Primordial Non-Gaussianity: Simulations

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w/ J. R. Bond, J. Braden, T. Morrison, G. Stein.

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Outline

- Introduction
1. Non-Gaussianity from inflationary models
2. Modeling non-Gaussianity for numerical simulations
3. Independent Component Analysis
- Conclusion

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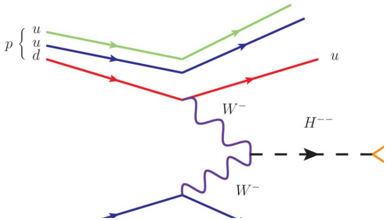
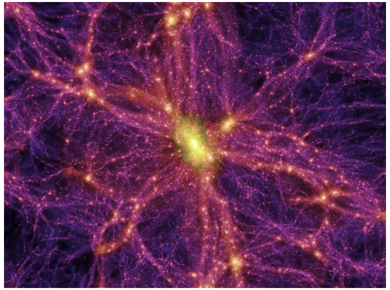
Why non-Gaussianity?

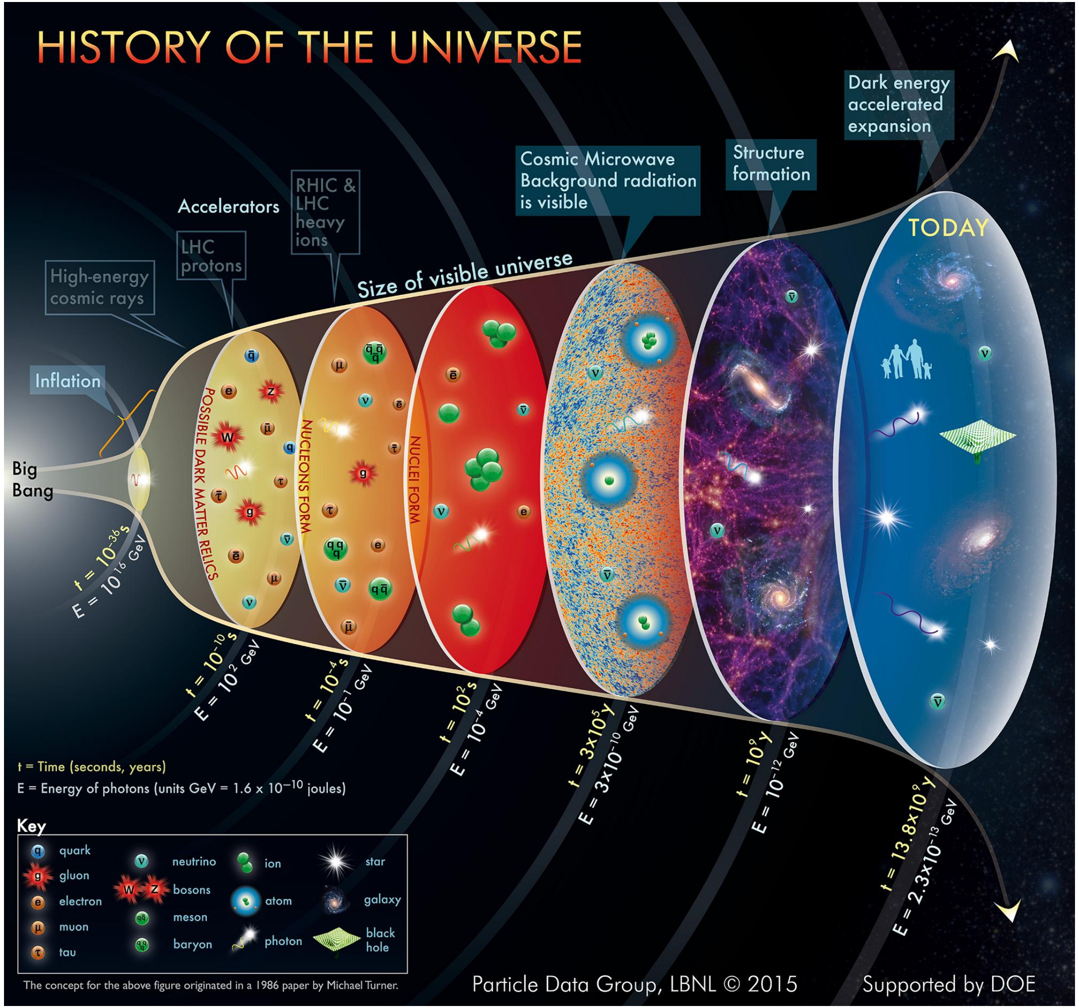
Detection of primordial non-Gaussianity is like:

- Detection of Higgs particles for the Standard Model

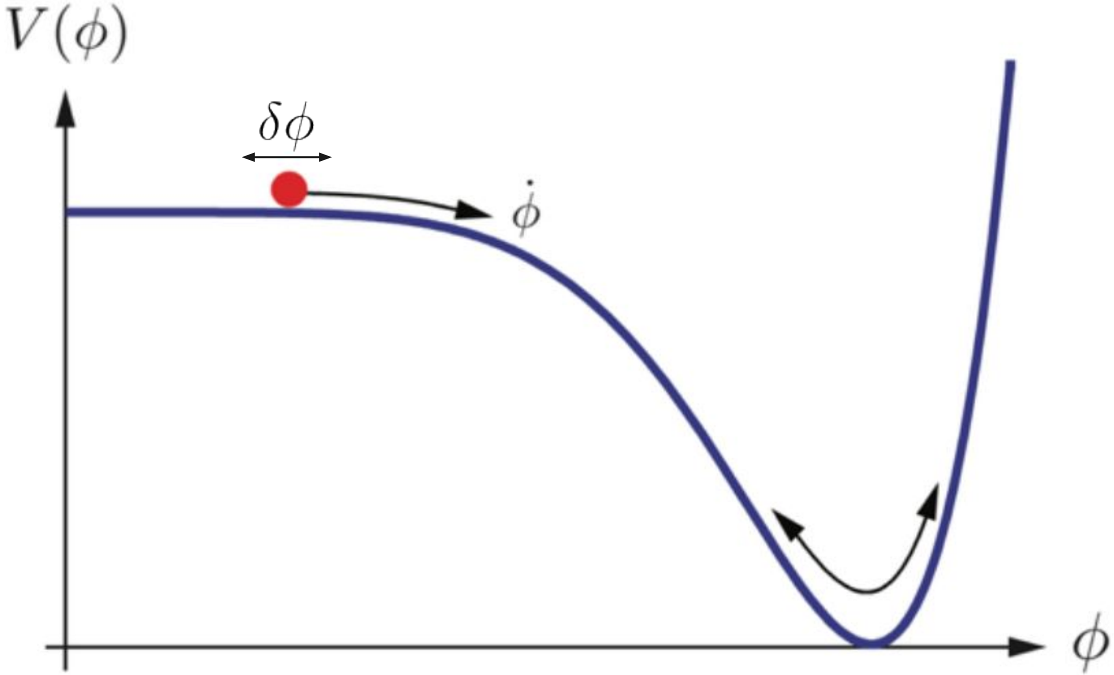


- Direct detection of Dark Matter





Credit: Particle Data Group at Lawrence Berkeley National Lab.



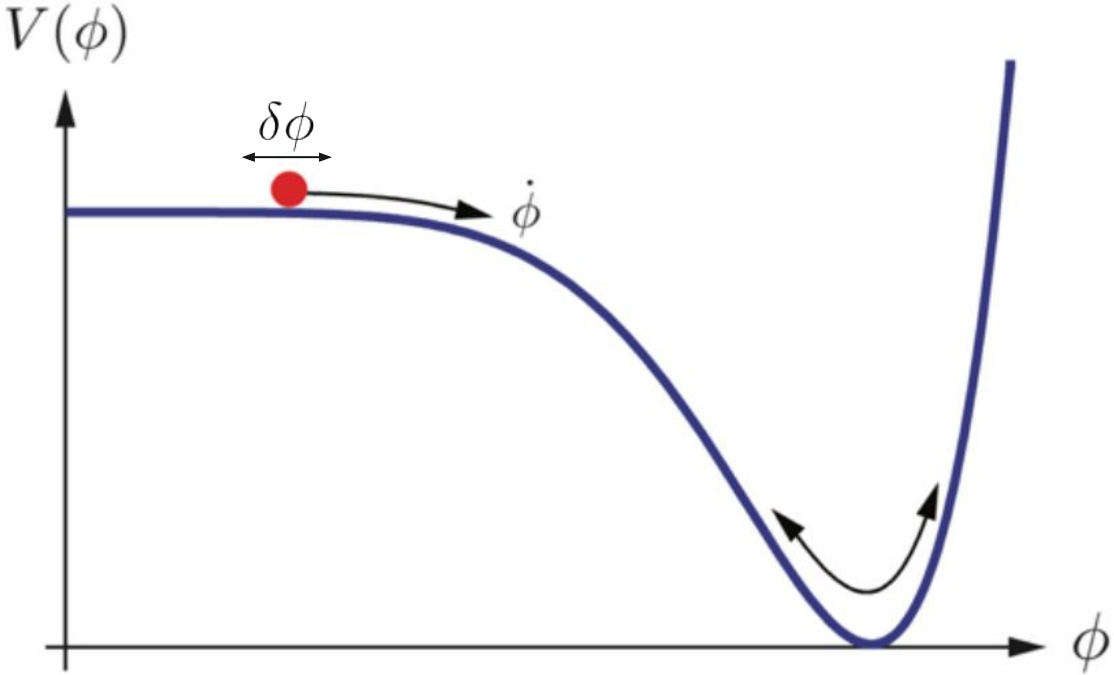
Credit: D. Bauman

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

- Inflaton motion drives the exponential expansion
- Predicts Gaussian and nearly scale invariant perturbations

Classical inflation

Single-field slow-roll



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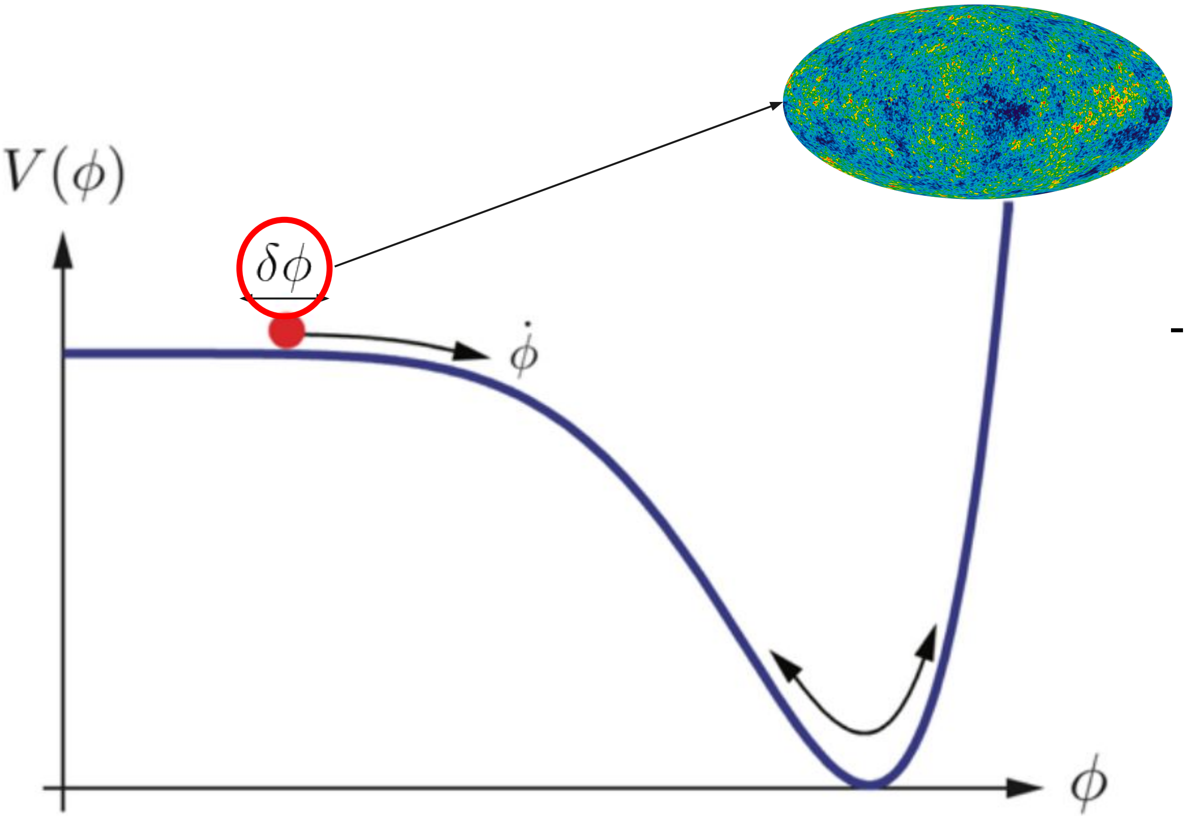
$$\mathcal{L}(\phi) = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$$

- Background evolution: solves flatness and horizon problems

Classical inflation

Single-field slow-roll



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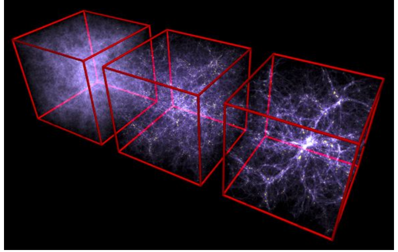

- Perturbations: seeds to the CMB and LSS

Classical inflation

Single-field slow-roll

Observable: Curvature Perturbations

$$ds_3^2 = a^2 e^{2\zeta} \delta_{ij} dx^i dx^j$$

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Physical variable: $\delta\phi(\mathbf{x}, t) \rightarrow \zeta(\mathbf{x}, t)$

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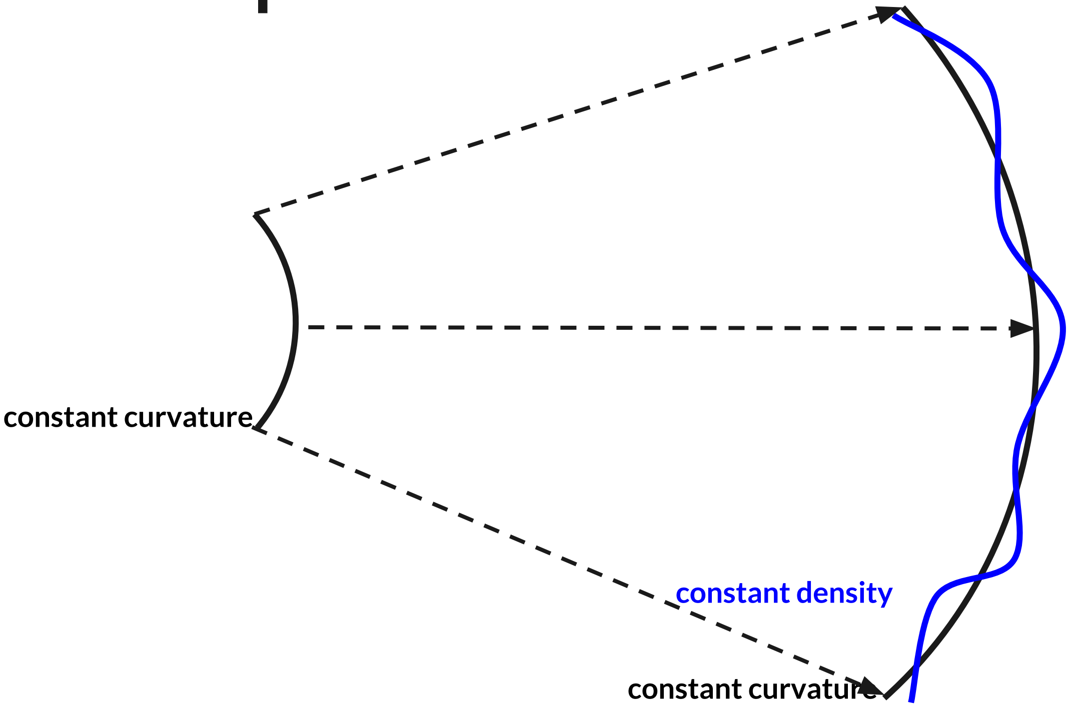
Classical inflation predicts:
$$P_\zeta \approx \frac{1}{2M_P^2 \epsilon} \left(\frac{H}{2\pi}\right)^2$$
$$\approx A_\phi \left(\frac{k}{k_0}\right)^{n_s - 1}$$

Measured by Planck

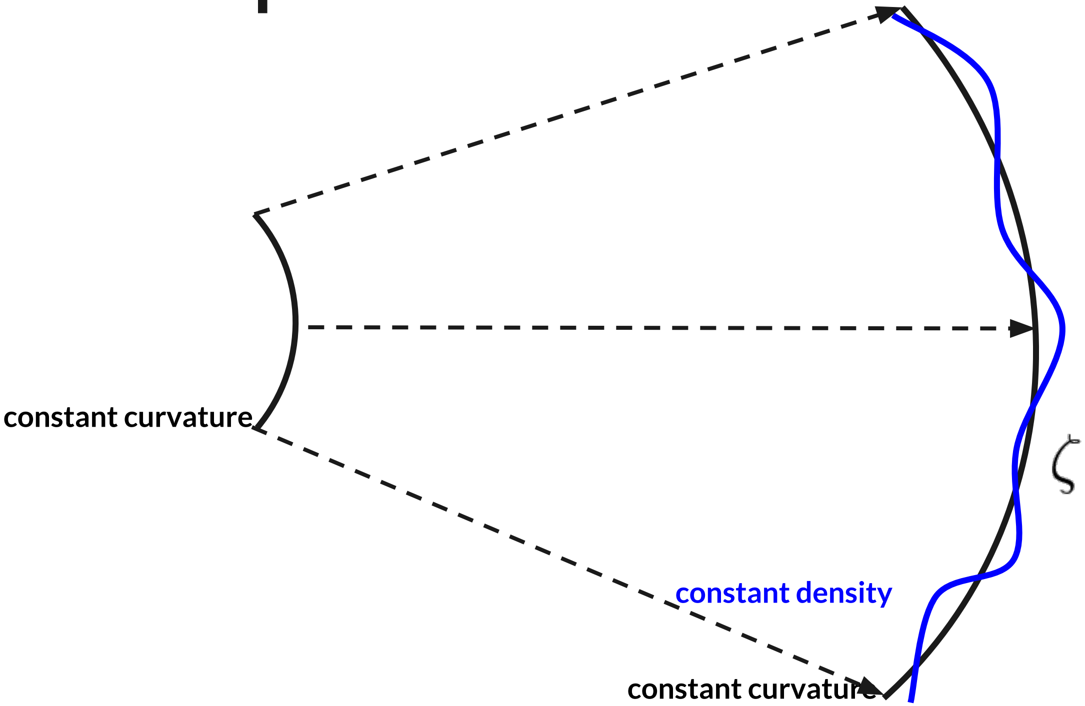
Curvature perturbations

constant curvature

Curvature perturbations



Curvature perturbations



Spatially modulated expansion

$$\zeta = \delta \ln a|_H = \delta \ln a|_\rho$$

Beyond classical inflation

Beyond classical inflation

- Gravity

$$\mathcal{S}_{EH} = \int d^4x\sqrt{-g}\left(\frac{R}{2} + \mathcal{L}_m\right) \to \mathcal{S}_{F(R)} = \int d^4x\sqrt{-g}\left(\frac{F(R)}{2} + \mathcal{L}_m\right)$$

$$\mathcal{S}_{EH} = \int d^4x\sqrt{-g}\left(\frac{R}{2} + \mathcal{L}_m\right) \to \mathcal{S}_{f(\phi)} = \int d^4x\sqrt{-g}\left(f(\phi)\frac{R}{2} + \mathcal{L}_m\right)$$

Beyond classical inflation

- Gravity
- Stochastic inflation

$$\phi(\mathbf{x},t) = \phi_0(t) + \delta\phi(\mathbf{x},t) \rightarrow \phi(\mathbf{x},t) = \phi_{cg}(\mathbf{x},t) + \phi_{fg}(\mathbf{x},t)$$

Beyond classical inflation

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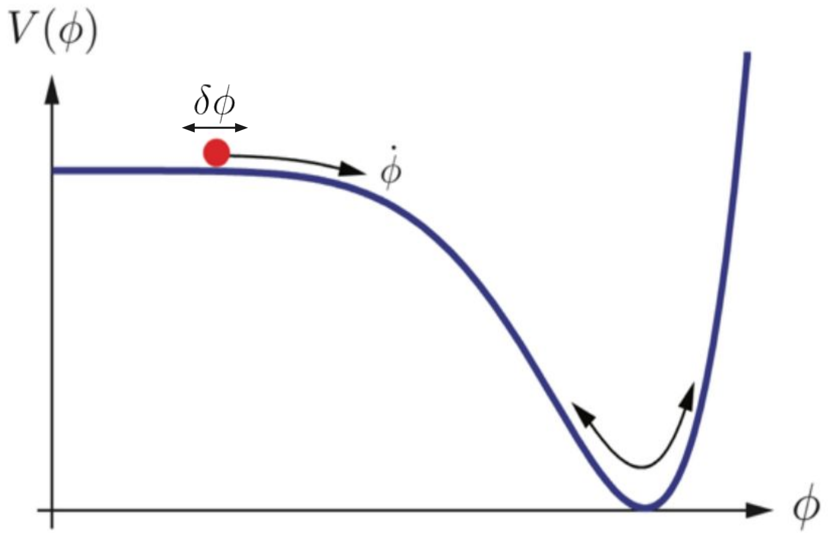

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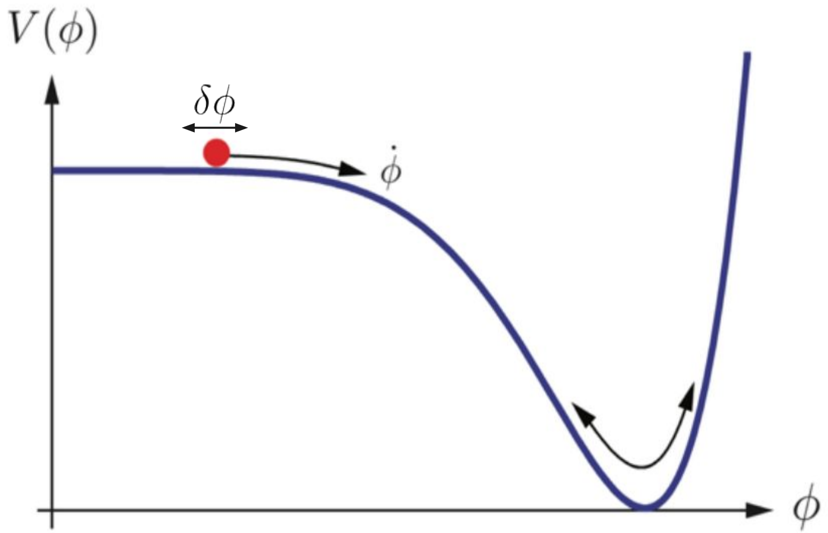

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$$\dot{\phi}_0 = -\frac{1}{3H}V'(\phi_0) \rightarrow \dot{\phi}_{cg} = -\frac{1}{3H}V'(\phi_{cg}) + \frac{H^{3/2}}{2\pi}\xi(t)$$

Beyond classical inflation

- Gravity
- Stochastic inflation
- Multiple fields

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) \rightarrow \mathcal{L} = \sum_i \frac{1}{2}(\partial\phi_i)^2 - V(\phi_1, \ldots, \phi_n)$$

Beyond classical inflation

- Gravity
- Stochastic inflation
- Multiple fields
- ...

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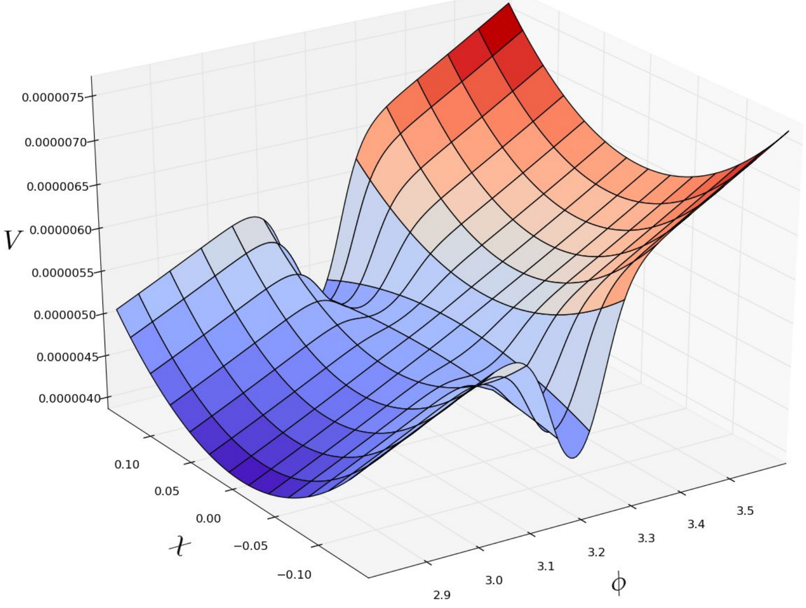
During inflation: potential instability

Two fields: $\phi + \chi$

ϕ-localized instability in the potential along χ

$$V(\chi, \phi) = \frac{1}{4}\lambda\phi^4 + \Delta V(\phi, \chi)$$



Credit: T. Morrison

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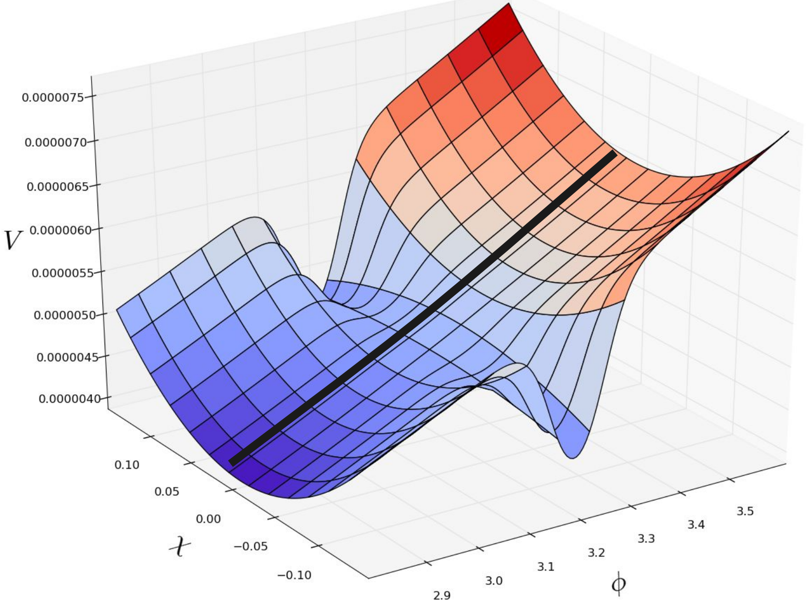

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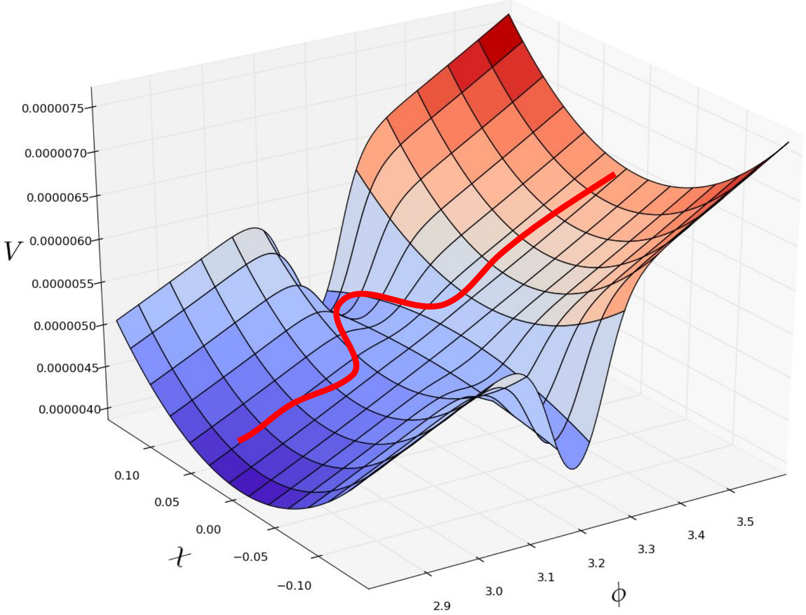
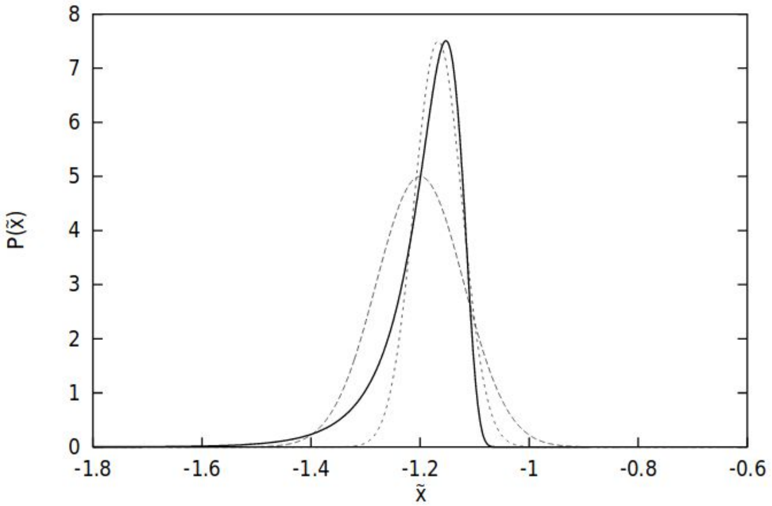


During inflation: stochastic inflation

The non-trivial inflaton trajectory may yield non-Gaussian contributions



Non-Gaussianity from stochastic inflation can also give more accurate PBH formation [Bullock, Primack ‘97]

Credit: Bullock, Primack

After inflation: preheating [Greene, Kofman, Linde, Starobinsky '97]

Fields: Inflaton ϕ + massless χ

Potential: $V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\chi^2\phi^2$

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\,\mathrm{cn}^2(\tilde{\tau}, 1/\sqrt{2})\right]\tilde{\chi}_k = 0, \quad \kappa^2 = \frac{k^2}{\lambda A_\phi^2}$$

$$\tilde{\chi}_k(\tilde{\tau}) = e^{\mu(\kappa, g^2/\lambda)\tilde{\tau}} f(\tilde{\tau})$$

After inflation: preheating [Greene, Kofman, Linde, Starobinsky '97]

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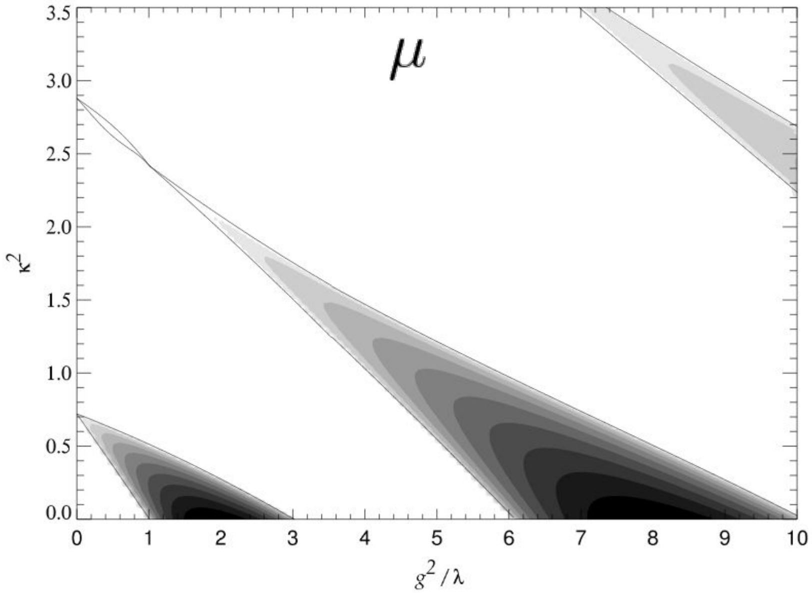
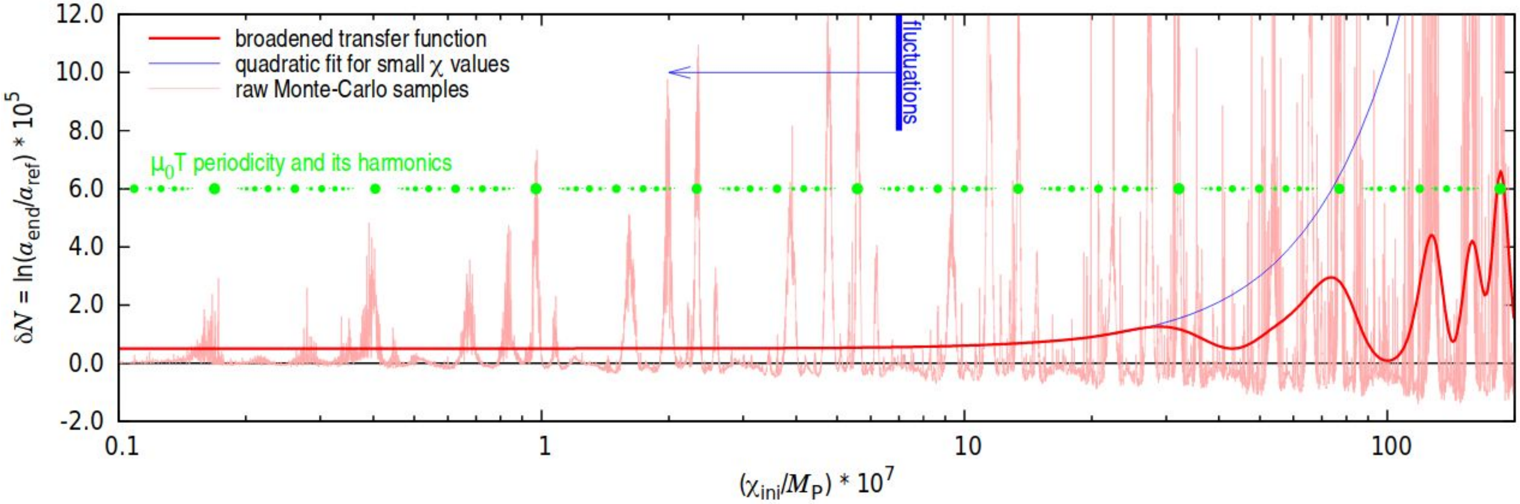


After inflation: preheating

[Bond, Frolov, Huang, Kofman ‘09]



Obtained from many highly accurate symplectic lattice simulations.

$$\zeta = \zeta_G + F_{NL}(\chi)$$

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Inflation: lattice simulations

Solve the the fields evolution equations in FLRW metric:

- Initial conditions: Bunch-Davies vacuum
- Equations: $\ddot{\phi}_i - \frac{1}{a^2}\Delta\phi_i + 3H\dot{\phi}_i + \frac{\partial V}{\partial \phi_i}(\phi_1, ..., \phi_n) = 0$ + Friedmann eqs.
- Curvature perturbations: $\dot{\zeta} = \frac{\dot{\rho} + 3H(\rho+p)}{3H(\rho+p)} = \frac{1}{a^2}\sum_i \frac{\nabla \cdot \left(\dot{\phi}_i \nabla \phi_i\right)}{3(\rho+p)}$

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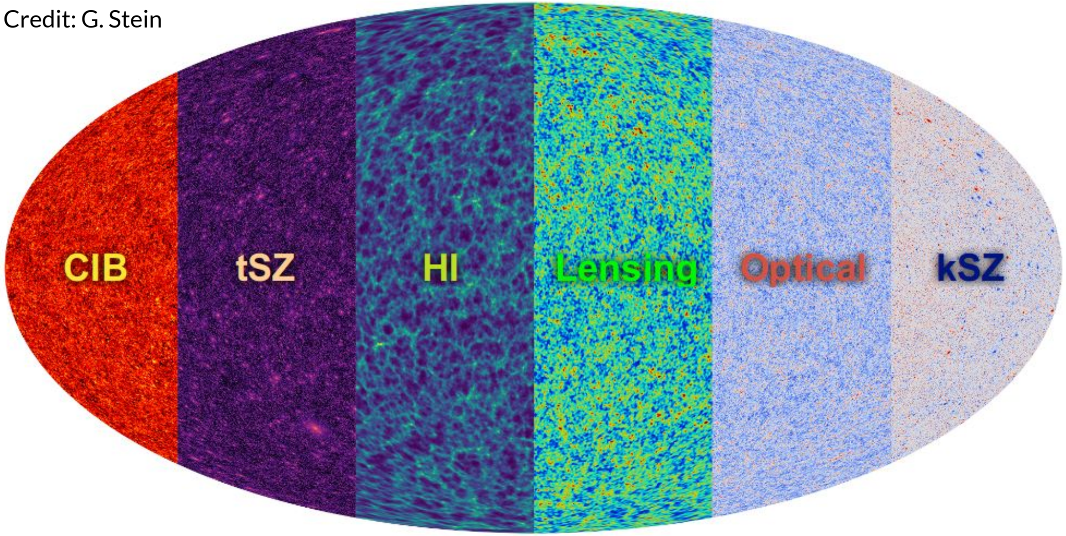
Non-Gaussianity with vanishing, e.g., bispectrum? (T. Morrison)

N-body approximation: Peak Patch

[Stein, Alvarez, Bond '18]

Credit: G. Stein



Fast generation of halo catalogues

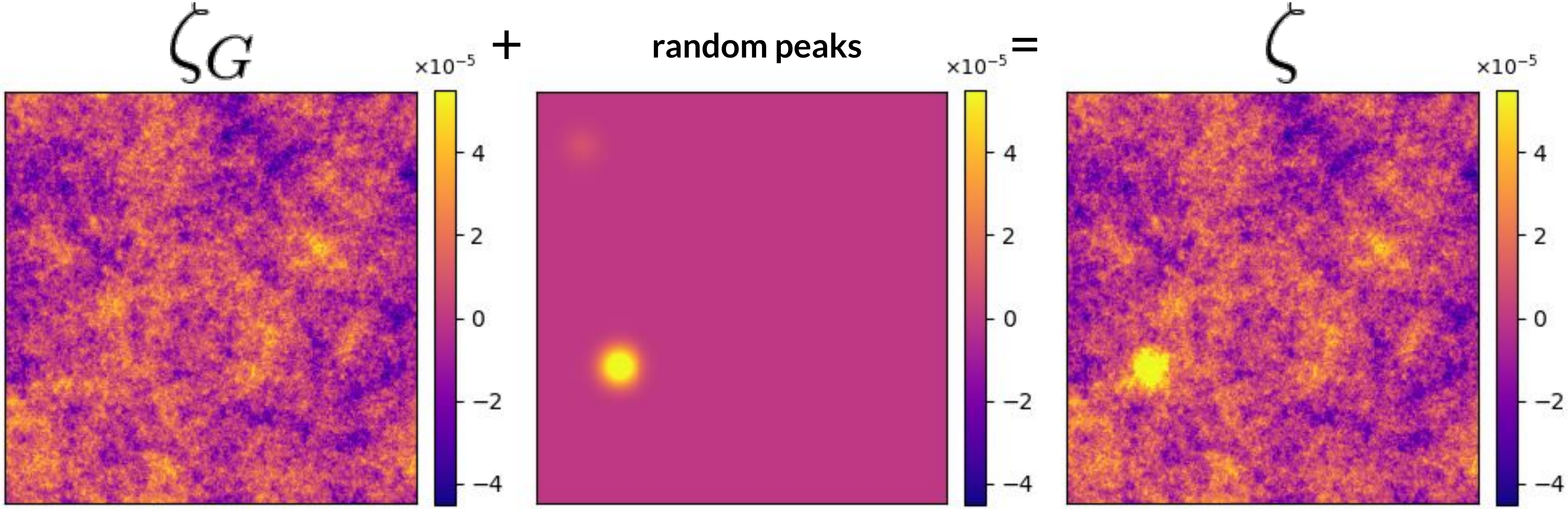
Full-sky extragalactic CMB mocks: WebSky

15.4Gpc/h, 12288^3 particles

https://mocks.cita.utoronto.ca/

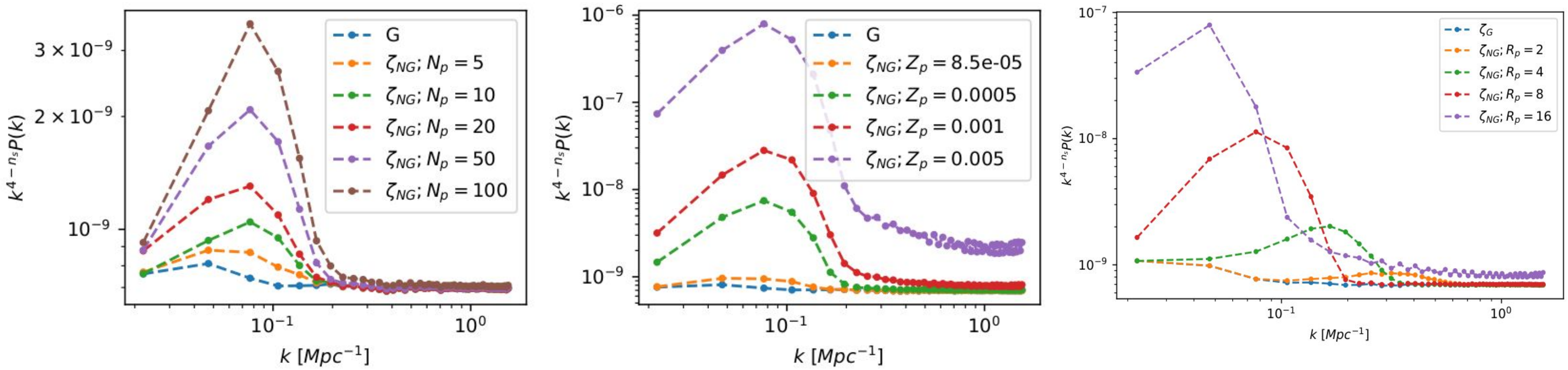
Mapmaking capabilities

Uncorrelated non-Gaussianity



$$P(k) = A_\phi \left(\frac{k}{k_0}\right)^{n_s-1}$$

Effects on statistics



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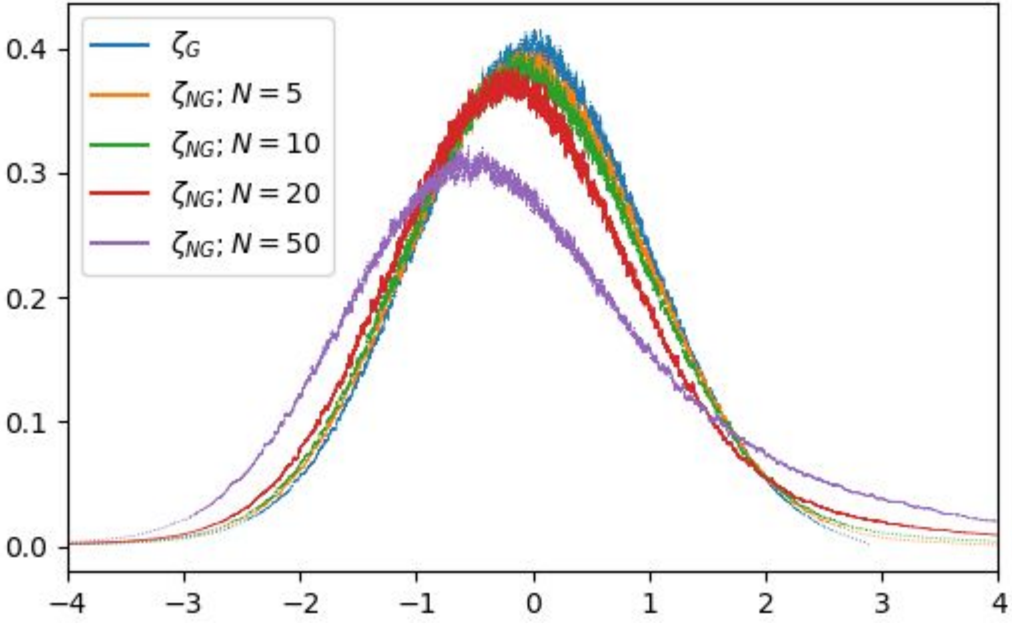
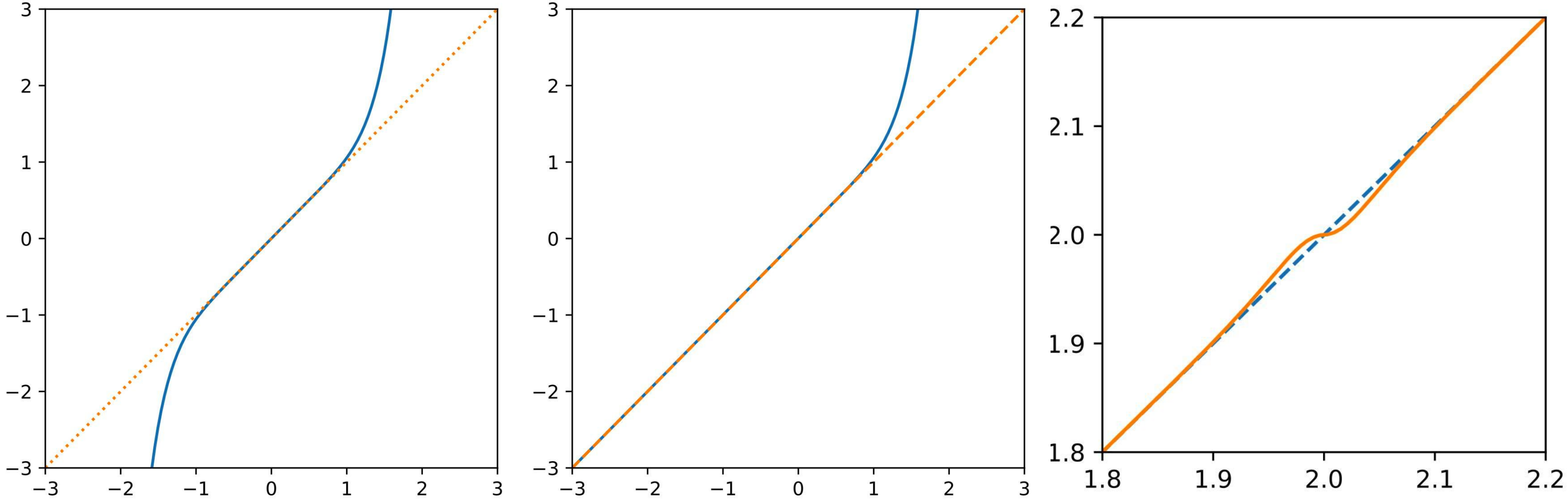
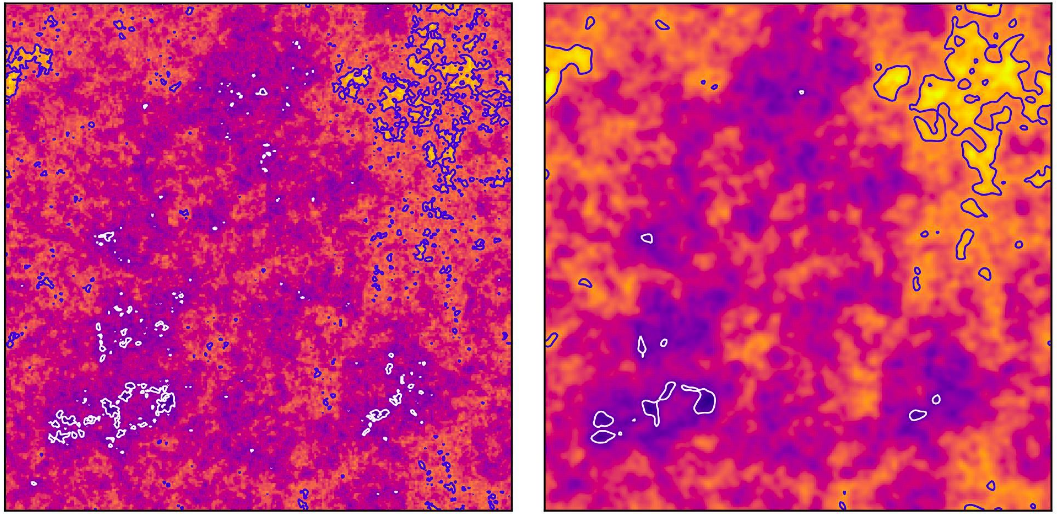


Correlated non-Gaussianity



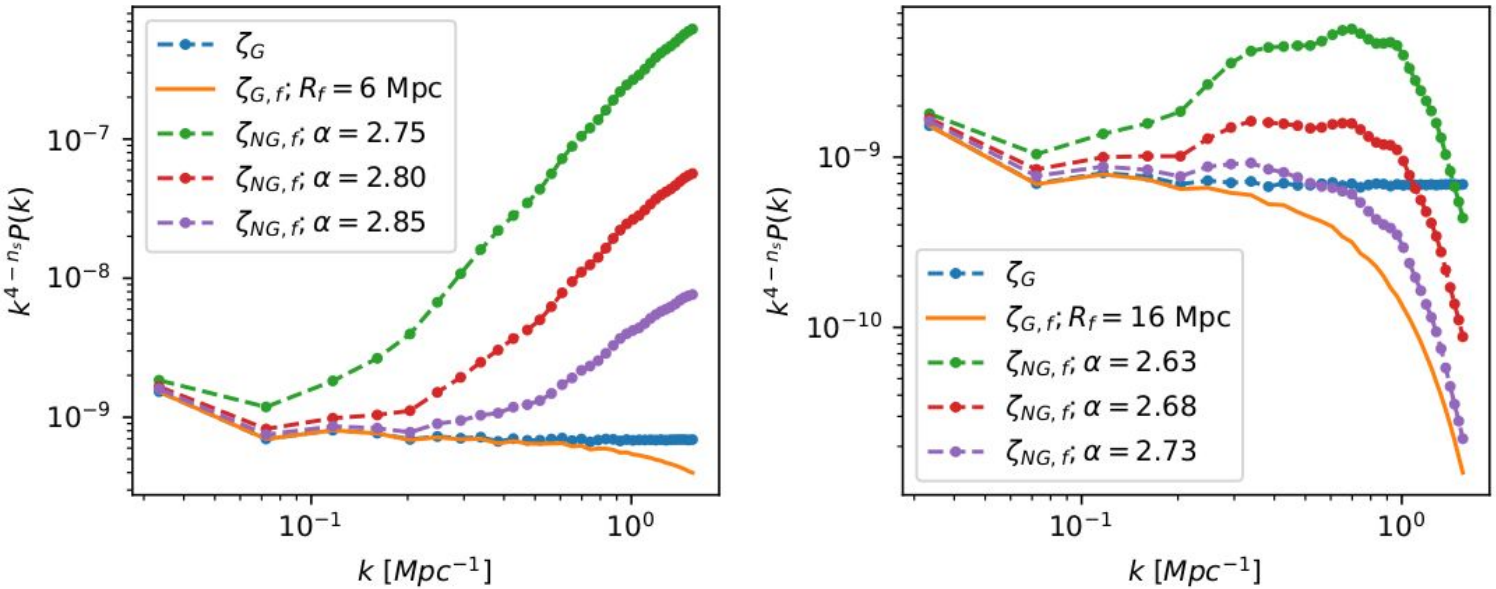
$$\zeta = F_{NL}(\zeta_G)$$

Smoothing

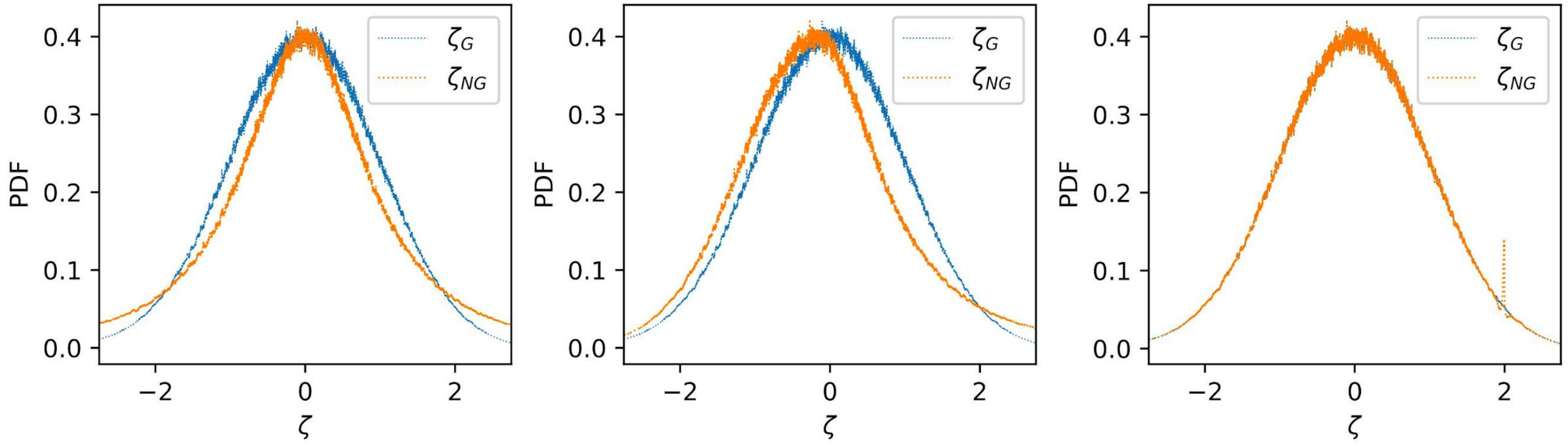


Smoothing → more compact structures
Also used after applying F_{NL}

Effects on statistics



Effects on statistics



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ICA: principle

$$\mathbf{x} = \mathbf{A}\mathbf{s}$$

Measured signal

Original signal (to be recovered)

ICA: principle

ICA-computed matrix

$$\mathbf{x} = \mathbf{A}\mathbf{s} \qquad\qquad \mathbf{s} \approx \mathbf{W}\mathbf{x}$$

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Non-Gaussianity = Independence

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Each component of **x** is a linear combination of the original signals

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Non-Gaussianity = Independence

Each component of **x** is a linear combination of the original signals
⇒ more Gaussian (Central Limit theorem)

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Measured signals

Original signals (to be recovered)

Recovered signals

Non-Gaussianity = Independence

Each component of **x** is a linear combination of the original signals
⇒ more Gaussian (Central Limit theorem)
⇒ each extracted component needs to maximize non-Gaussianity to match one of the source signals

Non-Gaussianity measures

Kurtosis

Negentropy

Non-Gaussianity measures

Kurtosis

$$\kappa_4 = \mathbb{E}\left[\left(\frac{X-\mu}{\sigma}\right)^4\right]$$

Negentropy

$$H(\mathbf{y}) = -\int f(\mathbf{y}') \ln f(\mathbf{y}') d\mathbf{y}'$$
$$J(\mathbf{y}) = H(\mathbf{y}_{\mathbf{G}}) - H(\mathbf{y})$$

Non-Gaussianity measures

Kurtosis

$$\kappa_4 = \mathbb{E}\left[\left(\frac{X-\mu}{\sigma}\right)^4\right]$$

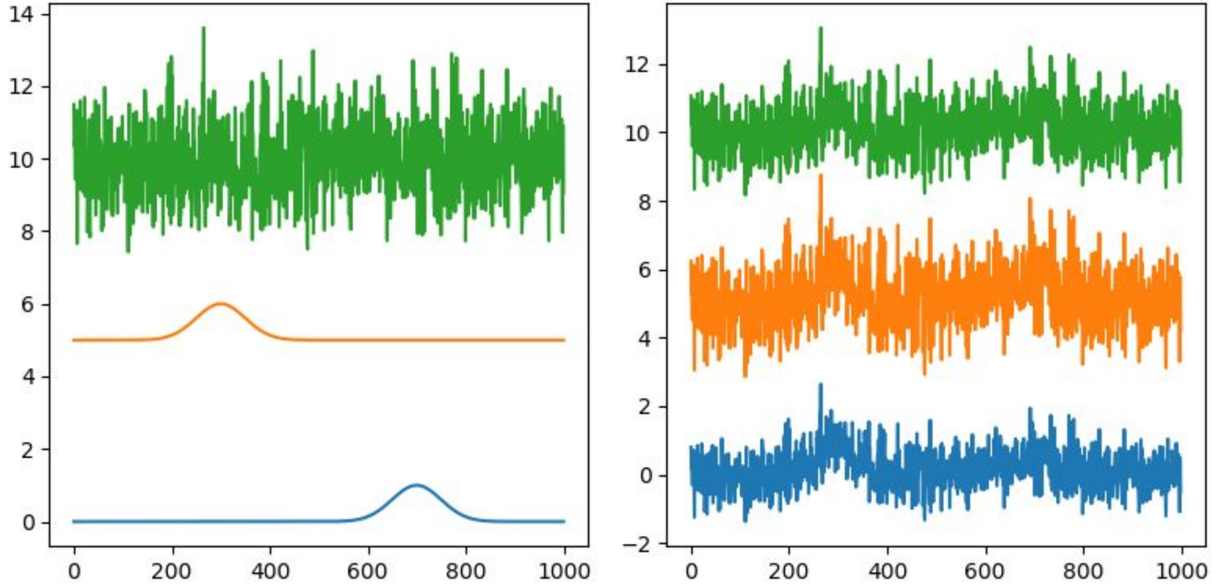
Easily computed from a sample
Sensitive to outliers

Negentropy

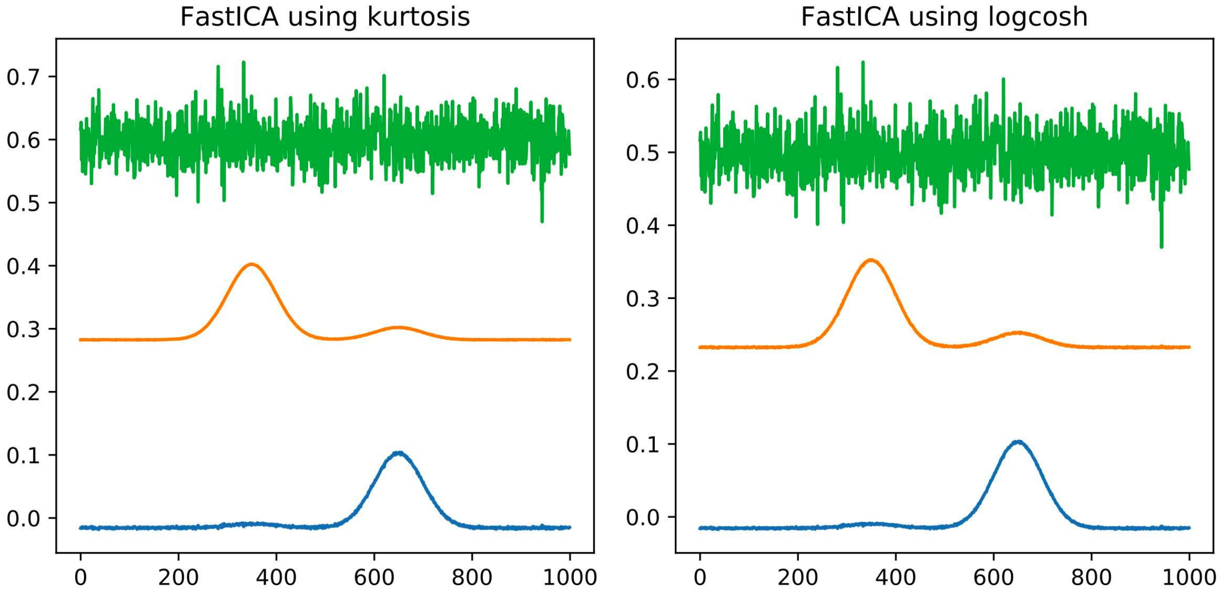
$$H(\mathbf{y}) = -\int f(\mathbf{y}')\ln f(\mathbf{y}')d\mathbf{y}'$$
$$J(\mathbf{y}) = H(\mathbf{y_G}) - H(\mathbf{y})$$

Only approximated in practice
Robust measure of non-Gaussianity

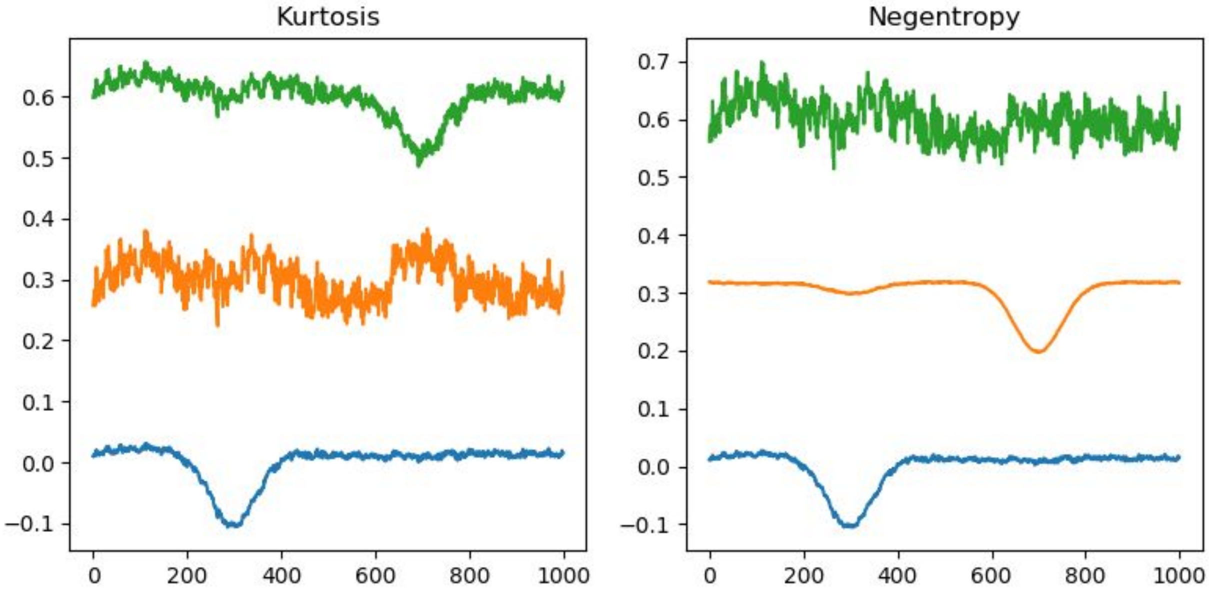
Sources and measurements



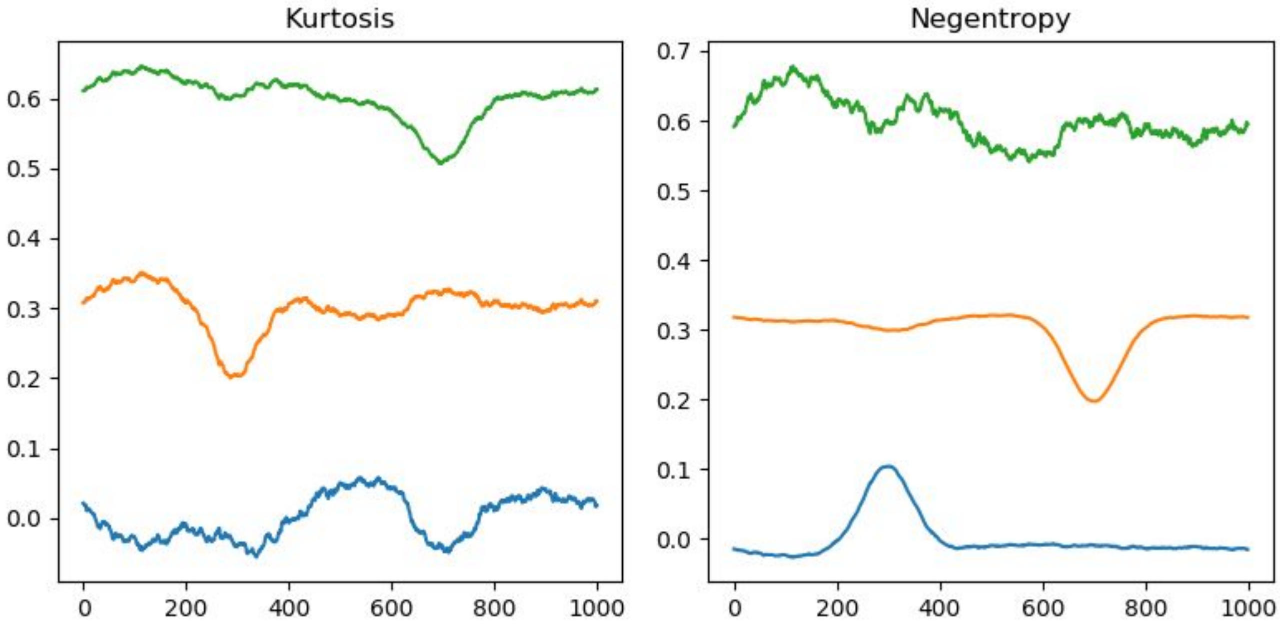
Test: white noise



Test: “pink noise”



Test: “brown noise”



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Conclusion

- Non-trivial non-Gaussianity may arise in (post-)inflationary models
- Lattice simulations can be used to understand complicated models
- Spatially-localized prominence, either correlated or uncorrelated, can skew the PDF and have a peaky contribution to the power spectrum.
- ICA, as a potential way to extract non-Gaussian contributions, gives better results with negentropy as a non-Gaussianity measure.