Strategic games and equilibrium concepts

Alexandros Voudouris

University of Oxford

Prisoner’s dilemma

- Two persons are caught by the police and are accused of a crime

Prisoner’s dilemma

- Two persons are caught by the police and are accused of a crime
- Each of them has two options:
  - **confess the crime**, or
  - **remain silent**

Prisoner’s dilemma

- Two persons are caught by the police and are accused of a crime
- Each of them has two options:
  - **confess the crime**, or
  - **remain silent**

- Every possible scenario (combination of actions of the two prisoners) yields some **payoff/cost** for them:

Prisoner’s dilemma

- Two persons are caught by the police and are accused of a crime
- Each of them has two options:
  - **confess the crime**, or
  - **remain silent**

- Every possible scenario (combination of actions of the two prisoners) yields some **payoff/cost** for them:
  - If both confess, they will go to prison for 3 years each

Prisoner's dilemma

- Two persons are caught by the police and are accused of a crime
- Each of them has two options:
  - **confess the crime**, or
  - **remain silent**

- Every possible scenario (combination of actions of the two prisoners) yields some **payoff/cost** for them:
  - If both confess, they will go to prison for 3 years each
  - If both remain silent, they will go to prison for only 1 year

Prisoner’s dilemma

- Two persons are caught by the police and are accused of a crime
- Each of them has two options:
  - **confess the crime**, or
  - **remain silent**

- Every possible scenario (combination of actions of the two prisoners) yields some **payoff/cost** for them:
  - If both confess, they will go to prison for 3 years each
  - If both remain silent, they will go to prison for only 1 year
  - If one confesses and the other remains silent, then the former will be set free and the latter will go to prison for 5 years

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?

- Assume that the column-prisoner confesses:

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?

- Assume that the column-prisoner confesses:
  - confessing yields 3 years in prison

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	-3, -3	0, -5
silent	-5, 0	-1, -1

- How does the row-prisoner think in order to find the best action?

- Assume that the column-prisoner confesses:
  – confessing yields 3 years in prison
  – remaining silent yields 5 years in prison

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?

- Assume that the column-prisoner confesses:
  - confessing yields 3 years in prison
  - remaining silent yields 5 years in prison
  - **best action = confess**

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?

- Assume that the column-prisoner remains silent:

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	-3, -3	0, -5
silent	-5, 0	-1, -1

- How does the row-prisoner think in order to find the best action?

- Assume that the column-prisoner remains silent:
  - confessing yields 0 years in prison

Prisoner’s dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	-3, -3	0, -5
silent	-5, 0	-1, -1

- How does the row-prisoner think in order to find the best action?
- Assume that the column-prisoner remains silent:
  - confessing yields 0 years in prison
  - remaining silent yields 1 year in prison

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?

- Assume that the column-prisoner remains silent:
  - confessing yields 0 years in prison
  - remaining silent yields 1 year in prison
  - **best action = confess**

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	**silent**
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?

- In any case, **confessing is the best action**, and the same holds for the column-prisoner due to symmetry

Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- How does the row-prisoner think in order to find the best action?
- In any case, **confessing is the best action**, and the same holds for the column-prisoner due to symmetry
- Confessing is a **dominant strategy** for both prisoners since, whatever the other prisoner does, this action is always better

Strategic games in general

Strategic games in general

- A set of **players**

Strategic games in general

- A set of **players**
- Each player has a set of possible **strategies** (actions)

Strategic games in general

- A set of **players**
- Each player has a set of possible **strategies** (actions)
- Each **state of the game** (defined by a strategy per player) yields a **payoff** (or **utility**) to each player

Strategic games in general

- A set of **players**
- Each player has a set of possible **strategies** (actions)
- Each **state of the game** (defined by a strategy per player) yields a **payoff** (or **utility**) to each player
- Given the strategies of the other players, each player aims to select its strategy in order to maximize its utility
  - Such a strategy is called a **best response**

Strategic games in general

- A set of **players**
- Each player has a set of possible **strategies** (actions)
- Each **state of the game** (defined by a strategy per player) yields a **payoff** (or **utility**) to each player
- Given the strategies of the other players, each player aims to select its strategy in order to maximize its utility
  - Such a strategy is called a **best response**
- A state consisting of best responses is stable, and called a **pure Nash equilibrium**: no player would like to deviate and select a different strategy

Back to prisoner's dilemma

- Players = the two prisoners
- Strategies = {confess, silent}
- Possible states = {(confess, confess), (confess, silent), (silent, confess), (silent, silent)}
- Utilities given by the bi-matrix:

	confess	silent
confess	**-3, -3**	**0, -5**
silent	**-5, 0**	**-1, -1**

- Confessing is a best response to any strategy of the other player
- (confess, confess) is a pure Nash equilibrium of the game

Battle of the sexes

- A couple (man and woman) want to decide what to do this evening; they can either attend a sports game or stay home and watch a movie
- They have different utilities for the two activities, but they would like to be together

Battle of the sexes

- A couple (man and woman) want to decide what to do this evening; they can either attend a sports game or stay home and watch a movie
- They have different utilities for the two activities, but they would like to be together

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

- How does the woman think?

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

- How does the woman think?
- If the man chooses sports, then she also prefers sports (3 vs. 2)

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

- How does the woman think?
- If the man chooses sports, then she also prefers sports (3 vs. 2)
- If the man chooses movie, then she also prefers movie (6 vs. 1)

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

- There is **no dominant strategy** for the woman (nor for the man)
- What is the equilibrium strategy profile then?

Battle of the sexes

		man: sports	man: movie
woman	sports	3, 6	1, 1
	movie	2, 2	6, 3

- Is the state (movie, sports) an equilibrium?

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

- Is the state (movie, sports) an equilibrium?
- No, the woman would prefer to **unilaterally change** her strategy to sports:
  - the state (sports, sports) gives her utility 3, while now she only gets utility 2

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

- Is the state (sports, sports) an equilibrium?

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

- Is the state (sports, sports) an equilibrium?
- Yes, none of the two players has incentive to unilaterally change its strategy:
  - a deviation to movie would give utility 1 to the man and 2 to the woman, compared to the utility of 6 and 3 they now get

Nash dynamics graph

- An easy way to graphically find Nash equilibria
- Built a graph containing a node per state
- A directed edge between two nodes represents the fact that there exists a player with a profitable unilateral deviation
- A node with only incoming edges corresponds to an equilibrium state: no player would like to deviate from there

Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

sports, sports

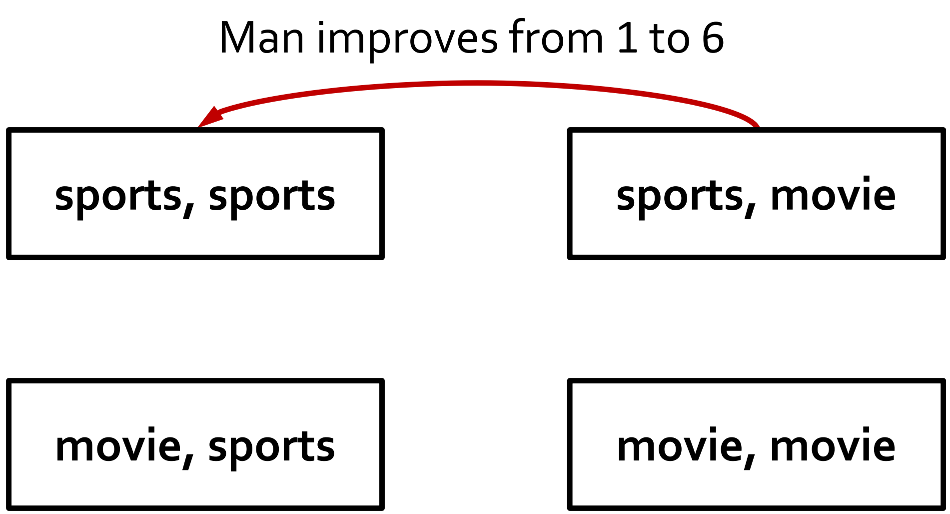
sports, movie

movie, sports

movie, movie

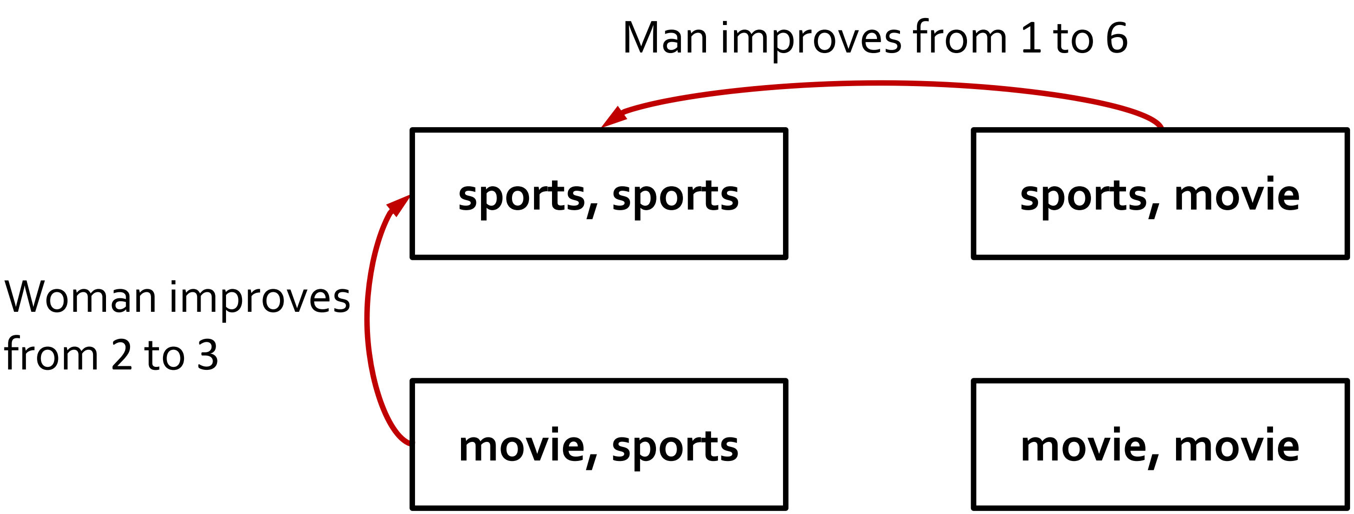
Battle of the sexes

		man	
		sports	movie
woman	sports	3, 6	1, 1
	movie	2, 2	6, 3



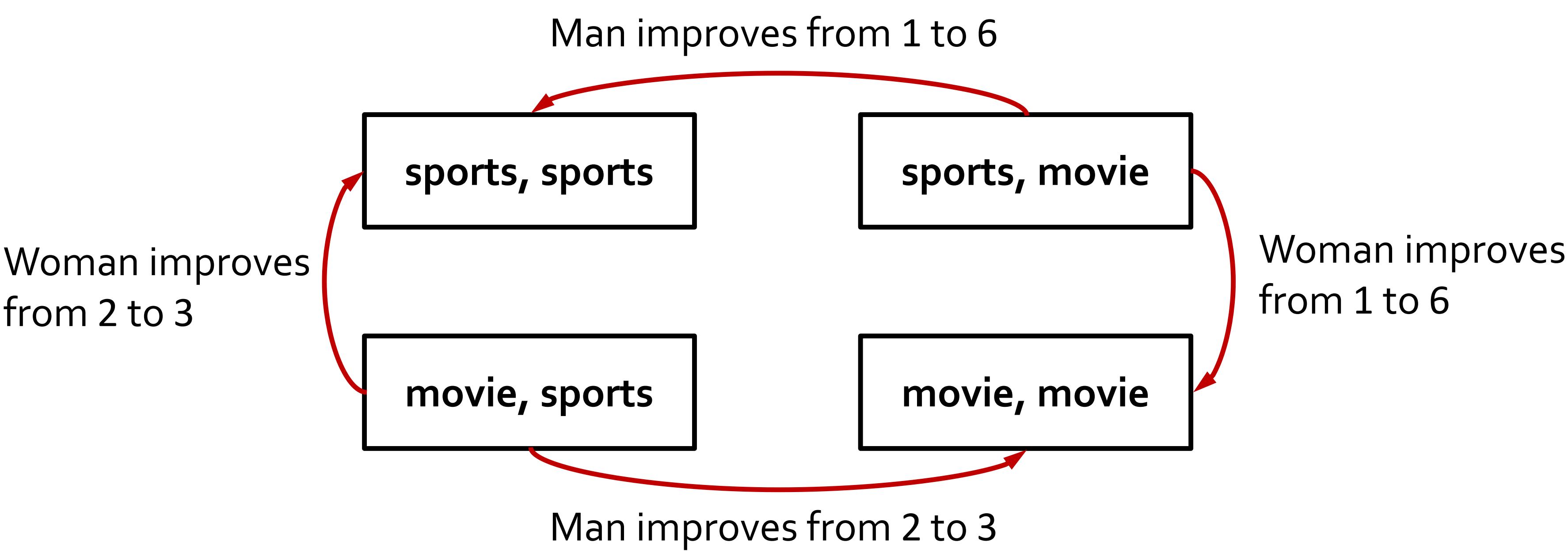
Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**



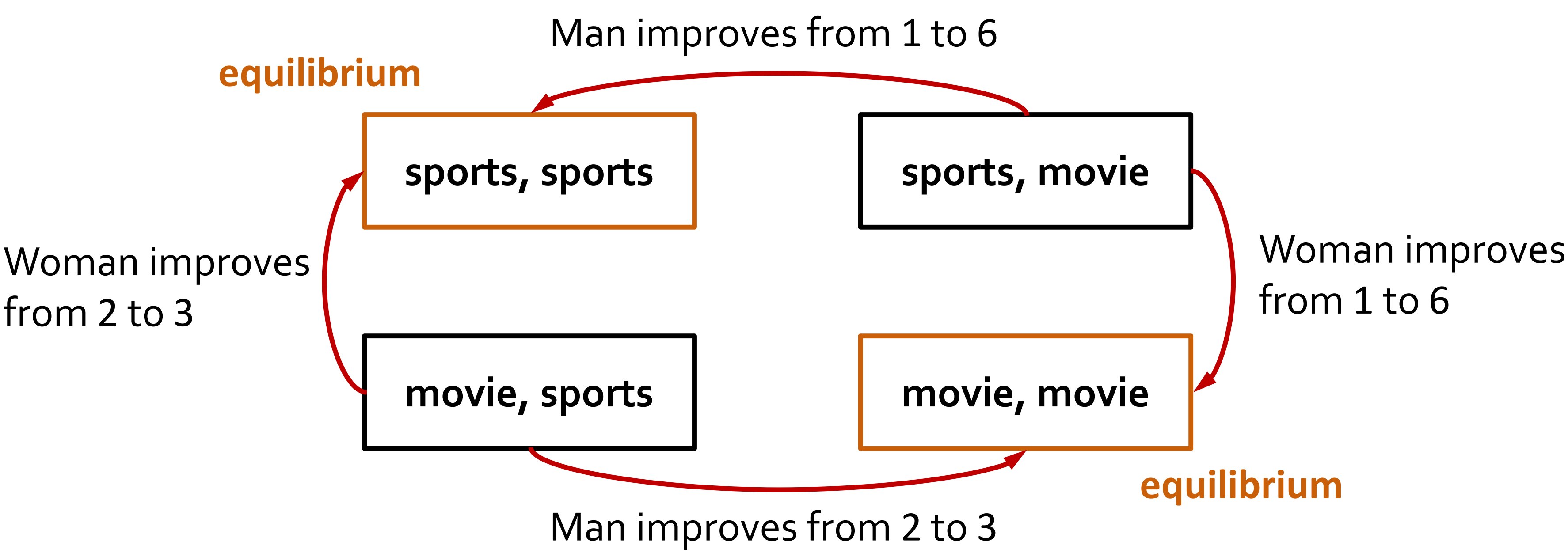
Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**



Battle of the sexes

		man	
		sports	**movie**
woman	**sports**	**3, 6**	**1, 1**
	movie	**2, 2**	**6, 3**

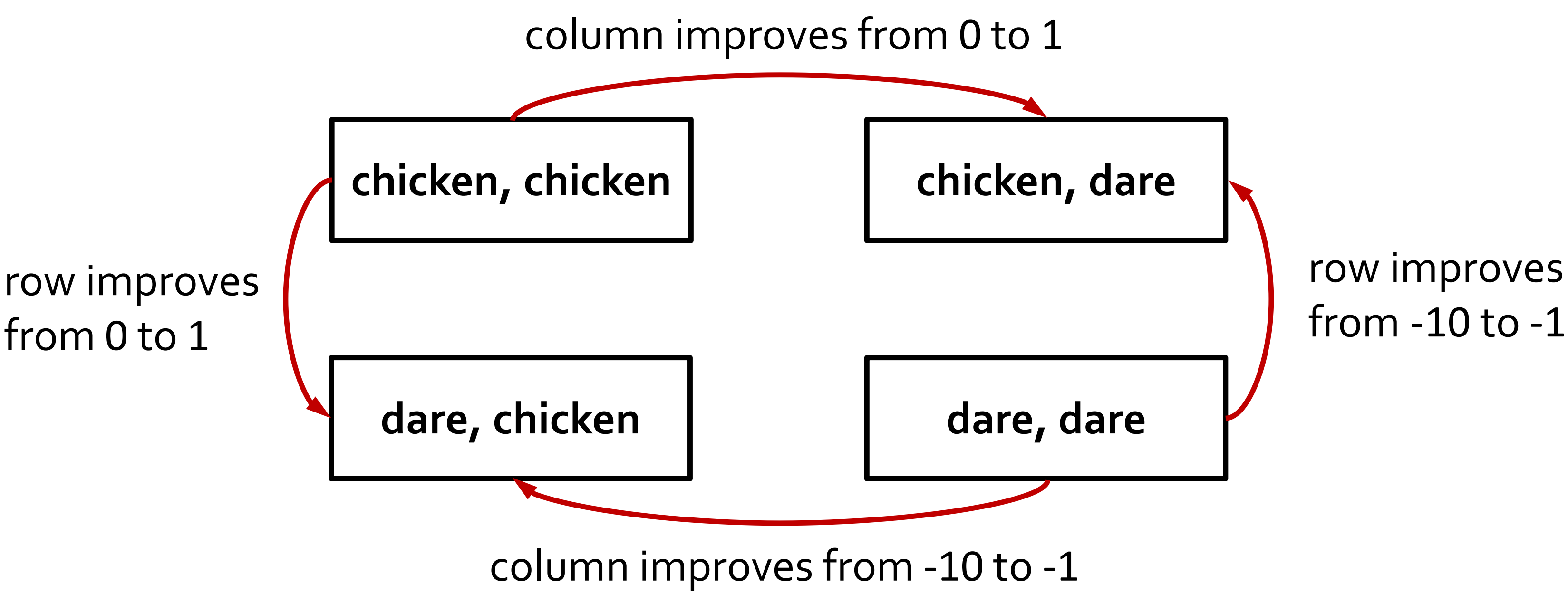


Chicken

		column-driver	
		chicken	**dare**
row-driver	**chicken**	**0, 0**	**-1, 1**
	dare	**1, -1**	**-10, -10**

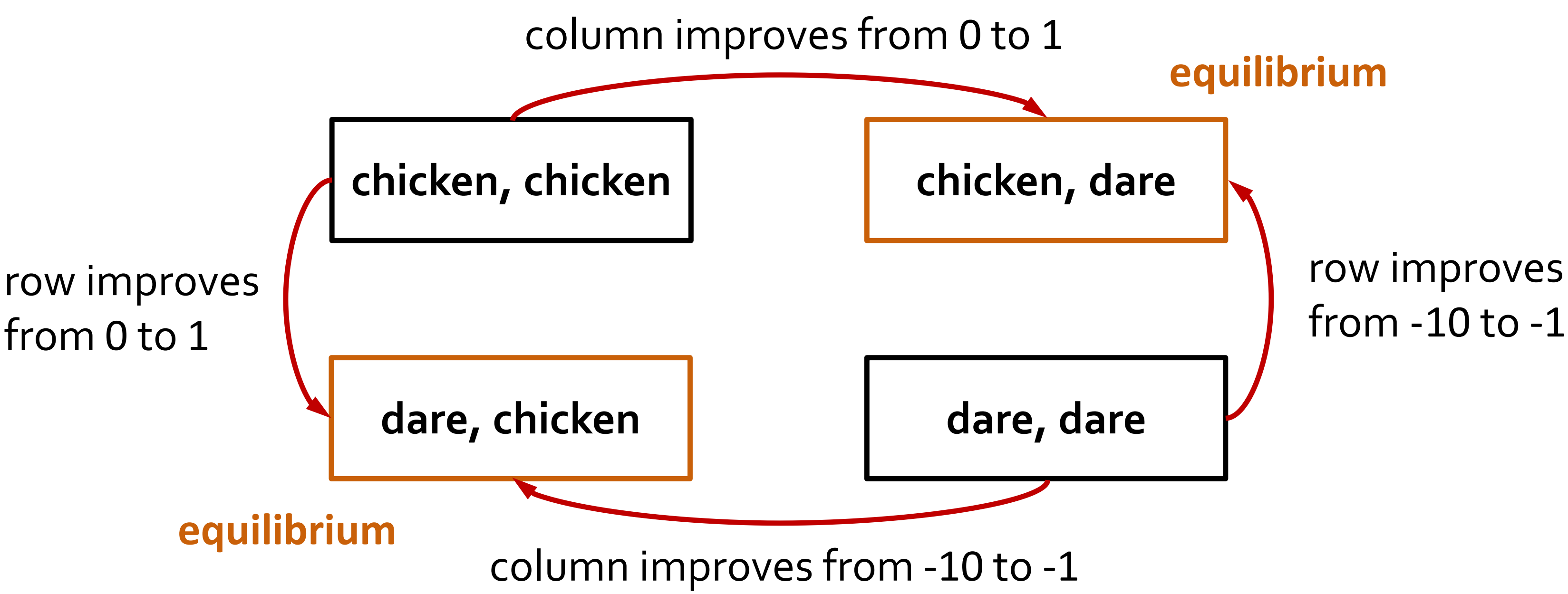
Chicken

		column-driver	
		chicken	**dare**
row-driver	**chicken**	**0, 0**	**-1, 1**
	dare	**1, -1**	**-10, -10**



Chicken

		column-driver	
		chicken	**dare**
row-driver	**chicken**	**0, 0**	**-1, 1**
	dare	**1, -1**	**-10, -10**

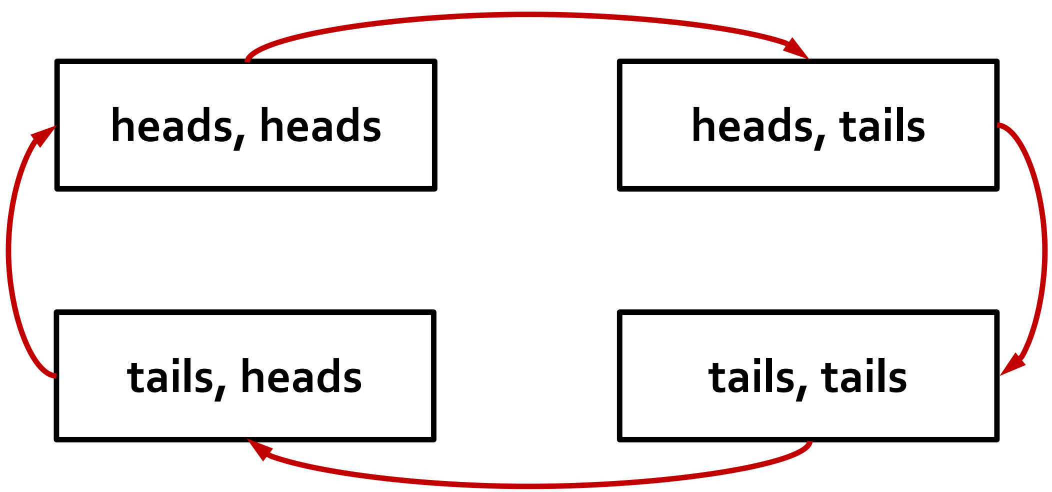


Matching pennies

		odd: heads	odd: tails
even	heads	1, -1	-1, 1
even	tails	-1, 1	1, -1

Matching pennies

			odd	
			heads	tails
even	heads		1, -1	-1, 1
	tails		-1, 1	1, -1



Mixed strategies

- **Not all games have pure equilibria**

Mixed strategies

- **Not all games have pure equilibria**
- What if we allow the players to randomize over their strategies?

Mixed strategies

- **Not all games have pure equilibria**
- What if we allow the players to randomize over their strategies?

- S_i = set of pure strategies for player i

Mixed strategies

- **Not all games have pure equilibria**
- What if we allow the players to randomize over their strategies?

- S_i = set of pure strategies for player i
- A **mixed strategy** for player i defines a probability $p_i(a)$ for each strategy $a \in S_i$ such that $\sum_{a \in S_i} p_i(a) = 1$

Mixed strategies

- **Not all games have pure equilibria**
- What if we allow the players to randomize over their strategies?

- S_i = set of pure strategies for player i
- A **mixed strategy** for player i defines a probability $p_i(a)$ for each strategy $a \in S_i$ such that $\sum_{a \in S_i} p_i(a) = 1$

- The game is at a state $\boldsymbol{s} = (s_1, s_2, \dots, s_n)$ with probability

$$p(\boldsymbol{s}) = p_1(s_1) \cdot p_2(s_2) \cdot \dots \cdot p_n(s_n) = \prod_i p_i(s_i)$$

Mixed strategies

- **Not all games have pure equilibria**
- What if we allow the players to randomize over their strategies?

- S_i = set of pure strategies for player i
- A **mixed strategy** for player i defines a probability $p_i(a)$ for each strategy $a \in S_i$ such that $\sum_{a \in S_i} p_i(a) = 1$

- The game is at a state $\boldsymbol{s} = (s_1, s_2, \dots, s_n)$ with probability

$$p(\boldsymbol{s}) = p_1(s_1) \cdot p_2\,(s_2) \cdot \dots \cdot p_n\,(s_n) = \prod_i p_i(s_i)$$

- The **expected utility** of player i is then

$$\mathbb{E}_p[u_i] = \sum_{\boldsymbol{s}} p(\boldsymbol{s}) \cdot u_i(\boldsymbol{s})$$

Matching pennies

		odd: heads	odd: tails	
even	heads	1, -1	-1, 1	0.8
	tails	-1, 1	1, -1	0.2
		0.4	0.6	

Matching pennies

		odd	
		heads	**tails**
even	**heads**	**1, -1**	**-1, 1**	**0.8**
	tails	**-1, 1**	**1, -1**	**0.2**
		0.4	**0.6**

- $p(\text{heads, heads}) = 0.8 \cdot 0.4 = 0.32$
- $p(\text{heads, tails}) = 0.8 \cdot 0.6 = 0.48$
- $p(\text{tails, heads}) = 0.2 \cdot 0.4 = 0.08$
- $p(\text{tails, tails}) = 0.2 \cdot 0.6 = 0.12$

Matching pennies

		odd: heads	odd: tails	
even	heads	1, -1	-1, 1	0.8
	tails	-1, 1	1, -1	0.2
		0.4	0.6	

- $p(\text{heads, heads}) = 0.8 \cdot 0.4 = 0.32$
- $p(\text{heads, tails}) = 0.8 \cdot 0.6 = 0.48$
- $p(\text{tails, heads}) = 0.2 \cdot 0.4 = 0.08$
- $p(\text{tails, tails}) = 0.2 \cdot 0.6 = 0.12$

- $\mathbb{E}_p[u_\text{e}] = 0.32 \cdot 1 + 0.48 \cdot (-1) + 0.08 \cdot (-1) + 0.12 \cdot 1 = -0.12$
- $\mathbb{E}_p[u_\text{o}] = 0.32 \cdot (-1) + 0.48 \cdot 1 + 0.08 \cdot 1 + 0.12 \cdot (-1) = 0.12$

Mixed equilibria

- **Mixed equilibrium:** A profile of *mixed* strategies such that each player maximizes its expected utility, given the strategies of the other players

Mixed equilibria

- **Mixed equilibrium:** A profile of *mixed* strategies such that each player maximizes its expected utility, given the strategies of the other players

<u>**Theorem**</u> [Nash, 1951]

Every finite strategic game of n players has at least one mixed equilibrium

Mixed equilibria

- **Mixed equilibrium:** A profile of *mixed* strategies such that each player maximizes its expected utility, given the strategies of the other players

Theorem [Nash, 1951]

Every finite strategic game of n players has at least one mixed equilibrium

- Every pure equilibrium is also a mixed equilibrium
  - Every pure strategy can be seen as a probability distribution over all strategies that assigns probability 1 to this one pure strategy

Matching Pennies: mixed equilibria

		odd	
		heads	**tails**
even	**heads**	**1, -1**	**-1, 1**
	tails	**-1, 1**	**1, -1**

- Even player selects heads with probability x and tails with $1 - x$
- Odd player selects heads with probability y and tails with $1 - y$

Matching Pennies: mixed equilibria

		odd	
		heads	**tails**
even	**heads**	**1, -1**	**-1, 1**
	tails	**-1, 1**	**1, -1**

- Even player selects heads with probability x and tails with $1 - x$
- Odd player selects heads with probability y and tails with $1 - y$

- $p(\text{heads, heads}) = xy$
- $p(\text{heads, tails}) = x(1 - y)$
- $p(\text{tails, heads}) = (1 - x)y$
- $p(\text{tails, tails}) = (1 - x)(1 - y)$

Matching Pennies: mixed equilibria

		odd		
		heads	**tails**	
even	**heads**	**1, -1**	**-1, 1**	x
	tails	**-1, 1**	**1, -1**	$1-x$
		y	$1-y$	

- $\mathbb{E}_p[u_{\mathrm{e}}]$
  $= xy \cdot 1 + x(1-y) \cdot (-1) + (1-x)y \cdot (-1) + (1-x)(1-y) \cdot 1$

Matching Pennies: mixed equilibria

		odd: heads	odd: tails	
even	heads	**1, -1**	**-1, 1**	x
	tails	**-1, 1**	**1, -1**	$1-x$
		y	$1-y$	

- $\mathbb{E}_p[u_{\mathrm{e}}]$
  $= xy \cdot 1 + x(1-y) \cdot (-1) + (1-x)y \cdot (-1) + (1-x)(1-y) \cdot 1$
  $= 4xy - 2x - 2y + 1$
  $= \boldsymbol{x(4y-2) - 2y + 1}$

Matching Pennies: mixed equilibria

		odd		
		heads	**tails**	
even	**heads**	**1, -1**	**-1, 1**	x
	tails	**-1, 1**	**1, -1**	$1-x$
		y	$1-y$	

- $\mathbb{E}_p[u_{\mathrm{e}}]$
  $= xy \cdot 1 + x(1-y) \cdot (-1) + (1-x)y \cdot (-1) + (1-x)(1-y) \cdot 1$
  $= 4xy - 2x - 2y + 1$
  $= \boldsymbol{x(4y-2) - 2y + 1}$

- $\mathbb{E}_p[u_{\mathrm{o}}]$
  $= xy \cdot (-1) + x(1-y) \cdot 1 + (1-x)y \cdot 1 + (1-x)(1-y) \cdot (-1)$
  $= \boldsymbol{y(2-4x) + 2x - 1}$

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathsf{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathsf{o}}] = y(2 - 4x) + 2x - 1$

- The expected utility of each player is a **linear function** in terms of her corresponding probability

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2 - 4x) + 2x - 1$

- The expected utility of each player is a **linear function** in terms of her corresponding probability
- To analyze how a player is going to act, we need to see whether the slope of the linear function is negative or positive

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_\mathrm{e}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_\mathrm{o}] = y(2 - 4x) + 2x - 1$

- The expected utility of each player is a **linear function** in terms of her corresponding probability
- To analyze how a player is going to act, we need to see whether the slope of the linear function is negative or positive

- **Negative:** the function is decreasing and the player aims to set a small value for the probability

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathsf{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathsf{o}}] = y(2-4x) + 2x - 1$

- The expected utility of each player is a **linear function** in terms of her corresponding probability
- To analyze how a player is going to act, we need to see whether the slope of the linear function is negative or positive

- **Negative:** the function is decreasing and the player aims to set a small value for the probability
- **Positive:** the function is increasing and the players aims to set a high value for the probability

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2 - 4x) + 2x - 1$

- **Even player:** the slope is $4y - 2$ and it depends on y, the probability with which the odd player selects heads

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y < 1/2}$

  ⇨ the slope $4y-2$ is **negative**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\text{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\text{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y < 1/2}$
  ⇨ the slope $4y-2$ is **negative**
  ⇨ the function $\mathbb{E}_p[u_{\text{e}}]$ is **decreasing in $\boldsymbol{x}$**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y < 1/2}$

  ⇨ the slope $4y-2$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_{\mathrm{e}}]$ is **decreasing in $\boldsymbol{x}$**

  ⇨ even player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_{\mathrm{e}}]$

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathsf{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathsf{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y < 1/2}$
  - ⇨ the slope $4y-2$ is **negative**
  - ⇨ the function $\mathbb{E}_p[u_{\mathsf{e}}]$ is **decreasing in $\boldsymbol{x}$**
  - ⇨ even player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_{\mathsf{e}}]$
  - ⇨ the slope $2-4x = 2$ of the odd player is **positive**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_\mathrm{e}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_\mathrm{o}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y < 1/2}$
  - ⇨ the slope $4y-2$ is **negative**
  - ⇨ the function $\mathbb{E}_p[u_\mathrm{e}]$ is **decreasing in $\boldsymbol{x}$**
  - ⇨ even player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_\mathrm{e}]$
  - ⇨ the slope $2-4x = 2$ of the odd player is **positive**
  - ⇨ the function $\mathbb{E}_p[u_\mathrm{o}]$ is **increasing in $\boldsymbol{y}$**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\text{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\text{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y < 1/2}$

  ⇨ the slope $4y-2$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_{\text{e}}]$ is **decreasing in $\boldsymbol{x}$**

  ⇨ even player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_{\text{e}}]$

  ⇨ the slope $2-4x = 2$ of the odd player is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\text{o}}]$ is **increasing in $\boldsymbol{y}$**

  ⇨ odd player sets $\boldsymbol{y = 1}$ to maximize $\mathbb{E}_p[u_{\text{o}}]$

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\text{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\text{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y < 1/2}$

  ⇨ the slope $4y-2$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_{\text{e}}]$ is **decreasing in** $\boldsymbol{x}$

  ⇨ even player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_{\text{e}}]$

  ⇨ the slope $2-4x = 2$ of the odd player is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\text{o}}]$ is **increasing in** $\boldsymbol{y}$

  ⇨ odd player sets $\boldsymbol{y = 1}$ to maximize $\mathbb{E}_p[u_{\text{o}}]$

  ⇨ **contradiction**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y > 1/2}$

  ⇨ the slope $4y-2$ is **positive**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y > 1/2}$

  ⇨ the slope $4y-2$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\mathrm{e}}]$ is **increasing in $\boldsymbol{x}$**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y > 1/2}$

  ⇨ the slope $4y-2$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\mathrm{e}}]$ is **increasing in $\boldsymbol{x}$**

  ⇨ even player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_{\mathrm{e}}]$

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y > 1/2}$
  ⇨ the slope $4y-2$ is **positive**
  ⇨ the function $\mathbb{E}_p[u_{\mathrm{e}}]$ is **increasing in $\boldsymbol{x}$**
  ⇨ even player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_{\mathrm{e}}]$
  ⇨ the slope $2-4x = -2$ of the odd player is **negative**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\text{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\text{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y > 1/2}$
  - ⇨ the slope $4y-2$ is **positive**
  - ⇨ the function $\mathbb{E}_p[u_{\text{e}}]$ is **increasing in $\boldsymbol{x}$**
  - ⇨ even player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_{\text{e}}]$
  - ⇨ the slope $2-4x = -2$ of the odd player is **negative**
  - ⇨ the function $\mathbb{E}_p[u_{\text{o}}]$ is **decreasing in $\boldsymbol{y}$**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y-2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2-4x) + 2x - 1$

- **Even player:** the slope is $4y-2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y > 1/2}$

  ⇨ the slope $4y-2$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\mathrm{e}}]$ is **increasing in $\boldsymbol{x}$**

  ⇨ even player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_{\mathrm{e}}]$

  ⇨ the slope $2-4x = -2$ of the odd player is **negative**

  ⇨ the function $\mathbb{E}_p[u_{\mathrm{o}}]$ is **decreasing in $\boldsymbol{y}$**

  ⇨ odd player sets $\boldsymbol{y = 0}$ to maximize $\mathbb{E}_p[u_{\mathrm{o}}]$

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_\mathrm{e}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_\mathrm{o}] = y(2 - 4x) + 2x - 1$

- **Even player:** the slope is $4y - 2$ and it depends on y, the probability with which the odd player selects heads
- $\boldsymbol{y > 1/2}$

  ⇨ the slope $4y - 2$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_\mathrm{e}]$ is **increasing in $\boldsymbol{x}$**

  ⇨ even player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_\mathrm{e}]$

  ⇨ the slope $2 - 4x = -2$ of the odd player is **negative**

  ⇨ the function $\mathbb{E}_p[u_\mathrm{o}]$ is **decreasing in $\boldsymbol{y}$**

  ⇨ odd player sets $\boldsymbol{y = 0}$ to maximize $\mathbb{E}_p[u_\mathrm{o}]$

  ⇨ **contradiction**

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2 - 4x) + 2x - 1$

- It must be $\boldsymbol{y = 1/2}$

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2 - 4x) + 2x - 1$

- It must be $\boldsymbol{y = 1/2}$
- Following the same reasoning for the odd player, we can see that it must also be $\boldsymbol{x = 1/2}$

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2 - 4x) + 2x - 1$

- It must be $\boldsymbol{y = 1/2}$
- Following the same reasoning for the odd player, we can see that it must also be $\boldsymbol{x = 1/2}$
- For these values of x and y both slopes are equal to 0 and the linear functions are maximized

Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2 - 4x) + 2x - 1$

- It must be $\boldsymbol{y = 1/2}$
- Following the same reasoning for the odd player, we can see that it must also be $\boldsymbol{x = 1/2}$
- For these values of x and y both slopes are equal to 0 and the linear functions are maximized
- The pair $(x, y) = (1/2, 1/2)$ corresponds to a mixed equilibrium, which is actually unique for this game

Unbalanced coordination

- Two players with two possible strategies A and B
- If both players select A, they get one point
- If both of them select B, they get two points
- If the select different strategies, they get zero points

		col player	
		A	**B**
row player	**A**	**1, 1**	**0, 0**
	B	**0, 0**	**2, 2**

Unbalanced coordination

- Two players with two possible strategies A and B
- If both players select A, they get one point
- If both of them select B, they get two points
- If the select different strategies, they get zero points

		col player A	col player B
row player	A	1, 1	0, 0
	B	0, 0	2, 2

- Easy to verify that (A, A) and (B, B) are pure equilibria
- Are there any other mixed equilibria?

Unbalanced coordination

		col player	
		A	B
row player	A	1, 1	0, 0
	B	0, 0	2, 2

- row player selects A with probability x and B with $1 - x$
- col player selects A with probability y and B with $1 - y$

- $p(\text{A}, \text{A}) = xy$
- $p(\text{A}, \text{B}) = x(1 - y)$
- $p(\text{B}, \text{A}) = (1 - x)y$
- $p(\text{B}, \text{B}) = (1 - x)(1 - y)$

Unbalanced coordination

			col player		
			A	**B**	
row player	**A**		**1, 1**	**0, 0**	x
	B		**0, 0**	**2, 2**	$1-x$
			y	$1-y$	

- $\mathbb{E}_p[u_{\mathrm{r}}]$
  $= xy \cdot 1 + x(1-y) \cdot 0 + (1-x)y \cdot 0 + (1-x)(1-y) \cdot 2$
  $= \boldsymbol{x(3y-2)+2-2y}$

- $\mathbb{E}_p[u_{\mathrm{c}}]$
  $= xy \cdot 1 + x(1-y) \cdot 0 + (1-x)y \cdot 0 + (1-x)(1-y) \cdot 2$
  $= \boldsymbol{y(3x-2)+2-2y}$

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x - 2) + 2 - 2x$

Unbalanced coordination

- $\mathbb{E}_p[u_\mathsf{r}] = x(3y-2) + 2 - 2y$
- $\mathbb{E}_p[u_\mathsf{c}] = y(3x-2) + 2 - 2x$

- $\boldsymbol{y < 2/3}$
  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is **negative**

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x - 2) + 2 - 2x$

- $\boldsymbol{y < 2/3}$
  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_{\mathsf{r}}]$ is **negative**
  ⇨ the function $\mathbb{E}_p[u_{\mathsf{r}}]$ is **decreasing in $\boldsymbol{x}$**

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathrm{r}}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathrm{c}}] = y(3x - 2) + 2 - 2x$

- $\boldsymbol{y < 2/3}$

  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_{\mathrm{r}}]$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_{\mathrm{r}}]$ is **decreasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_{\mathrm{r}}]$

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y-2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x-2) + 2 - 2x$

- $\boldsymbol{y < 2/3}$

  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_{\mathsf{r}}]$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_{\mathsf{r}}]$ is **decreasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_{\mathsf{r}}]$

  ⇨ the slope $3x - 2 = -2$ of $\mathbb{E}_p[u_{\mathsf{c}}]$ is **negative**

Unbalanced coordination

- $\mathbb{E}_p[u_\mathrm{r}] = x(3y-2)+2-2y$
- $\mathbb{E}_p[u_\mathrm{c}] = y(3x-2)+2-2x$

- $\boldsymbol{y < 2/3}$

  ⇨ the slope $3y-2$ of $\mathbb{E}_p[u_\mathrm{r}]$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_\mathrm{r}]$ is **decreasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_\mathrm{r}]$

  ⇨ the slope $3x-2=-2$ of $\mathbb{E}_p[u_\mathrm{c}]$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_\mathrm{c}]$ is **decreasing in $\boldsymbol{y}$**

Unbalanced coordination

- $\mathbb{E}_p[u_\mathsf{r}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_\mathsf{c}] = y(3x - 2) + 2 - 2x$

- $\boldsymbol{y < 2/3}$

  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_\mathsf{r}]$ is **decreasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_\mathsf{r}]$

  ⇨ the slope $3x - 2 = -2$ of $\mathbb{E}_p[u_\mathsf{c}]$ is **negative**

  ⇨ the function $\mathbb{E}_p[u_\mathsf{c}]$ is **decreasing in $\boldsymbol{y}$**

  ⇨ column player sets $\boldsymbol{y = 0}$ to maximize $\mathbb{E}_p[u_\mathsf{c}]$

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y-2)+2-2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x-2)+2-2x$

- $\boldsymbol{y < 2/3}$
  ⇨ the slope $3y-2$ of $\mathbb{E}_p[u_{\mathsf{r}}]$ is **negative**
  ⇨ the function $\mathbb{E}_p[u_{\mathsf{r}}]$ is **decreasing in $\boldsymbol{x}$**
  ⇨ row player sets $\boldsymbol{x = 0}$ to maximize $\mathbb{E}_p[u_{\mathsf{r}}]$
  ⇨ the slope $3x-2=-2$ of $\mathbb{E}_p[u_{\mathsf{c}}]$ is **negative**
  ⇨ the function $\mathbb{E}_p[u_{\mathsf{c}}]$ is **decreasing in $\boldsymbol{y}$**
  ⇨ column player sets $\boldsymbol{y = 0}$ to maximize $\mathbb{E}_p[u_{\mathsf{c}}]$

- $(x,y) = (0,0)$ is a mixed equilibrium
- We already knew that: it corresponds to the pure equilibrium (A, A)

Unbalanced coordination

- $\mathbb{E}_p[u_\mathsf{r}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_\mathsf{c}] = y(3x - 2) + 2 - 2x$

- $\boldsymbol{y > 2/3}$

  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is **positive**

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y-2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x-2) + 2 - 2x$

- $\boldsymbol{y > 2/3}$

  ⇨ the slope $3y-2$ of $\mathbb{E}_p[u_{\mathsf{r}}]$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\mathsf{r}}]$ is **increasing in $\boldsymbol{x}$**

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x - 2) + 2 - 2x$

- $\boldsymbol{y > 2/3}$

  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_{\mathsf{r}}]$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\mathsf{r}}]$ is **increasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_{\mathsf{r}}]$

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y-2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x-2) + 2 - 2x$

- $\boldsymbol{y > 2/3}$

  ⇨ the slope $3y-2$ of $\mathbb{E}_p[u_{\mathsf{r}}]$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_{\mathsf{r}}]$ is **increasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_{\mathsf{r}}]$

  ⇨ the slope $3x-2 = 1$ of $\mathbb{E}_p[u_{\mathsf{c}}]$ is **positive**

Unbalanced coordination

- $\mathbb{E}_p[u_\mathsf{r}] = x(3y-2) + 2 - 2y$
- $\mathbb{E}_p[u_\mathsf{c}] = y(3x-2) + 2 - 2x$

- $\boldsymbol{y > 2/3}$
  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is **positive**
  ⇨ the function $\mathbb{E}_p[u_\mathsf{r}]$ is **increasing in $\boldsymbol{x}$**
  ⇨ row player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_\mathsf{r}]$
  ⇨ the slope $3x - 2 = 1$ of $\mathbb{E}_p[u_\mathsf{c}]$ is **positive**
  ⇨ the function $\mathbb{E}_p[u_\mathsf{c}]$ is **increasing in $\boldsymbol{y}$**

Unbalanced coordination

- $\mathbb{E}_p[u_\mathsf{r}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_\mathsf{c}] = y(3x - 2) + 2 - 2x$

- $\boldsymbol{y > 2/3}$

  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_\mathsf{r}]$ is **increasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_\mathsf{r}]$

  ⇨ the slope $3x - 2 = 1$ of $\mathbb{E}_p[u_\mathsf{c}]$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_\mathsf{c}]$ is **increasing in $\boldsymbol{y}$**

  ⇨ column player sets $\boldsymbol{y = 1}$ to maximize $\mathbb{E}_p[u_\mathsf{c}]$

Unbalanced coordination

- $\mathbb{E}_p[u_\mathsf{r}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_\mathsf{c}] = y(3x - 2) + 2 - 2x$

- $\boldsymbol{y > 2/3}$

  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_\mathsf{r}]$ is **increasing in $\boldsymbol{x}$**

  ⇨ row player sets $\boldsymbol{x = 1}$ to maximize $\mathbb{E}_p[u_\mathsf{r}]$

  ⇨ the slope $3x - 2 = 1$ of $\mathbb{E}_p[u_\mathsf{c}]$ is **positive**

  ⇨ the function $\mathbb{E}_p[u_\mathsf{c}]$ is **increasing in $\boldsymbol{y}$**

  ⇨ column player sets $\boldsymbol{y = 1}$ to maximize $\mathbb{E}_p[u_\mathsf{c}]$

- $(x, y) = (1, 1)$ is a mixed equilibrium corresponding to the pure equilibrium (B, B)

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x - 2) + 2 - 2x$

- For $\boldsymbol{x < 2/3}$ and $\boldsymbol{x > 2/3}$ we will reach to the same conclusion

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y-2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x-2) + 2 - 2x$
- For $\boldsymbol{x < 2/3}$ and $\boldsymbol{x > 2/3}$ we will reach to the same conclusion
- It remains to see what is going on for $\boldsymbol{x = 2/3}$ and $\boldsymbol{y = 2/3}$

Unbalanced coordination

- $\mathbb{E}_p[u_\mathsf{r}] = x(3y-2) + 2 - 2y$
- $\mathbb{E}_p[u_\mathsf{c}] = y(3x-2) + 2 - 2x$

- For $\boldsymbol{x < 2/3}$ and $\boldsymbol{x > 2/3}$ we will reach to the same conclusion

- It remains to see what is going on for $\boldsymbol{x = 2/3}$ and $\boldsymbol{y = 2/3}$

- For $y = 2/3$ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is zero and $\mathbb{E}_p[u_\mathsf{r}]$ is maximized by any choice of x, including $x = 2/3$

Unbalanced coordination

- $\mathbb{E}_p[u_\mathrm{r}] = x(3y-2)+2-2y$
- $\mathbb{E}_p[u_\mathrm{c}] = y(3x-2)+2-2x$

- For $\boldsymbol{x < 2/3}$ and $\boldsymbol{x > 2/3}$ we will reach to the same conclusion

- It remains to see what is going on for $\boldsymbol{x = 2/3}$ and $\boldsymbol{y = 2/3}$

- For $y = 2/3$ the slope $3y-2$ of $\mathbb{E}_p[u_\mathrm{r}]$ is zero and $\mathbb{E}_p[u_\mathrm{r}]$ is maximized by any choice of x, including $x = 2/3$
- For $x = 2/3$ the slope $3x-2$ of $\mathbb{E}_p[u_\mathrm{c}]$ is zero and $\mathbb{E}_p[u_\mathrm{c}]$ is maximized by any choice of y, including $y = 2/3$

Unbalanced coordination

- $\mathbb{E}_p[u_{\mathsf{r}}] = x(3y - 2) + 2 - 2y$
- $\mathbb{E}_p[u_{\mathsf{c}}] = y(3x - 2) + 2 - 2x$

- For $\boldsymbol{x < 2/3}$ and $\boldsymbol{x > 2/3}$ we will reach to the same conclusion

- It remains to see what is going on for $\boldsymbol{x = 2/3}$ and $\boldsymbol{y = 2/3}$

- For $y = 2/3$ the slope $3y - 2$ of $\mathbb{E}_p[u_{\mathsf{r}}]$ is zero and $\mathbb{E}_p[u_{\mathsf{r}}]$ is maximized by any choice of x, including $x = 2/3$
- For $x = 2/3$ the slope $3x - 2$ of $\mathbb{E}_p[u_{\mathsf{c}}]$ is zero and $\mathbb{E}_p[u_{\mathsf{c}}]$ is maximized by any choice of y, including $y = 2/3$

- $(x, y) = (2/3, 2/3)$ is a fully mixed equilibrium of the game

Summary

Summary

- **Dominant strategy:** a strategy that is always the best response

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies
- **Mixed equilibrium:** every player selects a mixed strategy that is a best response to the mixed strategies of the other players

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies
- **Mixed equilibrium:** every player selects a mixed strategy that is a best response to the mixed strategies of the other players
- There is always at least one mixed equilibrium (finite games)

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies
- **Mixed equilibrium:** every player selects a mixed strategy that is a best response to the mixed strategies of the other players
- There is always at least one mixed equilibrium (finite games)
- Every pure equilibrium is a mixed equilibrium

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies
- **Mixed equilibrium:** every player selects a mixed strategy that is a best response to the mixed strategies of the other players
- There is always at least one mixed equilibrium (finite games)
- Every pure equilibrium is a mixed equilibrium
- **Computing mixed equilibria in 2x2 games:** define a parameterized probability distribution per player,

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies
- **Mixed equilibrium:** every player selects a mixed strategy that is a best response to the mixed strategies of the other players
- There is always at least one mixed equilibrium (finite games)
- Every pure equilibrium is a mixed equilibrium
- **Computing mixed equilibria in 2x2 games:** define a parameterized probability distribution per player, compute the probability distribution over the states of the game,

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies
- **Mixed equilibrium:** every player selects a mixed strategy that is a best response to the mixed strategies of the other players
- There is always at least one mixed equilibrium (finite games)
- Every pure equilibrium is a mixed equilibrium
- **Computing mixed equilibria in 2x2 games:** define a parameterized probability distribution per player, compute the probability distribution over the states of the game, compute the expected utility of each player and write it as a linear function of its parameter,

Summary

- **Dominant strategy:** a strategy that is always the best response
- **Pure equilibrium:** every player selects a best response, and has no incentive to deviate
- Pure equilibria are not guaranteed to exist
- **Mixed strategy:** a probability distribution over the set of strategies
- **Mixed equilibrium:** every player selects a mixed strategy that is a best response to the mixed strategies of the other players
- There is always at least one mixed equilibrium (finite games)
- Every pure equilibrium is a mixed equilibrium
- **Computing mixed equilibria in 2x2 games:** define a parameterized probability distribution per player, compute the probability distribution over the states of the game, compute the expected utility of each player and write it as a linear function of its parameter, argue about the slope (negative, positive, zero)

Bibliography

- Handbook of Algorithmic Game Theory
  - http://www.cs.cmu.edu/~sandholm/cs15-892F13/algorithmic-game-theory.pdf
- Networks, Crowds and Markets
  - http://www.cs.cornell.edu/home/kleinber/networks-book/networks-book.pdf
- Twenty Lectures on Algorithmic Game Theory
  - http://timroughgarden.org/notes.html