Group 6 CA2

Question 1:

```
[1] "Mean Vector of Rank 4 Approximation Error:"
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```
   V1    V2    V3    V4    V5    V6    V7    V8    V9   V10   V11   V12
 0.01 -0.01  0.13  0.02 -0.04  0.01  0.01 -0.02  0.06 -0.04  0.14 -0.04
```

Question 2:

```
[1] "Correlation Matrix of X:"
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       V1    V2    V3    V4    V5    V6    V7    V8    V9   V10   V11   V12
V1   1.00 -0.20  0.08 -0.25  0.17  0.05  0.33  0.52 -0.30 -0.26 -0.13  0.15
V2  -0.20  1.00  0.19  0.88  0.07 -0.06 -0.32 -0.37  0.07  0.67 -0.11  0.00
V3   0.08  0.19  1.00  0.28  0.16  0.12 -0.39 -0.12 -0.12 -0.07 -0.35  0.45
V4  -0.25  0.88  0.28  1.00  0.04 -0.02 -0.46 -0.36  0.02  0.54 -0.12  0.09
V5   0.17  0.07  0.16  0.04  1.00  0.50  0.15 -0.05 -0.06 -0.03 -0.03  0.10
V6   0.05 -0.06  0.12 -0.02  0.50  1.00  0.06 -0.04  0.03 -0.07 -0.14  0.13
V7   0.33 -0.32 -0.39 -0.46  0.15  0.06  1.00  0.24 -0.04 -0.08  0.26 -0.14
V8   0.52 -0.37 -0.12 -0.36 -0.05 -0.04  0.24  1.00  0.09 -0.20  0.09  0.28
V9  -0.30  0.07 -0.12  0.02 -0.06  0.03 -0.04  0.09  1.00  0.13 -0.16  0.13
V10 -0.26  0.67 -0.07  0.54 -0.03 -0.07 -0.08 -0.20  0.13  1.00  0.04  0.07
V11 -0.13 -0.11 -0.35 -0.12 -0.03 -0.14  0.26  0.09 -0.16  0.04  1.00 -0.22
V12  0.15  0.00  0.45  0.09  0.10  0.13 -0.14  0.28  0.13  0.07 -0.22  1.00
```

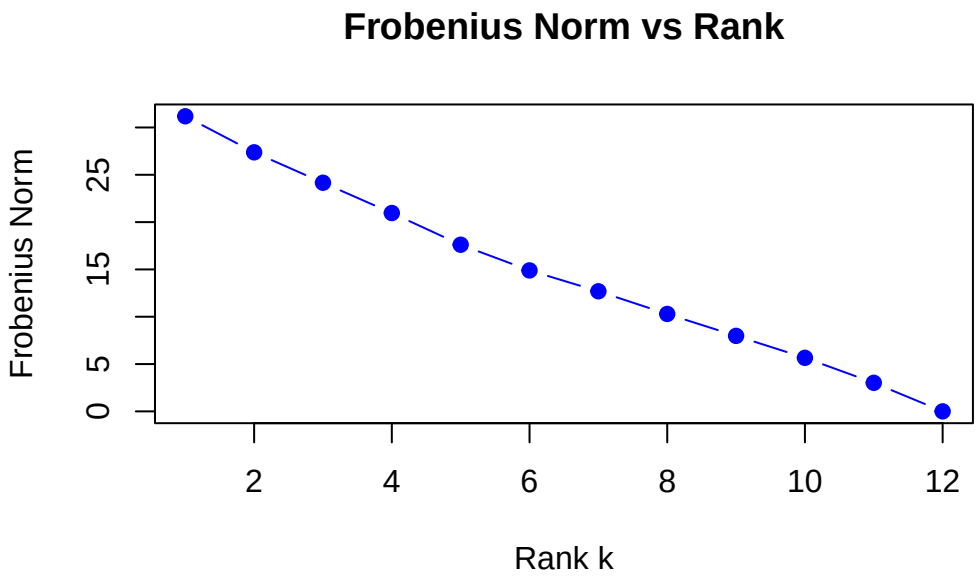
```
[1] "Correlation Matrix of X_2:"
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
[1,]	1.00	-0.65	0.34	-0.59	0.74	0.78	0.64	0.97	-0.59	-0.75	-0.36	0.64
[2,]	-0.65	1.00	0.50	1.00	0.04	-0.02	-1.00	-0.82	1.00	0.99	-0.48	0.18
[3,]	0.34	0.50	1.00	0.55	0.89	0.86	-0.50	0.09	0.56	0.36	-1.00	0.94
[4,]	-0.59	1.00	0.55	1.00	0.10	0.05	-1.00	-0.78	1.00	0.98	-0.54	0.24
[5,]	0.74	0.04	0.89	0.10	1.00	1.00	-0.04	0.54	0.11	-0.11	-0.89	0.99
[6,]	0.78	-0.02	0.86	0.05	1.00	1.00	0.02	0.59	0.05	-0.17	-0.86	0.98
[7,]	0.64	-1.00	-0.50	-1.00	-0.04	0.02	1.00	0.81	-1.00	-0.99	0.49	-0.18
[8,]	0.97	-0.82	0.09	-0.78	0.54	0.59	0.81	1.00	-0.78	-0.90	-0.11	0.42
[9,]	-0.59	1.00	0.56	1.00	0.11	0.05	-1.00	-0.78	1.00	0.98	-0.54	0.24
[10,]	-0.75	0.99	0.36	0.98	-0.11	-0.17	-0.99	-0.90	0.98	1.00	-0.35	0.03
[11,]	-0.36	-0.48	-1.00	-0.54	-0.89	-0.86	0.49	-0.11	-0.54	-0.35	1.00	-0.95
[12,]	0.64	0.18	0.94	0.24	0.99	0.98	-0.18	0.42	0.24	0.03	-0.95	1.00

Interpretation:

The rank-2 approximation exaggerates correlations, preserving major trends with values closer to -1 and 1. This suggests that the first two singular components capture the dominant variance, but finer variance details are lost (distorts smaller correlations), leading to over-simplified relationships.

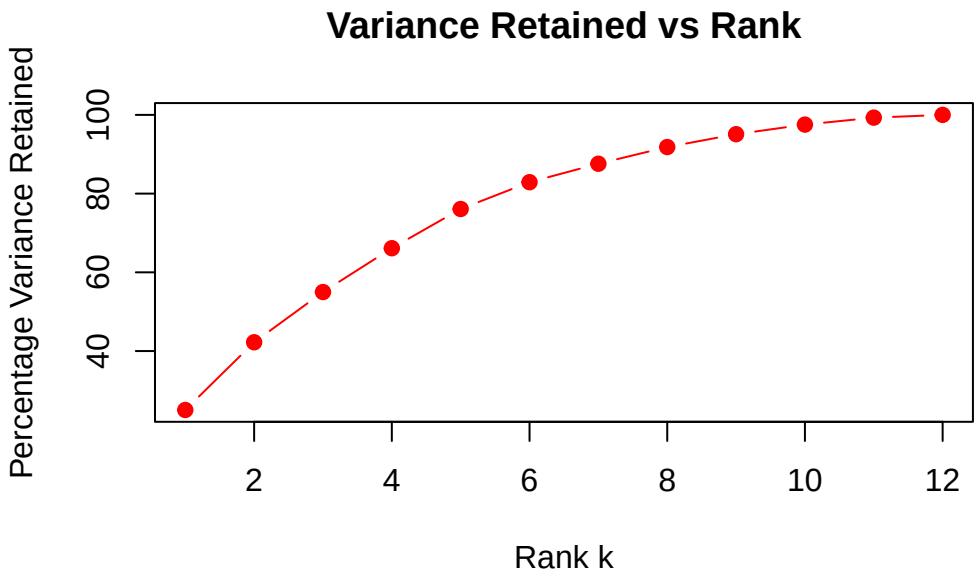
Question 3:



Interpretation:

The Frobenius norm decreases as K increases, indicating that higher-rank approximations provide better approximations of X. The error reduction is most significant for lower values of K (e.g., from K=1 to K=6), suggesting that the first few singular components capture most of the data's variance. After K=8, the reduction in the Frobenius norm slows down. This means additional components contribute less to improving the approximation. The norm approaches zero at K=12, confirming that the full-rank approximation perfectly reconstructs X.

Question 4:



Interpretation:

The first few singular values capture a large proportion of the total variance. As K increases, the additional variance retained slows down and beyond K=8, additional components contribute less marginal improvement. At K=12, nearly all the variation is captured, meaning again that the full-rank matrix perfectly reconstructs X.