



# Truthful Mechanisms for Two-Sided Markets via Prophet Inequalities

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CCS Concepts: • **Theory of computation** → **Algorithmic mechanism design**; *Stochastic approximation*.

Additional Key Words and Phrases: Mechanism Design; Two-Sided Markets; Double Auctions; Bilateral Trade; Prophet Inequalities

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## EXTENDED ABSTRACT

We design novel mechanisms for welfare-maximization in two-sided markets. That is, there are buyers willing to purchase items and sellers holding items initially, both acting rationally and strategically in order to maximize utility. Our mechanisms are designed based on a powerful correspondence between two-sided markets and prophet inequalities. They satisfy individual rationality, dominant-strategy incentive compatibility, budget-balance constraints and give constant-factor approximations to the optimal social welfare.

In order to illustrate our approach, consider the special case of bilateral trades. That is, there is one seller holding a single item and one buyer. We aim to (re-)allocate the item in order to maximize social welfare. Bilateral trade instances admit a very simple mechanism template: Let  $v_s$  denote the seller's value and  $v_b$  denote the buyer's value for the item. Both are drawn independently from some probability distributions. Fix a price  $p$  and trade the item if and only if  $v_b \geq p \geq v_s$ . Among others, Blumrosen and Dobzinski [1, 2] and Gerstgrasser et al. [9] set  $p$  to be the median of the seller's distribution which recovers an expected welfare of at least  $\frac{1}{2} \cdot \mathbb{E} [\max\{v_s, v_b\}]$ , so it is a  $\frac{1}{2}$ -approximation.

In contrast to previous work, we interpret this simple mechanism as a sequential posted-prices mechanism with price  $p$ : The mechanism first asks seller  $s$  if she would like to keep or try selling the item for price  $p$ . Afterwards, buyer  $b$  may purchase the item for price  $p$  if the seller accepted a trade. This setting is conceptually similar to posting a price  $p$  in a one-sided market with two buyers. The only difference is that if both values are below  $p$  the seller keeps the item whereas in the one-sided market it is assumed to be discarded.

Due to this correspondence, we can easily lower-bound the social welfare via *prophet inequalities*. In particular, one can also recover half of the optimal social welfare as follows: Instead of applying a pricing strategy via quantiles, one could also use a *balanced price* [6]  $p = \frac{1}{2} \cdot \mathbb{E} [\max\{v_s, v_b\}]$ . A proof of the approximation guarantee directly follows by the above considerations via standard prophet inequality results [8, 10] for  $n = 2$ .

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This observation immediately raises the question whether all problems in two-sided markets can be solved via posted-prices mechanisms and prophet inequalities in such a straightforward way. We might interpret sellers as buyers, consider them first and ask which items they would like to keep, afterwards offer the remaining items to buyers. Unfortunately, this brings about a number of issues, as e.g. budget balance is not guaranteed because the payments by the buyers will usually not match what we promise the sellers.

Nonetheless, based on the technique of balanced prices from prophet inequalities, we are able to design mechanisms for two-sided markets which improve the state-of-the-art approximation guarantees for several settings: Our main focus is on matroid double auctions [4, 7], where the set of buyers who obtain an item needs to be independent in a matroid. We construct two mechanisms, the first being a  $1/3$ -approximation of the optimal social welfare satisfying strong budget-balance and requiring the agents to trade in a customized order, the second being a  $1/2$ -approximation, weakly budget-balanced and able to deal with online arrival determined by an adversary. In addition, we construct constant-factor approximations in two-sided markets when buyers need to fulfill a knapsack constraint [7]. Also, in combinatorial double auctions [1, 5], where buyers have valuation functions over item bundles instead of being interested in only one item, using similar techniques, we design a mechanism which is a  $1/2$ -approximation of the optimal social welfare, strongly budget-balanced and can deal with online arrival of agents in an adversarial order.

Details can be found in the full version [3] on arxiv: <https://arxiv.org/abs/2105.15032>

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