Algorithmic Game Theory

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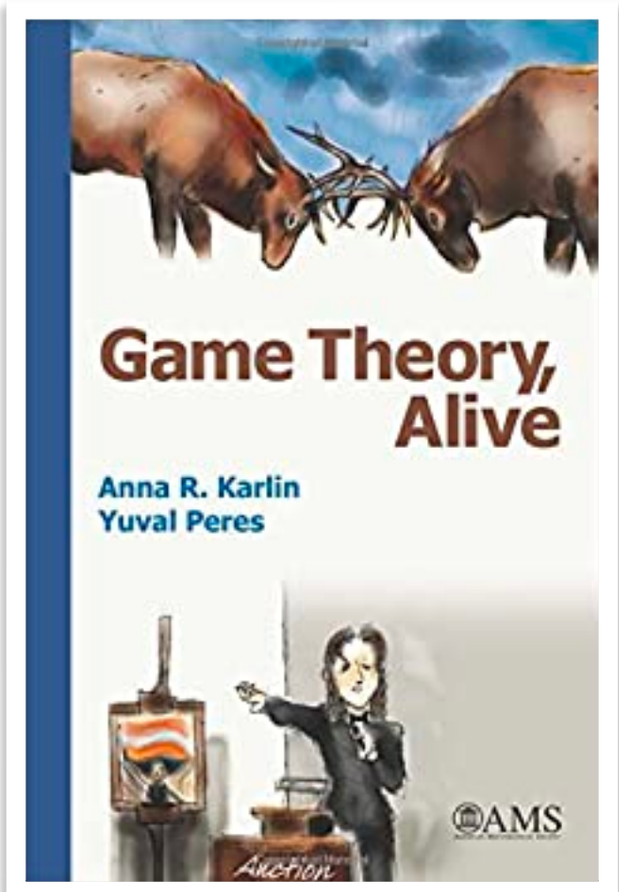
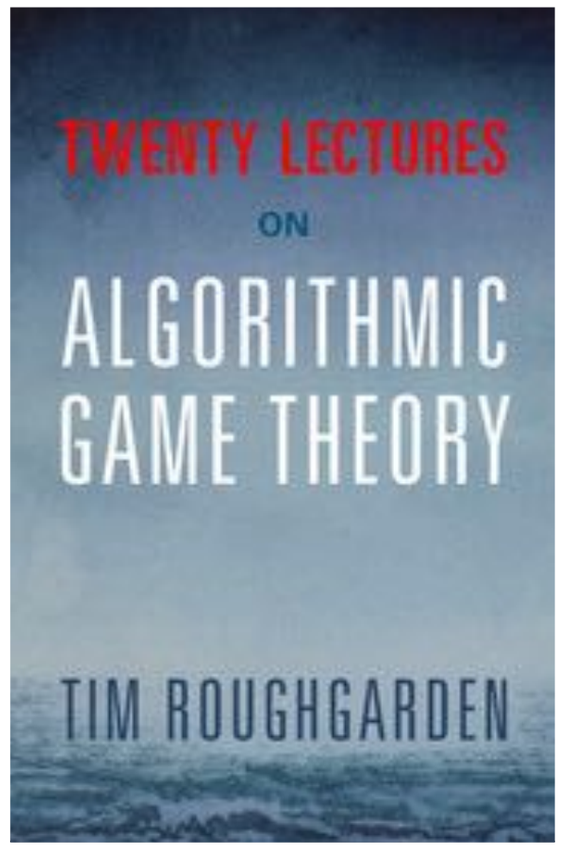
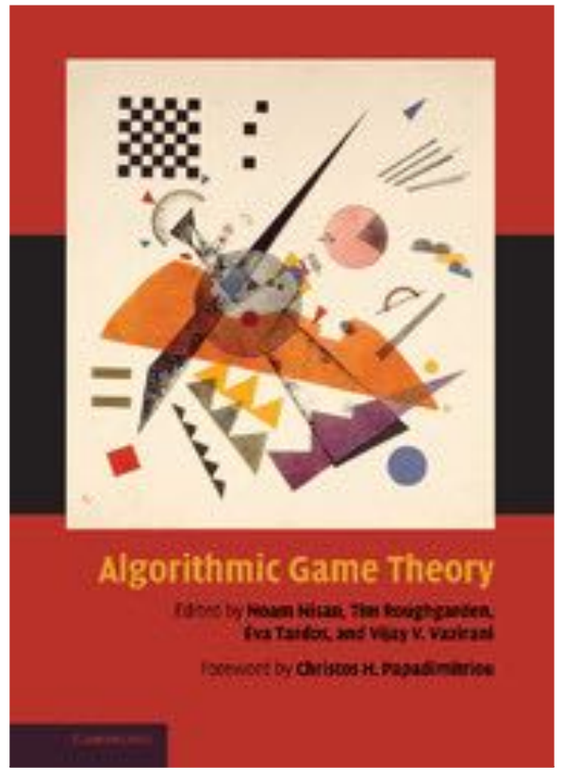
Administration

- Instructor: 刘正阳, zhengyang@bit.edu.cn
- TA: 陆皓霖, haolin@bit.edu.cn
- Time: Week 9-16 (Tue and Thu, 8:00 / 13:20)
- Location: F204, Wencui Building
- Grading: Homework (20*2) + Report (50) + Participation (10)
  - 100 - Of course, you can resolve research problems mentioned in class instead…
  - Or show me equivalent work…

Reference







(All these books have online versions.)

1. https://homes.cs.washington.edu/~karlin/GameTheoryBook.pdf
2. http://timroughgarden.org/f13/f13.pdf (Has Chinese version …)
3. https://www.cs.cmu.edu/~sandholm/cs15-892F13/algorithmic-game-theory.pdf
4. http://people.csail.mit.edu/costis/6853fa2011/

Game Theory

Incentive and Computation

- "We live in a highly *connected* world, with multiple *self-interested* agents interacting, leading to myriad opportunities for *conflict* and *cooperation*." [Preface in《Game Theory, Alive》]

- Games are thought experiments to help us learn how to predict rational behavior in situations of conflict or cooperation.

- Situation of conflict or cooperation: Everybody's actions affect others.

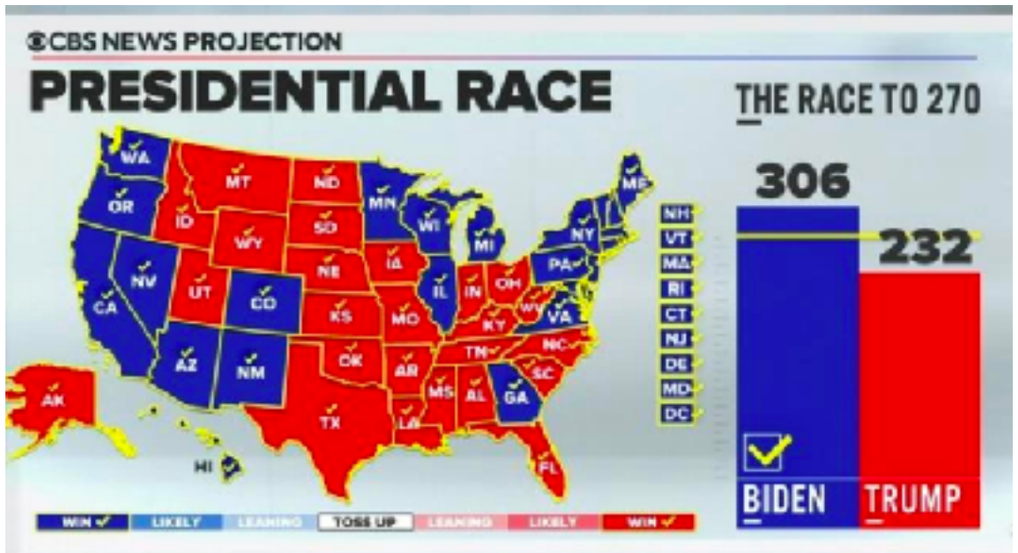
- Rational Behavior: The players want to maximize their own expected utility.

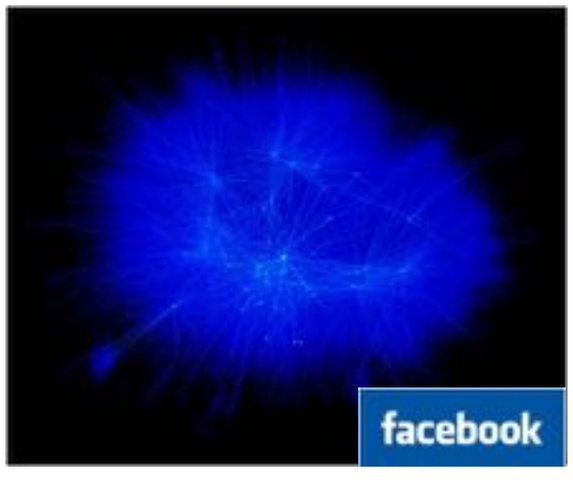
- Predict: We want to know what happens in a game. Such predictions are called solution concepts (e.g., Nash equilibrium).

Games v.s. Algorithms

Do you know/control everything in the system?

Game theory in real life?





Game Theory Basics

What is a game?

A set of Players/Agents

A set of Actions/Strategies

A set of Payoffs/Utilities

Nash Equilibrium

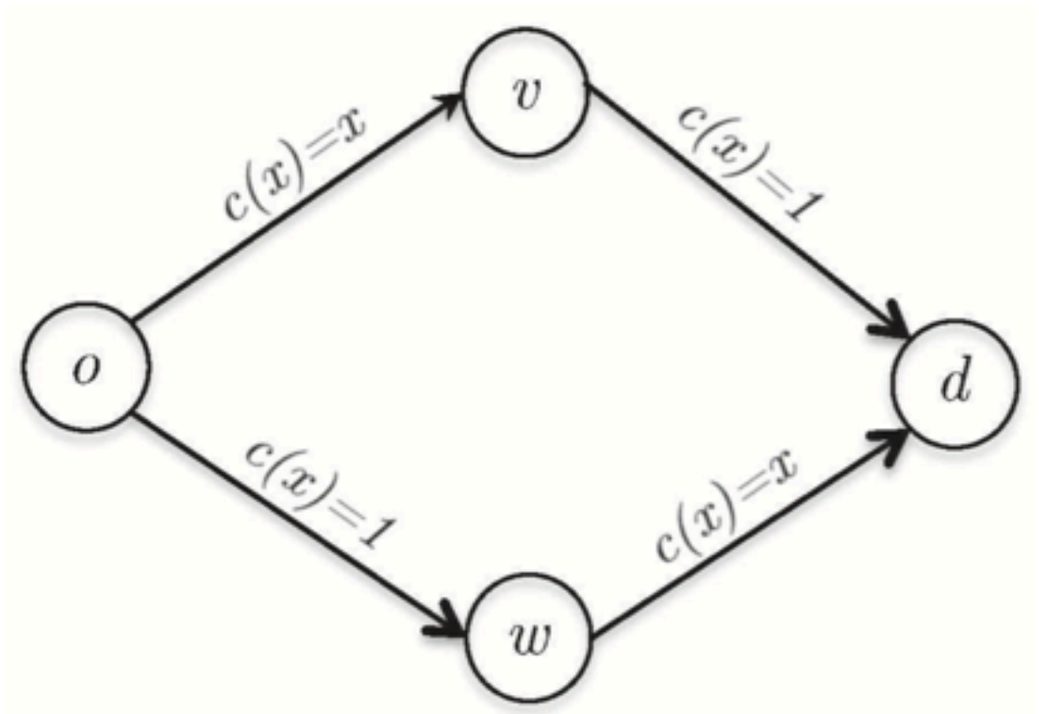
	S	P	R
S	0,0	1,-1	-1,1
P	-1,1	0,0	1,-1
R	1,-1	-1,1	0,0

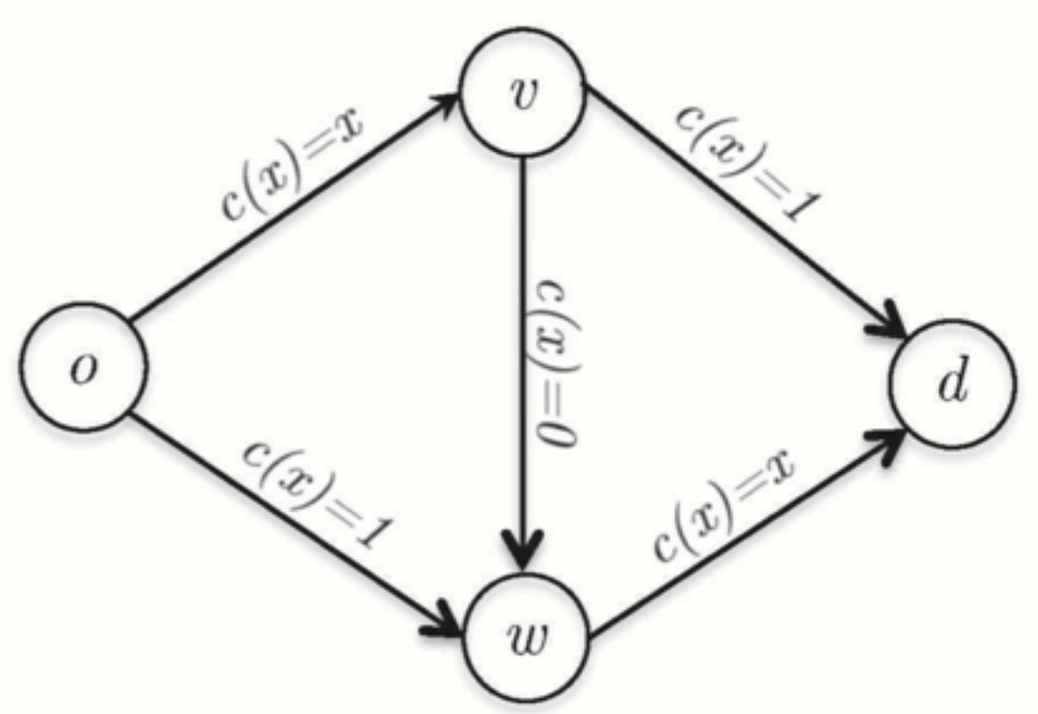
Informally, an equilibrium is a steady state. Is there a deterministic equilibrium?

We use randomness! Called mixed strategies.

Nash Equilibrium: A pair of strategies such that the strategy of the row player is a *Best Response* to the strategy of the column player and vice versa.

Traffic Routing





We define the price of anarchy (PoA) as the ratio between the system performance with strategic players and the best-possible system performance.

Mechanism Design

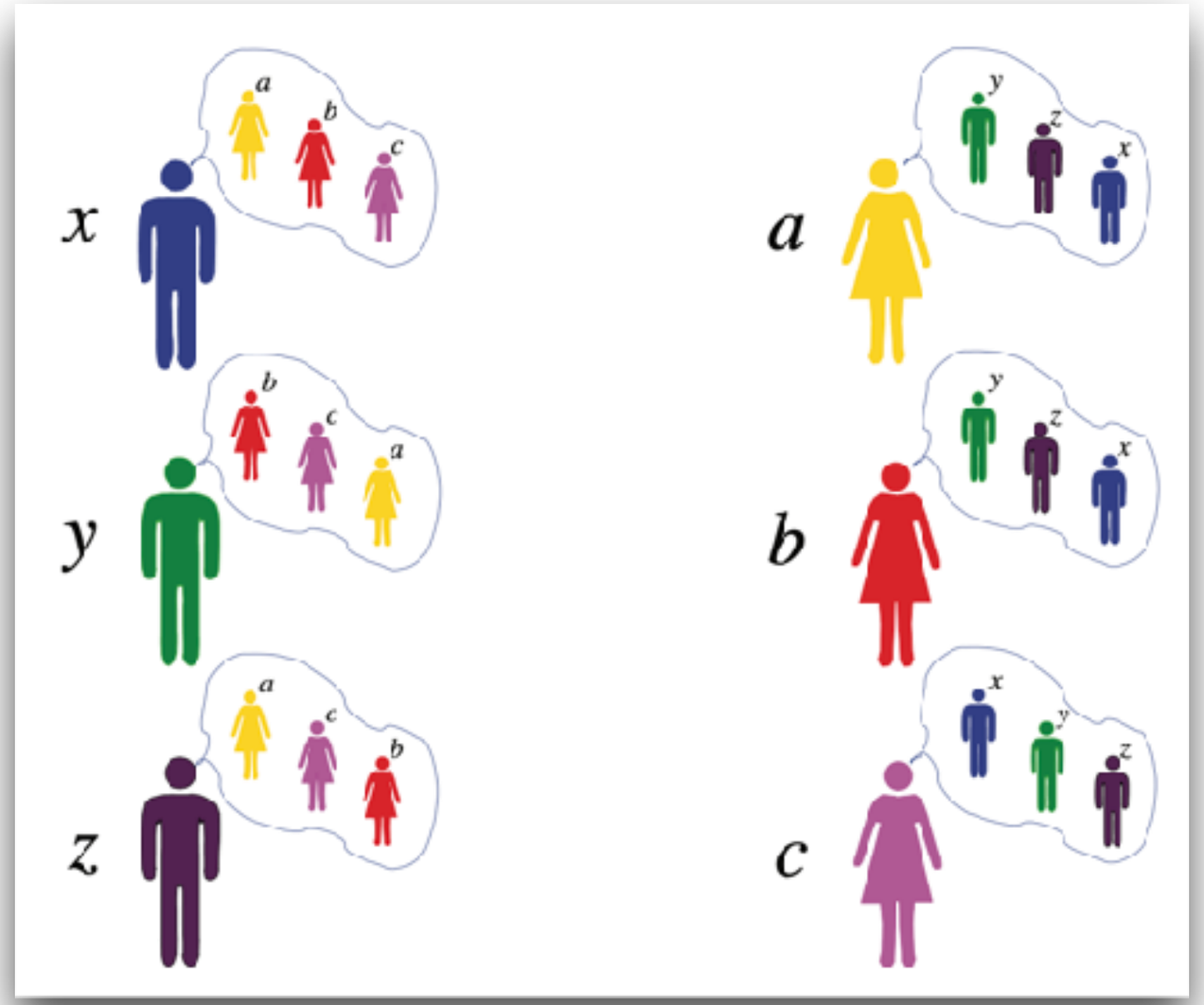
- In systems with strategic participants, the rules matters.
- Tian Ji's Horse Racing
- MD: to design rules so that strategic behaviors leads to a desirable outcome.
- Examples: internet Ads auctions, wireless spectrum auctions, the matching of medical residents to hospital, and even kidney exchange.

Stable Matching

- n men and n women
- Each one (man/woman) has a private preference list over women/men.
- Matching: one-to-one mapping between men and women, denoted by M.
- Unstable: two person who are not matched in M, but prefer each other to their partners in M, respectively. Otherwise, we say M is stable.

An unstable pair

An Example



$x \leftrightarrow a, y \leftrightarrow b, z \leftrightarrow c$ is an unstable matching!

Algorithms

Men-proposing version

1. Initially all men and women are unmatched.

2. Each man proposes to his most preferred woman who has not rejected him yet (or gives up if he's been rejected by all women).

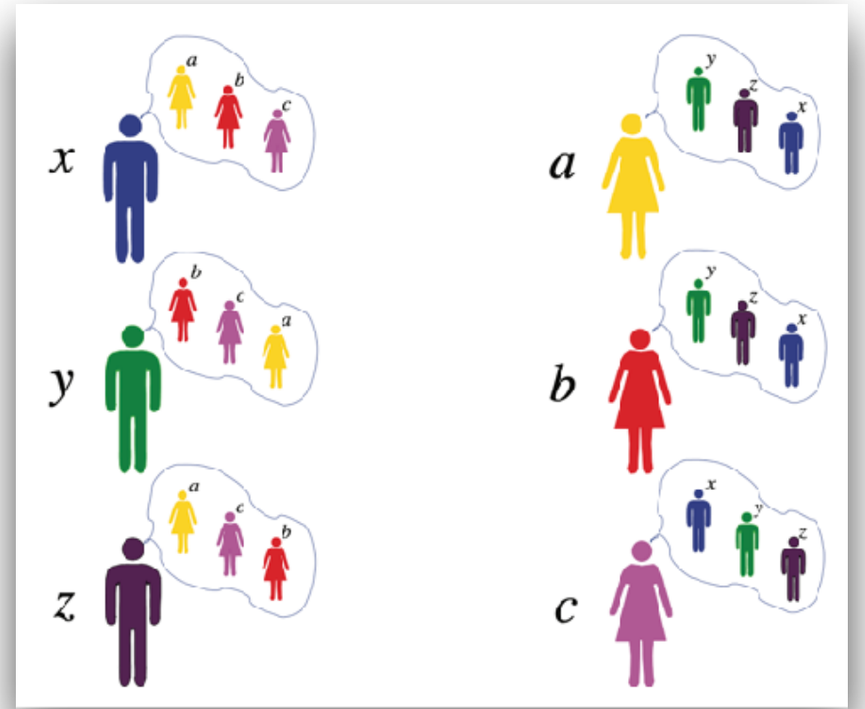
3. Each woman is tentatively matched to her favorite among her proposers and rejects the rest.

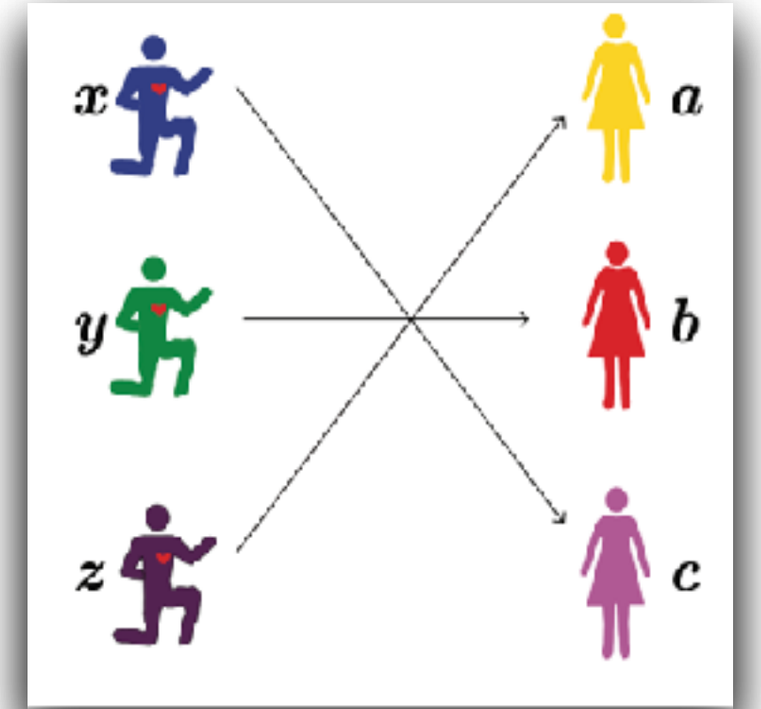
4. Repeat steps (2) and (3) until a round in which there are no rejections. At that point the tentative matches become final.

- If a man is tentatively matched to a woman in round k and the algorithm doesn't terminate, then he necessarily proposes to her again in round $k+1$.
- We can also describe the women-proposing algorithm.

Recall

The last example…

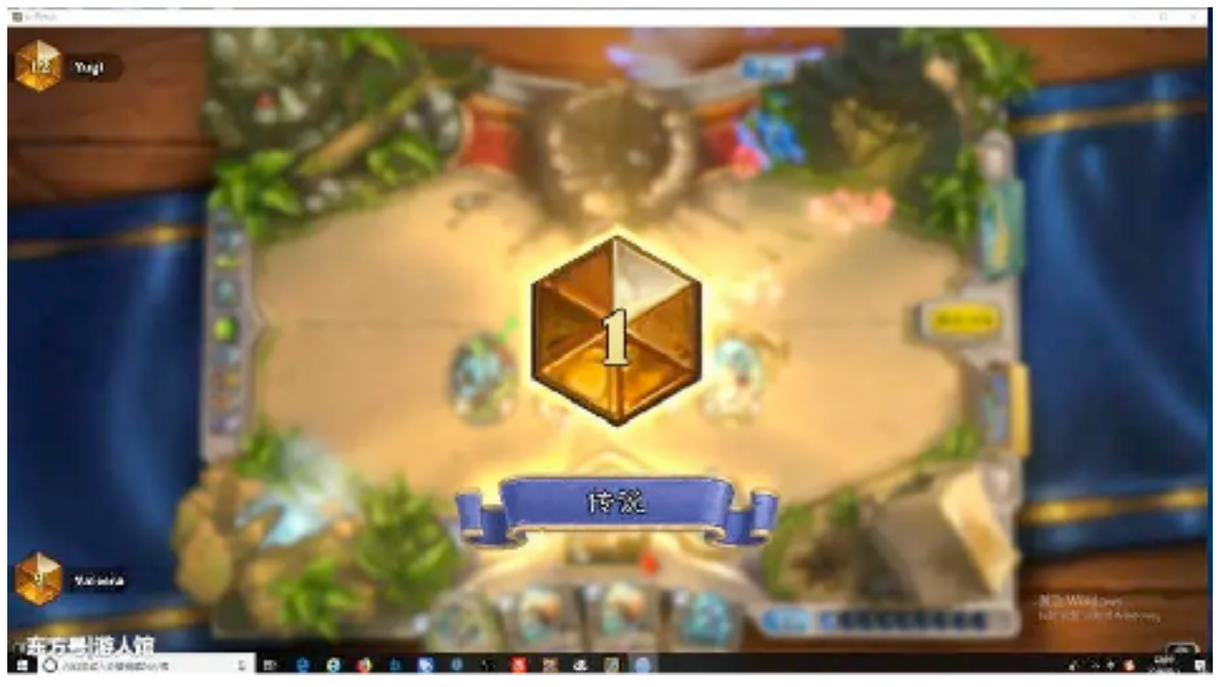


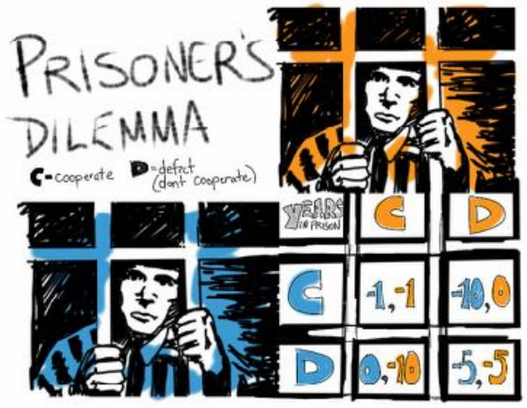


The men-proposing algorithm yields a stable matching.

Each men is matched to his most preferred attainable woman.

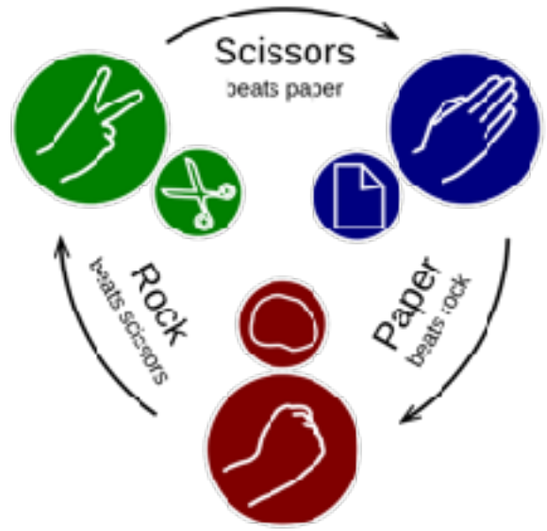
The goal of this class is not…

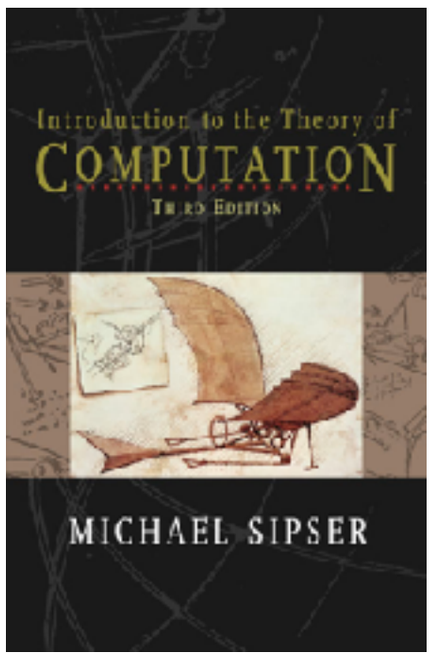




Incentive

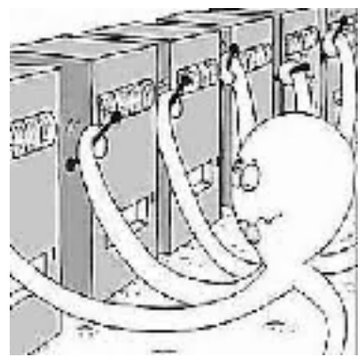
Games



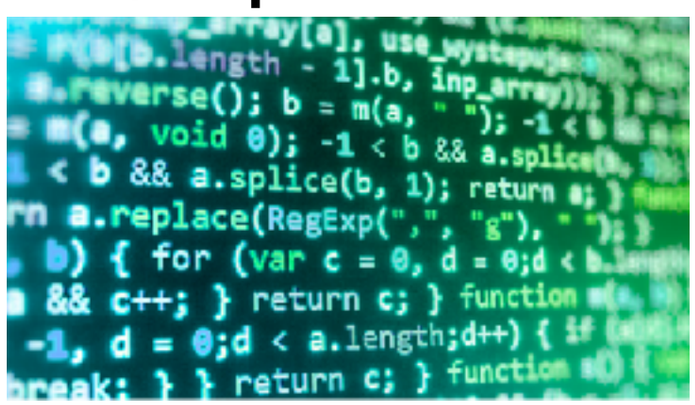


Algorithms

Learning



Computation



Randomness

Tentative Syllabus

- Strategic games and equilibrium concepts
- Congestion games
- Efficiency at equilibrium
- Proportional resource allocation
- Mechanism design: Single-parameter environemnts
- Mechanism design: Multi-parameter environments
- Computational social choice
- A utilitarian view of voting rules

From https://www.alexvoudouris.com/teaching.html

Q&A?

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