Primordial Black Holes (in Dark Matter clothing)

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See the jupyter notebook or plots folder for some visualisations.

1 Length Scales

1.1 Distribution of orbits

From Ref. [1] (Sec. 2), the probability distribution for the semi-major axis a and the eccentricity e of the binary PBH orbits is given by:

$$\mathrm{d}P(a,e) = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{1/2} e (1-e^2)^{-3/2}\, \mathrm{d}a\, \mathrm{d}e\,. \tag{1}$$

Here, f is the fraction of PBHs in DM and $\bar{x}$ is the mean physical separation of PBHs at matter-radiation equality $z = z_{\mathrm{eq}}$:

$$\bar{x} = \frac{1}{(1+z_{\mathrm{eq}}) f^{1/3}} \left(\frac{8\pi G}{3H_0^2} \frac{M_{\mathrm{BH}}}{\Omega_{DM}} \right)^{1/3} . \tag{2}$$

Note that there could be some extra factors of $4\pi/3$ in here, depending on your precise definition of $\bar{x}$ (see e.g. Eq. 2 in Ref. [2]).

1.2 Range of parameter values

The maximum eccentricity is given by (Ref. [1], Eq. 6):

$$e_{\mathrm{max}} = \sqrt{1 - f^{3/2}\left(\frac{a}{\bar{x}}\right)^{3/2}} . \tag{3}$$

The distance of closest approach of the PBHs - *peri-BH*, r_{peri} - is given by:

$$r_{\mathrm{peri}} = a(1-e)\,. \tag{4}$$

Then periBH lies in the range:

$$r_{\mathrm{peri}} \in a[1 - e_{\mathrm{max}}, 1]\,. \tag{5}$$

We note also that the requirement for forming a binary is that the physical separation of the BH is

$$x < f^{1/3}\bar{x}\,, \tag{6}$$

which means that the semi-major axis is limited to:

$$a \leq \alpha f^{1/3} \bar{x}\,, \tag{7}$$

where α is a numerical factor of $\mathcal{O}(1)$, set to 1 in Ref. [1].

1.3 Distribution of (dimensionless) periBH

We now change variables from eccentricity to periBH:

$$\mathrm{d}P(a, r_\mathrm{peri}) = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{-1/2} \left(1 - \frac{r_\mathrm{peri}}{a}\right) \left(\frac{2r_\mathrm{peri}}{a} - \frac{r_\mathrm{peri}^2}{a^2}\right)^{-3/2} \mathrm{d}a\, \mathrm{d}r_\mathrm{peri}\,. \tag{8}$$

Or, in terms of the dimensionless periBH $\mathfrak{r} = r_\mathrm{peri}/a$:

$$\mathrm{d}P(a, \mathfrak{r}) = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{1/2} \left(1 - \mathfrak{r}\right) \left(2\mathfrak{r} - \mathfrak{r}^2\right)^{-3/2} \mathrm{d}a\, \mathrm{d}\mathfrak{r}\,. \tag{9}$$

Note here, that the PDF is valid over the range:

$$a \in [0, f^{1/3}\bar{x}]\,, \tag{10}$$

$$\mathfrak{r} \in [1 - e_\mathrm{max}(a), 1]\,. \tag{11}$$

We can rearrange the limits:

$$a \in [0, \frac{\bar{x}}{f} \left(1 - (1-\mathfrak{r})^2\right)^{2/3}]\,, \tag{12}$$

$$\mathfrak{r} \in [0, 1]\,, \tag{13}$$

and then integrate over a:

$$\mathrm{d}P(\mathfrak{r}) = \frac{1}{2} \frac{(1-\mathfrak{r})}{\sqrt{1-(1-\mathfrak{r})^2}} \mathrm{d}\mathfrak{r}\,. \tag{14}$$

As a sanity check, we can integrate over all $\mathfrak{r} \in [0, 1]$ and we obtain $1/2$. This matches the original normalisation of the PDFs from Ref. [1] (my guess is that they normalised to $1/2$ because one PBH is one-half of a PBH binary).

1.4 Sensible units

The density of PBHs today is [3, 4]

$$\rho_\mathrm{PBH} = \rho_\mathrm{crit} f \Omega_\mathrm{DM} = 3.3 \times 10^{10}\, f M_\odot\, \mathrm{Mpc}^{-3}\,, \tag{15}$$

Taking matter-radiation equality to be at $z_\mathrm{eq} \approx 3400$ [4], we obtain

$$\bar{x} \approx 3 \times 10^{-1} \left(\frac{M_\mathrm{PBH}}{30\, M_\odot}\right)^{1/3} f^{-1/3}\, \mathrm{pc}\,. \tag{16}$$

The Schwarzschild radius of the PBH is:

$$r_s \approx 3 \times 10^{-12} \left(\frac{M_\mathrm{PBH}}{30\, M_\odot}\right) \mathrm{pc}\,. \tag{17}$$

1.5 Dark Matter Clothing

As an initial estimate of the size of a DM halo around a PBH, we can take the truncation radius at $z \approx z_\mathrm{eq}$ given by Eq. 1 of Ref. [5] (see also Refs. [6, 7]):

$$R_\mathrm{tr}(z) = 2 \times 10^{-2}\, \mathrm{pc} \left(\frac{M_\mathrm{PBH}}{30\, M_\odot}\right)^{1/3} \left(\frac{1 + z_{eq}}{1+z}\right)\,. \tag{18}$$

1.6 Classifying different regimes

We can classify 3 regimes:

- **Isolated binaries**, where $r_{\rm peri}$ is always much larger than the halo truncation radius ,
- **Common-halo binaries**, where $r_{\rm peri}$ and a are both smaller than the halo truncation radius ,
- **Close-passage binaries**, where $r_{\rm peri}$ is below the halo truncation radius, but a is above the halo truncation radius .

Check the jupyter notebook for some calculations and plots. The overall result seems to be that **close-passage binaries** are the dominant population (for $f \lesssim 0.1$).

References

[1] M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, *Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914*, *Phys. Rev. Lett.* **117** (2016) 061101, [1603.08338].

[2] Y. Ali-Haïmoud, E. D. Kovetz and M. Kamionkowski, *The merger rate of primordial-black-hole binaries*, 1709.06576.

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[4] Planck collaboration, P. A. R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13, [1502.01589].

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[7] M. Ricotti, *Bondi accretion in the early universe*, *Astrophys. J.* **662** (2007) 53–61, [0706.0864].