

Bayesian methods for quantifying global parameter tensions between cosmological datasets

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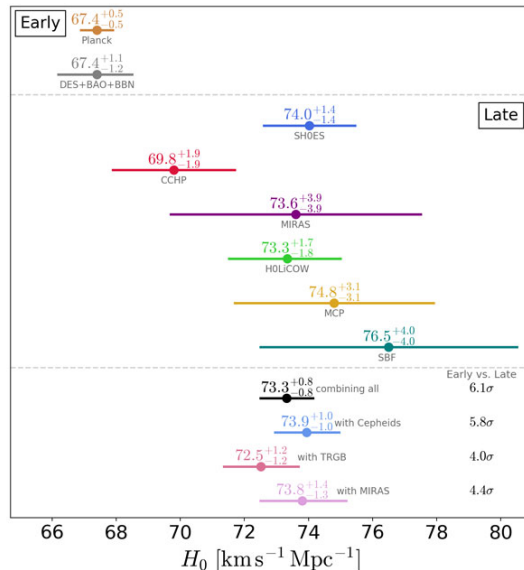


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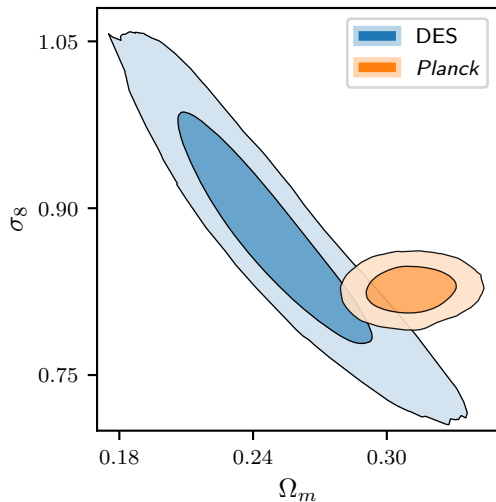
Cosmological parameter tensions

- ▶ Measurements H_0 differ between early and late time observations [1907.10625]
- ▶ “Tension” means a disagreement between different datasets on the inferred value of model parameters.
- ▶ The presence of tension indicates an error in the model or at least one of the datasets.
- ▶ It is statistically incorrect to combine datasets when they are in tension.



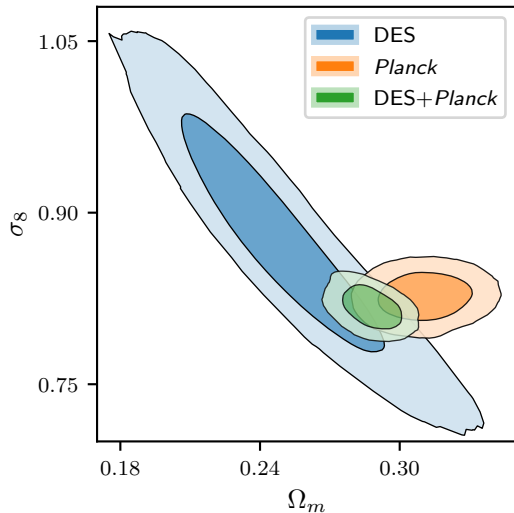
The importance of global measures of tension

- ▶ In other situations the discrepancy doesn't exist in a single interpretable parameter
- ▶ For example: DES+*Planck* [1902.04029]
- ▶ Are these two datasets in tension?
- ▶ Can we confidently combine them?
- ▶ There are a lot more parameters – are we sure that we've chosen wisely?



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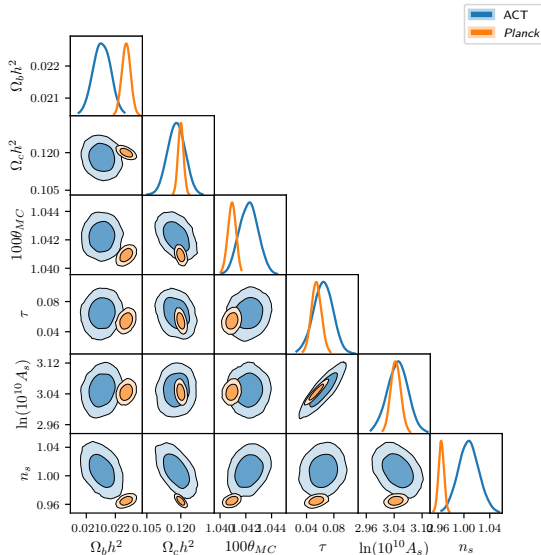


The perils of manual marginal inspection

- ▶ If you have enough parameters, then you might expect that tensions would naturally arise in some combinations by chance.
- ▶ For example, if you take ACT and *Planck*, and construct a linear combination of parameters in maximum tension:

$$t = -\Omega_b h^2 + 0.022\Omega_c h^2 + 34\theta_{MC} - 0.092\tau \\ + 0.05\ln(10^{10}A_s) + 0.067n_s$$

- ▶ In general you would expect such a parameter to be in $\sim \sqrt{d} - \sigma$ tension [2007.08496]

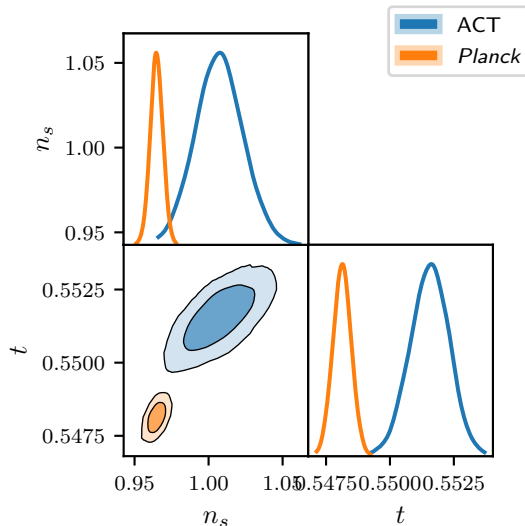


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Bayesian language

Notation

- ▶ Datasets A and B (e.g. *Planck* and DES)
- ▶ Model M (e.g. Λ CDM)
- ▶ Parameter(s) θ (e.g. (Ω_m, σ_8))
- ▶ Likelihoods \mathcal{L} : $P(A|\theta)$, $P(B|\theta)$
- ▶ Prior π : $P(\theta)$

Inference

- ▶ Evaluate representations (samples) from posteriors \mathcal{P} : $P(\theta|B)$, $P(\theta|A)$.
- ▶ Bayesian evidences $\mathcal{Z} = \langle \mathcal{L} \rangle_\pi$: $P(A)$, $P(B)$ using nested sampling
- ▶ Kullback–Leibler divergence $\mathcal{D} = \langle \log \mathcal{P} / \pi \rangle_{\mathcal{P}}$
- ▶ Bayesian model dimensionality $d = 2 \times \text{var}(\log \mathcal{L})$ [1903.06682]
- ▶ Tattoo-worthy equation:

$$\log \mathcal{Z} = \langle \log L \rangle_{\mathcal{P}} - \mathcal{D}$$

Suspiciousness [1902.04029]

- ▶ The natural Bayesian measure of tension is the Bayes ratio

$$\mathcal{R} = \frac{\mathcal{Z}_{AB}}{\mathcal{Z}_A \mathcal{Z}_B} = \frac{P(A, B)}{P(A)P(B)} = \frac{P(A|B)}{P(A)}$$

- ▶ \mathcal{R} is prior dependent, and can artificially reduce tension by drawing arbitrarily wide priors.
- ▶ Can remove this prior dependency by dividing out the KL dependent occam factor to give a “Suspiciousness”, computable from three MCMC chains:

$$\log S = \langle \log L_{AB} \rangle_{\mathcal{P}_{AB}} - \langle \log L_A \rangle_{\mathcal{P}_A} - \langle \log L_B \rangle_{\mathcal{P}_B}$$

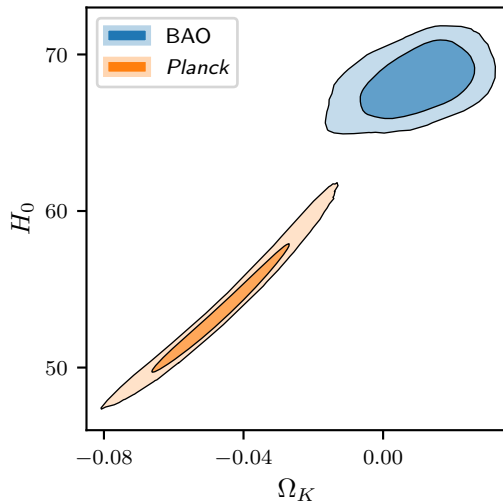
- ▶ Can be interpreted as the maximum tension allowed by reasonable priors.
- ▶ In the Gaussian case it is related to the usual Mahalanobis distance

$$\log S = \frac{d}{2} - \frac{1}{2}(\mu_A - \mu_B)^T (\Sigma_A + \Sigma_B)^{-1} (\mu_A - \mu_B)$$

which can be used to calibrate it into a tension probability and “ σ ” quantification.

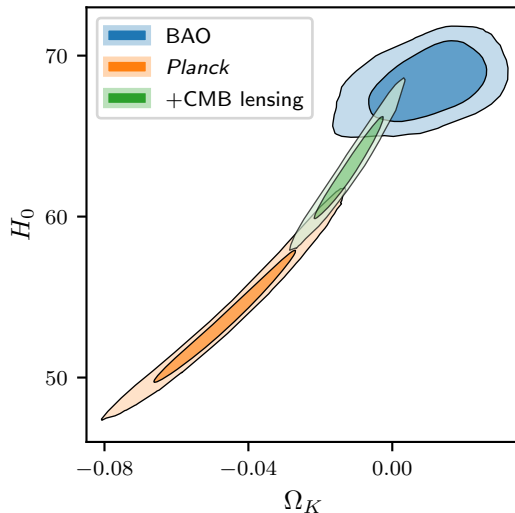
Curvature tension Ω_K

- ▶ Λ CDM assumes the universe is flat
- ▶ If you allow $\Omega_K \neq 0$, *Planck* (plikTTTEEE) has a moderate preference for closed universes (50:1)
- ▶ *Planck*+CMB lensing +BAO strongly prefer a flat universe
- ▶ But, *Planck* vs lensing is 2.5σ in tension, and *Planck* vs BAO is 3σ .
- ▶ This is reduced if plik \rightarrow camspec
 - ▶ Di Valentino et al [1911.02087]
 - ▶ Handley [1908.09139]
 - ▶ Efsthathiou & Gratton [2002.06892]
- ▶ BAO and lensing summary statistics and compression strategy assume Λ CDM.



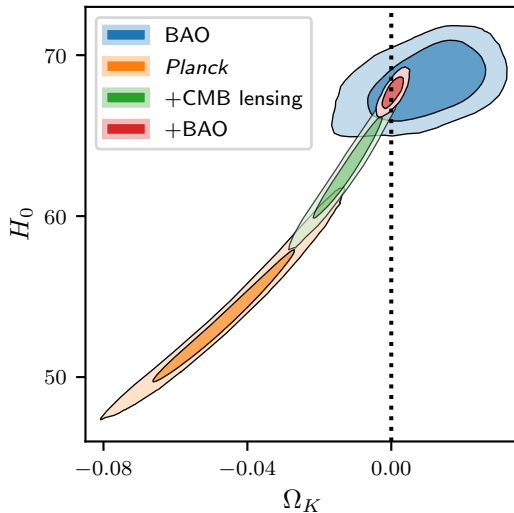
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Data	Model	Tension	Reference
DES vs Planck	Λ CDM	2.1σ	[2007.08496]
ACT vs Planck+SPT	Λ CDM	2.8σ	[2007.08496]
CMB lensing vs Planck	$K\Lambda$ CDM	2.5σ	[1908.09139]
BAO vs Planck	$K\Lambda$ CDM	3σ	[1908.09139]

1.

Slides, figures and plotting code available at:

https://github.com/williamjameshandley/talks/tree/tehran_2021