Inflation, curvature and kinetic dominance

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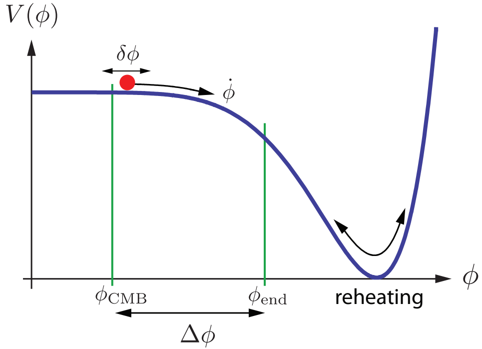
Lightning review of inflation

- Inflation explains observed present-day flatness and homogeneity.
- A primordial accelerated phase $\ddot{a} > 0$ shrinks horizon $1/aH$.
- Fill the universe with a homogeneous scalar field ϕ:

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{K}{a^2},$$
$$0 = \ddot{\phi} + 3H\dot{\phi} + V'(\phi).$$



- Slow roll solutions have $\dot{\phi}^2 \ll V(\phi) \Rightarrow H \approx H_* \Rightarrow a \propto e^{H_* t}$.
- Small δH generates $\mathcal{P} = A_s(k/k_*)^{n_s - 1}$ with $n_s \neq 1$.

The problem with eternal inflation

- The canonical view of inflation has an initially *eternal* exponential expansion phase $a \propto e^{H_* t}$ as $t \to -\infty$.
- This viewpoint is only compatible with the flat case ($K = 0$).
- In flat case, there is a rescaling symmetry $a \to \alpha a$.
- In curved case ($K \neq 0$), a is physically interpretable as (pseudo) radius.
- Inflation is limited by domination of curvature energy density $-\frac{K}{a^2}$.
- If one invokes inflation to flatten the universe, you cannot assume it is flat initially.

Primordial horizon evolution with curvature

Analytic approximation

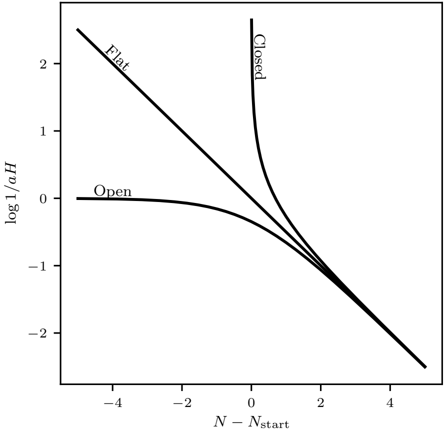
- Equation of motion of horizon:

$$\frac{\mathrm{d}}{\mathrm{d}N}\log\frac{1}{aH} = -1-\frac{K}{(aH)^2}+\frac{\dot\phi^2}{2H^2}.$$

- Assuming slow-roll: $\dot\phi^2 \ll H^2$

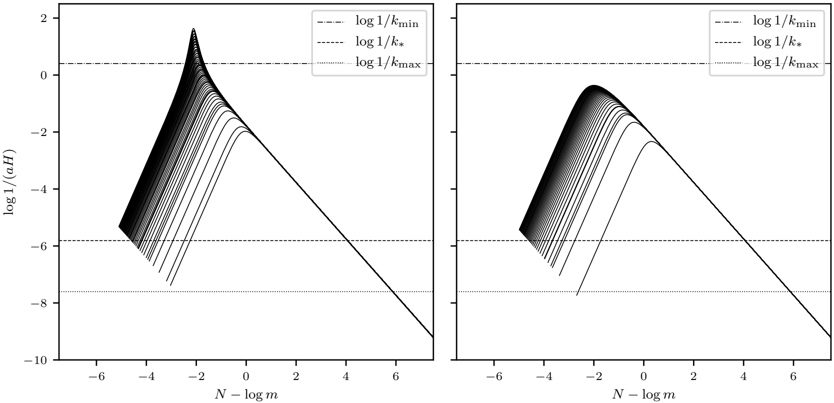
$$\log\frac{1}{aH} = -\frac{1}{2}\log\left(e^{N-N_\mathrm{start}} - K\right).$$

- Closed: Limit amount of inflation
- Open: Limit on Horizon size



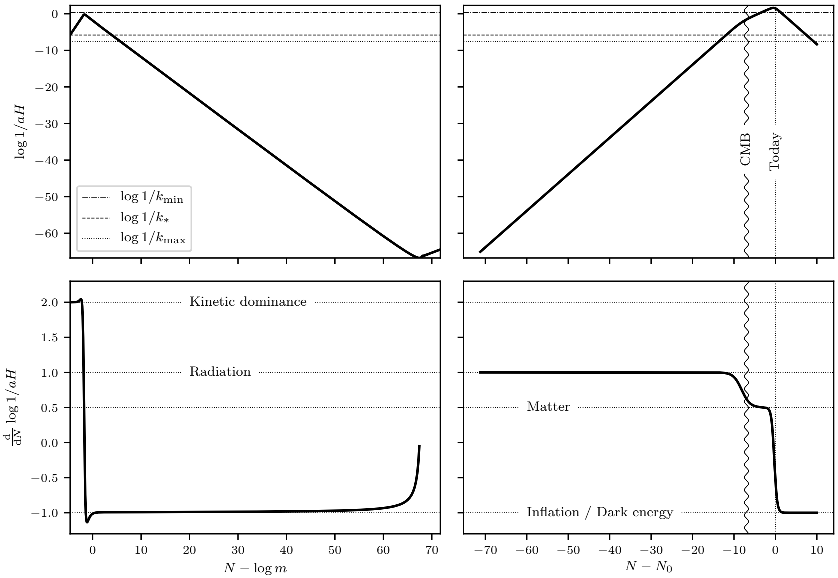
Primordial horizon evolution with curvature

Numerics



- Evolution for closed and open cases, such that $N_* = 50$.
- Inflation preceded by a kinetically dominated phase $\dot{\phi}^2 \gg V(\phi)$.

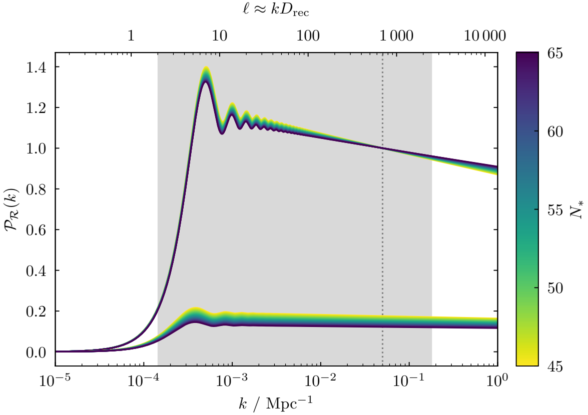
Primordial vs present-day curvature



Kinetically dominated power spectra

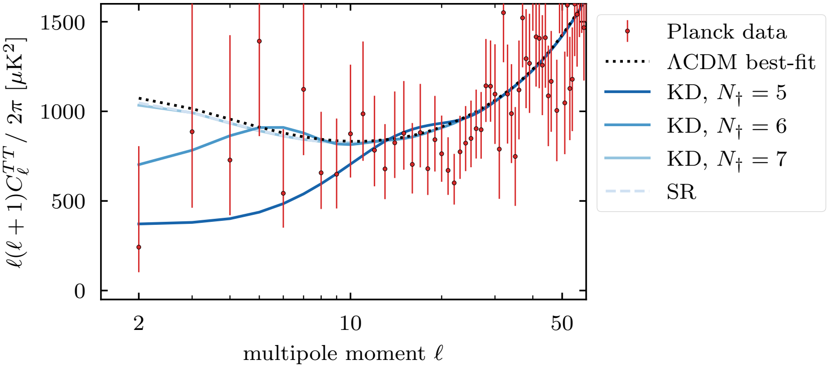
primordial power spectrum

- Eternal inflating models have $\mathcal{P} = A_s(k/k_*)^{n_s-1}$
- Finite amount of inflation introduces cutoff and oscillations.
- Hergt et al 2018 (arXiv:1809.07737)



Kinetically dominated power spectra

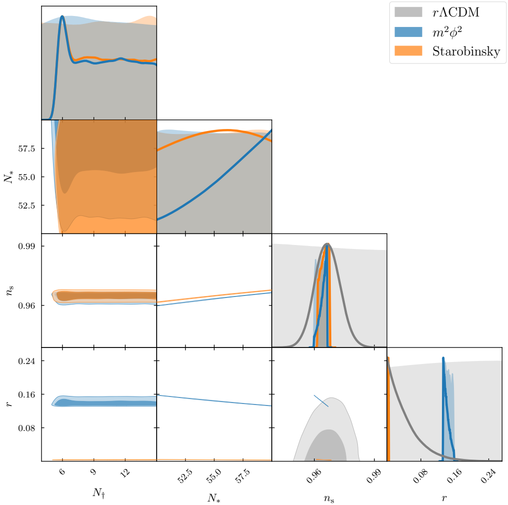
CMB power spectrum

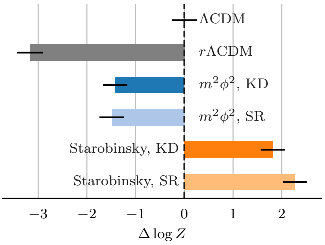


- Flat case can reproduce suppression of power
- Oscillations have wrong location to explain $\ell \sim 30$ feature.

Kinetically dominated power spectra

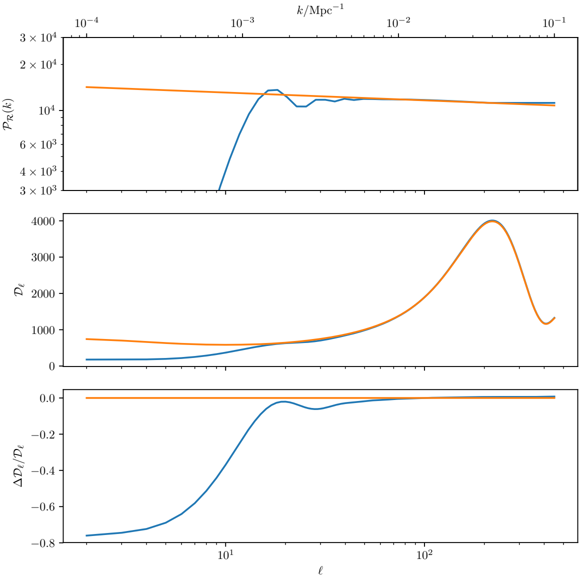
Parameter estimation and model comparison





Power spectra with primordial curvature.

- Primordial curvature is able to move oscillation to correct location
- Preliminary results: Need full constraint pipeline.
- Discretised PPS in closed case.



Computing the primordial power spectrum

- Comoving curvature perturbation $\mathcal{R}_\mathrm{k}$, Power spectrum $\mathcal{P}(k) \propto |\mathcal{R}_\mathrm{k}|^2$
- Mukhanov-Sasaki equation:

$$0 = {\mathcal{R}_\mathrm{k}}'' + 2\frac{z'}{z}{\mathcal{R}_\mathrm{k}}' + \mathrm{k}^2\mathcal{R}_\mathrm{k}$$

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$$0 = {\mathcal{R}_\mathsf{k}}'' + \left[2\frac{z'}{z} + 2K\mathcal{E}\frac{\mathcal{H} - \frac{z'}{z}}{\mathsf{k}^2 + K\mathcal{E}}\right]{\mathcal{R}_\mathsf{k}}' + \left[\mathsf{k}^2 + K\frac{\mathsf{k}^2 - K\mathcal{E} - \frac{2\mathsf{k}^2}{\mathcal{H}}\frac{z'}{z}}{\mathsf{k}^2 + K\mathcal{E}}\right]\mathcal{R}_\mathsf{k}$$

$$\mathcal{E} = \mathcal{H}/\dot{\phi}^2$$

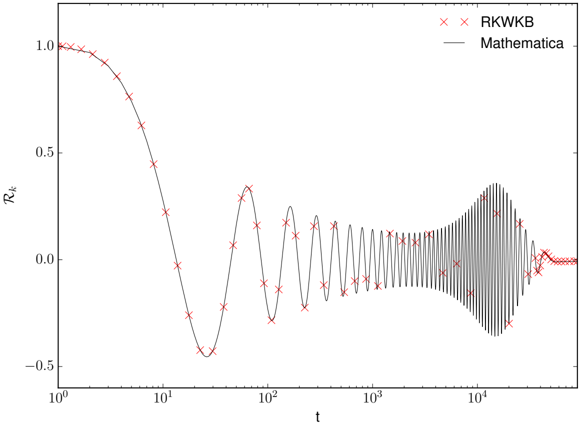
$$\begin{aligned}
\mathsf{k}^2 &= k(k+2) - 3 & \qquad K &> 0 \\
\mathsf{k}^2 &= k^2 & \qquad K &= 0 \\
\mathsf{k}^2 &= k^2 + 3 & \qquad K &< 0
\end{aligned}$$

Problems to be overcome

- How to set initial condition on $\mathcal{R}_\mathrm{k}$ for low-k modes?
  - Usually do so by invoking Bunch-Davies vacuum, which is tied to eternal inflation
  - Both when and how they are set becomes important
- Computing the MS equation numerically becomes bottleneck in computation: Need faster integrators.
- Must take care with properly discretised spectra

Runge-Kutta-Wentzel-Kramers-Brillioun methods

- Rapid solving of equations with oscillatory solutions.
- Runge-Kutta based on Taylor series
- Replace polynomials with oscillating solutions (e.g. Airy, Bessel or WKB).



Further reading

- Kinetic initial conditions: Handley et al. 2015 (arXiv:1401.2253)
- Quantum Kinetic Dominance: Handley et al. 2016 (arXiv:1607.04148)
- Kinetic dominance: Hergt et al. 2018 (arXiv:1809.07185)
- Kinetic constraints: Hergt et al. 2018 (arXiv:1809.07737)
- Mukhanov-Sasaki evolution: Haddadin et al.. 2018 (arXiv:1809.11095)