_g causing the rate dependence to be $1 - c_g \lesssim 10^{-15}$ [13, 14]. Thus, it is very unlikely that the speed of gravitational waves is less than the speed of light. In some MOND theories, the speed of gravitational waves depends on the gravitational potential and cannot generically be set to 1, thus making these theories inaccurate.

Regarding the second claim on the non-linearity of the gravitational wave dynamics, we can deduce the following. When these dynamics would indeed be non-linear, the GWs that originate from black hole mergers could interact with themselves. However, LIGO's observation of a black hole merger in 2014 showed no such interaction as the observed signal was consistent with predictions made by general relativity [15]. Therefore, we don't require such a scrambling effect, so we would expect that gravitational waves should satisfy linear equations of motions in the weak-field limit.

We can thus conclude that MOND theories must be heavily constrained by the principles arising from gravitational waves. This implies that it becomes questionable whether MOND is an applicable theory, let alone explain the problems that the theory of dark matter attempts to explain.

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