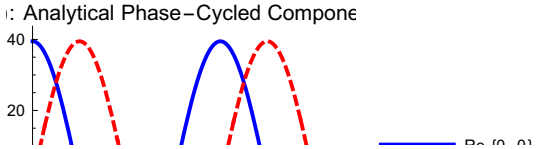
In[38]:=
```
(*Analytical Phase Cycling Component*)
phaseCycleComponentAnalytical[Pfunc_, lm_] := Module[{l, m}, {l, m} = lm;
   Integrate[Pfunc * Exp[-I (l phi1 + m phi2)], {phi1, 0, 2 Pi}, {phi2, 0, 2 Pi}] // Simplify];

(*Parameters*)
w = 1.0;
gamma = 0.1;
lmList = {{0, 0}, {1, 1}, {0, 1}, {1, 0}, {-1, 1}, {1, -1}};

(*Function to create plots for a given Pfunc and title*)

createPlots[Pfunc_, title_] := Module[{results, plots},
   results = Table[phaseCycleComponentAnalytical[Pfunc, lm], {lm, lmList}];
   Print["Analytical Results for " <> title];
   Print[TableForm[Table[{lmList[[i]], results[[i]]}, {i, Length[lmList]}],
     TableHeadings → {None, {"lm", "Analytical Result"}}]];
   plots = Table[Plot[{Re[results[[idx]]], Im[results[[idx]]]}, {t, 0, 10},
      PlotStyle → {{Blue, Thick}, {Red, Dashed, Thick}},
      PlotLegends → {"Re " <> ToString[lmList[[idx]]], "Im " <> ToString[lmList[[idx]]]},
      PlotLabel → title <> " " <> ToString[lmList[[idx]]], PlotRange → All,
      ImageSize → Small, AspectRatio → 1], {idx, 1, Length[lmList]}];
   GraphicsGrid[Partition[plots, 2], PlotLabel → Style[title, Bold, 14],
    ImageSize → 600, Spacings → {30, 30}]];

(*Test 1:exp(I w t)-function depending only on t*)
Pfunc1 = Exp[I w t];
Print["Test 1: exp(I w t)"];
createPlots[Pfunc1, "exp(I w t): Analytical Phase-Cycled Components"]

(*Test 2:Linear in phi1*)
Pfunc2 = phi1;
Print["Test 2: phi1"];
createPlots[Pfunc2, "phi1: Analytical Phase-Cycled Components"]

(*Test 3:Product phi1*phi2*)
Pfunc3 = phi1 * phi2;
Print["Test 3: phi1 * phi2"];
createPlots[Pfunc3, "phi1 * phi2: Analytical Phase-Cycled Components"]

(*Test 4:cos(phi1-w t)*)
Pfunc4 = Cos[phi1 - w t];
Print["Test 4: cos(phi1 - w t)"];
```

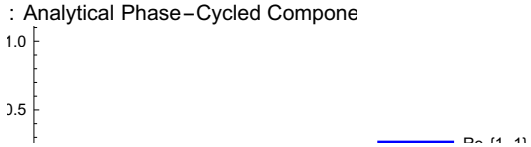
```
createPlots[Pfunc4, "cos(phi1 - w t): Analytical Phase-Cycled Components"]

(*Test 5:exp(phi1-I w t)*)
Pfunc5 = Exp[phi1 - I w t];
Print["Test 5: exp(phi1 - I w t)"];
createPlots[Pfunc5, "exp(phi1 - I w t): Analytical Phase-Cycled Components"]

(*Test 6:cos(|phi1-phi2|)*exp(-gamma t)*)
Pfunc6 = Cos[Abs[phi1 - phi2]] * Exp[-gamma t];
Print["Test 6: cos(|phi1 - phi2|) * exp(-gamma t)"];
createPlots[Pfunc6,
 "cos(|phi1 - phi2|) * exp(-gamma t): Analytical Phase-Cycled Components"]

(*Test 7:exp(-gamma t)*exp(I w t-I phi1+I phi2)*)
Pfunc7 = Exp[-gamma t] * Exp[I w t - I phi1 + I phi2];
Print["Test 7: exp(-gamma t) * exp(I w t - I phi1 + I phi2)"];
createPlots[Pfunc7,
 "exp(-gamma t) * exp(I w t - I phi1 + I phi2): Analytical Phase-Cycled Components"]

(*Test 8:exp(-gamma t)*exp(I w t+I phi1-I phi2)*)
Pfunc8 = Exp[-gamma t] * Exp[I w t + I phi1 - I phi2];
Print["Test 8: exp(-gamma t) * exp(I w t + I phi1 - I phi2)"];
createPlots[Pfunc8,
 "exp(-gamma t) * exp(I w t + I phi1 - I phi2): Analytical Phase-Cycled Components"]
```

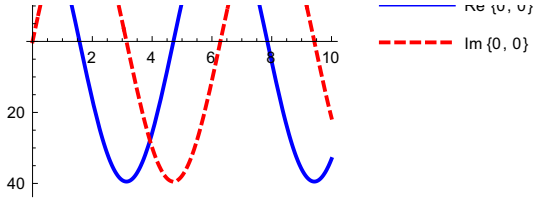
Test 1: exp(I w t)

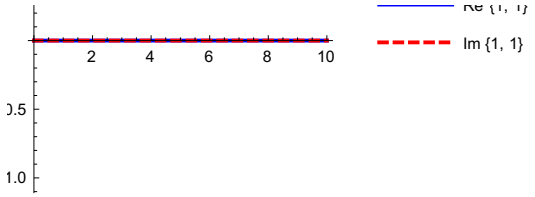
Analytical Results for exp(I w t): Analytical Phase-Cycled Components

lm	Analytical Result
0 0	$4\, e^{(0.+1.\, i)\, t}\, \pi^2$
1 1	0
0 1	0
1 0	$\left(-1.53894 \times 10^{-15} - 3.76931 \times 10^{-31}\, i\right) e^{(0.+1.\, i)\, t}$
-1 1	0
1 -1	0

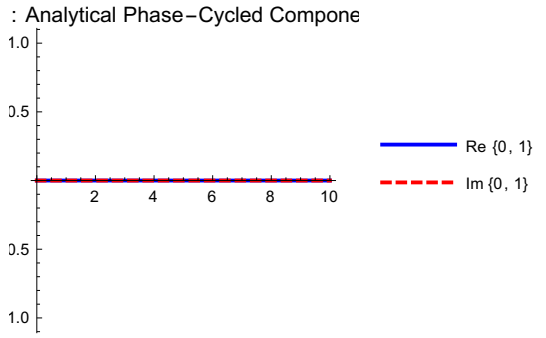


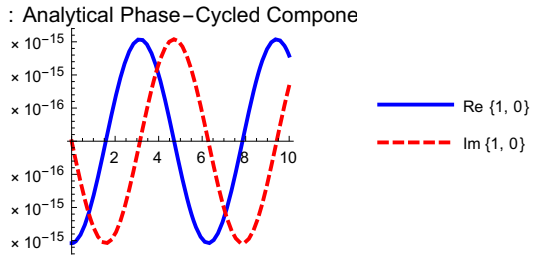


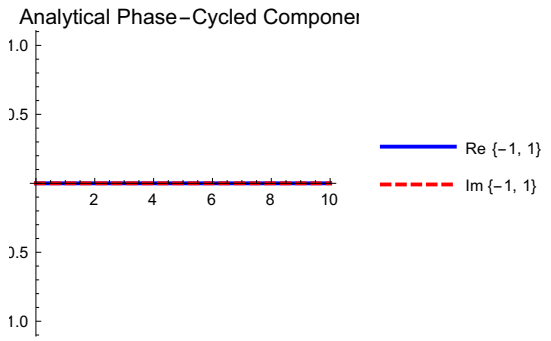


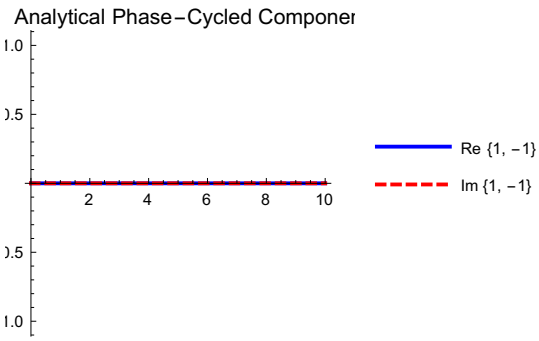


Out[45]=







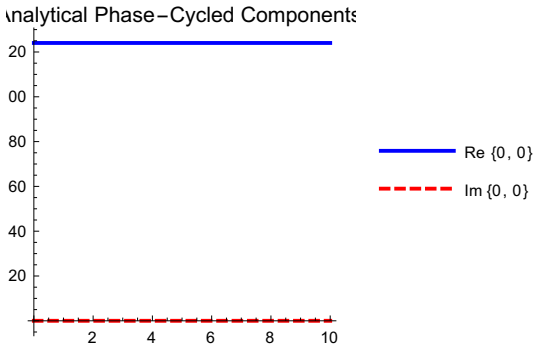


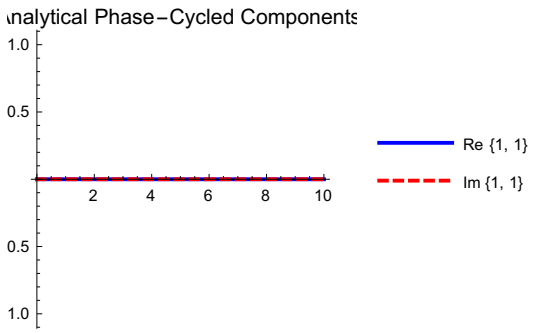
Test 2: phi1

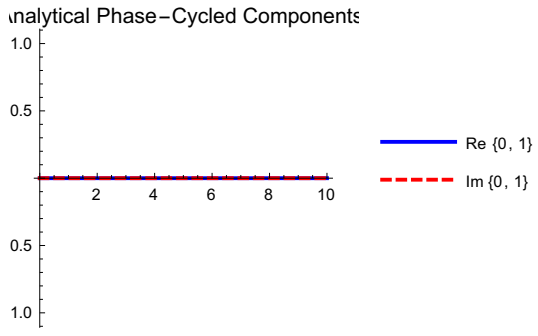
Analytical Results for phi1: Analytical Phase-Cycled Components

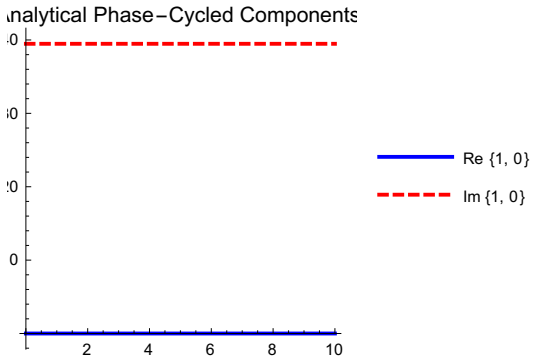
lm	Analytical Result
0 0	$4\,\pi^3$
1 1	0
0 1	0
1 0	$4\,i\,\pi^2$
−1 1	0
1 −1	0

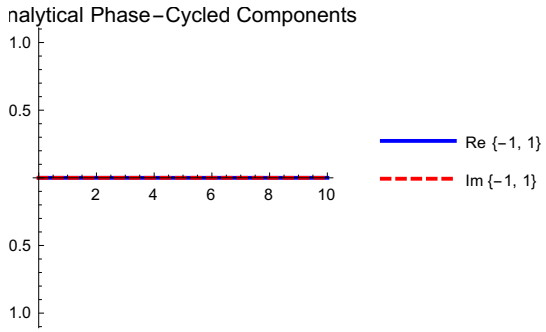
Out[48]=

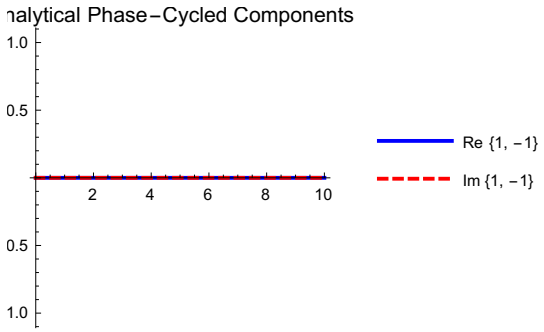








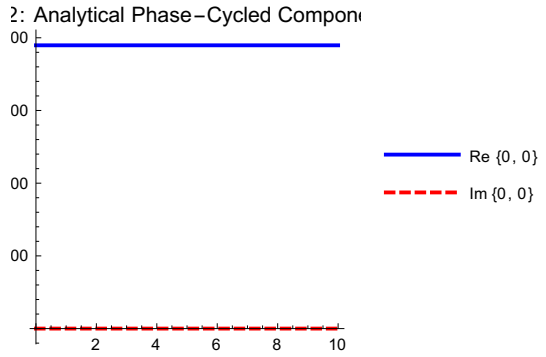


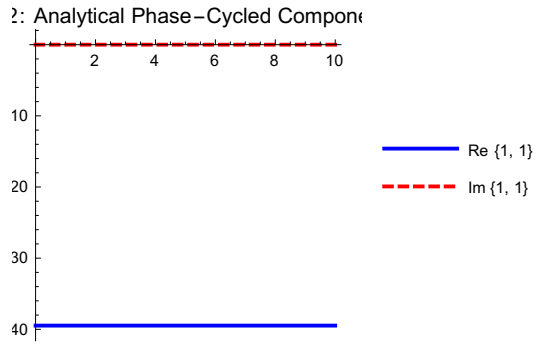


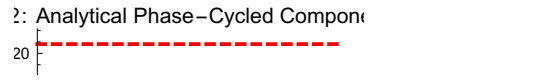
Test 3: phi1 * phi2

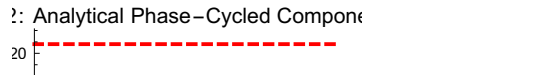
Analytical Results for phi1 * phi2: Analytical Phase-Cycled Components

lm	Analytical Result
0 0	$4\,\pi^4$
1 1	$-4\,\pi^2$
0 1	$4\,i\,\pi^3$
1 0	$4\,i\,\pi^3$
−1 1	$4\,\pi^2$
1 −1	$4\,\pi^2$

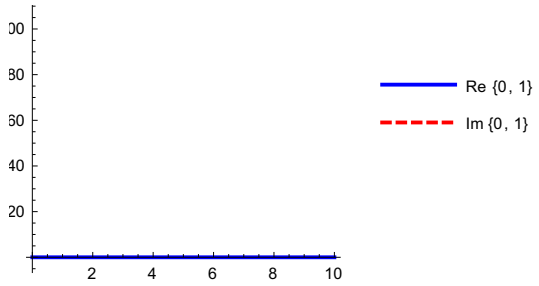


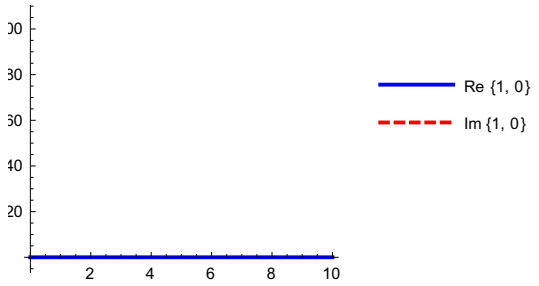


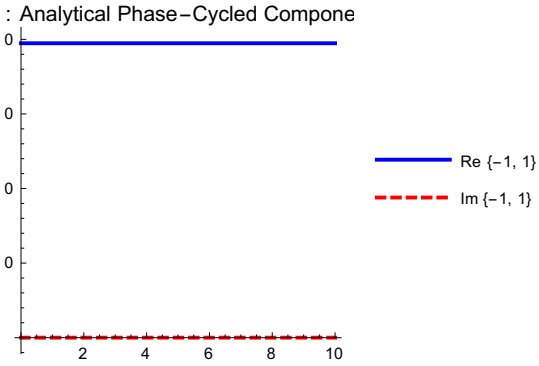


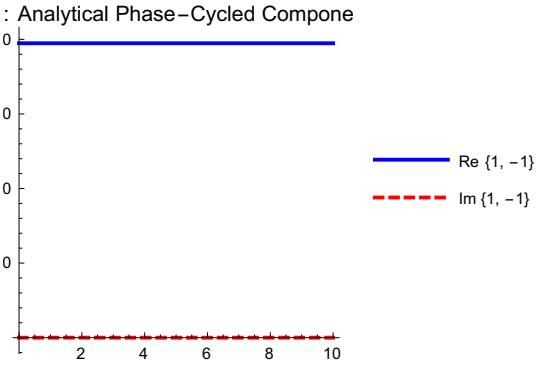


Out[51]=





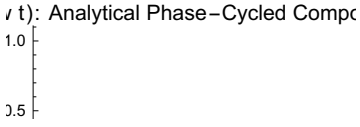


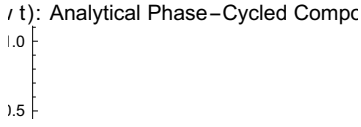


Test 4: cos(phi1 - w t)

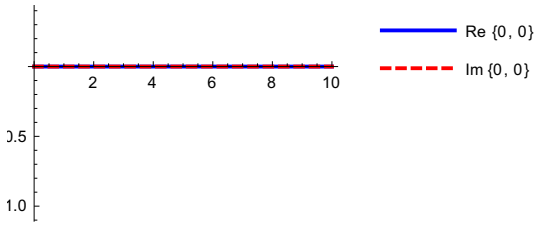
Analytical Results for cos(phi1 - w t): Analytical Phase-Cycled Components

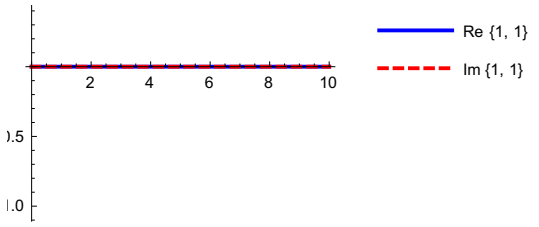
lm	Analytical Result
0 0	0.
1 1	0
0 1	0
1 0	$e^{(0.-1.\,i)\,t}\left(19.7392 - 7.69468 \times 10^{-16}\, e^{(0.+2.\,i)\,t}\right)$
−1 1	0
1 −1	0

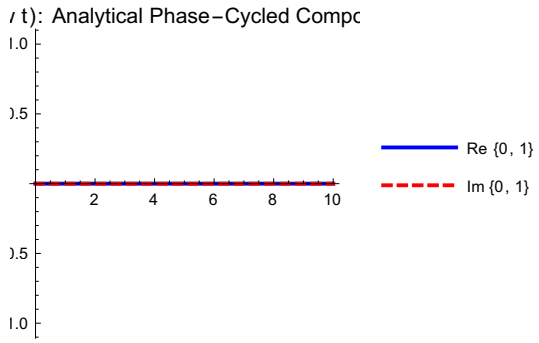


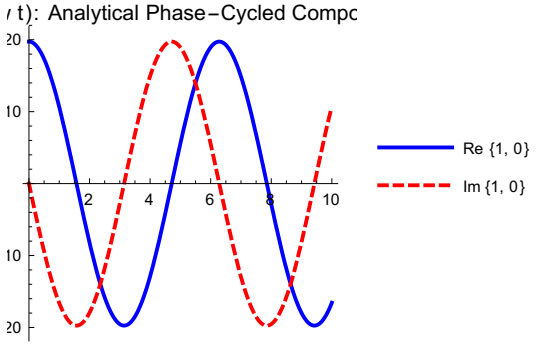


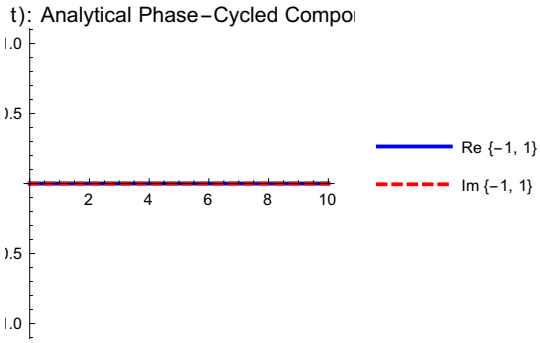
Out[54]=

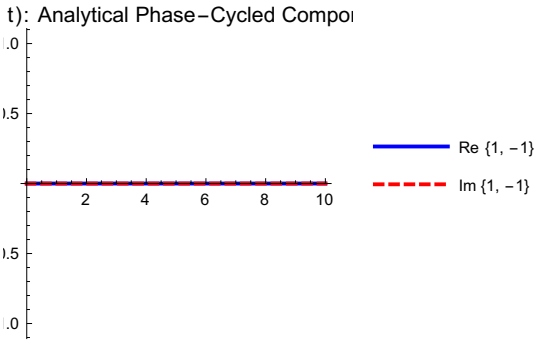










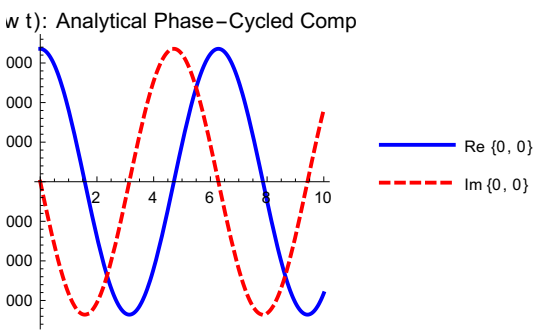


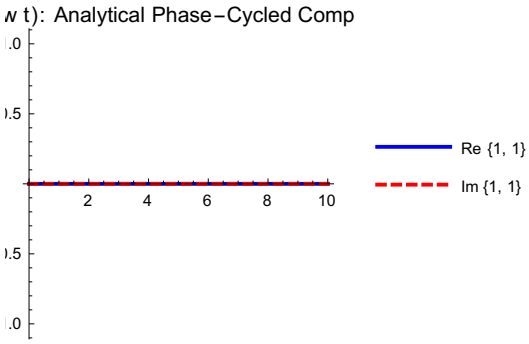
Test 5: exp(phi1 - I w t)

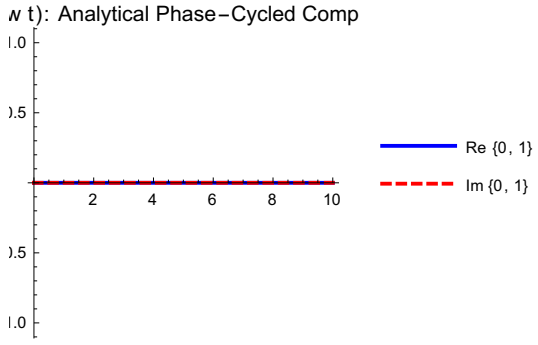
Analytical Results for exp(phi1 - I w t): Analytical Phase-Cycled Components

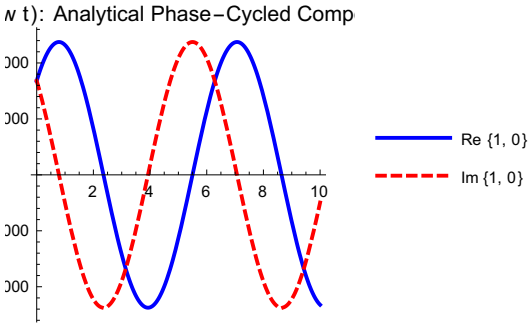
Out[57]=

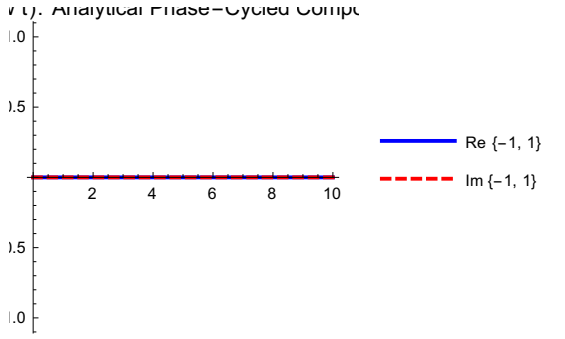
lm	Analytical Result
0 0	$3358.31\, e^{(0.-1.\, i)\, t}$
1 1	0
0 1	0
1 0	$(1679.16 + 1679.16\, i)\, e^{(0.-1.\, i)\, t}$
−1 1	0
1 −1	0

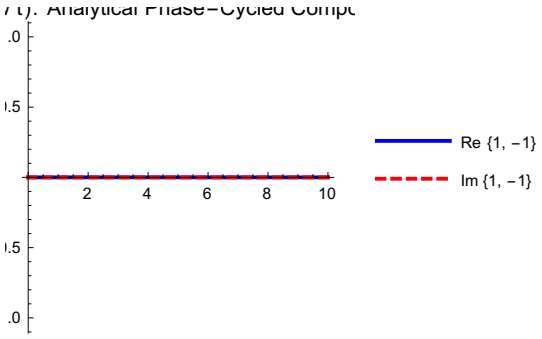








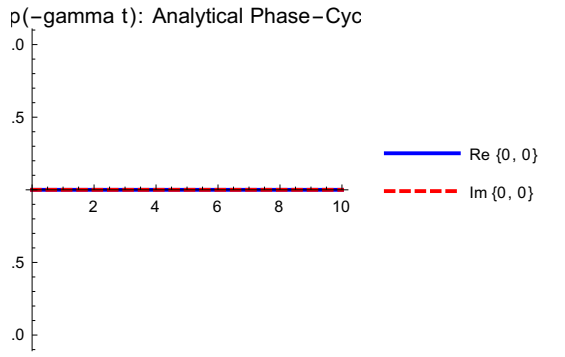


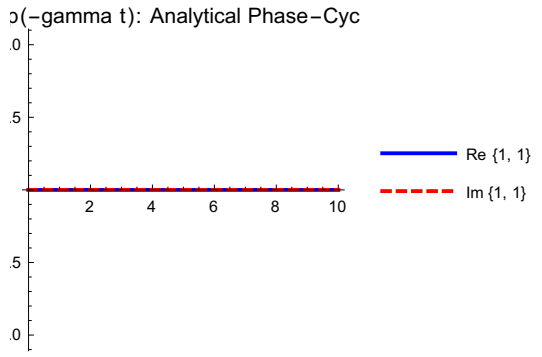


Test 6: cos(|phi1 - phi2|) * exp(-gamma t)

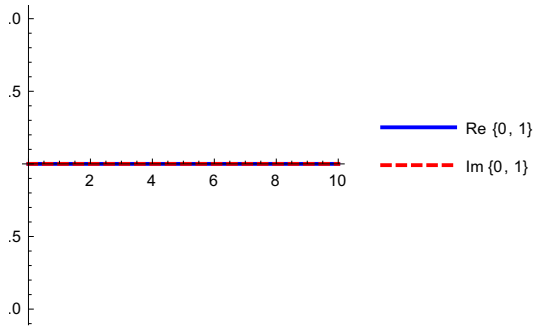
Analytical Results for cos(|phi1 -
  phi2|) * exp(-gamma t): Analytical Phase-Cycled Components

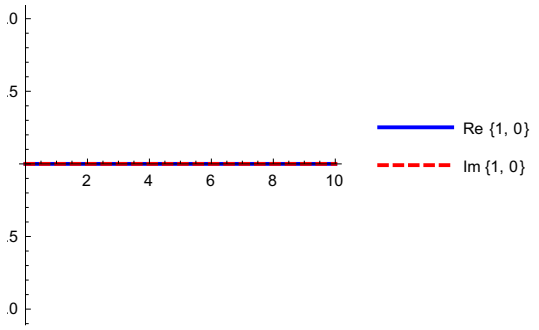
lm	Analytical Result
0 0	0.
1 1	0.
0 1	0.
1 0	0.
−1 1	$19.7392\, e^{-0.1\, t}$
1 −1	$19.7392\, e^{-0.1\, t}$

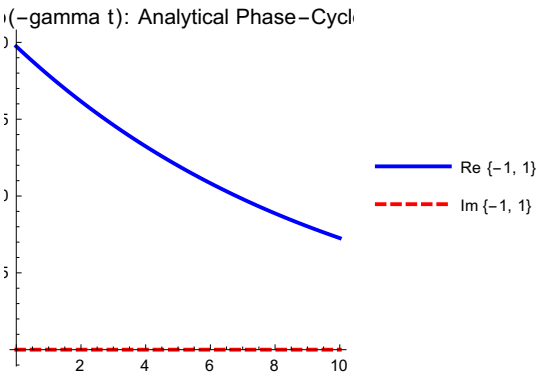


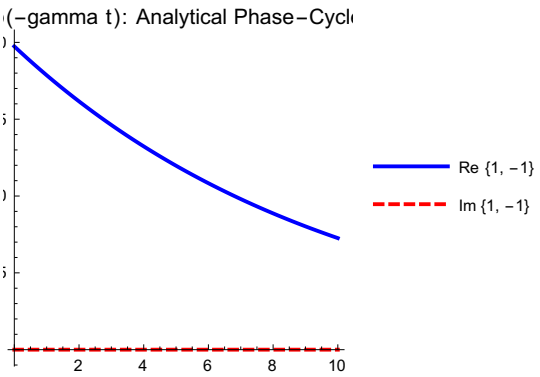


Out[60]=









Test 7: exp(-gamma t) * exp(I w t - I phi1 + I phi2)

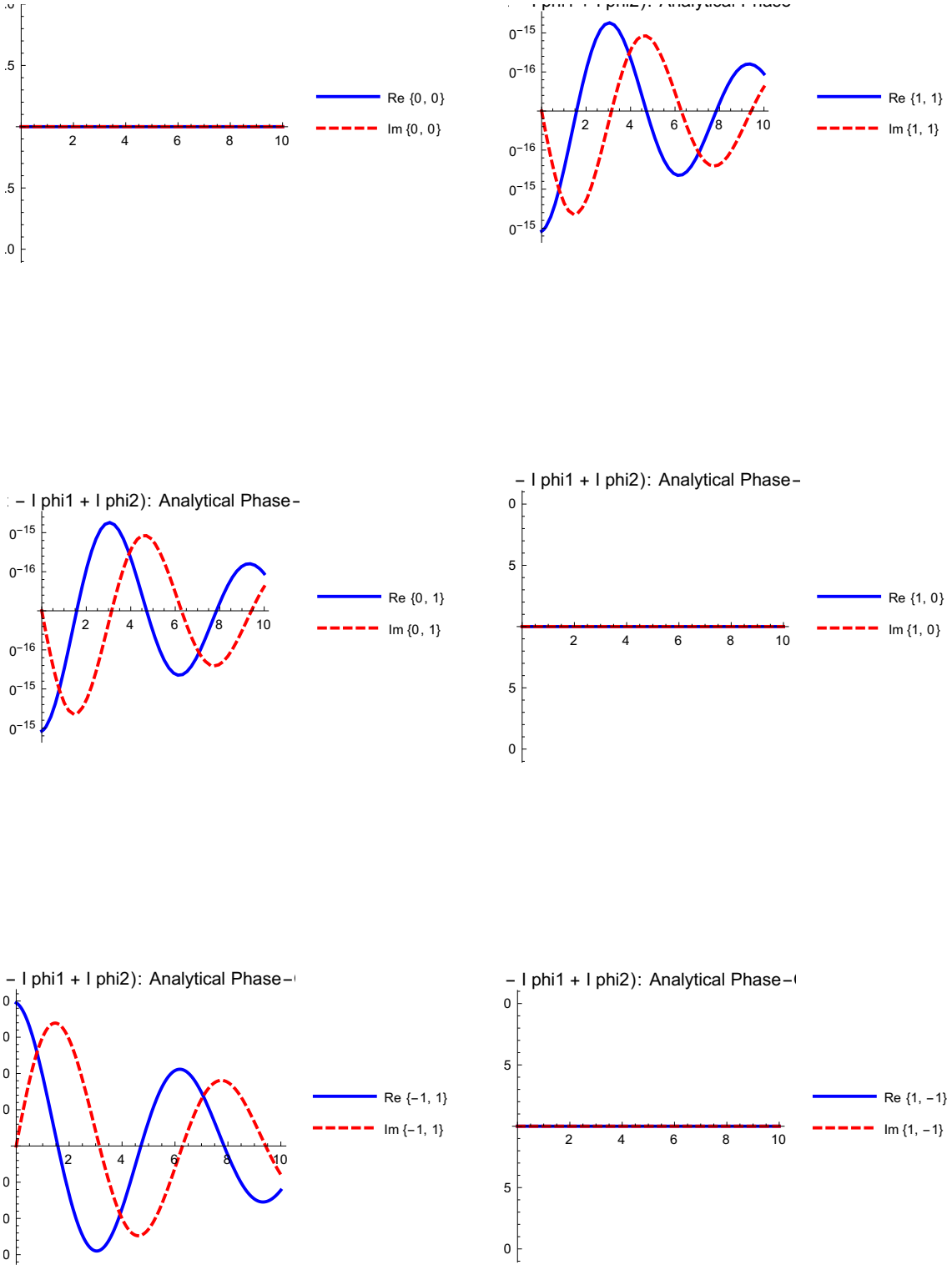
Analytical Results for exp(-gamma t) * exp(I
  w t - I phi1 + I phi2): Analytical Phase-Cycled Components

lm	Analytical Result
0 0	0
1 1	$\left(-1.53894 \times 10^{-15} - 7.53861 \times 10^{-31}\ i\right) e^{(-0.1+1.\ i)\ t}$
0 1	$\left(-1.53894 \times 10^{-15} - 3.76931 \times 10^{-31}\ i\right) e^{(-0.1+1.\ i)\ t}$
1 0	0
−1 1	$4\ e^{(-0.1+1.\ i)\ t}\ \pi^2$
1 −1	0

: − I phi1 + I phi2): Analytical Phase-

− I phi1 + I phi2): Analytical Phase-

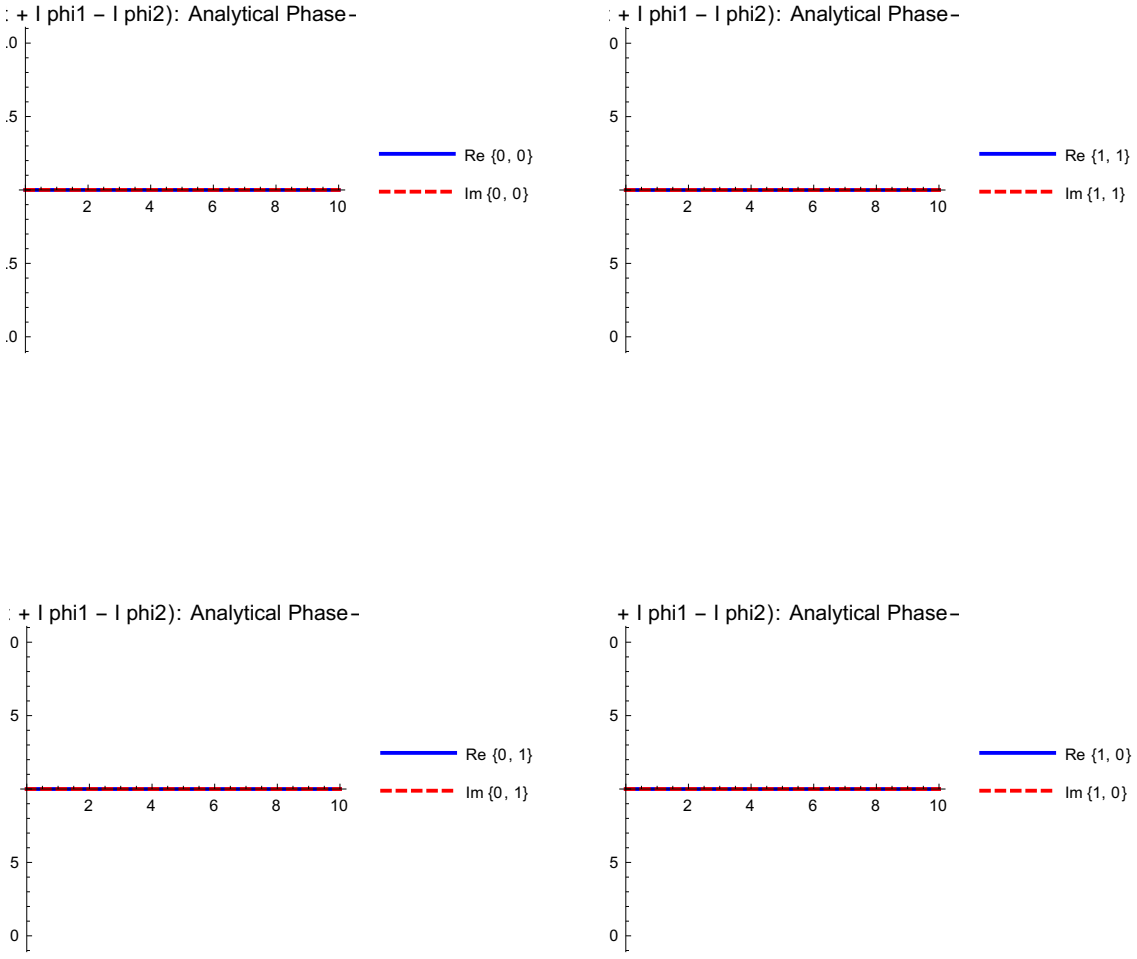
Out[63]=

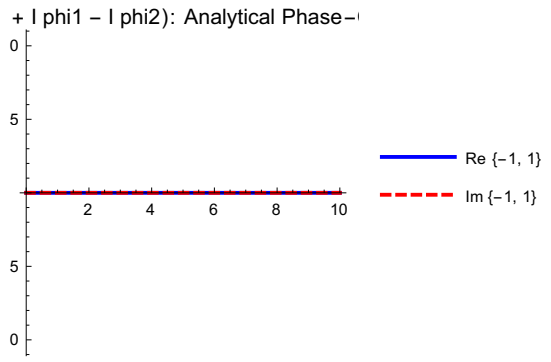


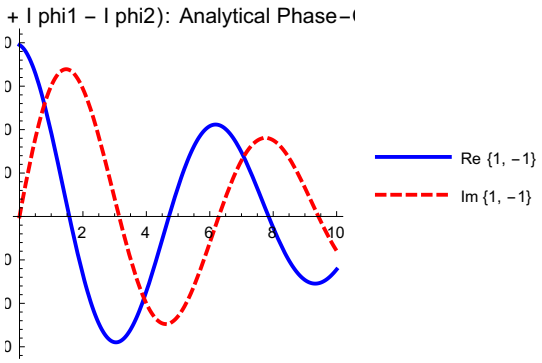
Test 8: exp(-gamma t) * exp(I w t + I phi1 - I phi2)

```
Analytical Results for exp(-gamma t) * exp(I
  w t + I phi1 - I phi2): Analytical Phase-Cycled Components

lm    Analytical Result
 0
 0              0
 1
 1              0
 0
 1              0
 1
 0              0
-1
 1              0
 1
-1        4 e^(-0.1+1. I) t π^2
```

Out[66]=




+ I phi1 − I phi2): Analytical Phase−
0
5
5
0
2
4
6
8
10
Re {−1, 1}
Im {−1, 1}


+ I phi1 − I phi2): Analytical Phase−
2
4
6
8
10
Re {1, −1}
Im {1, −1}