Fuzzy dark matter with Attractive Self-Interaction

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1 Quick Background

What if dark matter were an ultralight particle, $\sim 10^{-22}$ eV? Such a particle would exhibit wavelike behavior on kpc scales, thus affecting astrophysics and cosmology. This regime for dark matter is called 'Fuzzy Dark Matter' (FDM), because the particle de Broglie wavelength washes out features on kpc scales. FDM makes a lot of unique predictions for kpc-scale substructures: e.g. it predicts fewer subhalos than Cold Dark Matter (CDM), and it also predicts 'soliton' cores at halo centers (unlike 'cuspy' CDM halos), as well as interference fluctuations all throughout the halo. An overarching goal is to constrain the particle mass or rule out the model completely, using a variety of independent astrophysical arguments and observations.

For a detailed review of FDM, see Hui et al. (2017). For first cosmological simulations, and pictures to help visualize the small-scale features, see Nature paper Schive, Chiueh & Broadhurst (2014) and companion paper Schive et al. (2014)

For idealized simulations of merging soliton cores or individual halos, see Schwabe, Niemeyer & Engels (2016); Mocz et al. (2017).

The standard CDM is fantastic at describing the large-scale structure of the Universe. But there are a number of small-scale open challenges: e.g. the cusp-core problem or the too-big-to-fail problem, where CDM has a hard time predicting the number density, size and shapes of the dark matter halos of low-mass galaxies. This motivates the idea that dark matter's particle nature may be affecting astrophysical scales, as a possible solution to the problem. Furthermore, laboratory experiment searches for dark matter particle candidates like the WIMP have also been turning up negative and slowly ruling out the CDM parameter space, which motivates particle physicists to think about other dark matter particle models in new mass ranges.

One particle candidate for the FDM model is the *axion*, which will have an ultralight mass, m, which is the primary parameter to be constrained. A second key feature is that if FDM is the axion instead of just a general boson, then the particle also has a strong-CP symmetry breaking scale $f \sim 10^{15}$ GeV–10^{17} GeV. This leads to the dark matter effectively experiencing an additional attractive self-interaction. Though weak, this self-interaction may have astrophysical and cosmological consequences, e.g. see Desjacques, Kehagias & Riotto (2018).

The goal of the USRP project will be to explore and understand the role of this weak attractive self-interaction added to the FDM model through idealized simulations. The simulations will be an extension of those in Mocz et al. (2017) which look at halos in the 0 self-interaction case. The weak attractive self-interaction in the context of FDM has really not been studied in detail with numerical simulations before, which means we will be charting into new territory. It is expected to lead to interesting features, e.g. modification of soliton cores, where a phase-transition occurs above a critical mass (Chavanis, 2018). It'll be very exciting to lead some pioneering work to see what happens to idealized halos as the weak attractive self-interaction is added to the simulation and turned up.

2 Questions

- What happens to FDM halos as a function of attractive self-interaction strength?
- How is the soliton core modified due to the attractive self-interaction?

- Can we detect a phase transition of the soliton, and the threshold of onset? Is the new state stable?
- How does the core mass scale with halo mass?
- ...
- What are some astrophysical consequences of our findings?

3 Equations

The dark matter is governed by the Gross-Pitaesvkii-Poisson equations (which is the Schrödinger equation where the potential is the self-potential due to self-gravity, plus a nonlinear attractive self-interaction term:

$$i\hbar\left(\frac{\partial}{\partial t}\right)\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi - \frac{4\pi\hbar^2 a_s}{m^2}|\psi|^2\psi \tag{1}$$

$$\nabla^2 V = 4\pi G(\rho - \overline{\rho}) \tag{2}$$

Here ψ is the wavefunction that describes the dark matter, normalized to give the dark matter density $|\psi|^2 = \rho$. The attractive self-interaction strength is characterized by a_s, the s-scattering length.

In general a_s related to strong-CP energy decay constant f mentioned in the intro as:

$$f = \sqrt{\frac{\hbar c^3 m}{32\pi a_s}} \iff a_s = \frac{\hbar c^3 m}{32\pi f^2} \tag{3}$$

where c is the speed of light.

One can also add higher-order relativistic corrections to the theory: e.g., a repulsive term:

$$i\hbar\left(\frac{\partial}{\partial t}\right)\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi - \frac{4\pi\hbar^2 a_s}{m^2}|\psi|^2\psi + \frac{32\pi\hbar^4 a_s^2}{3m^5 c^2}|\psi|^4\psi \tag{4}$$

It will probably be useful to include this term as well. It is only expected to be important at very high densities.

Without any self-interaction, the soliton cores that form can be fit with an analytic profile:

$$\rho_{\mathrm{soliton}}(r) \simeq \rho_0\left[1 + 0.091 \times \left(\frac{r}{r_{\mathrm{c}}}\right)^2\right]^{-8} \tag{5}$$

(Schive et al., 2014), where r_{c} is the core radius and ρ_0 is the central density given by:

$$\rho_0 \simeq 1.9 \times 10^7 \left(\frac{10^{-22}\ \mathrm{eV}}{m}\right)^2 \left(\frac{\mathrm{kpc}}{r_{\mathrm{c}}}\right)^4 \frac{M_\odot}{\mathrm{kpc}^3} \tag{6}$$

Integrating this radial profile gives a total mass of:

$$M = \int_0^\infty \rho(r) 4\pi r^2\, dr = 11.6\rho_0 r_{\mathrm{c}}^3 \tag{7}$$

Note that (as $\rho_0 \propto r_{\mathrm{c}}^{-4}$) we have $M \propto r_{\mathrm{c}}^{-1}$. More massive solitons are smaller, which is a pretty unique feature; most equations of state yield objects that are bigger when they are more massive but the opposite is true here.

When you add self-interactions, the solitons are expected to go unstable above a critical mass (Chavanis, 2018)

$$M_{\max} = 1.012\hbar/\sqrt{Gma_s} \tag{8}$$

above this, they may form either a dense soliton, or a 'bosenova'. We will have to see what happens in our simulations!

One may also *estimate* a critical central density for collapse by substituting Eqn. 7 into Eqn. 8:

$$\rho_{\rm crit} = 1.2 \cdot 10^9 \left(\frac{m}{10^{-22}\ {\rm eV}}\right)^2 \left(\frac{f}{10^{15}\ {\rm GeV}}\right)^4 M_\odot\ {\rm kpc}^{-3} \tag{9}$$

Or you may estimate it in terms of a critical radius:

$$r_{\rm crit} \simeq 0.18 f_{15}^{-1} m_{22}^{-1}\ {\rm kpc} \tag{10}$$

Or in terms of the β parameter defined in Eqn. 13:

$$\beta_{\rm crit} \simeq 0.3 \tag{11}$$

4 Analytic Fitting Formula for non-collapsed case w/ SI

Once can numerically solve for the ground state of Eqn 1. The following is a fitting formula that approximates to solution with $< 2\%$ error:

$$\rho_{\rm soliton}(r) \simeq \rho_0 \left[1 + (1 + 2.60 i_1(\beta)) \times 0.091 \times \left(\frac{r\sqrt{1+\beta}}{r_{\rm c}}\right)^{\left(2 - \frac{1}{5} i_2(\beta)\right)}\right]^{\left(-8 + \frac{22}{5} i_3(\beta)\right)} \tag{12}$$

where the dimensionless constant β is

$$\beta \equiv 1.6 \times 10^{-12} \left(\frac{10^{-22}\ {\rm eV}}{m}\right) \left(\frac{\rho_0}{M_\odot\ {\rm kpc}^{-3}}\right)^{1/2} \times \frac{\hbar c^5}{32\pi G f^2} \tag{13}$$

and i_1, i_2, and i_3 are interpolating functions that smoothly transition from 0 to 1:

$$i_1(\beta) = \tanh(\beta/5) \tag{14}$$

$$i_2(\beta) = \tanh(\beta) \tag{15}$$

$$i_3(\beta) = \tanh(\sqrt{\beta})^2 \tag{16}$$

The constant ρ_0 is same as before (Eqn. 6).

Note that β is invariant under the scaling-symmetry of the non-linear SP equations which we discuss later (eqn. 29).

Note that in the limit $\beta \gg 1$, the equation approximately becomes:

$$\rho_{{\rm soliton},\beta\gg 1}(r) \simeq \rho_0 \left[1 + 3.60 \times 0.091 \times \left(\frac{r\sqrt{\beta}}{r_{\rm c}}\right)^{\frac{9}{5}}\right]^{-\frac{18}{5}} \tag{17}$$

although this solution is unstable because β is above $\beta_{\rm crit}$.

5 Predicting what may happen above critical mass

The attractive self-interaction potential term in Eqn. 1 can be expressed as a negative pressure term (via $-\nabla V = -\frac{\nabla P}{\rho}$):

$$P_2 = -\frac{2\pi a_s \hbar^2}{m^3} \rho^2 \tag{18}$$

Furthermore, the next-order repulsive term (Eqn. 4) becomes as *stabilizing* positive pressure:

$$P_4 = \frac{64\pi^2 a_s^2 \hbar^4}{9 m^6 c^2} \rho^3 \tag{19}$$

Note that the two pressures are equal at

$$\rho_{\rm eq} = \frac{9c^2m^3}{32a_s\hbar^2\pi} \tag{20}$$

and above this density, the positive P_4 dominates.

- If we assume that an object above $M_{\rm max}$ will collapse and the P_4 term will be dominant in supported the new core that forms against collapse from self-gravity, then we expect the new core will be a $\gamma = 3, n = 1/3$ polytrope. A polytrope is a star supported by a pressure of the form $P = K\rho^\gamma = K\rho^{(n+1)/n}$. A very famous object of study in astronomy (Lane-Embden equations) with known numerical solutions. Approximately, for polytrope, given a central density, ρ_c, the core size is approximately $\alpha = \sqrt{(n+1)K\rho_c^{1/n-1}/(4\pi G)}$

Let's consider a fiducial case: $m_{22} = m/(10^{-22}\text{ eV}) = 1$ and $f_{15} = m/(10^{15}\text{ GeV}) = 1$. Then the critical mass for collapse can be calculated to be: $M_{\rm max} = 6.26 \cdot 10^8\ M_\odot$. Also, $\rho_{\rm eq} = 3.1 \cdot 10^{18}\ M_\odot/\text{kpc}^3$.

Such a soliton, pre-collapse, would have radius: $r_c = 0.35$ kpc

Note that our simulation box resolution is $\Delta x = L_{\rm box}/N = 20\text{ kpc}/400 = 0.05$ kpc.

6 Main Equation in dimensionless variables

Sometimes it is useful to work in dimensionless variables, e.g.

$$\hat{\psi} = \frac{\hbar\sqrt{4\pi G}}{mc^2}\psi \tag{21}$$

$$\hat{r} = \frac{mc}{\hbar}r \tag{22}$$

$$\hat{t} = \frac{mc^2}{\hbar}t \tag{23}$$

$$\hat{V} = \frac{1}{c^2}V \tag{24}$$

$$\hat{a}_s = \frac{mc}{\hbar}a_s \tag{25}$$

So Eqn. 1 becomes:

$$i\left(\frac{\partial}{\partial\hat{t}}\right)\hat{\psi} = -\frac{1}{2}\nabla^2\hat{\psi} + \hat{V}\hat{\psi} - \hat{a}_s|\hat{\psi}|^2\hat{\psi}\frac{\hbar c}{m^2G} \tag{26}$$

$$\nabla^2\hat{V} = (\hat{\rho} - \overline{\hat{\rho}}) \tag{27}$$

Define:

$$\hat{\beta} \equiv \hat{a}_s\frac{\hbar c}{m^2G} = \frac{mc}{\hbar}\frac{\hbar c}{m^2G}a_s = \frac{\hbar c^5}{32\pi Gf^2} \tag{28}$$

Note the scaling symmetry of the non-linear SP equations, which allows one to rescale solutions to different values with scaling:

$$\{\hat{t}, \hat{x}, \hat{\psi}, \hat{\rho}, \hat{V}, \hat{\beta}\} \to \{\lambda^{-2}\hat{t}, \lambda^{-1}\hat{x}, \lambda^2\hat{\psi}, \lambda^4\hat{\rho}, \lambda^2\hat{V}, \lambda^{-2}\hat{\beta}\} \tag{29}$$

β does not scale like a length-scale due to the non-linearity in the Gross-Pitaevski equation.

References

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