

Primordial Black Holes (in Dark Matter clothing)

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1 Length Scales

Distribution of orbits

From Ref. [1] (Sec. 2), the probability distribution for the semi-major axis a and the eccentricity e of the binary PBH orbits is given by:

$$dP(a, e) = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{1/2} e (1 - e^2)^{-3/2} da de. \quad (1)$$

Here, f is the fraction of PBHs in DM and \bar{x} is the mean physical separation of PBHs at matter-radiation equality $z = z_{\text{eq}}$:

$$\bar{x} = \frac{1}{(1 + z_{\text{eq}}) f^{1/3}} \left(\frac{8\pi G M_{\text{BH}}}{3H_0^2 \Omega_{DM}} \right)^{1/3}. \quad (2)$$

Note that there could be some extra factors of $4\pi/3$ in here, depending on your precise definition of \bar{x} (see e.g. Eq. 2 in Ref. [2]).

Range of parameter values

The maximum eccentricity is given by (Ref. [1], Eq. 6):

$$e_{\text{max}} = \sqrt{1 - f^{3/2} \left(\frac{a}{\bar{x}} \right)^{3/2}}. \quad (3)$$

The distance of closest approach of the PBHs - *peri-BH*, r_{peri} - is given by:

$$r_{\text{peri}} = a(1 - e). \quad (4)$$

Then periBH lies in the range:

$$r_{\text{peri}} \in a[1 - e_{\text{max}}, 1]. \quad (5)$$

We note also that the requirement for forming a binary is that the physical separation of the BH is

$$x < f^{1/3} \bar{x}, \quad (6)$$

which means that the semi-major axis is limited to:

$$a \leq \alpha f^{1/3} \bar{x}, \quad (7)$$

where α is a numerical factor of $\mathcal{O}(1)$, set to 1 in Ref. [1].

Distribution of periBH

We now change variables from eccentricity to periBH:

$$dP(a, r_{\text{peri}}) = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{-1/2} \left(1 - \frac{r_{\text{peri}}}{a}\right) \left(\frac{2r_{\text{peri}}}{a} - \frac{r_{\text{peri}}^2}{a^2}\right)^{-3/2} da dr_{\text{peri}}. \quad (8)$$

Or, in terms of the dimensionless periBH $\mathfrak{r} = r_{\text{peri}}/a$:

$$dP(a, \mathfrak{r}) = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{1/2} (1 - \mathfrak{r}) (2\mathfrak{r} - \mathfrak{r}^2)^{-3/2} da d\mathfrak{r}. \quad (9)$$

Note here, that the PDF is valid over the range:

$$a \in [0, f^{1/3} \bar{x}], \quad (10)$$

$$\mathfrak{r} \in [1 - e_{\text{max}}(a), 1]. \quad (11)$$

We can rearrange the limits:

$$a \in [0, \frac{\bar{x}}{f} (1 - (1 - \mathfrak{r})^2)^{2/3}], \quad (12)$$

$$\mathfrak{r} \in [0, 1], \quad (13)$$

and then integrate over a :

$$dP(\mathfrak{r}) = \frac{1}{2} \frac{(1 - \mathfrak{r})}{\sqrt{1 - (1 - \mathfrak{r})^2}} d\mathfrak{r}. \quad (14)$$

As a sanity check, we can integrate over all $\mathfrak{r} \in [0, 1]$ and we obtain $1/2$. This matches the original normalisation of the PDFs from Ref. [1] (my guess is that they normalised to $1/2$ because one PBH is one-half of a PBH binary).

See the jupyter notebook or plots folder for some visualisations.

Sensible units

The density of PBHs today is [3, 4]

$$\rho_{\text{PBH}} = \rho_{\text{crit}} f \Omega_{\text{DM}} = 3.3 \times 10^{10} f M_{\odot} \text{Mpc}^{-3}, \quad (15)$$

Taking matter-radiation equality to be at $z_{\text{eq}} \approx 3400$ [4], we obtain

$$\bar{x} \approx 3 \times 10^{-7} \left(\frac{M_{\text{PBH}}}{30 M_{\odot}}\right)^{1/3} f^{-1/3} \text{Mpc}. \quad (16)$$

The Schwarzschild radius of the PBH is:

$$r_s \approx 3 \times 10^{-18} \left(\frac{M_{\text{PBH}}}{30 M_{\odot}}\right) \text{Mpc}. \quad (17)$$

References

- [1] M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, *Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914*, *Phys. Rev. Lett.* **117** (2016) 061101, [[1603.08338](#)].
- [2] Y. Ali-Haïmoud, E. D. Kovetz and M. Kamionkowski, *The merger rate of primordial-black-hole binaries*, [1709.06576](#).
- [3] O. Lahav and A. R. Liddle, *The Cosmological Parameters 2014*, [1401.1389](#).
- [4] PLANCK collaboration, P. A. R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13, [[1502.01589](#)].