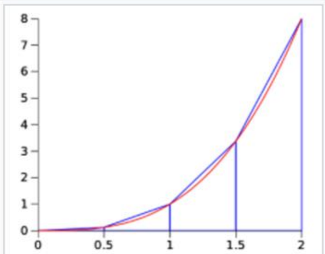
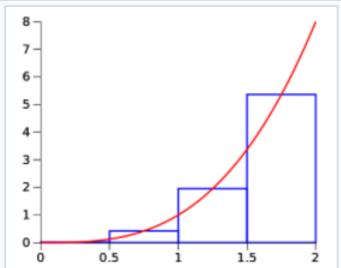
Trapezoidal Rule



Trapezoidal Riemann sum of x^3 over [0,2] using 4 subdivisions

$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{N} \frac{f(x_{k-1}) + f(x_k)}{2} \Delta x_k$$

Midpoint Rule



Midpoint Riemann sum of x^3 over [0,2] using 4 subdivisions

$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{N} f\left(\frac{x_{k-1} + x_k}{2}\right) \Delta x_k$$

The Trapezoidal Rule and Midpoint Rule are two methods for approximating the definite integral of a function with a sum.

The ‘function’ we are trying to integrate is a **weighted spectrum**. This is an intermediate step in calculating the weighted spectral average of a measured property. We can calculate the value of the weighted spectrum at a set of wavelengths for which we have measured, calculated or tabulated values of the following: P, the property of interest, S, the source weighting function and D, the detector weighting function (sometimes S or D are not required):

$$f(x_k) = P(x_k)S(x_k)D(x_k)$$

We can use the Trapezoidal Rule to approximate the integral of the weighted spectrum between the upper and lower wavelength limits by using the discrete values of the weighted spectrum in the Trapezoidal Rule sum above.

Alternatively, we can proceed by first estimating the value of the weighted spectrum at the midpoint of each wavelength interval and using these midpoint values in the Midpoint Rule sum above.

There are two ways you could get the midpoint values of the weighted spectrum:

Method 1: Estimate the midpoint value of the property, source and detector and *then* multiply them to estimate the value of the weighted spectrum at the midpoint:

$$f\left(\frac{x_{k-1} + x_k}{2}\right) = P\left(\frac{x_{k-1} + x_k}{2}\right) S\left(\frac{x_{k-1} + x_k}{2}\right) D\left(\frac{x_{k-1} + x_k}{2}\right)$$

Method 2: Estimate the midpoint value of the weighted spectrum by linear interpolation using the calculated values of the weighted spectrum at the original wavelengths:

$$f\left(\frac{x_{k-1}+x_k}{2}\right)=\frac{f(x_{k-1})+f(x_k)}{2}$$

Method 2 is exactly equivalent to the trapezoidal rule.

Method 1 will give slightly different results depending on the properties of P, S and D, because in general:

$$\frac{f(x_{k-1})+f(x_k)}{2}=\frac{P(x_{k-1})S(x_{k-1})D(x_{k-1})+P(x_k)S(x_k)D(x_k)}{2}$$
$$\neq P\left(\frac{x_{k-1}+x_k}{2}\right)S\left(\frac{x_{k-1}+x_k}{2}\right)D\left(\frac{x_{k-1}+x_k}{2}\right)$$

ISO 15099 specifies spectral averages consistent with using the midpoint rule for integration and Method 1 to obtain the midpoint values – for example, for visible / photopic averages:

$$\tau_{\mathrm{sl}}\left(\lambda_{\mathrm{w}\ j/j+1}\right)=\frac{1}{2}\tau_{\mathrm{sl}}\left(\lambda_{\mathrm{w}\ j}\right)+\frac{1}{2}\tau_{\mathrm{sl}}\left(\lambda_{\mathrm{w}\ j+1}\right) \quad (23)$$

Visible transmittance

Visible transmittance, τ_{vs}, is calculated using a weighting function that represents the photopic response of the eye, $R(\lambda_{\mathrm{w}})$. $R(\lambda_{\mathrm{w}j})$ is tabulated for N_{vs} values of $\lambda_{\mathrm{w}j}$. τ_{vs} is given by:

$$\tau_{\mathrm{vs}}=\frac{\sum_{j=1}^{N_{\mathrm{vs}-1}}\tau_{\mathrm{sl}}\left(\lambda_{\mathrm{w}\ j/j+1}\right)E_{\mathrm{vs}}\left(\lambda_{\mathrm{w}\ j/j+1}\right)R\left(\lambda_{\mathrm{w}\ j/j+1}\right)\Delta\lambda_{\mathrm{w}\ j}}{\sum_{j=1}^{N_{\mathrm{vs}-1}}E_{\mathrm{vs}}\left(\lambda_{\mathrm{w}\ j/j+1}\right)R\left(\lambda_{\mathrm{w}\ j/j+1}\right)\Delta\lambda_{\mathrm{w}\ j}}$$

$$\Delta\lambda_{\mathrm{w}\ j}=\lambda_{\mathrm{w}\ j+1}-\lambda_{\mathrm{w}\ j} \quad (24)$$

where

$$R\left(\lambda_{\mathrm{w}\ j/j+1}\right)=\frac{R\left(\lambda_{\mathrm{w}\ j}\right)+R\left(\lambda_{\mathrm{w}\ j+1}\right)}{2} \quad (25)$$

$$E_{\mathrm{vs}}\left(\lambda_{\mathrm{w}\ j/j+1}\right)=\frac{1}{2}E_{\mathrm{vs}}\left(\lambda_{\mathrm{w}\ j}\right)+\frac{1}{2}E_{\mathrm{vs}}\left(\lambda_{\mathrm{w}\ j+1}\right) \quad (26)$$

Values of $E_{\mathrm{v}}(\lambda_{\mathrm{w}})$ are given in ISO/CIE 10526.

and $\tau_{\mathrm{sl}}(\lambda_{\mathrm{w}j/j+1})$ is given by Equation (23).

The solar spectral average and other spectral averages are specified using the same calculation method.

NFRC 300 7.2.4 specifies using the Trapezoidal Rule for integration.

Conclusion:

Even if the source and detector spectra are identical, there will be some small differences in the spectral averages calculated according to NFRC 300 and ISO 15099 due to the different numerical integration method specified.