Bayesian methods for quantifying global parameter tensions between cosmological datasets

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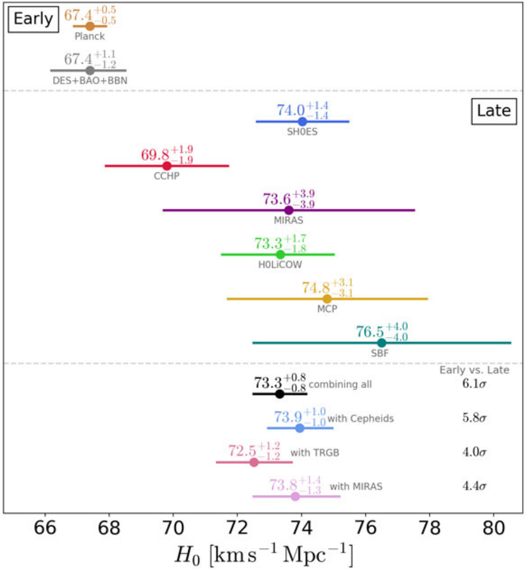
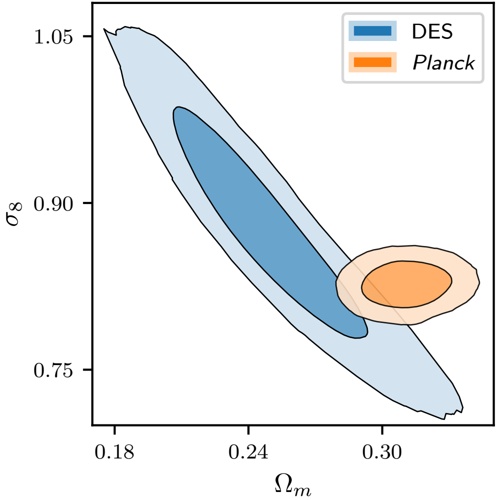
Cosmological parameter tensions

- Measurements H_0 differ between early and late time observations [1907.10625]
- "Tension" means a disagreement between different datasets on the inferred value of model parameters.
- The presence of tension indicates an error in the model and/or at least one of the datasets.
- It is statistically incorrect to combine datasets when they are in tension.



The importance of global measures of tension

- In other situations the discrepancy doesn't exist in a single interpretable parameter
- For example: DES+*Planck* [1902.04029]
- Are these two datasets in tension?
- Can we confidently combine them?
- There are a lot more parameters – are we sure that we've chosen wisely?



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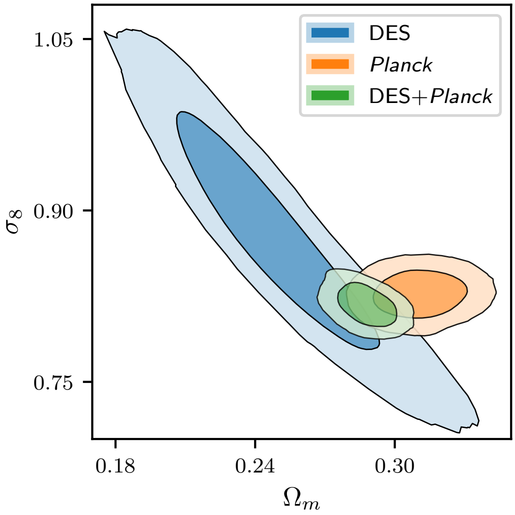
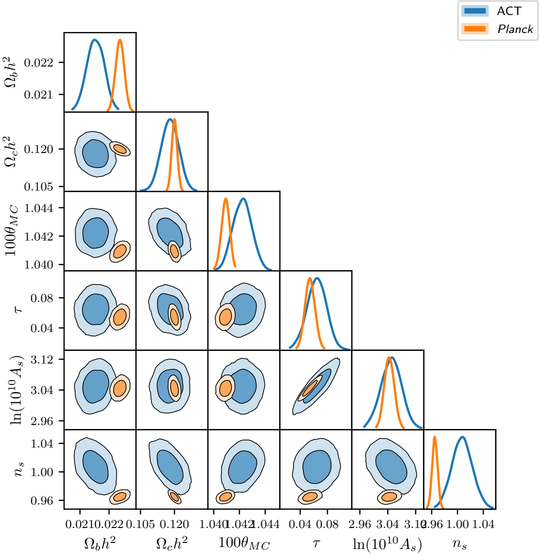


The perils of manual marginal inspection

- If you have enough parameters, then you might expect that tensions would naturally arise in some combinations by chance.
- For example, if you take ACT and *Planck*, and construct a linear combination of parameters in maximum tension:

$$t = -\Omega_b h^2 + 0.022\Omega_c h^2 + 34\theta_{MC} - 0.092\tau + 0.05\ln(10^{10}A_s) + 0.067n_s$$

- In general you would expect such a parameter to be in $\sim\sqrt{d}-\sigma$ tension [2007.08496]



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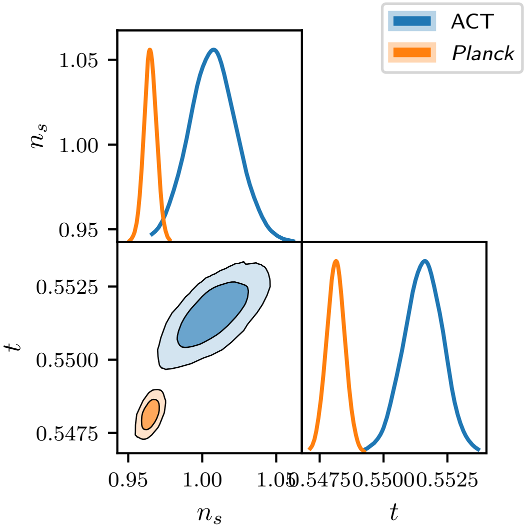


Bayesian language

Notation

Datasets:	A and B (e.g. *Planck* and DES)
Model:	M (e.g. ΛCDM)
Parameters:	θ (e.g. (Ω_m, σ_8))
Likelihoods:	$\mathcal{L}$: $P(A\|\theta)$, $P(B\|\theta)$

Inference

Prior:	π: $P(\theta)$	
Posteriors:	$\mathcal{P}$: $P(\theta\|B)$, $P(\theta\|A)$, (evaluate samples).	
Bayesian evidences:	$\mathcal{Z} = \langle\mathcal{L}\rangle_\pi$, $P(A)$, $P(B)$	[1506.00171]
Bayes theorem:	$\mathcal{L}\times\pi = \mathcal{P}\times\mathcal{Z}$	

Anatomy

Kullback–Leibler divergence:	$\mathcal{D} = \langle\log\mathcal{P}/\pi\rangle_\mathcal{P} \qquad \sim \log\mathrm{Vol}(\pi)/\mathrm{Vol}(\mathcal{P})$	[1902.04029]
Model dimensionality:	$d = 2\times\mathrm{var}(\log\mathcal{L})$	[1903.06682]
Occam's razor equation:	$\log\mathcal{Z} = \langle\log\mathcal{L}\rangle_\mathcal{P} - \mathcal{D}$	[2102.11511]

(Released today by Hergt et al)

Suspiciousness statistic [1902.04029] [2007.08496]

- The natural Bayesian measure of tension is the Bayes ratio

$$\mathcal{R} = \frac{\mathcal{Z}_{AB}}{\mathcal{Z}_A\mathcal{Z}_B} = \frac{P(A,B)}{P(A)P(B)} = \frac{P(A|B)}{P(A)} = \frac{P(B|A)}{P(B)} \tag{1}$$

- $\mathcal{R}$ is prior dependent, one can artificially reduce tension by drawing arbitrarily wide priors.
- Can remove this prior dependency by dividing out the KL-dependent Occam factor to give a "Suspiciousness", computable from three MCMC chains:

$$\log S = \langle \log L_{AB}\rangle_{\mathcal{P}_{AB}} - \langle \log L_A\rangle_{\mathcal{P}_A} - \langle \log L_B\rangle_{\mathcal{P}_B} \tag{2}$$

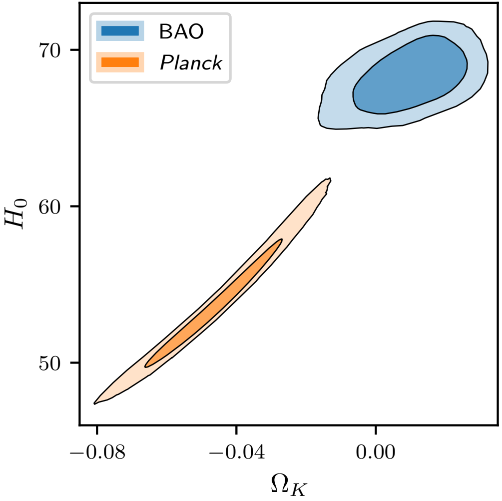
- Can be intepreted as the maximum Bayes ratio $\mathcal{R}$ allowed by reasonable priors.
- In the Gaussian case it is related to the usual Malhanobis distance

$$\log S = \frac{d}{2} - \frac{1}{2}(\mu_A - \mu_B)^T(\Sigma_A+\Sigma_B)^{-1}(\mu_A-\mu_B) \tag{3}$$

which can be used to calibrate it into a tension probability and "σ" quantification.

Curvature tension Ω_K

- ΛCDM assumes the universe is flat
- If you allow $\Omega_K \neq 0$, *Planck* (`plikTTTEEE`) has a moderate preference for closed universes (50:1)
- *Planck*+CMB lensing +BAO strongly prefer a flat universe
- But, *Planck* vs lensing is 2.5σ in tension, and Planck vs BAO is 3σ.
- This is reduced if `plik` $\to$ `camspec`
  - Di Valentino et al [1911.02087]
  - Handley [1908.09139]
  - Efsthathiou & Gratton [2002.06892]
- BAO and lensing summary statistics and compression strategy assume ΛCDM.



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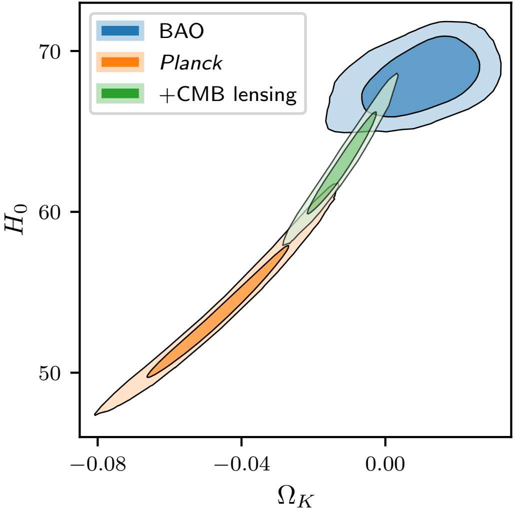

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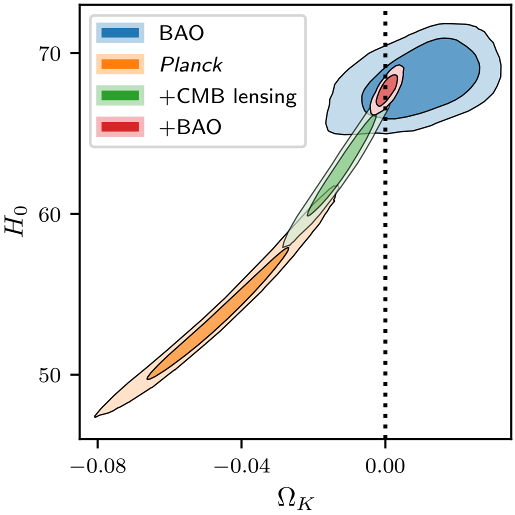

Summary

Data	Model	Tension	Reference
DES vs Planck	ΛCDM	2.1σ	[1902.04029]
ACT vs Planck+SPT	ΛCDM	2.8σ	[2007.08496]
CMB lensing vs Planck	$K\Lambda$CDM	2.5σ	[1908.09139]
BAO vs Planck	$K\Lambda$CDM	3σ	[1908.09139]

Slides, figures and plotting code available at:

`https://github.com/williamjameshandley/talks/tree/tehran_2021`