



Competitive Equilibrium with Chores: Combinatorial Algorithm and Hardness

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CCS Concepts: • **Theory of computation** → **Algorithmic game theory**.

Additional Key Words and Phrases: Competitive Equilibrium, Combinatorial Algorithm, PPAD-hardness, Chores

ACM Reference Format:

Bhaskar Ray Chaudhury, Jugal Garg, Peter McGlaughlin, and Ruta Mehta. 2022. Competitive Equilibrium with Chores: Combinatorial Algorithm and Hardness. In *Proceedings of the 23rd ACM Conference on Economics and Computation (EC '22), July 11–15, 2022, Boulder, CO, USA*. ACM, New York, NY, USA, 2 pages. <https://doi.org/10.1145/3490486.3538255>

We study the computational complexity of finding a competitive equilibrium (CE) with chores when agents have linear preferences. CE is one of the most preferred mechanisms for allocating a set of items among agents. CE with equal incomes (CEEI), Fisher, and Arrow-Debreu (exchange) are the fundamental economic models to study allocation problems, where CEEI is a special case of Fisher and Fisher is a special case of exchange. When the items are goods (giving utility), the CE set is convex even in the exchange model, facilitating several combinatorial polynomial-time algorithms (starting with the seminal work of Devanur, Papadimitriou, Saberi and Vazirani [2]) for all of these models. In sharp contrast, when the items are chores (giving disutility), the CE set is known to be non-convex and disconnected even in the CEEI model. Further, no combinatorial algorithms or hardness results are known for these models.

In this paper, we give two main results for CE with chores:

- A combinatorial algorithm to compute a $(1 - \epsilon)$ -approximate CEEI in time $\tilde{O}(n^4 m^2 / \epsilon^2)$, where n is the number of agents and m is the number of chores.
- PPAD-hardness of finding a $(1 - 1/\text{poly}(n))$ -approximate CE in the exchange model under a sufficient condition.

To the best of our knowledge, these results show the first separation between the CEEI and exchange models when agents have linear preferences, assuming $\text{PPAD} \neq \text{P}$. Furthermore, this is also the first separation between the two economic models when the CE set is non-convex in both cases.

Finally, we show that our new insight implies a straightforward proof of the existence of an allocation that is both envy-free up to one chore (EF1) and Pareto optimal (PO) in the discrete setting when agents have bivalued preferences. This result is recently obtained in [3, 4] using an involved analysis.

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EC '22, July 11–15, 2022, Boulder, CO, USA

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ACM ISBN 978-1-4503-9150-4/22/07.

<https://doi.org/10.1145/3490486.3538255>

The details can be found in the full version of the paper [1]

ACKNOWLEDGMENTS

Jugal Garg was supported by NSF Grant CCF-1942321 (CAREER) and Ruta Mehta was supported by NSF Grant CCF-1750436 (CAREER).

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