High − T Potential.

$$V = -\mu_H^2 h^2 + \lambda h^4 + \frac{1}{2}\mu_S^2 S^2 - ASh^2 + V_T$$

$$V_T = \frac{T^4}{2\pi^2}\left(\sum_{B,F} n_B J_B\left(\frac{m_B^2}{T^2}\right) + n_F J_F\left(\frac{m_F^2}{T^2}\right)\right)$$

$$V = -(\mu_H^2 - c_h)T^2 h^2 - ETh^3 + \lambda h^4 - A(h^2 + C_S T^2)S + \frac{1}{2}\mu_S^2 S^2 - E(h,S)T$$

$$J_B \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12}\frac{m^2}{T^2} - \frac{\pi}{6}\frac{m^3}{T^3} \cdots$$

$$J_F \simeq \frac{7\pi^4}{360} - \frac{\pi^2}{24}\frac{m^2}{T^2}$$

$$V_T = \frac{T^2}{24}\left(\sum n_B m_B^2\right) - \frac{T^2}{48}\left(\sum n_F m_F^2\right) - \frac{T}{12\pi}\sum n_B m_B^3$$

(neglect all field-independent parts)

$$m_W^2 = \frac{1}{2}g^2 h^2, \quad m_Z^2 = \frac{g^2 + g'^2}{2}h^2, \quad m_t^2 = y_t^2 h^2$$

$$m_\chi^2 = -2\mu_H^2 - 2AS + 4\lambda h^2$$

$$m_{S,h}^2 = -\mu_H^2 + \frac{1}{2}\mu_S^2 - AS + 6\lambda h^2 \pm \frac{1}{2}\sqrt{sqrt}$$

$$sqrt = 16A^2h^2 + 4A^2S^2 + 8\mu_H^2 AS + 4\mu_S^2 AS - 48\lambda ASh^2$$
$$+ 4\mu_H^4 + \mu_S^4 - 144\lambda^2 h^4 + 4\mu_H^2\mu_S^2 - 48\lambda\mu_H^2 h^2$$
$$- 24\lambda\mu_S^2 h^2$$

$$T^2: \frac{1}{24}\sum n_B m_B^2 = \frac{1}{24}\Big(6\cdot\frac{1}{2}g^2h^2 + 3\cdot\frac{g^2+g'^2}{2}h^2 + \frac{1}{2}\cdot 12 y_t^2 h^2$$
$$+3\left(-2\mu_H^2 - 2AS + 4\lambda h^2\right)$$
$$+2\left(-\mu_H^2 + \frac{1}{2}\mu_S^2 - AS + 6\lambda h^2\right)\Big)$$

$$= \frac{1}{48}\left(9g^2h^2 + 3g'^2h^2 + 12y_t^2h^2 - 16AS + 48\lambda h^2\right)$$

(field-independent part neglected)

$$= \underbrace{\frac{1}{16}\left(3g^2 + g'^2 + 4y_t^2 + 16\lambda\right)h^2}_{D} - \underbrace{\frac{1}{3}AS}_{C_S}$$

If neglect scalar contribution in C_h:

$$D = \frac{1}{24v^2}\left(6m_W^2 + 3m_Z^2 + 6m_t^2\right)$$

$$= \frac{1}{8v^2}\left(2m_W^2 + m_Z^2 + 2m_t^2\right) \quad \text{(Same as SM)}$$

$$T: \frac{1}{12\pi}\sum n_B m_B^3 = \frac{1}{12\pi}\Big(\underbrace{6\cdot\frac{1}{2\sqrt{2}}g^3h^3 + 3\cdot\frac{(g^2+g'^2)^{3/2}}{2\sqrt{2}}}_{E_{SM}}$$
$$\underbrace{+3\left(-2\mu_H^2 - 2AS + 4\lambda h^2\right)^{3/2} + m_{h}^3(h,s) + m_S^3(h,S)}_{E(h,S)}\Big)$$

$$E_{SM} = \frac{1}{4\pi v^3}\left(2m_W^3 + m_Z^3\right)$$

$$E(h,S) = \left(3 m_X^{3/2} + m_h^{3/2} + m_S^{3/2}\right)(h,S)$$

Phase transition

approx: $E(h,S) = 0$

$$V = D(T^2 - T_0^2)h^2 - E_{SM} T h^3 + \lambda h^4 + \frac{1}{2}\mu_S^2 S^2 - A(h^2 + c_S T^2)S$$

$$\langle S \rangle = \frac{A}{\mu_S^2}(h^2 + c_S T^2)$$

$$S^2 = \frac{A^2}{\mu_S^4}(h^2 + c_S T^2)^2 = \frac{A^2}{\mu_S^4}(h^4 + 2c_S T^2 h^2 + \underbrace{c_S^2 T^4}_{\text{neglect}})$$

$$\frac{1}{2}\mu_S^2 S^2 = \frac{A^2}{2\mu_S^2}(h^4 + 2c_S T^2 h^2)$$

$$-A(h^2 + c_S T^2)S = -\frac{A^2}{\mu_S^2}(h^2 + c_S T^2)^2 = -\frac{A^2}{\mu_S^2}(h^4 + 2c_S T^2 h^2)$$

$$V = \left(\left(D - \frac{A^2}{\mu_S^2}c_S\right)T^2 - \mu_H^2\right)h^2 - E_{SM} T h^3 + \left(\lambda - \frac{A^2}{2\mu_S^2}\right)h^4$$

$$D' \equiv D - \frac{A^2}{\mu_S^2}c_S, \quad \lambda' \equiv \lambda - \frac{A^2}{2\mu_S^2}, \quad T_0'^2 \equiv \frac{\mu_H^2}{D'}$$

$$V = D'(T^2 - T_0'^2)h^2 - E_{SM} T h^3 + \lambda' h^4$$

local min: $$\frac{\partial V}{\partial h} = 2D'(T^2 - T_0^2)h - 3E_{SM} T h^2 + 4\lambda' h^3$$

$$= 0$$

$$\langle h(T)\rangle : \quad 4\lambda' h^2 - 3E_{sm}Th + 2D'(T^2 - T_0^2) = 0$$

$$\langle h\rangle = \frac{3E_{sm}T \pm \sqrt{9E_{sm}^2T^2 - 32\lambda' D'(T^2 - T_0^2)}}{8\lambda'} \quad (\text{pick "+"})$$

Require : $V(\langle h\rangle) = 0 \Rightarrow D'(T^2 - T_0^2) - E_{sm}Th + \lambda' h^2 = 0$

$$h = \frac{1}{2\lambda'}\left(E_{sm}T \pm \sqrt{E_{sm}^2T^2 - 4\lambda' D'(T^2 - T_0^2)} \right)$$

degenerate : $E_{sm}^2T^2 = 4\lambda' D'(T^2 - T_0^2)$

$$4\lambda' D' T_0^2 = (4\lambda' D' - E_{sm}^2)T^2$$

$$T_c^2 = \frac{4\lambda' D'}{4\lambda' D' - E_{sm}^2} T_0^2$$

$$\frac{\langle h(T_c)\rangle}{T_c} = \frac{3E_{sm}}{8\lambda'} + \frac{1}{8\lambda'}\sqrt{9E_{sm}^2 - 32\lambda' D'\left(1 - \frac{T_0^2}{T_c^2}\right)}$$

$$= \frac{3E_{sm}}{8\lambda'} + \frac{1}{8\lambda'}\sqrt{9E_{sm}^2 - 32\lambda' D'\left(1 - \frac{4\lambda' D' - E_{sm}^2}{4\lambda' D'}\right)}$$

$$= \frac{3E_{sm}}{8\lambda'} + \frac{1}{8\lambda'}\sqrt{9E_{sm}^2 - 8E_{sm}^2} = \boxed{\frac{E_{sm}}{2\lambda'}}$$