Scalar Interactions

Fermion Coupling

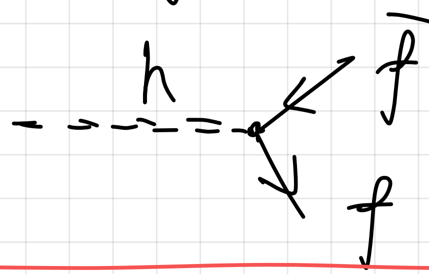
$$\mathcal{L} = -y \bar{F}_L H f_R + h.c. \qquad F = \begin{pmatrix} f_{L,u} \\ f_{L,d} \end{pmatrix} \text{ SU(2) doublet}$$

$$H = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\mathcal{L} = -y \bar{f}_L (v+h) f_R + h.c.$$

$$= -y v \bar{f} f - y h \bar{f} f$$

$m = yv$,



$$iy = i\frac{m_f}{v}$$

Electroweak coupling.

$$\mathcal{L} = (D_\mu H)^\dagger (D_\mu H), \quad H = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$D_\mu H = \partial_\mu H - i g W_\mu^a T^a H - \frac{1}{2} i g' B_\mu H$$

$$(D_\mu H)^\dagger (D_\mu H) = |\partial_\mu H|^2 + g^2 H^\dagger (W_\mu^a T^a + \frac{1}{2} B_\mu)^\dagger (W_\mu^a T^a + \frac{1}{2} B_\mu) H$$

$$= -h \Box h + \cdots$$

$$\cdots = \frac{1}{4} g^2 (v+h)^2 \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{g'}{g}B + W^3 & W^1 - iW^2 \\ W^1 + iW^2 & \frac{g'}{g}B - W^3 \end{pmatrix} \begin{pmatrix} \frac{g'}{g}B + W^3 & W^1 - iW^2 \\ W^1 + iW^2 & \frac{g'}{g}B - W^3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{g^2 (v+h)^2}{4} \left({W^1}^2 + {W^2}^2 + \left(\frac{g'}{g} B - W^3 \right)^2 \right)$$

$$B = c_w A - s_w Z, \quad W^3 = c_w Z + s_w A$$

$$t_w = \frac{g'}{g}$$

$$(t_w B - W^3)^2 = t_w^2 B^2 + {W^3}^2 - 2 t_w B W^3$$

$$B^2 = c_w^2 A^2 + s_w^2 Z^2 - 2 c_w s_w A Z$$

$${W^3}^2 = c_w^2 Z^2 + s_w^2 A^2 + 2 c_w s_w A Z$$

$$B W^3 = c_w^2 A Z + c_w s_w A^2 - c_w s_w Z^2 - s_w^2 A Z$$

$$\cancel{s_w^2 A^2} + t_w^2 s_w^2 Z^2 - \cancel{2 s_w^2 t_w A Z} + c_w^2 Z^2 + \cancel{s_w^2 A^2} + \cancel{2 c_w s_w A Z}$$
$$- \cancel{2 c_w s_w A Z} - \cancel{2 s_w^2 A^2} + 2 s_w^2 Z^2 + \cancel{2 t_w s_w^2 A Z}$$

$$= Z^2 \left(t_w^2 s_w^2 + c_w^2 + 2 s_w^2 \right)$$
$$= \frac{Z^2}{c_w^2} \left(s_w^4 + c_w^4 + 2 s_w^2 c_w^2 \right) = \frac{Z^2}{c_w^2} \left(c_w^2 + s_w^2 \right)^2 = \frac{1}{c_w^2} Z^2$$

$$W^{\pm} \equiv \frac{1}{\sqrt{2}}(W^1 \pm iW^2)$$

$$W^{1\,2} + W^{2\,2} = 2W^+W^-$$

$$\Rightarrow \mathcal{L} = -h\Box h + \frac{1}{4}g^2(v^2+h^2+2hv)\left(2W^+W^- + \frac{1}{c_W^2}Z^2\right)$$

$$= -h\Box h + \frac{1}{2}g^2v^2W^+W^- + \frac{1}{4}g^2\frac{1}{c_W^2}v^2Z^2$$

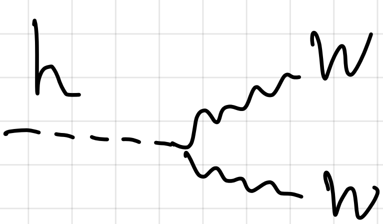
$$+ \frac{1}{2}g^2h^2W^+W^- + \frac{1}{4}g^2h^2\frac{1}{c_W^2}Z^2$$

$$+ g^2hv\,W^+W^- + \frac{1}{2}g^2hv\frac{1}{c_W^2}Z^2$$

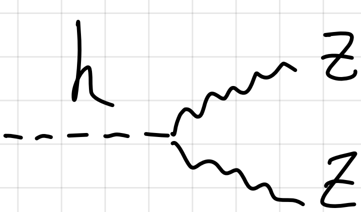
$$m_W^2W^+W^- + \frac{1}{2}m_Z^2Z^2 \Rightarrow m_W^2 = \frac{1}{2}g^2v^2,\ m_Z^2 = \frac{g^2v^2}{2c_W^2}$$

3-legs: $g^2v\,hW^+W^- = \frac{2m_W^2}{v}hW^+W^-$

$$\frac{1}{2}g^2\frac{v}{c_W^2}hZ^2 = \frac{1}{2}\frac{2m_Z^2}{v}hZ^2$$



$= i\cdot\frac{2m_W^2}{v}$



$= i\frac{2m_Z^2}{v}$

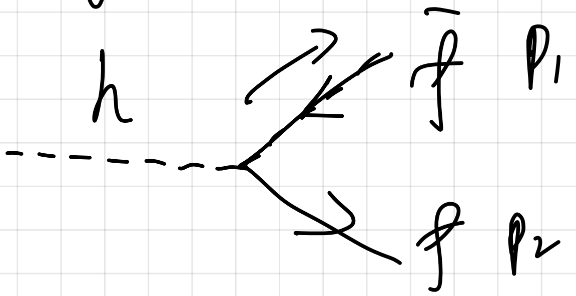
Mixing : $P\phi_g = \phi_m$ $\quad P = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & -c_\theta \end{pmatrix}$

$$\begin{pmatrix} h_g \\ s_g \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} h_m \\ s_m \end{pmatrix}$$

$$h_g = c_\theta h_m + s_\theta s_m$$

SZZ : $s_\theta \cdot \left(i \frac{2m_Z^2}{v} \right)$

Decay into fermions :



$$\mathcal{M} = \bar{u}(p_2)\, i \frac{m_f}{v}\, v(p_1)$$

$$|\mathcal{M}|^2 = \frac{1}{4}\frac{m_f^2}{v^2} \sum_{\text{spin}} \bar{v}(p_1) u(p_2) \bar{u}(p_2) v(p_1)$$

$$= \frac{1}{4}\frac{m_f^2}{v^2} \mathrm{Tr}\left((\not{p}_2 + m_f)(\not{p}_1 - m_f) \right)$$

$$= \frac{1}{4}\frac{m_f^2}{v^2} \mathrm{Tr}\left(\not{p}_2 \not{p}_1 - m_f^2 \right)$$

$$= \frac{m_f^2}{4v^2} \left(p_{1\alpha} p_{2\beta} \mathrm{Tr}(\gamma^\beta \gamma^\alpha) - 4 m_f^2 \right)$$

$$= \frac{m_f^2}{v^2} \left(p_1 p_2 - m_f^2 \right)$$

$$p_1 = \left(\frac{1}{2}m_h, \vec{p}\right) \quad p_2 = \left(\frac{1}{2}m_h, -\vec{p}\right)$$

$$p_1 \cdot p_2 = \frac{1}{4}m_h^2 + \vec{p}^{\,2}$$

And $\frac{1}{4}m_h^2 - \vec{p}^{\,2} = m_f^2 \Rightarrow \vec{p}^{\,2} = \frac{1}{4}m_h^2 - m_f^2 \Rightarrow m_f < \frac{1}{2}m_h$

$$\Rightarrow p_1 \cdot p_2 = \frac{1}{2}m_h^2 - m_f^2$$

$$\Rightarrow |M|^2 = \frac{m_f^2}{2v^2}\left(m_h^2 - 4m_f^2\right)$$

$$d\Gamma = \frac{1}{2m_h}|M|^2 \cdot (2\pi)^4 \delta^4(\Sigma p) \frac{1}{m_h} d^3p_1 \frac{1}{m_h} d^3p_2 \cdot \frac{1}{(2\pi)^6}$$

$$= \frac{1}{(2\pi)^2 2m_h^3} \frac{m_f^2}{2v^2}\left(m_h^2 - 4m_f^2\right) \delta^4(\Sigma p)\, d^3p_1\, d^3p_2$$

$$\Gamma_h = \frac{1}{4v^2(2\pi)^2 m_h^3} m_f^2 \left(m_h^2 - 4m_f^2\right) |\vec{p}|^4 \cdot 4\pi$$

$$= \frac{1}{64\pi v^2 m_h^3} m_f^2 \left(m_h^2 - 4m_f^2\right)^3$$

$$BR \propto m_f^2 \left(m_h^2 - 4m_f^2\right)^3$$

For S decay, $\Gamma_S = \sin^2\theta\, \Gamma_h$. BR not affected.