# PolyChord & the future of nested sampling

## Sampling, Parameter Estimation and Bayesian Model Comparison

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Astrophysics Group
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University of Cambridge

March 8, 2016

Parameter estimation & model comparison

Metropolis Hastings

Nested Sampling

PolyChord

Applications

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- Unlike traditional methods, it does not sample from the posterior directly.
- Instead it gradually compresses the prior onto the posterior.
- In doing so, it circumvents many issues (dimensionality, topology, geometry) that beset normal approaches.
- Similar to simulated annealing, but automatically picks the "correct" annealing schedule.

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- Markov-Chain Monte-Carlo (MCMC) can solve the first of these (kind of)
- Nested sampling (NS) promises to solve both simultaneously.

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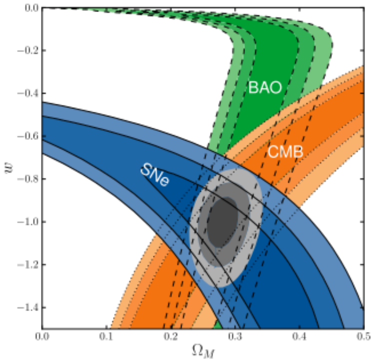

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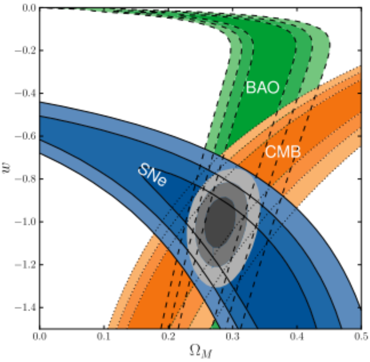

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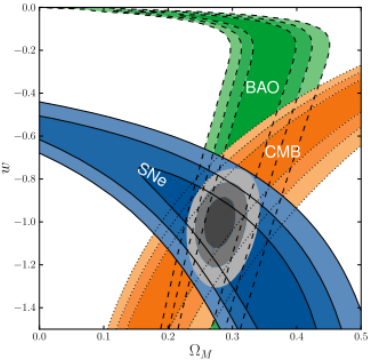

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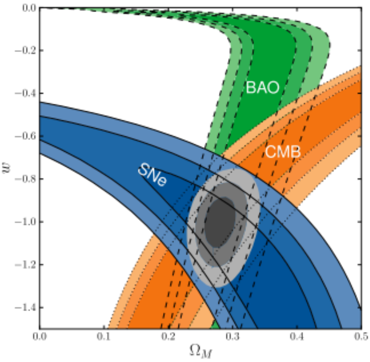


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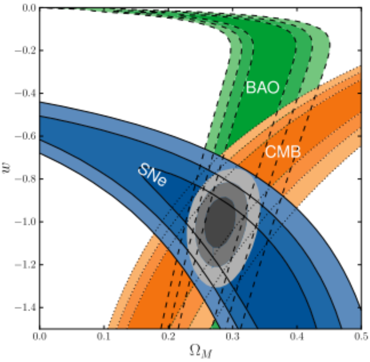


- Describing an *N*-dimensional posterior fully is impossible.
- *Sampling* the posterior is an excellent compression scheme.

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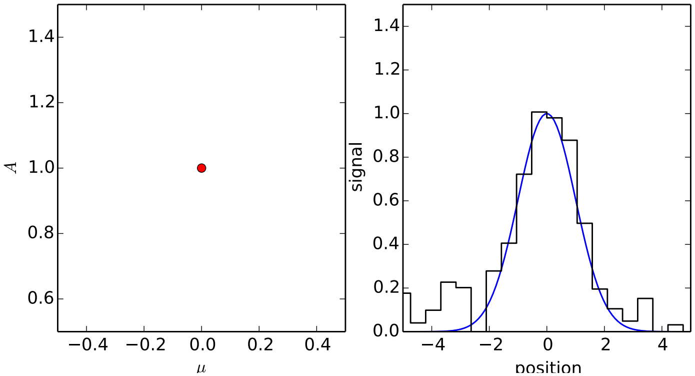

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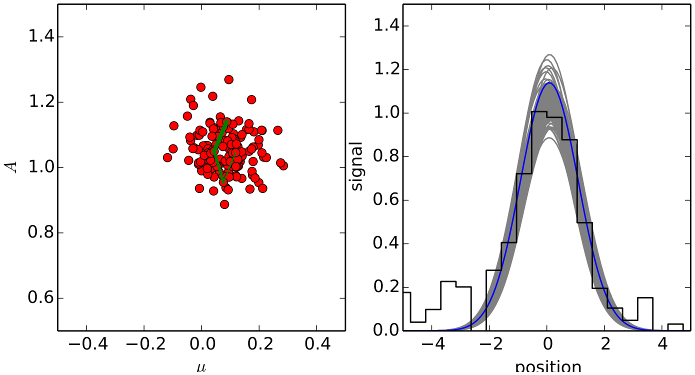

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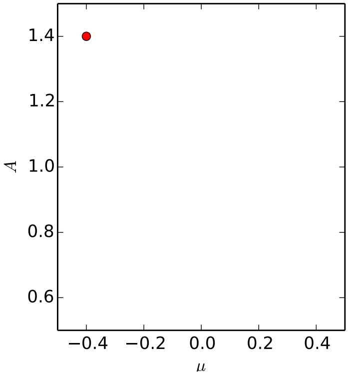


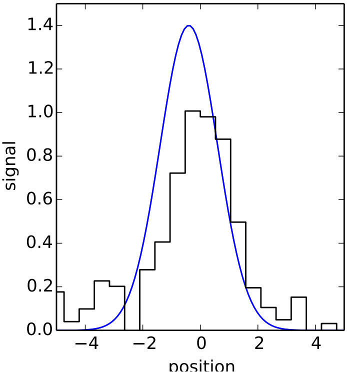

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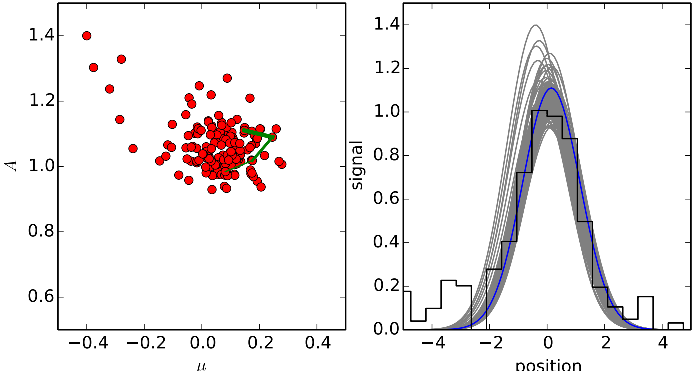

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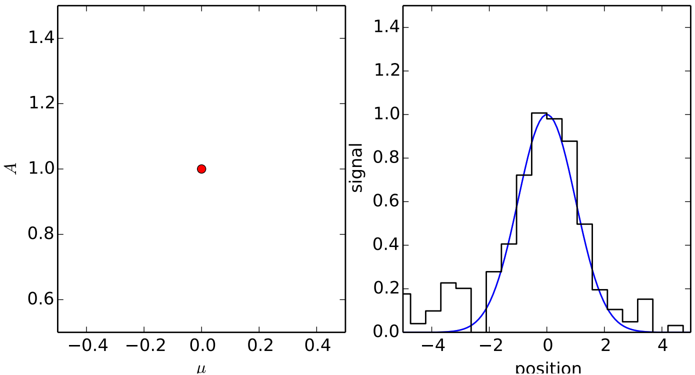

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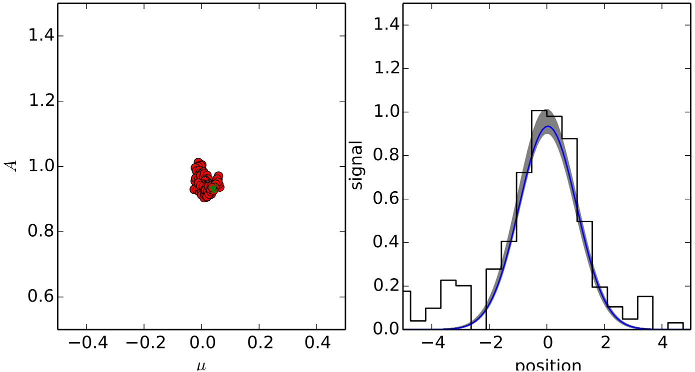

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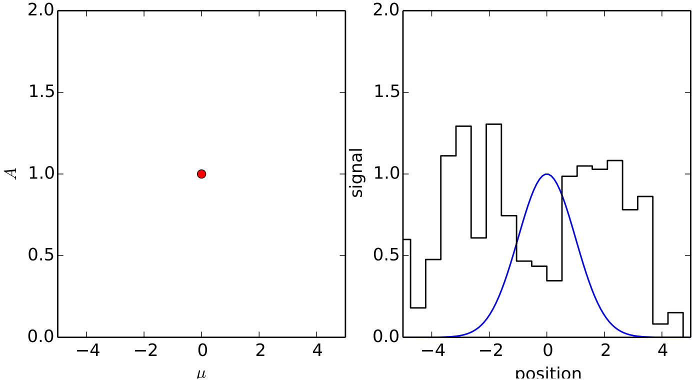

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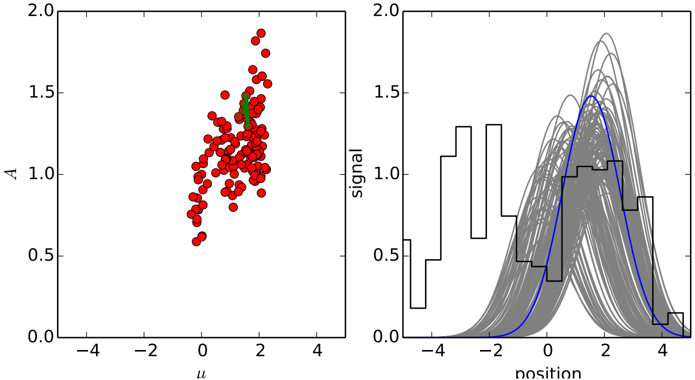

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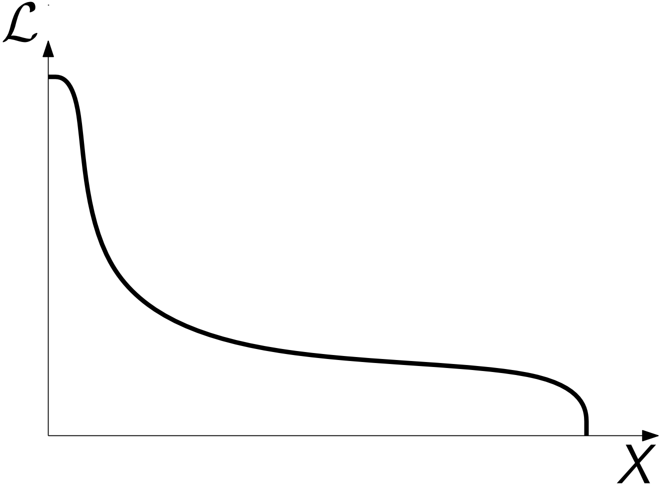
## Phase transitions



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  - Suffers from similar issues to MCMC, especially phase transitions

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Requires one to be able to sample from the prior, subject to a *hard likelihood constraint*.

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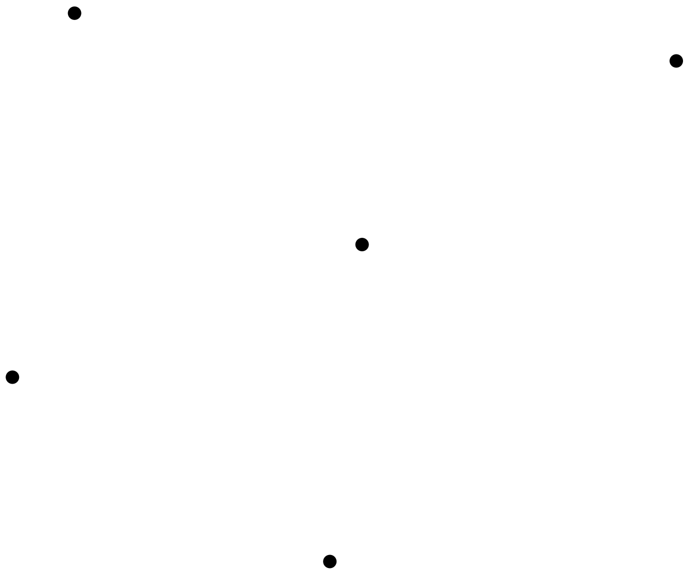

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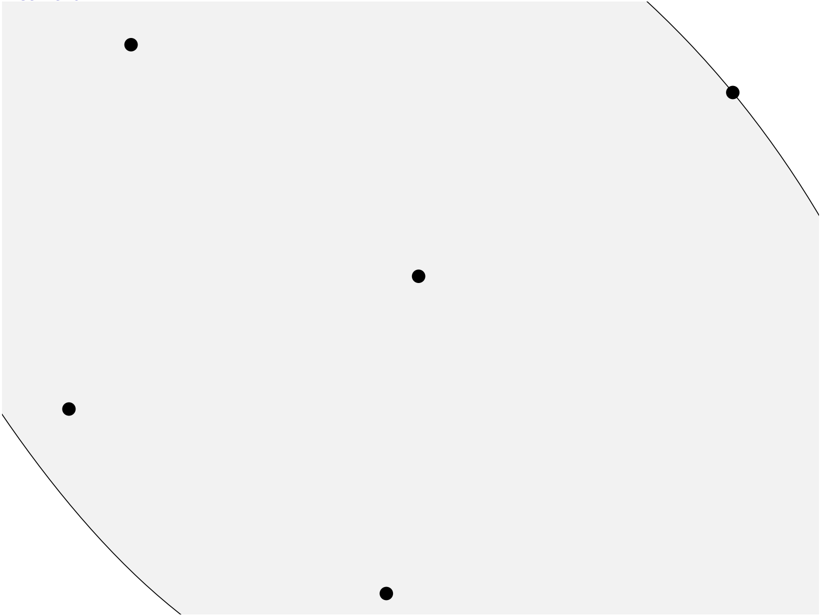

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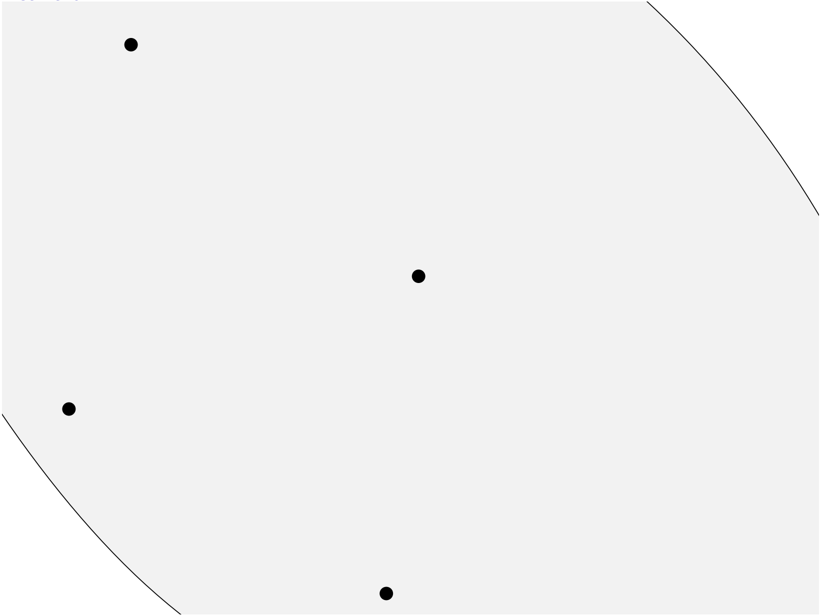

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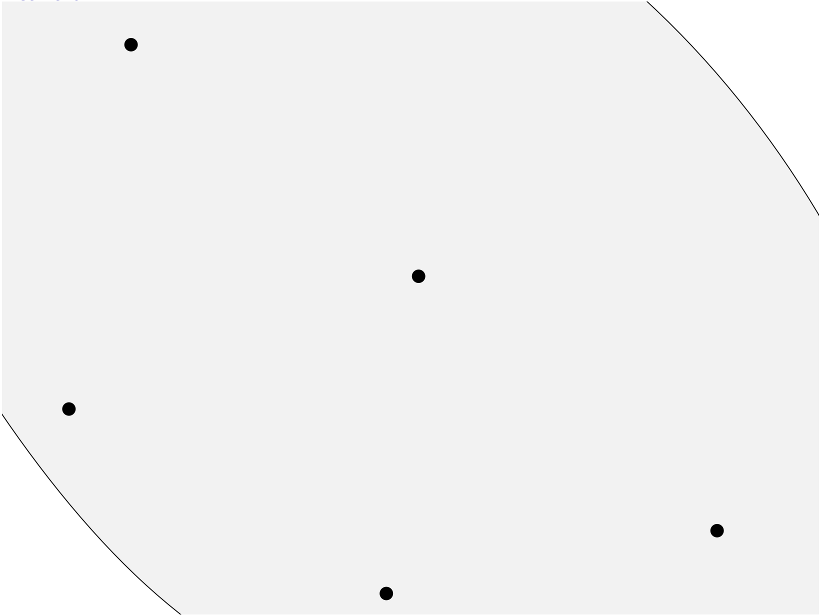

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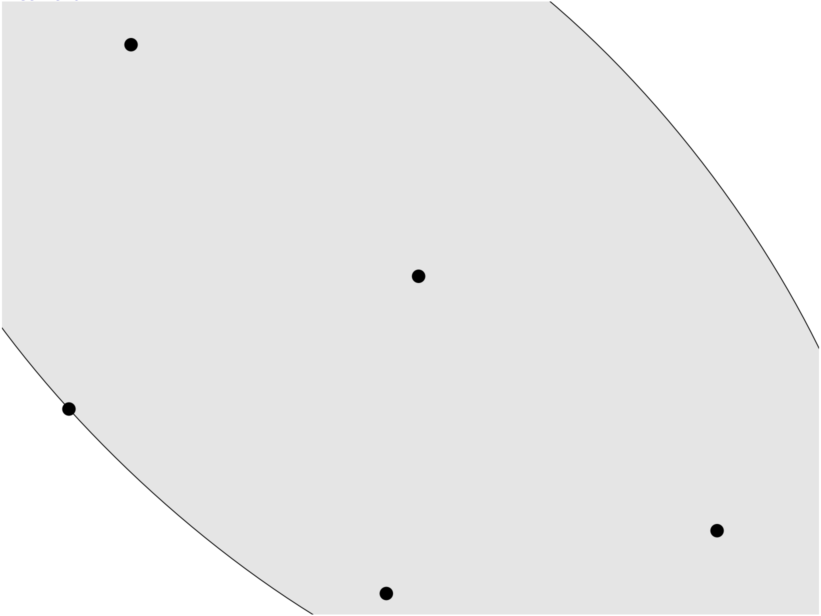

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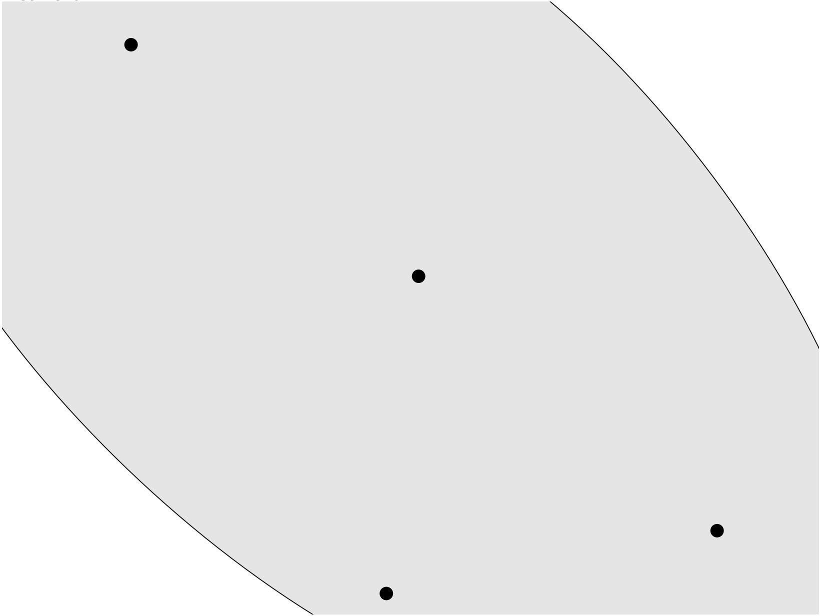

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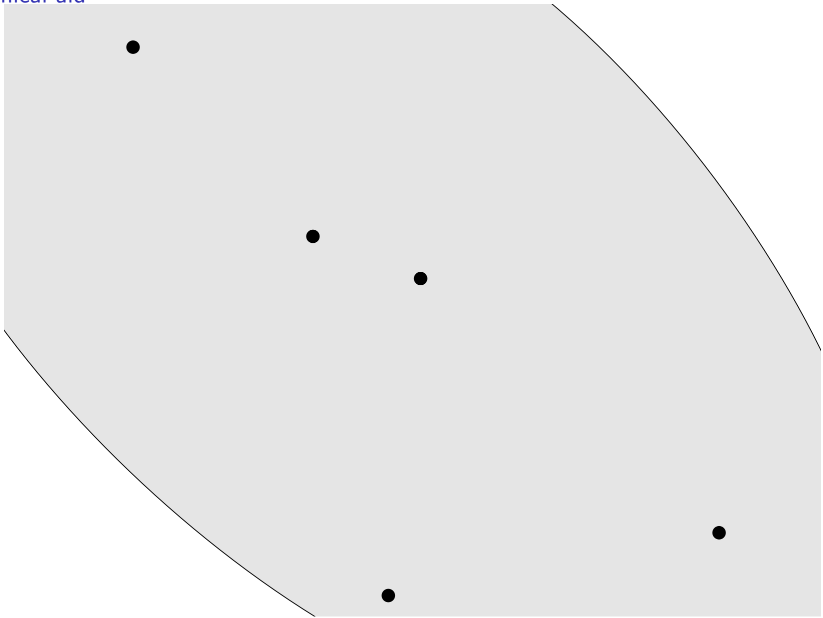

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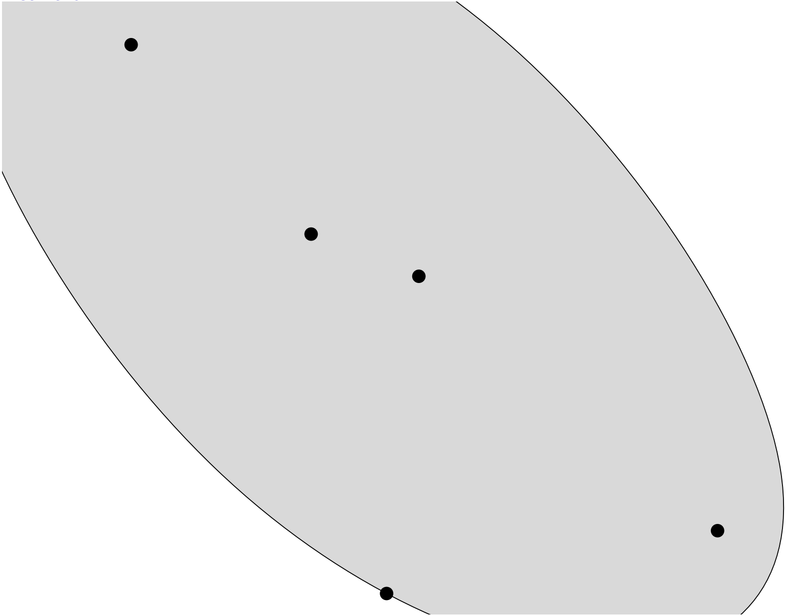

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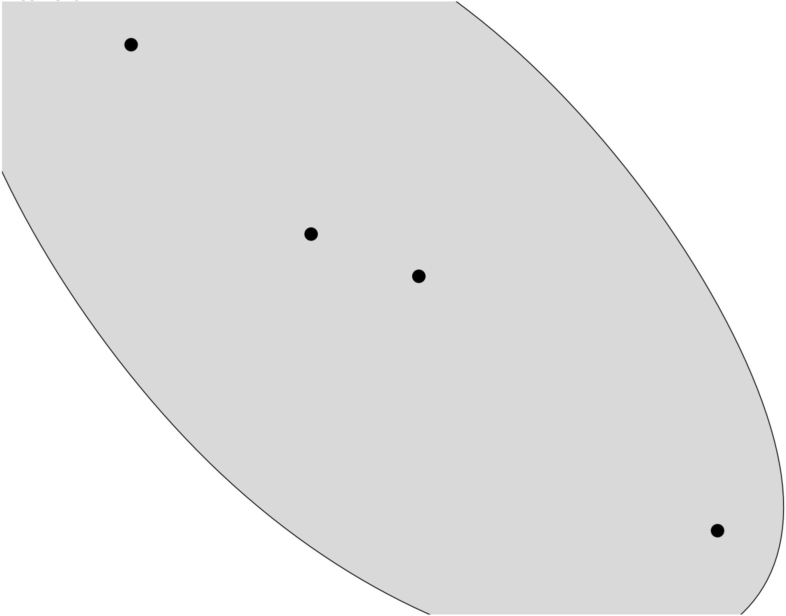

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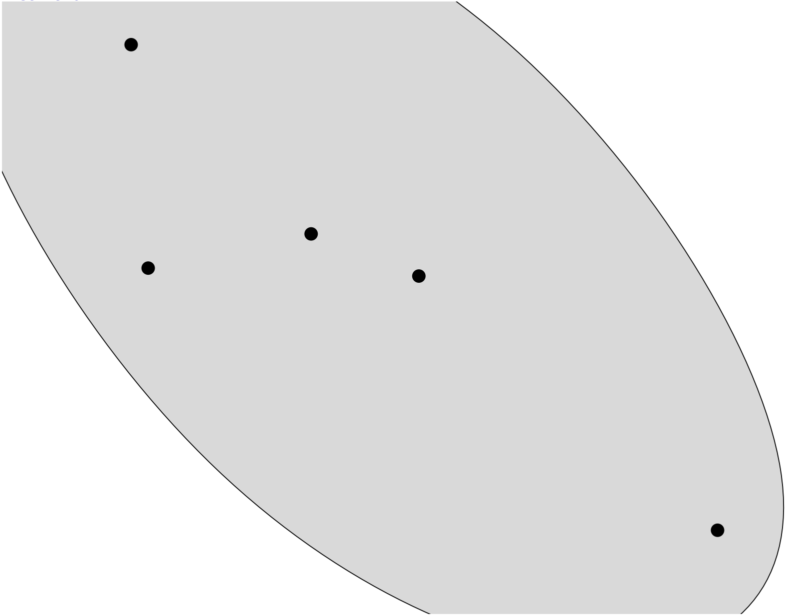

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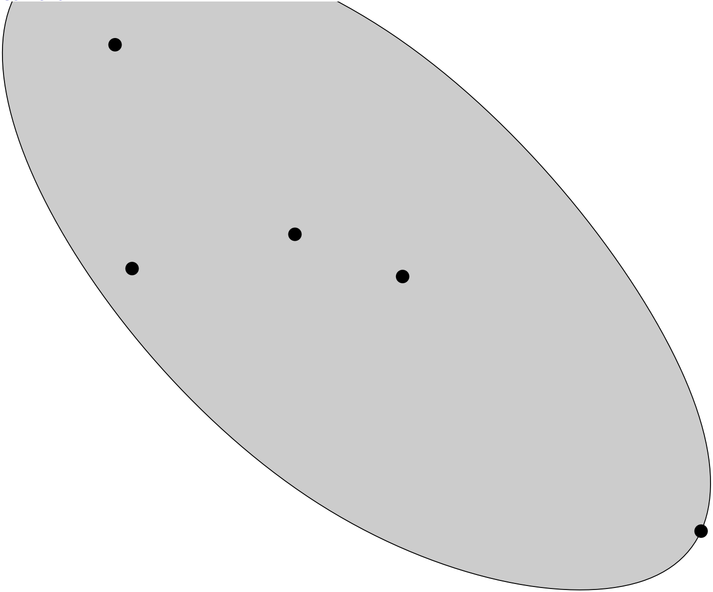

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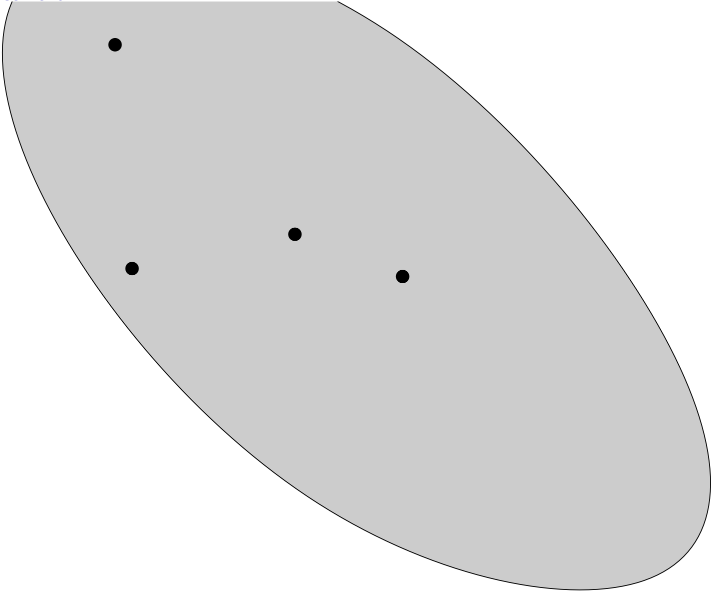

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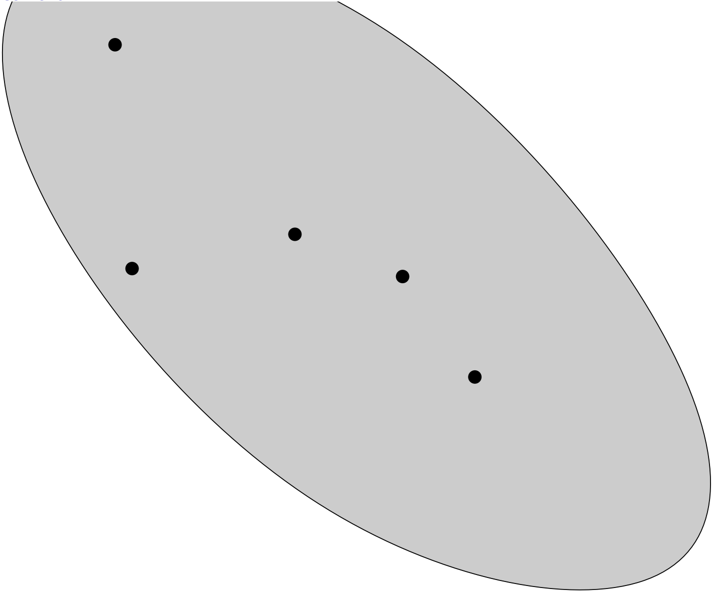

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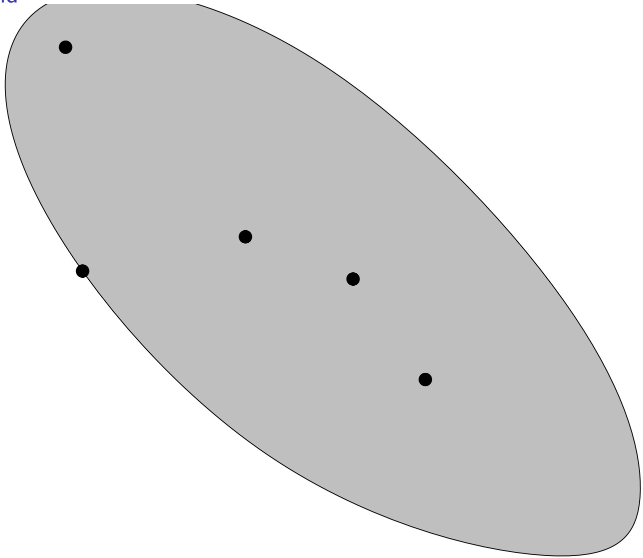

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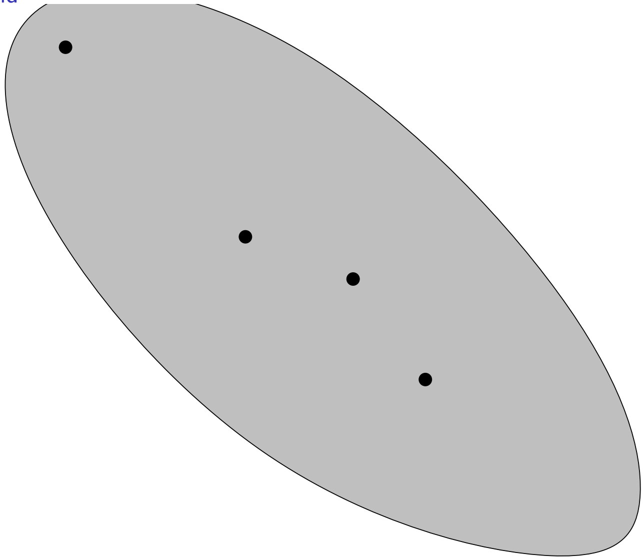

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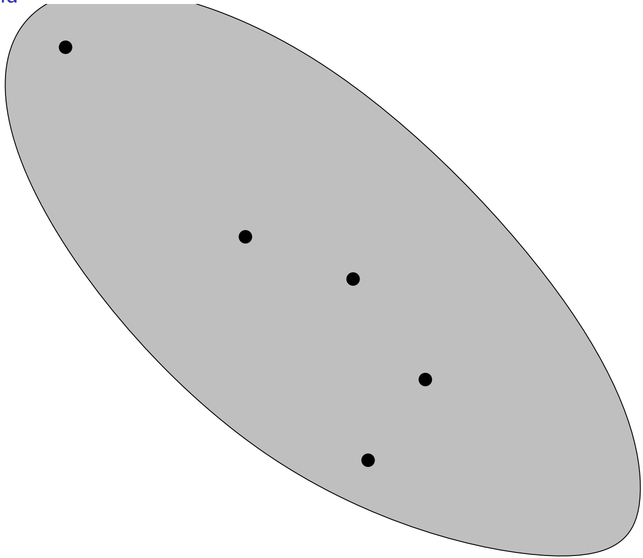

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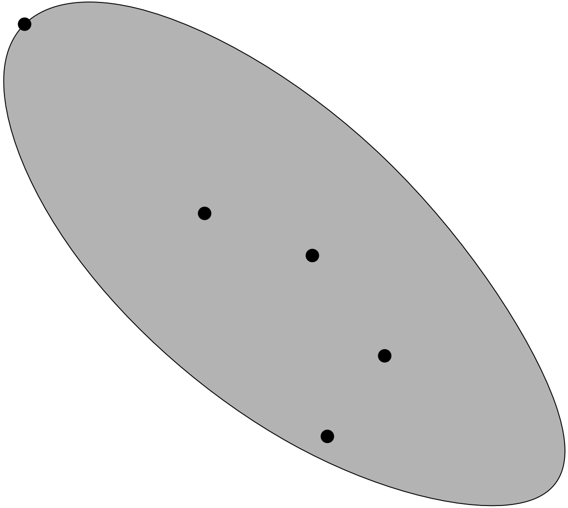

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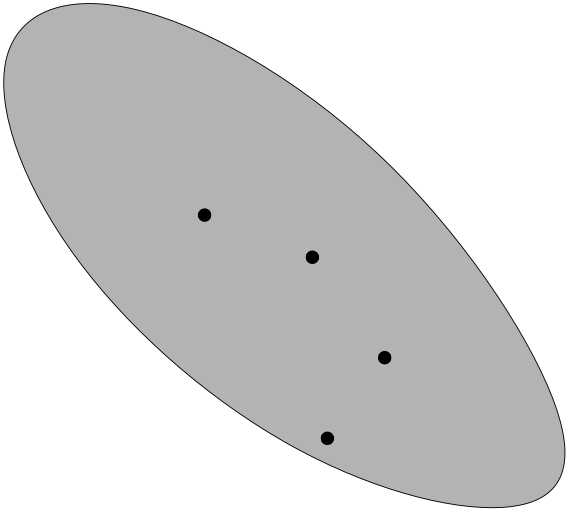

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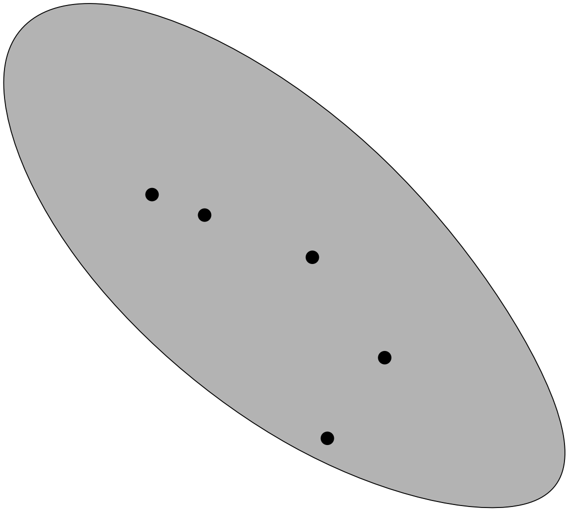

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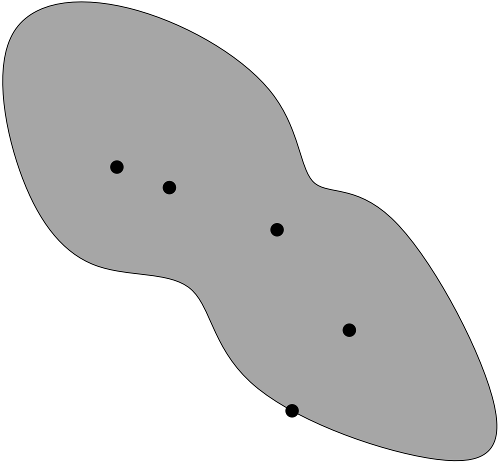

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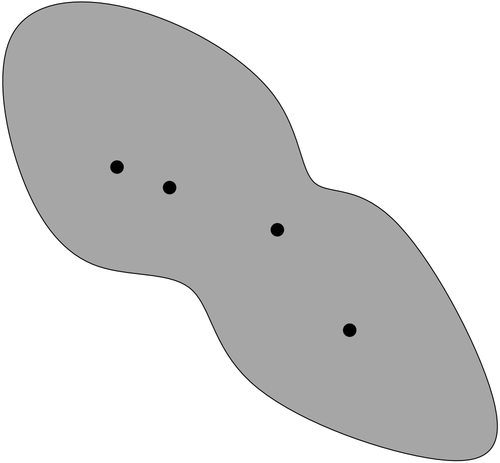

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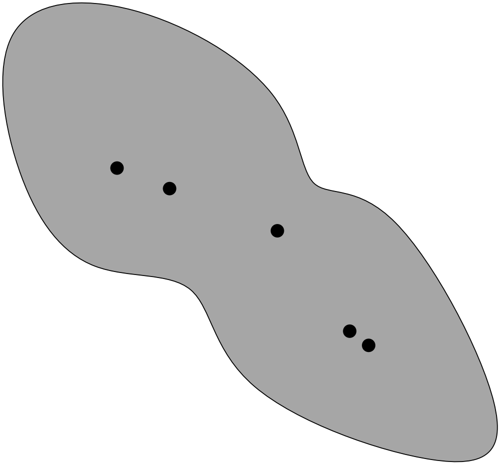

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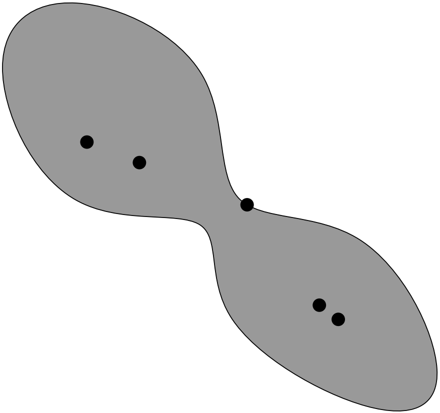

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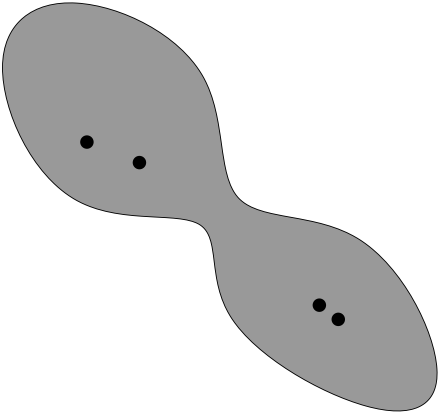

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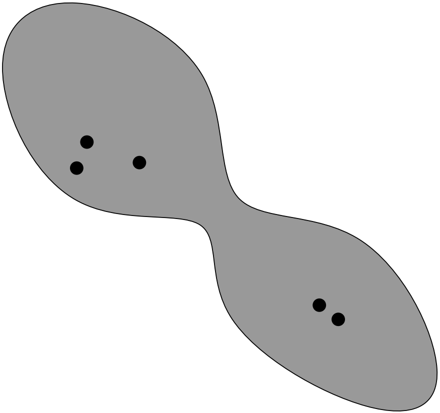

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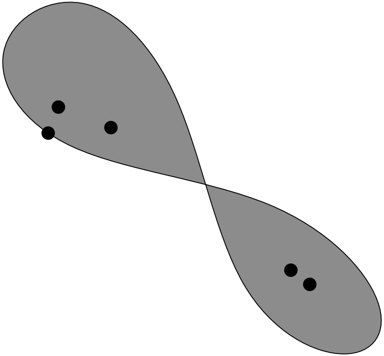

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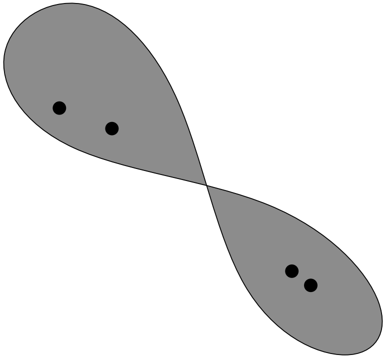

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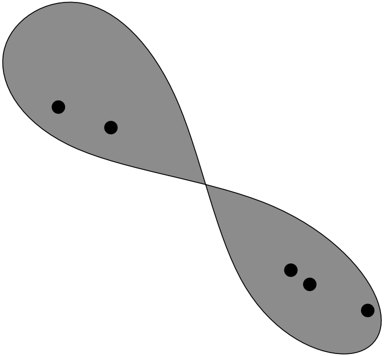

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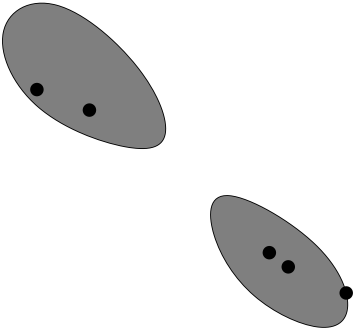

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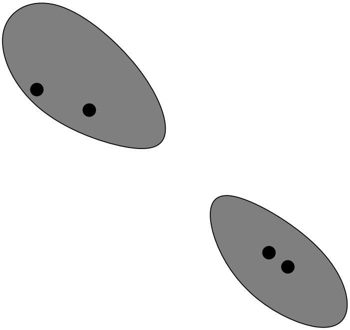

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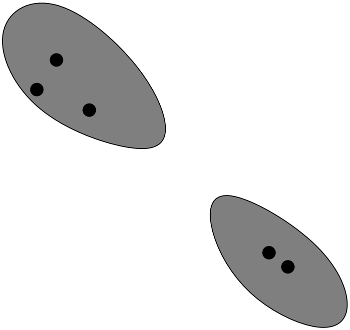

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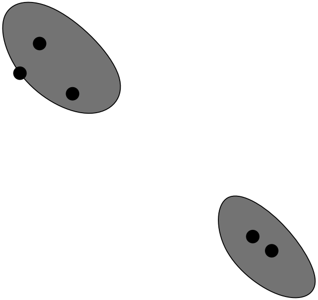

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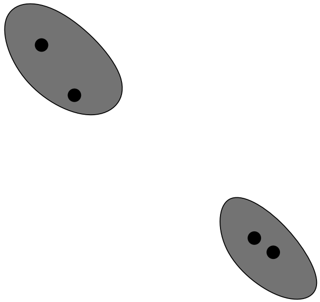

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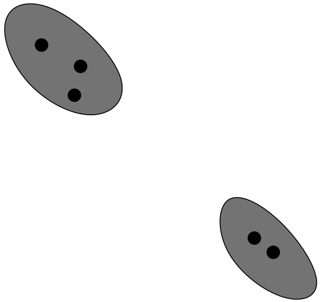

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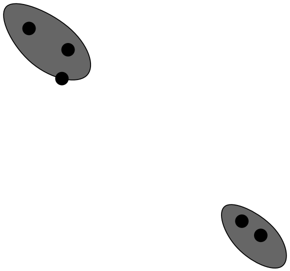

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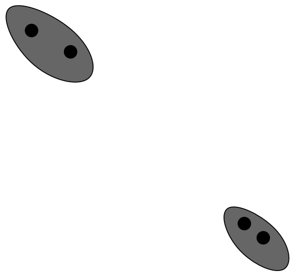

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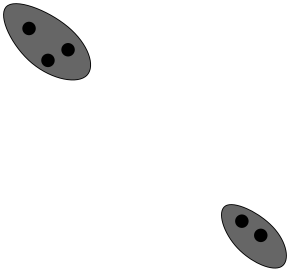

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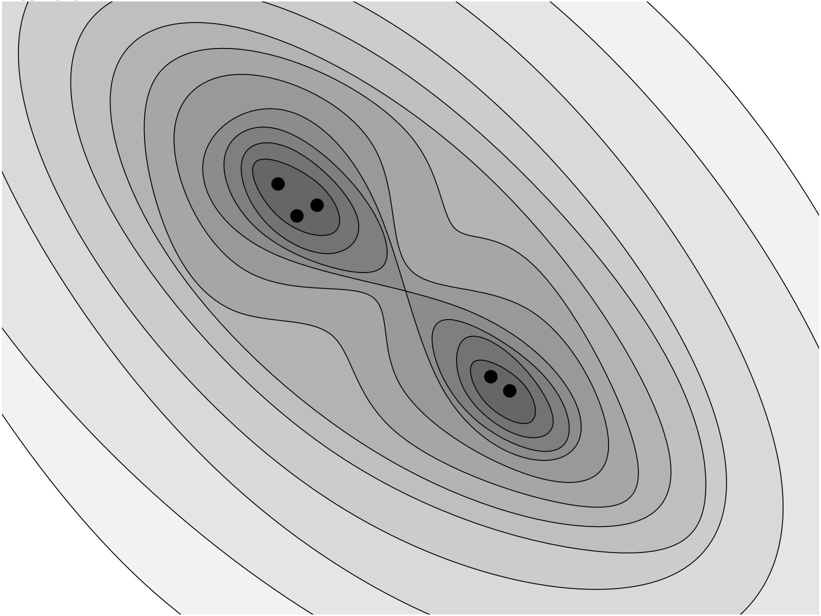

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- i.e. the fraction of the prior which the iso-likelihood contour $\mathcal{L}$ encloses.

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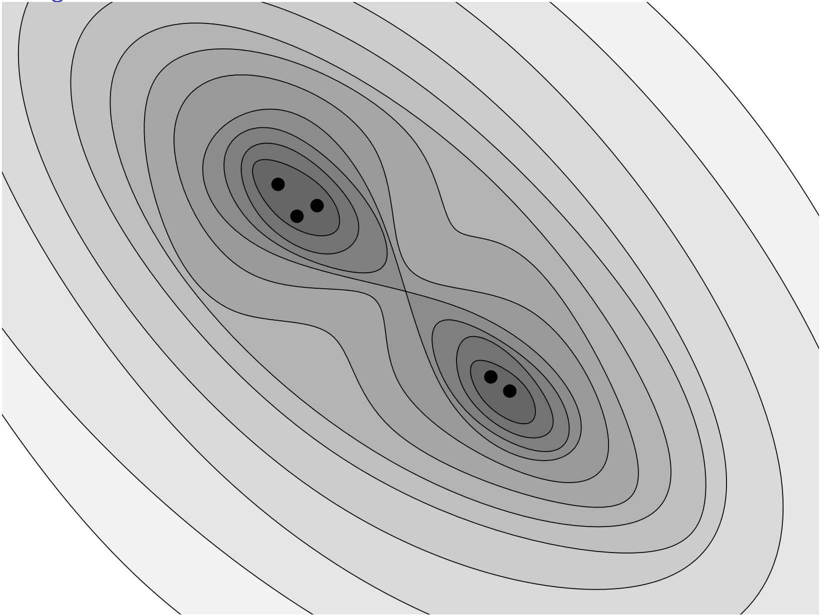

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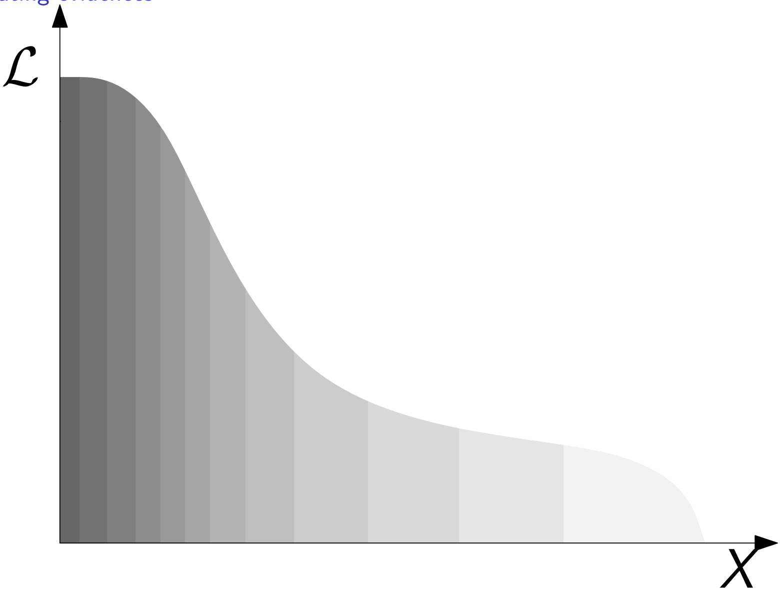

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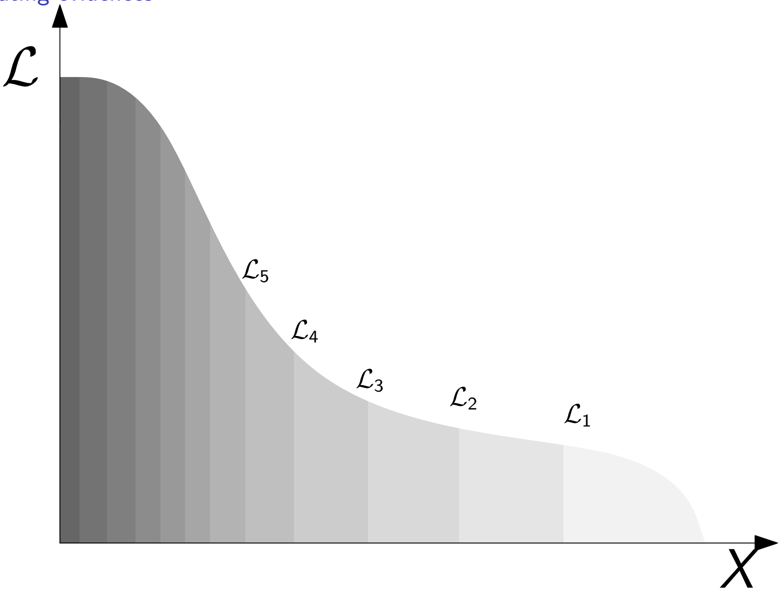

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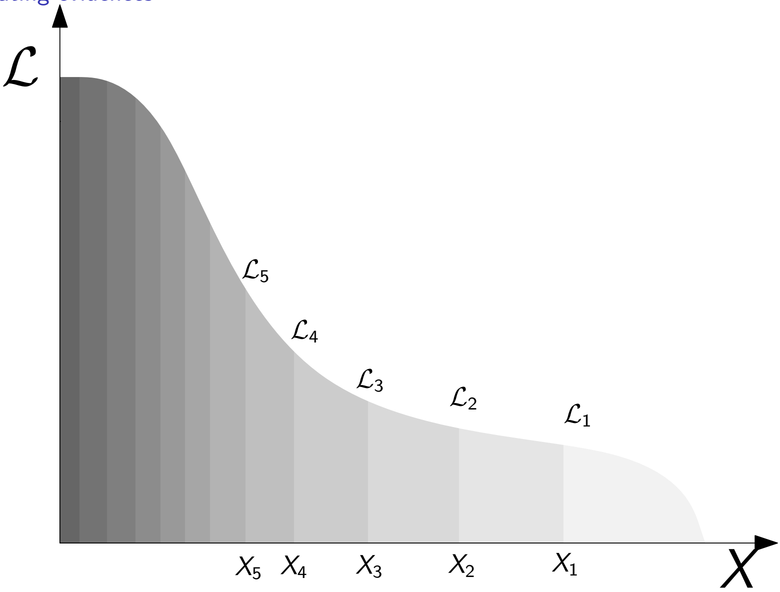

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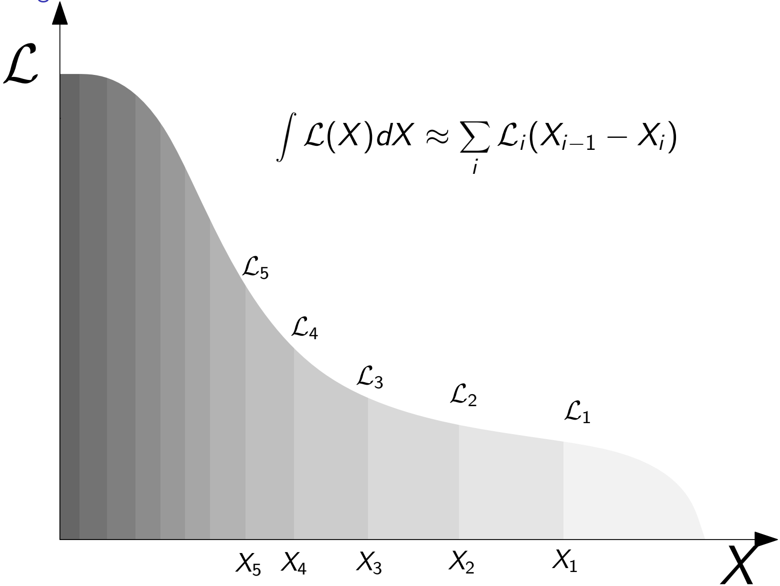

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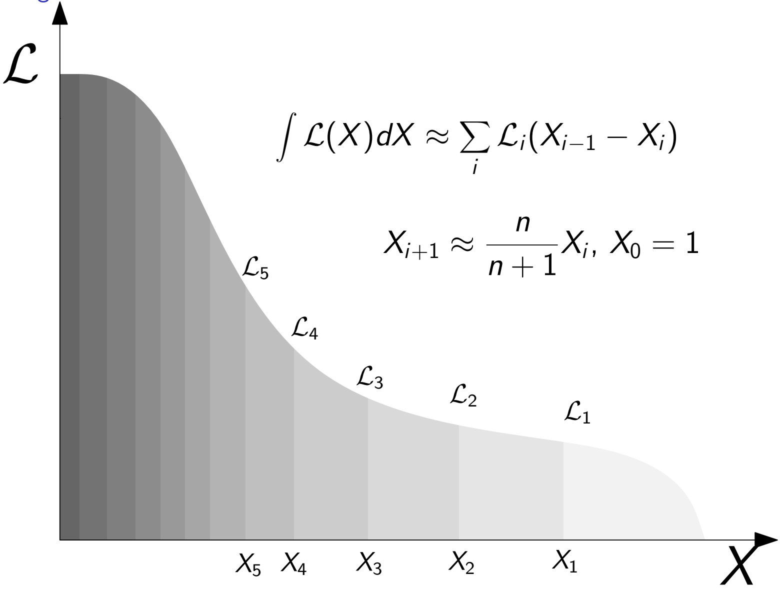


$$\int \mathcal{L}(X)dX \approx \sum_i \mathcal{L}_i(X_{i-1} - X_i)$$

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- The set of dead points are posterior samples with an appropriate weighting factor

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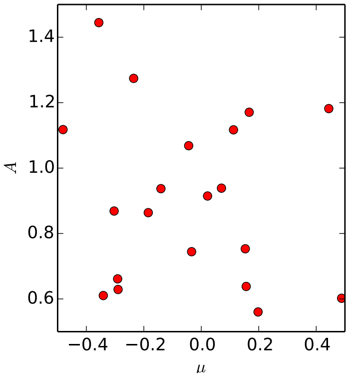


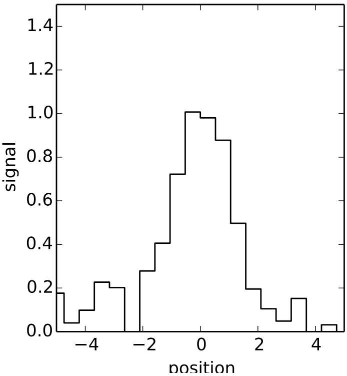

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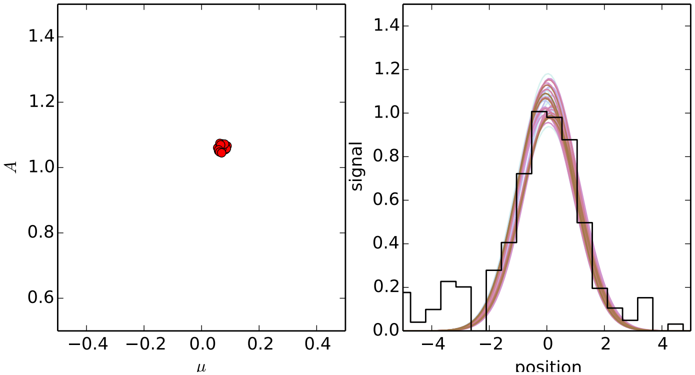

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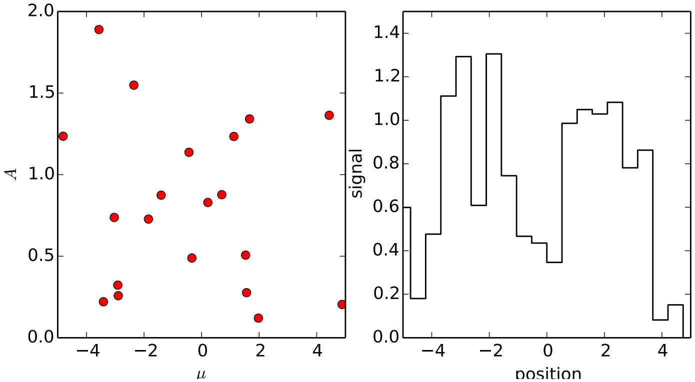

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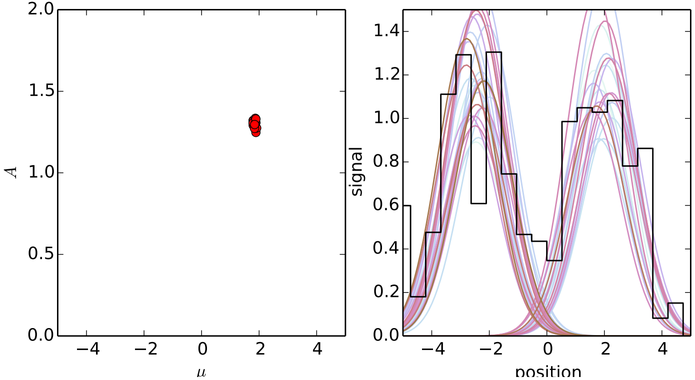

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- Most of the work in NS to date has been in attempting to implement a hard-edged sampler in the NS meta-algorithm.

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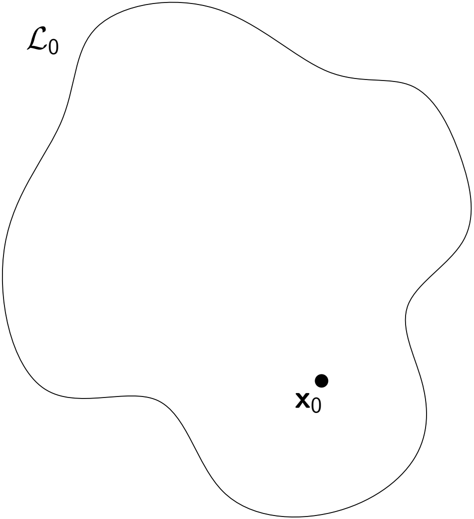

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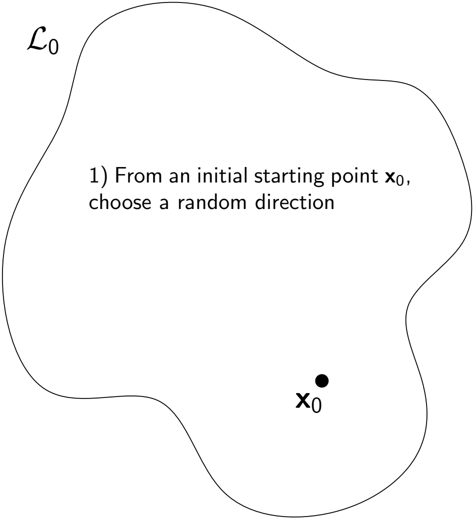

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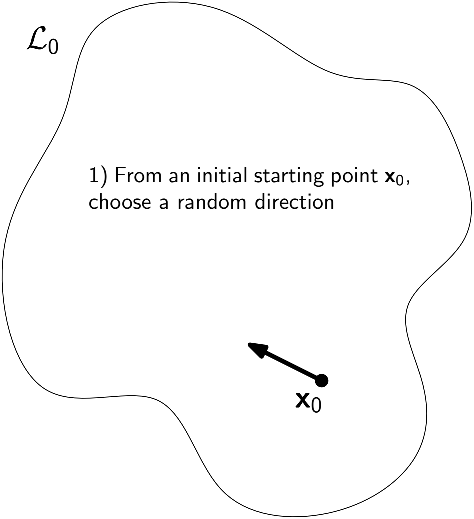

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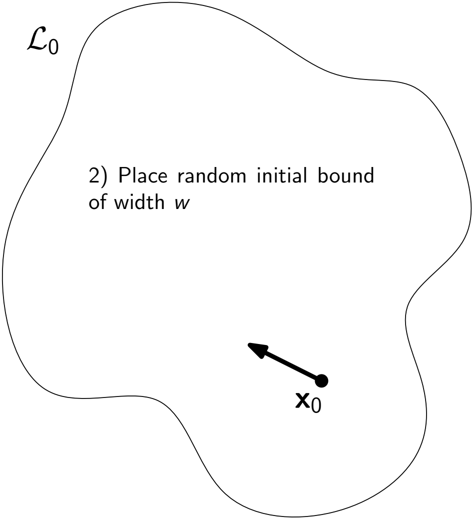

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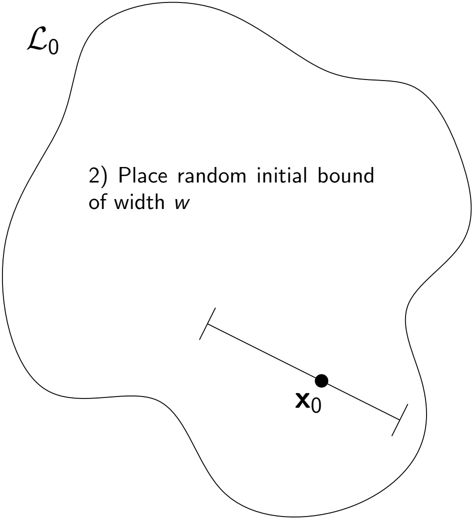

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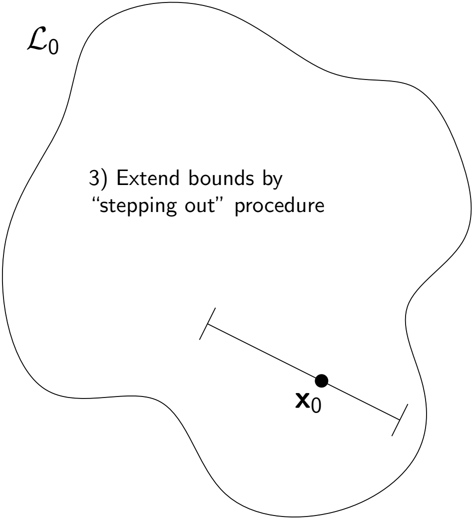

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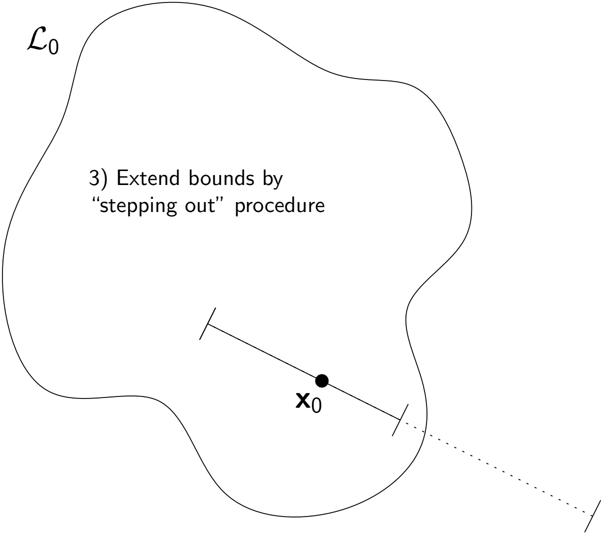

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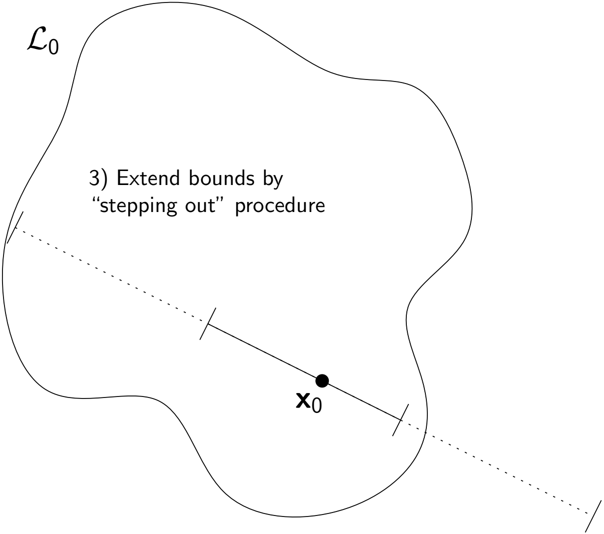

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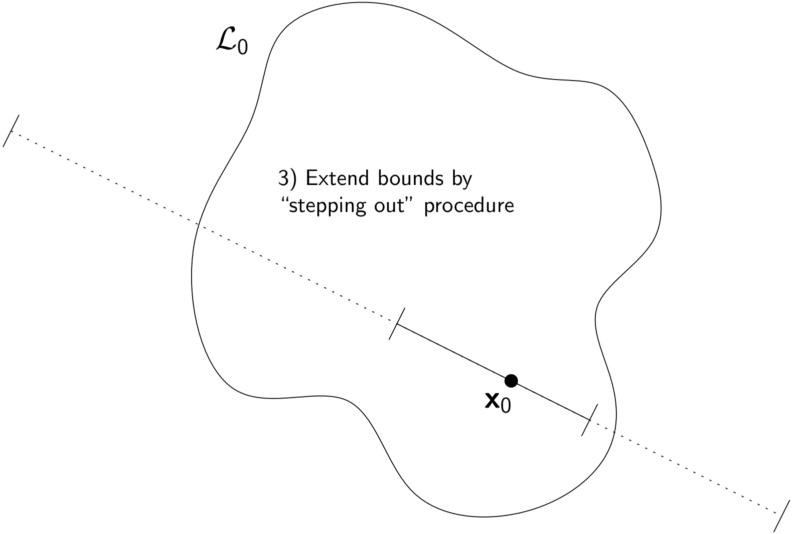

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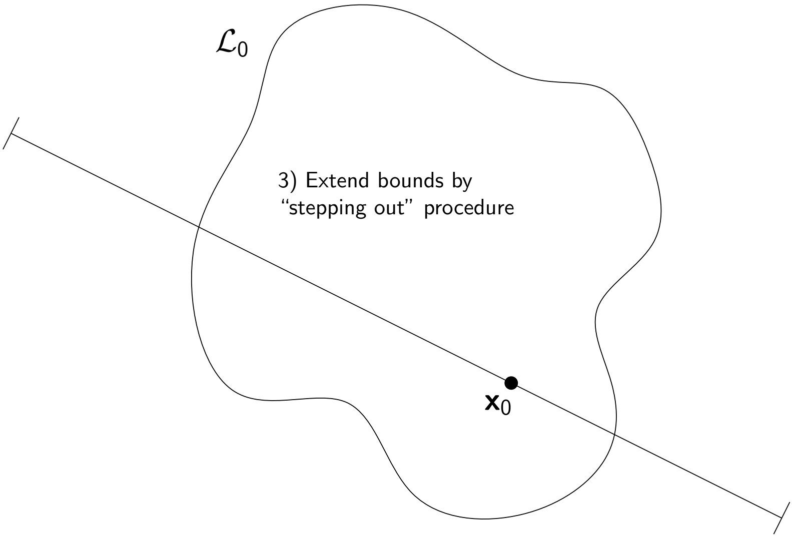

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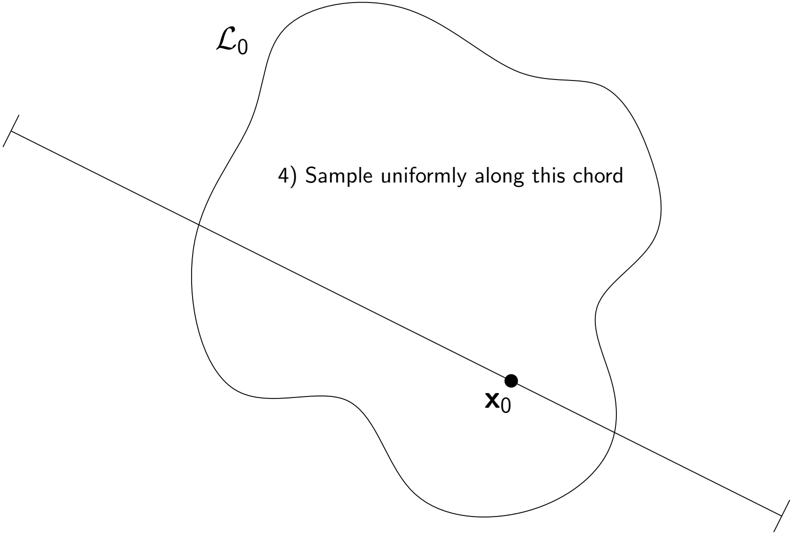

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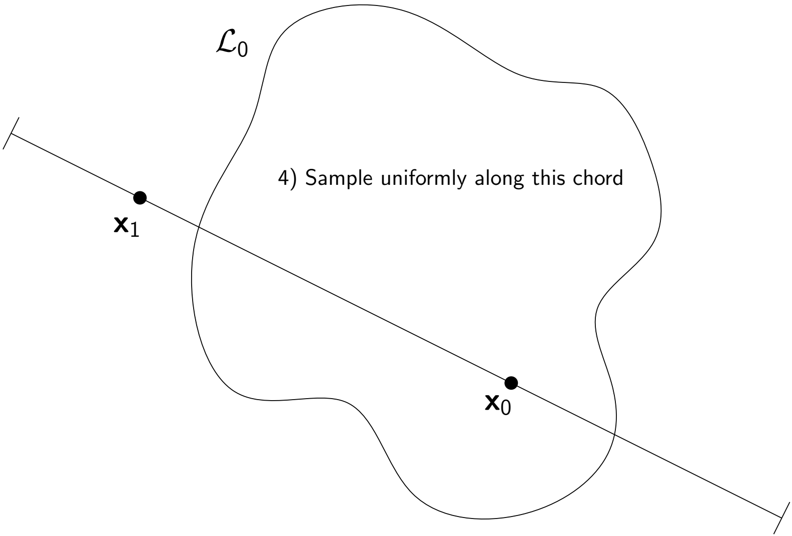

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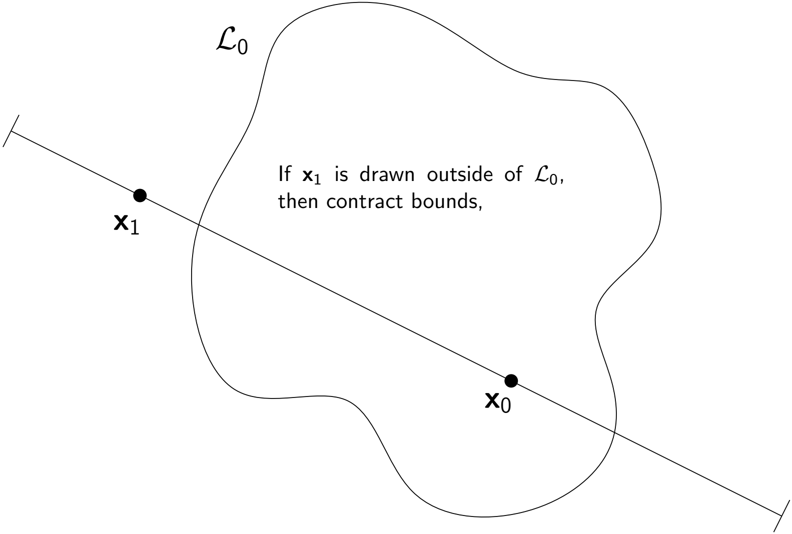

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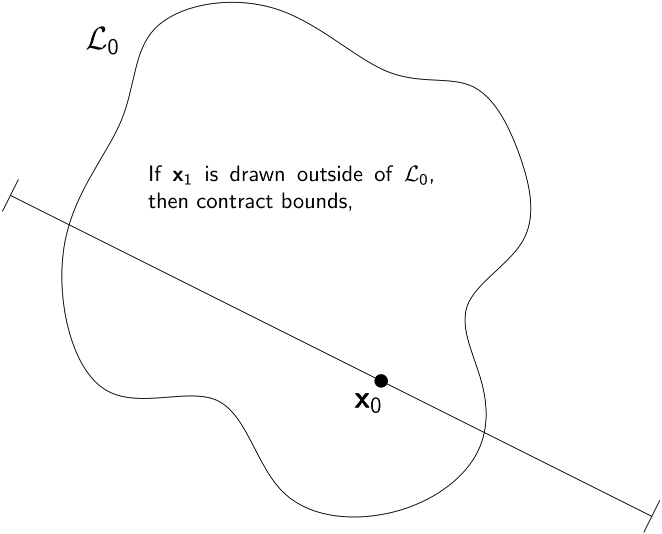

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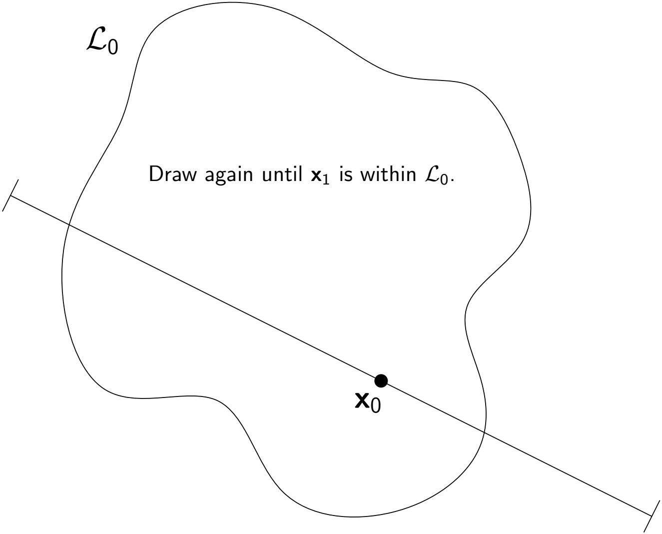

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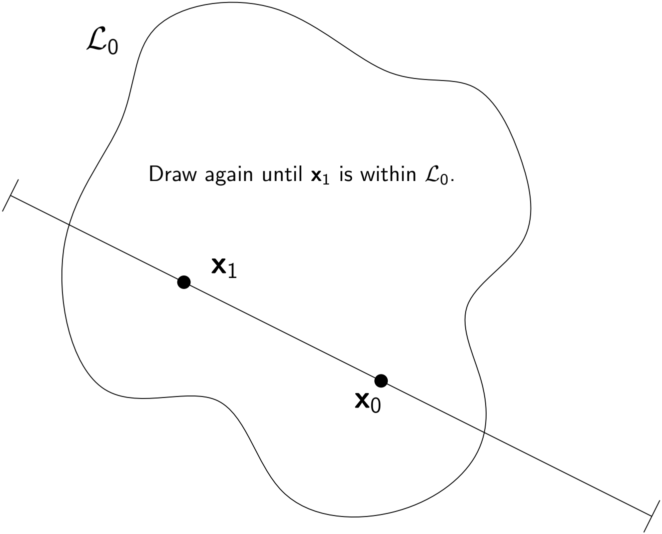

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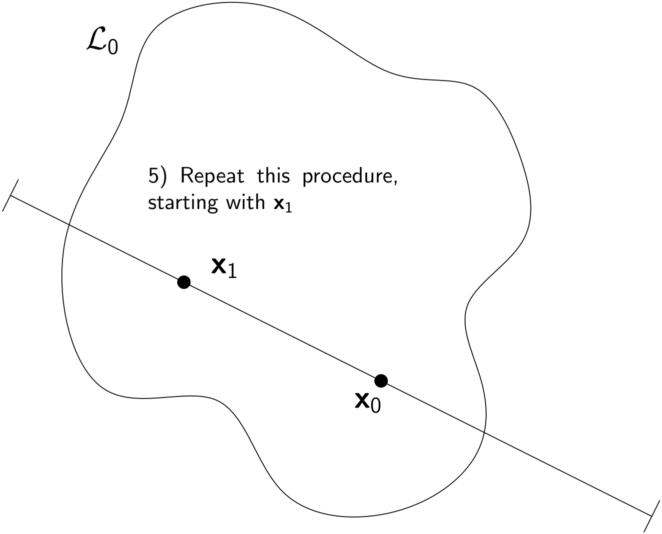

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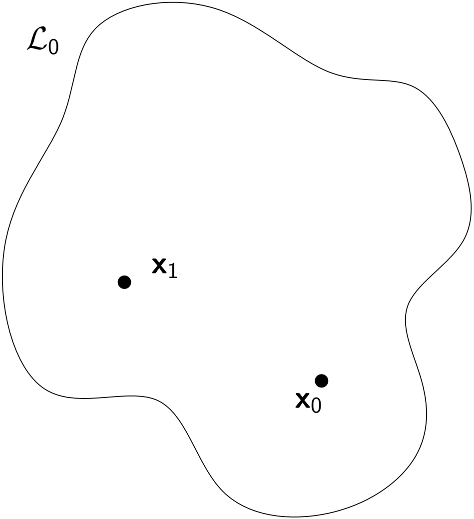

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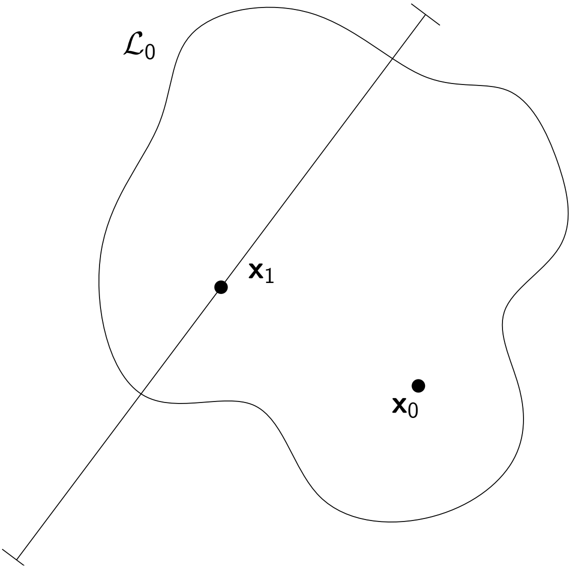

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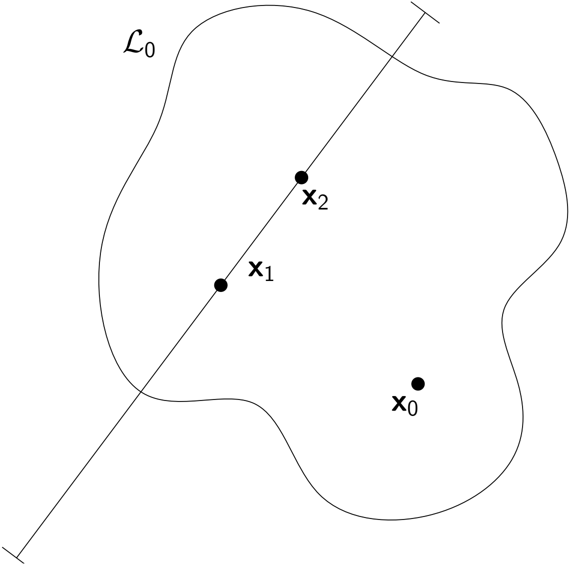

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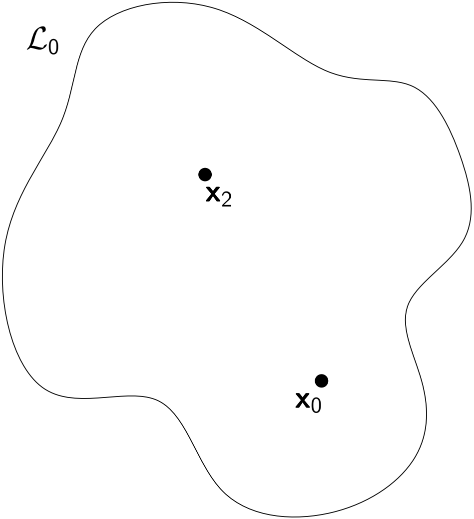

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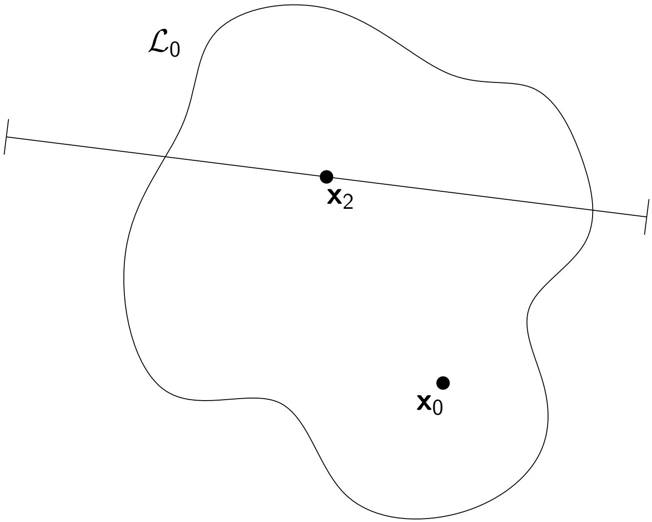

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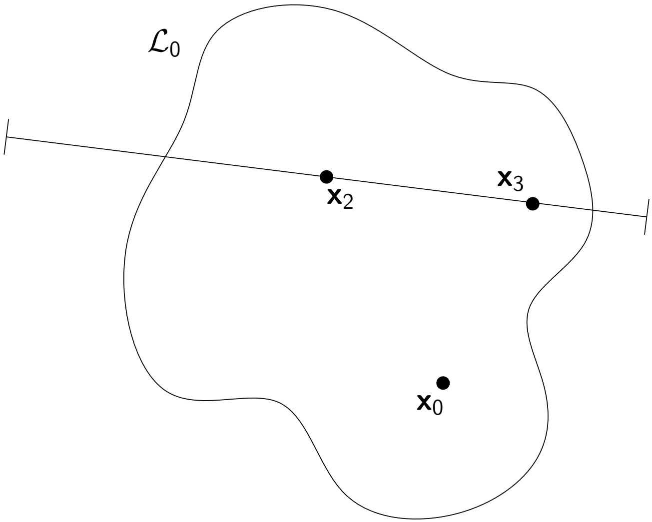

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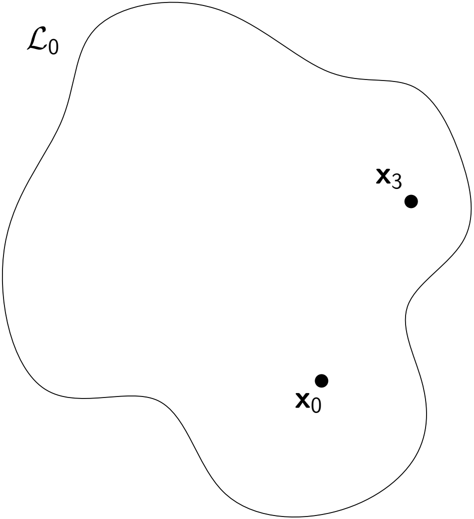

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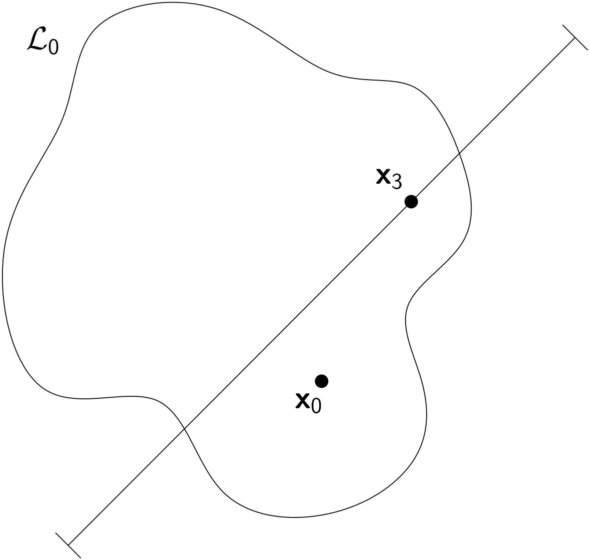

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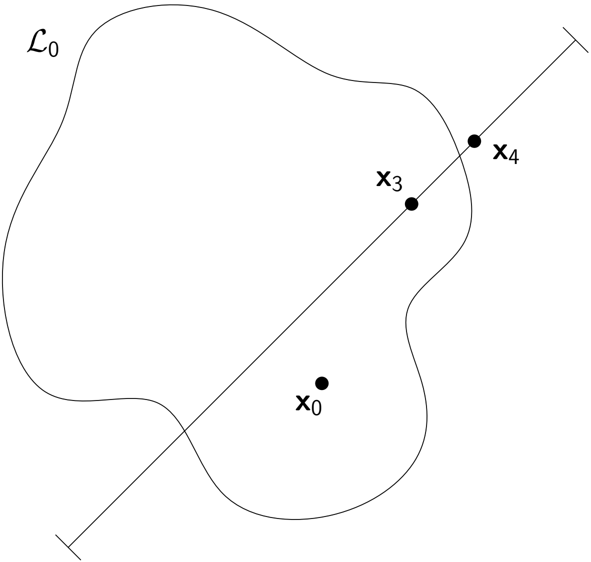

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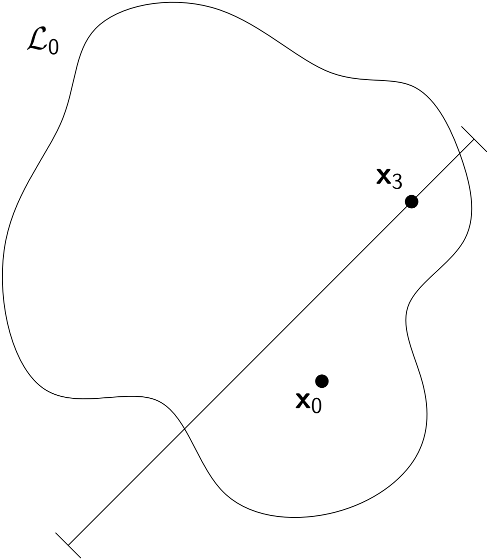

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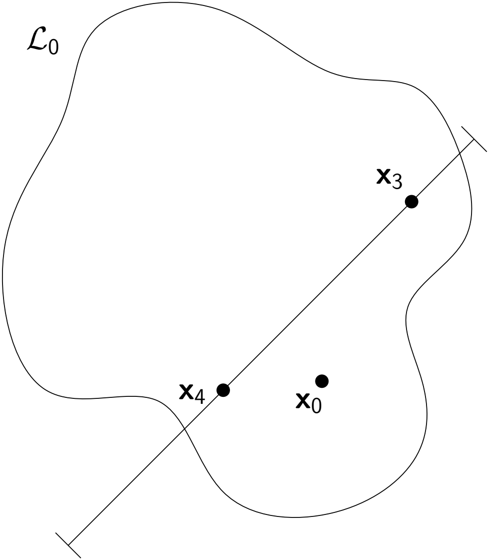

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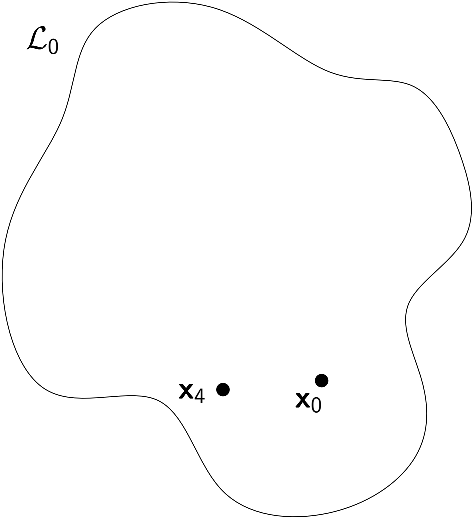

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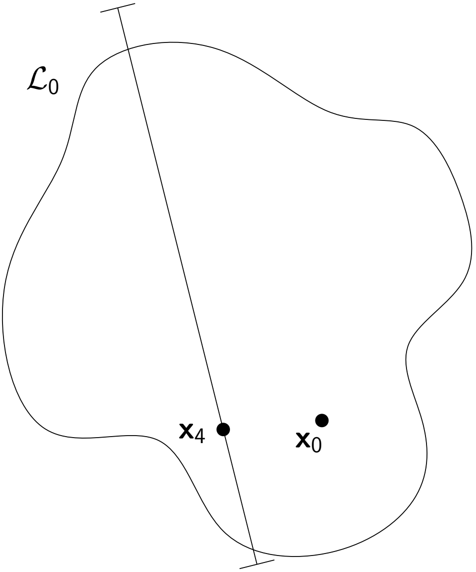

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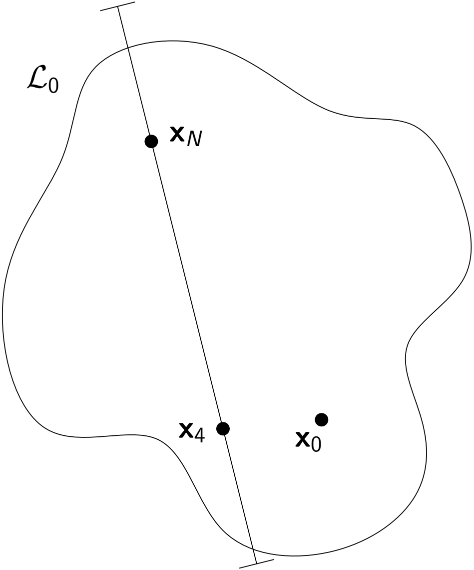

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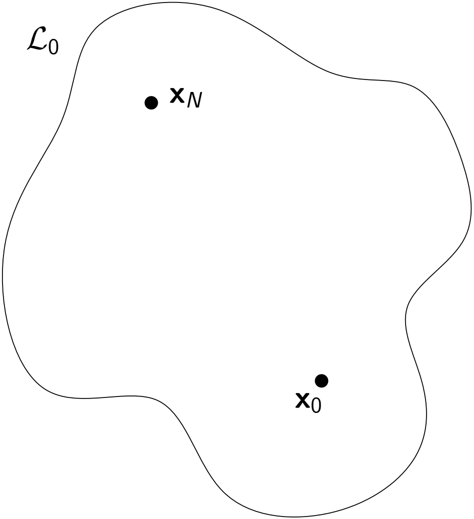

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- Need $N$ reasonably large $\sim \mathcal{O}(n_{\mathrm{dims}})$ so that $x_N$ is de-correlated from $x_1$.

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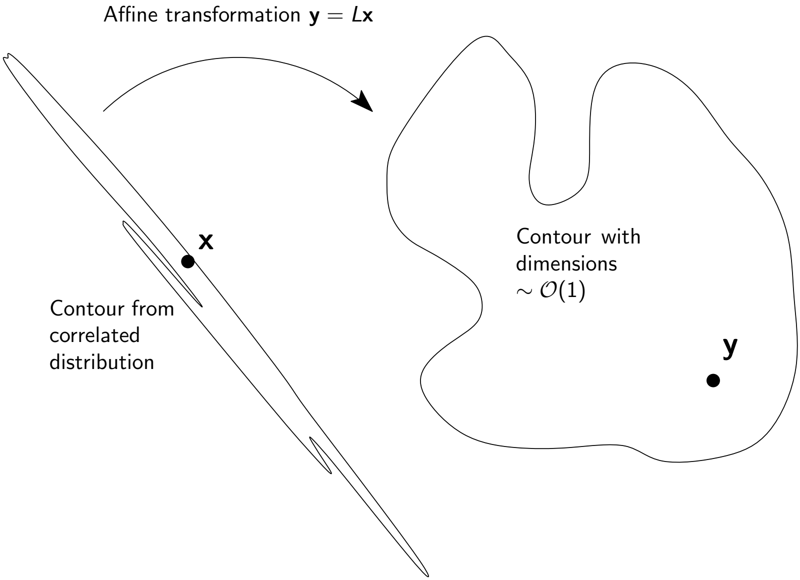
# PolyChord's solutions

## Correlated distributions



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- $w = 1$ in this transformed space

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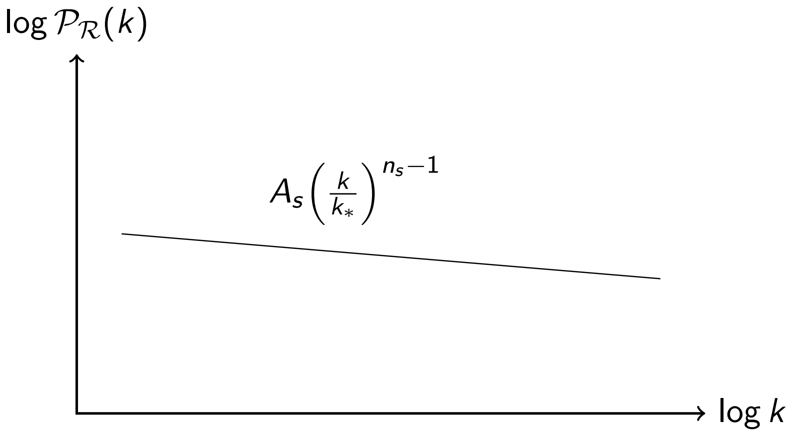
- Novel method for identifying and evolving modes separately.
- Implemented in CosmoMC, as "CosmoChord", with fast-slow parameters.

# PolyChord in action

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



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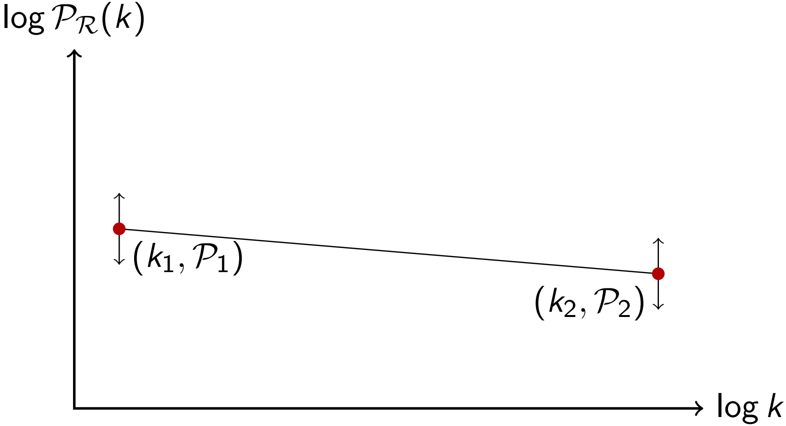

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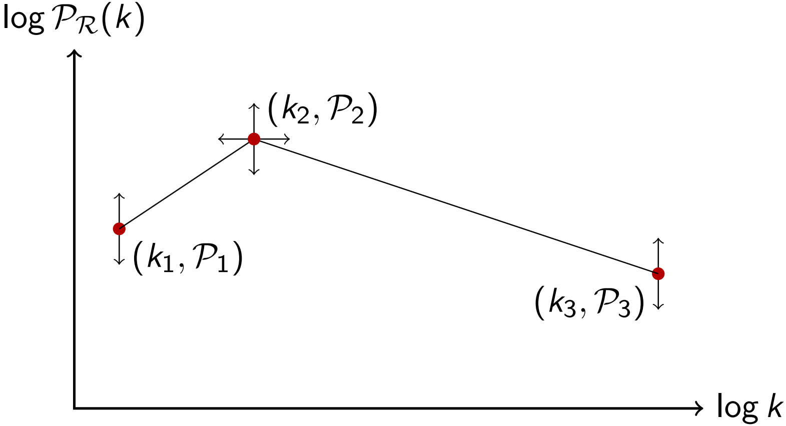

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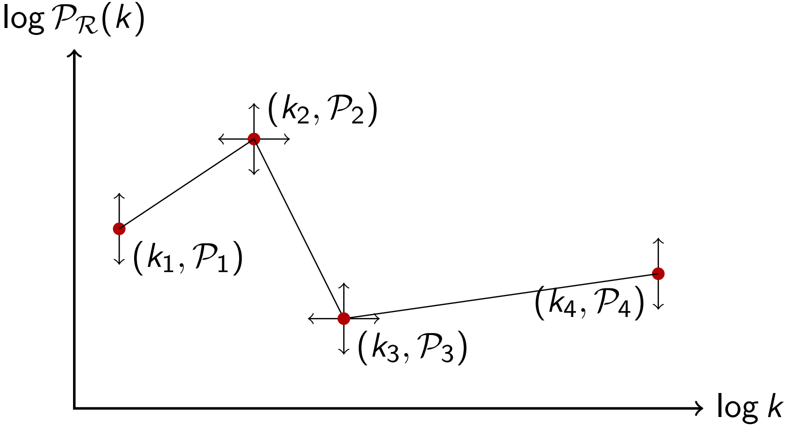

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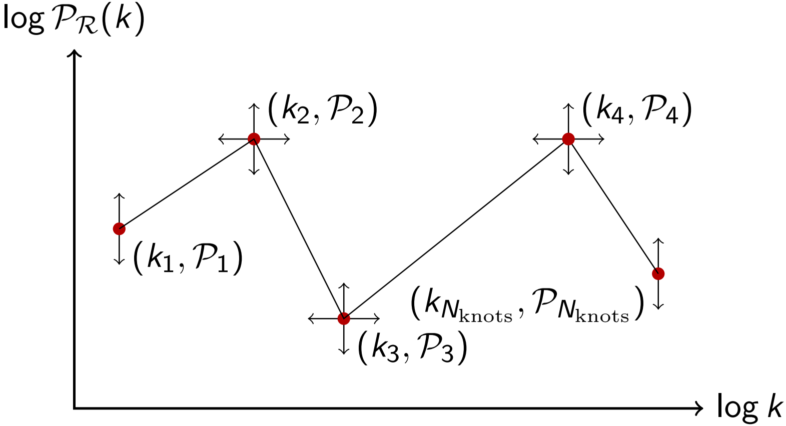

# 0 internal knots

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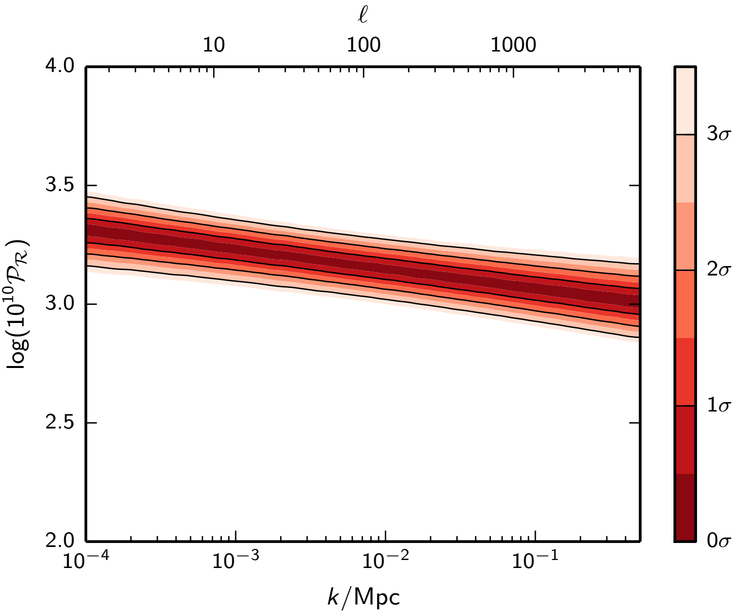

# 1 internal knots

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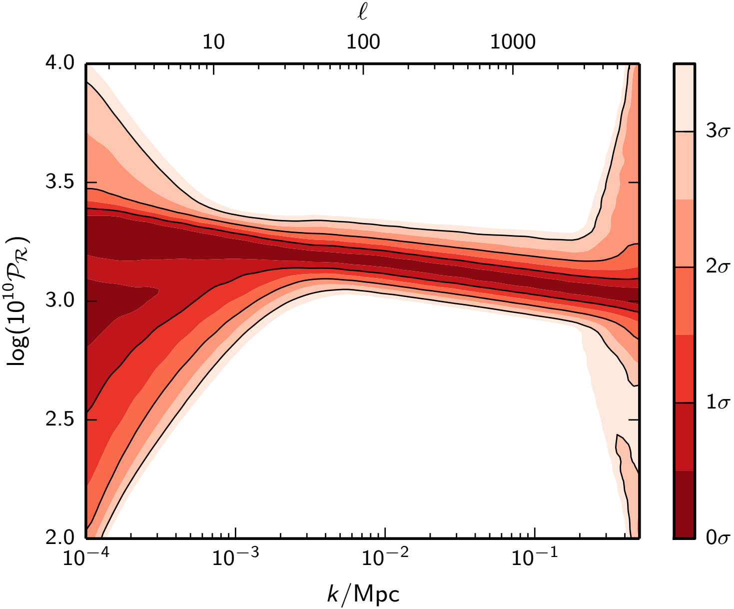
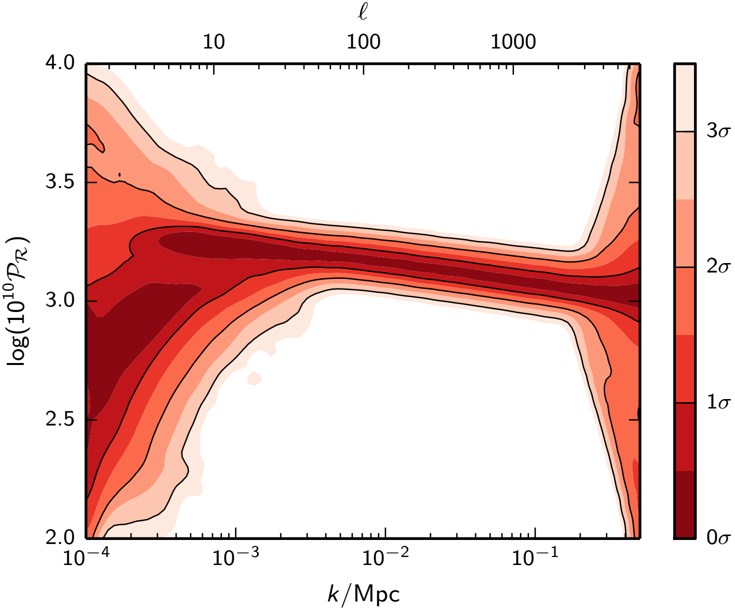


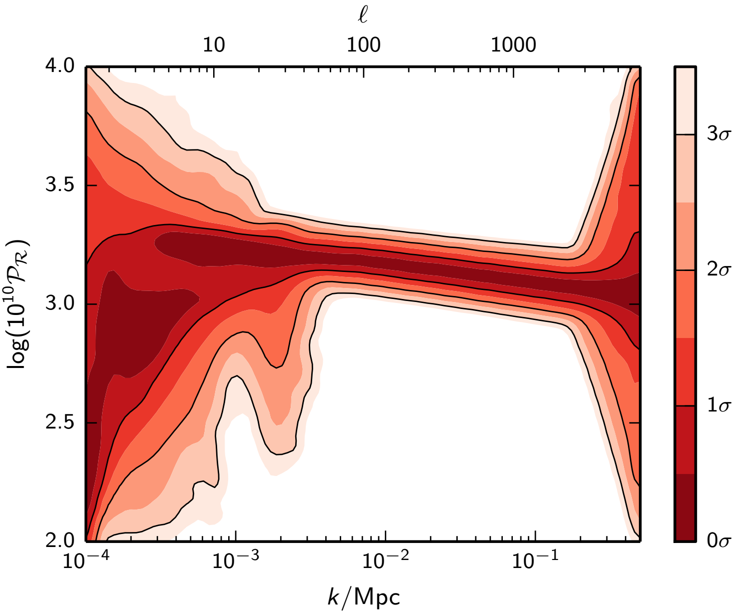
# 2 internal knots

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



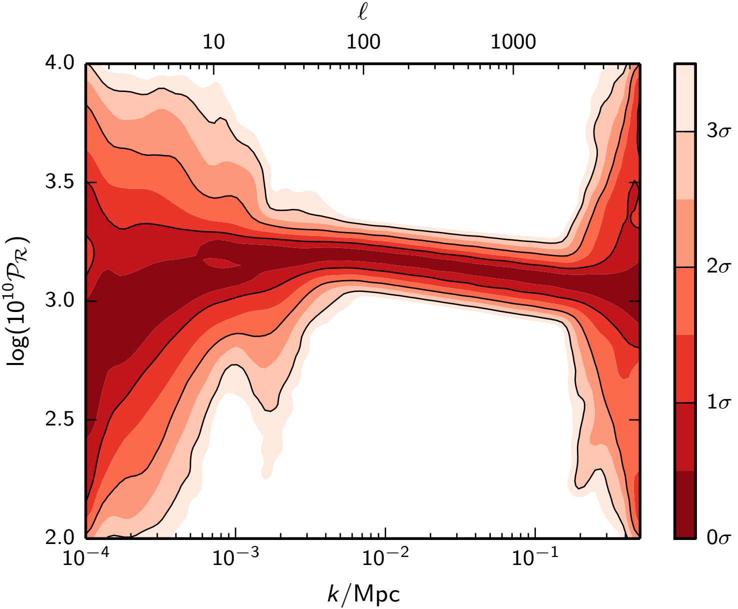
# 3 internal knots

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



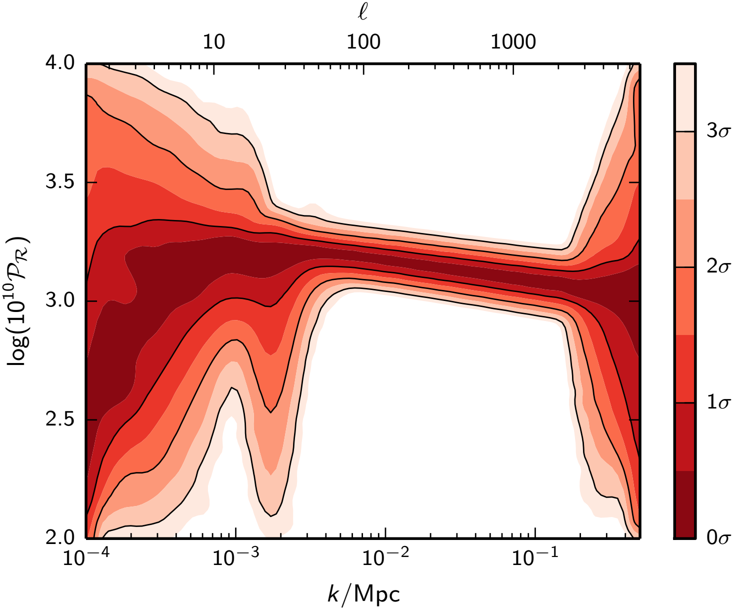
# 4 internal knots

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



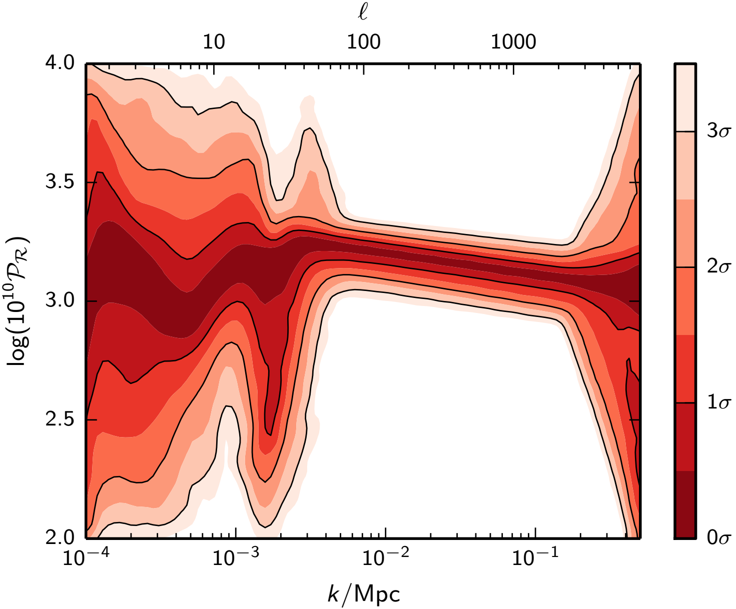
# 5 internal knots

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



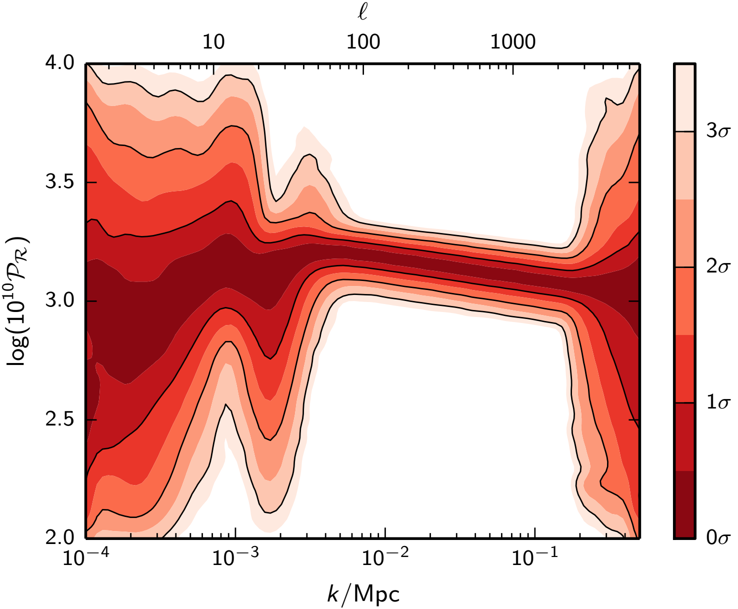
# 6 internal knots

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



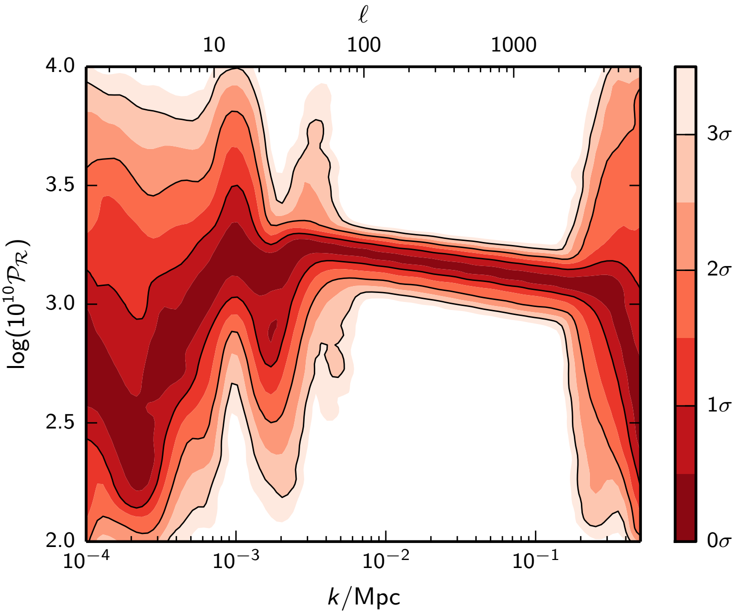
# 7 internal knots

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



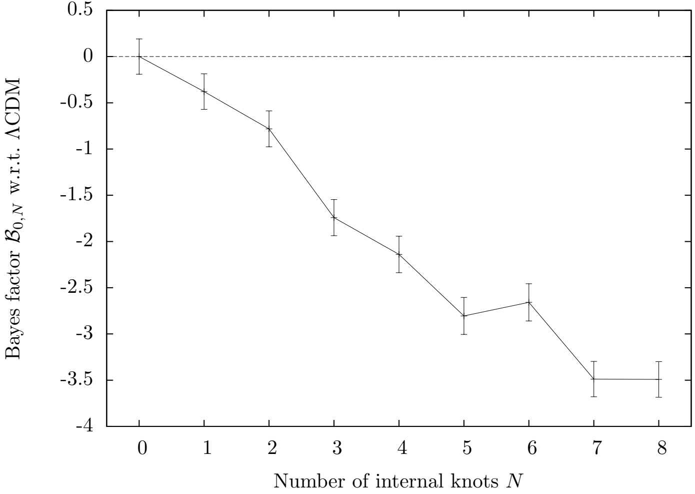
# 8 internal knots

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



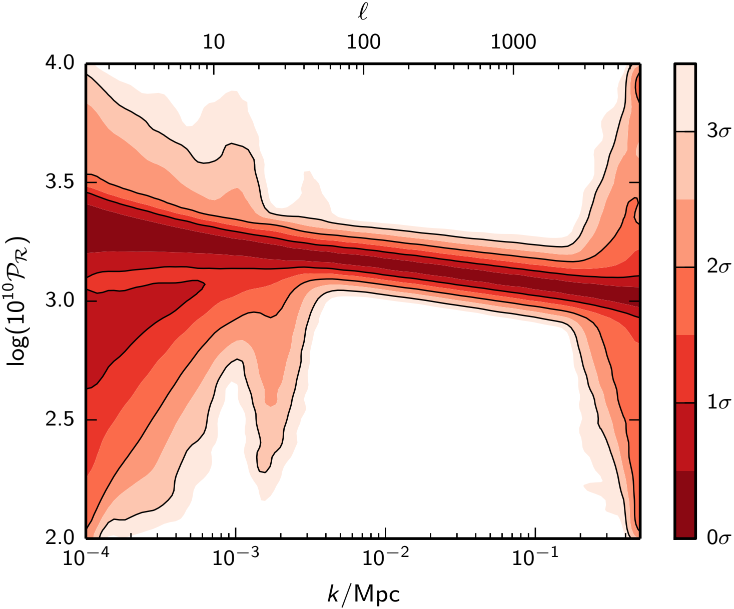
# Bayes Factors

## Primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$ reconstruction



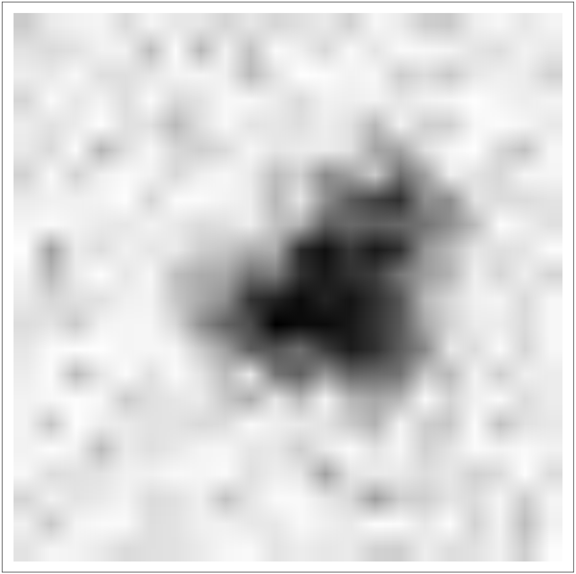
# Marginalised plot

## Primordial power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ reconstruction



# Object detection

## Toy problem



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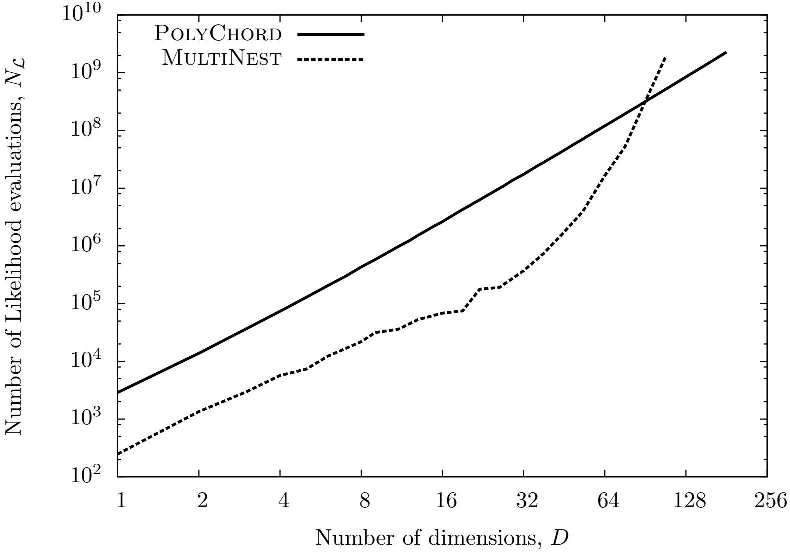
- odds ratio: $10^{-60} : 10^{-19} : 1 : 0.04 : 10^{-10}$

# PolyChord vs. MultiNest

## Gaussian likelihood



# PolyChord vs. MultiNest

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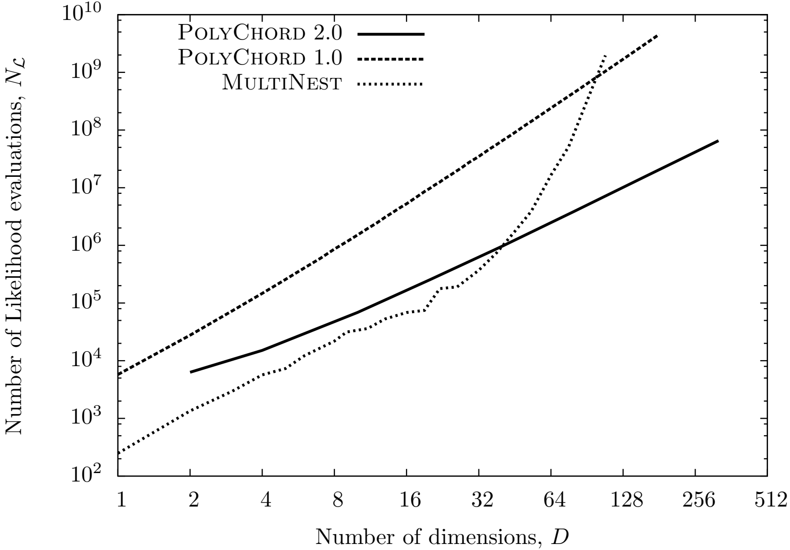

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- Nested sampling is really the first in a new class of "probabilistic integration" algorithms.

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- arXiv:1506.00171
- http://ccpforge.cse.rl.ac.uk/gf/project/polychord/