PRIMAT *(PRImordial MATter)*

Short description

*PRIMAT is a *Mathematica* code which computes the abundances of elements at the end of the big-bang nucleosynthesis (BBN).
It can be downloaded by registering at www2.iap.fr/users/pitrou/primat.htm.

*The implementation follows the presentation of Pitrou, Coc, Uzan, Vangioni, Physics Reports, 04, (2018) 005.
All equation numbers, when non specified, refer to this companion paper, in its arXiv version (arXiv:1801.08023).

*It is based on a previous Fortran code written by Alain Coc.

*The user can choose the number of nuclear reactions involved in the reactions network.
-The minimal set of equations is made of 12 reactions involving selected isotopes of Hydrogen, Helium, Lithium and Berylium.
-The maximum set of equation is made of 423 reactions (including decay channels), with isotopes up to Z=11 (Na).

*The user can modify several parameters which are in the preambule of the code :

a) The reaction rates involved in the nuclear reaction network are tabulated in an external file which can be easily modified.
b) The corrections to the weak-interaction reactions (n + ν <-> p^+ + e^- and its related reactions), can be turned on and off with booleans.
The detail of these corrections is given in the companion paper.
c) Cosmological parameters, that is essentially the number of baryons (Ω_b) and the number of neutrino generations N_ν can also be modified.

*The numerical integration is performed in two steps. First the cosmology and the thermodynamics of the plasma are solved,
and then the nuclear reactions are computed, the backreaction of the latter on the former being negligible (see companion paper).

*The results are given as a set of interpolating functions of time for the abundances of individual species.
If one is interested in final abundances only, these are also directly accessible by evaluating these functions at final time.

*A very basic Monte-Carlo exploration of uncertainties is provided at the end of the code. It is used in associated example notebooks present in the Example folder.
For each reaction, an uncertainty factor variance is provided and it is possible to run the code with these uncertainty parameters taking random values according to the variances (see [Coc et al. 2014]). A parallelization is possible for this Monte-Carlo exploration and the analysis of the results can be output in an external file and analyzed separatly.

*Several examples and applications are gathered in the ‘Examples’ folder.

Basic usage

*In the Evaluation menu, select ‘Evaluate notebook’. If asked the question ‘evaluate initialization cells first ?’, answer no.
Then Mathematica proceeds in evaluating all cells, in order, and it should reach the end of the notebook (with the plots and results) in less than one minute.

*If the user erases the precomputed weak rates which are precomputed and stored in the ‘Interpolation’ folder, or asks for these rates to be re-precomputed, this can take considerably longer, typically a few tens of minutes.

*Furthermore, when PRIMAT-Main.nb (this notebook) is opened and saved in Mathematica, the cells which are' initialization cells' are saved into the file PRIMAT-Main.m.
This file contains then all the principal definitions and functions and variables which can be loaded from another notebook to perform BBN computations and analysis of results.
The 'Examples' Folder contains several typical applications which work exactly like that (first it loads all necessary definitions stored in PRIMAT-Main.m and then it performs a few useful computations for each selected example).

Preambule

Information

Authors

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Dates and versions

Version 0.1.1 (07/09/2018)

```
Date[]
{2018, 9, 10, 10, 36, 17.549149}
```

Mathematica version used :

```
$Version
10.4.1 for Mac OS X x86 (64-bit) (April 11, 2016)
```

GPL

Bibliography

Herefater we use the following shorthands for the references cited.

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[Gnedin 1997] N. Y. Gnedin, O. Y. Gnedin, AP. J. **509**, 11-15 (1998), astro-ph/9712199.

Reaction rates

They are listed in references of [Coc. et. al 2012] (see Table 4). We use the following acronyms :

NACRE (Angulo et al. 1999)
NACRE II (Xu et al. 2010, 2011)
DAACV04 (Descouvemont et al. 2004)
ILCCF10 (Iliadis et al. 2010)
CF88 (Caughlan& Fowler 1988)
MF89 (Malaney& Fowler 1989)
Boy93 (Boyd et al. 1993)
Bal95 (Balbes et al. 1995)
Hei98 (Heil et al. 1998)
Rau94 (Rauscher et al. 1994)
Des99 (Descouvemont 1999)
Bea01 (Beaumel et al. 2001)
Tan03 (Tang et al. 2003)
Wan91 (Wang et al. 1991)
Efr96 (Efros et al. 1996)
Wie87 (Wiescher et al. 1987 Wiescher,Harms,Goerres,Thielemann& Rybarcyk ApJ 316 (1997) 162 1001.2053)
Bar97 (Bardayan& Smith 1997)
Koe91 (Koehler& Graff 1991)
And06 (Ando et al. 2006)
Ser04 (Serpico et al. 2004)
Wag69 (Wagoner 1969)
Has09 (Hashimoto et al.2009)
Wie89 (Wiescher et al.1989)
FK90 (Fukugita& Kajino 1990)
Bru91 (Brune et al.1991)
Bec92 (Becchetti et al.1992)
Iga95 (Igashira et al.1995)
Cyb08 (Cyburt& Davids 2008)

Miz00 (Mizoi et al. 2000)
Nag06 (Nagai et al. PRC 74 (2006) 025804 AC2010)
Has09c (Hashimoto et al.PLB 674 (2009) 27)
FK90 (Fukugita& Kajino,PRD 42 (1990) 4251)
Rau94 (C. Rauscher et al.ApJ 429 (1994) 499)
Men12 (C Mendes et al.PRC 86 (2012) 064321)
Tang03 (Tang et al.PRC 67 (2003) 015804)
Bar97C (Bardayan& Smith PRC 56 (1997) 1647)
Kaw91 (Kawano,Fowler,Kavanagh,Malaney ApJ 372 (1991) 1-7)
Cam08 (Camargo et al.Phys.Rev.C 78,034605 (2008))
Ili16 (Iliadis et al. 2016)

Numerical values

[Planck 2015 XIII] Ade et al. A.&A. 594, A13 (2016).
[PDG] Particle Data Group 2017.

Options

Directory set up

We set the directory.

```
SetDirectory[NotebookDirectory[]]
```

```
/Users/pitrou/Dropbox/iap/notebooks/BBN
```

To check what is the directory of your notebook :

```
Print["The current Directory is ", Directory[]]
```

```
The current Directory is /Users/pitrou/Dropbox/iap/notebooks/BBN
```

Numerical options

■ Miscellaneous options

```
$InterpolateAnalytics = True;
```

It is slightly faster to interpolate the analytic expressions of nuclear reaction rates. Setting $InterpolateAnalytics to True is recommended.

```
$HistoryLength = 10;
```

This is to avoid *Mathematica* to store too many results in memory. Only the past $HistoryLength results are kept in computer memory (this is standard Mathematica option).

```
$PaperPlots = False;
$ResultsPlots = False;
```

If $PaperPlots is set to True, the most important plots are constructed and they are output in pdf in the ‘Plots’ subfolder.
Unless interested in reproducing the plots of the companion paper, this should be set to False to avoid any loss of time in the code.

If $ResultsPlots is True the results for the abundances are plotted at the end. Similarly, to avoid loss of time this shouldbe set to False.

Numerical precision

```
$CompileNDSolve = True;
```

If $CompileNDSolve is set to True (advised), then the differential equation solver uses compiled functions. This is slightly faster.

```
$BDFOrder = 2;
```

Order of numerical scheme for Backward Differentiation Scheme (BDS) integration.
-Order 1 works well but it is slow.
-Order 2 (advised) is faster. Higher orders result in numerical crash.

```
PrecisionNDSolve = 2;(*TODO Put 2 here !!!! *)
```

PrecisionNDSolve is a precision parameter used in the differential equation solver. It is tuned such that 0 gives reasonable results, 1 gets very good results, and 2 gets excellent results and 3 gets super dupper precise results (10^{-5} precision).

```
AccuracyNDSolve := 15 + PrecisionNDSolve;
```

We slave AccuracyNDSolve to PrecisionNDSolve. This is roughly the number of digits of precision behind the dot, so increasing it will lead to always better results but this can crash if the accuracy required is too strong.

```
NTemperaturePoints = 1200; (*1000 is enough*)
```

Number of points in discretization of temperature between the
highest temperature (10^{12} K) and the lowest ($\sim 10^{7.5}$ K). 1000 is enough.
Sampling is performed with NTemperaturePoints + 1 points. Spacing is performed logarithmically, with Log_{10}.

```
InterpOrder = 3;
```

Order of polynomials used in interpolations of reaction rates. Usually with Spline functions.

```
$FastPENRatesIntegrals = True;
```

If $FastPENRatesIntegrals is set to True, it uses a Simpson method for numerical integrals. Otherwise it uses the Mathematica function NIntegrate which is more accurate (adaptative refinement of integral) but much slower.

```
$PENRatesIntegralsPoints = 300;(*200 is enough *)
```

In case $FastPENRatesIntegrals is set to True, $PENRatesIntegralsPoints is the number of points used to perform the numerical integrals. 200 is enough. 400 is ultra precise.

```
SetOptions[SelectedNotebook[], PrintPrecision → 8]
```

We increase the number of digits which are displayed

BBN

Nuclear rates

Most nuclear reactions rates are tabulated in a separate file. The rest of the rates are given as analytic fits.
The external file loaded with all reactions definitions and rates is given by the name TabulatedReactionsFile.

We can choose to use this file but only keep a subnumber of equations defined by NumberNuclearReactions.

The full network corresponds to NumberNuclearReactions=423.
A small network including Li7 and Li6 corresponds to NumberNuclearReactions=17;
A very small network including Li7 but not Li6 corresponds to NumberNuclearReactions=12;

```
TabulatedReactionsFile = "BBNRatesAlainCoc2018.dat";
NumberNuclearReactions = 423;
```

We can also decide to keep only a subset of the equations by specifying the maximum nuclear mass. The default value is Infinity, meaning that we do not cut the network.

```
MaximumNuclearMass = Infinity;
```

■ Monte-Carlo uncertainty estimation options

```
$RandomNuclearRates = False;
$MaxVariationRate = 1000;
```

If $RandomNuclearRates is True, then each time we generate the equations we generate rates with a log normal distribution according to the ‘f’ specified (see Eq. 4.5 of [Coc et al. 2014] for definition of f).
We limit f to the values 1/$MaxVariationRate<f<$MaxVariationRate .

■ Rescaling of some rates

This is a rescaling factor for the d + p -> He3 + γ reaction. It can be varied so as to obtain constraints on this reaction rates from measured abundance.

```
dpTOHe3gFactor = 1;
```

Corrections for weak rates

Since the weak interaction rates, that is the rates of weak reactions of the type n + ν <-> p + e and associated reactions, depend only on temperature,
these can be computed once and for all so as to explore the dependence in cosmological parameters or other parameters.

If $RecomputeWeakRates is set to True, the code recomputes the weak rates.
Otherwise it reads them from files previously stored. If the file does not exist it recomputes the rates.

If $ParallelWeakRates=True this is done using parallelization over the various CPUs that Mathematica can detect.

```
$RecomputeWeakRates = False;
$ParallelWeakRates = True;
```

There are several booleans corresponding to the different types of corrections which can be considered in these weak rates.
The name of the file used for storing the rates is built out of these booleans.

Since the corrections do not add linearly, there are in principle many choices of corrections, but the only meaningful
choices are those without any corrections and those with all corrections included.
Or maybe those with just one correction added, so as to check its amplitude.

```
$RadiativeCorrections = True;

$ResummedLogsRadiativeCorrections = True;
$RelativisticFermiFunction = True;
```

1) If $RadiativeCorrections is set to True, we use Coulomb and radiative corrections (see section III.E of companion paper. The corrections are implemented in Eqs. 101 and 104.).

If $ResummedLogsRadiativeCorrections is set to True we use Eq. 15 of [Czarnecki et al 2004] which amounts to resumming some higher order radiative corrections, and this is also Eq. B35 of the companion paper.

If $ResummedLogsRadiativeCorrections is False we use simply Eq. 7 of the same reference, which corresponds to Eq. 103 and B30 of the companion paper.
If $RelativisticFermiFunction is True we use the relativistic Fermi function (Eq. 5 in [Ivanov 2012]), which corresponds to Eq. 100 of the companion paper. Otherwise we use the standard non-relativistic Fermi function, given by Eq. 99 of the companion paper.

```
$RadiativeThermal = True;
$CorrectionBremsstrahlung = True;
```

2) If $RadiativeThermal set to True, the thermal radiative corrections are taken into account.
The first expressions date back from [Dicus et al.]. However other expressions were derived subsequently in [Lopez&Turner 1998], [Cambier et al.] and [Esposito et al. 1999] among other references.
[Kernan] pointed typos and compared the differing results of [Cambier et al.] and [Dicus et al.].
These were correctly computed in [Brown&Sawyer].
In the companion paper, the thermal radiative corrections are given by Eq. 108 with the various contributions given by Eqs. 109-113. However, it is easier to compute the first contribution of 108 using Eqs. 109 (with definition B41), but to compute the second and third contributions of 108 using Eqs. B50-B51.

We showed in companion paper that Bremsstrahlung corrections need also to be added to be fully consistent with [Brown&Sawyer]. This is controlled by the boolean $CorrectionBremsstrahlung. If $RadiativeThermal is set to True, then these bremsstrahlung corrections (corresponding to Eqs. 107a and 107b in companion paper with the definitions B48-B49) are also incorporated if $CorrectionBremsstrahlung is True.

```
$FiniteNucleonMass = True;
```

3) If $FiniteNucleonMass is set to True, we take into account the finite mass of nucleons by keeping corrections in $1/M_n$ in the collision integrals of the weak rates.
Our method is described in the companion paper and differs from earlier literature.
There is a suboption for these finite mass corrections which is

```
$CoupledFMandRC = True;
```

If $CoupledFMandRC is False, finite mass corrections are computed from the Born results with no radiative null temperature corrections. If this is set to True, the finite mass corrections are applied to the rates on which the Coulomb and radiative corrections are taken into account. True is the advised value.

The expressions implemented are Eqs. 114 when $CoupledFMandRC is False, with the definition B23. If $CoupledFMandRC is True, then we use Eqs. 115 with the Fermi and radiative corrections corresponding to the above choices for radiative corrections.

4) Mass shifts due to QED plasma effects are **ALREADY** taken into account in thermal radiative corrections.
However it is possible to turn the option $QEDMassShift to true to check how this affects the rate when taken individually.
Apart to satisfy this curiosity, **this option should be set to False** in all cases.

```
$QEDMassShift = False;
```

Plasma corrections

```
$QEDPlasmaCorrections = True;
$CompleteQEDPressure = True;
```

If $QEDPlasmaCorrections is set to True, the QED corrections to the pressure and the energy density are taken into account. This affects for instance the expansion rate via the Friedmann equation.
Expressions can be found in [Lopez&Turner 1998], [Mangano et al. 2001] or in [Heckler 1994]. See also the companion paper where Eqs. 48-52 are used inside Eq. 55 to modify the entropy and Eq. 58 to modify the energy density.
If $CompleteQEDPressure is False, the subdominant term is ignored. Otherwise it is included. We checked it is so subdominant that it does not change the results.

```
$IncompleteNeutrinoDecoupling = True;
```

If $IncompleteNeutrinoDecoupling is set to True we use a fit for the heating function of the neutrinos due to incomplete decoupling. Indeed if we consider the details of the decoupling of neutrinos, it is found that decoupling is incomplete by the time electrons and positrons annihilate into photons. This results in a slightly overheating of neutrinos and cooling of the electrons/photons plasma. In principle, one should also take into account the spectral distortions that this heating induces on the neutrinos. But neglecting it we can use a fit of the neutrinos heating functions and assume that neutrinos always stay thermalized. This allows to graps most of the effect on the abundance of Helium. The fitting functions which is used is the one given in [PArthENoPE]. It is used in Eq. 63 of the companion paper.
The advised value for $IncompleteNeutrinoDecoupling is True.

```
$RecomputePlasmaCorrections = False;
```

Since the QED effects depend on temperature, they need to be computed only once for all and they are stored on a file. But we can force the recomputation of these corrections by setting $RecomputePlasmaCorrections to True;

Degenerate Neutrinos

```
$DegenerateNeutrinos = False;
μOverTν = 0.0;
```

If neutrinos have a chemical potential, then $DegenerateNeutrinos = True. Standard BBN is with non-degenerate neutrinos and when $DegenerateNeutrinos = False the value of μOverTν is ignored. Note that in the case of degenerate neutrinos, one must run the code with RecomputeWeakRates = True because this modifies the weak rates. And for every value of μOverTν one must recompute the weak rates.
μOverTν is also noted ξ_ν in the companion paper. See section VI.C.

Initial definitions

Temperature eras

We choose to split the BBN numerical calculations in three eras.
1) First the high temperature era between Tstart = 10^{11} K and TMiddle. Only neutrons and protons abundances are tracked and this is ruled by weak interactions.
2) The intermediary era, between TMiddle and T18, where only a small network of reactions (17

nuclear reactions plus weak interactions or possibly less) is used.
3) A low temperature region, between T18 and Tend, where Tend is usually slightly lower than $10^8\,K$, typically Tend = $6 \times 10^7\,K$.

So we have Tstart > TMiddle > T18 > Tf.

```
Kelvin = 1;
Tstart = 10^11 Kelvin;
TMiddle := 0.9999 * 10^10 Kelvin;
T18 := 1.25 * 10^9 Kelvin;
Tend = 6. * 10^7 Kelvin;
```

Temperature sampling

We choose to sample temperature starting from
$10^{12}\,K$. This is interesting to check high T behaviour of some effects.

```
Ti = 10^12 Kelvin;
Tf = 10^7 Kelvin;
LogTi = 1. Log10[Ti];
LogTf = 1. Log10[Tf];
```

We first build the list of LogT points (ListLogT) and then the list of T points (ListT).

```
ListLogT = Sort@DeleteDuplicates@
     Join[{10.}, Table[i, {i, LogTf, LogTi, (LogTi - LogTf) / NTemperaturePoints}]];
ListT = 1. × 10^ListLogT;
```

```
ListTRange[T1_, T2_] := Module[
   {len = Length@ListT, imindown, imaxup, Tmin = Min[T1, T2], Tmax = Max[T1, T2]},
   imindown = Max[1, -1 + Position[ListT, SelectFirst[ListT, # > Tmin &]][[1, 1]]];
   imaxup = Min[len, Position[ListT, SelectFirst[ListT, # >= Tmax &]][[1, 1]]];
   ListT[[imindown ;; imaxup]]
  ]
```

ListTRange is a function to select a sublist in this list of temperature, according to a range of temperature which makes sure to have either the points on the boundary or at least one point beyond (to avoid problems with interpolating functions).
If T = 10^{10} is not in the list, we add it to avoid problems with interpolations of reactions rates which all start at 10^10 and below.

Constants of Physics

■ cgs system

We use cgs system with Kelvin. (For instance an erg is 1 g $\mathrm{cm}^2\,\mathrm{s}^{-2}$).
By definition these are set to one in these units. Any change
of system of units can be made by modifying these variables only.

For instance if we want to use the m/kg/s system we need only put cm =
0.01 and gram = 0.001 below.

As a check,
final results should not depend on these conventions since abundance rates are dimensionless.

```
second = 1;
cm = 1;
gram = 1;
```

When taking values from the kg/m/s system, we use the factors

```
kg = 10^3 gram;
meter  =  10^2 cm;
km = 10^3 meter;
Joule  = kg meter^2 / second^2; (* This gives 10^7 ergs *)
DensityUnit = gram / cm^3;
Hz = 1 / second;
```

```
Giga = 10^9;
Mega = 10^6;
Kilo = 10^3;
```

Fundamental constants

```
kB  = 1.3806488 × 10^-23 Joule / Kelvin;(* Boltzmann constant in J/K *)
clight = 2.99792458 * 10^8 * meter / second; (* speed of light in cm/s *)
hbar = 6.62606957/(2 π) 10^-34 (*1.054571596 10^-34*) Joule second;
Avogadro = 6.0221415 × 10^23;
```

When using masses of particles, we use eV and MeV that we convert in the cgs system

```
eV = 1.60217653 × 10^-19 Joule;
keV  =  Kilo eV;
MeV = Mega eV;
GeV = Giga eV;
```

Interactions constants

```
GN = 6.67384 × 10^-11 meter^3 / kg / second^2; (* Gravitation constant *)
GF = 1.1663787 * 10^-5 / (GeV)^2; (* Fermi Constant*)
gA = 1.2723;
(* Axial current constant of structure of the
 nucleons Particle data group : 1.2723(+-23) PDG2016 *)
(* However post 2002 data suggest 1.2755(11) as advised by William Marciano*)
```

```
fWM = 3.7058 / 2(*1.853*); (* Weak magnetism see 1212.0332*)
radiusproton = 0.841 * 10^-15 meter (*(arXiv:1212.0332)*)
```

```
8.41 × 10^-14
```

```
fWM
```

```
1.8529
```

`fWM` is the weak magnetism constant. See Eq. [Horowitz&Li] for definition with the value given by its Table 1.
Note that all expression in [Seckel 1993] seem to have a factor 2 difference. That is all interaction rates involving the weak magnetism in [Seckel 1993] are underestimated by a factor 2. Expressions in [Lopez et al. 1997] seem correct however.

```
αFS = 1 / 137.03599911;(* Fine structure constant =e^2/(4π) *)
```

Particle masses

Throughout, masses stand always for mc^2 so that they
are in fact energies. This avoids putting unnecessary c^2 factors

```
me = 0.510998918 MeV;
mn = 939.565360 MeV;
mp = 938.272029 MeV;
Q = mn - mp; (* Mass difference between neutrons and protons *)
```
m_{Nucleon} = mn;

m_W = 80.385 GeV; (* Mass of the W Boson. *)
m_Z = 91.1876 GeV;

The energy difference between neutron and proton in Mev is

```
Q / MeV
```

```
1.293331
```

- Cosmology constants

h is the Hubble rate in units of 100 km/s/Mpc.

pc = 3.0856777807 × 10^{16} meter; (* The parsec *)
```
Mpc = Mega pc;
H0 = 100 h km / second / Mpc; (* Hubble constant today *)
H100 = 100 km / second / Mpc;
(*Fake Hubble rate given by 100 km/s/Mpc so that h = H0/H100 *)
```

We define two critical densities. One for the actual Hubble rate, and one for the rate at 100km/s/Mpc.

$\rho_{\text{crit}} = \frac{3.}{8\,\pi\,\text{GN}}\,(\text{H0})^2$ (* in g cm^{-3} by construction *)

$\rho\text{crit100} = \frac{3.}{8\,\pi\,\text{GN}}\,(\text{H100})^2$ (* in g cm^{-3} by construction *)

$1.8784708 \times 10^{-29}\,h^2$

$1.8784708 \times 10^{-29}$

Neutron life time (Particle Data Group 2017).

```
Meanτneutron := 879.5(*880.2second+-1.1s was previous value from PDG2017 *);
(* Now we use 1712.05663 Section 11
 which includes recente 2017 measurements.*)
στneutron := 0.8 second;
τneutron = Meanτneutron;
```

Cosmological Parameters

Neutrinos generations and possible neutrino degeneracy (neutrino chemical potential).

```
NeutrinosGenerations := 3.;
ξν := If[$DegenerateNeutrinos, μOverTν, 0];
```

$\rho\text{FD}[\text{c\_}] = \frac{1}{2\,\pi^2}\int_0^{\text{Infinity}} \frac{y^3}{\left(e^{y-c}+1\right)}\,dy;$

$\text{nFD}[\text{c\_}] = \frac{1}{2\,\pi^2}\int_0^{\text{Infinity}} \frac{y^2}{\left(e^{y-c}+1\right)}\,dy;$

ρFDNonDegenerate = ρFD[0];

Information

$\text{Series}\left[\frac{\rho\text{FD}[c] + \rho\text{FD}[-c]}{2\,\rho\text{FDNonDegenerate}}, \{c, 0, 4\}\right];$

```
ην[c_] = (nFD[c] - nFD[-c]) / (2 Zeta[3] / π^2);
Series[ην[c], {c, 0, 3}];
```

Effective number of neutrinos generation due to chemical potential
(this is different from N_{eff} which takes into account also QED and incomplete neutrino decoupling)

```
Nneu := NeutrinosGenerations * (ρFD[ξν] + ρFD[-ξν]) / (2 ρFDNonDegenerate)
```

CMB temperature today

```
TCMB0 := 2.7255 Kelvin;
σTCMB0 := 0.0006 Kelvin; (* [Planck 2015 XIII] *)
```

Temperature of CMB today in Kelvin. We consider the case where QED effects are ignored or taken into account. This is the implementation of (the inverse of) Eq. 56 in companion paper.

```
FourOverElevenQED := 4/11 (1 + 25 αFS/(22 π));
FourOverElevenNoQED := 4/11;
FourOverEleven :=
  If[$QEDPlasmaCorrections, FourOverElevenQED, FourOverElevenNoQED];

Tν0 = (FourOverEleven)^(1/3) TCMB0;

(1/FourOverEleven)^(1/3)
1.3997891
```

Temperature of neutrinos today is lower than photons because they decoupled earlier and electron/-positron annihilation has only reheated photons. This leads to the famous ratio of 4/11 between the T^3 of the neutrinos and photons. However, since decoupling is slightly incomplete, this in principle should be corrected like in [Mangano et al. 2005] or [Grohs et al. 2012].

Additionally, there is another source of modification to this 4/11 ratio which comes from the QED corrections to the plasma thermodynamic quantities (modification of pressure and energy density and thus of entropy). Taking the high temperature modification leads to the correction added above in the variable FourOverEleven. See e.g. Eq. 41 of [Lopez&Turner 1998] and/or the companion paper (Eq. 56). The effect of incomplete neutrino decoupling is considered further below.

Hubble rate in units of 100 km/s/Mpc.

```
h := 0.6727; (*+-0.0066 *) (*[Planck 2015 XIII]*)
```

Baryons and Cold Dark Matter density fraction. This is $\Omega_b h^2$ and $\Omega_c h^2$.

```
Meanh2Ωb0Planck = 0.02225; (*[Planck 2015 XIII TT and ET and EE]*)
σh2Ωb0Planck = 0.00016; (* Standard deviation*)

Meanh2Ωb0 = Meanh2Ωb0Planck;
σh2Ωb0 = σh2Ωb0Planck;
h2Ωb0 = Meanh2Ωb0;

ReSetCosmology := (
   Meanh2Ωb0 = Meanh2Ωb0Planck;
   NeutrinosGenerations = 3;
  );
```

```
Meanh2Ωc0 = 0.1198;(* [Planck 2015 XIII]*)
σh2Ωc0 = 0.0015;
h2Ωc0 = Meanh2Ωc0;
```

Cosmological constant fraction Ω_Λ. Obtained by summing baryons and cold dark matter, given that radiation is negligible today.
This is just for information and it is not used since the cosmological constant is totally negligible for its influence in the expansion rate during BBN.

$$1 - (\text{h2}\Omega\text{b0} + \text{h2}\Omega\text{c0}) / h^2$$

```
0.68609489
```

Density of photons and neutrinos

The Black Body constant is defined as

$$\text{aBB} = \frac{\pi^2}{15\,\text{hbar}^3\,(\text{clight})^5}$$

$$2.3167363 \times 10^{28}$$

Energy density and number density of CMB today (See appendix A1 in companion paper)

$$\rho_{\text{CMB0}} := \text{aBB}\,(\text{kB TCMB0})^4\,;(*\text{ in g cm}^{-3}*)$$

$$n_{\text{CMB0}} := \frac{2\,\text{Zeta}[3]}{\pi^2\,\text{hbar}^3\,(\text{clight})^3}\,(\text{kB TCMB0})^3$$

We recover the number of photons per cubic centimeter (410) :

$$n_{\text{CMB0}}$$

```
410.72678
```

The fraction of energy content due to photons is simply

$$\Omega_{\gamma 0} := \rho_{\text{CMB0}} / \rho_{\text{crit}};$$

For neutrinos, we must take into account the temperature of neutrinos today, the number of neutrinos, and the fact that they are Fermions.
See companion paper for details.

$$\Omega_{\nu 0} := \text{Nneu} * \frac{7}{8} * (\text{FourOverEleven})^{1/3}\,\Omega_{\gamma 0};$$

The contribution to the energy content is obtained by the ratio between energy densities and critical density. We check that today it is around 0.1%.

$$\frac{\Omega_{\gamma 0}\,h^2}{\text{h2}\Omega\text{b0}}$$

```
0.0011113728
```

Density of baryons

```
ma = 931.494061 MeV;(* Audi2012 *)
He4Overma = 4.0026032541; (* Audi2012 *)
H1Overma = 1.00782503223; (* Audi2012 *)
```

The atomic mass, the Helium4 mass (in units of atomic mass) and Hydrogen mass (in units of atomic mass).

```
x_He4 = 0.24709 ;(* Chemical composition at the end of BBN. In
  principle one should account for He4 produced by stars...*)
x_H1 = 1 - x_He4;
mbaryon0 = (x_H1 H1Overma + x_He4 He4Overma / 4) ma;
```

This is the (average) mass of baryons today (that is of nucleons), taking into account that part is in Hydrogen and part in Helium. We use the current chemical composition with 24.75 % of Helium but this is subject to controversy. Indeed the abundance of baryons is measured with CMB, and thus refers to an epoch (z ~1100) where the composition was the same as the one at the end of BBN. See Eqs. C5 C6 in companion paper.

```
mbaryon0 / ma
ma / mbaryon0
```

```
1.0060524
```

```
0.99398406
```

$$\frac{\left(\frac{\text{He4Overma}}{4} - \text{H1Overma}\right)}{\text{H1Overma}}$$

$\% * x_{\text{He4}}$

```
-0.0071185161
```

```
-0.0017589141
```

The number density of baryons, that is of nucleons is then given by the baryons mass density divided by the average mass of baryons.

$\rho\text{B0} := \text{h2}\Omega\text{b0} * \rho\text{crit100};$

$$\text{nbaryons0} := \frac{\rho\text{B0}}{\left(\text{mbaryon0} / (\text{clight})^2\right)}$$

The ratio between baryons number and photons number is by definition the η parameter and its value for the parameters chosen is

$$\frac{\text{nbaryons0}}{n_{\text{CMB0}}}$$

```
6.0913257 × 10^-10
```

It is convenient to define the ratio between $\Omega_b\, h^2$ and this η parameter.

$$\Omega\text{bh2Over}\eta := \frac{n_{\text{CMB0}}}{\rho\text{crit100}} \frac{\text{mbaryon0}}{(\text{clight})^2}$$

$\Omega\text{bh2Over}\eta$

```
3.6527352 × 10^7
```

The η parameter is then obtained from the baryon density fraction as

$$\eta\text{factor} := \frac{\text{h2}\Omega\text{b0}}{\Omega\text{bh2Over}\eta}$$

Baryons density is obtained from its valued today scaled by dilution (no thermal effects, so it is only the energy density due to rest mass of baryons).

```
ρB[av_] := ρB0/av^3;
nB[av_] := nbaryons0/av^3;
```

For nuclear reactions, the mass density of baryons is in fact a number density of species multiplied by the atomic mass (see appendix C1 of companion paper for a detailed discussion).
This differs slightly from the mass density of baryons and we take this into account.

If $CorrectBaryonsEnergyDensityinBBNRRates is set to False, then we use the baryons density naively in nuclear rates.
Otherwise we take into account that the baryons density is in fact the number density times the atomic mass as explained in App. C1 of the companion paper.

```
$CorrectBaryonsEnergyDensityinBBNRRates = True;
ρBForBBN[av_] :=
  ρB[av] If[$CorrectBaryonsEnergyDensityinBBNRRates, ma / mbaryon0, 1];
 (* This is Eq. C8 of the companion paper *)
```

Distribution functions

Basic Fermi - Dirac (FD) and Bose - Einstein (BE) functions. x here $1/(k_B T)$.

```
FD[EoverT_] = 1/(Exp[EoverT] + 1);(* Fermi Dirac Distribution *)
FD[Energy_, x_] = 1/(Exp[x Energy] + 1);
BE[EoverT_] = 1/(Exp[EoverT] - 1); (* Bose Einstein Distribution *)
BE[Energy_, x_] = 1/(Exp[x Energy] - 1);

(* For neutrinos with a chemical potential *)
FDν[Energy_, ϕ_, x_] = 1/(Exp[x Energy - ϕ] + 1);
```

Derivatives of FD wrt to energy.

```
FDp[Energy_, x_] = D[1/(Exp[x Energy] + 1), Energy];
```

Customized Mathematica tools

This function NP displays a certain number of digits for a given real number

```
NP[number_] := NumberForm[number, 8]
```

This function displays a table in grid form, that is with lines between the entries

```
MyGrid[Table_List] := Grid[Table, Frame → All]
```

This function performs interpolation on a list of points (x, f(x)) to the required order.

```
MyInterpolation[Tab_List] :=
  Interpolation[Tab, InterpolationOrder → InterpOrder];

(* Does not work to interpolate the log
 of rates because it fails when rates vanish !!!*)
MyInterpolationLog[Tab_List] :=
  Function[{x}, Exp[Interpolation[{#[[1]], Log[#[[2]]]} & /@ Tab,
      InterpolationOrder → InterpOrder][x]]];

$InterpolateLogRate = False;
MyInterpolationRate[Tab_List] :=
 If[$InterpolateLogRate, MyInterpolationLog[Tab], MyInterpolation[Tab]]
```

Tools to avoid too small numbers in numerics.
MyChop chops small numbers and replaces them by 0.

```
MyChop[el_?NumericQ] := (Chop[el, $MinMachineNumber]);
SetAttributes[MyChop, Listable];
```

Redefinition of Set to allow to set values to quantities already set

```
MySet[Hold[expr_], value_] := (expr = value);
MySetDelayed[Hold[expr_], value_] := (expr := value);
```

Personal simple integral with second order polynomial interpolation (Simpson method).

```
TableSimpsonC = Compile[
   {{a, _Real}, {b, _Real}, {Np, _Integer}}, With[{h = 1. (b - a) / Np, n2 = Np / 2},
    With[{h3 = h / 3.}, Join[{{a, h3}}, Table[{a + 2. j h, 2 h3}, {j, 1, n2 - 1}],
      Table[{a + (2. j - 1) h, 4 h3}, {j, 1, n2}], {{b, h3}}]]],
   CompilationTarget → "C", "RuntimeOptions" → "Speed"];
```

Generic compilation of a function and of its integration.

```
MyCompile[LV_List, Body_] :=
 Compile[LV, Evaluate[Body], "RuntimeOptions" → "Speed", CompilationTarget → "C",
  CompilationOptions → {"InlineExternalDefinitions" → True},
  RuntimeAttributes → {Listable}]
```

Compilation of a scalar product.

```
V1dotV2 = Compile[{{V1, _Real, 1}, {V2, _Real, 1}},
   V1.V2, CompilationTarget → "C", "RuntimeOptions" → "Speed"];
```

Compiled version of the Simpson integral

```
IntegrateFunction[fun_, pemin_, pemax_, Np_] :=
  With[{interv = (pemax - pemin) / (Np), tab = TableSimpsonC[pemin, pemax, Np]},
   V1dotV2[tab[[All, 2]], MyChop[fun[tab[[All, 1]]]]]];
```

A function to import an external file and which returns an error and quits if the file does not exist.

```
SafeImport[args__] := Module[{out}, out = Catch[Check[Import[args],
     Print["File ", {args}[[1]], " not found. Quiting Kernel."];
     Throw[$Failed];, Import::nffil]];
  If[out === $Failed, Quit[]];
  out]
```

Tools for plots. Some useful grid of ticks

```
MyFrameTicksLog = {{Automatic, Automatic},
   {{{Log[10^8], "10^8"}, {Log[10^8.5], "10^8.5"}, {Log[10^9], "10^9"},
     {Log[10^9.5], "10^9.5"}, {Log[10^10], "10^10"}, {Log[10^10.5], "10^10.5"},
     {Log[10^11], "10^11"}, {Log[10^11.5], "10^11.5"}}, Automatic}};

MyFrameTicks =
  {{Automatic, Automatic}, {{{10^8, "10^8"}, {10^8.5, "10^8.5"}, {10^9, "10^9"},
     {10^9.5, "10^9.5"}, {10^10, "10^10"}, {10^10.5, "10^10.5"},
     {10^11, "10^11"}, {10^11.5, "10^11.5"}}, Automatic}};
```

Thermodynamics of the plasma

Thermodynamic integrals

Defined in appendix A1 of companion paper. These are the integrals needed to obtain the thermodynamic quantities of FD or BE distributions and correspond to Eqs. A5 in companion paper.

```
Clear[Imn]

Imn[sgn_][m_, n_][x_] := NIntegrate[(pe^2 + x^2)^((m-1)/2) pe^(n+1) / (Exp[Sqrt[pe^2 + x^2]] + sgn),
  {pe, 0, Infinity}, Method → {Automatic, "SymbolicProcessing" → 0}]
ImnT[sgn_][m_, n_][T_] := Imn[sgn][m, n][me/(kB T)]

(* Interpolations *)
ImnI[sgn_][m_, n_] := ImnI[sgn][m, n] =
  Interpolation@Table[{me/(kB Tv), Imn[sgn][m, n][me/(kB Tv)]}, {Tv, ListT}]
ImnIT[sgn_][m_, n_][T_] := ImnI[sgn][m, n][me/(kB T)]
```

QED corrections to plasma thermodynamics

QED mass corrections

From this, using Eq. 12 and 13 of [Mangano.et.al 2001] (or Eq .35 of [Lopez & Turner 1998] for the mass of the electron), we get the modification to the mass of the electron and of the photon. For this we ignore the last term in Eq. 12 of [Mangano.et.al 2001] or Eq. 35 of [Lopez&Turner 1998].
See also companion paper (Eqs. 44 and 46).

The mass shift is expressed in units of the electron mass so as to be dimensionless. So what we define as dme2 is really $\delta(m_e)^2/(m_e)^2$ and similarly for dmγ2 it is $\delta(m_\gamma)^2/(m_e)^2$

```
dme2[T_] := (kB T/me)^2 (2 π αFS/3 + 4 αFS/π ImnT[1][0, 1][T])
(* Only main part of mass shift *)
dmγ2[T_] := 8 αFS/π ImnT[1][0, 1][T] (kB T/me)^2
```

We perform interpolations of these mass shifts over the relevant range of temperatures. We store it on disk in the files dme2.dat and dmg2.dat if they have never been computed.
If they have already been computed we just load the results (unless the Boolean option $RecomputePlasmaCorrections is set to True).

```
dme2Tab = Check[Import["Interpolations/dme2.dat", "TSV"],
   Print["Precomputed data not found. We recompute and store the data."];
   $Failed, Import::nffil];

dmg2Tab = Check[Import["Interpolations/dmg2.dat", "TSV"],
   Print["Precomputed data not found. We recompute and store the data."];
   $Failed, Import::nffil];


Timing[If[dme2Tab == $Failed || dmg2Tab == $Failed || $RecomputePlasmaCorrections,

   dme2Tab = Table[{T, dme2[T]}, {T, ListT}];
   dmg2Tab = Table[{T, dmγ2[T]}, {T, ListT}];

   Export["Interpolations/dme2.dat", dme2Tab, "TSV"];
   Export["Interpolations/dmg2.dat", dmg2Tab, "TSV"];
  ];]
{0.000019, Null}
```

Once having the table of values for the mass shift as a function of temperature, we perform an interpolation

```
dme2I = MyInterpolation@ToExpression@dme2Tab;
dmγ2I = MyInterpolation@ToExpression@dmg2Tab;
```

We define a function which gives a value for all temperature and not just in the range of the interpolation so as to avoid any numerical problem.
TODO eventually this is useless. Simplify as much as possible for readability.

```
dme2N[T_?NumericQ] := Which[T < Tf, 0, T ≤ Ti , dme2I[T], T > Ti, dme2I[Ti]];
dmγ2N[T_?NumericQ] := Which[T < Tf, 0, T ≤ Ti , dmγ2I[T], T > Ti, dme2I[Ti]];
```

We also define these interpolations in terms of the inverse temperature (in units of electron mass, that is the quantity $x = m_e / (k_B T)$)

```
dme2x[x_] := dme2N[me / (kB x)];
```

We plot the result for illustration purposes (only if option $PaperPlots is True).

```
If[$PaperPlots, LogLogPlot[Abs@dme2N[Tv] / Tv^2, {Tv, 10^8, 10^12}, Frame → True,
  FrameLabel → {"T  (K)", "δme^2/T^2"}, PlotStyle → {Black, Thickness[0.0035]}]]
If[$PaperPlots, Export["Plots/Plotdme2.pdf",
   Style[%, Magnification → 1], "PDF"];]
```

QED pressure corrections

Pressure corrections are obtained from Eq. 13 of [Heckler 1994] when including only electron mass shift, or Eq. 16 of [Mangano et al. 2001] for both electron mass and photon mass shifts. See also companion paper (around Eqs. 48 and 49) It is made of the dominant term dPa, and the subdominant terms dPb which are the two contributions of Eq. 48 in companion paper.

$$\texttt{dPa[T\_] := dPa[T]} = \frac{\alpha FS}{\pi} (kB\,T)^4 \left(-\frac{2}{3}\,\texttt{ImnT[1][0, 1][T]} - \frac{2}{\pi^2} (\texttt{ImnT[1][0, 1][T]})^2\right);$$

For the subdominant contribution we use reduced variables. But contrary to the rest of the code where p stands for p/me here it stands for p/T.

```
Fdp1dp2 = Compile[{{p1, _Real}, {p2, _Real}, {x, _Real}}, Evaluate[With[
      {e1 = Sqrt[p1^2 + x^2], e2 = Sqrt[p2^2 + x^2]},
      (αFS/π^3) (x^2 p1^2 p2^2)/(p1 p2 e1 e2) Log[Abs[(p1 + p2)/(p1 - p2)]] 1/((Exp[e1] + 1) (Exp[e2] + 1))
     ]], "RuntimeOptions" → "Speed", CompilationTarget → "C"];


Fdp1dp2N[p1_?NumericQ, p2_?NumericQ, x_] := Fdp1dp2[p1, p2, x];


Clear[dPb]
dPb[Tv_] := dPb[Tv] = (kB Tv)^4 With[{x = me / (kB Tv)},
      0.5 NIntegrate[
         Fdp1dp2N[(p1pp2 + p1mp2) / 2, (p1pp2 - p1mp2) / 2, x]
          + Fdp1dp2N[(p1pp2 - p1mp2) / 2, (p1pp2 + p1mp2) / 2, x],
         {p1mp2, 0.0001, Max[20, 20 * x]}, {p1pp2,
          0.0001 + Abs[p1mp2], Max[20, 20 * x] + Abs[p1mp2]}, PrecisionGoal → 4]
     ];

If[$PaperPlots, PlotdPadPb = ListLogLogPlot[
    {Table[{Tv, Abs@dPa[Tv] / (kB Tv)^4}, {Tv, ListTRange[10^8.5, 10^11]}],
     Table[{Tv, Abs@dPb[Tv] / (kB Tv)^4}, {Tv, ListTRange[10^8.5, 10^11]}]},
   FrameLabel → {"T  (K)", "δP/(kBT)^4"}, LabelStyle → {FontSize → 12},
   FrameTicks → MyFrameTicksLog,
   PlotStyle → {{Red, Thickness[0.0035]}, {Blue, Dashed, Thickness[0.0035]}},
   Frame → True, FrameStyle → Thickness[0.004],
   Joined → True, PlotRange → {10^-10, 10^-2}]]

If[$PaperPlots,
  Export["Plots/PlotdPadPb.pdf", Style[PlotdPadPb, Magnification → 1], "PDF"];]
```

The pressure is then obtained (restoring the correct dimensions)

```
dP[T_] := dP[T] = dPa[T] + If[$CompleteQEDPressure, dPb[T], 0]

dPI := dPI = Interpolation@Table[{Tv, dP[Tv]}, {Tv, ListT}]
```

We check the high temperature limit, which is given in Eq. 30 of [Lopez&Turner 1998] or Eq. 1 of [Heckler 2013]. See also companion paper.

```
dPa[10^12] / (kB 10^12)^4
dPb[10^12] / (kB 10^12)^4
dP[10^12] / (kB 10^12)^4
-(5/288) 4 π αFS
```

```
-0.0015919089

4.1712453 × 10^-9

-0.0015919047

-0.0015920354
```

QED energy density corrections

Energy density corrections are obtained from the thermodynamic identity ρ = -P + T dP/dT. See Eq. 50 of companion paper.

```
Clear[dρ]
dρ[T_] := dρ[T] = -dP[T] + T dPI'[T]
```

QED modified relativistic degrees of freedom

The modified relativistic degrees of freedom (see [Lopez & Turner 1998] for definition) are [see also Eq. 52 of companion paper]

```
dgP[T_] := dP[T] 90/(π^2 (kB T)^4);
dgρ[T_] := dρ[T] 30/(π^2 (kB T)^4);
```

We check the high temperature limits (Eq. 54 of companion paper)

```
dgP[10^12]
(*3dgρ[10^12]*)
-25 αFS/(4 π)
-0.01451643

-0.014517622
```

We interpolate these relativistic degrees of freedom and we store them in a file ‘dg.dat’. If this file is already present we do not recompute unless the option $RecomputePlasmaCorrections is set to True.

```
dgρdgP = Check[Import["Interpolations/dg.dat", "TSV"],
   Print["Precomputed data not found. We recompute and store the data."];
   $Failed, Import::nffil];

Timing[If[dgρdgP == $Failed || $RecomputePlasmaCorrections,

   dgρTab = Table[{T, dgρ[T]}, {T, ListT}];
   dgPTab = Table[{T, dgP[T]}, {T, ListT}];

   dgρdgP = {dgρTab, dgPTab};
   Export["Interpolations/dg.dat", dgρdgP, "TSV"];
  ];]
{8. × 10^-6, Null}
```

We perform an interpolation in time of the modified relativistic degrees of freedom

```
dgρI = MyInterpolation@ToExpression[dgρdgP[[1]]];
dgPI = MyInterpolation@ToExpression[dgρdgP[[2]]];
```

We also define functions which are valid everywhere so as to avoid numerical problems (indeed at very low and very large temperature $\delta g_\rho$and δg_P are constants).

```
dgρN[T_?NumericQ] := Which[T < Tf, 0, T ≤ Ti , dgρI[T], T > Ti, dgρI[Ti]];
dgPN[T_?NumericQ] := Which[T < Tf, 0, T ≤ Ti , dgPI[T], T > Ti, dgPI[Ti]];
```

We define the relativistic degrees of freedom in function of inverse temperature (in units of electron mass), that is a functions of $x = m_e/(k_B T)$.

```
dgρx[x_] := dgρN[me/(kB x)];
dgPx[x_] := dgPN[me/(kB x)];
```

We reproduce Fig. 14 of [Lopez & Turner 1998]

```
If[$PaperPlots, PlotdPdρ =
  LogLinearPlot[{Abs@dgPN[Tv], Abs@dgρN[Tv], 25 αFS / (4 π)}, {Tv, 10^8.5, 10^11},
   Frame → True, FrameLabel → {"T (K)", "-2δP/P        -2δρ/ρ"},
   LabelStyle → {FontSize → 12}, FrameTicks → MyFrameTicks,
   FrameStyle → Thickness[0.004], PlotStyle → {{Thickness[0.004], Red},
     {Blue, Thickness[0.004], Dashing[{0.018}]}, {Black, Thickness[0.003]}}]]

If[$PaperPlots,
 Export["Plots/PlotdPdrho.pdf", Style[PlotdPdρ, Magnification → 1], "PDF"];]
```

Entropy and energy density of the plasma

We compute the thermodynamics using thermodynamical equilibrium. Indeed if we assume total neutrino decoupling then the collision rates inside the electrons/protons/photons plasma are so high that it is always both at thermal and chemical equilibrium.

Furthermore there are so many more photons than baryons today that the chemical potential of electrons and positrons can be ignored. See companion paper for a discussion on the chemical potentials of electrons/psoitrons.

We have two functions to tabulate.

The first function, gives the extra amount of entropy at high temperature due to electrons and positrons, in units of the entropy of photons.
It is noted $\mathcal{S}$ in the companion paper (Eq. 30b). We distinguish the case with and without QED plasma corrections (See Eq. 50 for QED plasma corrections).

```
DSTNoQED = MyInterpolation@
   Table[{T, With[{x = me / (kB T)}, 1 + 45/(2 π^4) (1/3 Imn[1][0, 3][x] + Imn[1][2, 1][x])]},
    {T, ListT}];
DSTQED[Tv_] := (3 dgρN[Tv] + dgPN[Tv]) / 8 + DSTNoQED[Tv];


DST[Tv_] := If[$QEDPlasmaCorrections, DSTQED[Tv], DSTNoQED[Tv]]
DSTN[T_?NumericQ] = Which[T < Tf, 1, T ≤ Ti , DST[T], T > Ti, DST[Ti]];
```

The second function, gives the extra amount of energy density at high temperature due to electrons and positrons, in units of the energy density of photons.
It is noted $\mathcal{E}$ in the companion paper in Eq. 41b. We distinguish the case with and without QED plasma corrections (see Eq. 58 QED plasma corrections).

```
DρTNoQED = MyInterpolation@
  Table[{T, With[{x = me / (kB T)}, 30/π^4 (Imn[1][2, 1][x])]}, {T, ListT}];
DρT[T_] := If[$QEDPlasmaCorrections, dgρN[T]/2, 0] + DρTNoQED[T];
```

We check that the ratio of entropy long before and long after electron/psoitrons annihilation is the famous 4/11, possibly corrected by the QED corrections.

```
DST[10^8] / DST[10^12]
FourOverEleven // N
0.36459959

0.36459621
```

```
If[$PaperPlots,
 LogLinearPlot[{DSTNoQED[T] - 1, DρTNoQED[T]},
  {T, 10^8, 10^12}, Frame → True, FrameStyle → Thickness[0.004],
  FrameLabel → {"T(K)", "𝒮-1       ℰ-1"}, LabelStyle → {FontSize → 12},
  GridLines → {{{me / kB, {Darker@Gray, Thickness[0.005]}}}, {}}, PlotStyle →
   {{Red, Thickness[0.0035]}, {Blue, Thickness[0.0035], Dashing[0.01]}}]]
If[$PaperPlots, Export["Plots/PlotCalSCalE.pdf",
   Style[%, Magnification → 1], "PDF"];]
```

Incomplete decoupling of neutrinos

We follow [PArthENoPE]. We use the function $\mathcal{N}$(z) (Eqs. A.24 – A 25 in [PArthENoPE]). It is found from the full numerical integration of neutrinos, and then a fit is given

```
Listnl = {-10.21703221236002, 61.24438067531452,
   -340.3323864212157, 1057.2707914654834, -2045.577491331372,
   2605.9087171012848, -2266.1521815470196, 1374.2623075963388,
   -586.0618273295763, 174.87532902234145, -35.715878215468045,
   4.7538967685808755, -0.3713438862054167, 0.012908416591272199};

𝒩[z_] := If[z ≥ 4, 0, Exp[Plus @@ Table[Listnl[[i + 1]] z^i, {i, 0, 13}]]]
```

By construction this is the heat rate transfered in unit of Hubble rate. Or more precisely, the volumic heat rate (the source on the r.h.s of dρ/dt equation) is dq/dt = H (kB T)^4 $\mathcal{N}$ with plus sign for neutrinos and minus sign for electron/photons plasma.

See companion paper for more details in section II.F.
We transform it to a function of temperature or Log[T].

```
𝒩T[Tv_] := 𝒩[me / (kB Tv)];
𝒩lT[lTv_] := 𝒩[me / (kB Exp@lTv)];
```

$$\text{DS2lTQED[lTv\_]} := \frac{2 * 2\,\pi^2}{45}\ \text{DSTQED[Exp@lTv];}$$

$$\text{DST2QED[Tv\_]} := \frac{2 * 2\,\pi^2}{45}\ \text{DSTQED[Tv]}$$

$$\text{DS2lTNoQED[lTv\_]} := \frac{2 * 2\,\pi^2}{45}\ \text{DSTNoQED[Exp@lTv];}$$

$$\text{DST2NoQED[Tv\_]} := \frac{2 * 2\,\pi^2}{45}\ \text{DSTNoQED[Tv]}$$

Visualization of the heating period

```
If[$PaperPlots, LogLogPlot[𝒩T[Tv], {Tv, Ti, 10^9}, Frame → True,
  FrameStyle → Thickness[0.004], FrameLabel → {"T (K)", "𝒩(T)"},
  LabelStyle → {FontSize → 12}, PlotStyle → {Black, Thickness[0.003]}]]
If[$PaperPlots, Export["Plots/PlotCalN.pdf",
   Style[%, Magnification → 1], "PDF"];]
```

We first solve d ln(aT) / d ln(T) so as to get a(T). This is computed using the fact that there is a cooling of electron/photon plasma due to interactions with neutrinos.
We distinguish the case with and without QED corrections.

The equation solved is Eq. 62 of companion paper. SolveaOFTwhenID calls the solver and can be recalled anytime we have varied parameters.

```
SolveaOFTwhenID :=
 (laTCQED = NDSolveValue[{laTCN'[lTv] == (𝒩lT[lTv] - DS2lTQED'[lTv]) / (𝒩lT[lTv] + 3 * DS2lTQED[lTv]),
      laTCN[Log@Tf] == Log[TCMB0 / DSTQED[Tf]^(1/3)]}, {laTCN},
     {lTv, Log@Ti, Log@Tf}, PrecisionGoal → 40, AccuracyGoal → 9][[1]];

   laTCNoQED = NDSolveValue[{laTCNNoQED'[lTv] == (𝒩lT[lTv] - DS2lTNoQED'[lTv]) / (𝒩lT[lTv] + 3 * DS2lTNoQED[lTv]),
      laTCNNoQED[Log@Tf] == Log[TCMB0 / DSTNoQED[Tf]^(1/3)]}, {laTCNNoQED},
     {lTv, Log@Ti, Log@Tf}, PrecisionGoal → 40, AccuracyGoal → 9][[1]];
  );
```

We call SolveaOFTwhenID at first evaluation

```
SolveaOFTwhenID
```

```
aTCQED[Tv_] := Exp[laTCQED[Log@Tv]];
aCQED[Tv_] := aTCQED[Tv] / Tv;
```

```
aCQED[Tf] Tf / TCMB0
aCQED[Ti] Ti / TCMB0
```

```
1.
```

```
0.71536904
```

```
aTCNoQED[Tv_] := Exp[laTCNoQED[Log@Tv]];
aCNoQED[Tv_] := aTCNoQED[Tv] / Tv;
```

```
aCNoQED[Tf] Tf / TCMB0
```

```
1.
```

We invert numerically a(T) to obtain T(a). We also do it for z=AT as a function of a. This is operation is performed when we call the wrapping function InvertaofTwhenID.

```
InvertaofTwhenID :=
  (TofaCQED = Interpolation@Table[{aCQED[T], T}, {T, ListT}];
   TofaCNoQED = Interpolation@Table[{aCNoQED[T], T}, {T, ListT}];

   aTofaCQED = Interpolation@Table[{aCQED[T], aTCQED[T]}, {T, ListT}];
   aTofaCNoQED = Interpolation@Table[{aCNoQED[T], aTCNoQED[T]}, {T, ListT}];
  );
```

We call it at first evaluation

```
InvertaofTwhenID
```

```
aC[T_] := If[$QEDPlasmaCorrections, aCQED[T], aCNoQED[T]]
```

We now solve d (ρ_v)/d ln (a)
(we have to pay attention to units, hence the hbar and clight placed in the system).

In fact we solve for the evolution of $a^4\ \rho_v$ as a function of Log[a]. This is
Eq. 63 of companion paper. The function Solve*ρv*OFawhenID calls the solver.

```
SolveρνOFawhenID :=
  (Timing[a4ρνLogaQED = NDSolveValue[{barρaNQED'[lav] == 1/(hbar^3 clight^5)
            (kB aTofaCQED[Exp@lav])^4 𝒩T[TofaCQED[Exp@lav]],
          barρaNQED[Log@aCQED@Ti] == aBB (kB aTCQED[Ti])^4 7/8 Nneu},
        {barρaNQED}, {lav, Log[aCQED[Ti]], Log[aCQED[Tf]]},
        Method → "StiffnessSwitching", PrecisionGoal → 12][[1]];];

   Timing[a4ρνLogaNoQED = NDSolveValue[{barρaNNoQED'[lav] == 1/(hbar^3 clight^5)
            (kB aTofaCNoQED[Exp@lav])^4 𝒩T[TofaCNoQED[Exp@lav]],
          barρaNNoQED[Log@aCNoQED@Ti] == aBB (kB aTCNoQED[Ti])^4 7/8 Nneu},
        {barρaNNoQED}, {lav, Log[aCNoQED[Ti]], Log[aCNoQED[Tf]]},
        Method → "StiffnessSwitching", PrecisionGoal → 12][[1]];];
  );
```

We call Solve*ρv*OFawhenID at first evaluation

```
SolveρνOFawhenID
```

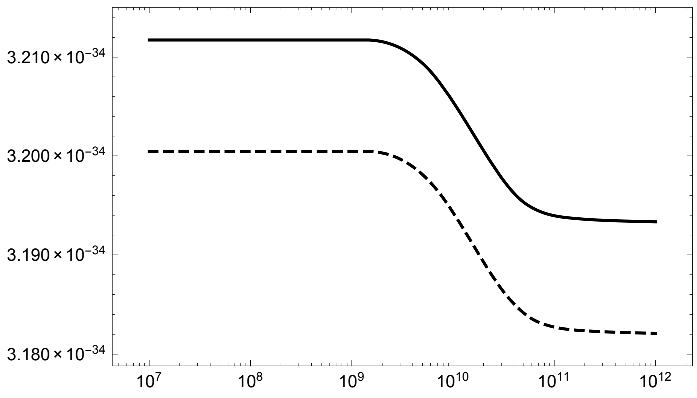
```
a4ρνCQED[av_] := a4ρνLogaQED[Log@av];
ρνCQED[av_] := a4ρνCQED[av] / av^4;

a4ρνCNoQED[av_] := a4ρνLogaNoQED[Log@av];
ρνCNoQED[av_] := a4ρνCNoQED[av] / av^4;
```

```
aTofaCNoQED[a[10^12]] / TCMB0
```

```
0.36690516
```

```
InterpolatingFunction[ Domain: {{1.95×10^-12, 2.73×10^-7}}
                       Output: scalar ][a[1 000 000 000 000]]
```

```
(*ρνC[av_]:=If[$QEDPlasmaCorrections,ρνCQED[av],ρνCNoQED[av]];*)

ρνIncompleteDecoupling[av_] :=
  If[$QEDPlasmaCorrections, ρνCQED[av], ρνCNoQED[av]];

LogLogPlot[{a4ρνCQED[aC[Tv]], a4ρνCNoQED[aC[Tv]]},
 {Tv, Ti, Tf}, Frame → True, PlotStyle → {Black, {Dashed, Black}}]
```



We gather all operations needed to compute incomplete neutrino decoupling. Hence the function RecomputeIncompleteNeutrinoDecoupling can be called whenever we change some parameters so as to modify the incomplete decoupling of neutrinos.

```
RecomputeIncompleteNeutrinoDecoupling := (
  SolveaOFTwhenID;
  InvertaofTwhenID;
  SolveρνOFawhenID;
 )
```

Neutrino Temperature

We extract the corresponding temperature of neutrinos

For decoupling it is easy and deduced from entropy conservation

We choose the correct temperature ratio depending on options chosen and we build the corresponding energy density

```
TνoverTDecoupling[T_] := (FourOverEleven DST[T])^(1/3);
```

$$\text{ρνDecoupling[Tv\_] := aBB } \left(\text{kB TνoverTDecoupling[Tv] Tv}\right)^4 \frac{7}{8}\text{ Nneu;}$$

When considering incomplete decoupling of neutrinos, we find the temperature variation, which is defined as a brightness temperature. See Eq. 64 of companion paper.

$$\text{TνoverTIncompleteDecouplingQED[Tv\_] := } \left(\frac{\text{ρνCQED[aCQED[Tv]]}}{\text{aBB (kB Tv)}^4\ \frac{7}{8}\text{ Nneu}}\right)^{(1/4)};$$

$$\text{TνoverTIncompleteDecouplingNoQED[Tv\_] := } \left(\frac{\text{ρνCNoQED[aCNoQED[Tv]]}}{\text{aBB (kB Tv)}^4\ \frac{7}{8}\text{ Nneu}}\right)^{(1/4)};$$

```
TνoverTIncompleteDecoupling[T_] := If[$QEDPlasmaCorrections,
  TνoverTIncompleteDecouplingQED[T], TνoverTIncompleteDecouplingNoQED[T]]
```

So that now we can decide to use either the full decoupling or the incomplete decoupling neutrino temperature, depending on the options chosen

```
If[$IncompleteNeutrinoDecoupling,
  TνoverT[Tv_] := TνoverTIncompleteDecoupling[Tv];,
  TνoverT[Tv_] := TνoverTDecoupling[Tv];];
```

Effective Description of Neutrinos

We check the translation into equivalent number of neutrinos. See section II.G of companion paper for the definition of Neff.

```
TνoverTDecouplingNoQED[T_] := (FourOverElevenNoQED * DSTNoQED[T])^(1/3);
TνoverTDecouplingQED[T_] := (FourOverElevenQED * DSTQED[T])^(1/3);
```

$$\text{EffectiveNeutrinosQED[Tv\_]} := 3\left(\frac{\text{TνoverTIncompleteDecouplingQED[Tv]}}{\text{TνoverTDecouplingNoQED[Tv]}}\right)^4;$$

$$\text{EffectiveNeutrinosNoQED[Tv\_]} := 3\left(\frac{\text{TνoverTIncompleteDecouplingNoQED[Tv]}}{\text{TνoverTDecouplingNoQED[Tv]}}\right)^4;$$

z (z is defined as z=aT with the convention z=1 deep before BBN).

$$\text{zOFTDecouplingNoQED[T\_]} := \left(\frac{\text{DSTNoQED[Ti]}}{\text{DSTNoQED[T]}}\right)^{(1/3)};$$

$$\text{zOFTDecouplingQED[T\_]} := \left(\frac{\text{DSTQED[Ti]}}{\text{DSTQED[T]}}\right)^{(1/3)};$$

$$\text{zOFTIncompleteDecouplingNoQED[T\_]} := \frac{\text{aCNoQED[T] T}}{\text{aCNoQED[Ti] Ti}};$$

$$\text{zOFTIncompleteDecouplingQED[T\_]} := \frac{\text{aCQED[T] T}}{\text{aCQED[Ti] Ti}};$$

The various z at the end depending on the physics. See table I in companion paper.

```
zendQED = zOFTIncompleteDecouplingQED[Tf]
zendNoQED = zOFTIncompleteDecouplingNoQED[Tf]

zendDecouplingQED = zOFTDecouplingQED[Tf]
zendDecouplingNoQED = zOFTDecouplingNoQED[Tf]
```

$$\left(\frac{11.}{4}\right)^{(1/3)}$$

```
1.3978799

1.3991121

1.3997848

1.4010185

1.4010197
```

Effective number of neutrinos

```
EffectiveNeutrinosQED[10^8]
EffectiveNeutrinosNoQED[10^8]
```

$$3\left(\frac{\left(\frac{11.}{4}\right)^{(1/3)}}{\text{zendDecouplingQED}}\right)^4$$

```
3.0444955

3.0337988

3.0106002
```

z_ν at the end of integration (Only computed in the case of incomplete decoupling because otherwise it remains unity)

```
zνOFTQED[T_] :=
  TνoverTIncompleteDecouplingQED[T] * zOFTIncompleteDecouplingQED[T];
zνOFTNoQED[T_] := TνoverTIncompleteDecouplingNoQED[T] *
   zOFTIncompleteDecouplingNoQED[T];
zνOFT[T_] := If[$QEDPlasmaCorrections, zνOFTQED[T], zνOFTNoQED[T]]
```

```
zνendQED = zνOFTQED[10^8]
zνendNoQED = zνOFTNoQED[10^8]
1.0014382

1.0014394
```

The total energy density increase in neutrinos is

```
zνendQED^4
zνendNoQED^4
1.0057652

1.0057699
```

$N_{\rm eff}$ is by definition $(z_\nu\, z_{\rm dec} / z)^4$. See notation in companion paper.
So we recheck that we find the $N_{\rm eff}$ given in the companion paper in Table I.

$$3*\left(\text{z}\nu\text{endQED} * \frac{\text{zendDecouplingNoQED}}{\text{zendQED}}\right)^4$$

$$3*\left(\text{z}\nu\text{endNoQED} * \frac{\text{zendDecouplingNoQED}}{\text{zendNoQED}}\right)^4$$

$$3*\left(\frac{\text{zendDecouplingNoQED}}{\text{zendDecouplingQED}}\right)^4$$

```
3.0444856

3.033789

3.0105904
```

```
If[$PaperPlots, ListLogLinearPlot[
  Table[{Tv, TνoverTIncompleteDecouplingQED[Tv] / TνoverTDecouplingQED[Tv]},
   {Tv, ListTRange[10^8, 10^12]}], Joined → True,
  PlotStyle → Black, FrameStyle → Thickness[0.004],
  Frame → True, FrameLabel → {"T(K)", "T_ν^ID / T_ν^dec"}]]
If[$PaperPlots, Export["Plots/PlotTnuvariation.pdf",
   Style[%, Magnification → 1], "PDF"];]

If[$PaperPlots,
 ListLogLinearPlot[Table[{Tv, TνoverT[Tv]}, {Tv, ListTRange[10^8, 10^12]}],
  Joined → True, PlotStyle → Black, FrameStyle → Thickness[0.004],
  Frame → True, FrameLabel → {"T(K)", "T_ν^ID / T_ν^dec"}]]
If[$PaperPlots, Export["Plots/PlotTnuOverT.pdf",
   Style[%, Magnification → 1], "PDF"];]

If[$PaperPlots, ListLogLinearPlot[
  {Table[{Tv, EffectiveNeutrinosQED[Tv]}, {Tv, ListTRange[10^8, 10^12]}],
   Table[{Tv, EffectiveNeutrinosNoQED[Tv]}, {Tv, ListTRange[10^8, 10^12]}],
   Table[{Tv, 3 (zOFTDecouplingNoQED[Tv] / zOFTDecouplingQED[Tv])^4},
    {Tv, ListTRange[10^8, 10^12]}] (*,
   Table[{Tv,3(zνOFT[Tv])^4},{Tv,ListTRange[10^8,10^12]}]*)},
  Joined → True, PlotStyle → {Black, {Red, Dashed}, {Blue, Dotted}},
  Frame → True, FrameLabel → {"T(K)", "N_eff"}, LabelStyle → {FontSize → 12},
  FrameStyle → Thickness[0.004], PlotRange → {2.99, 3.05}]]
If[$PaperPlots, Export["Plots/PlotNeff.pdf",
   Style[%, Magnification → 1], "PDF"];]
```

Scale factor determination

If neutrino decoupling is total, then the total entropy is conserved and so it is only a function of temperature (entropy density) and scale factor (volume), since S= s a^3.
So this can be solved without a differential equation in time. If neutrinos would still be interacting, they would exchange energy and this would violate entropy conservation, so entropy density would be sourced and it would require to solve the evolution in time of the entropy (see section above).

This is the function which gives the scale factor as a function of the temperature. If Incomplete Neutrino decoupling is taken into account, we use the result aC previously obtained.

```
If[$IncompleteNeutrinoDecoupling,
  a[T_] := aC[T],
  a[T_] := TCMB0 / (T DST[T]^(1/3))];

(*LogLinearPlot[{a[Tv], TCMB0/Tv},{Tv,10^9,10^10},
 Frame→True,FrameLabel→{"T (K)","a(T)/a0       TCMB0/TCMB"}]*)
```

Just for simplicity we define again the z and z_ν variables which are z = aT and z = aT_ν respectively

```
zT[T_] := (a[T] T) / (a[Ti] Ti);
znuT[T_] := (a[T] TνoverT[T] T) / (a[Ti] TνoverT[Ti] Ti);
```

And again we check how they varied

```
zT[Tf] // NP
znuT[Tf] // NP
```

```
1.3978799
```

```
1.0014382
```

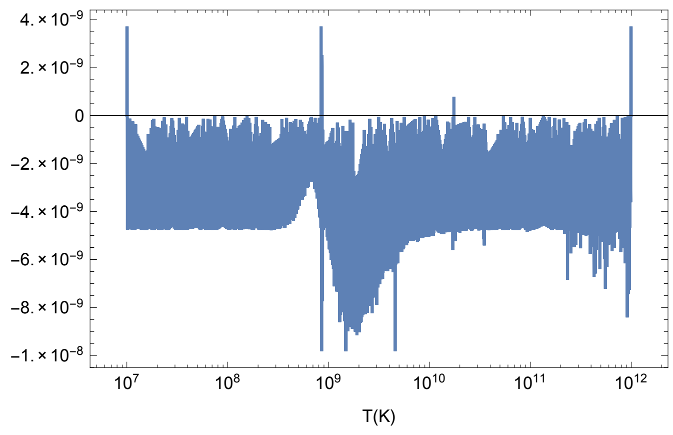
We build a table with (a, T) so as to obtain the inverse T(a) via an interpolation.
In the incomplete neutrino case we have already performed such inversion so there is a slight loss of time and computer energy.

```
InvertaOFT := (Tofa = Interpolation@Table[{a[T], T}, {T, ListT}];);
(*aI=Interpolation@Table[{T,a[T]},{T,ListT}];*)
```

```
InvertaOFT
```

We can check how accurate is this inversion by checking how a(T(a)) is the identity. The error is below 10^{-8} so our numerics are precise enough.

```
LogLinearPlot[{Tofa[a[T]] / T - 1},
 {T, Ti, Tf}, Frame → True, FrameLabel → {"T(K)", ""}]
```



η factor with temperature dependence (because photons density has evolved due to electron positron recombination)

$$\eta\text{factorT[Tv\_]} := \text{nB[a[Tv]]} * \frac{\pi^2}{2\ \text{Zeta[3]}} \left(\frac{\text{hbar clight}}{\text{kB Tv}}\right)^3;$$

Another way to obtain this quantity

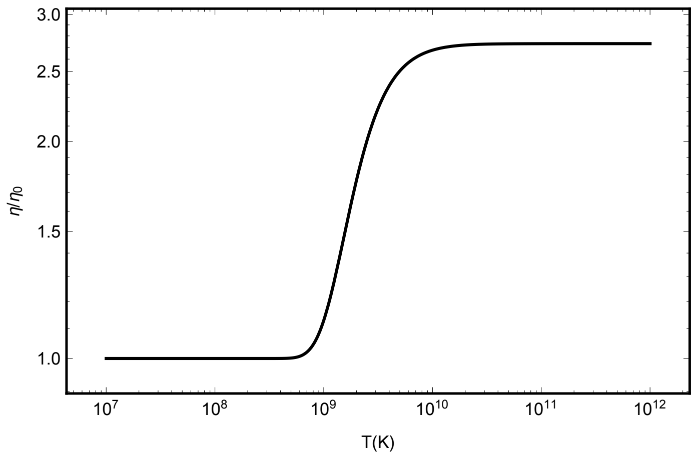
$$\eta\text{factorTBis[Tv\_]} := \eta\text{factor} * (\text{zT[Tf]} / \text{zT[Tv]})^3;$$

```
ηfactorTBis[Ti] // NP
ηfactorT[Ti] // NP
```

1.6638777×10^{-9}

1.6638777×10^{-9}

```
LogLogPlot[ηfactorT[Tv] / ηfactor, {Tv, Ti, Tf}, Frame → True,
 FrameStyle → Thickness[0.004], PlotStyle → Black, FrameLabel → {"T(K)", "η/η0"}]
If[$PaperPlots, Export["Plots/PlotEta.pdf", Style[%, Magnification → 1], "PDF"];]
```



Check of normalization. We chose that today the scale factor is unity.

```
a[Tf] Tf / TCMB0 // NP
1.
```

Weak reactions n+ν ↔ p+e

Distribution functions derivatives

These Fermi-Dirac function and its derivatives are needed further. en stands for energy. x for $1/(k_B\,T)$.

FDeipj means that it is the Fermi Dirac distribution multiplied by Energy^i and derived j times wrt to Energy. See Def B25 in companion paper.

```
FDe2p0[en_, x_] = Simplify[FD[en, x] en^2];
FDe3p0[en_, x_] = Simplify[FD[en, x] en^3];
FDe2p2[en_, x_] = Simplify@D[D[FD[en, x] en^2, en], en];

FDe3p2[en_, x_] = Simplify@D[D[FD[en, x] en^3, en], en];
FDe4p2[en_, x_] = Simplify@D[D[FD[en, x] en^4, en], en];
FDe2p1[en_, x_] = Simplify@D[FD[en, x] en^2, en];
FDe3p1[en_, x_] = Simplify@D[FD[en, x] en^3, en];
FDe4p1[en_, x_] = Simplify@D[FD[en, x] en^4, en];

FDve2p0[en_, ϕ_, x_] = Simplify[FDν[en, ϕ, x] en^2];
FDve3p0[en_, ϕ_, x_] = Simplify[FDν[en, ϕ, x] en^3];
FDve2p2[en_, ϕ_, x_] = Simplify@D[D[FDν[en, ϕ, x] en^2, en], en];

FDve3p2[en_, ϕ_, x_] = Simplify@D[D[FDν[en, ϕ, x] en^3, en], en];
FDve4p2[en_, ϕ_, x_] = Simplify@D[D[FDν[en, ϕ, x] en^4, en], en];
FDve2p1[en_, ϕ_, x_] = Simplify@D[FDν[en, ϕ, x] en^2, en];
FDve3p1[en_, ϕ_, x_] = Simplify@D[FDν[en, ϕ, x] en^3, en];
FDve4p1[en_, ϕ_, x_] = Simplify@D[FDν[en, ϕ, x] en^4, en];
```

λ_0

Born approximation λ_0

See e.g. Eq. 13 of [Lopez & Turner 1998]. Or Eq. 92 of companion paper.

The constant λ_0 is a proxy for $\tau_n^{-1} = \lambda_0 * \left[\cos^2(\theta_c)\, G_F^2 \left(C_V^2 + 3\, C_A^2\right)/\left(2\,\pi^3\right)\right]$

```
λBORN = With[{q = Q / me}, NIntegrate[en (en - q)^2 Sqrt[en^2 - 1] , {en, 1, q}]]
```

```
1.6360874
```

Corrections to λ_0

Radiative corrections to λ_0

1) If $ResummedLogsRadiativeCorrections=False, we use Eq. 7 of [Czarnecki et al. 2004] which is B30 of companion paper.
 Combined with Eq 20b of [Sirlin 1967] for Sirlin’s universal function, that is Eq. B32 in companion paper.

```
AgCzarnecki = -0.34;
CCzarnecki = 0.891;
mA = 1.2 GeV;
ConstantSirlin  = 4 Log[mZ / mp] + Log[mp / mA] + 2 CCzarnecki + AgCzarnecki;
```

For information, this quantity is evaluated to 40 in [Dicus et al.]

```
ConstantSirlin + 3 Log[mp / (me)] - 3 / 4
```

```
41.298783
```

```
Rd[x_] := ArcTanh[x] / x;
```

NB : ArcTanh[x] = 1/2 Log[$\frac{(1+x)}{(1-x)}$]

```
Lfun[x_] = Integrate[Log[1 - t] / t, {t, 0, x}, Assumptions → x < 1 && x > 0]
  (* Lfun is called the Spence function *)
```

```
-PolyLog[2, x]
```

It can be computed from a Taylor expansion (this used to be done in the past), but a direct evaluation by Mathematica is much better

```
LfunSeries[b_] = Normal@Series[-1 / 4 * (1 + b)^6 * 4/b Lfun[2 b/(1 + b)], {b, 0, 12(*22*)}]
```

$$2 + 11\,b + \frac{224\,b^2}{9} + \frac{89\,b^3}{3} + \frac{1496\,b^4}{75} + \frac{596\,b^5}{75} + \frac{128\,b^6}{49} + \frac{68\,b^7}{49} + \frac{3704\,b^8}{3969} + \frac{21\,988\,b^9}{33\,075} + \frac{2\,016\,208\,b^{10}}{4\,002\,075} + \frac{946\,628\,b^{11}}{2\,401\,245} + \frac{43\,024\,472\,b^{12}}{135\,270\,135}$$

Sirlin universal function (Eq 20b of [Sirlin 1967]) on which we add the constants of Eq. 7 of [Czarnecki et al. 2004]) is obtained as (this is B32 of companion paper)

```
$SeriesSpenceFunction = False;
```

```
SirlinGFunction[b_, y_, en_] := (3 Log[mp / (me)] - 3 / 4 +
    4 (Rd[b] - 1) (y/(3 en) - 3 / 2 + Log[2 y]) + Rd[b] (2 (1 + b^2) + y^2/(6 en^2) - 4 b Rd[b]) +
    If[$SeriesSpenceFunction, -4 / (1 + b) ^6 * LfunSeries[b], 4/b Lfun[2 b/(1 + b)]]);
Cd[b_, y_, en_] := (ConstantSirlin + SirlinGFunction[b, y, en]);
```

NB : In [Dicus et al. 1982] Eq 2.14 (or in [Lopez&Turner 1998] Eq. 17) they use the approximation
$\frac{4}{b}\,\mathrm{Lfun}\left[\frac{2\,b}{1+b}\right] = -4\left(2 + 11\,b + \frac{224}{9}\,b^2 + \frac{89}{3}\,b^3 + \frac{1496}{75}\,b^4 + \frac{596}{75}\,b^5 + \frac{128}{49}\,b^6\right) \Big/ (1+b)^6$
However there is a typo and some incorrect coefficients. In [Kernan] there are approximate coefficients which are nearly correct.

2) if $ResummedLogsRadiativeCorrections = True, we use a resummed version of all the Log[m_Z/m_P]. It consists in using Eq. 15 of [Czarnecki 2004] or B35 in companion paper.
See also [Esposito et al. 1998])

We build the multiplicative factor for inner radiative corrections (Eq. 15 of [Czarnecki et al. 2004]), that is B35 of companion paper.

```
LFactor = 1.02094;
SFactor = 1.02248;
δfactor = -0.00043 * 2 Pi / αFS;
NLL = -0.0001;
```

```
RadiativeCorrectionsResummed[b_, y_, en_] :=
  (1 + αFS/(2 π) (SirlinGFunction[b, y, en] - 3 Log[mp/(2 Q)])) *
   (LFactor + αFS/π CCzarnecki + αFS/(2 π) δfactor) *
   (SFactor + 1/(134 * 2 * Pi) * (Log[mp/mA] + AgCzarnecki) + NLL);
```

Finally we define a function which selects either choice depending on option

```
RadiativeCorrections[b_, y_, en_] := If[$ResummedLogsRadiativeCorrections,
   RadiativeCorrectionsResummed[b, y, en], (1 + αFS/(2 π) Cd[b, y, en])]
```

- Fermi function for Coulomb interactions of electron and proton (if on the same side of the interaction).
  Either the relativistic or the non-relativistic depending on option $RelativisticFermiFunction. That is either Eq. 99 or Eq. 100 of companion paper depending on option chosen.

```
FermiRelat[b_] := With[{γ = Sqrt[1 - αFS^2] - 1, λCompton = 1 / (me / (hbar clight))},
   (1 + γ / 2) * 4 (2 radiusproton b/λCompton)^(2 γ) *
    1/Gamma[3 + 2 γ]^2 Exp[π αFS/b] * 1/(1 - b^2)^γ Abs[Gamma[1 + γ + I αFS/b]]^2];
```

```
FermiNonRelat[b_] := (2 π αFS / b)/(1 - Exp[-2 π αFS / b]);
```

```
If[$RelativisticFermiFunction,

 Fermi[b_] := FermiRelat[b];
 bFermi[b_] := b Fermi[b];,

 Fermi[b_] := FermiNonRelat[b];
 bFermi[b_] := 2 π αFS / (1 - Exp[-2 π αFS / b]);]

If[$PaperPlots,
 DFermi = Plot[{100 * (FermiRelat[b] / FermiNonRelat[b] - 1)}, {b, 0, 1},
   Frame → True, FrameStyle → Thickness[0.004], PlotStyle → {Black}, FrameLabel →
    {"x", "100 x (F^rel (x) / F^non rel (x)  -1) "}, LabelStyle → {FontSize → 12}]]
If[$PaperPlots, Export["Plots/PlotDeltaFermi.pdf",
    Style[DFermi, Magnification → 1], "PDF"];];
```

λ_0 when taking only Fermi function and not radiative corrections

```
λFermiOnly = With[{q = Q / me , b = Sqrt[en^2 - 1] / en, y = Q / me - en},
  NIntegrate[en (en - q)^2 en * bFermi[b], {en, 1.0000001, q}]]
1.6923295
```

This already a 3.44 % correction

```
λFermiOnly / (λBORN)
1.034376
```

Adding the inner radiative corrections, the constant involved in the decay of the neutron is

```
λRad = With[{q = Q / me , b = Sqrt[en^2 - 1] / en, y = Q / me - en},
  NIntegrate[
   en (en - q)^2 en (RadiativeCorrections[b, y, en]) * bFermi[b], {en, 1.0000001, q}]]
1.758373
```

Note that in companion paper, Eq. 106, the value 1.75767 is quoted when it should be the value found here 1.758373.

This adds another 3.90 % correction as found in Eq 16 of [Czarnecki et al. 2004].

```
λRad / λFermiOnly
1.0390252
```

Finite mass corrections to λ_0

See companion papers for expression of finite nucleon mass corrections.
We take the general form of finite mass correction and remove all terms which involve temperature.
So we consider Eq. 118 of companion paper.

```
IntegrateCorrectionNeutronDecay[fun_] :=
  NIntegrate[fun[pe],
   {pe, 0.0000001, Sqrt[(Q / me)^2 - 1]}, WorkingPrecision → MachinePrecision];
```

```
χFMNeutronDecay[en_, pe_] :=
  With[{M = mp / me, enu = en - Q / me,
```
$$f1 = \frac{(1 + gA)^2 + 4\, fWM\, gA}{(1 + 3\, gA^2)},\quad f2 = \frac{(1 - gA)^2 - 4\, fWM\, gA}{(1 + 3\, gA^2)},\quad f3 = \frac{(gA^2 - 1)}{(1 + 3\, gA^2)}\},$$

$$f1 * enu^2 \left(\frac{pe^2}{M * en}\right)$$
$$+ f2 * enu\,\hat{}\,3 \left(-\frac{1}{M}\right)$$
$$+ (f1 + f2 + f3)\ \frac{1}{2\,M} * (4\, enu^3 + 2\, enu\, pe^2)$$
$$+ f3 * \frac{1}{3\,M}\ 3\, enu^2\ \frac{pe^2}{(en)}$$
```
  ];
```

Without coupling to radiative corrections we get

```
IλFMBasic[pe_] := With[{en = Sqrt[pe^2 + 1]}, With[{b = pe / en}, pe² *
      (χFMNeutronDecay[en, pe])
   ]];
```

```
IntegrateCorrectionNeutronDecay[IλFMBasic] / λBORN
```

```
-0.0020676269
```

However we couple to radiative corrections if $CoupledFMandRC = True

```
IλFM[pe_] := With[{en = √(pe² + 1)}, With[{b = pe / en}, pe² *
      (χFMNeutronDecay[en, pe] * If[$RadiativeCorrections && $CoupledFMandRC,
         (RadiativeCorrections[b, Abs[en - Q / me ], en]) Fermi[b], 1])
   ]];
```

```
λFM = If[$FiniteNucleonMass, IntegrateCorrectionNeutronDecay[IλFM], 0]
```

```
-0.0036333381
```

When taking only into account Fermi function and finite mass effect on should get 1.6887. See Eq. 6 of [Czarnecki et al. 2004]

```
λFermiOnly + λFM
1.6887 / % // NP
```

```
1.6886962
```

```
1.0000023
```

We compare the correction to the Born result

```
CorrectionRate = λFM / λBORN
```

```
-0.0022207482
```

If Coulomb and Radiative corrections are set to False (0) this is exactly (-0.00206) what is found by [Lopez & Turner 1997] (see three lines after Eq. 23, in the text).
See also the value -0.00201 found in Eq. 20 of [Seckel 1993].

We also reproduce for comparison the exact finite mass correction to the neutron decay rate as computed by Eq. 19 of [Lopez & Turner 1998], which uses two-dimensional integrals. Again we find the -0.00206 correction so our finite mass method based one-dimensional integrals is reliable.

```
λexact = With[{pe = Sqrt[en² - 1]},
    With[{enu = ((mn ² - mp²) / me² + 1 - 2 mn / me en) / (2 (mn / me - en + pe Cnu))},
     With[{Ep = mn / me - enu - en, f1 = ((1 + gA) ² + 4 fWM gA) / (1 + 3 gA²),
       f2 = ((1 - gA) ² - 4 fWM gA) / (1 + 3 gA²), f3 = (gA² - 1) / (1 + 3 gA²)},
      With[{J = 1 + (enu + pe Cnu) / (Ep),
        M2 = f1 mn / me enu (Ep en - (-pe^2 - pe enu Cnu)) + f2 mn / me en
            (Ep enu - (-pe enu Cnu - enu^2)) + f3 mn / me Ep (enu en - Cnu enu pe) },
       NIntegrate[1 / 2 * M2 pe enu / (mn / me Ep J), {en, 1,
          (Q - (Q² - me ²) / (2 mn)) / me }, {Cnu, -1, 1}, PrecisionGoal → 10
         (*,Method→{Automatic,"SymbolicProcessing"→0}*)]
      ]]]];
```

```
(λexact - λBORN) / λBORN // NP
```

```
-0.0020636766
```

Total correction

The total correction for the neutron decay constant λ_0 is the sum of the Radiative corrected one plus the finite mass effects

We recall the result from radiative corrections (again we stress that Eq. 106 in companion paper should report that value).

```
λRad
```

```
1.758373
```

The result from mass corrections (see text before Eq. 120 in companion paper.)

```
λFM
```

```
-0.0036333381
```

And we sum them to get Eq. 120 of companion paper.

```
λRadandFM = λRad + λFM
```

```
1.7547397
```

This gives a total correction which is

```
λRadandFM / λBORN
```

```
1.072522
```

If we compare with [Cooper et al 2010] (Phys. Rev. C 81, 035503 (2010)). They find that this parameter should be

```
λCooper = 1.03887 * 1.6887
λCzarnecki = 1.0390 * 1.6887
  (* = (1+RC)*f with f=1.6887 and RC = 0.0390(8) [Czarnecki et al. 2004]] *)
```

```
1.7543398
```

```
1.7545593
```

We compute the ratio to see how far we are

```
λRadandFM / λCooper
λRadandFM / λCzarnecki
```

```
1.000228

1.0001028
```

We check that it is in agreement with the theoretical formula. This expression below should reproduce the neutron decay time. It is amazingly close !

```
MixingCosAngle = 0.97420;(* (+-16) Value taken from CKM particle data group
 2017. More precisely from the review on Vud Vus of the PDG 2017.*)
MyK = MixingCosAngle^2 (GF)^2 (1 + 3 (gA)^2) / (2 π^3) * (me )^5 /hbar
1 / MyK / λRadandFM
1 / MyK / λCzarnecki
1 / MyK / λCooper
```

```
0.0006454297

882.95456

883.04534

883.15584
```

We recall the neutron lifetime.

```
τneutron
```

```
879.5
```

We see that we are extremely close but still not quite satisfactory.
Actually this would work much better if we used the post-2000 average for g_A which is 1.2755(11) (Marciano, private communication).

```
gAbetter = 1.2755;
MyKbetter = MixingCosAngle^2 (GF)^2 (1 + 3 (gAbetter)^2) / (2 π^3) * (me )^5 /hbar

1 / MyKbetter / λRadandFM
1 / MyKbetter / λCzarnecki
1 / MyKbetter / λCooper
```

```
0.00064812537

879.28219

879.37259

879.48264
```

Let us also compare with Eq. 17 of [Czarnecki et al. 2004] to check how close we are in including all corrections. See text after Eq. 120 in companion paper as well.

```
ConstantVud = hbar * (2 π^3) / (me)^5 / (GF)^2 / λRadandFM
ConstantVud2 = hbar * (2 π^3) / (me )^5 / (GF)^2 / λCzarnecki
ConstantVud3 = hbar * (2 π^3) / (me )^5 / (GF)^2 / λCooper
```

```
4907.4243

4907.9288

4908.543
```

Born rates

We compute in this section the Born rates for n -> p and p -> n.

Tools to perform integration on electron momentum

```
pemin = 0.00001;
pemiddle[x_] := Sqrt[Max[pemin^2, (Q / me )^2 - 1 - If[$QEDMassShift, dme2x[x], 0]]];
pemaxC[x_] := Max[7, 30 / x];
pemax[x_] := Max[7, 30 / x];
```

$TnuEqualT is some cheat to check detailed balance. Should be False. When it is True neutrinos have always the plasma temperature.

```
$TnuEqualT = False;

IntegratedpNpoints[fun_, sgnq_, Tv_, Npoints_] :=
 With[{x = me/(kB Tv), znu = me/(kB Tv TvoverT[Tv])},
  If[$FastPENRatesIntegrals,
   IntegrateFunction[
    fun[#, x, If[$TnuEqualT, x, znu], sgnq] &, pemin, pemaxC[x], Npoints],
   NIntegrate[fun[pe, x, If[$TnuEqualT, x, znu], sgnq],
    {pe, pemin, pemiddle[x], pemax[x]}]
  ]]

IntegrateRatedp[fun_, sgnq_, Tv_] :=
  IntegratedpNpoints[fun, sgnq, Tv, $PENRatesIntegralsPoints];
```

Energy as a function of momentum, taking into account QED mass shifts.
This is only used if the option is set to True but it is useless because it is taken into account in finite temperature corrections

```
enOFpe[pe_, x_] := Sqrt[pe^2 + 1 + If[$QEDMassShift, dme2x[x], 0]];
```

Functions to build integrands in electron momentum. Without and with CCR corrections. This is then passed to the integrating routine IntegrateRatedp.

```
IPENdpFromχNoCCR[en_, pe_, x_, znu_, sgnq_, χfunction_] :=
  With[{q = Q / me }, With[{b = pe / en},
    pe^2 * (χfunction[en, pe, x, znu, sgnq] + χfunction[-en, pe, x, znu, sgnq])
   ]];

Fermi[sgnq_, signE_, b_?NumericQ] := If[sgnq signE > 0, Fermi[b], 1];
SetAttributes[Fermi, Listable];

IPENdpFromχCCR[en_, pe_, x_, znu_, sgnq_, χfunction_] :=
  With[{q = Q / me , b = pe / en},
   pe^2 * (χfunction[en, pe, x, znu, sgnq]
        (RadiativeCorrections[b, Abs[sgnq Q / me - en], en])
        Fermi[sgnq, 1, b] + χfunction[-en, pe, x, znu, sgnq]
        (RadiativeCorrections[b, Abs[sgnq Q / me + en], en]) Fermi[sgnq, -1, b])
  ];
```

Eq 2.29 in [Brown & Sawyer] for the Born approximation. It is Eq. 79 of companion paper

```
χ[en_, pe_, x_, znu_, sgnq_] :=
  With[{q = Q / me }, FDν[en - sgnq q, sgnq ξν, znu] FD[-en, x] (en - sgnq q)^2];
```

Eq 2.30 in [Brown & Sawyer], that is the integration of Eq. 78 in companion paper

```
IPENdp[pe_, x_, znu_, sgnq_] :=
 IPENdpFromχNoCCR[enOFpe[pe, x], pe, x, If[$TnuEqualT, x, znu], sgnq, χ]
```

We also define a function which is incorrect in which we force the neutrinos to have the temperature of photons. This is to check detailed balance. Indeed, detailed balance is satisfied only if all species have the same temperature.

```
IPENdpCheatNeutrinoTemperature[pe_, x_, znu_, sgnq_] :=
 IPENdpFromχNoCCR[Sqrt[pe^2 + 1], pe, x, x, sgnq, χ]
```

Born rates are given by Eq 2.30 in [Brown & Sawyer].

```
λnTOpBORN[Tv_] := IntegrateRatedp[IPENdp, 1, Tv];
λpTOnBORN[Tv_] := IntegrateRatedp[IPENdp, -1, Tv];
```

```
λnTOpBORNCheatNeutrino[10^7]
```

```
λnTOpBORNCheatNeutrino[10 000 000]
```

Born rates where the neutrino temperature is cheated to be equal to photons temperature are then given by

```
λnTOpBORNCheatNeutrino[Tv_] :=
  IntegrateRatedp[IPENdpCheatNeutrinoTemperature, 1, Tv];
λpTOnBORNCheatNeutrino[Tv_] :=
  IntegrateRatedp[IPENdpCheatNeutrinoTemperature, -1, Tv];
```

Finite mass effects

For finite mass effects, we use a (Fokker-Planck) expansion which leads to one-dimensional integrals.
We include also the weak-magnetism which is important indeed. This is B23 of companion paper.

```
χFM[en_, pe_, x_, znu_, sgnq_] :=
  With[{ϕ = sgnq ξν, q = Q / me , M = (mp + mn  - sgnq Q)/(2 me ),
    Mp = mp / me , Mn = mn  / me , enu = en - sgnq Q / me,
    f1 = ((1 + sgnq gA)^2 + 4 fWM sgnq gA)/(1 + 3 gA^2),
    f2 = ((1 - sgnq gA)^2 - 4 fWM sgnq gA)/(1 + 3 gA^2), f3 = (gA^2 - 1)/(1 + 3 gA^2)},
   f1 * FDνe2p0[enu, ϕ, znu] FD[-en, x] (pe^2/(M * en))
    + f2 * FDνe3p0[enu, ϕ, znu] FD[-en, x] (-1/M)
    + (f1 + f2 + f3) 1/(2 x M) *
      (FDνe4p2[enu, ϕ, znu] FD[-en, x] + FDνe2p2[enu, ϕ, znu] FD[-en, x] pe^2)
    + (f1 + f2 + f3) 1/(2 M) * (FDνe4p1[enu, ϕ, znu] FD[-en, x] +
       FDνe2p1[enu, ϕ, znu] FD[-en, x] pe^2)
    - (f1 + f2) 1/(x M) * (FDνe3p1[enu, ϕ, znu] FD[-en, x] +
       FDνe2p1[enu, ϕ, znu] FD[-en, x] pe^2 / (-en))
    - f3 * 3/(x M) FDνe2p0[enu, ϕ, znu] FD[-en, x] (* This term
     seems to give very small corrections *)
    + f3 * 1/(3 M) FDνe3p1[enu, ϕ, znu] FD[-en, x] pe^2/(en)
    + f3 * 2/(2 x * 3 M) FDνe3p2[enu, ϕ, znu] FD[-en, x] pe^2/(en)
    - (f1 + f2 + f3) * 3/(2 x) * (1 - (Mn/Mp)^sgnq) * (FDνe2p1[enu, ϕ, znu] FD[-en, x])
  ];
```

Integrand for finite mass corrections. We couple it to radiative corrections.

```
IPENdpFMNoCCR[pe_, x_, znu_, sgnq_] :=
 IPENdpFromχNoCCR[enOFpe[pe, x], pe, x, If[$TnuEqualT, x, znu], sgnq, χFM]
IPENdpFMCCR[pe_, x_, znu_, sgnq_] :=
 IPENdpFromχCCR[enOFpe[pe, x], pe, x, If[$TnuEqualT, x, znu], sgnq, χFM]
```

Integrand for finite mass corrections when neutrinos are forced to have the same temperature as photons.

```
IPENdpFMCheatNeutrinoTemperature[pe_, x_, znu_, sgnq_] :=
 IPENdpFromχNoCCR[enOFpe[pe, x], pe, x, x, sgnq, χFM]
```

```
Clear[λnTOpFMCCR, λpTOnFMCCR, λnTOpFMNoCCR,
 λpTOnFMNoCCR, λnTOpCheatNeutrinoFM, λpTOnCheatNeutrinoFM]
```

Finite mass corrections using the λ_0 which is corrected with radiative corrections. The computation below implements Eqs. 114 of companion paper.

```
λnTOpFMCCR[Tv_] := IntegrateRatedp[IPENdpFMCCR, 1, Tv];
λpTOnFMCCR[Tv_] := IntegrateRatedp[IPENdpFMCCR, -1, Tv];
```

Finite mass corrections using the λ_0 which is computed with the Born infinite mass expression (we do not advise to use it).

```
λnTOpFMNoCCR[Tv_] := IntegrateRatedp[IPENdpFMNoCCR, 1, Tv];
λpTOnFMNoCCR[Tv_] := IntegrateRatedp[IPENdpFMNoCCR, -1, Tv];
```

Finite mass corrections cheating with the neutrino
  temperature and setting it equal to photons temperature

```
λnTOpCheatNeutrinoFM[Tv_] :=
  IntegrateRatedp[IPENdpFMCheatNeutrinoTemperature, 1, Tv];
λpTOnCheatNeutrinoFM[Tv_] := IntegrateRatedp[
   IPENdpFMCheatNeutrinoTemperature, -1, Tv];
```

Examples of neutron decay corrections for some low temperatures:

```
λnTOpFMCCR[10^8] / λnTOpBORN[10^8] // Timing
λnTOpFMCCR[10^7.5] / λnTOpBORN[10^7.5] // Timing
```

```
{0.081506, -0.0022357456}

{0.253562, -0.0022260612}
```

```
λnTOpFMCCR[.8 × 10^10] / λnTOpBORN[.8 × 10^10] // Timing
λnTOpFMCCR[10^10] / λnTOpBORN[10^10] // Timing
λnTOpFMCCR[10^10.5] / λnTOpBORN[10^10.5] // Timing
(*λpTOnFMCCR[.8 10^10]/λpTOnBORN[.8 10^10]//Timing
    λpTOnFMCCR[10^10]/λpTOnBORN[10^10]//Timing
   λpTOnFMCCR[10^10.5]/λpTOnBORN[10^10.5]//Timing*)
```

```
{0.032203, -0.0091500489}

{0.030255, -0.010911997}

{0.032497, -0.030525244}
```

Plots of the finite mass corrections

We plot the amplitude of the finite mass corrections. This is essentially similar to Fig 8.1 of [Kernan] but the result is a little different since here we have put all the finite nucleon mass effects, and [Kernan] did not.

```
If[$PaperPlots,
 TabδλFMnTOp = Table[
   {T, (λnTOpFMCCR[T] * (τneutron λRadandFM)^-1) / (λnTOpBORN[T] * (τneutron λBORN)^-1)}, {T, ListTRange[10^9, 10^11]}];
 TabδλFMpTOn = Table[{T, (λpTOnFMCCR[T] * (τneutron λRadandFM)^-1) / (λpTOnBORN[T] * (τneutron λBORN)^-1)},
   {T, ListTRange[10^9, 10^11]}];

TFreeze = 0.8 MeV / kB;

PlotdeltaGammaFM =
  Show[ListLogLinearPlot[{TabδλFMnTOp, TabδλFMpTOn}, Frame → True,
    FrameStyle → Thickness[0.004], Joined → True, PlotRange → {-10^-1, 10^-2},
    FrameLabel → {"T  (K)", "δΓ/Γ"}, LabelStyle → {FontSize → 12}, PlotStyle →
     {{Black, Thickness[0.003]}, {Black, Dashing[{0.01}], Thickness[0.003]}},
    GridLines → {{{TFreeze, {Gray, Thickness[0.005]}}}, {}},
    FrameTicks → MyFrameTicksLog],
   Graphics[{Rotate[Text[Style["0.8  MeV", FontSize → 10, Black],
       {Log@TFreeze - .1, -0.06}], 90 Degree]}]]
]
If[$PaperPlots, Export["Plots/PlotdeltaGammaFM.pdf",
    Style[PlotdeltaGammaFM, Magnification → 1], "PDF"];];

If[$PaperPlots,
 TabδλFMnTOp2 = Table[{T, Identity[(λnTOpFMCCR[T] * (τneutron λRadandFM)^-1) /
       (λnTOpBORN[T] * (τneutron λBORN)^-1)]}, {T, ListTRange[5 × 10^8, 10^10.5]}];
 TabδλFMpTOn2 = Table[{T, Identity[(λpTOnFMCCR[T] * (τneutron λRadandFM)^-1) /
       (λpTOnBORN[T] * (τneutron λBORN)^-1)]}, {T, ListTRange[5 × 10^8, 10^10.5]}];

 PlotdeltaGammaFM2 = Show[ListPlot[{TabδλFMnTOp2, TabδλFMpTOn2}, Frame → True,
    FrameStyle → Thickness[0.004], Joined → True, PlotRange → {-3 * 10^-2, 10^-2},
    FrameLabel → {"T  (10^10K)", "δΓ/Γ"}, LabelStyle → {FontSize → 12}, PlotStyle →
     {{Red, Thickness[0.003]}, {Blue, Dashing[{0.01}], Thickness[0.003]}},
    FrameTicks → {{Automatic, Automatic}, {{{0.5 * 10^10, ".5"}, {10^10, "1"},
        {1.5 * 10^10, "1.5"}, {2 × 10^10, "2"}, {2.5 × 10^10, "2.5"}}, Automatic}},
    GridLines → {{{TFreeze, {Gray, Thickness[0.005]}}}, {}}],
   Graphics[{Rotate[Text[Style["0.8  MeV", FontSize → 10, Black],
       {TFreeze 0.93, -0.02}], 90 Degree]}]]
]

If[$PaperPlots, Export["Plots/PlotdeltaGammaFM2.pdf",
    Style[PlotdeltaGammaFM2, Magnification → 1], "PDF"];];
```

Checking detailed balance

Born approximation detailed balance

```
DetailedBalanceRatio0[T_] := Exp[- Q/(kB T) - ξν];
```

At Born order, detailed balance is of course satisfied by construction.

```
DetailedBalance0[T_] :=
  (λnTOpBORN[T]) / (λpTOnBORN[T]) * DetailedBalanceRatio0[T];
DetailedBalanceCheatNeutrino0[T_] := (λnTOpBORNCheatNeutrino[T]) /
    (λpTOnBORNCheatNeutrino[T]) * DetailedBalanceRatio0[T];
```

If we enforce $T_\nu = T$ the precision is insane. It is numerical precision because detailed balance is rooted in our method.

```
If[$PaperPlots,
 ListLogLinearPlot[{Table[
    {T, DetailedBalanceCheatNeutrino0[T] - 1}, {T, ListTRange[10^9, 10^12]}]},
  Joined → True, PlotStyle → {Black, {Black, Dashed}}, Frame → True]
]
```

Including corrections in Q / mp,
detailed balance is given by (where α should be set to 0).

```
DetailedBalanceRatio[T_] := Exp[- Q/(kB T) - ξν] (1 + (1 + α) Q/mp)^(3/2);
```

[Lopez&Turner 1997] define some quantity to parameterize how good detailed balance is obtained, this is the parameter α.
α should be 0 so we use it to estimate the failure of detailed balance

We first solve for α correctly, and then we do it when forcing the temperature of neutrinos to be the one of photons, in which case detailed balance must be true. Indeed if we do not force the temperature of neutrinos to be the one of photons, then it is only true before electrons/positrons annihilate, and at low temperature detailed balance is not satisfied.

```
Clear[αLopez, αLopezCheatNeutrino]
αLopez[T_] := αLopez[T] =
  α /. Solve[Normal@Series[(λnTOpBORN[T] + λnTOpFMNoCCR[T]) / (λpTOnBORN[T] +
           λpTOnFMNoCCR[T]) * DetailedBalanceRatio[T], {α, 0, 1}] == 1, α][[1]]

αLopezCheatNeutrino[T_] := αLopezCheatNeutrino[T] = α /.
   Solve[Normal@Series[(λnTOpBORNCheatNeutrino[T] + λnTOpCheatNeutrinoFM[T]) /
        (λpTOnBORNCheatNeutrino[T] + λpTOnCheatNeutrinoFM[T]) *
       DetailedBalanceRatio[T], {α, 0, 1}] == 1, α][[1]]
```

We plot something similar to Fig. 4 of [Lopez&Turner 1997]. The remaining error should come from second order corrections. For instance at low temperature it remains an effect of order $(Q/mn)^2$ so α is expected to differ from a null value by something of order Q/mn =0.002. We also see that below 5x10^{10} K, detailed balance does not work because neutrinos no longer have the temperature of photons but it works if we force neutrinos to be at the temperature of photons.

```
If[$PaperPlots, PlotDetailedBalance = Show[
   ListLogLinearPlot[{Table[{T, αLopez[T]}, {T, ListTRange[10^8.5, 10^11.5]}],
     Table[{T, αLopezCheatNeutrino[T]}, {T, ListTRange[10^8.5, 10^11.5]}]},
    Frame → True, FrameTicks → MyFrameTicksLog,
    GridLines → {{{TFreeze, {Gray, Thickness[0.005]}}}, {}},
    Joined → True, FrameLabel → {"T (K)", "α"},
    FrameStyle -> Thickness[0.003], LabelStyle → {FontSize → 12},
    PlotRange → {-1, 1}, PlotStyle → {{Red}, {Black, Dashed, Blue}}],
   Graphics[{Rotate[Text[Style["0.8 MeV", FontSize → 10, Black],
       {Log@TFreeze - 0.2, 0.5}], 90 Degree]}]]
]

If[$PaperPlots, Export["Plots/PlotDetailedBalance.pdf",
    Style[PlotDetailedBalance, Magnification → 1], "PDF"];];
```

Radiative Corrections (T=0)

The CCR corrected rates are described in details in the companion paper.

When electron and protons are on the same side we use the Fermi function to apply the Coulomb corrections.

For all processes the radiative correction are a factor $(1+\frac{\alpha}{2\pi}C)$, (similarly to Eq. 18 of [Lopez&Turner 1998] or Eq. 2.13 of [Dicus .et. al]) as described in companion paper.

```
IPENdpCCR[pe_, x_, znu_, sgnq_] :=
 IPENdpFromχCCR[enOFpe[pe, x], pe, x, If[$TnuEqualT, x, znu], sgnq, χ]

λnTOpCCR[Tv_] := IntegrateRatedp[IPENdpCCR, 1, Tv];
λpTOnCCR[Tv_] := IntegrateRatedp[IPENdpCCR, -1, Tv];
```

Plots of the radiative corrections

```
If[$PaperPlots, TabδλnTOp = Table[{T, (λnTOpCCR[T] * (τneutron λRadandFM)^-1)/(λnTOpBORN[T] (τneutron λBORN)^-1) - 1},
   {T, ListTRange[10^8.5, 10^10.5]}];
 TabδλpTOn = Table[{T, (λpTOnCCR[T] * (τneutron λRadandFM)^-1)/(λpTOnBORN[T] (τneutron λBORN)^-1) - 1},
   {T, ListTRange[10^8.5, 10^10.5]}];

 PlotdeltaGammaCCR = Show[ListLogLinearPlot[{TabδλnTOp, TabδλpTOn}, Frame → True,
    FrameStyle → Thickness[0.004], Joined → True, FrameLabel → {"T  (K)", "δΓ/Γ"},
    LabelStyle → {FontSize → 12}, PlotStyle → {{Red}, {Blue, Dashing[{0.01}]}},
    GridLines → {{{TFreeze, {Gray, Thickness[0.004]}}}, {}},
    FrameTicks → {{Automatic, Automatic}, {{{Log[10^8.5], "10^8.5"},
        {Log[10^9], "10^9"}, {Log[10^9.5], "10^9.5"}, {Log[10^10], "10^10"},
        {Log[10^10.5], "10^10.5"}, {Log[10^11], "10^11"}}, Automatic}}],
   Graphics[{Rotate[Text[Style["0.8 MeV", FontSize → 10, Black],
       {Log@TFreeze - 0.1, -0.01}], 90 Degree]}]]
]

If[$PaperPlots, Export["Plots/PlotdeltaGammaCCR.pdf",
    Style[PlotdeltaGammaCCR, Magnification → 1], "PDF"];];

(*ListPlot[{TabδλnTOp,TabδλpTOn},Frame→True,Joined→True,
   FrameLabel→{"T  (K)","δΓ/Γ"},PlotStyle→{Black,{Black,Dashing[{0.01}]}}]
  Export["Plots/NoLogPlotdeltaGammaCCR.pdf",Style[%,Magnification→1],"PDF"];*)
```

Finite-temperature Radiative Corrections

Method

We use exclusively [Brown&Sawyer] for the finite temperature radiative corrections,
but the equation are gathered and summarized in companion paper essentially in section III.F and related appendix. The various terms are in Eq. 108.

Note also that on top of that, we also need to add what we called Brehmstrahlung corrections (Eqs. 107 in companion paper)

Real photons processes integrand

Bose Einstein distribution function (and if sq = -1 it is stimulated emission)

```
BEQ[en_, sq_] := sq BE[sq en];
```

$\tilde{\chi}$ is defined in companion paper in Eq. B45.

```
χtilde[en_, znu_, sgnq_] :=
 With[{q = Q / me }, FDν[en - sgnq q, sgnq ξν, znu] (en - sgnq q) ^2]
```

We first compute real photons processes (absorption or stimulated emission)
We put Fermi function everywhere to be consistent.
The integrand is (Eq. 109 of companion paper)

```
IPENCCRT[en_, k_, x_, znu_, sgnq_] := With[{p = Sqrt[en^2 - 1]},
   With[{b = p / en, A = (2 en^2 + k^2) Log[(en + p)/(en - p)] - 4 p en, B = 2 en Log[(en + p)/(en - p)] - 4 p},
    αFS/(2 π) * (BE[x k]/k) * (A (FD[-en, x] Fermi[sgnq, 1, b] (χtilde[en - k, znu, sgnq] +
             χtilde[en + k, znu, sgnq] - 2 χtilde[en, znu, sgnq]) +
          FD[en, x] Fermi[sgnq, -1, b] (χtilde[-en + k, znu, sgnq] +
             χtilde[-en - k, znu, sgnq] - 2 χtilde[-en, znu, sgnq]))
       - k B * (FD[-en, x] Fermi[sgnq, 1, b] (χtilde[en - k, znu, sgnq] -
             χtilde[en + k, znu, sgnq]) + FD[en, x] Fermi[sgnq, -1, b]
           (χtilde[-en + k, znu, sgnq] - χtilde[-en - k, znu, sgnq]))
      )
   ]];
```

```
(* Compiled version to compute the integrals slightly faster *)
IPENCCRTC =
 MyCompile[{{en, _Real}, {k, _Real}, {x, _Real}, {znu, _Real}, {sgnq, _Integer}},
  Evaluate[IPENCCRT[en, k, x, znu, sgnq]]];
IPENCCRTCN[en_?NumericQ, k_, x_, znu_, sgnq_] := IPENCCRTC[en, k, x, znu, sgnq]
```

Bremsstrahlung corrections (needed to obtain all real photons processes and eventually detailed balance).
The integrand is (Eqs. B48 and B49 using definitions B41)

```
Clear[IPENCCRDiffBremsstrahlungCN,
 IPENCCRDiffBremsstrahlungC, IPENCCRDiffBremsstrahlung]
IPENCCRDiffBremsstrahlung[en_, k_, x_, znu_, sgnq_] :=
  With[{p = Sqrt[en^2 - 1], q = Q / me},
   With[{b = p / en, A = (2 en^2 + k^2) Log[(en + p)/(en - p)] - 4 p en, B = 2 en Log[(en + p)/(en - p)] - 4 p},
    With[{Fp = A + k B, Fm = A - k B},
     αFS/(2 π k) ((FD[-en, x] Fermi[sgnq, 1, b] (Fp χtilde[en + k, znu, sgnq] - If[k <
              Abs[en - sgnq q], Fp FD[en - sgnq q, znu] (Abs[en - sgnq q] - k)^2, 0]))
         + (FD[en, x] Fermi[sgnq, -1, b] (Fm χtilde[-en + k, znu, sgnq] - If[k < Abs[
               en + sgnq q], Fp FD[-en - sgnq q, znu] (Abs[en + sgnq q] - k) ^2, 0]))
      )
    ]]];

(* We compile for the integration *)
IPENCCRDiffBremsstrahlungC =
 MyCompile[{{en, _Real}, {k, _Real}, {x, _Real}, {znu, _Real}, {sgnq, _Integer}},
  Evaluate[IPENCCRDiffBremsstrahlung[en, k, x, znu, sgnq]]];
IPENCCRDiffBremsstrahlungCN[en_?NumericQ, k_, x_, znu_, sgnq_] :=
 IPENCCRDiffBremsstrahlungC[en, k, x, znu, sgnq]
```

The expression IPENFiveBodyT0 implements the integrand of 5.21 of [Brown&Sawyer] (and not 5.20. Careful).
We showed in companion paper that this is only a part of the bremsstrahlung corrections needed.

```
IPENFiveBodyT0[en_, k_, x_, znu_, sgnq_] := With[{p = Sqrt[en^2 - 1]},
  With[{A = (2 en^2 + k^2) Log[(en + p)/(en - p)] - 4 p en, B = 2 en Log[(en + p)/(en - p)] - 4 p},
   αFS/(2 π k) (FD[en, x]) χtilde[-en + k, znu, sgnq] (A - k B)]]


(* Compiled version *)
IPENFiveBodyT0C =
  Compile[{{en, _Real}, {k, _Real}, {x, _Real}, {znu, _Real}, {sgnq, _Integer}},
   Evaluate[With[{p = Sqrt[en^2 - 1]},
     With[{A = (2 en^2 + k^2) Log[(en + p)/(en - p)] - 4 p en, B = 2 en Log[(en + p)/(en - p)] - 4 p},
      αFS/(2 π k) (-BE[-k x]) * (FD[en, x]) χtilde[-en + k, znu, sgnq] (A - k B)]]],
   "RuntimeOptions" → "Speed", CompilationTarget → "C"];
IPENFiveBodyT0CN[en_?NumericQ, k_, x_, znu_, sgnq_] :=
 IPENFiveBodyT0C[en, k, x, znu, sgnq]
```

Mass shift and pe+ee integrand

We finally consider the mass shift and ep + ee corrections

We split the contributions of the last two terms of 5.14 in [Brown and Sawyer] in two parts.
Indeed these are given by 5.15 [Brown and Sawyer]. The first part has only one integral, while the second part has two integrals.
The integrand of first part with only one integral is C1dE, and the integrand of the second part with two integrals is C2dE1dE2 (see 5.16 [Brown and Sawyer]).

This is the simple integration in Eq. B50a of companion paper (more precisely the integrand and the integration itself is performed further below)

$$\mathtt{C1dE[en\_,\ x\_,\ znu\_,\ sgnq\_]\ :=\ With}\Big[\Big\{\mathtt{pe} = \sqrt{\mathtt{en}^2 - 1}\ ,\ \mathtt{q} = \mathtt{Q}/\mathtt{me}\Big\},\\ -\frac{\alpha\mathtt{FS\ en}}{2\,\pi\,\mathtt{pe}} * \frac{2\,\pi^2}{3\,\mathtt{x}^2}\,(\chi[\mathtt{en,\ pe,\ x,\ znu,\ sgnq}] + \chi[-\mathtt{en,\ pe,\ x,\ znu,\ sgnq}])\Big];$$

NB : L is given by 5.18 [Brown and Sawyer]

And this is the double integration integrand of Eq.B50a of companion paper

$$\mathtt{C2dE1dE2[e1\_,\ e2\_,\ x\_,\ znu\_,\ sgnq\_]\ :=}\\ \mathtt{With}\Big[\Big\{\mathtt{p1} = \sqrt{\mathtt{e1}^2 - 1}\ ,\ \mathtt{p2} = \sqrt{\mathtt{e2}^2 - 1}\ ,\ \mathtt{q} = \mathtt{Q}/\mathtt{me}\Big\},\ \mathtt{With}\Big[\Big\{\mathtt{L} = \mathtt{Log}\Big[\frac{\mathtt{e1\,e2\ + p1\,p2\ + 1}}{\mathtt{e1\,e2\ - p1\,p2\ + 1}}\Big]\Big\},\\ \frac{\alpha\mathtt{FS}}{2\,\pi}\,(\chi[\mathtt{e1,\ p1,\ x,\ znu,\ sgnq}] + \chi[-\mathtt{e1,\ p1,\ x,\ znu,\ sgnq}])\\ *\\ \left(-\frac{1}{4}\,\mathtt{Log}\Big[\left(\frac{\mathtt{p1+p2}}{\mathtt{p1-p2}}\right)^2\Big]^2 * \left(\mathtt{FDp[e2,\ x]}\,\frac{\mathtt{p2}}{\mathtt{p1}}\,\frac{\mathtt{e1}^2}{\mathtt{e2}}\,(\mathtt{e1+e2}) + \mathtt{FD[e2,\ x]}\,\frac{\mathtt{e1}^2}{\mathtt{p1\,p2}}\left(\mathtt{e2} + \frac{\mathtt{e1}}{\mathtt{e2}^2}\right)\right)\right.\\ + \mathtt{Log}\Big[\left(\frac{\mathtt{p1+p2}}{\mathtt{p1-p2}}\right)^2\Big]\left(\mathtt{FDp[e2,\ x]}\left(\mathtt{p2}^2\,\frac{\mathtt{e1}}{\mathtt{e2}}\left(\frac{1}{\mathtt{p1}^2}+2\right) - \mathtt{e1}^2\,\frac{\mathtt{p2}}{\mathtt{p1}}\,\mathtt{L}\right) + \right.\\ \left.\mathtt{FD[e2,\ x]}\left(\frac{\mathtt{e1}}{\mathtt{p1}^2\,\mathtt{e2}^2}\left(\mathtt{e2}^2 + 2\,\mathtt{p1}^2 + 1\right) - \frac{\left(\mathtt{e1}^2+\mathtt{e2}^2\right)}{(\mathtt{e1+e2})} - \frac{\mathtt{e1}^2\,\mathtt{e2}}{\mathtt{p1\,p2}}\,\mathtt{L}\right)\right)\\ \left.-\mathtt{FD[e2,\ x]}\left(4\,\mathtt{e1}\,\frac{\mathtt{p2}}{\mathtt{p1}} + 2\,\mathtt{e2\,L}\right)\right)\\ \Big]\Big];$$

```
(* Compiled version *)
C2dE1dE2C =
  Compile[{{e1, _Real}, {e2, _Real}, {x, _Real}, {znu, _Real}, {sgnq, _Integer}},
```

$$\mathtt{Evaluate}\Big[\mathtt{With}\Big[\Big\{\mathtt{p1} = \sqrt{\mathtt{e1}^2 - 1}\ ,\ \mathtt{p2} = \sqrt{\mathtt{e2}^2 - 1}\ ,\ \mathtt{q} = \mathtt{Q}/\mathtt{me}\Big\},\\ \mathtt{With}\Big[\Big\{\mathtt{L} = \mathtt{Log}\Big[\frac{\mathtt{e1\,e2\ + p1\,p2\ + 1}}{\mathtt{e1\,e2\ - p1\,p2\ + 1}}\Big]\Big\},\\ \frac{\alpha\mathtt{FS}}{2\,\pi}\,(\chi[\mathtt{e1,\ p1,\ x,\ znu,\ sgnq}] + \chi[-\mathtt{e1,\ p1,\ x,\ znu,\ sgnq}])\\ * \left(-\frac{1}{4}\,\mathtt{Log}\Big[\left(\frac{\mathtt{p1+p2}}{\mathtt{p1-p2}}\right)^2\Big]^2 * \right.\\ \left(\mathtt{FDp[e2,\ x]}\,\frac{\mathtt{p2}}{\mathtt{p1}}\,\frac{\mathtt{e1}^2}{\mathtt{e2}}\,(\mathtt{e1+e2}) + \mathtt{FD[e2,\ x]}\,\frac{\mathtt{e1}^2}{\mathtt{p1\,p2}}\left(\mathtt{e2} + \frac{\mathtt{e1}}{\mathtt{e2}^2}\right)\right)\\ + \mathtt{Log}\Big[\left(\frac{\mathtt{p1+p2}}{\mathtt{p1-p2}}\right)^2\Big]\left(\mathtt{FDp[e2,\ x]}\left(\mathtt{p2}^2\,\frac{\mathtt{e1}}{\mathtt{e2}}\left(\frac{1}{\mathtt{p1}^2}+2\right) - \mathtt{e1}^2\,\frac{\mathtt{p2}}{\mathtt{p1}}\,\mathtt{L}\right) + \right.\\ \left.\mathtt{FD[e2,\ x]}\left(\frac{\mathtt{e1}}{\mathtt{p1}^2\,\mathtt{e2}^2}\left(\mathtt{e2}^2 + 2\,\mathtt{p1}^2 + 1\right) - \frac{\left(\mathtt{e1}^2+\mathtt{e2}^2\right)}{(\mathtt{e1+e2})} - \frac{\mathtt{e1}^2\,\mathtt{e2}}{\mathtt{p1\,p2}}\,\mathtt{L}\right)\right)\\ \left.-\mathtt{FD[e2,\ x]}\left(4\,\mathtt{e1}\,\frac{\mathtt{p2}}{\mathtt{p1}} + 2\,\mathtt{e2\,L}\right)\right)\\ \Big]\Big]\Big],\ \mathtt{"RuntimeOptions"} \to \mathtt{"Speed"},\ \mathtt{CompilationTarget} \to \mathtt{"C"}\Big];$$

```
C2dE1dE2CN[e1_?NumericQ, e2_?NumericQ, x_, znu_, sgnq_] :=
  C2dE1dE2C[e1, e2, x, znu, sgnq];
```

Integrations on momenta

We perform all integrations now that we have expressed their integrands.

TruePhoton -> real photon processes
Diffbremsstrahlung -> bremsstrahlung corrections
Thermal -> mass shift and pe+ee corrections

Let us start with n -> p processes

```
Clear[λnTOpThermal, λnTOpThermalTruePhoton,
 λpTOnThermalTruePhoton, λnTOp5bodies, λpTOnThermal,
 λnTOpThermalDiffBremsstrahlung, λpTOnThermalDiffBremsstrahlung]
```

```
λnTOpThermalTruePhoton[Tv_] := (*λnTOpThermalTruePhoton[Tv]=*)
  With[{x = me/(kB Tv), znu = me/(kB Tv TνoverT[Tv]), q = Q / me},
   NIntegrate[IPENCCRTCN[en, k, x, If[$TnuEqualT, x, znu], 1],
    {k, 0.001, Max[10, 20 / x]}, {en, 1.001, Max[10, 20 / x]}, PrecisionGoal → 4]
  ];
```

```
λnTOpThermalDiffBremsstrahlung[Tv_] := (*λnTOpThermalDiffBremsstrahlung[Tv]=*)
  With[{x = me/(kB Tv), znu = me/(kB Tv TνoverT[Tv]), q = Q / me},
   NIntegrate[IPENCCRDiffBremsstrahlungCN[en, k, x, If[$TnuEqualT, x, znu], 1],
    {en, 1.001, Max[10, 20 / x]},
    {k, 0.001, Abs[en - q], Abs[en + q], Max[10, 20 / x]}, PrecisionGoal → 4]
  ];
```

The global 1/2 factor is Jacobian of the change of variables (we integrate in the sum and difference of energies).

```
λnTOpThermal[Tv_] := (*λnTOpThermal[Tv]=*)
  With[{x = me/(kB Tv), znu = me/(kB Tv TνoverT[Tv]), q = Q / me},
   NIntegrate[C1dE[en, x, If[$TnuEqualT, x, znu], 1], {en, 1, Max[25, 150 / x]}]
    + NIntegrate[1 / 2 C2dE1dE2CN[(e1pe2 + e1me2) / 2, (e1pe2 - e1me2) / 2,
       x, If[$TnuEqualT, x, znu], 1], {e1me2, -Max[10, 15 / x], -0.001},
     {e1pe2, 2.001 + Abs[e1me2], 2 + Abs[e1me2] + Max[10, 15 / x]},
     PrecisionGoal → 3, Exclusions → {0}]
    + NIntegrate[1 / 2 C2dE1dE2CN[(e1pe2 + e1me2) / 2, (e1pe2 - e1me2) / 2, x,
       If[$TnuEqualT, x, znu], 1], {e1me2, 0.001, Max[10, 15 / x]}, {e1pe2,
      2.001 + Abs[e1me2], 2 + Abs[e1me2] + Max[10, 15 / x]}, PrecisionGoal → 3]
  ];
```

The 5 body is just for comparison with [Brown & Sawyer]

```
λnTOp5bodies[Tv_] := (*λnTOp5bodies[Tv]=*)
  With[{x = me/(kB Tv), znu = me/(kB Tv TνoverT[Tv]), q = Q / me},
   NIntegrate[IPENFiveBodyTOCN[en, k, x, If[$TnuEqualT, x, znu], 1],
    {en, 1, Max[20, 20 / x]}, {k, en + q, en + q + Max[20, 20 / x]}, PrecisionGoal → 4]
  ];
```

Let us finish with p -> n processes

When we have computed the corrections to n -> p rates, the converse are obtained from detailed balance arguments. That is we perform the replacement Q -> -Q as argued in [Brown&Sawyer]. This is also explained in details in the companion paper.

```
λpTOnThermalTruePhoton[Tv_] := (*λpTOnThermalTruePhoton[Tv]=*)If[Tv < 10^8.2
   (* When the temperature is too low it is better to put 0 *), 0,
   With[{x = me/(kB Tv), znu = me/(kB Tv TνoverT[Tv]), q = Q / me},
    NIntegrate[IPENCCRTCN[en, k, x, If[$TnuEqualT, x, znu], -1],
     {k, 0.001, Max[10, 20 / x]}, {en, 1.001, Max[10, 20 / x]}, PrecisionGoal → 4]
   ]];

λpTOnThermalDiffBremsstrahlung[Tv_] :=
  (*λpTOnThermalDiffBremsstrahlung[Tv]=*)If[Tv < 10^8.2
   (* When the temperature is too low it is better to put 0 *), 0,
   With[{x = me/(kB Tv), znu = me/(kB Tv TνoverT[Tv]), q = Q / me},
    NIntegrate[IPENCCRDiffBremsstrahlungCN[en, k, x, If[$TnuEqualT, x, znu], -1],
     {en, 1.001, Max[10, 20 / x]},
     {k, 0.001, Abs[en - q], Abs[en + q], Max[10, 20 / x]}, PrecisionGoal → 4]
   ]];


λpTOnThermal[Tv_] := (*λpTOnThermal[Tv]=*)If[Tv < 10^8.2
   (* When the temperature is too low it is better to put 0 *), 0,
   With[{x = me/(kB Tv), znu = me/(kB Tv TνoverT[Tv]), q = Q / me},
    NIntegrate[C1dE[en, x, If[$TnuEqualT, x, znu], -1], {en, 1, Max[25, 150 / x]}]
     + NIntegrate[1 / 2 C2dE1dE2CN[(e1pe2 + e1me2) / 2, (e1pe2 - e1me2) / 2, x,
        If[$TnuEqualT, x, znu], -1], {e1me2, -Max[10, 15 / x], -0.001}, {e1pe2,
       2.001 + Abs[e1me2], 2 + Abs[e1me2] + Max[10, 15 / x]}, PrecisionGoal → 3]
     + NIntegrate[1 / 2 C2dE1dE2CN[(e1pe2 + e1me2) / 2, (e1pe2 - e1me2) / 2, x,
        If[$TnuEqualT, x, znu], -1], {e1me2, 0.001, Max[10, 15 / x]}, {e1pe2,
       2.001 + Abs[e1me2], 2 + Abs[e1me2] + Max[10, 15 / x]}, PrecisionGoal → 3]
   ]];
```

Collecting all finite-temperature corrections

We collect the various contributions of Eq. 108. The TruePhoton contribution refers to the first term on the rhs of Eq. 108, and the Thermal contribution to the second and third. The Bremsstrahlung corrections are Eqs. 107.

```
λnTOpCCRTh[Tv_] :=
  (λnTOpThermal[Tv] + λnTOpThermalTruePhoton[Tv] + If[$CorrectionBremsstrahlung,
     λnTOpThermalDiffBremsstrahlung[Tv], λnTOp5bodies[Tv]]);

λpTOnCCRTh[Tv_] := (λpTOnThermal[Tv] + λpTOnThermalTruePhoton[Tv] +
    If[$CorrectionBremsstrahlung, λpTOnThermalDiffBremsstrahlung[Tv], 0]);
```

We gather all corrections according to the Booleans chosen at the beginning

```
λ0 := If[$RadiativeCorrections, λRad, λBORN] + If[$FiniteNucleonMass, λFM, 0]
```

```
λnTOpCCRTh[10^9]
λnTOpCCR[10^9]
λnTOpBORN[10^9]
```

```
NIntegrate::slwcon :
  Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration
     is 0, highly oscillatory integrand, or WorkingPrecision too small. >>
```

```
0.00020896899

1.777733

1.6546137
```

Gathering all corrections

We add them depending on options

```
Clear[λnTOp, λpTOn, λnTOpNormalized, λpTOnNormalized];
λnTOpNormalized[Tv_] := (λ0)^-1 (
     If[$RadiativeCorrections, λnTOpCCR[Tv], λnTOpBORN[Tv]]
      + If[$RadiativeThermal, λnTOpCCRTh[Tv], 0]
      + If[$FiniteNucleonMass,
        If[$CoupledFMandRC, λnTOpFMCCR[Tv], λnTOpFMNoCCR[Tv]], 0]
    );

λpTOnNormalized[Tv_] := (λ0)^-1 (
     If[$RadiativeCorrections, λpTOnCCR[Tv], λpTOnBORN[Tv]]
      + If[$RadiativeThermal, λpTOnCCRTh[Tv], 0]
      + If[$FiniteNucleonMass,
        If[$CoupledFMandRC, λpTOnFMCCR[Tv], λpTOnFMNoCCR[Tv]], 0]);

Clear[λnTOp]
λnTOp[Tv_] := 1 / τneutron λnTOpNormalized[Tv];
λpTOn[Tv_] := 1 / τneutron λpTOnNormalized[Tv];
```

Detailed balance check

```
Tt = 10^10.8;
(λnTOpThermalTruePhoton[Tt] + 1 λnTOpThermalDiffBremsstrahlung[Tt]) /
  (λpTOnThermalTruePhoton[Tt] + 1 λpTOnThermalDiffBremsstrahlung[Tt])
λnTOp[Tt] / λpTOn[Tt]
λnTOpCCR[Tt] / λpTOnCCR[Tt]
λnTOpBORN[Tt] / λpTOnBORN[Tt]
λnTOpThermal[Tt] / λpTOnThermal[Tt]
Exp[Q / kB / Tt]
```

```
1.2685403

1.2656748

1.268542

1.2685418

1.2685381

1.2685423
```

Precomputation and storage of rates

We precompute the weak rates the first time and then we save them on the disk.
If the options $RecomputeWeakRates is set to True, the computation is forced.

We first build the name of the file to store the PEN rates. It includes as a postfix the values of the booleans for the various effects taken or not taken into account.

```
LetterFromBoolean[Bool_] := If[Bool, "T", "F"];
StringFromBoolean[BoolList_List] :=
  StringJoin[LetterFromBoolean /@ BoolList];
BooleanSuffix = StringFromBoolean[
  {$RadiativeCorrections, $RadiativeThermal, $FiniteNucleonMass,
   $CoupledFMandRC, $QEDPlasmaCorrections, $IncompleteNeutrinoDecoupling}]
```

```
TTTTTT
```

```
NamePENFilenp = "Interpolations/PENRatenp" <> BooleanSuffix <> ".dat";
NamePENFilepn = "Interpolations/PENRatepn" <> BooleanSuffix <> ".dat";
```

```
$BornBool = Not[$RadiativeThermal] &&
   Not[$RadiativeCorrections] && Not[$FiniteNucleonMass];
```

We store the rate but without the division by τneutron . So that
  we can use the same fit for the reaction rates, and still vary τneutron .

The rates in the files, once loaded, are interpolated once the division by τneutron has been added.

```
MyTableWeakRate :=
 If[$ParallelWeakRates, ParallelEvaluate[Off[NIntegrate::slwcon];];
  ParallelTable, Table]

PreComputeWeakRates := (
   Off[NIntegrate::slwcon];
   λnTOpTab = MyTableWeakRate[{T, λnTOpNormalized[T]}, {T, ListT}];
   λpTOnTab = MyTableWeakRate[{T, λpTOnNormalized[T]}, {T, ListT}];
   TabRatenp = λnTOpTab;
   TabRatepn = λpTOnTab;
   On[NIntegrate::slwcon];
   λnTOpI = MyInterpolationRate[ToExpression[TabRatenp]];
   λpTOnI = MyInterpolationRate[ToExpression[TabRatepn]];
  );
```

Importing the rates previsouly stored if they exist, and recompute if not.

```
TabRatenp = Check[Import[NamePENFilenp, "TSV"],
   Print["Precomputed n -> p rate not found. We recompute
      the rates and store them. This can take very long"];
   $Failed, Import::nffil];

TabRatepn = Check[Import[NamePENFilepn, "TSV"],
   Print["Precomputed p -> n rate not found. We recompute
      the rates and store them. This can take very long"];
   $Failed, Import::nffil];

Timing[If[TabRatenp === $Failed || TabRatepn === $Failed || $RecomputeWeakRates,
   PreComputeWeakRates;
   Export[NamePENFilenp, TabRatenp, "TSV"];
   Export[NamePENFilepn, TabRatepn, "TSV"];,
   λnTOpI = MyInterpolationRate[ToExpression[TabRatenp]];
   λpTOnI = MyInterpolationRate[ToExpression[TabRatepn]];
  ];]
```

```
{0.008053, Null}
```

We give standard names that are used in the network of reactions later. The factor $1/\tau_n$ is the one appearing in the constant defined in Eq. 91 of companion paper.

```
LnTOp[Tv_] := 1 / τneutron * λnTOpI[Tv];
LpTOn[Tv_] := 1 / τneutron * λpTOnI[Tv];
LbarnTOp[Tv_] := LpTOn[Tv];
```

We check that the rate for neutron decay is what we expect at low temperature, that is it is τneutron only. At 10^8 K it should be the case.

```
1 / LnTOp[Tf]
```

```
879.50275
```

Plots of finite-temperature corrections

This is very long so I comment this section but to get the plot of finite temperature corrections of the companion paper, this should be uncommented.

```
(*

Off[NIntegrate::slwcon];
TabdlambdanTOp=
 MyTableWeakRate[{T, (λRadandFM* (λnTOpThermal[T]+λnTOpThermalTruePhoton[T]+
        λnTOpThermalDiffBremsstrahlung[T]))/
    (λBORN*λnTOpBORN[T])},{T,ListTRange[1 10^9,10^11]}];
Print[TabdlambdanTOp];

TabdlambdapTOn=
 MyTableWeakRate[{T, (λRadandFM* (λpTOnThermal[T]+λpTOnThermalTruePhoton[T]+
        λpTOnThermalDiffBremsstrahlung[T]))/
    (λBORN*λpTOnBORN[T])},{T,ListTRange[1 10^9,10^11]}];

Export["Interpolations/TabdlambdanTOp.dat",TabdlambdanTOp,"TSV"];
Export["Interpolations/TabdlambdapTOn.dat",TabdlambdapTOn,"TSV"];

TabdlambdanTOpBrown=MyTableWeakRate[
  {T, λRadandFM*(λnTOpThermal[T]+λnTOpThermalTruePhoton[T])/(λBORN*λnTOpBORN[T])},{T,ListTRange[1 10^9,10^11]}];
TabdlambdanTOpBrown5Bodies=MyTableWeakRate[
  {T, (λRadandFM* (λnTOpThermal[T]+λnTOpThermalTruePhoton[T]+λnTOp5bodies[T]))/
    (λBORN*λnTOpBORN[T])},{T,ListTRange[1 10^9,10^11]}];
TabdlambdapTOnBrown=MyTableWeakRate[{T, λRadandFM*(λpTOnThermal[T]+λpTOnThermalTruePhoton[T])/(λBORN*λpTOnBORN[T])},
  {T,ListTRange[1 10^9,10^11]}];

Export["Interpolations/TabdlambdanTOpBrown.dat",TabdlambdanTOpBrown,"TSV"];
Export["Interpolations/TabdlambdanTOpBrown5Bodies.dat",
 TabdlambdanTOpBrown5Bodies,"TSV"];
Export["Interpolations/TabdlambdapTOnBrown.dat",TabdlambdapTOnBrown,"TSV"];

TabdlambdanTOpBrehm=MyTableWeakRate[
  {T, λRadandFM*(λnTOpThermalDiffBremsstrahlung[T])/(λBORN*λnTOpBORN[T])},{T,ListTRange[1 10^9,10^11]}];
TabdlambdapTOnBrehm=MyTableWeakRate[{T, λRadandFM*(λpTOnThermalDiffBremsstrahlung[T])/(λBORN*λpTOnBORN[T])},
  {T,ListTRange[1 10^9,10^11]}];
On[NIntegrate::slwcon];


Export["Interpolations/TabdlambdanTOpBrehm.dat",TabdlambdanTOpBrehm,"TSV"];
Export["Interpolations/TabdlambdapTOnBrehm.dat",TabdlambdapTOnBrehm,"TSV"];*)

If[$PaperPlots,
 TabδλnTOp = Import["Interpolations/TabdlambdanTOp.dat"];
 TabδλpTOn = Import["Interpolations/TabdlambdapTOn.dat"];

 TabδλnTOpBrown = Import["Interpolations/TabdlambdanTOpBrown.dat"];
 TabδλnTOpBrown5Bodies =
  Import["Interpolations/TabdlambdanTOpBrown5Bodies.dat"];
 TabδλpTOnBrown = Import["Interpolations/TabdlambdapTOnBrown.dat"];

 TabδλnTOpBrehm = Import["Interpolations/TabdlambdanTOpBrehm.dat"];
 TabδλpTOnBrehm = Import["Interpolations/TabdlambdapTOnBrehm.dat"];
]
```

```
If[$PaperPlots,
 TFreeze = 0.8 MeV / kB;
 RCT = ListLogLinearPlot[{TabδλnTOp, TabδλpTOn, TabδλnTOpBrown, TabδλpTOnBrown,
     TabδλnTOpBrehm, TabδλpTOnBrehm, TabδλnTOpBrown5Bodies}, Frame → True,
    FrameStyle → Thickness[0.004], Joined → True, FrameLabel → {"T (K)", "δΓ/Γ"},
    LabelStyle → {FontSize → 12}, GridLines → {{{TFreeze, {Gray, Thickness[
           0.005]}}}, {}}, PlotStyle → {{Darker@Darker@Green, Thickness[0.005]},
      {Darker@Darker@Green, Thickness[0.005], Dashing[{0.02}]},
      {Red, Thickness[0.006]}, {Red, Thickness[0.006], Dashing[{0.02}]},
      {Blue, Thickness[0.004]}, {Blue, Thickness[0.004], Dashing[{0.02}]},
      {Red, Thickness[0.002], Thickness[0.003]}}, FrameTicks → MyFrameTicksLog]]

If[$PaperPlots, Export["Plots/LogPlotdeltaGammaCCRT.pdf",
   Style[RCT, Magnification → 1], "PDF"]];
```

Nuclear reactions network

Nuclear Species

Names and (N,Z)

The is the list of short names with their (neutron, proton) weights. So by definition a neutron has (1, 0) and a proton has (0, 1) and He4 has (2, 2) and so on.

```
NamesWithWeightsAll = {{"n", {1, 0}}, {"p", {0, 1}}, {"d", {1, 1}}, {"t", {2, 1}},
   {"He3", {1, 2}}, {"a", {2, 2}}, {"He5", {3, 2}}, {"He6", {4, 2}},
   {"Li6", {3, 3}}, {"Li7", {4, 3}}, {"Li8", {5, 3}}, {"Li9", {6, 3}},
   {"Be7", {3, 4}}, {"Be8", {4, 4}}, {"Be9", {5, 4}},
   {"Be10", {6, 4}}, {"Be11", {7, 4}}, {"Be12", {8, 4}},
   {"B8", {3, 5}}, {"B9", {4, 5}}, {"B10", {5, 5}}, {"B11", {6, 5}},
   {"B12", {7, 5}}, {"B13", {8, 5}}, {"B14", {9, 5}}, {"B15", {10, 5}},
   {"C9", {3, 6}}, {"C10", {4, 6}}, {"C11", {5, 6}}, {"C12", {6, 6}},
   {"C13", {7, 6}}, {"C14", {8, 6}}, {"C15", {9, 6}}, {"C16", {10, 6}},
   {"N12", {5, 7}}, {"N13", {6, 7}}, {"N14", {7, 7}},
   {"N15", {8, 7}}, {"N16", {9, 7}}, {"N17", {10, 7}},
   {"O13", {5, 8}}, {"O14", {6, 8}}, {"O15", {7, 8}}, {"O16", {8, 8}},
   {"O17", {9, 8}}, {"O18", {10, 8}}, {"O19", {11, 8}}, {"O20", {12, 8}},
   {"F17", {8, 9}}, {"F18", {9, 9}}, {"F19", {10, 9}}, {"F20", {11, 9}},
   {"Ne18", {8, 10}}, {"Ne19", {9, 10}}, {"Ne20", {10, 10}},
   {"Ne21", {11, 10}}, {"Ne22", {12, 10}}, {"Ne23", {13, 10}},
   {"Na20", {9, 11}}, {"Na21", {10, 11}}, {"Na22", {11, 11}}, {"Na23", {12, 11}}};
```

Let us vizuallize the nuclides used in as a table in (Z,N).

```
TableNZNucleons = Table[" ", {i, 0, 13}, {j, 0, 11}];
Map[(TableNZNucleons[[Sequence @@ (#[[2]] + {1, 1})]] = #[[1]]) &,
  NamesWithWeightsAll];
```

```
Grid[Transpose@TableNZNucleons, Frame → All]
(* This is table III in companion paper*)
```

	n												
p	d	t											
	He3	a	He5	He6									
			Li6	Li7	Li8	Li9							
			Be7	Be8	Be9	Be10	Be11	Be12					
			B8	B9	B10	B11	B12	B13	B14	B15			
			C9	C10	C11	C12	C13	C14	C15	C16			
					N12	N13	N14	N15	N16	N17			
					O13	O14	O15	O16	O17	O18	O19	O20	
								F17	F18	F19	F20		
								Ne18	Ne19	Ne20	Ne21	Ne22	Ne23
									Na20	Na21	Na22	Na23	

The list of the species names only

```
ShortNamesAll = NamesWithWeightsAll[[All, 1]];
```

The list of {n, p} pairs only.

```
ListNPPairs = NamesWithWeightsAll[[All, 2]];
```

Functions to check if a name exists

```
ExistName[name_] := MemberQ[ShortNamesAll, name];
ExistPair[pair_List] := MemberQ[ListNPPairs, pair];
```

This function selects the names of the species which all have the same mass number A

```
NamesMassNumberAll[A_] :=
 Select[NamesWithWeightsAll, ((Plus @@ (#[[2]])) == A) &][[All, 1]]
```

Dictionaries between names and numbers

We define dictionaries to handle species. We associate a number to each species
It is a simple correspondance between names and position of the species in the list, or between the names and the pair (neutron,proton).

```
KeySpecies = Association@ (Rule @@@ NamesWithWeightsAll)
```

```
<|n → {1, 0}, p → {0, 1}, d → {1, 1}, t → {2, 1}, He3 → {1, 2}, a → {2, 2}, He5 → {3, 2},
 He6 → {4, 2}, Li6 → {3, 3}, Li7 → {4, 3}, Li8 → {5, 3}, Li9 → {6, 3}, Be7 → {3, 4},
 Be8 → {4, 4}, Be9 → {5, 4}, Be10 → {6, 4}, Be11 → {7, 4}, Be12 → {8, 4},
 B8 → {3, 5}, B9 → {4, 5}, B10 → {5, 5}, B11 → {6, 5}, B12 → {7, 5}, B13 → {8, 5},
 B14 → {9, 5}, B15 → {10, 5}, C9 → {3, 6}, C10 → {4, 6}, C11 → {5, 6}, C12 → {6, 6},
 C13 → {7, 6}, C14 → {8, 6}, C15 → {9, 6}, C16 → {10, 6}, N12 → {5, 7}, N13 → {6, 7},
 N14 → {7, 7}, N15 → {8, 7}, N16 → {9, 7}, N17 → {10, 7}, O13 → {5, 8}, O14 → {6, 8},
 O15 → {7, 8}, O16 → {8, 8}, O17 → {9, 8}, O18 → {10, 8}, O19 → {11, 8},
 O20 → {12, 8}, F17 → {8, 9}, F18 → {9, 9}, F19 → {10, 9}, F20 → {11, 9},
 Ne18 → {8, 10}, Ne19 → {9, 10}, Ne20 → {10, 10}, Ne21 → {11, 10}, Ne22 → {12, 10},
 Ne23 → {13, 10}, Na20 → {9, 11}, Na21 → {10, 11}, Na22 → {11, 11}, Na23 → {12, 11}|>
```

We have a reverse dictionary if we want

```
KeyNucleons = Association@(Rule @@@ (Reverse /@ NamesWithWeightsAll))
```

```
⟨|{1, 0} → n, {0, 1} → p, {1, 1} → d, {2, 1} → t, {1, 2} → He3, {2, 2} → a, {3, 2} → He5,
 {4, 2} → He6, {3, 3} → Li6, {4, 3} → Li7, {5, 3} → Li8, {6, 3} → Li9, {3, 4} → Be7,
 {4, 4} → Be8, {5, 4} → Be9, {6, 4} → Be10, {7, 4} → Be11, {8, 4} → Be12,
 {3, 5} → B8, {4, 5} → B9, {5, 5} → B10, {6, 5} → B11, {7, 5} → B12, {8, 5} → B13,
 {9, 5} → B14, {10, 5} → B15, {3, 6} → C9, {4, 6} → C10, {5, 6} → C11, {6, 6} → C12,
 {7, 6} → C13, {8, 6} → C14, {9, 6} → C15, {10, 6} → C16, {5, 7} → N12, {6, 7} → N13,
 {7, 7} → N14, {8, 7} → N15, {9, 7} → N16, {10, 7} → N17, {5, 8} → O13, {6, 8} → O14,
 {7, 8} → O15, {8, 8} → O16, {9, 8} → O17, {10, 8} → O18, {11, 8} → O19,
 {12, 8} → O20, {8, 9} → F17, {9, 9} → F18, {10, 9} → F19, {11, 9} → F20,
 {8, 10} → Ne18, {9, 10} → Ne19, {10, 10} → Ne20, {11, 10} → Ne21, {12, 10} → Ne22,
 {13, 10} → Ne23, {9, 11} → Na20, {10, 11} → Na21, {11, 11} → Na22, {12, 11} → Na23|⟩
```

N, Z, A from name

```
Ni["Bm"] := 1;
Ni["Bp"] := -1;
Ni["g"] := 0;

Zi["Bm"] := -1;
Zi["Bp"] := 1;
Zi["g"] := 0;

Ai["Bm"] := 0;
Ai["Bp"] := 0;
Ai["g"] := 0;
```

```
Ni[key_] := KeySpecies[key][[1]]
Zi[key_] := KeySpecies[key][[2]]
Ai[key_] := Zi[key] + Ni[key]
```

```
Ai["Li7"]
Ni["Li7"]
Zi["Li7"]
Ni["Bm"]
```

```
7

4

3

1
```

Binding energies and spins of nuclei

Tools to reshape the file "nubase2016.asc"

```
SpinFromCharList[charlist_List] := StringReplace[StringJoin @@ charlist,
   {"(" → "", ")" → "", "," → "", "+" → "", "-" → "", " " → "", "#" → ""}]
MassFromCharList[charlist_List] :=
  StringReplace[StringJoin @@ charlist, {" " → "", "#" → ""}]
```

We load the file "nubtab03.asc"

```
StringListParticles = #[[1]] & /@ Import[(*"nubtab03.asc"*) "nubase2016.asc"];
NubTabChar = Select[Characters /@ StringListParticles, Length[#] ≥ 93 &];
```

NUBASE Syntax :
The first three characters of each line are the A.
Characters from 5 to 8 are 10*Z

From 20 to 29 it is mass excess.
From 80 to 93 it is some information on spin and parity.

```
Alist = ToExpression /@ StringJoin /@ (Take[#, {1, 3}] & /@ NubTabChar);
Zlist = # / 10 & /@ ToExpression /@ StringJoin /@ (Take[#, {5, 8}] & /@ NubTabChar);
MassExcessesString = MassFromCharList /@ (Take[#, {20, 29}] & /@ NubTabChar);
Spins = SpinFromCharList /@ (Take[#, {80, 93}] & /@ NubTabChar);
Nlist = Alist - Zlist;
```

We gather all this information in a table

```
MyGrid[ListNPBindingSpinName =
  Flatten[{KeyNucleons[{#[[1]], #[[2]]}], #}] & /@ Select[Transpose[
     {Nlist, Zlist, MassExcessesString, Spins}], ExistPair[{#[[1]], #[[2]]}] &]]
```

n	1	0	8071.3171	1/2
p	0	1	7288.9706	1/2
d	1	1	13135.7217	1
t	2	1	14949.8099	1/2
He3	1	2	14931.2179	1/2
a	2	2	2424.9156	0
He5	3	2	11231	3/2
He6	4	2	17592.10	0
Li6	3	3	14086.8789	1
Li7	4	3	14907.105	3/2
Be7	3	4	15769.00	3/2
Li8	5	3	20945.80	2
Be8	4	4	4941.67	0
B8	3	5	22921.6	2
Li9	6	3	24954.90	3/2
Be9	5	4	11348.45	3/2
B9	4	5	12416.5	3/2
C9	3	6	28911.0	3/2
Be10	6	4	12607.49	0
B10	5	5	12050.609	3
C10	4	6	15698.67	0
Be11	7	4	20177.17	1/2
B11	6	5	8667.707	3/2
C11	5	6	10649.40	3/2
Be12	8	4	25077.8	0
B12	7	5	13369.4	1
C12	6	6	0.0	0
N12	5	7	17338.1	1
B13	8	5	16561.9	3/2
C13	7	6	3125.0088	1/2
N13	6	7	5345.48	1/2
O13	5	8	23115	3/2
B14	9	5	23664	2
C14	8	6	3019.893	0
N14	7	7	2863.4167	1
O14	6	8	8007.781	0
B15	10	5	28958	3/2
C15	9	6	9873.1	1/2
N15	8	7	101.4387	1/2
O15	7	8	2855.6	1/2
C16	10	6	13694	0
N16	9	7	5683.9	2
O16	8	8	-4737.0013	0
N17	10	7	7870	1/2
O17	9	8	-808.7635	5/2
F17	8	9	1951.70	5/2
O18	10	8	-782.8156	0
F18	9	9	873.1	1
Ne18	8	10	5317.6	0
O19	11	8	3332.9	5/2
F19	10	9	-1487.4442	1/2
Ne19	9	10	1752.05	1/2
O20	12	8	3796.2	0
F20	11	9	-17.463	2
Ne20	10	10	-7041.9305	0
Na20	9	11	6850.6	2
Ne21	11	10	-5731.78	3/2
Na21	10	11	-2184.63	3/2
Ne22	12	10	-8024.719	0
Na22	11	11	-5181.51	3
Ne23	13	10	-5154.05	5/2
Na23	12	11	-9529.8525	3/2

We define a dictionary for excess masses (in keV)

```
ExcessMassKeys =
 Association[{#[[1]] → ToExpression[#[[4]]]} & /@ListNPBindingSpinName]
```

```
⟨|n → 8071.3171, p → 7288.9706, d → 13 135.722, t → 14 949.81, He3 → 14 931.218,
 a → 2424.9156, He5 → 11 231, He6 → 17 592.1, Li6 → 14 086.879, Li7 → 14 907.105,
 Be7 → 15 769., Li8 → 20 945.8, Be8 → 4941.67, B8 → 22 921.6, Li9 → 24 954.9,
 Be9 → 11 348.45, B9 → 12 416.5, C9 → 28 911., Be10 → 12 607.49, B10 → 12 050.609,
 C10 → 15 698.67, Be11 → 20 177.17, B11 → 8667.707, C11 → 10 649.4,
 Be12 → 25 077.8, B12 → 13 369.4, C12 → 0., N12 → 17 338.1, B13 → 16 561.9,
 C13 → 3125.0088, N13 → 5345.48, O13 → 23 115, B14 → 23 664, C14 → 3019.893,
 N14 → 2863.4167, O14 → 8007.781, B15 → 28 958, C15 → 9873.1, N15 → 101.4387,
 O15 → 2855.6, C16 → 13 694, N16 → 5683.9, O16 → -4737.0013, N17 → 7870,
 O17 → -808.7635, F17 → 1951.7, O18 → -782.8156, F18 → 873.1, Ne18 → 5317.6,
 O19 → 3332.9, F19 → -1487.4442, Ne19 → 1752.05, O20 → 3796.2, F20 → -17.463,
 Ne20 → -7041.9305, Na20 → 6850.6, Ne21 → -5731.78, Na21 → -2184.63,
 Ne22 → -8024.719, Na22 → -5181.51, Ne23 → -5154.05, Na23 → -9529.8525|⟩
```

And a dictionary for spins

```
SpinKeys =
 Association[{#[[1]] -> ToExpression[#[[5]]]} & /@ListNPBindingSpinName]
```

```
⟨|n → 1/2, p → 1/2, d → 1, t → 1/2, He3 → 1/2, a → 0, He5 → 3/2, He6 → 0, Li6 → 1, Li7 → 3/2,
 Be7 → 3/2, Li8 → 2, Be8 → 0, B8 → 2, Li9 → 3/2, Be9 → 3/2, B9 → 3/2, C9 → 3/2, Be10 → 0,
 B10 → 3, C10 → 0, Be11 → 1/2, B11 → 3/2, C11 → 3/2, Be12 → 0, B12 → 1, C12 → 0, N12 → 1,
 B13 → 3/2, C13 → 1/2, N13 → 1/2, O13 → 3/2, B14 → 2, C14 → 0, N14 → 1, O14 → 0, B15 → 3/2,
 C15 → 1/2, N15 → 1/2, O15 → 1/2, C16 → 0, N16 → 2, O16 → 0, N17 → 1/2, O17 → 5/2, F17 → 5/2,
 O18 → 0, F18 → 1, Ne18 → 0, O19 → 5/2, F19 → 1/2, Ne19 → 1/2, O20 → 0, F20 → 2,
 Ne20 → 0, Na20 → 2, Ne21 → 3/2, Na21 → 3/2, Ne22 → 0, Na22 → 3, Ne23 → 5/2, Na23 → 3/2|⟩
```

From excess masses we can find binding energies (in keV). WE only need the excess mass of proton and neutron and the (Z,A,N) of the nuclide.

```
Eneutron := ExcessMassKeys["n"];
Eproton := ExcessMassKeys["p"];

BindingEnergy[name_] := Module[{Pair, A, Z, N},
  Pair = KeySpecies[name];
  Z = Pair[[2]];
  N = Pair[[1]];
  A = Z + N;
  N Eneutron + Z Eproton - ExcessMassKeys[name]]

Mass[name_] := Module[{Pair, A, Z, N},
  Pair = KeySpecies[name];
  Z = Pair[[2]];
  N = Pair[[1]];
  A = Z + N;
  A ma + keV ExcessMassKeys[name] - Z me]
```

We check a few binding energies (in keV)

```
BindingEnergy["n"]
BindingEnergy["p"]
BindingEnergy["d"]
BindingEnergy["a"]
0.

0.

2224.566

28 295.66
```

```
Mass["n"] / MeV
mn / MeV
939.56538

939.56536
```

```
Mass["p"] / MeV
mp / MeV
938.27203

938.27203
```

```
Mass["d"] / MeV
1875.6128
```

Nuclear Statistical Equilibrium

This is Eq. A24 of companion paper.

```
YNSE[name_, Yn_, Yp_, Tv_] := Module[{Pair, N, A, Z, mN, A32Overmn},
   mN = (mn + mp) / 2;
   Pair = KeySpecies[name];
   Z = Pair[[2]];
   N = Pair[[1]];
   A = Z + N;
   A32Overmn = (Mass[name] / (mn^(A-Z) * mp^Z))^(3/2);
   (2 * SpinKeys[name] + 1) Zeta[3]^(A-1) π^((1-A)/2) 2^((3 A-5)/2) A32Overmn
    (kB Tv)^(3 (A-1)/2) (ηfactorT[Tv])^(A-1) Yp^Z Yn^(A-Z) Exp[(BindingEnergy[name] * keV) / (kB Tv)]
  ]
```

Reverse reaction information

The reverse reaction depends on three constants (α, β, γ) defined in companion paper in Eq. 142. From the Spin, mass and binding energy we can find these constants.

```
Qreaction[ListIn_, ListOut_] :=
  Module[{Ni = Length@ListIn, Nf = Length@ListOut, factorin, factorout, Units},
   factorin = Plus @@ ((BindingEnergy[#]) & /@ListIn);
   factorout = Plus @@ ((BindingEnergy[#]) & /@ListOut);
   Units = keV;
   -Units (factorin - factorout)
  ];

PowerT9[ListIn_, ListOut_] := Module[{Ni = Length@ListIn, Nf = Length@ListOut},
   3 / 2. * (Ni - Nf)
  ];

FactorInverseReaction[ListIn_, ListOut_] :=
  Module[{Ni = Length@ListIn, Nf = Length@ListOut, factorin, factorout, Units},
   factorin = Times @@
     ((((2 SpinKeys[#[[1]]] + 1) (2 Pi / Mass[#[[1]]] / (kB 10^9)) ^ (-3 / 2)) ^
          (#[[2]]) / (#[[2]] !)) & /@ (Tally@ListIn));
   factorout = Times @@ ((((2 SpinKeys[#[[1]]] + 1) (2 Pi / Mass[#[[1]]] / (kB 10^9)) ^
                (-3 / 2)) ^ (#[[2]]) / (#[[2]] !)) & /@ (Tally@ListOut));
   Units = ((ma / clight^2) / (hbar clight) ^3) ^ (Ni - Nf);
   factorin / factorout Units
  ];

GatherInfoReac[ListIn_, ListOut_] :=
  {Qreaction[ListIn, ListOut] / MeV, FactorInverseReaction[ListIn, ListOut],
   PowerT9[ListIn, ListOut], -Qreaction[ListIn, ListOut] / kB / 10^9};

RemoveNonNuclear[Species_List] :=
  Select[Species, # =!= "g" && # =!= "Bm" && # =!= "Bp" &];
InfoReaction[{ListIn_, ListOut_}] :=
  GatherInfoReac[RemoveNonNuclear@ListIn, RemoveNonNuclear@ListOut];
InfoReaction[ListIn_, ListOut_] := InfoReaction[{ListIn, ListOut}]
```

For a given reaction, defined by the list of initial particles and final particles, we get these constants with the function InfoReaction.
For example

```
InfoReaction[{"n", "p"}, {"d", "g"}]
```

```
{2.224566, 4.7161407 × 10^9, 1.5, -25.815019}
```

Check reaction coherence (formal conservation of N and Z)

```
CheckReaction[{ListIn_, ListOut_}] := Module[{Znet, Nnet, Anet},
  Znet = -Plus @@ (Zi /@ListIn) + Plus @@ (Zi /@ListOut);
  Nnet = -Plus @@ (Ni /@ListIn) + Plus @@ (Ni /@ListOut);
  Anet = -Plus @@ (Ai /@ListIn) + Plus @@ (Ai /@ListOut);
  (*Print[ListIn," ",ListOut," ",Znet,Nnet,Anet];*)
  If[Znet =!= 0 || Nnet =!= 0 || Anet =!= 0,
   Print["ERROR! This reaction ", ListIn, " -> ", ListOut,
    " is not possible.\nThe net result for Z, N and A are ",
    Znet, " ", Nnet, " ", Anet];
   Print["We abort the evaluation !"];
   Quit[];
   (*TODO Maybe a better handling of errors than juts a violent Quit[]... *)
  ];
 ]

CheckReaction[ListIn_, ListOut_] := CheckReaction[{ListIn, ListOut}]

(*CheckReaction[{"n"},{"p","Bm"}]*)
```

Nuclear Reaction rates

Random Number Generation for nuclear rates uncertainties

Generator of random number according to Normal distribution. But we make sure to use always the same sequence to avoid noise in Monte-Carlo.
This is crucial because this reduces Monte-Carlo noise when evaluating uncertainty in rates.

So for a given seed we precompute a list of 1000 random numbers.
Then we call several times MyNormalRandom[seed] which gives successively the random numbers which were generated with the seed.

```
Clear[TableRandom, MyNormalRandom]
$NRandomPoints = 1000;
(* We put something larger than the max number of reactions *)
TableRandom[seed_] := TableRandom[seed] = (SeedRandom[seed];
   Table[RandomVariate[NormalDistribution[]], {i, 1, $NRandomPoints}])

InitializeRandom[seed_] := (IndexRandom[seed] = 1);
RandomFromTable[seed_] := With[{r = TableRandom[seed][[IndexRandom[seed]]]},
  IndexRandom[seed] = IndexRandom[seed] + 1;
  r]
MyNormalRandom[seed_] := RandomFromTable[seed]

$Seed := 0;
Initialize[$Seed];
NormalRealisation := If[$RandomNuclearRates, MyNormalRandom[$Seed], 0];
```

Importation of reactions from external files (336 reactions)

We collect tools to read the reactions from the external file. This is low level code... because we need to deal with syntax.

This function constructs the reverse reaction. Its arguments are the name of the reverse reaction, the front factor, the power on T9 and the Q of the reaction.

```
ReverseReaction[Name_, FrontFactor_, PoweronT9_, Qoverkb_] :=
  With[{Reversname = ToExpression["Hold@Lbar" <> Name],
    name = Evaluate[Symbol["L" <> Name]]},
   If[FrontFactor > 0,
    MySet[Reversname, Function[{Tvr}, With[{T9 = Tvr / Giga},
        FrontFactor (T9)^PoweronT9 * Exp[Qoverkb/T9] * name[Tvr]]]];,
    MySet[Reversname, Function[{Tvr}, 0]];
   ]];
```

For a line (the list of elements of this line more precisely) describing a reaction we build the rates and inverse rates, and we output a formal description of the reaction in terms of initial and final particles

```
TreatData[Data_] := Module[{reac, constants, ReferencePaper,
    dat, rest, list, resultat, reacreshaped, replacements},
   resultat = {};
   list = Data;

   While[Length@list > 0,
    reac = list[[1]];
    ReferencePaper = StringDrop[list[[2, 1]], 2];
    (*Print[ReferencePaper];*)
    (*Print[reac];*)
    constants = list[[3]];
    rest = Drop[list, 3];
    dat = {};
    While[rest =!= {} && NumericQ[rest[[1, 1]]],
     dat = Append[dat, rest[[1]]];
     rest = Rest@rest;
    ];
    list = rest;
    reacreshaped = Append[{Select[reac, (# =!= "+" && # =!= "*-") &],
       constants, ReferencePaper}, dat];
    resultat = Append[resultat, reacreshaped];
   ];
   replacements = {"He4" → "a"};
   resultat /. replacements
  ];

TruncateRateVariation[rate_] := Min[$MaxVariationRate, rate]

TreatReactionLine[line_] :=
  Module[{rescalefactor, reac, constants, interpfunction, data, len,
    table, Tmin, rmin, wedgeposition, colonposition, InitialParticles,
    FinalParticles, BoolenFileData, Q, FrontFactor, PoweronT9,
    Qoverkb, Name, Lname, rv, ReferencePaper, InfoFromAudi2017},
   reac = line[[1]];
   (*Print["Treating reaction : ",reac];*)
   constants = line[[2]];
   ReferencePaper = line[[3]];
   data = line[[4]];

   len = Length@line;
   wedgeposition = Position[reac, ">"][[1, 1]];
```

```
colonposition = Position[reac, ";"][[1, 1]];
InitialParticles = Take[reac, {1, wedgeposition - 1}];
FinalParticles = Take[reac, {wedgeposition + 1, colonposition - 1}];

(* We quit if the reaction does not conserve formally Z or N,
that is if it cannot exist *)
CheckReaction[InitialParticles, FinalParticles];


Name = StringJoin @@ ToString /@ InitialParticles <>
  "TO" <> StringJoin @@ ToString /@ FinalParticles;

Q = constants[[1]];
FrontFactor = constants[[2]];
PoweronT9 = constants[[3]];
Qoverkb = constants[[4]];


(* We check the constants used in reverse rates *)
InfoFromAudi2017 = InfoReaction[InitialParticles, FinalParticles];
(*Print[InitialParticles," ",FinalParticles," ",InfoFromAudi2017];*)

If[Abs[FrontFactor / InfoFromAudi2017[[2]] - 1] > 0.001,
 Print[Name, " WARNING. We use α=", FrontFactor,
  " but we should use ", InfoFromAudi2017[[2]]]
];

If[Abs[Qoverkb / InfoFromAudi2017[[4]] - 1] > 0.001,
 Print[Name, " WARNING. We use Q/kB=",
  Qoverkb, " but we should use ", InfoFromAudi2017[[4]]]
];

If[PoweronT9 =!= InfoFromAudi2017[[3]],
 Print[Name, " WARNING. We use power on T9 =",
  PoweronT9, " but we should use ", InfoFromAudi2017[[3]]]
];
(* ****************)


(* *** *)
(*For exploration of parameters we can redefine some front
 factors to recales reactions. For instance the DPG reaction*)
(* Added on request of Antony Lewis *)

rescalefactor = 1;
(*Print[Name,FullForm[Name]];*)

If[Name === "dpTOHe3g",
 rescalefactor = dpTOHe3gFactor;
 If[rescalefactor =!= 1, Print["dpTOHe3g reaction is rescaled by ",
    dpTOHe3gFactor, " New front factor is ", rescalefactor];];];
(* *** *)



table = Map[{Giga #[[1]], #[[2]] Hz, #[[3]]} &, data];
Tmin = Last[table][[1]];
rmin = Last[table][[2]];
```

```
   Lname = ToExpression["Hold@L" <> Name];
   rv = NormalRealisation;
   MySet[Lname, MyInterpolationRate[{#[[1]], Identity[rescalefactor #[[2]] *
          If[$RandomNuclearRates, TruncateRateVariation[#[[3]]^rv], 1]]} & /@
      table]];
   (* We do not rescale the reverse because it is computed
    FROM the forward rate. So rescaling the forward
    rate by rescalefactor rescales them both *)
   ReverseReaction[Name, FrontFactor, PoweronT9, Qoverkb];

   {Name, InitialParticles, FinalParticles, rv, ReferencePaper}

  ];

SetAttributes[TreatReactionLine, SequenceHold]
```

Lists of analytic reactions (87 reactions)

We have a list of 87 reactions for which we use analytic fits from the literature
In principle these reactions could be tabulated and incorporated into the external file but we prefer to keep their analytic forms.

- We need a few tools (this is painful low level code)

```
ListTWagoner =
  {0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009, 0.01, 0.011,
    0.012, 0.013, 0.014, 0.015, 0.016, 0.018, 0.02, 0.025, 0.03, 0.04, 0.05, 0.06,
    0.07, 0.08, 0.09, 0.1, 0.11, 0.12, 0.13, 0.14, 0.15, 0.16, 0.18,
    0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.6, 0.7, 0.8, 0.9, 1., 1.25,
    1.5, 1.75, 2., 2.5, 3., 3.5, 4., 5., 6., 7., 8., 9., 10.} * 10^9;

$ListTWagoner = False;

TableInterpolationTemperature =
  If[$ListTWagoner, ListTWagoner, ListTRange[0.9 Tf, 10^10]];
```

```
SimplifyReactionStringRules =
  {"+" → " + ", ">" → " > ", ";" → " ; ", "2n" → " n + n ",
   "2p" → " p + p ", "2g" → " g + g ", "2a" → " a + a ", "He4" → " a "};

ReshapheReactionString[string_String] :=
  Select[StringSplit[StringReplace[string, SimplifyReactionStringRules], " "],
   (# =!= "+" && # =!= "*-" && # =!= "") &];

TreatReactionString[reac_String, source_String, f_] :=
 Module[{reacshaped, wedgeposition,
   colonposition, InitialParticles, FinalParticles, Name},
  reacshaped = ReshapheReactionString[reac];
  wedgeposition = Position[reacshaped, ">"][[1, 1]];
  colonposition = Position[reacshaped, ";"][[1, 1]];
  InitialParticles = Take[reacshaped, {1, wedgeposition - 1}];
  FinalParticles = Take[reacshaped, {wedgeposition + 1, colonposition - 1}];

  (* We check tha the reaction is possible,
  that is it should conserve N and Z*)
  (* If not the case, the code will violently
   quit after spitting out warning messages.*)
  CheckReaction[InitialParticles, FinalParticles];

  Name = StringJoin @@ ToString /@ InitialParticles <>
    "TO" <> StringJoin @@ ToString /@ FinalParticles;
  {Name, InitialParticles, FinalParticles, NormalRealisation, source, f}
 ]



PostTreatT9[var_, funT9_] := If[$InterpolateAnalytics,
   MyInterpolationRate[Table[
     {i, var MyChop[funT9[i / 10^9]]}, {i, TableInterpolationTemperature}]],
   (var MyChop[funT9[# / 10^9]]) &];

GenRateT9[var_, funT9_] := PostTreatT9[var, funT9];
```

■ This is the actual function where all analytic rates are defined. And it outputs the list of reactions.

```
DefineAnalyticRates :=
  Module[{f, Var, Name, λReac, λbarReac, treatedreac, source, reac,
    analyticforward, AddReaction, initialparticles, finalparticles,
    InfoFromAudi2017, FrontFactor, Qoverkb, PoweronT9, forward},

   (* Most recent implementation
    with automatic computation of reverse rate *)
   AddReaction[reac_String, source_String, f_, ForwardT9_, BoolBackward_] := (
     treatedreac = TreatReactionString[reac, source, f];
     Name = treatedreac[[1]];

     (* Building the backward ratio *)
     initialparticles = treatedreac[[2]];
     finalparticles = treatedreac[[3]];
     InfoFromAudi2017 = InfoReaction[initialparticles, finalparticles];
     (*Print[InitialParticles," ",FinalParticles," ",InfoFromAudi2017];*)
     FrontFactor = InfoFromAudi2017[[2]];
     Qoverkb = InfoFromAudi2017[[4]];
     PoweronT9 = InfoFromAudi2017[[3]];
    (* End of building backward ratio *)

    λReac = ToExpression["Hold@L" <> Name];
    λbarReac = ToExpression["Hold@Lbar" <> Name];
    Sow[treatedreac];
    Var = f^treatedreac[[4]];

    MySet[λReac, GenRateT9[Var, ForwardT9]];
    (*MySet[λbarReac,GenRateT9[Var,BackwardT9 ]];*)
    If[BoolBackward,
     MySet[λbarReac, GenRateT9[Var,
         (FrontFactor * #^PoweronT9 * Exp[Qoverkb / #] * ForwardT9[#]) & ]];,
     MySet[λbarReac, GenRateT9[0, 0 & ]];(* No backward reaction *)
    ];

    treatedreac);


Reap[

    (* This is where all extra analytic reactions must be listed.*)
    (* TODO. Explain syntax better, but it si now rather transparent *)
    (* For each reactions added analytically we need to specify
     a String source which is the paper in which it is found *)
    (* Then we give a string reac which is the reaction considered.*)
    (* The factor of incertainty for Monte-Carlo is then given*)
    (* The analytic function forward[T9_],
    which is a function of T9 (that is the temperature in GK)*)
    (* With all these definitions we call AddReaction. The
     last argument is a boolean. If True it computes also
     the reverse rate from detailed balance arguments,
    and if False it does not do so. This is essentially for pure decay
     reactions that there is no need to compute the reverse rates.*)

    (**=================================================================
     *4He,3He,D,7Li (Extra reactions)
      ============================================================
       =======*)
    source = "Nag06";
    reac = " d + n  > t + g ; dng";
    f = 1.40;
    forward[T9_] := With[{T923 = T9^(2 / 3)}, (214. T9^0.075 + 7.42 T9 + T923)];
    AddReaction[reac, source, f, forward, True];
    (* End of first reaction added analytically *)


    source = "Nag06";
    reac = "t+t>a+n+n;ttn";
    f = 3.;
    forward[T9_] := With[{T923 = T9^(2 / 3), T913 = T9^(1 / 3),
       T943 = T9^(4 / 3), T953 = T9^(5 / 3)}, (1/T923 1.67*^9 e^(-4.872/T913)
        (1. - 0.272 T9 + 0.086 T913 - 0.455 T923 + 0.148 T943 + 0.225 T953))];
    AddReaction[reac, source, f, forward, True];
```

```
source = "Wag69";
reac = "He3 +n > He4 + g ; hng";
f = 3.;
forward[T9_] := 6.62 * (1 + 905 * T9);
AddReaction[reac, source, f, forward, True];


source = "CF88";
reac = "He3 + t > He4 + d ; htd";
f = 3.;
forward[T9_] := With[{T9A = T9 / (1. + 0.128 * T9), T932 = T9^(3 / 2)},
  With[{T9A13 = T9A^(1. / 3.), T9A56 = T9A^(5. / 6.)},
   5.46*^9 * T9A56 / T932 * Exp[-7.733 / T9A13]
  ]];
AddReaction[reac, source, f, forward, True];



source = "CF88";
reac = "He3 + t > He4 + n + p ; htp";
f = 3.;
forward[T9_] := With[{T9A = T9 / (1. + 0.115 * T9), T932 = T9^(3 / 2)},
  With[{T9A13 = T9A^(1. / 3.), T9A56 = T9A^(5. / 6.)},
   7.71*^9 * T9A56 / T932 * Exp[-7.733 / T9A13]
  ]];
AddReaction[reac, source, f, forward, True];


source = "NACRE";
reac = "a + a + n > Be9 + g ; aang";
f = 1.25;
forward[T9_] := With[{T932 = T9^(3 / 2), T923 = T9^(2 / 3), T913 = T9^(1 / 3)},
  With[{he4abe8 = 2.43*^9 * (1. + 74.5 * T9) / T923 *
       Exp[-13.49 / T913 - (T9 / 0.15)^2] + 6.09*^5 / T932 * Exp[-1.054 / T9]},
   If[T9 < 0.03,
    (he4abe8) * 6.69*^-12 * (1. - 192 * T9 + 2.48*^4 * T9^2 -
       1.50*^6 * T9^3 + 4.13*^7 * T9^4 - 3.90*^8 * T9^5),
    (he4abe8) * 2.42*^-12 * (1. - 1.52 * Log10[T9] +
       0.448 * (Log10[T9])^2 + 0.435 * (Log10[T9])^3)]]];
AddReaction[reac, source, f, forward, True];


source = "CF88&MF89";
reac = "Li7 + t > a + a + n + n; li7ta";
f = 3.;
forward[T9_] := With[{T923 = T9^(2 / 3), T913 = T9^(1 / 3)},
  8.81*^+11 / T923 * Exp[-11.333 / T913]];
AddReaction[reac, source, f, forward, True];
(* Problem T93 not divided
 in Coc's code. TODO Make sure to correct it.*)


source = "CF88&MF89";
reac = "Li7 + He3 > a + a + n + p; li7haa";
f = 3.;
forward[T9_] := With[{T923 = T9^(2 / 3), T913 = T9^(1 / 3)},
  1.11*^+13 / T923 * Exp[-17.989 / T913]];
```

```
AddReaction[reac, source, f, forward, True];
(* Idem problem T93 not divided in Coc *)


(* TODO Check because the 74 at the end is strange *)
source = "Bal95";
reac = " Li8 + d > Li9 + p ; li8dp";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3)},
  9.63*^6/T923 * Exp[-10.324/T913] * (1. + 0.404 * T913) * 74.];
AddReaction[reac, source, f, forward, True];


source = "Has09c";
reac = " Li8 + d > Li7 + t ; li8dt";
f = 3.;
forward[T9_] := With[
  {T923 = T9^(2/3), T913 = T9^(1/3)}, (3.02*^8/T9^0.624 * Exp[-3.51/T9] +
    5.82*^11/T923 * Exp[-19.72/T913] * (1.0 + 0.280 * T913))];
AddReaction[reac, source, f, forward, True];


source = "CF88";
reac = "Be7 + d > a + a + p ; be7dp";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3)},
  1.07*^+12/T923 * Exp[-12.428/T913]];
AddReaction[reac, source, f, forward, True];


source = "CF88&MF89";
reac = "Be7 + t > a + a + n + p ; be7t";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3)},
  2.91*^+12/T923 * Exp[-13.729/T913]];
AddReaction[reac, source, f, forward, True];
(* Idem problem in Coc's Fortran code. *)


source = "CF88&MF89";
reac = "Be7 + He3 > 2a + p + p  ; be7h";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3)},
  6.11*^+13/T923 * Exp[-21.793/T913]];
AddReaction[reac, source, f, forward, True];
(* Idem problem in COC since here
 it is a division by T93 to get the revsre reaction *)


source = "Wie89";
reac = "C9 + a > N12 + p ; c9an";
f = 3.;
forward[T9_] :=
 With[{T923 = T9^(2/3), T932 = T9^(3/2), T913 = T9^(1/3), T943 = T9^(4/3),
   T953 = T9^(5/3)}, (1.668*^+15/T923 * Exp[-31.272/T913 - (T9/.307)^2] *
     (1. + 1.33*^-2 * T913 - 6.42 * T923 - .599 * T9 + 14.4 * T943 + 3.42 * T953) +
    56.8/T932 * Exp[-5.292/T9] + 1.7*^+5/T932 * Exp[-14.08/T9] +
    6.52*^7 / T932 * Exp[-23.09 / T9]) ];
AddReaction[reac, source, f, forward, True];



(* *=================================================================
 *6Li (Extra reactions)
 *==========================================================
    ==========*)

source = "CF88";
f = 3.;
(*(* TODO  Change this because it is presented as being
 endothermic. That would be better to do the opposite? *)
reac="t+a>Li6+n;tan";
forward[T9_]:=
 With[{T9A=T9/(1.+49.18*T9)},With[{ T9A32=T9A^(3./2.),T932=T9^(3/2)},
   (1.80*^8*Exp[-55.494/T9]*(1.-.261*T9A32/T932)+
     2.72*^9/T932*Exp[-57.884/T9])
  ]];*)


(* Here is the same reaction but presented backward,
such that it is exothermic in the forward direction *)
reac = "Li6+n>t+a;tan";
forward[T9_] := With[{T9A = T9 / (1. + 49.18 * T9)},
  With[{ T9A32 = T9A^ (3. / 2.), T932 = T9^ (3 / 2)},
   (1.80*^+8 * (1. - .261 * T9A32 / T932) * .935 +
     2.72*^9 / T932 * Exp[(55.494 - 57.884) / T9] * .935)
  ]];
AddReaction[reac, source, f, forward, True];


source = "FK90";
reac = "He3 + t > Li6 + g ; htg";
f = 3.;
forward[T9_] :=
 With[{T92 = T9^2, T923 = T9^ (2 / 3), T932 = T9^ (3 / 2), T913 = T9^ (1 / 3),
   T943 = T9^ (4 / 3), T953 = T9^ (5 / 3)}, 2.21*^5 / T923 * Exp[-7.720 / T913] *
   (1. + 2.68 * T923 + 0.868 * T9 + 0.192 * T943 + 0.174 * T953 + 0.044 * T92)];
AddReaction[reac, source, f, forward, True];


 source = "CF88";
reac = "a + n + p > Li6 + g ; anpg";
f = 3.;
forward[T9_] :=
 If[T9 > 1, 4.62*^-6 / T9^2 * (1. + 0.075 * T9) * Exp[-19.353 / T9], 0];
AddReaction[reac, source, f, forward, True];


 source = "MF89";
reac = "Li6 + n > Li7 + g ; li6ng";
f = 3.;
forward[T9_] := 5.10*^3;
AddReaction[reac, source, f, forward, True];
```

```
 source = "MF89";
reac = "Li6 + d > Li7 + p ; li6dp";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3)},
   1.48*^12/T923 * Exp[-10.135/T913]];
AddReaction[reac, source, f, forward, True];

 source = "MF89";
reac = "Li6 + d > Be7 + n ; li6dn";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3)},
   1.48*^12/T923 * Exp[-10.135/T913]];
AddReaction[reac, source, f, forward, True];


(**=====================================================================
   *Berylium& Boron (Main reactions)
  *===========================================================
      ==========*)
source = "CF88";
reac = "Li6 + a > B10 + g ; li6ag";
f = 3.;
forward[T9_] :=
 With[{T923 = T9^(2/3), T932 = T9^(3/2), T913 = T9^(1/3), T943 = T9^(4/3),
   T953 = T9^(5/3)}, (4.06*^06/T923 * Exp[-18.79/T913 - (T9/1.326)^2] *
      (1. + 0.022 * T913 + 1.54 * T923 + 0.239 * T9 + 2.2 * T943 + 0.869 * T953)
     + 1.91*^3/T932 * Exp[-3.484/T9] + 1.01*^4/T9 * Exp[-7.269/T9])];
AddReaction[reac, source, f, forward, True];


source = "NACRE";
reac = " Li7 + a > B10 + n ; li7an / b10na";
f = 1.08;
forward[T9_] :=
 1.325 * 1.66*^7 * (1. + 1.064 * T9) * 1/1.3242 * Exp[-32.3755/T9];
AddReaction[reac, source, f, forward, True];

(* C MF89 remplace Wiescher et al.ApJ 464 (1989) 464.C Voir Blackmon
   et al.PRC 54 (1996) 383& Heil et al.ApJ 507 (1998) 997.*)
      (*%MF89Hei98   *)
source = "MF89&Hei98";
reac = "Li7+n>Li8+g;li7ng";
f = 3.;
forward[T9_] :=
 With[{T932 = T9^(3/2)}, (6.015*^3 + 1.141*^4/T932 * Exp[-2.576/T9])];
AddReaction[reac, source, f, forward, True];


(*Replace by exothermic reaction ?*)
source = "MF89";
reac = "Li7 + d > Li8 + p ; li7dp ! Q<0 !";
f = 3.;
forward[T9_] := With[{T932 = T9^(3/2)}, 8.31*^8/T932 * Exp[-6.998/T9]];
AddReaction[reac, source, f, forward, True];


source = "Rau94";
reac = "Li8 + n > Li9 + g ; li8ng";
```

```
f = 3.;
forward[T9_] :=
 With[{T932 = T9^(3/2)}, (3.260*^3 + 6.328*^4/T932 * Exp[-2.866/T9])];
AddReaction[reac, source, f, forward, True];


source = "Men12";
reac = "Li8 + p > a + a + n ; li8pn";
forward[T9_] := With[{T932 = T9^(3/2), T913 = T9^(1/3),
   T923 = T9^(2/3), T92 = T9^2, T93 = T9^3, T94 = T9^4, T95 = T9^5},
  If[T9 < 5, (
    5.36*^8/T932 * Exp[-4.41/T9] + 1.99*^8/T932 * Exp[-7.08/T9] +
     5.85*^10/T923 * Exp[-8.50/T913] * (1. - 1.70 * T9 +
        0.849 * T92 - 0.175 * T93 + 1.62*^-2 * T94 - 5.60*^-4 * T95)),
   7.777*^7]];
AddReaction[reac, source, f, forward, True];


source = "Bal95";
reac = "Li8 + d > Be9 + n ; li8dn";
f = 3.;
forward[T9_] := With[{T913 = T9^(1/3), T923 = T9^(2/3)},
  9.63*^6/T923 * Exp[-10.324/T913] * (1. + 0.404 * T913) * 188.];
AddReaction[reac, source, f, forward, True];

(**=================================================================
 *Beryllium& Boron (Extra reactions)
 *=============================================================
    ========== *)
source = "Rau94";
reac = "Be9 + n > Be10 + g ; be9ng";
f = 3.;
forward[T9_] :=
 With[{T913 = T9^(1/3), T923 = T9^(2/3), T932 = T9^(3/2)}, (1.01*^3 +
    1.01*^4/T932 * Exp[-6.487/T9] + 5.41*^4/T932 * Exp[-8.471/T9])];
AddReaction[reac, source, f, forward, True];

source = "NACRE";
reac = "Be9 + p > a + a + p + n ; be9pn";
f = 1.05;
forward[T9_] := 5.06*^7 * Exp[-21.479/T9] * (1. + 1.26 * T9 - 0.0302 * T9^2);
AddReaction[reac, source, f, forward, True];
(* I find that it is division by
 T93 by Coc used multiplication by T93 ! Carefull !!!*)

source = "NACRE";
reac = "B11 + p > C11 + n ; b11pn ! Q < 0 !";
f = 1.1;
forward[T9_] := 1.36*^8 * Exp[-32.085/T9] *
  (1. + 0.963 * T9 - 0.285 * T9^2 + 3.36*^-2 * T9^3 - 1.37*^-3 * T9^4);
AddReaction[reac, source, f, forward, True];


source = "Rau94";
reac = " Be10 + n > Be11 + g ; be10ng";
f = 3.;
forward[T9_] :=
 With[{T932 = T9^(3/2)}, (5.96*^2 + 6.67*^5/T932 * Exp[-14.85/T9]) ];
AddReaction[reac, source, f, forward, True];


source = "Rau94";
reac = "Be11 + n > Be12 + g ; be11ng";
f = 3.;
forward[T9_] := With[{T932 = T9^(3/2)}, 3.56*^2 ];
AddReaction[reac, source, f, forward, True];


source = "Des99Bea01";
reac = "B8 + p > C9 + g ; b8pg";
f = 3.;
forward[T9_] := With[{T932 = T9^(3/2), T913 = T9^(1/3), T92 = T9^2},
  6.253*^5 * Exp[-11.971/T913] * (1. - 7.03*^-2 * T9 + 6.25*^-3 * T92)];
AddReaction[reac, source, f, forward, True];

(* ======================================================================
    *Leaks to CNO
 !*=========================================================
    =============*)

source = "NACRE";
reac = "a + a + a > C12 + 2g ; aaag";
f = 1.15;
forward[T9_] := With[{T932 = T9^(3/2), T923 = T9^(2/3), T913 = T9^(1/3)},
  With[{he4abe8 = 2.43*^9 * (1. + 74.5 * T9) / T923 *
        Exp[-13.49/T913 - (T9/0.15)^2] + 6.09*^5/T932 * Exp[-1.054/T9],
     be8agc12 =  2.76*^7 * (1. + 5.47 * T9 + 326 * T9^2) /T923 *
        Exp[-23.570/T913 - (T9/0.4)^2] +
       130.7/T932 * Exp[-3.338/T9] + 2.51*^4/T932 * Exp[-20.307/T9]},
    If[T9 < 0.03,
     he4abe8 * be8agc12 * 3.07*^-16 * (1. - 29.1 * T9 + 1308 * T9^2),
     he4abe8 * be8agc12 * 3.44*^-16 * (1. + 0.0158/T9^0.65)]]];
AddReaction[reac, source, f, forward, True];

source = "Tang03";
reac = "C11+p>N12+g;c11pg";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T932 = T9^(3/2),
    T913 = T9^(1/3), T943 = T9^(4/3), T953 = T9^(5/3)},
   (1.670*^2 * Exp[-4.166/T9] / T932 + 2.148*^5 * Exp[-13.281/T913] / T923 *
      (1. + 4.639 * T913 - 2.641 * T923 - 1.543 * T9 + 2.030 * T943 + 4.657 * T953))];
AddReaction[reac, source, f, forward, True];


source = "CF88";
reac = "B10 + a > N13 + n ; b10an";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3)},
  1.2*^13/T923 * Exp[-27.989/T913 - (T9/9.589)^2] ];
AddReaction[reac, source, f, forward, True];

source = "Wan91";
reac = "B11+a>C14+p;b11ap";
f = 3.;
```

```
forward[T9_] :=
 With[{T923 = T9^(2/3), T932 = T9^(3/2), T913 = T9^(1/3), T943 = T9^(4/3),
   T953 = T9^(5/3)}, (8.403*^15 * Exp[-31.914 / T913 - (T9 / 0.3432)^2] *
     (1. + 0.022 * T913 + 5.712 * T923 + 0.642 * T9 + 15.982 * T943 + 4.062 * T953)
    + 5.44*^-3 / T932 * Exp[-2.868 / T9] + 2.419*^2 / T932 * Exp[-5.147 / T9] +
    4.899*^2 / T932 * Exp[-5.157 / T9] +
    4.944*^6 / T9^(3/5) * Exp[-11.26 / T9])];
AddReaction[reac, source, f, forward, True];


source = "Rau94";
reac = "C11+n>C12+g;c11ng";
f = 3.;
forward[T9_] := With[{T932 = T9^(3/2)}, (3.18*^4 +
    3.30*^3 / T932 * Exp[-0.917 / T9] + 1.05*^6 / T932 * Exp[-5.57 / T9])];
AddReaction[reac, source, f, forward, True];


(*=======================================================================*)
(*       Decay Rates                                                   *)
(*=======================================================================*)
(* %Aud03  *)
(* All decay rates from %Aud03  *)

source = "Aud03";
reac = "He6>Li6+Bm;";
forward[T9_] := Log[2] / 8.0670*^-1 ;
AddReaction[reac, source, 1, forward, False];
(* The 1 is because we do not put uncertainty on decays,
and the False because we do not put reverse reactions on decays *)

reac = "Li8>2a+Bm;";
forward[T9_] := Log[2] / 8.4030*^-1 ;
AddReaction[reac, source, 1, forward, False];

reac = "Li9>Be9+Bm;";
forward[T9_] := Log[2] / 1.7830*^-1 * 0.492 ;
AddReaction[reac, source, 1, forward, False];

reac = "Li9>a+a+n+Bm;";
forward[T9_] := Log[2] / 1.7830*^-1 * 0.508 ;
AddReaction[reac, source, 1, forward, False];

reac = "Be11>B11+Bm;";
forward[T9_] := Log[2] / (1.3810*^1) ;
AddReaction[reac, source, 1, forward, False];

reac = "Be12>B12+Bm;";
forward[T9_] := Log[2] / (2.15*^-2) ;
AddReaction[reac, source, 1, forward, False];

reac = "B8>a+a+Bp;";
forward[T9_] := Log[2] / (7.70*^-1) ;
AddReaction[reac, source, 1, forward, False];

reac = "B12>C12+Bm;";
forward[T9_] := Log[2] / (2.02*^-2) ;
```

```
AddReaction[reac, source, 1, forward, False];

reac = "B13>C13+Bm;";
(* !04/11/2010 *)
forward[T9_] := Log[2] / (1.733*^-2) ;
AddReaction[reac, source, 1, forward, False];

reac = "B14>C14+Bm;";
(* !04/11/2010 *)
forward[T9_] := Log[2] / (1.25*^-2) ;
AddReaction[reac, source, 1, forward, False];

 reac = "B15>C15+Bm;";
(* !04/11/2010 *)
forward[T9_] := Log[2] / (9.87*^-3) ;
AddReaction[reac, source, 1, forward, False];

reac = "C9>a+a+p+Bp;";
forward[T9_] := Log[2] / (1.26*^-1) ;
AddReaction[reac, source, 1, forward, False];

reac = "C10>B10+Bp;";
forward[T9_] := Log[2] / (19.29) ;
AddReaction[reac, source, 1, forward, False];

reac = "C11>B11+Bp;";
forward[T9_] := Log[2] / 1.2234*^3 ;
AddReaction[reac, source, 1, forward, False];

 reac = "C15>N15+Bm;";
(*28/10/2010*)
forward[T9_] := Log[2] / 2.449 ;
AddReaction[reac, source, 1, forward, False];

reac = "C16>N16+Bm;";
(*14/01/2011*)
forward[T9_] := Log[2] / 7.4700*^-1 ;
AddReaction[reac, source, 1, forward, False];

reac = "N12>C12+Bp;";
forward[T9_] := Log[2] / 1.100*^-2 ;
AddReaction[reac, source, 1, forward, False];

reac = "N13>C13+Bp;";
(*14/01/2011*)
forward[T9_] := Log[2] / 5.979*^2 ;
AddReaction[reac, source, 1, forward, False];

reac = "N16>O16+Bm;";
(*14/01/2011*)
forward[T9_] := Log[2] / 7.13 ;
AddReaction[reac, source, 1, forward, False];

reac = "N17>O16+n+Bm;";
(*14/01/2011*)
forward[T9_] := Log[2] / 4.1730 ;
AddReaction[reac, source, 1, forward, False];
```

```
reac = "O13>N13+Bp;";
(*14/01/2011*)
forward[T9_] := Log[2] / 8.58*^-3 ;
AddReaction[reac, source, 1, forward, False];

reac = "O14>N14+Bp;";
(*14/01/2011*)
forward[T9_] := Log[2] / 70.598 ;
AddReaction[reac, source, 1, forward, False];

reac = "O15>N15+Bp;";
(*14/01/2011*)
forward[T9_] := Log[2] / 122.24;
AddReaction[reac, source, 1, forward, False];

reac = "O19>F19+Bm;";
(*14/01/2011*)
forward[T9_] := Log[2] / 26.464;
AddReaction[reac, source, 1, forward, False];

reac = "O20>F20+Bm;";
(*14/01/2011*)
forward[T9_] := Log[2] / 13.51;
AddReaction[reac, source, 1, forward, False];

reac = "F17>O17+Bp;";
(*04/11/2010*)
forward[T9_] := Log[2] / 64.49;
AddReaction[reac, source, 1, forward, False];

reac = "F18>O18+Bp;";
(*04/11/2010*)
forward[T9_] := Log[2] / 6.5863*^3;
AddReaction[reac, source, 1, forward, False];

reac = "F20>Ne20+Bm;";
(*04/11/2010*)
forward[T9_] := Log[2] / 11.1630;
AddReaction[reac, source, 1, forward, False];

reac = "Ne18>F18+Bp;";
(*04/11/2010*)
forward[T9_] := Log[2] / 1.6720;
AddReaction[reac, source, 1, forward, False];

reac = "Ne19>F19+Bp;";
(*04/11/2010*)
forward[T9_] := Log[2] / 17.296;
AddReaction[reac, source, 1, forward, False];

reac = "Ne23>Na23+Bm;";
(*04/11/2010*)
forward[T9_] := Log[2] / 37.240;
AddReaction[reac, source, 1, forward, False];

reac = "Na20>Ne20+Bp;";
(*14/01/2011*)
forward[T9_] := Log[2] / 4.4790*^-1;
AddReaction[reac, source, 1, forward, False];
```

```
reac = "Na21>Ne21+Bp;";
(*04/11/2010*)
forward[T9_] := Log[2] / 22.49;
AddReaction[reac, source, 1, forward, False];

(* *==================================================================
   *New reactions following Thomas,Schramm et al.1993;1994
 *==========================================================
   =======*)

source = "Efr96";
reac = "He4 + 2n  > He6 + g ;";
f = 3.;
forward[T9_] := If[T9 < 2,
   (2.65*^-3 * T9^2.555 * Exp[0.181 / Max[T9, .1]]),
   (2.93*^-1 * T9^ (-3.51*^-1) * Exp[-5.24 / T9])];
AddReaction[reac, source, f, forward, True];


source = "Iga95";
reac = "O16 + n  > O17 + g ;";
f = 3.;
forward[T9_] := (2.7*^1 + 1.38*^4 * T9 );
AddReaction[reac, source, f, forward, True];


source = "CF88";
reac = "N14 + n  > C14 + p ;";
f = 3.;
forward[T9_] :=
 With[{T912 = T9^ (1 / 2)}, ( 7.19*^5 * (1. + .361 * T912 + .502 * T9) +
      3.34*^8 / T912 * Exp[-4.983 / T9]) * .333];
AddReaction[reac, source, f, forward, True];


source = "CF88";
reac = "O14 + n  > N14 + p ;";
f = 3.;
forward[T9_] :=
 With[{T912 = T9^ (1 / 2)}, (6.74*^7 * (1. + 0.658 * T912 + 0.379 * T9) * 2.99 )];
AddReaction[reac, source, f, forward, True];


source = "Wie87";
reac = "O14 + a  > Ne18 + g ;";
f = 3.;
forward[T9_] := With[{T932 = T9^ (3 / 2)}, ( 1.16*^-1 / T932 * Exp[-11.73 / T9] +
     3.40*^1 / T932 * Exp[-22.61 / 79] + 9.10*^-3 * T9^5 * Exp[-12.159]) ];
AddReaction[reac, source, f, forward, True];


source = "NACRE";
reac = "C11 + a  > N14 + p ;";
f = 2.;
forward[T9_] := With[
  {T913 = T9^ (1 / 3), T92 = T9^2}, (0.2719 * 3.01*^16 * Exp[-31.884 / T913] *
    Exp[-1.379 * T9 + .215 * T92 - 2.13*^-2 * T92 * T9 + 8*^-4 * T92 * T92] *
    (1. + 0.14 * Exp[-.275 / T9 - .210 * T9]) )];
AddReaction[reac, source, f, forward, True];


source = "Bar97C";
reac = "O14 + a  > F17 + p ;";
f = 3.;
forward[T9_] := With[{T932 = T9^(3/2), T923 = T9^(2/3),
   T913 = T9^(1/3), T943 = T9^(4/3), T953 = T9^(5/3)},
  With[{offset = 1.330*^5 / T932 * Exp[-11.86 / T9] +
       8.42*^-47 * T932 * Exp[-0.453 / T9] + 6.74*^4 / T932 * Exp[-13.60 / T9] +
       1.21*^7 / T932 * Exp[-22.51 / T9] + 1.26*^8 / T932 * Exp[-26.00 / T9]},
   (offset + If[T9 < 1,
       7.906*^15 / T923 * Exp[-40.33 / T913] * (1. - 1.884*^1 * T913 + 2.446*^2 *
            T923 - 7.735*^2 * T9 + 9.485*^2 * T943 - 3.961*^2 * T953), 0])]];
AddReaction[reac, source, f, forward, True];


source = "Koe91";
reac = " O17 + n > C14 + a ;";
f = 3.;
forward[T9_] := With[{T932 = T9^(3/2)}, ( 3.11*^4 +
    9.18*^5 / T932 * Exp[-1.961 / T9] + 7.02*^7 / T932 * Exp[-2.759 / T9])];
AddReaction[reac, source, f, forward, True];


source = "NACRE";
(* Check this one because there seems to be a typo in the
 exponential. Or maybe this is correct but this is strange. *)
reac = "F17 + n > N14 + a ;";
f = 1.05;
forward[T9_] := (1.38*^8 * T9^0.053 * Exp[-(55.0 - 54.943) / T9] *
   (1. + .039 * Exp[-.012 / T9 + .217 * T9]) / 1.478 );
AddReaction[reac, source, f, forward, True];


source = "CF88";
reac = "F18 + n > N15 + a ;";
f = 3.;
forward[T9_] :=
 With[{T912 = T9^(1/2)}, ( 3.14*^8 * (1. - 0.641 * T912 + 0.108 * T9) * 2.)];
AddReaction[reac, source, f, forward, True];


source = "Kaw91";
reac = "C14 + d  > N15 + n ;";
f = 3.;
forward[T9_] := With[{T923 = T9^(2/3)}, (4.27*^13 / T923 * Exp[-16.939])];
AddReaction[reac, source, f, forward, True];


source = "CF88";
reac = "p + p + n > d + p ;";
f = 3.;
forward[T9_] :=
 With[{T923 = T9^(2/3), T913 = T9^(1/3)}, (1.35*^7 * Exp[-3.720 / T913] *
    (1. + 0.784 * T913 + 0.346 * T923 + 0.690 * T9) / 2.3590*^9)];
    AddReaction[reac, source, f, forward, True];


    source = "Kaw91";
    reac = "C14  +  n  >  C15  +  g  ;";
    f = 3.;
    forward[T9_] := (3240. * T9 );
    AddReaction[reac, source, f, forward, True];


    source = "CF88";
    reac = "  O16  +  p   >  N13  +  a  ;";
    f = 3.;
    forward[T9_] := With[{T953 = T9^(5/3), T932 = T9^(3/2)},
      With[{T9A =
          T9 / (1. + 7.76*^-2 * T9 + 2.64*^-2 * T953 / (1. + 7.76*^-2 * T9)^(2./3.))},
        With[{T9A13 = T9A^(1./3.), T9A56 = T9A^(5./6.)},
          With[{SVRev = 1.88*^18 * T9A56 / T932 * Exp[-35.829 / T9A13] * 1.7232*^-1},
            With[{SVDir = SVRev / 0.172255 * Exp[-60.5573 / T9]},
             SVDir]]]]];
    AddReaction[reac, source, f, forward, True];



     (* %TUNL&Cam08   !Camargo et al.Phys.Rev.C 78,034605 (2008) pour DC
      !Tilley (TUNL) Table 9.5 pour la res.a 87 keV (dominante)   *)
    source = "TUNL&Cam08";
    reac = "Li8  +  p  >  Be9  +  g  ;";
    f = 3.;
    forward[T9_] := With[{T923 = T9^(2/3), T913 = T9^(1/3), T932 = T9^(3/2)},
      (3.516*^6 / T923 * Exp[-8.5155 / T913] + 2.669*^4 / T932 * Exp[-1.010 / T9] )];
    AddReaction[reac, source, f, forward, True];


    source = "Wan91";
    reac = "B11  +  a   >  N15  +  g  ;";
    f = 3.;
    forward[T9_] := With[{T932 = T9^(3/2)}, (643. / T932 * Exp[-5.1526 / T9] )];
    AddReaction[reac, source, f, forward, True];

   ][[2, 1]]

  (* The output is the list of reactions in
   standard format (Name,List initial,List final,f factor) which
   is then used by the differential equation constructor *)

];
```

Collecting all reaction rates

We collect the description of all rates in a single table (ListReactions). This include the weak rate (1 reaction), all the reactions from the external file (generated by the function LoadRates), and the reactions which were given analytically by the function DefineAnalyticRates.

```
ReactionPEN = {"nTOp", {"n"}, {"p"}, 0, "Companion Paper"};
(* Format is name, List of initial particles,
List of final particles, f factor for uncertainty*)

TabulatedReactions :=
  (Select[SafeImport[TabulatedReactionsFile],
    (NumericQ[#[[1]]] || "*-" == #[[1]] || StringMatchQ[#[[1]], "\\*%" ~~ __]) &]);
ReshapedTabulatedReactions := TreatData[TabulatedReactions];

$TabulatedAnalyticReactions = False;
TabulatedReactionsAnalyticFile = "BBNRatesFromAnalytic.dat";

LoadRates := Module[{len},
   len = Length@ReshapedTabulatedReactions;
   ListReactionsFile = TreatReactionLine /@
     (Take[ReshapedTabulatedReactions, Min[NumberNuclearReactions, len]]);
   (* If the number is larger than the file,
   we also dig into the analytic expressions *)
   If [NumberNuclearReactions > len,

    If[$TabulatedAnalyticReactions,

     TabulatedReactionsAnalytic =
      Select[SafeImport[TabulatedReactionsAnalyticFile], (NumericQ[#[[1]]] ||
          "*-" == #[[1]] || StringMatchQ[#[[1]], "\\*%" ~~ __]) &];
     ExtraAnalyticReactions = TreatReactionLine /@
       TreatData[TabulatedReactionsAnalytic];,

     ExtraAnalyticReactions = DefineAnalyticRates;];

    ListReactions = Take[Join[{ReactionPEN}, ListReactionsFile,
       ExtraAnalyticReactions], NumberNuclearReactions + 1],
    ListReactions = Join[{ReactionPEN}, ListReactionsFile]]
  ];
```

LoadRates is the function which does all that. Let us call it. It will stop and quit if one of the reactions is inconsistent (not conservation of N nor Z).

```
LoadRates;
```

We now restrict the nuclides up to a maximum mass. Useless if MaximumNuclearMass has been set to Infinity.

```
SpeciesUpToMaximumMass[A_Integer] :=
  SpeciesUpToMaximumMass[A] = Union @@ Table[NamesMassNumberAll[i], {i, A}];
ReactionUpToMaximumMass[A_Integer][Reaction_List] :=
  And @@ (MemberQ[SpeciesUpToMaximumMass[A], #] & /@ Flatten@Reaction[[2 ;; 3]]);
ListReactionsUpToMass[A_Integer, ListReactions_List] :=
  Select[ListReactions, ReactionUpToMaximumMass[A][#] &];
ListReactionsUpToMass[Infinity, ListReactions_List] := ListReactions;
ListReactionsUpToChosenMass :=
  ListReactionsUpToMass[MaximumNuclearMass, ListReactions];
```

For information let us print the list of reactions which are taken into account.

```
ReactionWithArrow[name_String] := StringReplace[name, "TO" → " -> "]
NiceDisplayReaction[reaction_List] :=
 Join[{ReactionWithArrow[First[reaction]]}, Rest[reaction]]
```

```
MyGrid[
 Join[{{"Reaction Name", "Initial species", "Final Species", "Uncertainty",
    "Reference"}}, NiceDisplayReaction /@ ListReactionsUpToChosenMass]]
```

Reaction Name	Initial species	Final Species	Uncertainty	Reference	
n -> p	{n}	{p}	0	Companion Paper	
np -> dg	{n, p}	{d, g}	0	And06	
dp -> He3g	{d, p}	{He3, g}	0	Ili16	
dd -> He3n	{d, d}	{He3, n}	0	Gom17	
dd -> tp	{d, d}	{t, p}	0	Gom17	
tp -> ag	{t, p}	{a, g}	0	Ser04	
td -> an	{t, d}	{a, n}	0	DAACV04	
ta -> Li7g	{t, a}	{Li7, g}	0	DAACV04	
He3n -> tp	{He3, n}	{t, p}	0	DAACV04	
He3d -> ap	{He3, d}	{a, p}	0	DAACV04	
He3a -> Be7g	{He3, a}	{Be7, g}	0	Ili16	
Be7n -> Li7p	{Be7, n}	{Li7, p}	0	DAACV04	
Li7p -> aa	{Li7, p}	{a, a}	0	DAACV04	
Li7p -> aag	{Li7, p}	{a, a, g}	0	NACRE	
Be7n -> aa	{Be7, n}	{a, a}	0	Bar16	
da -> Li6g	{d, a}	{Li6, g}	0	Ham10	
Li6p -> Be7g	{Li6, p}	{Be7, g}	0	NACRE	
Li6p -> He3a	{Li6, p}	{He3, a}	0	NACRE	
Be9t -> B11n	{Be9, t}	{B11, n}	0	TALYS2	
O18n -> O19g	{O18, n}	{O19, g}	0	TALYS2	
Li9p -> He6a	{Li9, p}	{He6, a}	0	=li7pa!!	
Li9d -> Be10n	{Li9, d}	{Be10, n}	0	TALYS2	
Be10a -> C14g	{Be10, a}	{C14, g}	0	TALYS2	
N12n -> C12p	{N12, n}	{C12, p}	0	TALYS2	
Li9p -> Be9n	{Li9, p}	{Be9, n}	0	TALYS2	
Li9a -> B12n	{Li9, a}	{B12, n}	0	New2011	
Li9p -> Be10g	{Li9, p}	{Be10, g}	0	TALYS2	
N13n -> N14g	{N13, n}	{N14, g}	0	TALYS2	
B10a -> N14g	{B10, a}	{N14, g}	0	TALYS2	
B8a -> N12g	{B8, a}	{N12, g}	0	TALYS2	
B12p -> Be9a	{B12, p}	{Be9, a}	0	TALYS2	
Be10p -> B11g	{Be10, p}	{B11, g}	0	TALYS2	
Be10p -> Li7a	{Be10, p}	{Li7, a}	0	TALYS2	
Be11p -> Li8a	{Be11, p}	{Li8, a}	0	TALYS2	
Be11p -> B11n	{Be11, p}	{B11, n}	0	TALYS2	
B8n -> aap	{B8, n}	{a, a, p}	0	TALYS2	
B10n -> B11g	{B10, n}	{B11, g}	0	TALYS2	
B10a -> C13p	{B10, a}	{C13, p}	0	TALYS2	
O17n -> O18g	{O17, n}	{O18, g}	0	TALYS2	
F17n -> O17p	{F17, n}	{O17, p}	0	TALYS2	
F18n -> O18p	{F18, n}	{O18, p}	0	TALYS2	
Be10a -> C13n	{Be10, a}	{C13, n}	0	TALYS2	
Be11a -> C14n	{Be11, a}	{C14, n}	0	TALYS2	
N14a -> F18g	{N14, a}	{F18, g}	0	ILCCF10	
N15a -> F19g	{N15, a}	{F19, g}	0	ILCCF10	
O15a -> Ne19g	{O15, a}	{Ne19, g}	0	ILCCF10	
O16p -> F17g	{O16, p}	{F17, g}	0	ILCCF10	
O16a -> Ne20g	{O16, a}	{Ne20, g}	0	ILCCF10	
O17p -> F18g	{O17, p}	{F18, g}	0	ILCCF10	
O18p -> F19g	{O18, p}	{F19, g}	0	ILCCF10	
O18a -> Ne22g	{O18, a}	{Ne22, g}	0	ILCCF10	
F17p -> Ne18g	{F17, p}	{Ne18, g}	0	ILCCF10	
F18p -> Ne19g	{F18, p}	{Ne19, g}	0	ILCCF10	
Ne19p -> Na20g	{Ne19, p}	{Na20, g}	0	ILCCF10	
O17p -> N14a	{O17, p}	{N14, a}	0	ILCCF10	
O18p -> N15a	{O18, p}	{N15, a}	0	ILCCF10	

F18p -> O15a	{F18, p}	{O15, a}	0	ILCCF10	
C14a -> O18g	{C14, a}	{O18, g}	0	ILCCF10	
C14p -> N15g	{C14, p}	{N15, g}	0	ILCCF10	
Be12p -> Li9a	{Be12, p}	{Li9, a}	0	TALYS2	
Li6He3 -> aap	{Li6, He3}	{a, a, p}	0	TALYS2	
Li6t -> Be9g	{Li6, t}	{Be9, g}	0	TALYS2	
Li6t -> aan	{Li6, t}	{a, a, n}	0	TALYS2	
Li6t -> Li8p	{Li6, t}	{Li8, p}	0	TALYS2	
Li7d -> Be9g	{Li7, d}	{Be9, g}	0	New2011	
Li7He3 -> B10g	{Li7, He3}	{B10, g}	0	TALYS2	
Li7He3 -> Li6a	{Li7, He3}	{Li6, a}	0	TALYS2	
Li7t -> Be10g	{Li7, t}	{Be10, g}	0	TALYS2	
Li8a -> B12g	{Li8, a}	{B12, g}	0	TALYS2	
Li8a -> B11n	{Li8, a}	{B11, n}	0	New2011	
Li8d -> Be10g	{Li8, d}	{Be10, g}	0	TALYS2	
Li8He3 -> B11g	{Li8, He3}	{B11, g}	0	TALYS2	
Li8He3 -> B10n	{Li8, He3}	{B10, n}	0	TALYS2	
Li8He3 -> Be10p	{Li8, He3}	{Be10, p}	0	TALYS2	
Li8He3 -> Li7a	{Li8, He3}	{Li7, a}	0	TALYS2	
Li8t -> Be11g	{Li8, t}	{Be11, g}	0	TALYS2	
Li8t -> Be10n	{Li8, t}	{Be10, n}	0	TALYS2	
Li9a -> B13g	{Li9, a}	{B13, g}	0	TALYS2	
Li9d -> Be11g	{Li9, d}	{Be11, g}	0	TALYS2	
Li9He3 -> B12g	{Li9, He3}	{B12, g}	0	TALYS2	
Li9He3 -> B11n	{Li9, He3}	{B11, n}	0	TALYS2	
Li9He3 -> Be11p	{Li9, He3}	{Be11, p}	0	TALYS2	
Li9He3 -> Li8a	{Li9, He3}	{Li8, a}	0	TALYS2	
Li9t -> Be12g	{Li9, t}	{Be12, g}	0	TALYS2	
Li9t -> Be11n	{Li9, t}	{Be11, n}	0	TALYS2	
Be7He3 -> C10g	{Be7, He3}	{C10, g}	0	TALYS2	
Be7t -> B10g	{Be7, t}	{B10, g}	0	TALYS2	
Be7t -> Be9p	{Be7, t}	{Be9, p}	0	New2011	
Be7t -> Li6a	{Be7, t}	{Li6, a}	0	TALYS2	
Be9a -> C13g	{Be9, a}	{C13, g}	0	TALYS2	
Be9d -> B11g	{Be9, d}	{B11, g}	0	TALYS2	
Be9d -> B10n	{Be9, d}	{B10, n}	0	TALYS2	
Be9d -> Be10p	{Be9, d}	{Be10, p}	0	TALYS2	
Be9d -> Li7a	{Be9, d}	{Li7, a}	0	TALYS2	
Be9He3 -> C12g	{Be9, He3}	{C12, g}	0	TALYS2	
Be9He3 -> C11n	{Be9, He3}	{C11, n}	0	TALYS2	
Be9He3 -> B11p	{Be9, He3}	{B11, p}	0	TALYS2	
Be9He3 -> aaa	{Be9, He3}	{a, a, a}	0	TALYS2	
Be9t -> B12g	{Be9, t}	{B12, g}	0	TALYS2	
Be9t -> Li8a	{Be9, t}	{Li8, a}	0	TALYS2	
Be10d -> B12g	{Be10, d}	{B12, g}	0	TALYS2	
Be10d -> B11n	{Be10, d}	{B11, n}	0	TALYS2	
Be10d -> Li8a	{Be10, d}	{Li8, a}	0	TALYS2	
Be10He3 -> C13g	{Be10, He3}	{C13, g}	0	TALYS2	
Be10He3 -> C12n	{Be10, He3}	{C12, n}	0	TALYS2	
Be10He3 -> B12p	{Be10, He3}	{B12, p}	0	TALYS2	
Be10He3 -> Be9a	{Be10, He3}	{Be9, a}	0	TALYS2	
Be10t -> B13g	{Be10, t}	{B13, g}	0	TALYS2	
Be10t -> B12n	{Be10, t}	{B12, n}	0	TALYS2	
Be10t -> Li9a	{Be10, t}	{Li9, a}	0	TALYS2	

Be11a -> C15g	{Be11, a}	{C15, g}	0	TALYS2	
Be11d -> B13g	{Be11, d}	{B13, g}	0	TALYS2	
Be11d -> B12n	{Be11, d}	{B12, n}	0	TALYS2	
Be11d -> Be12p	{Be11, d}	{Be12, p}	0	TALYS2	
Be11d -> Li9a	{Be11, d}	{Li9, a}	0	TALYS2	
Be11He3 -> C14g	{Be11, He3}	{C14, g}	0	TALYS2	
Be11He3 -> C13n	{Be11, He3}	{C13, n}	0	TALYS2	
Be11He3 -> B13p	{Be11, He3}	{B13, p}	0	TALYS2	
Be11He3 -> Be10a	{Be11, He3}	{Be10, a}	0	TALYS2	
Be11p -> B12g	{Be11, p}	{B12, g}	0	TALYS2	
Be11t -> B14g	{Be11, t}	{B14, g}	0	TALYS2	
Be11t -> B13n	{Be11, t}	{B13, n}	0	TALYS2	
Be12a -> C16g	{Be12, a}	{C16, g}	0	TALYS2	
Be12a -> C15n	{Be12, a}	{C15, n}	0	TALYS2	
Be12d -> B14g	{Be12, d}	{B14, g}	0	TALYS2	
Be12d -> B13n	{Be12, d}	{B13, n}	0	TALYS2	
Be12He3 -> C15g	{Be12, He3}	{C15, g}	0	TALYS2	
Be12He3 -> C14n	{Be12, He3}	{C14, n}	0	TALYS2	
Be12He3 -> B14p	{Be12, He3}	{B14, p}	0	TALYS2	
Be12He3 -> Be11a	{Be12, He3}	{Be11, a}	0	TALYS2	
Be12p -> B13g	{Be12, p}	{B13, g}	0	TALYS2	
Be12p -> B12n	{Be12, p}	{B12, n}	0	TALYS2	
Be12t -> B15g	{Be12, t}	{B15, g}	0	TALYS2	
Be12t -> B14n	{Be12, t}	{B14, n}	0	TALYS2	
B8a -> C11p	{B8, a}	{C11, p}	0	TALYS2	
B8d -> C10g	{B8, d}	{C10, g}	0	TALYS2	
B8He3 -> C10p	{B8, He3}	{C10, p}	0	TALYS2	
B8t -> C11g	{B8, t}	{C11, g}	0	TALYS2	
B8t -> C10n	{B8, t}	{C10, n}	0	TALYS2	
B8t -> B10p	{B8, t}	{B10, p}	0	TALYS2	
B8t -> Be7a	{B8, t}	{Be7, a}	0	TALYS2	
B10d -> C12g	{B10, d}	{C12, g}	0	TALYS2	
B10d -> C11n	{B10, d}	{C11, n}	0	TALYS2	
B10d -> B11p	{B10, d}	{B11, p}	0	TALYS2	
B10d -> aaa	{B10, d}	{a, a, a}	0	TALYS2	
B10He3 -> N13g	{B10, He3}	{N13, g}	0	TALYS2	
B10He3 -> N12n	{B10, He3}	{N12, n}	0	TALYS2	
B10He3 -> C12p	{B10, He3}	{C12, p}	0	TALYS2	
B10n -> Be10p	{B10, n}	{Be10, p}	0	TALYS2	
B10t -> C13g	{B10, t}	{C13, g}	0	TALYS2	
B10t -> C12n	{B10, t}	{C12, n}	0	TALYS2	
B10t -> B12p	{B10, t}	{B12, p}	0	TALYS2	
B10t -> Be9a	{B10, t}	{Be9, a}	0	TALYS2	
B11d -> C13g	{B11, d}	{C13, g}	0	TALYS2	
B11d -> C12n	{B11, d}	{C12, n}	0	New2011	
B11d -> B12p	{B11, d}	{B12, p}	0	New2011	
B11d -> Be9a	{B11, d}	{Be9, a}	0	TALYS2	
B11He3 -> N14g	{B11, He3}	{N14, g}	0	TALYS2	
B11He3 -> N13n	{B11, He3}	{N13, n}	0	TALYS2	
B11He3 -> C13p	{B11, He3}	{C13, p}	0	TALYS2	
B11He3 -> B10a	{B11, He3}	{B10, a}	0	TALYS2	
B11t -> C14g	{B11, t}	{C14, g}	0	TALYS2	
B11t -> C13n	{B11, t}	{C13, n}	0	TALYS2	
B11t -> Be10a	{B11, t}	{Be10, a}	0	TALYS2	

B12a -> N16g	{B12, a}	{N16, g}	0	TALYS2	
B12p -> C12n	{B12, p}	{C12, n}	0	TALYS2	
B12a -> N15n	{B12, a}	{N15, n}	0	TALYS2	
B12d -> C14g	{B12, d}	{C14, g}	0	TALYS2	
B12d -> C13n	{B12, d}	{C13, n}	0	TALYS2	
B12d -> B13p	{B12, d}	{B13, p}	0	TALYS2	
B12d -> Be10a	{B12, d}	{Be10, a}	0	TALYS2	
B12He3 -> N15g	{B12, He3}	{N15, g}	0	TALYS2	
B12He3 -> N14n	{B12, He3}	{N14, n}	0	TALYS2	
B12He3 -> C14p	{B12, He3}	{C14, p}	0	TALYS2	
B12He3 -> B11a	{B12, He3}	{B11, a}	0	TALYS2	
B12n -> B13g	{B12, n}	{B13, g}	0	TALYS2	
B12p -> C13g	{B12, p}	{C13, g}	0	TALYS2	
B12t -> C15g	{B12, t}	{C15, g}	0	TALYS2	
B12t -> C14n	{B12, t}	{C14, n}	0	TALYS2	
B12t -> Be11a	{B12, t}	{Be11, a}	0	TALYS2	
C9a -> O13g	{C9, a}	{O13, g}	0	TALYS2	
C9d -> C10p	{C9, d}	{C10, p}	0	TALYS2	
C9n -> C10g	{C9, n}	{C10, g}	0	TALYS2	
C9t -> N12g	{C9, t}	{N12, g}	0	TALYS2	
C9t -> C11p	{C9, t}	{C11, p}	0	TALYS2	
C9t -> B8a	{C9, t}	{B8, a}	0	TALYS2	
C11d -> N13g	{C11, d}	{N13, g}	0	TALYS2	
C11d -> C12p	{C11, d}	{C12, p}	0	New2011	
C11He3 -> O14g	{C11, He3}	{O14, g}	0	TALYS2	
C11He3 -> N13p	{C11, He3}	{N13, p}	0	TALYS2	
C11He3 -> C10a	{C11, He3}	{C10, a}	0	TALYS2	
C11t -> N14g	{C11, t}	{N14, g}	0	TALYS2	
C11t -> N13n	{C11, t}	{N13, n}	0	TALYS2	
C11t -> C13p	{C11, t}	{C13, p}	0	TALYS2	
C11t -> B10a	{C11, t}	{B10, a}	0	TALYS2	
C12a -> O16g	{C12, a}	{O16, g}	0	NACRE	
C12d -> N14g	{C12, d}	{N14, g}	0	TALYS2	
C12d -> C13p	{C12, d}	{C13, p}	0	TALYS2	
C12He3 -> O15g	{C12, He3}	{O15, g}	0	TALYS2	
C12He3 -> N14p	{C12, He3}	{N14, p}	0	TALYS2	
C11a -> O15g	{C11, a}	{O15, g}	0	TALYS2	
C12He3 -> C11a	{C12, He3}	{C11, a}	0	TALYS2	
C12n -> C13g	{C12, n}	{C13, g}	0	TALYS2	
C12p -> N13g	{C12, p}	{N13, g}	0	NACRE	
C12t -> N15g	{C12, t}	{N15, g}	0	TALYS2	
C12t -> N14n	{C12, t}	{N14, n}	0	TALYS2	
C12t -> C14p	{C12, t}	{C14, p}	0	TALYS2	
C12t -> B11a	{C12, t}	{B11, a}	0	TALYS2	
C13a -> O17g	{C13, a}	{O17, g}	0	TALYS2	
C13d -> N15g	{C13, d}	{N15, g}	0	TALYS2	
C13d -> N14n	{C13, d}	{N14, n}	0	TALYS2	
C13d -> C14p	{C13, d}	{C14, p}	0	TALYS2	
C13d -> B11a	{C13, d}	{B11, a}	0	TALYS2	
C13He3 -> O16g	{C13, He3}	{O16, g}	0	TALYS2	
C13He3 -> O15n	{C13, He3}	{O15, n}	0	TALYS2	
C13He3 -> N15p	{C13, He3}	{N15, p}	0	TALYS2	
C13He3 -> C12a	{C13, He3}	{C12, a}	0	TALYS2	
C13n -> C14g	{C13, n}	{C14, g}	0	TALYS2	
C13p -> N14g	{C13, p}	{N14, g}	0	NACRE	
C13t -> N16g	{C13, t}	{N16, g}	0	TALYS2	
C13t -> N15n	{C13, t}	{N15, n}	0	TALYS2	
C13t -> C15p	{C13, t}	{C15, p}	0	TALYS2	
C13t -> B12a	{C13, t}	{B12, a}	0	TALYS2	
C14d -> N16g	{C14, d}	{N16, g}	0	TALYS2	
C14d -> B12a	{C14, d}	{B12, a}	0	TALYS2	
C14He3 -> O17g	{C14, He3}	{O17, g}	0	TALYS2	
C14He3 -> O16n	{C14, He3}	{O16, n}	0	TALYS2	
C14He3 -> N16p	{C14, He3}	{N16, p}	0	TALYS2	
C14He3 -> C13a	{C14, He3}	{C13, a}	0	TALYS2	
C14t -> N17g	{C14, t}	{N17, g}	0	TALYS2	

C14t -> N16n	{C14, t}	{N16, n}	0	TALYS2	
C15a -> O19g	{C15, a}	{O19, g}	0	TALYS2	
C15a -> O18n	{C15, a}	{O18, n}	0	TALYS2	
C15n -> C16g	{C15, n}	{C16, g}	0	TALYS2	
C15p -> N16g	{C15, p}	{N16, g}	0	TALYS2	
C15p -> N15n	{C15, p}	{N15, n}	0	TALYS2	
C15p -> B12a	{C15, p}	{B12, a}	0	TALYS2	
N12a -> O15p	{N12, a}	{O15, p}	0	TALYS2	
N12n -> N13g	{N12, n}	{N13, g}	0	TALYS2	
N12p -> O13g	{N12, p}	{O13, g}	0	TALYS2	
N13a -> F17g	{N13, a}	{F17, g}	0	TALYS2	
N13n -> C13p	{N13, n}	{C13, p}	0	TALYS2	
N13p -> O14g	{N13, p}	{O14, g}	0	NACRE	
N14n -> N15g	{N14, n}	{N15, g}	0	TALYS2	
N14p -> O15g	{N14, p}	{O15, g}	0	NACRE	
N15n -> N16g	{N15, n}	{N16, g}	0	TALYS2	
N15p -> O16g	{N15, p}	{O16, g}	0	NACRE	
O14n -> O15g	{O14, n}	{O15, g}	0	TALYS2	
O14n -> C11a	{O14, n}	{C11, a}	0	TALYS2	
O15n -> O16g	{O15, n}	{O16, g}	0	TALYS2	
O15n -> N15p	{O15, n}	{N15, p}	0	TALYS2	
O15n -> C12a	{O15, n}	{C12, a}	0	TALYS2	
O17a -> Ne21g	{O17, a}	{Ne21, g}	0	CF88	
O17a -> Ne20n	{O17, a}	{Ne20, n}	0	NACRE	
O19a -> Ne23g	{O19, a}	{Ne23, g}	0	TALYS2	
O19a -> Ne22n	{O19, a}	{Ne22, n}	0	TALYS2	
O19n -> O20g	{O19, n}	{O20, g}	0	TALYS2	
O19p -> F20g	{O19, p}	{F20, g}	0	TALYS2	
O19p -> F19n	{O19, p}	{F19, n}	0	TALYS2	
O19p -> N16a	{O19, p}	{N16, a}	0	TALYS2	
F17a -> Na21g	{F17, a}	{Na21, g}	0	TALYS2	
F17a -> Ne20p	{F17, a}	{Ne20, p}	0	TALYS2	
F17n -> F18g	{F17, n}	{F18, g}	0	TALYS2	
F18a -> Na22g	{F18, a}	{Na22, g}	0	TALYS2	
F18a -> Ne21p	{F18, a}	{Ne21, p}	0	TALYS2	
F18n -> F19g	{F18, n}	{F19, g}	0	TALYS2	
F19a -> Na23g	{F19, a}	{Na23, g}	0	TALYS2	
F19a -> Ne22p	{F19, a}	{Ne22, p}	0	TALYS2	
F19n -> F20g	{F19, n}	{F20, g}	0	TALYS2	
F19p -> Ne20g	{F19, p}	{Ne20, g}	0	NACRE	
F19p -> O16a	{F19, p}	{O16, a}	0	NACRE	
B8n -> Li6He3	{B8, n}	{Li6, He3}	0	TALYS2	
Li9p -> Li7t	{Li9, p}	{Li7, t}	0	TALYS2	
B8n -> Be7d	{B8, n}	{Be7, d}	0	TALYS2	
C9n -> Be7He3	{C9, n}	{Be7, He3}	0	TALYS2	
B10n -> aat	{B10, n}	{a, a, t}	0	TALYS2	
Be10p -> aat	{Be10, p}	{a, a, t}	0	TALYS2	
Be11p -> Be9t	{Be11, p}	{Be9, t}	0	TALYS2	
Be11p -> Be10d	{Be11, p}	{Be10, d}	0	TALYS2	
Be12p -> Be10t	{Be12, p}	{Be10, t}	0	TALYS2	
C9n -> B8d	{C9, n}	{B8, d}	0	TALYS2	
N13n -> C12d	{N13, n}	{C12, d}	0	TALYS2	
B10a -> C12d	{B10, a}	{C12, d}	0	TALYS2	
O14n -> C12He3	{O14, n}	{C12, He3}	0	TALYS2	
C15p -> C14d	{C15, p}	{C14, d}	0	TALYS2	
Ne18n -> O15a	{Ne18, n}	{O15, a}	0	TALYS2	
Ne19n -> O16a	{Ne19, n}	{O16, a}	0	TALYS2	
Na20n -> F17a	{Na20, n}	{F17, a}	0	TALYS2	
Ne18n -> F18p	{Ne18, n}	{F18, p}	0	TALYS2	
Ne19n -> F19p	{Ne19, n}	{F19, p}	0	TALYS2	
Li7He3 -> Be9p	{Li7, He3}	{Be9, p}	0	TALYS2	
Li6t -> Li7d	{Li6, t}	{Li7, d}	0	TALYS2	
Li6He3 -> Be7d	{Li6, He3}	{Be7, d}	0	TALYS2	
Li7He3 -> aad	{Li7, He3}	{a, a, d}	0	TALYS2	
Li8He3 -> Be9d	{Li8, He3}	{Be9, d}	0	TALYS2	

Li8He3 -> aat	{Li8, He3}	{a, a, t}	0	TALYS2	
Li9d -> Li8t	{Li9, d}	{Li8, t}	0	TALYS2	
Li9He3 -> Be10d	{Li9, He3}	{Be10, d}	0	TALYS2	
Li9He3 -> Be9t	{Li9, He3}	{Be9, t}	0	TALYS2	
Be7t -> aad	{Be7, t}	{a, a, d}	0	TALYS2	
Be7t -> Li7He3	{Be7, t}	{Li7, He3}	0	TALYS2	
Be9d -> aat	{Be9, d}	{a, a, t}	0	TALYS2	
Be9t -> Be10d	{Be9, t}	{Be10, d}	0	TALYS2	
Be9He3 -> B10d	{Be9, He3}	{B10, d}	0	TALYS2	
Be10He3 -> B11d	{Be10, He3}	{B11, d}	0	TALYS2	
Be10He3 -> B10t	{Be10, He3}	{B10, t}	0	TALYS2	
B8d -> Be7He3	{B8, d}	{Be7, He3}	0	TALYS2	
B8t -> aaHe3	{B8, t}	{a, a, He3}	0	TALYS2	
B10p -> aaHe3	{B10, p}	{a, a, He3}	0	TALYS2	
B10t -> B11d	{B10, t}	{B11, d}	0	TALYS2	
B10He3 -> C11d	{B10, He3}	{C11, d}	0	TALYS2	
B11t -> B13p	{B11, t}	{B13, p}	0	TALYS2	
B11He3 -> C12d	{B11, He3}	{C12, d}	0	TALYS2	
N12n -> C11d	{N12, n}	{C11, d}	0	TALYS2	
C11t -> C12d	{C11, t}	{C12, d}	0	TALYS2	
C11t -> B11He3	{C11, t}	{B11, He3}	0	TALYS2	
Be7He3 -> ppaa	{Be7, He3}	{p, p, a, a}	0	TALYS2	
dd -> ag	{d, d}	{a, g}	0	NACRE	
He3He3 -> app	{He3, He3}	{a, p, p}	0	NACRE	
Li7a -> B11g	{Li7, a}	{B11, g}	0	NACRE	
Be7p -> B8g	{Be7, p}	{B8, g}	0	NACRE	
Be7a -> C11g	{Be7, a}	{C11, g}	0	NACRE	
Be9p -> B10g	{Be9, p}	{B10, g}	0	NACRE	
Be9p -> aad	{Be9, p}	{a, a, d}	0	NACRE	
Be9p -> Li6a	{Be9, p}	{Li6, a}	0	NACRE	
Be9a -> C12n	{Be9, a}	{C12, n}	0	NACRE	
B10p -> C11g	{B10, p}	{C11, g}	0	NACRE	
B10p -> Be7a	{B10, p}	{Be7, a}	0	NACRE	
B11p -> C12g	{B11, p}	{C12, g}	0	NACRE	
B11p -> aaa	{B11, p}	{a, a, a}	0	NACRE	
B11a -> N14n	{B11, a}	{N14, n}	0	NACRE	
C13a -> O16n	{C13, a}	{O16, n}	0	NACRE	
N15p -> C12a	{N15, p}	{C12, a}	0	NACRE	
Li7t -> Be9n	{Li7, t}	{Be9, n}	0	New2011	
B11n -> B12g	{B11, n}	{B12, g}	0	New2011	
C11n -> aaa	{C11, n}	{a, a, a}	0	New2011	
Li7d -> aan	{Li7, d}	{a, a, n}	0	New2011	
dn -> tg	{d, n}	{t, g}	0	Nag06	1.4
tt -> ann	{t, t}	{a, n, n}	0	Nag06	3.
He3n -> ag	{He3, n}	{a, g}	0	Wag69	3.
He3t -> ad	{He3, t}	{a, d}	0	CF88	3.
He3t -> anp	{He3, t}	{a, n, p}	0	CF88	3.
aan -> Be9g	{a, a, n}	{Be9, g}	0	NACRE	1.25
Li7t -> aann	{Li7, t}	{a, a, n, n}	0	CF88&MF89	3.
Li7He3 -> aanp	{Li7, He3}	{a, a, n, p}	0	CF88&MF89	3.
Li8d -> Li9p	{Li8, d}	{Li9, p}	0	Bal95	3.
Li8d -> Li7t	{Li8, d}	{Li7, t}	0	Has09c	3.
Be7d -> aap	{Be7, d}	{a, a, p}	0	CF88	3.
Be7t -> aanp	{Be7, t}	{a, a, n, p}	0	CF88&MF89	3.
Be7He3 -> aapp	{Be7, He3}	{a, a, p, p}	0	CF88&MF89	3.
C9a -> N12p	{C9, a}	{N12, p}	0	Wie89	3.
Li6n -> ta	{Li6, n}	{t, a}	0	CF88	3.
He3t -> Li6g	{He3, t}	{Li6, g}	0	FK90	3.
anp -> Li6g	{a, n, p}	{Li6, g}	0	CF88	3.
Li6n -> Li7g	{Li6, n}	{Li7, g}	0	MF89	3.

Li6d -> Li7p	{Li6, d}	{Li7, p}	0	MF89	3.
Li6d -> Be7n	{Li6, d}	{Be7, n}	0	MF89	3.
Li6a -> B10g	{Li6, a}	{B10, g}	0	CF88	3.
Li7a -> B10n	{Li7, a}	{B10, n}	0	NACRE	1.08
Li7n -> Li8g	{Li7, n}	{Li8, g}	0	MF89&Hei98	3.
Li7d -> Li8p	{Li7, d}	{Li8, p}	0	MF89	3.
Li8n -> Li9g	{Li8, n}	{Li9, g}	0	Rau94	3.
Li8p -> aan	{Li8, p}	{a, a, n}	0	Men12	3.
Li8d -> Be9n	{Li8, d}	{Be9, n}	0	Bal95	3.
Be9n -> Be10g	{Be9, n}	{Be10, g}	0	Rau94	3.
Be9p -> aapn	{Be9, p}	{a, a, p, n}	0	NACRE	1.05
B11p -> C11n	{B11, p}	{C11, n}	0	NACRE	1.1
Be10n -> Be11g	{Be10, n}	{Be11, g}	0	Rau94	3.
Be11n -> Be12g	{Be11, n}	{Be12, g}	0	Rau94	3.
B8p -> C9g	{B8, p}	{C9, g}	0	Des99Bea01	3.
aaa -> C12gg	{a, a, a}	{C12, g, g}	0	NACRE	1.15
C11p -> N12g	{C11, p}	{N12, g}	0	Tang03	3.
B10a -> N13n	{B10, a}	{N13, n}	0	CF88	3.
B11a -> C14p	{B11, a}	{C14, p}	0	Wan91	3.
C11n -> C12g	{C11, n}	{C12, g}	0	Rau94	3.
He6 -> Li6Bm	{He6}	{Li6, Bm}	0	Aud03	1
Li8 -> aaBm	{Li8}	{a, a, Bm}	0	Aud03	1
Li9 -> Be9Bm	{Li9}	{Be9, Bm}	0	Aud03	1
Li9 -> aanBm	{Li9}	{a, a, n, Bm}	0	Aud03	1
Be11 -> B11Bm	{Be11}	{B11, Bm}	0	Aud03	1
Be12 -> B12Bm	{Be12}	{B12, Bm}	0	Aud03	1
B8 -> aaBp	{B8}	{a, a, Bp}	0	Aud03	1
B12 -> C12Bm	{B12}	{C12, Bm}	0	Aud03	1
B13 -> C13Bm	{B13}	{C13, Bm}	0	Aud03	1
B14 -> C14Bm	{B14}	{C14, Bm}	0	Aud03	1
B15 -> C15Bm	{B15}	{C15, Bm}	0	Aud03	1
C9 -> aapBp	{C9}	{a, a, p, Bp}	0	Aud03	1
C10 -> B10Bp	{C10}	{B10, Bp}	0	Aud03	1
C11 -> B11Bp	{C11}	{B11, Bp}	0	Aud03	1
C15 -> N15Bm	{C15}	{N15, Bm}	0	Aud03	1
C16 -> N16Bm	{C16}	{N16, Bm}	0	Aud03	1
N12 -> C12Bp	{N12}	{C12, Bp}	0	Aud03	1
N13 -> C13Bp	{N13}	{C13, Bp}	0	Aud03	1
N16 -> O16Bm	{N16}	{O16, Bm}	0	Aud03	1
N17 -> O16nBm	{N17}	{O16, n, Bm}	0	Aud03	1
O13 -> N13Bp	{O13}	{N13, Bp}	0	Aud03	1
O14 -> N14Bp	{O14}	{N14, Bp}	0	Aud03	1
O15 -> N15Bp	{O15}	{N15, Bp}	0	Aud03	1
O19 -> F19Bm	{O19}	{F19, Bm}	0	Aud03	1
O20 -> F20Bm	{O20}	{F20, Bm}	0	Aud03	1
F17 -> O17Bp	{F17}	{O17, Bp}	0	Aud03	1
F18 -> O18Bp	{F18}	{O18, Bp}	0	Aud03	1
F20 -> Ne20Bm	{F20}	{Ne20, Bm}	0	Aud03	1
Ne18 -> F18Bp	{Ne18}	{F18, Bp}	0	Aud03	1
Ne19 -> F19Bp	{Ne19}	{F19, Bp}	0	Aud03	1
Ne23 -> Na23Bm	{Ne23}	{Na23, Bm}	0	Aud03	1
Na20 -> Ne20Bp	{Na20}	{Ne20, Bp}	0	Aud03	1
Na21 -> Ne21Bp	{Na21}	{Ne21, Bp}	0	Aud03	1
ann -> He6g	{a, n, n}	{He6, g}	0	Efr96	3.
O16n -> O17g	{O16, n}	{O17, g}	0	Iga95	3.
N14n -> C14p	{N14, n}	{C14, p}	0	CF88	3.
O14n -> N14p	{O14, n}	{N14, p}	0	CF88	3.
O14a -> Ne18g	{O14, a}	{Ne18, g}	0	Wie87	3.
C11a -> N14p	{C11, a}	{N14, p}	0	NACRE	2.
O14a -> F17p	{O14, a}	{F17, p}	0	Bar97C	3.
O17n -> C14a	{O17, n}	{C14, a}	0	Koe91	3.
F17n -> N14a	{F17, n}	{N14, a}	0	NACRE	1.05
F18n -> N15a	{F18, n}	{N15, a}	0	CF88	3.
C14d -> N15n	{C14, d}	{N15, n}	0	Kaw91	3.
ppn -> dp	{p, p, n}	{d, p}	0	CF88	3.

C14n -> C15g	{C14, n}	{C15, g}	0	Kaw91	3.
O16p -> N13a	{O16, p}	{N13, a}	0	CF88	3.
Li8p -> Be9g	{Li8, p}	{Be9, g}	0	TUNL&Cam08	3.
B11a -> N15g	{B11, a}	{N15, g}	0	Wan91	3.

Constants for reverse reactions

```
MyGrid[
 Join[{{"Reaction Name", "Q (MeV)", "Front Factor", "T9 power", "Q/kB/10^9"}},
  Join[{ReactionWithArrow[#[[1]]]}, InfoReaction[#[[2]], #[[3]]]] & /@
   Rest@ListReactions]]
```

Reaction Name	Q (MeV)	Front Factor	T9 power	Q/kB/10^9
np -> dg	2.224566	4.7161407×10^{9}	1.5	-25.815019
dp -> He3g	5.4934744	1.6335104×10^{10}	1.5	-63.749128
dd -> He3n	3.2689084	1.7318296	0.	-37.93411
dd -> tp	4.0326629	1.7349209	0.	-46.797113
tp -> ag	19.813865	2.6105753×10^{10}	1.5	-229.93037
td -> an	17.589299	5.5354059	0.	-204.11535
ta -> Li7g	2.4676205	1.1132989×10^{10}	1.5	-28.635549
He3n -> tp	0.7637545	1.001785	0.	-8.8630036
He3d -> ap	18.353053	5.5452865	0.	-212.97836
He3a -> Be7g	1.5871335	1.1128945×10^{10}	1.5	-18.417921
Be7n -> Li7p	1.6442415	1.0021491	0.	-19.080632
Li7p -> aa	17.346244	4.6898011	0.	-201.29482
Li7p -> aag	17.346244	4.6898011	0.	-201.29482
Be7n -> aa	18.990486	4.6998798	0.	-220.37546
da -> Li6g	1.4737584	1.5305259×10^{10}	1.5	-17.102257
Li6p -> Be7g	5.6068495	1.1877778×10^{10}	1.5	-65.064792
Li6p -> He3a	4.019716	1.067287	0.	-46.646871
Be9t -> B11n	9.5592358	3.8284908	0.	-110.93033
O18n -> O19g	3.9556015	3.0716044×10^{9}	1.5	-45.902853
Li9p -> He6a	12.226855	1.8556823	0.	-141.88677
Li9d -> Be10n	17.411815	14.484998	0.	-202.05573
Be10a -> C14g	12.012513	4.7767564×10^{10}	1.5	-139.39943
N12n -> C12p	18.120447	3.0129978	0.	-210.27907
Li9p -> Be9n	12.824104	1.0004549	0.	-148.81755
Li9a -> B12n	5.9390985	3.4308601	0.	-68.920382
Li9p -> Be10g	19.636381	6.8313289×10^{10}	1.5	-227.87075
N13n -> N14g	10.55338	1.1930596×10^{10}	1.5	-122.4669
B10a -> N14g	11.612108	1.1144706×10^{11}	1.5	-134.75293
B8a -> N12g	8.0084156	7.1824543×10^{10}	1.5	-92.933811
B12p -> Be9a	6.885005	0.29160469	0.	-79.897172
Be10p -> B11g	11.228754	4.3271615×10^{9}	1.5	-130.30429
Be10p -> Li7a	2.56444	0.10767485	0.	-29.759093
Be11p -> Li8a	4.095425	0.16270048	0.	-47.525437
Be11p -> B11n	10.727117	0.49984572	0.	-124.48303
B8n -> aap	18.854115	3.6007303×10^{-10}	-1.5	-218.79294
B10n -> B11g	11.454219	3.0347568×10^{10}	1.5	-132.9207
B10a -> C13p	4.0615452	9.3625853	0.	-47.132279
O17n -> O18g	8.0453692	1.1010406×10^{11}	1.5	-93.36264
F17n -> O17p	3.54281	1.002282	0.	-41.112606
F18n -> O18p	2.4382621	3.0065131	0.	-28.294859
Be10a -> C13n	3.8360797	1.3349807	0.	-44.51586
Be11a -> C14n	11.510876	5.5178002	0.	-133.57817
N14a -> F18g	4.4152323	5.419721×10^{10}	1.5	-51.236647
N15a -> F19g	4.0137985	5.5418274×10^{10}	1.5	-46.578201
O15a -> Ne19g	3.5284656	5.5418803×10^{10}	1.5	-40.946146
O16p -> F17g	0.6002693	3.034455×10^{9}	1.5	-6.9658365

O16a -> Ne20g	4.7298448	5.6527359×10^{10}	1.5	-54.887574
O17p -> F18g	5.6071071	3.6621845×10^{10}	1.5	-65.067781
O18p -> F19g	7.9935992	9.1999498×10^{9}	1.5	-92.761874
O18a -> Ne22g	9.666819	5.8490811×10^{10}	1.5	-112.17879
F17p -> Ne18g	3.9230706	1.0985029×10^{11}	1.5	-45.525348
F18p -> Ne19g	6.4100206	2.7596293×10^{10}	1.5	-74.385206
Ne19p -> Na20g	2.1904206	7.387221×10^{9}	1.5	-25.418778
O17p -> N14a	1.1918748	0.67571458	0.	-13.831134
O18p -> N15a	3.9798007	0.16600931	0.	-46.183673
F18p -> O15a	2.881555	0.49795902	0.	-33.43906
C14a -> O18g	6.2276242	5.4206925×10^{10}	1.5	-72.268584
C14p -> N15g	10.207425	8.9988545×10^{9}	1.5	-118.45226
Be12p -> Li9a	4.986955	0.097115699	0.	-57.871214
Li6He3 -> aap	16.879295	7.24625×10^{-10}	-1.5	-195.8761
Li6t -> Be9g	17.688239	4.2265092×10^{10}	1.5	-205.2635
Li6t -> aan	16.115541	$7.2333386 \times 10^{-10}$	-1.5	-187.0131
Li6t -> Li8p	0.8019182	2.0175581	0.	-9.305875
Li7d -> Be9g	16.694377	5.8104623×10^{10}	1.5	-193.73021
Li7He3 -> B10g	17.787714	3.4631848×10^{10}	1.5	-206.41786
Li7He3 -> Li6a	13.326528	2.1970664	0.	-154.64795
Li7t -> Be10g	17.249425	2.4244986×10^{11}	1.5	-200.17128
Li8a -> B12g	10.001316	7.1839162×10^{10}	1.5	-116.06046
Li8a -> B11n	6.6316915	3.0721835	0.	-76.95759
Li8d -> Be10g	21.474032	3.0330298×10^{11}	1.5	-249.19581
Li8He3 -> B11g	27.209311	8.0344821×10^{10}	1.5	-315.75097
Li8He3 -> B10n	15.755092	2.6474879	0.	-182.83026
Li8He3 -> Be10p	15.980557	18.567558	0.	-185.44668
Li8He3 -> Li7a	18.544997	1.999259	0.	-215.20577
Li8t -> Be11g	15.71844	1.6045283×10^{11}	1.5	-182.40494
Li8t -> Be10n	15.216803	18.534474	0.	-176.58368
Li9a -> B13g	10.817916	4.5613945×10^{10}	1.5	-125.53671
Li9d -> Be11g	17.913452	1.2539654×10^{11}	1.5	-207.87699
Li9He3 -> B12g	26.516718	8.9725059×10^{10}	1.5	-307.71376
Li9He3 -> B11n	23.147094	3.8370693	0.	-268.61089
Li9He3 -> Be11p	12.419977	7.6765072	0.	-144.12786
Li9He3 -> Li8a	16.515402	1.2489714	0.	-191.6533
Li9t -> Be12g	14.82691	2.6881084×10^{11}	1.5	-172.05916
Li9t -> Be11n	11.656223	7.6628292	0.	-135.26486
Be7He3 -> C10g	15.001548	2.4232017×10^{11}	1.5	-174.08575
Be7t -> B10g	18.668201	3.4644435×10^{10}	1.5	-216.63549
Be7t -> Be9p	12.081389	3.5583331	0.	-140.19871
Be7t -> Li6a	14.207015	2.197865	0.	-164.86558
Be9a -> C13g	10.648357	9.1155457×10^{10}	1.5	-123.56906
Be9d -> B11g	15.816465	6.2650426×10^{10}	1.5	-183.54247
Be9d -> B10n	4.3622456	2.0644299	0.	-50.621762
Be9d -> Be10p	4.5877111	14.478412	0.	-53.238181
Be9d -> Li7a	7.1521511	1.5589608	0.	-82.997274
Be9He3 -> C12g	26.279668	2.6899784×10^{11}	1.5	-304.96291
Be9He3 -> C11n	7.5589508	3.8265847	0.	-87.717989
Be9He3 -> B11p	10.32299	3.8353246	0.	-119.79334
Be9He3 -> aaa	19.004921	1.3427309×10^{-9}	-1.5	-220.54297
Be9t -> B12g	12.92886	8.9524462×10^{10}	1.5	-150.0332
Be9t -> Li8a	2.9275443	1.2461791	0.	-33.972744
Be10d -> B12g	12.373812	2.1455096×10^{10}	1.5	-143.59213
Be10d -> B11n	9.0041876	0.91752172	0.	-104.48927
Be10d -> Li8a	2.3724961	0.2986546	0.	-27.531676

Be10He3 -> C13g	24.413699	3.4912885×10^{10}	1.5	-283.30924
Be10He3 -> C12n	19.467391	3.9395002	0.	-225.90971
Be10He3 -> B12p	6.8803373	1.3134349	0.	-79.843005
Be10He3 -> Be9a	13.765342	0.38300378	0.	-159.74018
Be10t -> B13g	10.9954	1.7431255×10^{10}	1.5	-127.59633
Be10t -> B12n	6.1165828	1.3110946	0.	-70.980002
Be10t -> Li9a	0.1774843	0.38214751	0.	-2.0596199
Be11a -> C15g	12.728986	4.9701694×10^{10}	1.5	-147.71376
Be11d -> B13g	16.750992	3.2950193×10^{10}	1.5	-194.3872
Be11d -> B12n	11.872175	2.478354	0.	-137.77088
Be11d -> Be12p	0.9461211	7.4382519	0.	-10.97928
Be11d -> Li9a	5.9330761	0.72237104	0.	-68.850495
Be11He3 -> C14g	32.088495	1.4430345×10^{11}	1.5	-372.37155
Be11He3 -> C13n	23.912062	4.0329107	0.	-277.48798
Be11He3 -> B13p	11.257517	2.0171401	0.	-130.63807
Be11He3 -> Be10a	20.075982	3.0209505	0.	-232.97212
Be11p -> B12g	14.096741	1.1688266×10^{10}	1.5	-163.58589
Be11t -> B14g	11.46298	2.8798749×10^{10}	1.5	-133.02237
Be11t -> B13n	10.493763	2.0135459	0.	-121.77507
Be12a -> C16g	13.808716	5.1410643×10^{10}	1.5	-160.2435
Be12a -> C15n	9.5582985	1.4168163	0.	-110.91946
Be12d -> B14g	14.549522	1.3434218×10^{10}	1.5	-168.8402
Be12d -> B13n	13.580305	0.93929136	0.	-157.5929
Be12He3 -> C15g	30.135918	3.7053079×10^{10}	1.5	-349.71283
Be12He3 -> C14n	28.917808	4.1135717	0.	-335.57725
Be12He3 -> B14p	9.0560473	0.82241401	0.	-105.09107
Be12He3 -> Be11a	17.406932	0.74550937	0.	-201.99908
Be12p -> B13g	15.804871	4.4298302×10^{9}	1.5	-183.40792
Be12p -> B12n	10.926053	0.33319038	0.	-126.7916
Be12t -> B15g	11.06961	1.8492882×10^{10}	1.5	-128.4575
Be12t -> B14n	8.2922928	0.82094863	0.	-96.22807
B8a -> C11p	7.408145	3.078465	0.	-85.96796
B8d -> C10g	20.358652	3.0326013×10^{11}	1.5	-236.25236
B8He3 -> C10p	14.865177	18.564934	0.	-172.50323
B8t -> C11g	27.22201	8.0365646×10^{10}	1.5	-315.89833
B8t -> C10n	14.101423	18.531855	0.	-163.64023
B8t -> B10p	18.53183	2.6542225	0.	-215.05298
B8t -> Be7a	19.677494	2.0000464	0.	-228.34786
B10d -> C12g	25.186331	4.5131921×10^{11}	1.5	-292.27525
B10d -> C11n	6.4656136	6.4201674	0.	-75.030336
B10d -> B11p	9.2296531	6.434831	0.	-107.10569
B10d -> aaa	17.911584	2.252807×10^{-9}	-1.5	-207.85532
B10He3 -> N13g	21.636347	2.4429649×10^{11}	1.5	-251.0794
B10He3 -> N12n	1.5724098	9.1698685	0.	-18.247059
B10He3 -> C12p	19.692856	27.628793	0.	-228.52613
B10n -> Be10p	0.2254655	7.0132737	0.	-2.6164187
B10t -> C13g	23.87541	2.4441734×10^{11}	1.5	-277.06265
B10t -> C12n	18.929102	27.579564	0.	-219.66312
B10t -> B12p	6.3420483	9.1950656	0.	-73.59642
B10t -> Be9a	13.227053	2.6813243	0.	-153.49359
B11d -> C13g	18.67842	1.317967×10^{11}	1.5	-216.75408
B11d -> C12n	13.732112	14.871676	0.	-159.35455
B11d -> B12p	1.1450581	4.9582379	0.	-13.287849
B11d -> Be9a	8.0300631	1.4458454	0.	-93.18502
B11He3 -> N14g	20.735508	9.6040733×10^{10}	1.5	-240.6256
B11He3 -> N13n	10.182128	8.0499526	0.	-118.1587
B11He3 -> C13p	13.184945	8.068311	0.	-153.00495
B11He3 -> B10a	9.1234003	0.86176102	0.	-105.87267
B11t -> C14g	20.597624	2.8818158×10^{11}	1.5	-239.02552
B11t -> C13n	12.421191	8.0539349	0.	-144.14195

B11t -> Be10a	8.5851113	6.0329971	0.	-99.626088
B12a -> N16g	10.110416	3.0822334×10^{10}	1.5	-117.32651
B12p -> C12n	12.587054	2.9993874	0.	-146.0667
B12a -> N15n	7.6215598	4.2481819	0.	-88.444536
B12d -> C14g	23.485229	2.0167336×10^{11}	1.5	-272.53478
B12d -> C13n	15.308796	5.6362523	0.	-177.65121
B12d -> B13p	2.6542511	2.8190831	0.	-30.801307
B12d -> Be10a	11.472716	4.2219728	0.	-133.13535
B12He3 -> N15g	28.199179	1.1109995×10^{11}	1.5	-327.23791
B12He3 -> N14n	17.365884	4.1071575	0.	-201.52273
B12He3 -> C14p	17.991754	12.34601	0.	-208.78566
B12He3 -> B11a	17.207995	1.1183986	0.	-199.69051
B12n -> B13g	4.8788171	1.3295192×10^{10}	1.5	-56.616326
B12p -> C13g	17.533362	2.6581359×10^{10}	1.5	-203.46623
B12t -> C15g	18.44611	1.1100878×10^{11}	1.5	-214.05823
B12t -> C14n	17.228	12.324012	0.	-199.92265
B12t -> Be11a	5.7171243	2.2335009	0.	-66.344478
C9a -> O13g	8.2209156	4.5605338×10^{10}	1.5	-95.399772
C9d -> C10p	19.059081	14.516402	0.	-221.17147
C9n -> C10g	21.283647	6.8461396×10^{10}	1.5	-246.98649
C9t -> N12g	26.52271	8.9753787×10^{10}	1.5	-307.78329
C9t -> C11p	25.922439	3.8469286	0.	-300.81744
C9t -> B8a	18.514294	1.2496256	0.	-214.84948
C11d -> N13g	18.439642	1.3179715×10^{11}	1.5	-213.98317
C11d -> C12p	16.496151	14.905643	0.	-191.4299
C11He3 -> O14g	17.572837	2.8803067×10^{11}	1.5	-203.92432
C11He3 -> N13p	12.946167	8.0683386	0.	-150.23405
C11He3 -> C10a	7.4570323	6.0305815	0.	-86.535273
C11t -> N14g	22.735793	9.6088573×10^{10}	1.5	-263.83795
C11t -> N13n	12.182413	8.0539624	0.	-141.37104
C11t -> C13p	15.18523	8.0723299	0.	-176.2173
C11t -> B10a	11.123685	0.86219027	0.	-129.08502
C12a -> O16g	7.1619169	5.1331451×10^{10}	1.5	-83.110601
C12d -> N14g	10.272305	2.2368183×10^{10}	1.5	-119.20516
C12d -> C13p	2.7217423	1.8791345	0.	-31.58451
C12He3 -> O15g	12.075618	3.6955521×10^{10}	1.5	-140.13174
C12He3 -> N14p	4.7788306	1.3693321	0.	-55.456031
C11a -> O15g	10.218716	9.933586×10^{10}	1.5	-118.58328
C12He3 -> C11a	1.8569023	0.37202599	0.	-21.548458
C12n -> C13g	4.9463083	8.8622626×10^{9}	1.5	-57.399529
C12p -> N13g	1.9434906	8.8420976×10^{9}	1.5	-22.553274
C12t -> N15g	14.848371	3.697488×10^{10}	1.5	-172.30821
C12t -> N14n	4.0150761	1.3668922	0.	-46.593027
C12t -> C14p	4.6409463	4.1088429	0.	-53.85595
C12t -> B11a	3.8571873	0.3722113	0.	-44.760803
C13a -> O17g	6.3586879	1.7616091×10^{10}	1.5	-73.789515
C13d -> N15g	16.159292	6.8274508×10^{10}	1.5	-187.52081
C13d -> N14n	5.3259967	2.523981	0.	-61.80563
C13d -> C14p	5.9518669	7.5870221	0.	-69.068553
C13d -> B11a	5.1681079	0.68729211	0.	-59.973407
C13He3 -> O16g	22.793228	1.5147804×10^{11}	1.5	-264.50445
C13He3 -> O15n	7.1293096	4.1699872	0.	-82.732209
C13He3 -> N15p	10.665817	4.1796188	0.	-123.77168
C13He3 -> C12a	15.631311	2.9509791	0.	-181.39385
C13n -> C14g	8.1764329	3.5781463×10^{10}	1.5	-94.883571
C13p -> N14g	7.5505627	1.190345×10^{10}	1.5	-87.620649
C13t -> N16g	12.390919	3.0270846×10^{10}	1.5	-143.79065
C13t -> N15n	9.9020629	4.1721715	0.	-114.90868
C13t -> C15p	0.9127481	4.176189	0.	-10.592003

C13t -> B12a	2.2805031	0.98210754	0.	-26.464142
C14d -> N16g	10.471715	1.3844041×10^{10}	1.5	-121.51921
C14d -> B12a	0.3612991	0.44915617	0.	-4.1927023
C14He3 -> O17g	18.759874	1.2875443×10^{10}	1.5	-217.69932
C14He3 -> O16n	14.616795	4.2334221	0.	-169.62088
C14He3 -> N16p	4.9782403	0.84750249	0.	-57.770084
C14He3 -> C13a	12.401186	0.73089104	0.	-143.9098
C14t -> N17g	10.099703	3.8603637×10^{10}	1.5	-117.20219
C14t -> N16n	4.2144858	0.84599241	0.	-48.907081
C15a -> O19g	8.9651156	1.8484803×10^{10}	1.5	-104.03585
C15a -> O18n	5.0095141	6.0179635	0.	-58.133002
C15n -> C16g	4.2504171	3.6286032×10^{10}	1.5	-49.324046
C15p -> N16g	11.478171	7.2484378×10^{9}	1.5	-133.19865
C15p -> N15n	8.9893148	0.99903799	0.	-104.31667
C15p -> B12a	1.367755	0.23516836	0.	-15.872139
N12a -> O15p	9.618445	4.2576249	0.	-111.61743
N12n -> N13g	20.063937	2.664122×10^{10}	1.5	-232.83234
N12p -> O13g	1.5120706	1.3264752×10^{10}	1.5	-17.546852
N13a -> F17g	5.8186956	1.7616067×10^{10}	1.5	-67.523164
N13n -> C13p	3.0028177	1.0022806	0.	-34.846255
N13p -> O14g	4.6266696	3.5698883×10^{10}	1.5	-53.690276
N14n -> N15g	10.833295	2.7050325×10^{10}	1.5	-125.71518
N14p -> O15g	7.2967873	2.698799×10^{10}	1.5	-84.675707
N15n -> N16g	2.4888558	7.2554175×10^{9}	1.5	-28.881975
N15p -> O16g	12.127411	3.624207×10^{10}	1.5	-140.73277
O14n -> O15g	13.223498	9.0194085×10^{9}	1.5	-153.45234
O14n -> C11a	3.0047825	0.090797105	0.	-34.869056
O15n -> O16g	15.663918	3.632578×10^{10}	1.5	-181.77224
O15n -> N15p	3.5365078	1.0023097	0.	-41.039472
O15n -> C12a	8.5020015	0.70767101	0.	-98.661638
O17a -> Ne21g	7.3479321	8.6333607×10^{10}	1.5	-85.269218
O17a -> Ne20n	0.5867655	18.586092	0.	-6.8091314
O19a -> Ne23g	10.911866	5.9348197×10^{10}	1.5	-126.62695
O19a -> Ne22n	5.7112175	19.04243	0.	-66.275932
O19n -> O20g	7.6080171	1.1107561×10^{11}	1.5	-88.287379
O19p -> F20g	10.639334	2.217699×10^{10}	1.5	-123.46435
O19p -> F19n	4.0379977	2.995161	0.	-46.859021
O19p -> N16a	2.513055	0.39212956	0.	-29.162795
F17a -> Na21g	6.5612456	8.6331898×10^{10}	1.5	-76.1401
F17a -> Ne20p	4.1295755	18.628505	0.	-47.921738
F17n -> F18g	9.1499171	3.6705415×10^{10}	1.5	-106.18039
F18a -> Na22g	8.4795256	2.5065777×10^{10}	1.5	-98.400816
F18a -> Ne21p	1.740825	2.3574347	0.	-20.201437
F18n -> F19g	10.431861	2.7659769×10^{10}	1.5	-121.05673
F19a -> Na23g	10.467324	2.9670847×10^{10}	1.5	-121.46826
F19a -> Ne22p	1.6732198	6.3577315	0.	-19.416911
F19n -> F20g	6.6013359	7.4042731×10^{9}	1.5	-76.605328
F19p -> Ne20g	12.843457	3.6967382×10^{10}	1.5	-149.04214
F19p -> O16a	8.1136121	0.65397327	0.	-94.154566
B8n -> Li6He3	1.9748203	0.49690948	0.	-22.91684
Li9p -> Li7t	2.3869557	0.28176254	0.	-27.699473
B8n -> Be7d	2.0881954	0.36131883	0.	-24.232503
C9n -> Be7He3	6.2820992	0.28252455	0.	-72.90074
B10n -> aat	0.322285	1.3566045×10^{-10}	-1.5	-3.7399624
Be10p -> aat	0.0968195	1.9343385×10^{-11}	-1.5	-1.1235437
Be11p -> Be9t	1.1678807	0.13055947	0.	-13.552694
Be11p -> Be10d	1.7229289	0.54477808	0.	-19.993761
Be12p -> Be10t	4.8094707	0.25413145	0.	-55.811594

C9n -> B8d	0.9249954	0.22575139	0.	-10.734127
N13n -> C12d	0.2810754	0.53337351	0.	-3.2617448
B10a -> C12d	1.3398029	4.9823924	0.	-15.547768
O14n -> C12He3	1.1478802	0.24406119	0.	-13.320598
C15p -> C14d	1.0064559	0.52357817	0.	-11.679437
Ne18n -> O15a	8.1084015	0.16638821	0.	-94.0941
Ne19n -> O16a	12.135453	0.65547753	0.	-140.82609
Na20n -> F17a	10.545302	0.26925106	0.	-122.37315
Ne18n -> F18p	5.2268465	0.33414036	0.	-60.65504
Ne19n -> F19p	4.0218407	1.0023002	0.	-46.671527
Li7He3 -> Be9p	11.200902	3.5570403	0.	-129.98108
Li6t -> Li7d	0.9938621	0.72739637	0.	-11.533292
Li6He3 -> Be7d	0.1133751	0.72713209	0.	-1.3156635
Li7He3 -> aad	11.85277	$2.8709955 \times 10^{-10}$	-1.5	-137.5457
Li8He3 -> Be9d	11.392846	1.2824306	0.	-132.2085
Li8He3 -> aat	16.077377	$3.5915942 \times 10^{-10}$	-1.5	-186.57023
Li9d -> Li8t	2.1950118	0.78151655	0.	-25.472056
Li9He3 -> Be10d	14.142906	4.1819929	0.	-164.12162
Li9He3 -> Be9t	13.587858	1.0022407	0.	-157.68056
Be7t -> aad	12.733257	2.872039×10^{-10}	-1.5	-147.76332
Be7t -> Li7He3	0.880487	1.0003635	0.	-10.217628
Be9d -> aat	4.6845306	$2.8006149 \times 10^{-10}$	-1.5	-54.361725
Be9t -> Be10d	0.5550482	4.1726432	0.	-6.4410674
Be9He3 -> B10d	1.0933372	0.59602569	0.	-12.687652
Be10He3 -> B11d	5.7352792	0.26489954	0.	-66.555157
Be10He3 -> B10t	0.538289	0.14284128	0.	-6.246585
B8d -> Be7He3	5.3571038	1.2514853	0.	-62.166613
B8t -> aaHe3	18.090361	$3.5943146 \times 10^{-10}$	-1.5	-209.92994
B10p -> aaHe3	-0.4414695	$1.3541873 \times 10^{-10}$	-1.5	5.1230412
B10t -> B11d	5.1969902	1.8545027	0.	-60.308572
B10He3 -> C11d	3.1967052	1.8535794	0.	-37.096226
B11t -> B13p	-0.2333537	4.0283348	0.	2.7079575
B11He3 -> C12d	10.463203	4.2936315	0.	-121.42044
N12n -> C11d	1.6242954	0.20213806	0.	-18.849167
C11t -> C12d	12.463488	4.2957702	0.	-144.63279
C11t -> B11He3	2.000285	1.0004981	0.	-23.212345
Be7He3 -> ppaa	11.272446	$1.2201356 \times 10^{-19}$	-3.	-130.81131
dd -> ag	23.846528	4.5291416×10^{10}	1.5	-276.72749
He3He3 -> app	12.859579	$3.3947053 \times 10^{-10}$	-1.5	-149.22923
Li7a -> B11g	8.6643136	4.01873×10^{10}	1.5	-100.54519
Be7p -> B8g	0.1363706	1.3052574×10^{10}	1.5	-1.5825152
Be7a -> C11g	7.5445156	4.0181891×10^{10}	1.5	-87.550475
Be9p -> B10g	6.5868116	9.7361417×10^{9}	1.5	-76.436781
Be9p -> aad	0.6518677	$8.0713045 \times 10^{-11}$	-1.5	-7.5646111
Be9p -> Li6a	2.1256261	0.61766701	0.	-24.666869
Be9a -> C12n	5.7020485	10.2858	0.	-66.16953
B10p -> C11g	8.6901796	3.0278413×10^{10}	1.5	-100.84535
B10p -> Be7a	1.145664	0.75353379	0.	-13.29488
B11p -> C12g	15.956678	7.0136917×10^{10}	1.5	-185.16957
B11p -> aaa	8.6819308	$3.5009576 \times 10^{-10}$	-1.5	-100.74963
B11a -> N14n	0.1578888	3.6723556	0.	-1.8322236
C13a -> O16n	2.2156086	5.7921384	0.	-25.711072
N15p -> C12a	4.9654937	0.70604024	0.	-57.622166
Li7t -> Be9n	10.437148	3.5507024	0.	-121.11808
B11n -> B12g	3.3696241	2.3383747×10^{10}	1.5	-39.102867
C11n -> aaa	11.44597	$3.5089537 \times 10^{-10}$	-1.5	-132.82498
Li7d -> aan	15.121678	9.94415×10^{-10}	-1.5	-175.47981
dn -> tg	6.2572289	1.6364262×10^{10}	1.5	-72.612132
tt -> ann	11.33207	$3.3826187 \times 10^{-10}$	-1.5	-131.50322

He3n -> ag	20.577619	2.6152351×10^{10}	1.5	-238.79338
He3t -> ad	14.32039	1.5981381	0.	-166.18124
He3t -> anp	12.095824	3.3886566×10^{-10}	-1.5	-140.36623
aan -> Be9g	1.5726983	5.843096×10^{19}	3.	-18.250407
Li7t -> aann	8.8644495	1.2153497×10^{-19}	-3.	-102.86767
Li7He3 -> aanp	9.628204	6.0875952×10^{-20}	-3.	-111.73068
Li8d -> Li9p	1.8376511	4.4398826	0.	-21.325057
Li8d -> Li7t	4.2246068	1.2509926	0.	-49.02453
Be7d -> aap	16.76592	9.9655209×10^{-10}	-1.5	-194.56044
Be7t -> aanp	10.508691	6.0898077×10^{-20}	-3.	-121.9483
Be7He3 -> aapp	11.272446	1.2201356×10^{-19}	-3.	-130.81131
C9a -> N12p	6.708845	3.4380846	0.	-77.85292
Li6n -> ta	4.7834705	1.0691921	0.	-55.509875
He3t -> Li6g	15.794149	2.4459918×10^{10}	1.5	-183.2835
anp -> Li6g	3.6983244	7.2181753×10^{19}	3.	-42.917276
Li6n -> Li7g	7.251091	1.1903305×10^{10}	1.5	-84.145424
Li6d -> Li7p	5.026525	2.5239503	0.	-58.330405
Li6d -> Be7n	3.3822835	2.5185377	0.	-39.249773
Li6a -> B10g	4.4611855	1.5762768×10^{10}	1.5	-51.769912
Li7a -> B10n	-2.7899055	1.3242346	0.	32.375512
Li7n -> Li8g	2.0326221	1.3081022×10^{10}	1.5	-23.587602
Li7d -> Li8p	-0.1919439	2.7736709	0.	2.2274166
Li8n -> Li9g	4.0622171	2.0939111×10^{10}	1.5	-47.140076
Li8p -> aan	15.313622	3.5851946×10^{-10}	-1.5	-177.70722
Li8d -> Be9n	14.661755	4.4419024	0.	-170.14261
Be9n -> Be10g	6.8122771	6.8282226×10^{10}	1.5	-79.053199
Be9p -> aapn	-1.5726983	1.7114215×10^{-20}	-3.	18.250407
B11p -> C11n	-2.7640395	0.99772122	0.	32.075349
Be10n -> Be11g	0.5016371	8.6569942×10^{9}	1.5	-5.8212573
Be11n -> Be12g	3.1706871	3.5079842×10^{10}	1.5	-36.794299
B8p -> C9g	1.2995706	2.089086×10^{10}	1.5	-15.080892
aaa -> C12gg	7.2747468	2.0033638×10^{20}	3.	-84.419938
C11p -> N12g	0.6002706	2.3331285×10^{10}	1.5	-6.9658516
B10a -> N13n	1.0587275	9.3412819	0.	-12.286023
B11a -> C14p	0.783759	11.039007	0.	-9.0951462
C11n -> C12g	18.720717	7.0297109×10^{10}	1.5	-217.24492
He6 -> Li6Bm	2.7228746	0.33369189	0.	-31.59765
Li8 -> aaBm	15.313622	3.5926101×10^{-10}	-1.5	-177.70722
Li9 -> Be9Bm	12.824104	1.0025242	0.	-148.81755
Li9 -> aanBm	11.251405	1.7157415×10^{-20}	-3.	-130.56715
Be11 -> B11Bm	10.727117	0.50087959	0.	-124.48303
Be12 -> B12Bm	10.926053	0.33387954	0.	-126.7916
B8 -> aaBp	18.854115	3.5932981×10^{-10}	-1.5	-218.79294
B12 -> C12Bm	12.587054	3.0055912	0.	-146.0667
B13 -> C13Bm	12.654545	2.0034564	0.	-146.84991
B14 -> C14Bm	19.861761	5.0121716	0.	-230.48618
B15 -> C15Bm	18.302553	2.0042068	0.	-212.39233
C9 -> aapBp	17.554545	1.7200336×10^{-20}	-3.	-203.71205
C10 -> B10Bp	4.4304075	0.14292924	0.	-51.412748
C11 -> B11Bp	2.7640395	1.0002152	0.	-32.075349
C15 -> N15Bm	8.9893148	1.0011044	0.	-104.31667
C16 -> N16Bm	7.2277535	0.20017152	0.	-83.874603
N12 -> C12Bp	18.120447	3.0067786	0.	-210.27907
N13 -> C13Bp	3.0028177	1.0002118	0.	-34.846255
N16 -> O16Bm	9.6385548	5.0055055	0.	-111.85079
N17 -> O16nBm	3.7533377	1.0969484×10^{-10}	-1.5	-43.555679
O13 -> N13Bp	18.551867	2.0042765	0.	-215.28549

O14 -> N14Bp	5.9267108	0.33351101	0.	-68.776628
O15 -> N15Bp	3.5365078	1.0002409	0.	-41.039472
O19 -> F19Bm	4.0379977	3.0013561	0.	-46.859021
O20 -> F20Bm	3.0313165	0.20006966	0.	-35.17697
F17 -> O17Bp	3.54281	1.0002132	0.	-41.112606
F18 -> O18Bp	2.4382621	3.0003074	0.	-28.294859
F20 -> Ne20Bm	6.242121	5.0030359	0.	-72.436812
Ne18 -> F18Bp	5.2268465	0.33345067	0.	-60.65504
Ne19 -> F19Bp	4.0218407	1.0002313	0.	-46.671527
Ne23 -> Na23Bm	3.593456	1.5005136	0.	-41.700329
Na20 -> Ne20Bp	14.674877	5.0053916	0.	-170.29489
Na21 -> Ne21Bp	4.3294965	1.000233	0.	-50.241725
ann -> He6g	0.9754498	1.0837999×10^{20}	3.	-11.319626
O16n -> O17g	4.1430793	3.0413795×10^{9}	1.5	-48.078443
N14n -> C14p	0.6258702	3.0059743	0.	-7.2629227
O14n -> N14p	5.9267108	0.33420083	0.	-68.776628
O14a -> Ne18g	5.1150966	5.4207018×10^{10}	1.5	-59.358236
C11a -> N14p	2.9219283	3.6807432	0.	-33.907573
O14a -> F17p	1.192026	0.49346269	0.	-13.832888
O17n -> C14a	1.817745	2.0311806	0.	-21.094056
F17n -> N14a	4.7346848	0.67725653	0.	-54.94374
F18n -> N15a	6.4180628	0.49910918	0.	-74.478532
C14d -> N15n	7.9828589	1.9080971	0.	-92.637238
ppn -> dp	2.224566	2.3580703×10^{9}	1.5	-25.815019
C14n -> C15g	1.2181101	9.0075198×10^{9}	1.5	-14.135582
O16p -> N13a	-5.2184263	0.17225497	0.	60.557327
Li8p -> Be9g	16.886321	2.0948637×10^{10}	1.5	-195.95763
B11a -> N15g	10.991184	9.9338413×10^{10}	1.5	-127.5474

List of reactions

```
ListReactionsNames = ListReactionsUpToChosenMass[[All, 1]];
```

```
ListNuclearReactionsNames = Select[ListReactionsNames, # =!= "nTOp" &];
```

We can define an association (a dictionary) between reaction names and number

```
KeyNuclearReaction =
  Association[# → Position[ListNuclearReactionsNames, #][[1, 1]] & /@
    ListNuclearReactionsNames];
KeyReaction = Association[# → Position[ListReactionsNames, #][[1, 1]] & /@
    ListReactionsNames];
```

Let us collect all the species involved in the reactions. These are the species whose abundance we are going to solve numerically.

```
VariablesInEquations =
 Union@Flatten[Join[RemoveNonNuclear[#[[2]]], RemoveNonNuclear[#[[3]]]] & /@
    ListReactionsUpToChosenMass]
```

```
{a, B10, B11, B12, B13, B14, B15, B8, Be10, Be11, Be12, Be7, Be9, C10, C11,
 C12, C13, C14, C15, C16, C9, d, F17, F18, F19, F20, He3, He6, Li6, Li7,
 Li8, Li9, n, N12, N13, N14, N15, N16, N17, Na20, Na21, Na22, Na23, Ne18,
 Ne19, Ne20, Ne21, Ne22, Ne23, O13, O14, O15, O16, O17, O18, O19, O20, p, t}
```

We can check the number of species (59 for the full network of 423 equations).

```
NumberVariable = Length@VariablesInEquations
```

```
59
```

List of chemical species names used

Let us collect all the species used together with their weights in terms of neutrons and protons

```
NamesWithWeights =
 Select[NamesWithWeightsAll, MemberQ[VariablesInEquations, #[[1]]] &]
```

```
{{n, {1, 0}}, {p, {0, 1}}, {d, {1, 1}}, {t, {2, 1}}, {He3, {1, 2}}, {a, {2, 2}},
 {He6, {4, 2}}, {Li6, {3, 3}}, {Li7, {4, 3}}, {Li8, {5, 3}}, {Li9, {6, 3}},
 {Be7, {3, 4}}, {Be9, {5, 4}}, {Be10, {6, 4}}, {Be11, {7, 4}}, {Be12, {8, 4}},
 {B8, {3, 5}}, {B10, {5, 5}}, {B11, {6, 5}}, {B12, {7, 5}}, {B13, {8, 5}},
 {B14, {9, 5}}, {B15, {10, 5}}, {C9, {3, 6}}, {C10, {4, 6}}, {C11, {5, 6}},
 {C12, {6, 6}}, {C13, {7, 6}}, {C14, {8, 6}}, {C15, {9, 6}}, {C16, {10, 6}},
 {N12, {5, 7}}, {N13, {6, 7}}, {N14, {7, 7}}, {N15, {8, 7}}, {N16, {9, 7}},
 {N17, {10, 7}}, {O13, {5, 8}}, {O14, {6, 8}}, {O15, {7, 8}}, {O16, {8, 8}},
 {O17, {9, 8}}, {O18, {10, 8}}, {O19, {11, 8}}, {O20, {12, 8}}, {F17, {8, 9}},
 {F18, {9, 9}}, {F19, {10, 9}}, {F20, {11, 9}}, {Ne18, {8, 10}}, {Ne19, {9, 10}},
 {Ne20, {10, 10}}, {Ne21, {11, 10}}, {Ne22, {12, 10}}, {Ne23, {13, 10}},
 {Na20, {9, 11}}, {Na21, {10, 11}}, {Na22, {11, 11}}, {Na23, {12, 11}}}
```

WeightsNuclear is the list of nuclear weights for the variables (their A in nuclear physics notation). It is obtained by summing the (n,p) numbers

```
WeightsNuclear = (Plus @@ (#[[2]])) & /@ NamesWithWeights
```

```
{1, 1, 2, 3, 3, 4, 6, 6, 7, 8, 9, 7, 9, 10, 11, 12, 8, 10, 11, 12, 13, 14,
 15, 9, 10, 11, 12, 13, 14, 15, 16, 12, 13, 14, 15, 16, 17, 13, 14, 15, 16,
 17, 18, 19, 20, 17, 18, 19, 20, 18, 19, 20, 21, 22, 23, 20, 21, 22, 23}
```

We build a function which selects all species having the same mass number A or Z number

```
NamesMassNumber[A_] :=
 Select[NamesWithWeights, ((Plus @@ (#[[2]])) == A) &][[All, 1]]
NamesAtomicNumber[Z_] := Select[NamesWithWeights, (#[[2, 2]] == Z) &][[All, 1]]
```

Shorthand notation for abundances of species

We use two ways of noting the abundance of a species. One notation is Ynipj where i is the number of neutrons and j the number of protons. For instance Yn2p2 is He4.

But we also want to use short names. For instance He4 for Helium4. So we want to relate automatically YHe4 to Yn2p2.

When the function ShortString is evaluated, it defines the relation between the Yshortname and the Ynipj notation

```
StackY[name_] := "Y" <> ToString[name];

YName[PostString_][n_, p_] :=
  ToExpression@StackY["n" <> ToString[n] <> "p" <> ToString[p] <> PostString];
ShortString[nameshort_, np_List] :=
  (Evaluate@ToExpression["Y" <> nameshort] := YName[""] @@ np;);

ShortString @@@ NamesWithWeights;
```

Some examples to see how these short names have been related ot the correct names

```
Ya
Yp
Yt
YLi7
```

```
Yn2p2

Yn0p1

Yn2p1

Yn4p3
```

The list of all shortnames available is ShortNames, and we associate a number through a dictionary KeyVal

```
ShortNames = NamesWithWeights[[All, 1]]
KeyVal = Association[# → Position[ShortNames, #][[1, 1]] & /@ ShortNames]
```

```
{n, p, d, t, He3, a, He6, Li6, Li7, Li8, Li9, Be7, Be9, Be10, Be11, Be12,
 B8, B10, B11, B12, B13, B14, B15, C9, C10, C11, C12, C13, C14, C15, C16,
 N12, N13, N14, N15, N16, N17, O13, O14, O15, O16, O17, O18, O19, O20, F17,
 F18, F19, F20, Ne18, Ne19, Ne20, Ne21, Ne22, Ne23, Na20, Na21, Na22, Na23}
```

```
<|n → 1, p → 2, d → 3, t → 4, He3 → 5, a → 6, He6 → 7, Li6 → 8, Li7 → 9, Li8 → 10, Li9 → 11,
 Be7 → 12, Be9 → 13, Be10 → 14, Be11 → 15, Be12 → 16, B8 → 17, B10 → 18, B11 → 19,
 B12 → 20, B13 → 21, B14 → 22, B15 → 23, C9 → 24, C10 → 25, C11 → 26, C12 → 27, C13 → 28,
 C14 → 29, C15 → 30, C16 → 31, N12 → 32, N13 → 33, N14 → 34, N15 → 35, N16 → 36,
 N17 → 37, O13 → 38, O14 → 39, O15 → 40, O16 → 41, O17 → 42, O18 → 43, O19 → 44,
 O20 → 45, F17 → 46, F18 → 47, F19 → 48, F20 → 49, Ne18 → 50, Ne19 → 51, Ne20 → 52,
 Ne21 → 53, Ne22 → 54, Ne23 → 55, Na20 → 56, Na21 → 57, Na22 → 58, Na23 → 59|>
```

The list of variable in standard Ynipj forms are

```
VarList = ToExpression /@ (StackY /@ ShortNames)
```

```
{Yn1p0, Yn0p1, Yn1p1, Yn2p1, Yn1p2, Yn2p2, Yn4p2, Yn3p3, Yn4p3, Yn5p3, Yn6p3,
 Yn3p4, Yn5p4, Yn6p4, Yn7p4, Yn8p4, Yn3p5, Yn5p5, Yn6p5, Yn7p5, Yn8p5, Yn9p5,
 Yn10p5, Yn3p6, Yn4p6, Yn5p6, Yn6p6, Yn7p6, Yn8p6, Yn9p6, Yn10p6, Yn5p7,
 Yn6p7, Yn7p7, Yn8p7, Yn9p7, Yn10p7, Yn5p8, Yn6p8, Yn7p8, Yn8p8, Yn9p8,
 Yn10p8, Yn11p8, Yn12p8, Yn8p9, Yn9p9, Yn10p9, Yn11p9, Yn8p10, Yn9p10,
 Yn10p10, Yn11p10, Yn12p10, Yn13p10, Yn9p11, Yn10p11, Yn11p11, Yn12p11}
```

Abstract abundance functions

We build list of abundance function and abundance function derivatives which are used later to build the system of equations. These stay at an abstract level and they are used only to build the differential system of equations which is later solved by NDSolve.

```
SetTimeDependence[list_List, tv_] := #[tv] & /@ list;
```

```
FunList[tv_] = SetTimeDependence[VarList, tv]
FunPrimeList[tv_] = FunList'[tv]
```

```
{Yn1p0[tv], Yn0p1[tv], Yn1p1[tv], Yn2p1[tv], Yn1p2[tv], Yn2p2[tv], Yn4p2[tv],
 Yn3p3[tv], Yn4p3[tv], Yn5p3[tv], Yn6p3[tv], Yn3p4[tv], Yn5p4[tv], Yn6p4[tv],
 Yn7p4[tv], Yn8p4[tv], Yn3p5[tv], Yn5p5[tv], Yn6p5[tv], Yn7p5[tv], Yn8p5[tv],
 Yn9p5[tv], Yn10p5[tv], Yn3p6[tv], Yn4p6[tv], Yn5p6[tv], Yn6p6[tv], Yn7p6[tv],
 Yn8p6[tv], Yn9p6[tv], Yn10p6[tv], Yn5p7[tv], Yn6p7[tv], Yn7p7[tv], Yn8p7[tv],
 Yn9p7[tv], Yn10p7[tv], Yn5p8[tv], Yn6p8[tv], Yn7p8[tv], Yn8p8[tv],
 Yn9p8[tv], Yn10p8[tv], Yn11p8[tv], Yn12p8[tv], Yn8p9[tv], Yn9p9[tv],
 Yn10p9[tv], Yn11p9[tv], Yn8p10[tv], Yn9p10[tv], Yn10p10[tv], Yn11p10[tv],
 Yn12p10[tv], Yn13p10[tv], Yn9p11[tv], Yn10p11[tv], Yn11p11[tv], Yn12p11[tv]}
```

```
{Yn1p0′[tv], Yn0p1′[tv], Yn1p1′[tv], Yn2p1′[tv], Yn1p2′[tv], Yn2p2′[tv], Yn4p2′[tv],
 Yn3p3′[tv], Yn4p3′[tv], Yn5p3′[tv], Yn6p3′[tv], Yn3p4′[tv], Yn5p4′[tv],
 Yn6p4′[tv], Yn7p4′[tv], Yn8p4′[tv], Yn3p5′[tv], Yn5p5′[tv], Yn6p5′[tv],
 Yn7p5′[tv], Yn8p5′[tv], Yn9p5′[tv], Yn10p5′[tv], Yn3p6′[tv], Yn4p6′[tv],
 Yn5p6′[tv], Yn6p6′[tv], Yn7p6′[tv], Yn8p6′[tv], Yn9p6′[tv], Yn10p6′[tv],
 Yn5p7′[tv], Yn6p7′[tv], Yn7p7′[tv], Yn8p7′[tv], Yn9p7′[tv], Yn10p7′[tv],
 Yn5p8′[tv], Yn6p8′[tv], Yn7p8′[tv], Yn8p8′[tv], Yn9p8′[tv], Yn10p8′[tv],
 Yn11p8′[tv], Yn12p8′[tv], Yn8p9′[tv], Yn9p9′[tv], Yn10p9′[tv], Yn11p9′[tv],
 Yn8p10′[tv], Yn9p10′[tv], Yn10p10′[tv], Yn11p10′[tv], Yn12p10′[tv],
 Yn13p10′[tv], Yn9p11′[tv], Yn10p11′[tv], Yn11p11′[tv], Yn12p11′[tv]}
```

Numerical abundance functions

We also need the list of functions to store the results of the numerical integrations.
We have divided our BBN period in 3 eras.

High temperature (HT),
Middle temperature (MT)
Low temperature (LT).

So for instance the abundance of a species, say Lithium7 whose shortname is ‘Li7’, during the HT period at a given cosmological time t, is

YHT[“Li7”][t].

Eventually we are interested in the values YLT[“species”][tend] where tend is the final time of our numerical integration.

```
KeyQ[key_] := MemberQ[NamesWithWeightsAll[[All, 1]], key];

YHT[key_?KeyQ][t_] := Y["HT"][key][t];
YMT[key_?KeyQ][t_] := Y["MT"][key][t];
YLT[key_?KeyQ][t_] := Y["LT"][key][t];
```

We make a list of the Y_i for a given period at a given
time. This is typically used for initial conditions in Differential Solver.

```
YPeriodTime[period_String][tv_] := Y[period][#][tv] & /@ ShortNames;
```

We also define a function which can choose between previsouly numerically solved function in a given era, or equilibrium values for a list of species

```
NumericalValueOrThermalEquilibriumValue[
   period_, ListThermal_, name_, Tv_, tv_] :=
  Module[{Yv = Y[period][name][tv], Yn = Y[period]["n"][tv],
    Yp = Y[period]["p"][tv]},
   If[ MemberQ[ListThermal, name], YNSE[name, Yn, Yp, Tv], Yv]];
```

We make a list of the Y_i for a given period at a given time. If there
  is no known numerical value it takes the thermostatistical equilibrium.
    This is only used for the initial conditions in the middle era,
where all species considered start at thermodynamical equilibrium or not very far from it except
  for neutrons and protons which are computed numerically from the high temperature era.

```
YPeriodTimeOrStateEquilibrium[period_String, ListThermal_List][Tv_, tv_] :=
  NumericalValueOrThermalEquilibriumValue[period, ListThermal, #, Tv, tv] & /@
   ShortNames;
```

CNO

List of CNO nuclei. This includes all N and O, but only C with A>=12. C11 decays into B11 and C10 into B10 and C9 into a+2p.

```
CNONuclei = KeyNucleons /@ Join[Table[{i, 6}, {i, 6, 10}],
   Table[{i, 7}, {i, 5, 10}], Table[{i, 8}, {i, 5, 12}]]
{C12, C13, C14, C15, C16, N12, N13, N14,
 N15, N16, N17, O13, O14, O15, O16, O17, O18, O19, O20}

YLT["CNO"][t_] := Plus @@ ((YLT[#][t] &) /@ CNONuclei)
```

Coupled systems of differential reactions

Formal construction of r.h.s

This is a function which takes the list of reactions, with the specification of initial and final particles and their multiplicity, and builds the rhs of the differential system.
It builds the rhs of Eq 138 in companion paper.

```
FillReactionMatrix[listreac_List] :=
 Module[{Tab, i, j, nvar, TreatReaction, FactorInitialElements},
  nvar = Length@VarList;
  Tab = Table[0, {ii, 1, nvar}];
  FactorInitialElements[el_List] :=
   Times @@ ((AρB / DensityUnit Y[KeyVal[#[[1]]]]) ^#[[2]] / (#[[2]] !) & /@ el);
  TreatReaction[reaction_List] := Module[{
     InitialParticles = Tally[RemoveNonNuclear@reaction[[2]]],
     FinalParticles = Tally[RemoveNonNuclear@reaction[[3]]],
     ReactionForward, ReactionBackward,
     FactorInitialForward, FactorInitialBackward},

    ReactionForward = L[KeyReaction[reaction[[1]]]];
    ReactionBackward = Lbar[KeyReaction[reaction[[1]]]];
    FactorInitialForward = FactorInitialElements@InitialParticles;
    (* This computes the product Yi^ni/ni! for initial particles*)
    FactorInitialBackward = FactorInitialElements@FinalParticles;

    (Tab[[KeyVal[#[[1]]]]] = Tab[[KeyVal[#[[1]]]]] - ReactionForward
          FactorInitialForward #[[2]] / AρB * DensityUnit) & /@ InitialParticles;
    (Tab[[KeyVal[#[[1]]]]] = Tab[[KeyVal[#[[1]]]]] + ReactionForward
          FactorInitialForward #[[2]] / AρB * DensityUnit) & /@ FinalParticles;
    (Tab[[KeyVal[#[[1]]]]] = Tab[[KeyVal[#[[1]]]]] - ReactionBackward
          FactorInitialBackward #[[2]] / AρB * DensityUnit) & /@ FinalParticles;
    (Tab[[KeyVal[#[[1]]]]] = Tab[[KeyVal[#[[1]]]]] + ReactionBackward
          FactorInitialBackward #[[2]] / AρB * DensityUnit) & /@ InitialParticles;
   ];
  TreatReaction /@ listreac;
  Tab
 ]
```

To check the list of reactions used at this stage, just evaluate the next cell after uncommenting it.

```
(*Print[ListReactionsUpToChosenMass];*)
```

FormalReactions is the list of rhs of differential equation in an abstract level.
-The species are given by Y[1], Y[2],Y[3], and we need the dictionary KeyVal to know to which species it corresponds.
-The reactions are L[1], L[2],L[3] and we also need a dictionary (KeyReaction) to know to which reaction it corresponds.

```
FormalReactions = FillReactionMatrix@ListReactionsUpToChosenMass;
```

To see how this works we look at a few elements of FormalReactions. For instance the source of Li7 due to nuclear reactions is one element of FormalReactions. It is

```
FormalReactions[[KeyVal["Li7"]]]
```

$$
\begin{aligned}
&\text{A}\rho\text{B}\,\text{L[8] Y[4] Y[6]} + \frac{1}{2}\text{A}\rho\text{B}\,\text{Lbar[13] Y[6]}^2 + \frac{1}{2}\text{A}\rho\text{B}\,\text{Lbar[14] Y[6]}^2 + \\
&\frac{1}{2}\text{A}\rho\text{B}^2\,\text{Lbar[337] Y[1] Y[6]}^2 + \frac{1}{4}\text{A}\rho\text{B}^3\,\text{Lbar[344] Y[1]}^2\,\text{Y[6]}^2 + \\
&\frac{1}{2}\text{A}\rho\text{B}^3\,\text{Lbar[345] Y[1] Y[2] Y[6]}^2 + \frac{1}{2}\text{A}\rho\text{B}^2\,\text{Lbar[294] Y[3] Y[6]}^2 + \\
&\text{A}\rho\text{B L[355] Y[1] Y[8]} + \text{A}\rho\text{B L[356] Y[3] Y[8]} + \text{A}\rho\text{B L[292] Y[4] Y[8]} + \\
&\text{A}\rho\text{B Lbar[67] Y[6] Y[8]} - \text{Lbar[8] Y[9]} - \text{Lbar[355] Y[9]} - \text{A}\rho\text{B L[360] Y[1] Y[9]} - \\
&\text{A}\rho\text{B L[13] Y[2] Y[9]} - \text{A}\rho\text{B L[14] Y[2] Y[9]} - \text{A}\rho\text{B Lbar[12] Y[2] Y[9]} - \\
&\text{A}\rho\text{B Lbar[356] Y[2] Y[9]} - \text{A}\rho\text{B L[65] Y[3] Y[9]} - \text{A}\rho\text{B L[337] Y[3] Y[9]} - \\
&\text{A}\rho\text{B L[361] Y[3] Y[9]} - \text{A}\rho\text{B Lbar[292] Y[3] Y[9]} - \text{A}\rho\text{B L[68] Y[4] Y[9]} - \\
&\text{A}\rho\text{B L[334] Y[4] Y[9]} - \text{A}\rho\text{B L[344] Y[4] Y[9]} - \text{A}\rho\text{B Lbar[273] Y[4] Y[9]} - \\
&\text{A}\rho\text{B Lbar[347] Y[4] Y[9]} - \text{A}\rho\text{B L[66] Y[5] Y[9]} - \text{A}\rho\text{B L[67] Y[5] Y[9]} - \\
&\text{A}\rho\text{B L[291] Y[5] Y[9]} - \text{A}\rho\text{B L[294] Y[5] Y[9]} - \text{A}\rho\text{B L[345] Y[5] Y[9]} - \\
&\text{A}\rho\text{B Lbar[301] Y[5] Y[9]} - \text{A}\rho\text{B L[320] Y[6] Y[9]} - \text{A}\rho\text{B L[359] Y[6] Y[9]} - \\
&\text{A}\rho\text{B Lbar[33] Y[6] Y[9]} - \text{A}\rho\text{B Lbar[75] Y[6] Y[9]} - \text{A}\rho\text{B Lbar[94] Y[6] Y[9]} + \\
&\text{Lbar[360] Y[10]} + \text{A}\rho\text{B Lbar[361] Y[2] Y[10]} + \text{A}\rho\text{B L[347] Y[3] Y[10]} + \\
&\text{A}\rho\text{B L[75] Y[5] Y[10]} + \text{A}\rho\text{B L[273] Y[2] Y[11]} + \text{A}\rho\text{B L[12] Y[1] Y[12]} + \\
&\text{A}\rho\text{B L[301] Y[4] Y[12]} + \text{Lbar[65] Y[13]} + \text{A}\rho\text{B Lbar[334] Y[1] Y[13]} + \\
&\text{A}\rho\text{B Lbar[291] Y[2] Y[13]} + \text{A}\rho\text{B L[94] Y[3] Y[13]} + \text{Lbar[68] Y[14]} + \\
&\text{A}\rho\text{B L[33] Y[2] Y[14]} + \text{Lbar[66] Y[18]} + \text{A}\rho\text{B Lbar[359] Y[1] Y[18]} + \text{Lbar[320] Y[19]}
\end{aligned}
$$

* AρB is an abstract variable which needs to be replaced with the appropriate ρB
  (taking into account that this is not exactly ρ_B but $n_B\, m_a$, see appendix of companion paper).

* The reactions L[i] and Lbar[i] are also the reactions,
with i being the i – th line of ListReactions. For instance reaction L[8] is

```
NiceDisplayReaction[ListReactions[[8]]]
```

```
{ta -> Li7g, {t, a}, {Li7, g}, 0, DAACV04}
```

The Y[i] are the abstract abundances, corresponding to the i-th elements in VariableList. For instance 9 corresponds to Li7 and so Y[9] is the abstract abundance of Li7.

```
KeyVal["Li7"]
ShortNames[[9]]
```

```
9

Li7
```

We also build formally a small network of reaction which is used in the middle temperature era. It is the same thing but with a reduced number of equations.

```
NReactionsSmallNetwork = Min[NumberNuclearReactions + 1, 18];
FormalReactions18 = FillReactionMatrix@Take[ListReactionsUpToChosenMass,
    Min[NReactionsSmallNetwork, Length@ListReactionsUpToChosenMass]];
```

We check that formally nucleons are conserved. So we compute formally the
$\Sigma_i A_i\, \mathrm{d}Y_i/\mathrm{d}t$ and check that it is 0.

```
WeightsNuclear.FormalReactions // Simplify
```

```
0
```

We also build a differential system with just neutrons and protons and the weak interactions for the high temperature era

```
FormalReactionsOnlyPEN = FillReactionMatrix[{ReactionPEN}];
```

Actual r.h.s of differential system

Now we can build the rhs of the differential equation. This is obtained from its formal expression “FormalReactions” in which time dependence is added thanks to some replacement rules.

First replacement rules for the abstract abundances.

```
Yi := Y[KeyVal[#]] & /@ShortNames;

RulesY[tv_] := Thread[Rule[Yi, FunList[tv]]];
```

Let us visualize few of these rules to understand

```
Take[RulesY[tv], 8]
```

```
{Y[1] → Yn1p0[tv], Y[2] → Yn0p1[tv], Y[3] → Yn1p1[tv], Y[4] → Yn2p1[tv],
 Y[5] → Yn1p2[tv], Y[6] → Yn2p2[tv], Y[7] → Yn4p2[tv], Y[8] → Yn3p3[tv]}
```

Then for the abstract reactions, we also define rules to replace the abstract reaction L[i] or its reverse Lbar[i] by the actual rate.

```
Li = L[KeyReaction[[#]]] & /@ListReactionsNames;
Lbari = Lbar[KeyReaction[[#]]] & /@ListReactionsNames;


RulesλRHS[Tv_] := Symbol["L" <> #][Tv] & /@ListReactionsNames;
RulesλRHS[n_, Tv_] := Symbol["L" <> #][Tv] & /@Take[ListReactionsNames, n];

RulesλbarRHS[Tv_] := Symbol["Lbar" <> #][Tv] & /@ListReactionsNames;
RulesλbarRHS[n_, Tv_] := Symbol["Lbar" <> #][Tv] & /@Take[ListReactionsNames, n];

Rulesλ[Tv_] := Thread[Rule[Li, RulesλRHS[Tv]]];
Rulesλbar[Tv_] := Thread[Rule[Lbari, RulesλbarRHS[Tv]]];
```

Let us visualize a few of these rules.

```
Take[Rulesλ[Tv], 4]
```

```
{L[1] → 0.0011370097 InterpolatingFunction[ ⊞ Domain: {{1.×10^7, 1.×10^12}} Output: scalar ][Tv],

 L[2] → InterpolatingFunction[ ⊞ Domain: {{1.×10^6, 1.×10^10}} Output: scalar ][Tv],

 L[3] → InterpolatingFunction[ ⊞ Domain: {{1.×10^6, 1.×10^10}} Output: scalar ][Tv],

 L[4] → InterpolatingFunction[ ⊞ Domain: {{1.×10^6, 1.×10^10}} Output: scalar ][Tv]}
```

Let us apply these rules to form the r.h.s

Here is the r.h.s of the differential system for nuclear reactions. We distinguish between high, middle and low temperature system of equations.

```
DYOnlyPEN[Temp_, ρB_, time_] :=
  (FormalReactionsOnlyPEN) /. Dispatch@Rulesλ[Temp] /.
     Dispatch@Rulesλbar[Temp] /. Dispatch@RulesY[time] /. AρB → ρB;

DY18[Temp_, ρB_, time_] :=
  (FormalReactions18) /. Dispatch@Rulesλ[Temp] /. Dispatch@Rulesλbar[Temp] /.
    Dispatch@RulesY[time] /. AρB → ρB;

DY[Temp_, ρB_, time_] :=
  (FormalReactions) /. Dispatch@Rulesλ[Temp] /. Dispatch@Rulesλbar[Temp] /.
    Dispatch@RulesY[time] /. AρB → ρB;


(*DY[Tv,rv,tv][[5]];//Timing*)
```

We define compiled version which are used if the $CompileNDSolve option is set to True. This is much faster.
Otherwise if $CompileNDSolve=False, the DY, DY18 and DYPEN are called in DefineEquations further below when it si associated with the l.h.s to form the differential system.

```
CompileFromFormal[FormalReactions_List] := ReleaseHold[
  Hold[Compile[{{AρB, _Real}, {L, _Real, 1}, {Lbar, _Real, 1}, {Y, _Real, 1}},
     inside, CompilationTarget → "C", "RuntimeOptions" → "Speed",
     CompilationOptions → {"InlineExternalDefinitions" → True}]] //.
   {inside → FormalReactions, Y[m_] :→ Y[[m]], L[m_] :→ L[[m]],
    Lbar[m_] :→ Lbar[[m]]}]

Timing[If[$CompileNDSolve,
   DYC = CompileFromFormal[FormalReactions];
   DY18C = CompileFromFormal[FormalReactions18];
   DYCN[AρB_?NumericQ, L_, Lbar_, Y_] := DYC[AρB, L, Lbar, Y];
   DY18CN[AρB_?NumericQ, L_, Lbar_, Y_] := DY18C[AρB, L, Lbar, Y];
  ];]
{2.738546, Null}
```

Time integration of Cosmology and BBN

Friedmann equation

The Friedmann equation gives the Hubble expansion rate.
For completeness we put baryons and CDM even though it makes a difference of order 10^{-5} which is below what we can achieve anyway with homogeneous computations.

$H^2 = \frac{8\pi G \rho}{3} = \frac{\rho}{\rho_{\rm crit}}$ so we build first the energy density.

We select the neutrino energy density depending on options for decoupling

```
ρν[T_] := If[$IncompleteNeutrinoDecoupling,
   ρνIncompleteDecoupling[a[T]], ρνDecoupling[T]];
```

Two methods to get the total energy density. Either from energy density of neutrinos or from temperature. This should be (and it is...) totally equivalent.

```
ρtot1[T_] := (aBB (kB T)^4 (1 + DρT[T]) + ρν[T]
      + nbaryons0 (mbaryon0 * (1 + h2Ωc0 / h2Ωb0) + 3/2 (kB T)) / (clight)^2 / (a[T])^3);

ρtot2[T_] := (aBB (kB T)^4 (1 + 7/8 Nneu (TνoverT[T])^4 + DρT[T])
      + nbaryons0 (mbaryon0 * (1 + h2Ωc0 / h2Ωb0) + 3/2 (kB T)) / (clight)^2 / (a[T])^3);

ρtot[T_] := ρtot1[T];

ρtot2[10^10]
ρtot1[10^10]
442 752.36

442 752.36
```

We check that we have the correct asymptotic behaviours for the degrees of freedom (Beware that QED corrections alter a bit the results)

```
ρtot[T] / (aBB (kB T)^4) /. T → 10^12 // NP
(2 + 4 * 7 / 8 + 3 * 2 * 7 / 8) / 2 // N // NP
5.367737

5.375

ρtot[T] / (aBB (kB T)^4) /. T → 10^7.5 // NP
(2 + 3 * 2 * 7 / 8 * (FourOverEleven) ^ (4 / 3)) / 2 // N // NP
1.6919223

1.6837209
```

We plot the energy density as a function of temperature.

```
If[$PaperPlots, ListLogLogPlot[
  Table[{T, 1/T^4 ρtot[T]}, {T, ListT}], Frame → True, Joined → True,
  PlotStyle → Black, FrameLabel → {"T (K)", "T^-4 ρ(T)    g cm^-3 K^-4"}]]
If[$PaperPlots, Export["Plots/PlotρT.pdf", Style[%, Magnification → 1], "PDF"];]
```

Hubble function from Friedmann equation in flat FL space-time is then immediate

```
H[a_] := (8 π GN / 3 ρtot[Tofa[a]])^(1/2);
```

Time and scale factor

This is the solver for dt/da giving t(a). Direct integration of the Friedmann equation

```
Computetofa :=
 (tofa = NDSolveValue[{tv'[av] == 1/(av H[av]), tv[a[Ti]] == 1/(2 H[a[Ti]])},
     tv, {av, a[Ti], a[Tf]}, PrecisionGoal → 8, AccuracyGoal → 10];)
```

We check that it is quick enough.

```
Timing@Computetofa
```

```
{0.045016, Null}
```

This is the solver for the inverse function, that is for da/dt, giving a(t)

```
Computeaoft :=
 (aoft = NDSolveValue[{av'[tv] == 1/tofa'[av[tv]], av[tofa@a[Ti]] == a[Ti]}, av,
     {tv, tofa@a[Ti], tofa@a[Tf]}, PrecisionGoal → 75, AccuracyGoal → 20];)
```

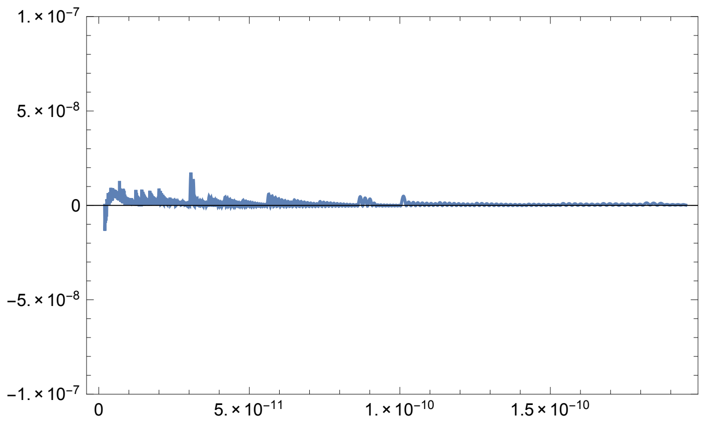
We check that it is quick enough.

```
Timing@Computeaoft
```

```
{0.02735, Null}
```

We also check that computing a(t(a)) gives negligible error to identity. It is of order 10^{-9} so we are totally OK.

```
Plot[1 / av * aoft@tofa@av - 1, {av, a[Ti], a[Ti] * 100},
 PlotRange → {-10^-7, 10^-7}, Frame → True]
```



We define the temperature as a function of time. Since we have T(a) (conserved entropy) and a(t) (Friedman equation), we just combine both.
In case of incomplete decoupling of neutrinos, we could not use conservation of entropy to get T(a) but we used the fit for the heating rate which allowed us to obtain numericall T(a) and a(T).

```
Toft[tv_] := Tofa[aoft[tv]];
```

Initial nuclear conditions

We use equilibrium solution for the initial condition. This equation A15 of compantion paper.

```
Yni[Tv_] := 1 / (1 + (1 - 3/2 Q/mn) Exp[Q/(kB Tv)]);
Ypi[Tv_] := 1 - Yni[Tv];
```

Other initial conditions based on the equation and the rates. It is better when including corrections because these are the correct initial conditions whatever the corrections.

```
Yn2i[Tv_] := LpTOn[Tv] / (LpTOn[Tv] + LnTOp[Tv]);
Yp2i[Tv_] := 1 - Yn2i[Tv];
```

We see that the difference is very small

```
Yni[10^11.5]
Yn2i[10^11.5]
```

```
0.48865343
```

```
0.48896618
```

Actual list of initial conditions. Vanishing for everything except protons and neutrons

```
CIList[Tv_] :=
 Table[Which[i == 1, Yni[Tv], i == 2, Ypi[Tv], i ≥ 3, 0], {i, 1, NumberVariable}]
```

Construction of differential equations

The function DefineEquations wraps the definition of differential equations. This must be called any time we regenerate the probabilities on the reaction rates when including uncertainties on reaction rates in a Monte-Carlo analysis.

```
DefineEquations := (

   (*We build the differential system for the High temperatures *)
   (* So we associate the r.h.s which is constructed thanks to DYOnlyPEN,
  with the l.h.s made of abundances derivatives *)
  SystemEquationsHT[tv_] = Thread[Equal[FunPrimeList[tv],
      (DYOnlyPEN[Tv, ρBv, tv] (*/.Dispatch@ReactionProbabilities*))]] /.
    {Tv → Toft@tv, ρBv -> ρBForBBN@a@Toft@tv};

   (* For middle and low temperature we distinguish
    between the compiled and the uncompiled method *)

  If[$CompileNDSolve,

    (* If $CompileNDSolve=True,
    we reinterpolate the rates for the middle and the low temperatures. *)
    (* The system is a matrix system in this case and
     it is built directly in NDSolve below *)
    RulesλRHSI = MyInterpolationRate@
      Table[{Tv, MyChop@RulesλRHS[Tv]}, {Tv, ListTRange[Tf, T18]}];
    RulesλbarRHSI = MyInterpolationRate@
      Table[{Tv, MyChop@RulesλbarRHS[Tv]}, {Tv, ListTRange[Tf, T18]}];
   RulesλRHS18I = MyInterpolationRate@Table[{Tv,
        RulesλRHS[NReactionsSmallNetwork, Tv]}, {Tv, ListTRange[T18, TMiddle]}];
    RulesλbarRHS18I = MyInterpolationRate@Table[{Tv, RulesλbarRHS[
         NReactionsSmallNetwork, Tv]}, {Tv, ListTRange[T18, TMiddle]}];,

    (* If $CompileNDSolve=False,
    we (re-)define the systems of equations for Middle and Low temperatures*)
    (* We associate the r.h.s formed thanks to DY18 and DY,
    with the l.h.s made of derivatives *)
    SystemEquationsMT[tv_] = Thread[Equal[FunPrimeList[tv],
        (DY18[Tv, ρBv, tv])]] /. {Tv → Toft@tv, ρBv -> ρBForBBN@a@Toft@tv};
    SystemEquationsLT[tv_] = Thread[Equal[FunPrimeList[tv], (DY[Tv, ρBv, tv])]] /.
      {Tv → Toft@tv, ρBv -> ρBForBBN@a@Toft@tv};
   ]
 )
```

```
DefineEquations; // Timing
```

```
{5.242376, Null}
```

Time delimitation of low, middle and high temperature eras

The time delimitation corresponding to Temperature delimitations of high, middle and low temperature eras.

```
t0 := tofa@a[Tstart];
tmiddle := tofa@a[TMiddle];
t18 := tofa@a[T18];
tend := tofa@a[Tend];
```

Let us check the values in seconds or t0< tmiddle < t18 < tend

```
{t0, tmiddle, t18, tend}
```

```
{0.0099481199, 1.006982, 99.706252, 49 227.544}
```

High temperature integration (n and p only)

The initial conditions at high temperature are found from thermal and chemical equilibrium. This is used to integrate from 10^{11} K to 10^{10} K. We keep track only of neutrons and protons.

```
HoldYNames[period_String] :=
  ToExpression /@ ("Hold@Y[\"" <> period <> "\"][\"" <> # <> "\"]" & /@ ShortNames);
```

Initial conditions

```
InitialConditionsHT[tv_] := Thread[Equal[FunList[tv], CIList[Toft[tv]]]];
```

Actual solver. It solves the system and affects the results to the functions YHT["n"] (neutrons) and YHT["p"] (protons) which are functions of time.

```
SolveValueHighTemperatures := (Thread[MySet[Evaluate[HoldYNames["HT"]],
      NDSolveValue[
       Flatten@Join[SystemEquationsHT[tv], InitialConditionsHT[t0]],
       VarList, {tv, t0, tmiddle},
       PrecisionGoal → 8 + PrecisionNDSolve,
       AccuracyGoal → 11, InterpolationOrder → InterpOrder]]];
   tHT = Y["HT"]["n"][[3, 1]];
  );
```

This period is very quick to solve as can be checked. The variable tHT stores the time steps used by the solver in case we are interested.

```
AbsoluteTiming[SolveValueHighTemperatures;]
```

```
{0.10601, Null}
```

We can also extend this integration with only weak interactions to much later times to see what would happen without nuclear reactions.
This is only to perform plots in the paper.

```
SolveValueHighTemperaturesYnOnly :=
  (Thread[MySet[Evaluate[HoldYNames["WeakInteractionsOnly"]],
      NDSolveValue[
        Flatten@Join[SystemEquationsHT[tv], InitialConditionsHT[t0]],
        VarList,
        {tv, t0, tend},
        PrecisionGoal → 8 + PrecisionNDSolve, AccuracyGoal → 11(* 11*),
        InterpolationOrder → InterpOrder]]];
  );

If[$ResultsPlots, SolveValueHighTemperaturesYnOnly;];
```

Middle temperature integration (n,p,d,t,He3,He4,Be7,Li7,Li6)

The end values for neutrons and protons at high temperatures are used as initial conditions for the middle temperature era.
We use thermostatistical equilibrium for the species defined in the list ListThermalValuesMT below, and 0 otherwise.

```
ListThermalValuesMT = {"d", "t", "He3", "a", "Be7", "Li7", "Li6"};
ListThermalValuesUsedMT =
 Intersection[VariablesInEquations, ListThermalValuesMT]
{a, Be7, d, He3, Li6, Li7, t}
```

We check the initial value used for the middle temperature era.

```
YPeriodTimeOrStateEquilibrium["HT", ListThermalValuesUsedMT][TMiddle, tmiddle]
```

$\{0.24027546, 0.75972454, 8.8811153 \times 10^{-13}, 3.2251155 \times 10^{-23},$
$4.2102962 \times 10^{-23}, 1.5800019 \times 10^{-26}, 0., 8.8012607 \times 10^{-51}, 1.3924708 \times 10^{-60},$
$0., 0., 6.5451706 \times 10^{-61}, 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,$
$0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,$
$0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.\}$

```
InitialConditionsMT[Tv_, tv_] := Thread[Equal[FunList[tv],
    YPeriodTimeOrStateEquilibrium["HT", ListThermalValuesUsedMT][Tv, tv]]];
```

We then have the differential equation solver. There are two possibilities depending on if we use the compiled version or not.
The values are then affected to the YMT[“n”], YMT[“n”], YMT[“d”] etc... which are the abundances of species in this era.

```
SolveValueMiddleTemperatures := (If[$CompileNDSolve,
    (* Compiled version.*)
    resMT = NDSolveValue[
      {Ytab'[tv] == DY18CN[ρBForBBN@a@Toft@tv,
          RulesλRHS18I[Toft@tv], RulesλbarRHS18I[Toft@tv], Ytab[tv]],
       Ytab[tmiddle] == YPeriodTimeOrStateEquilibrium["HT",
          ListThermalValuesUsedMT][TMiddle, tmiddle]},
      Ytab, {tv, tmiddle, t18},
      Method → {"BDF", "MaxDifferenceOrder" → $BDFOrder},
      PrecisionGoal → 7 + PrecisionNDSolve, AccuracyGoal → 11,
      InterpolationOrder → InterpOrder, Compiled → Automatic];
    Y["MT"][key_][tv_?NumericQ] := resMT[tv][[KeyVal[key]]];
    tMT = resMT[[3, 1]];,

    (* Non compiled version. Slightly slower *)
    Thread[MySet[Evaluate[HoldYNames["MT"]], NDSolveValue[
       Flatten@
        Join[SystemEquationsMT[tv], InitialConditionsMT[TMiddle, tmiddle]],
       VarList, {tv, tmiddle, t18},
       Method → {"BDF", "MaxDifferenceOrder" → $BDFOrder},
       PrecisionGoal → 7 + PrecisionNDSolve, AccuracyGoal → 11,
       InterpolationOrder → InterpOrder, Compiled → False]]];
    tMT = Y["MT"]["n"][[3, 1]];
  ]);
```

NB: tMT stores the time steps used. Can be used to check the behaviour of the integrator.

```
AbsoluteTiming[SolveValueMiddleTemperatures;]
```

```
{0.572228, Null}
```

For information we plot the result of the integration

```
If[$ResultsPlots, LogLogPlot[Evaluate[YPeriodTime["MT"][tv]], {tv, tmiddle, t18},
  Frame → True, PlotRange → {10^-40, 10}, FrameLabel → {"t(s)", "Yi"}]]
```

Low temperature integration (All 59 isotopes)

The end values at middle temperatures are then used as initial conditions for the low temperature era.
Now the full system of equations is used.

```
InitialConditionsLT[tv_] := Thread[Equal[FunList[tv], YPeriodTime["MT"][tv]]];
```

```
SolveValueLowTemperatures := (If[$CompileNDSolve,
    (* Compiled version*)
    resLT = NDSolveValue[
      {Ytab'[tv] == DYCN[ρBForBBN@a@Toft@tv,
         RulesλRHSI[Toft@tv], RulesλbarRHSI[Toft@tv], Ytab[tv]],
       Ytab[t18] == YPeriodTime["MT"][t18]},
      Ytab, {tv, t18, tend},
      Method → {"BDF", "MaxDifferenceOrder" → $BDFOrder},
      PrecisionGoal → 5 + PrecisionNDSolve, AccuracyGoal → AccuracyNDSolve,
      InterpolationOrder → InterpOrder,
      StartingStepSize → 10^-4, MaxStepSize → 500];
    Y["LT"][key_][tv_?NumericQ] := resLT[tv][[KeyVal[key]]];
    tLT = resLT[[3, 1]];,

    (* Uncompiled version. Slower. *)
    Thread[MySet[Evaluate[HoldYNames["LT"]], NDSolveValue[
       Flatten@Join[SystemEquationsLT[tv], InitialConditionsLT[t18]],
       VarList, {tv, t18, tend},
       Method → {"BDF", "MaxDifferenceOrder" → $BDFOrder,
          "EquationSimplification" → "Solve"},
       PrecisionGoal → 5 + PrecisionNDSolve, AccuracyGoal → AccuracyNDSolve,
       InterpolationOrder → InterpOrder, StartingStepSize → 10^-4]]];
    tLT = Y["LT"]["n"][[3, 1]];
   ];)
```

```
AbsoluteTiming[SolveValueLowTemperatures;]
```

```
{33.494036, Null}
```

We can plot the results

```
If[$ResultsPlots, LogLogPlot[Evaluate[YPeriodTime["LT"][tv]], {tv, t18, tend},
  Frame → True, PlotRange → {10^-40, 10}, FrameLabel → {"t(s)", "Yi"}]]
```

Gathering integrations on all periods in one function

We define an interpolation of the results. We join the results from high, middle and low temperature eras. This is joined in the function

YI[“key”][time] where key is the name of the nuclide (e.g. “a” for He4, “t” for tritium and “d” for deuterium but otherwise “Li7”, “C12” etc...).

```
InterpolateResults = (
   Clear[Yall, YI];
   Yall[key_?KeyQ] := Yall[key] = Function[{tv},
      Piecewise[{{Y["HT"][key][tv], tv < tmiddle}, {Y["MT"][key][tv],
          tv < t18 && tv >= tmiddle}, {Y["LT"][key][tv], tv <= tend && tv >= t18}}]];

   YI[key_?KeyQ] := YI[key] = Interpolation[Table[{tv, Yall[key][tv]},
       {tv, Join[tHT, Rest@tMT, Rest@tLT]}], InterpolationOrder → 1];);
```

```
InterpolateResults;
```

The function RunNumericalIntegrals below performs the integrations of incomplete neutrino decoupling
and then the Friedmann equation integration.
Then it defines the nuclear reactions, possibly having introduced uncertainty on rates depending on options,

and solves for the high, middle and low temperature era.
This is the Driver of PRIMAT which needs to be called whenever we rerun PRIMAT with new parameters (e.g. exploring dependence in baryons or neutrinos).

```
RunNumericalIntegralsNuclearReactions := (
   (* Middle temperature integration *)
   SolveValueMiddleTemperatures;

   (* Low temperature integration *)
   SolveValueLowTemperatures;
  );

RunNumericalIntegrals := (

   (* In case of incomplete neutrino decoupling,
   we recompute all the integrations a(T) then inversion T(a), then ρν(a).*)
   If[$IncompleteNeutrinoDecoupling, RecomputeIncompleteNeutrinoDecoupling;];

   (*In case the plasma conditions have changed in a MC exploration,
   we recompute the inversion of a[T]*)
   (* This is needed if we have recomputed the neutrino decoupling,
   but I am wondering if this is always needed. *)
   InvertaOFT;

   (* scale factor integration from Friedmann equation. *)
   Computetofa;
   Computeaoft;

   (* Build equations. Needed since rate
    are modified randomly by the f factor of each reaction*)
   LoadRates;
   DefineEquations;

   (* High temperature integration with only PEN reactions *)
   SolveValueHighTemperatures;

   (* Middle and Low temperature WITH nuclear reactions*)
   RunNumericalIntegralsNuclearReactions;

   InterpolateResults;
  );
```

Gathering the numerical results

We define a pseudo-mass fraction as a function of time using the interpolated results

```
XI[key_?KeyQ][t_] := Ai[key] YI[key][t]
```

Final abundances are obtained by evaluation at t = tend.

We define a shorthand for the final abundances (Yf) and pseudo mass fraction (Xf).

```
Yf[key_] := YLT[key][tend]
Xf[key_] := Ai[key] Yf[key]
```

And a shorthand notation for Y_i / H

```
YfH[key_] := Yf[key] / Yf["p"]
```

Results and plots

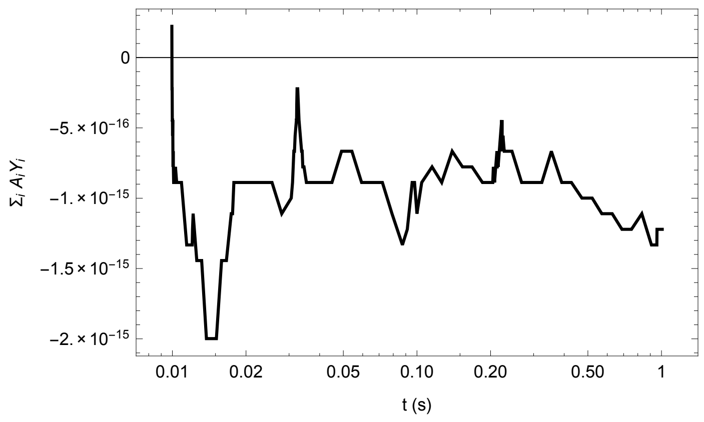
Checks of the conservation of total number of baryons

The total number MUST be conserved. We build it from the nuclear weights of species. At high temperatures we check visually. By construction the quantity Ytot = $\Sigma_i A_i Y_i$ should be 1 and conserved. We define it and plot the difference with unity.

```
Ytot[period_String] :=
  Function[{tv}, Plus @@ (WeightsNuclear * YPeriodTime[period][tv])];
```

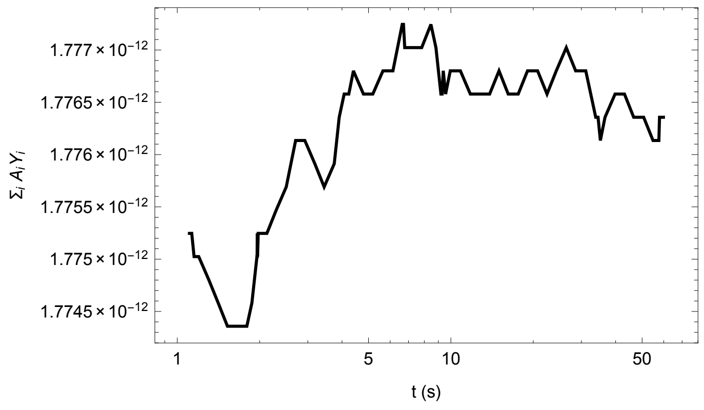
High temperature era

```
LogLinearPlot[Ytot["HT"][tv] - 1, {tv, t0, tmiddle},
 Frame → True, FrameLabel → {"t  (s)", "Σi  AiYi"}, PlotStyle → Black]
```



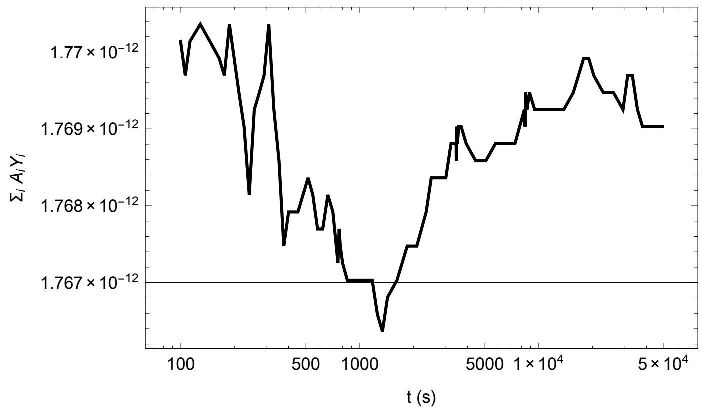
Middle temperature era

```
LogLinearPlot[Ytot["MT"][tv] - 1, {tv, tmiddle * 1.1, 0.6 t18},
 Frame → True, FrameLabel → {"t  (s)", "Σi  AiYi"}, PlotStyle → Black]
```



Low temperature era with the full network.

```
LogLinearPlot[Ytot["LT"][t] - 1, {t, t18, tend},
 Frame → True, FrameLabel → {"t (s)", "Σi AiYi"}, PlotStyle → Black]
```



Time evolution of abundances

Early thermodynamical equilibrium

Estimate of $T_{\rm nuc}$ (see companion paper)

```
TFreeze = 0.8 MeV / kB;
tFreeze = tofa@a@TFreeze
YnF[tv_] := 1 / (1 + Exp[Q / kB / TFreeze]) Exp[- (tv - tFreeze) / τneutron];
YpF[tv_] := 1 - YnF[tv];
```

```
1.1706326
```

```
tnuc =
 FindRoot[YNSE["d", YnF[tv], YpF[tv], Toft[tv]] == YnF[tv], {tv, 100}][[1, 2]]
Tnuc = Toft[tnuc]
```

```
296.85471
```

7.6940866×10^{8}

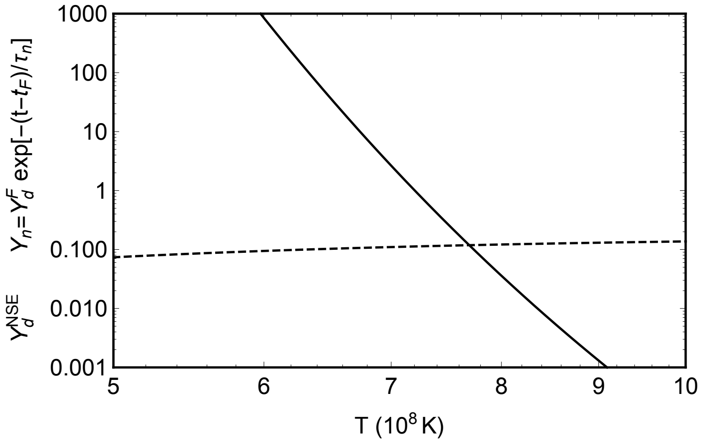
Abundance of neutrons at $T_{\rm nuc}$ and $T_{\rm nuc}$ in MeV

```
YnF[tnuc]
kB * Tnuc / MeV
```

```
0.11836523
```

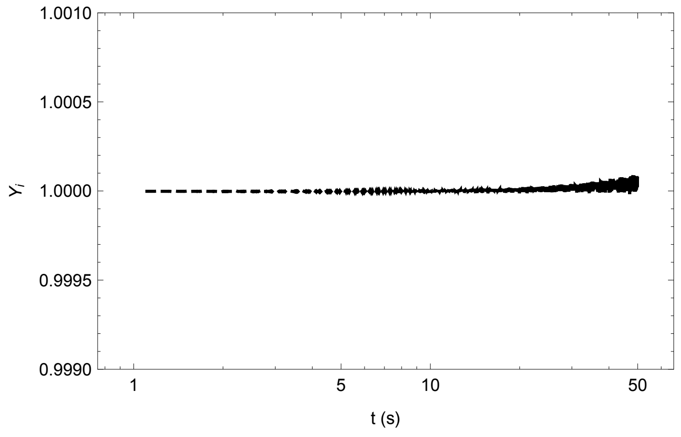
```
0.066302503
```

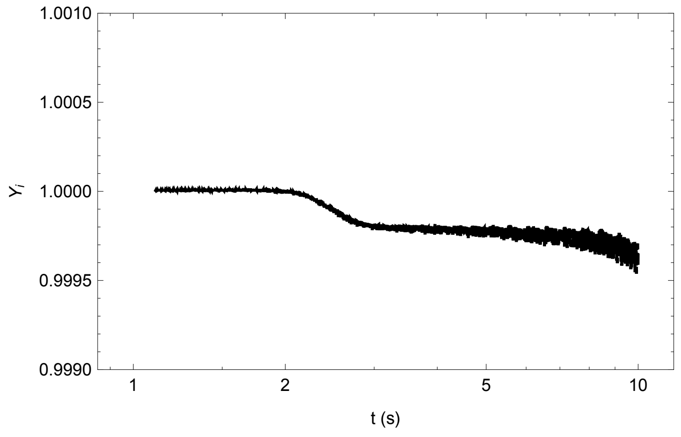
```
PlotDeutEq =
 Show[LogLogPlot[{YnF[tofa@a[10^8 Tv]], YNSE["d", YnF[tofa@a[10^8 Tv]],
      YpF[tofa@a[10^8 Tv]], 10^8 Tv]}, {Tv, 0.05 * 10^2, 0.1 * 10^2},
    GridLines → {{{Tnuc, {Gray, Thickness[0.005]}}}, {}}, Frame → True,
    PlotRange → {{0.05 * 10^2, 0.1 * 10^2}, {10^-3, 10^3}},
    PlotRangePadding → None, FrameStyle → Thickness[0.004],
    PlotStyle → {{Black, Thickness[0.004], Dashing[0.01]},
      {Black, Thickness[0.004]}}, PlotRange → {10^-3, 1000},
    FrameLabel → {"T  (10⁸ K)", "Y_d^NSE         Y_n=Y_d^F exp[-(t-t_F)/τ_n]"},
    LabelStyle → {FontSize → 12}],
  Graphics[{Rotate[Text[Style["0.066 MeV", FontSize → 10, Black],
       {Log@Tnuc - 0.015, 2}], 90 Degree]}]]

If[$ResultsPlots,
 Export["Plots/PlotDeutEq.pdf", Style[PlotDeutEq, Magnification → 1], "PDF"];]
```



Checks of thermo equilibrium at early times.
We check the accuracy of thermal equilibrium for “d” “t” “a” “Li7”.
Most important is deuterium because it determines the final Helium abundance.

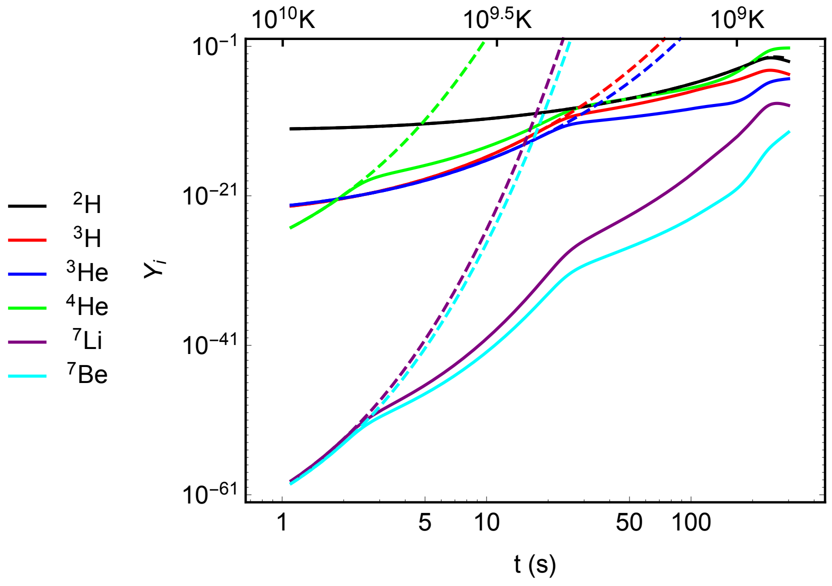
```
LogLogPlot[{YI["d"][tv] / YNSE["d", YI["n"][tv], YI["p"][tv], Toft[tv]]},
 {tv, tmiddle * 1.1, 50}, Frame → True, FrameLabel → {"t (s)", "Yᵢ"},
 PlotStyle → {Black, {Black, Dashed}},
 ImagePadding → {{50, 10}, {40, 25}}, PlotRange → {0.999, 1.001}]

LogLogPlot[{YI["t"][tv] / YNSE["t", YI["n"][tv], YI["p"][tv], Toft[tv]]},
 {tv, tmiddle * 1.1, 10}, Frame → True, FrameLabel → {"t (s)", "Yᵢ"},
 PlotStyle → {Black, {Black, Dashed}},
 ImagePadding → {{50, 10}, {40, 25}}, PlotRange → {0.999, 1.001}]

(*LogLogPlot[{YI["a"][tv]/YNSE["a",YI["n"][tv],YI["p"][tv],Toft[tv]]},
  {tv,tmiddle*1.1,1.5},Frame→True,FrameLabel→{"t (s)","Yᵢ"},
  PlotStyle→{Black,{Black,Dashed}},FrameTicks→MyTicks,
  ImagePadding→{{50,10},{40,25}},PlotRange→{0.999,1.001}]

 LogLogPlot[{YI["Li7"][tv]/YNSE["Li7",YI["n"][tv],YI["p"][tv],Toft[tv]]},
  {tv,tmiddle*1.1,1.5},Frame→True,FrameLabel→{"t (s)","Yᵢ"},
  PlotStyle→{Black,{Black,Dashed}},FrameTicks→MyTicks,
  ImagePadding→{{50,10},{40,25}},PlotRange→{0.999,1.001}]*)
```





We plot early values together with thermal equilibrium in dashes

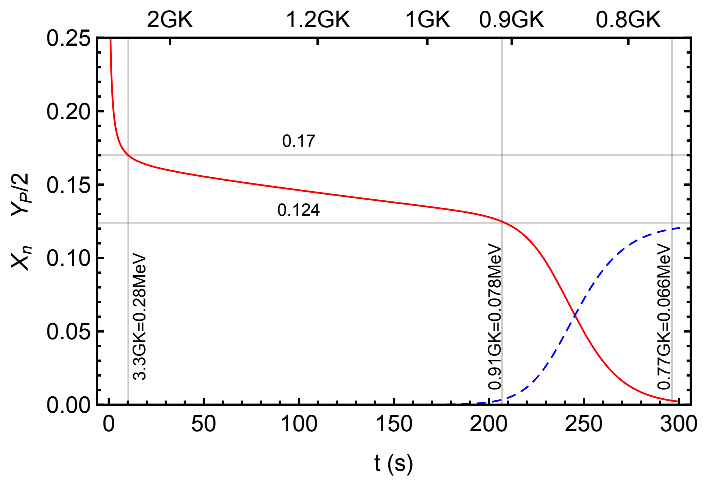
```
MyTickst = {{Automatic, Automatic},
    {Automatic, {{tofa@a[10^11], "10^11K"}, {tofa@a[10^10.5], "10^10.5K"},
      {tofa@a[10^10], "10^10K"}, {tofa@a[10^9.5], "10^9.5K"}, {tofa@a[10^9],
       "10^9K"}, {tofa@a[10^8.5], "10^8.5K"}, {tofa@a[10^8], "10^8K"}}}};

PL1 = LogLogPlot[{YI["d"][tv], YI["t"][tv], YI["He3"][tv], YI["a"][tv],
    YI["Li7"][tv], YI["Be7"][tv], YNSE["d", YI["n"][tv], YI["p"][tv], Toft[tv]],
    YNSE["t", YI["n"][tv], YI["p"][tv], Toft[tv]],
    YNSE["He3", YI["n"][tv], YI["p"][tv], Toft[tv]],
    YNSE["a", YI["n"][tv], YI["p"][tv], Toft[tv]],
    YNSE["Li7", YI["n"][tv], YI["p"][tv], Toft[tv]],
    YNSE["Be7", YI["n"][tv], YI["p"][tv], Toft[tv]]},
   {tv, tmiddle * 1.1, 300}, Frame → True, FrameLabel → {"t (s)", "Yᵢ"},
   FrameTicks → MyTickst, LabelStyle → {FontSize → 13},
   FrameStyle → Thickness[0.004], PlotRange → {10^-62, 1}, AspectRatio → .8,
   PlotStyle → {Black, Red, Blue, Green, Purple, Cyan, {Black, Dashed},
     {Red, Dashed}, {Blue, Dashed}, {Green, Dashed}, {Purple, Dashed},
     {Cyan, Dashed}}, (*ImagePadding→{{50,10},{40,25}},*)
   PlotLegends → Placed[LineLegend[{"²H", "³H", "³He", "⁴He", "⁷Li", "⁷Be"},
      LegendLayout → (Grid[#, Frame → None] &)], Left]]

If[$ResultsPlots,
 Export["Plots/PlotEarlyEquilibrium.pdf", Style[PL1, Magnification → 1], "PDF"];]
```



Neutrons only evolution

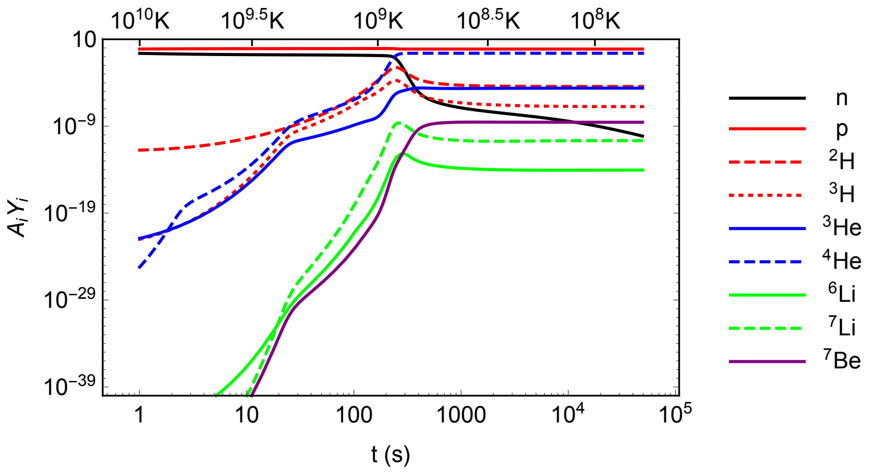
```
YnCoc =
 Show[Plot[{YI["n"][tv], Y["WeakInteractionsOnly"]["n"][tv], YI["a"][tv] * 2
     (*,1/(1+Exp[Q/kB/Toft[tv]])*)}, {tv, t0, 300}, Frame → True,
   FrameTicks → {{Automatic, Automatic}, {Automatic, {{tofa@a[10^9], "1GK"},
       {tofa@a[.8 * 10^9], "0.8GK"}, {tofa@a[.9 * 10^9], "0.9GK"},
       {tofa@a[1.2 * 10^9], "1.2GK"}, {tofa@a[2 × 10^9], "2GK"}}}},
   FrameStyle → Thickness[0.004], FrameLabel → {"t (s)", "Xn     YP/2"},
   LabelStyle → {FontSize → 12},
   PlotStyle → {{Red, Thickness[0.003]}, {Red, Thickness[0.003], Dotted},
     {Blue, Thickness[0.003], Dashed}}, PlotRange → {0, 0.25},
   GridLines → {{{tofa@a[3.3 Giga Kelvin], {Gray, Thickness[0.003]}},
      {tofa@a[0.91 Giga Kelvin], {Gray, Thickness[0.003]}},
      {tofa@a[0.77 × 10^9], {Gray, Thickness[0.003]}}},
     {{0.124, {Gray, Thickness[0.003]}}, {0.17, {Gray, Thickness[0.003]}}}}],
  Graphics[{Rotate[Text[Style["3.3GK=0.28MeV", FontSize → 9, Black], {16, 0.06}],
     90 Degree], Rotate[Text[Style["0.91GK=0.078MeV", FontSize → 9, Black],
      {203, 0.06}], 90 Degree],
    Rotate[Text[Style["0.77GK=0.066MeV", FontSize → 9, Black], {292, 0.06}],
     90 Degree],
    Rotate[Text[Style["0.17", FontSize → 9, Black], {100, 0.18}], 0 Degree],
    Rotate[Text[Style["0.124", FontSize → 9, Black], {100, 0.133}], 0 Degree]}]]
```

```
If[$ResultsPlots,
 Export["Plots/PlotYnCoc.pdf", Style[YnCoc, Magnification → 1], "PDF"];]
```



Main species of the small network

```
BBNsmall = LogLogPlot[{YI["n"][tv], YI["p"][tv], 2 YI["d"][tv], 3 YI["t"][tv],
    3 YI["He3"][tv], 4 YI["a"][tv], YI["Li6"][tv], YI["Li7"][tv], 7 YI["Be7"][tv]},
   {tv, tmiddle, tend}, Frame → True, FrameStyle → Thickness[0.003],
   FrameLabel → {"t  (s)", "AᵢYᵢ"}, LabelStyle → {FontSize → 12},
   PlotRange → {10^-40, 10}, PlotStyle → {Black, Red, {Red, Dashed},
     {Red, Dotted}, Blue, {Blue, Dashed}, Green, {Green, Dashed}, Purple},
   FrameTicks → MyTickst, ImagePadding → {{50, 10}, {40, 25}}, PlotLegends →
    Placed[LineLegend[{"n", "p", "²H", "³H", "³He", "⁴He", "⁶Li", "⁷Li", "⁷Be"},
      LegendLayout → (Grid[#, Frame → None] &)], Right]]
```



```
If[$ResultsPlots, Export["Plots/BBNsmall.pdf", BBNsmall, "PDF"];]
```

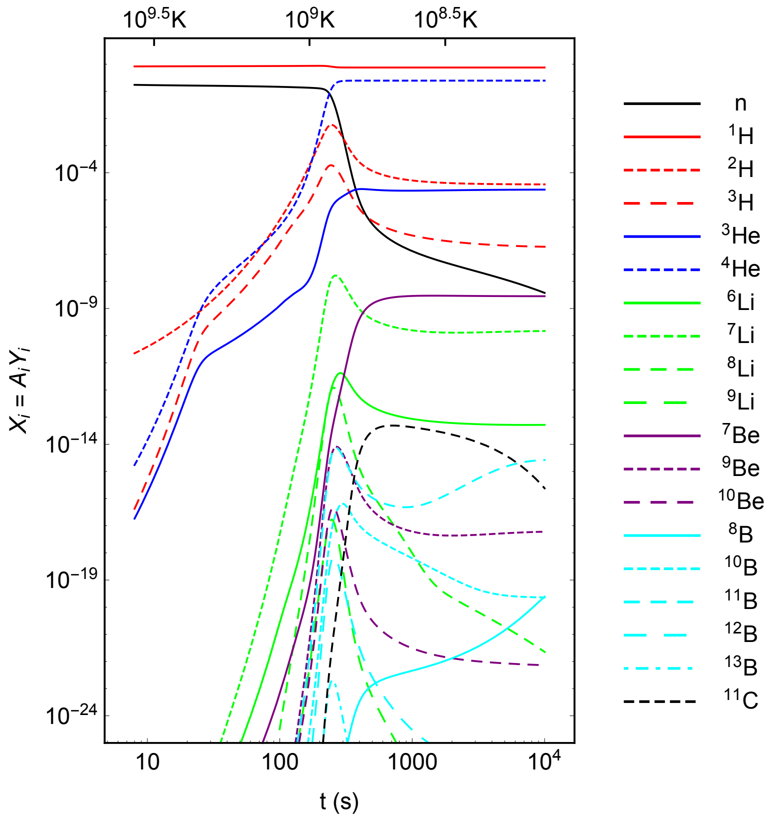
From Hydrogen to Borron

Custom colors for plots of a given element

```
ListColor[Color_, n_] := Take[Join[{{Color, Thickness[0.004]}},
    Table[{Color, Thickness[0.004], Dashing[(i) 0.01]}, {i, 1, 3}],
    Table[{Color, Thickness[0.004],
      Dashing[{0, 0.015 * i, (i) 0.015, i 0.015}]}, {i, 1, 4}]], n]
```

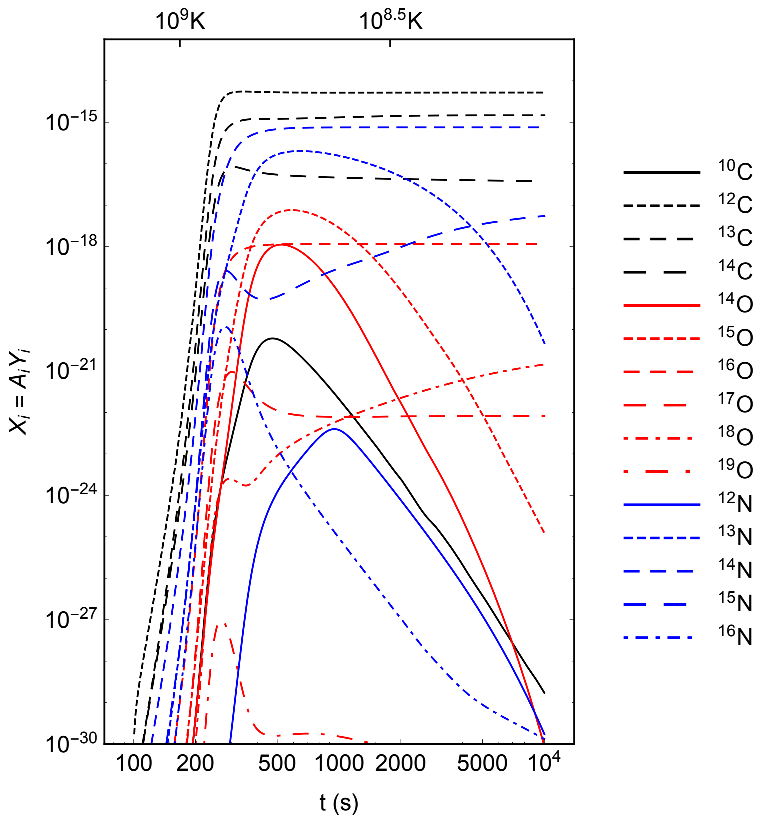
```
BBNsmall2 =
 LogLogPlot[{XI["n"][tv], XI["p"][tv], XI["d"][tv], XI["t"][tv], XI["He3"][tv],
   XI["a"][tv], XI["Li6"][tv], XI["Li7"][tv], XI["Li8"][tv], XI["Li9"][tv],
   XI["Be7"][tv], XI["Be9"][tv], XI["Be10"][tv], XI["B8"][tv], XI["B10"][tv],
   XI["B11"][tv], XI["B12"][tv], XI["B13"][tv], XI["C11"][tv]}, {tv, 8, 10^4},
  Frame → True, FrameLabel → {"t (s)", "Xᵢ = AᵢYᵢ"}, LabelStyle → {FontSize → 12},
  FrameStyle → Thickness[0.004], PlotRange → {10^-25, 9},
  PlotStyle → Join[ListColor[Black, 1], ListColor[Red, 3],
    ListColor[Blue, 2], ListColor[Green, 4], ListColor[Purple, 3],
    ListColor[Cyan, 5], {{Black, Thickness[0.004], Dashed}}],
  AspectRatio → 1.5, FrameTicks → MyTickst, ImagePadding → {{50, 10}, {40, 25}},
  PlotLegends → Placed[LineLegend[{"n", "¹H", "²H", "³H", "³He", "⁴He", "⁶Li",
       "⁷Li", "⁸Li", "⁹Li", "⁷Be", "⁹Be", "¹⁰Be", "⁸B", "¹⁰B", "¹¹B", "¹²B",
       "¹³B", "¹¹C"}, LegendLayout → (Grid[#, Frame → None] &)], Right]]
```

```
If[$ResultsPlots,
 Export["Plots/PlotBBNLight.pdf", Style[BBNsmall2, Magnification → 1], "PDF"];]
```



From Carbon to Oxygen

```
BBNCNO = LogLogPlot[
  {XI["C10"][tv], XI["C12"][tv], XI["C13"][tv], XI["C14"][tv], XI["O14"][tv],
   XI["O15"][tv], XI["O16"][tv], XI["O17"][tv], XI["O18"][tv], XI["O19"][tv],
   XI["N12"][tv], XI["N13"][tv], XI["N14"][tv], XI["N15"][tv], XI["N16"][tv]},
  {tv, 1.01 t18, 10^4}, Frame → True, FrameLabel → {"t  (s)", "Xi  = AiYi"},
  LabelStyle → {FontSize → 12}, FrameStyle → Thickness[0.004],
  PlotRange → {10^-30, 10^-13}, PlotStyle → Join[ListColor[Black, 4],
    ListColor[Red, 6], ListColor[Blue, 5], ListColor[Purple, 1]],
  AspectRatio → 1.5, FrameTicks → MyTickst, ImagePadding → {{50, 10}, {40, 25}},
  PlotLegends → Placed[LineLegend[{"10C", "12C", "13C", "14C", "14O",
      "15O", "16O", "17O", "18O", "19O", "12N", "13N", "14N", "15N", "16N"},
     LegendLayout → (Grid[#, Frame → None] &)], Right]]

If[$ResultsPlots,
  Export["Plots/PlotBBNHeavy.pdf", Style[BBNCNO, Magnification → 1], "PDF"];];
```



Final abundances

Main results

Standard abundances as reported in most BBN papers.
Note the definition Y_P =
 4 Y_{He4}. Since the atomic mass of Helium is not exactly 4 this is not exactly Helium mass abundance

```
MyGrid@Transpose[
  {{"H", "YP=4YHe", "D/H x10^5", "^3He/H x10^5", "T/H x10^8", "(^3He+T)/H x10^5",
    "^7Li/H x10^11", "^7Be/H x10^10", "(^7Li+^7Be)/H x10^10", "^6Li/H x10^14",
    "^9Be/H x10^19", "^10B/H x10^21", "^11B/H x10^16", "CNO/H x10^16"},
   {Yf["p"], 4 Yf["a"], YfH["d"] 10^5, YfH["He3"] 10^5, YfH["t"] 10^8,
    (YfH["t"] + YfH["He3"]) 10^5, YfH["Li7"] 10^11, YfH["Be7"] 10^10,
    (YfH["Li7"] + YfH["Be7"]) 10^10, YfH["Li6"] 10^14, YfH["Be9"] 10^19,
    YfH["B10"] 10^21, YfH["B11"] 10^16, YfH["CNO"] 10^16}}]
```

H	0.75284554
$Y_P=4Y_{He}$	0.24709317
D/H x10^5	2.4591922
^{3}He/H x10^5	1.0660997
T/H x10^8	7.9615184
(^{3}He+T)/H x10^5	1.0740612
^{7}Li/H x10^{11}	2.8886826
^{7}Be/H x10^{10}	5.3815144
(^{7}Li+^{7}Be)/H x10^{10}	5.6703826
^{6}Li/H x10^{14}	1.1922717
^{9}Be/H x10^{19}	9.1902956
^{10}B/H x10^{21}	2.9915441
^{11}B/H x10^{16}	3.2732284
CNO/H x10^{16}	8.029231

All final abundances

```
MyGrid[{#, Yf[#]} & /@ ShortNames]
```

n	7.2859746×10^{-11}
p	0.75284554
d	0.000018513919
t	5.9937937×10^{-8}
He3	8.0260837×10^{-6}
a	0.061773292
He6	4.5866394×10^{-44}
Li6	8.9759645×10^{-15}
Li7	2.1747318×10^{-11}
Li8	5.1217648×10^{-26}
Li9	3.4391581×10^{-41}
Be7	4.0514491×10^{-10}
Be9	6.9188731×10^{-19}
Be10	6.7736275×10^{-24}
Be11	2.6022296×10^{-38}
Be12	1.1095567×10^{-55}
B8	1.0518128×10^{-23}
B10	2.2521706×10^{-21}
B11	2.4642354×10^{-16}
B12	1.8338598×10^{-32}
B13	1.6836348×10^{-48}
B14	2.7360579×10^{-63}
B15	1.558024×10^{-81}
C9	2.0850331×10^{-40}
C10	1.1255758×10^{-36}

C11	4.6825969×10^{-27}
C12	4.3421296×10^{-16}
C13	1.1328406×10^{-16}
C14	2.4615236×10^{-18}
C15	5.4618347×10^{-34}
C16	2.0876886×10^{-49}
N12	1.5475608×10^{-45}
N13	1.4868096×10^{-28}
N14	5.3846083×10^{-17}
N15	5.9995409×10^{-19}
N16	3.2610731×10^{-34}
N17	2.6070908×10^{-44}
O13	-1.0607526×10^{-58}
O14	3.2700219×10^{-43}
O15	6.6362049×10^{-32}
O16	7.236541×10^{-20}
O17	4.778956×10^{-24}
O18	1.3263278×10^{-22}
O19	6.6978833×10^{-36}
O20	6.3940755×10^{-49}
F17	5.5386729×10^{-36}
F18	8.6266749×10^{-25}
F19	6.3017734×10^{-26}
F20	3.8590069×10^{-38}
Ne18	3.7552566×10^{-49}
Ne19	1.2905113×10^{-41}
Ne20	6.8140496×10^{-28}
Ne21	5.3213096×10^{-30}
Ne22	8.8196188×10^{-32}
Ne23	-2.9864385×10^{-74}
Na20	8.3712062×10^{-64}
Na21	-8.7052063×10^{-78}
Na22	1.7258425×10^{-36}
Na23	1.0828535×10^{-37}

Tools for Monte-Carlo on nuclear rates

We gather some tools for Monte-Carlo estimation of uncertainties from nuclear rates.
Some Examples are provided in the Example folder.

There are 3 booleans which control what variables are varied randomly.
Nuclear rates
Neutron lifetime
And possibly baryons abundance according to [Planck 2015] results when this is interesting to vary it as well.

```
$ParallelBool = True;
$Randomτneutron = True;
$Randomh2Ωb = True;
```

Initialize Kernels (if parallelization this is called. It just distributes definitions)

```
InitializeKernels := (
   LaunchKernels[];
   Print["Number of Kernels ", $KernelCount];
   DistributeDefinitions[ReshapedTabulatedReactions,
    ListReactionsUpToChosenMass, LoadRates, DefineEquations,
    SystemEquationsHT, SystemEquationsMT, SystemEquationsLT,
    LoadRates, DY, DY18, DYOnlyPEN, LbarnTOp, LnTOp];
  );
```

We define a function which launches the Monte-Carlo on subKernels and collects the results

```
RunPRIMATMonteCarlo[number_] := Module[{res, time, Abundances,
    mytabfunctions, sss, RandomVariables, CosmoParametersList},
   If[number > 1, Print["Running a Monte-Carlo with ", number, " points."];];
   Off[CompiledFunction::cfta];
   mytabfunctions = If[$ParallelBool, ParallelTable, Table];
   If[$ParallelBool, InitializeKernels;
    ParallelEvaluate[$HistoryLength = 0;]];

   (* We always use the same seed so that we always use the same
    sequence of random number as advocated in [Cyburt et al. 2015].*)

   res = mytabfunctions[
     $Seed := i;
     (* We use a different seed so that for each MC
      point we have a different sequence of reaction rates *)
     InitializeRandom[$Seed];
     (* We restart our random list from the beginning *)

     h2Ωb0 = Meanh2Ωb0 + If[$Randomh2Ωb, σh2Ωb0 NormalRealisation, 0];
     τneutron =
      Meanτneutron + If[$Randomτneutron, στneutron NormalRealisation, 0];
     CosmoParametersList = {h2Ωb0, τneutron};

     time = AbsoluteTiming[RunNumericalIntegrals][[1]];
     RandomVariables = Rest@ListReactionsUpToChosenMass[[All, 4]];
     Share[];
     Print["Iteration ", i, "  Memory usage = ",
      MemoryInUse[], " time = ", time, "  Kernel : ", $KernelID];
     Abundances = YPeriodTime["LT"][tend];
     ClearSystemCache[];
     If[$Verbose, Print[Abundances(*," ",RandomVariables*)]];
     {Abundances, RandomVariables, CosmoParametersList}, {i, 1, number}];
   If[$ParallelBool, CloseKernels[]];

   h2Ωb0 = Meanh2Ωb0;
   τneutron = Meanτneutron;
   MC = res[[All, 1]];
   RV = res[[All, 2]];
   Cosmo = res[[All, 3]];

   res];

RunPRIMAT := RunPRIMATMonteCarlo[1];
```

The results of RunPRIMAT are gathered in the files MC (abundances) RV (rates variations) Cosmo (List of cosmological paramters, limited to baryons abundances and neutron lifetime)

We define functions to dump the results of a Monte - Carlo in a file and also the converse to load the result so as to analyze and use it.

```
Clear[LoadMC, DumpMC]

DumpMC[File_String] := (
   Print["Exporting ", "MonteCarlo/MC" <> File <> ".dat"];
   Export["MonteCarlo/MC" <> File <> ".dat", MC];

   Print["Exporting ", "MonteCarlo/RV" <> File <> ".dat"];
   Export["MonteCarlo/RV" <> File <> ".dat", RV];

   Print["Exporting ", "MonteCarlo/Cosmo" <> File <> ".dat"];
   Export["MonteCarlo/Cosmo" <> File <> ".dat", Cosmo];);

LoadMC[File_String] := (
   MCfile = "MonteCarlo/MC" <> File <> ".dat";
   RVfile = "MonteCarlo/RV" <> File <> ".dat";
   Cosmofile = "MonteCarlo/Cosmo" <> File <> ".dat";
   MC = Import[MCfile];
   RV = Import[RVfile];
   Cosmo = Import[Cosmofile];
   TMC = Transpose[MC];
  );
```

Whenever a Monte-Carlo is finished, or loaded, we can obtain the table of values for a given element, or a given reaction

```
ElementColumn[el_] := MC[[All, KeyVal[[el]]]];
ReactionColumn[el_] := RV[[All, KeyNuclearReaction[el]]];
h2Ωb0List := Cosmo[[All, 1]];
τneutronList := Cosmo[[All, 2]];
```