Computational social choice

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Making decisions

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- Each of the friends has preferences over the restaurants:
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  - Bob prefers White Rabbit the most, and Zizzi to Franco Manca
  - Carol prefers Franco Manca the most, and White Rabbit to Zizzi

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- How should they decide where to go?

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  - Bob prefers White Rabbit the most, and Zizzi to Franco Manca
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- How should they decide where to go?
- They can vote!

Making decisions

- There are many ways to vote however

Making decisions

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- One way is for everyone to vote only for their favourite restaurant, and then choose the restaurant with the most votes:
  - Alice and Carol vote Franco Manca, and Bob votes White Rabbit
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Making decisions

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- One way is for everyone to vote only for their favourite restaurant, and then choose the restaurant with the most votes:
  - Alice and Carol vote Franco Manca, and Bob votes White Rabbit
  - Franco Manca is chosen
- But, observe that Bob really doesn't like Franco Manca
- Another way is for everyone to veto their most disliked restaurant, and then choose the restaurant with the least vetos
  - Alice and Carol veto Zizzi, and Bob vetos Franco Manca
  - White Rabbit is chosen

Making decisions

- One more way is to count for each restaurant the number of restaurants it beats in pairwise comparisons, and then choose the restaurant with the most wins:
  - Franco Manca beats both White Rabbit and Zizzi twice
  - White Rabbit beats Franco Manca once, and Zizzi three times
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- Franco Manca and White Rabbit have 4 wins each

- The decision depends on how this tie is broken
- For example, using the pairwise comparison between these two restaurants, Franco Manca is finally chosen

Our setting

- A set of n **agents**: $N = \{1,2, \dots , n\}$
- A set of m **alternatives**: $A = \{a_1, a_2, \dots , a_m\}$

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agent	ranking			
1	b	d	a	c
2	d	a	c	b
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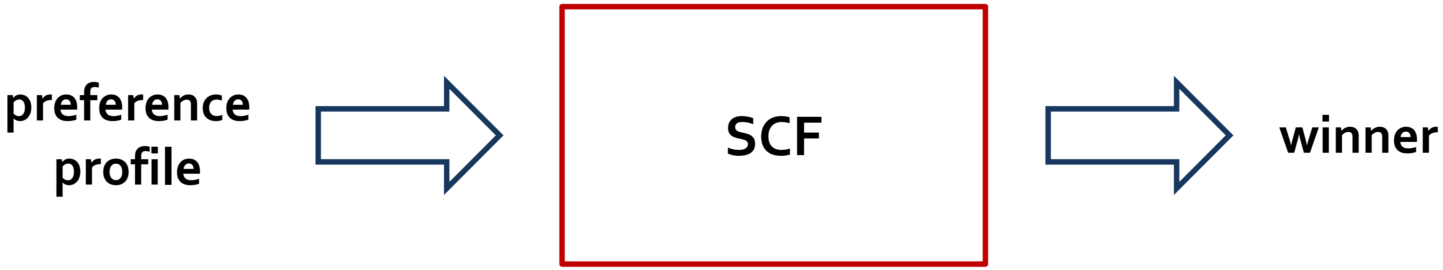
- Our **goal** is to select an alternative or come up with a ranking over all alternatives, by taking into account the preferences of the agents

Social choice and welfare functions

- A social choice function (SCF) takes as input a preference profile, and outputs a winning alternative



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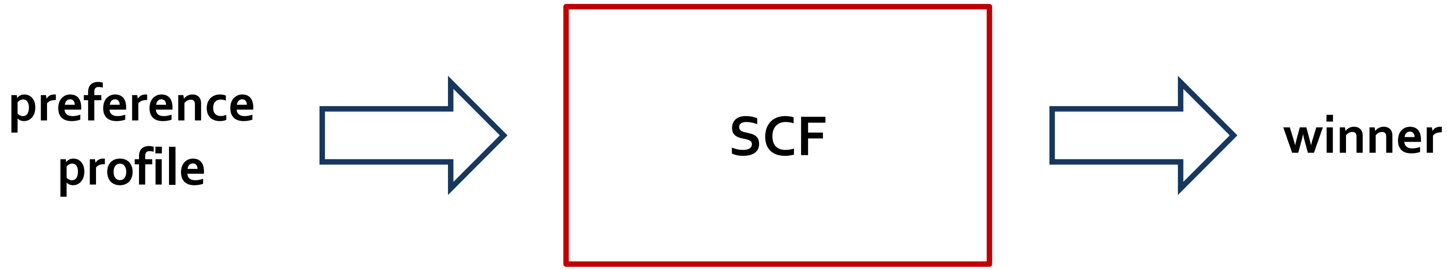
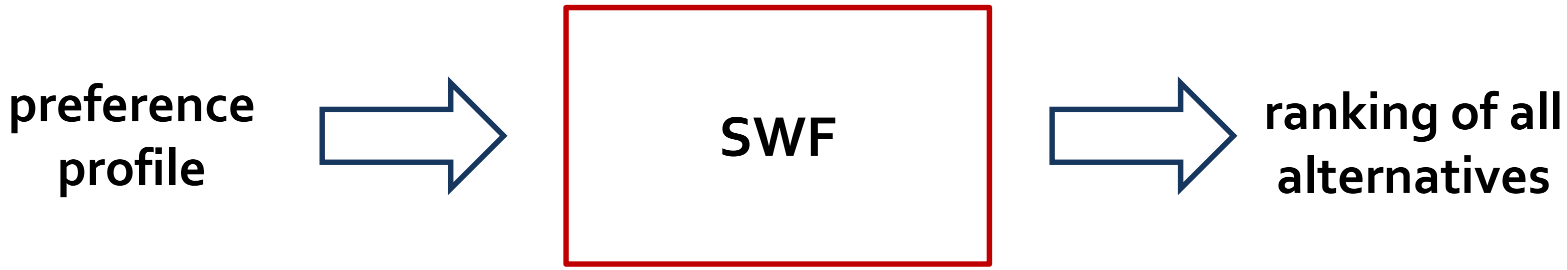


- A social welfare function (SWF) takes as input a preference profile, and outputs a complete ranking of all alternatives



Positional scoring rules

- A PSR is defined by a scoring vector of size m: $\boldsymbol{s} = (s_1, s_2, \dots, s_m)$
- For every agent, the alternative that is ranked k-th gets s_k points
- The alternatives are ranked according to their total points

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$\boldsymbol{s}$	4	2	1	0

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alternative	points
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1	$\boldsymbol{b}$	d	a	c
2	$\boldsymbol{d}$	a	c	b
3	$\boldsymbol{d}$	c	a	b
4	$\boldsymbol{a}$	b	c	d

$\boldsymbol{s}$	**4**	2	1	0

alternative	points
a	4
b	4
c	0
d	8

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agent	ranking			
1	b	$\boldsymbol{d}$	a	c
2	d	$\boldsymbol{a}$	c	b
3	d	$\boldsymbol{c}$	a	b
4	a	$\boldsymbol{b}$	c	d
$\boldsymbol{s}$	4	**2**	1	0

alternative	points
a	6
b	6
c	2
d	10

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2	d	a	$\boldsymbol{c}$	b
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$\boldsymbol{s}$	4	2	**1**	0

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$\boldsymbol{s}$	4	2	1	**0**

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3	d	c	a	b
4	a	b	c	d
$\boldsymbol{s}$	4	2	1	0

alternative	points	
a	8	
b	6	
c	4	
d	10	winner!

Positional scoring rules

- **Plurality:** give a point to the favourite alternative of each agent, and rank the alternatives in terms of total score
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  - $\mathbf{VE} = (1,1, \dots, 1, 0)$

- **Borda:** give a point to an alternative for every pairwise win against another alternative, and rank the alternatives in terms of total score
  - $\mathbf{B} = (m-1, m-2, \dots, 1, 0)$

Copeland

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1	$\boldsymbol{b}$	d	$\boldsymbol{a}$	c
2	d	$\boldsymbol{a}$	c	$\boldsymbol{b}$
3	d	c	$\boldsymbol{a}$	$\boldsymbol{b}$
4	$\boldsymbol{a}$	$\boldsymbol{b}$	c	d

alternative	points
a	1
b	0
c	0
d	0

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alternative	points
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alternative	points
a	2
b	0
c	0
d	1

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alternative	points
a	2
b	1
c	0.5
d	2.5

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1	b	d	a	c
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alternative	points	
a	2	
b	1	
c	0.5	
d	2.5	**winner!**

Ranked pairs

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- We can model the execution of this process by a directed graph, where each node represents an alternative and an edge from some alternative x to an alternative y represents the fact that x is ranked higher than y
- So, we successively add edges to this graph following the ranking of pairs as long as no cycle is created

Ranked pairs

agent	ranking			
1	b	d	a	c
2	d	a	c	b
3	d	a	c	b
4	a	b	c	d

pair	victories
(a, c)	4
(a, b)	3
(d, c)	3
(d, a)	3
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
(c, d)	1
(c, a)	0

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agent	ranking			
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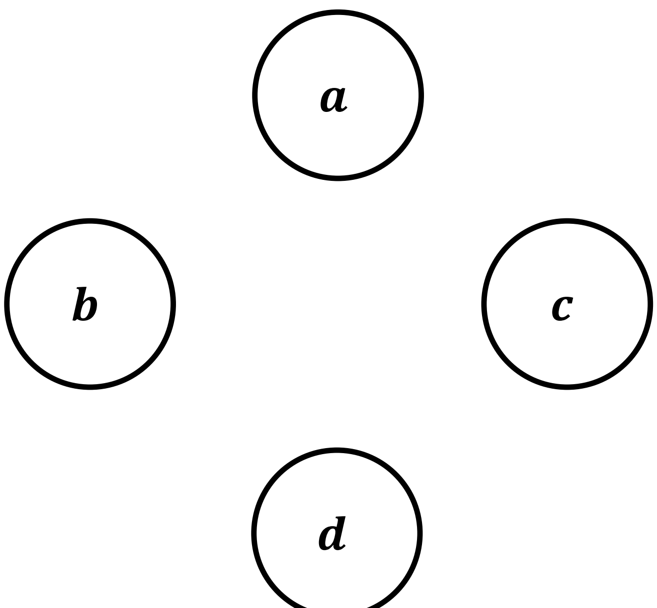

pair	victories
(a,c)	4
(a,b)	3
(d,c)	3
(d,a)	3
(c,b)	2
(b,d)	2
(b,c)	2
(d,b)	2
(a,d)	1
(b,a)	1
(c,d)	1
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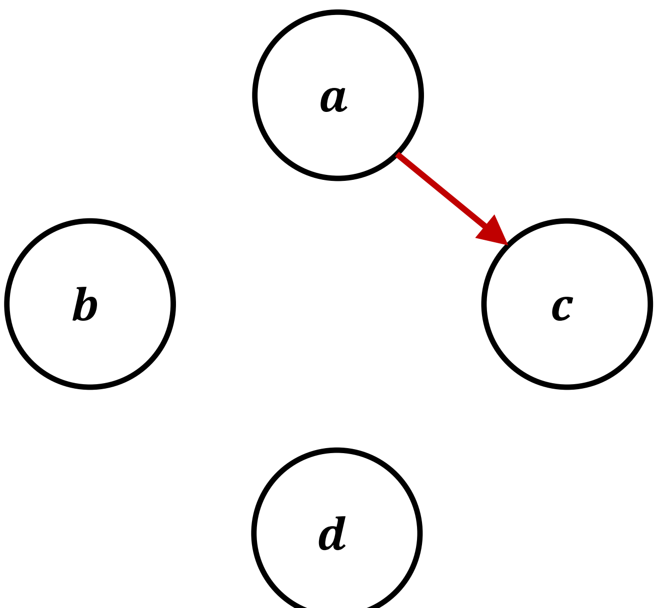

pair	victories
$(\boldsymbol{a}, \boldsymbol{c})$	**4**
(a, b)	3
(d, c)	3
(d, a)	3
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
(c, d)	1
(c, a)	0

Ranked pairs

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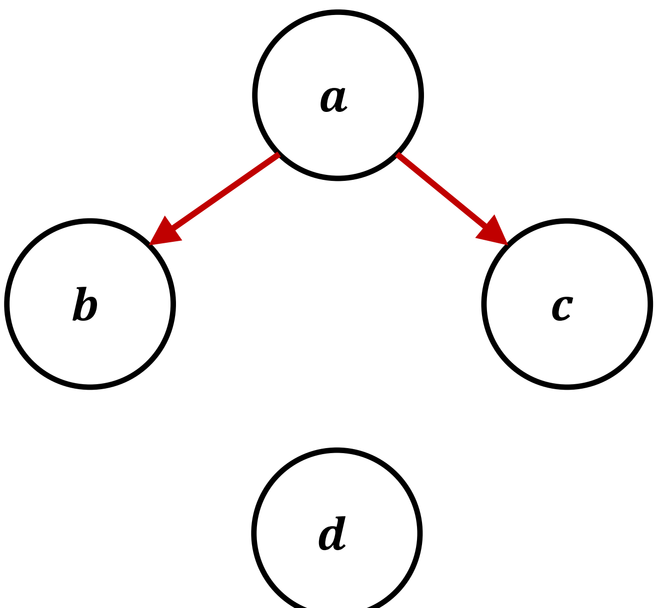

pair	victories
(a, c)	4
(a, b)	3
(d, c)	3
(d, a)	3
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
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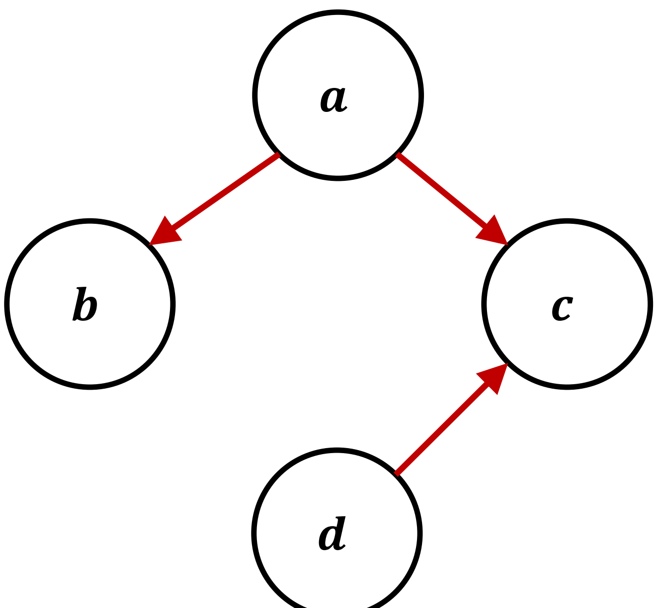

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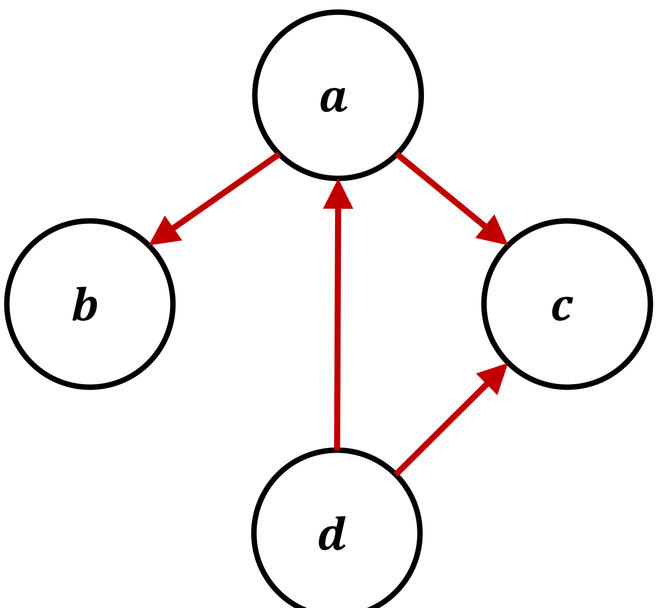

pair	victories
(a, c)	4
(a, b)	3
(d, c)	3
$\boldsymbol{(d, a)}$	**3**
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
(c, d)	1
(c, a)	0

Ranked pairs

agent	ranking			
1	b	d	a	c
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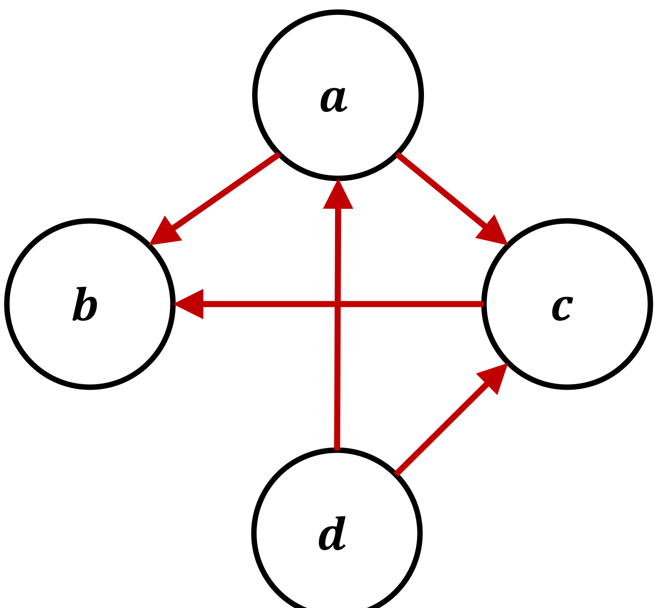
3	d	a	c	b
4	a	b	c	d



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Ranked pairs

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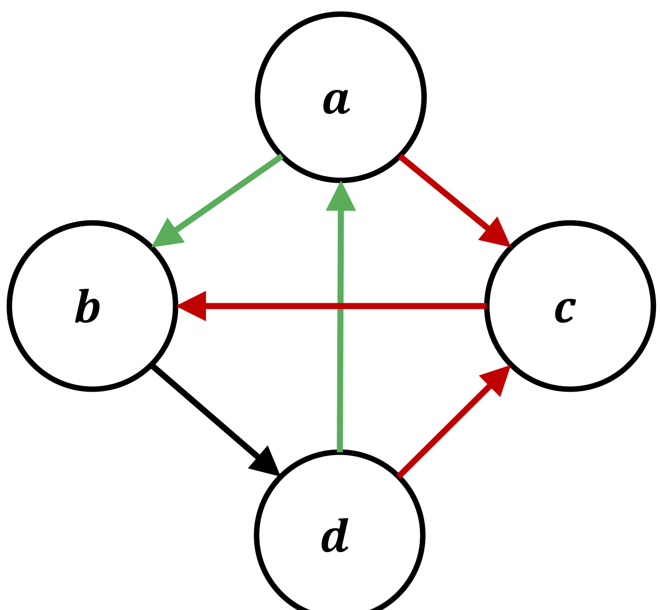

pair	victories
(a, c)	4
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$\boldsymbol{(b, d)}$	**2**
(b, c)	2
(d, b)	2
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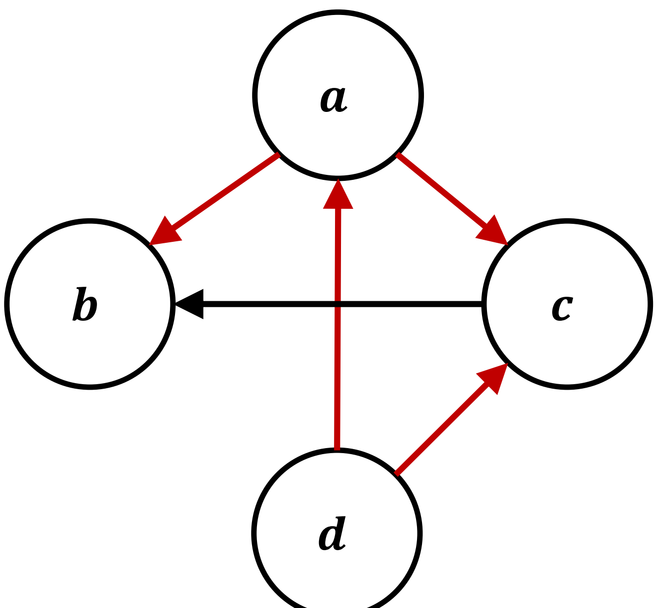

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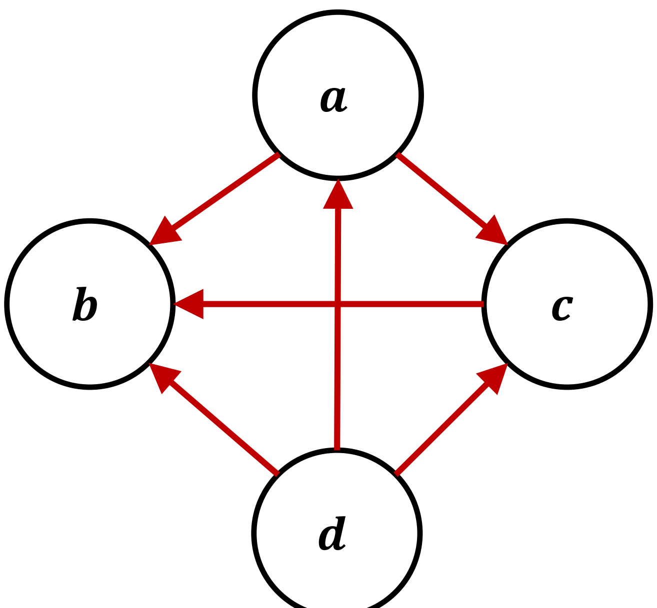

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(b, c)	2
$\boldsymbol{(d, b)}$	**2**
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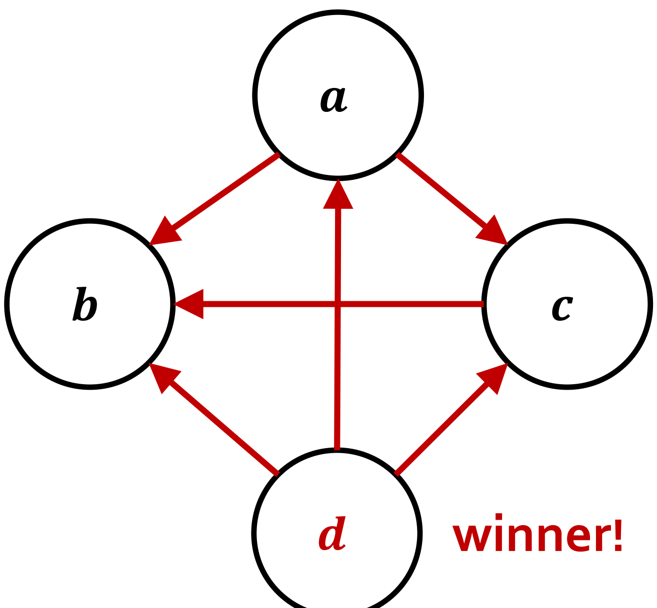

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(b, c)	2
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(a, d)	1
(b, a)	1
(c, d)	1
(c, a)	0

Dictatorship

- The simplest and most unfair voting rule
- The output is the favourite alternative or the whole preference of a particular agent
- Naturally, this agent is called the dictator

Some desired properties

- **Unanimity:** If all agents have exactly the same preferences over the alternatives, then the output should be what everyone wants

Some desired properties

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agent	ranking			
1	a	b	c	d
2	a	b	c	d
3	a	b	c	d
4	a	b	c	d

Some desired properties

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agent	ranking			
1	d	c	b	a
2	a	c	d	b
3	a	d	b	c
4	b	a	c	d

agent	ranking			
1	c	b	d	a
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agent	ranking			
1	d	c	$\boldsymbol{b}$	$\boldsymbol{a}$
2	$\boldsymbol{a}$	c	d	$\boldsymbol{b}$
3	$\boldsymbol{a}$	d	$\boldsymbol{b}$	c
4	$\boldsymbol{b}$	$\boldsymbol{a}$	c	d

agent	ranking			
1	c	$\boldsymbol{b}$	d	$\boldsymbol{a}$
2	$\boldsymbol{a}$	$\boldsymbol{b}$	c	d
3	c	d	$\boldsymbol{a}$	$\boldsymbol{b}$
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- Unanimity and IIA seem to be two very natural properties to request from a voting rule to satisfy

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- But, …

Theorem [Arrow, 1951]
For at least three alternatives, any unanimous and IIA social welfare function must be a dictatorship

Manipulations

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2	d	c	a	b
3	d	c	b	a

alternative	Borda score
a	4
b	3
c	5
$\boldsymbol{d}$	**6**

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2	*d*	*c*	*a*	*b*
3	*d*	*c*	*b*	*a*

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agent	ranking			
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2	d	c	a	b
3	d	c	b	a

alternative	Borda score
a	3
b	2
$\boldsymbol{c}$	**7**
d	6

More than two options

Beyond the Majority Rule

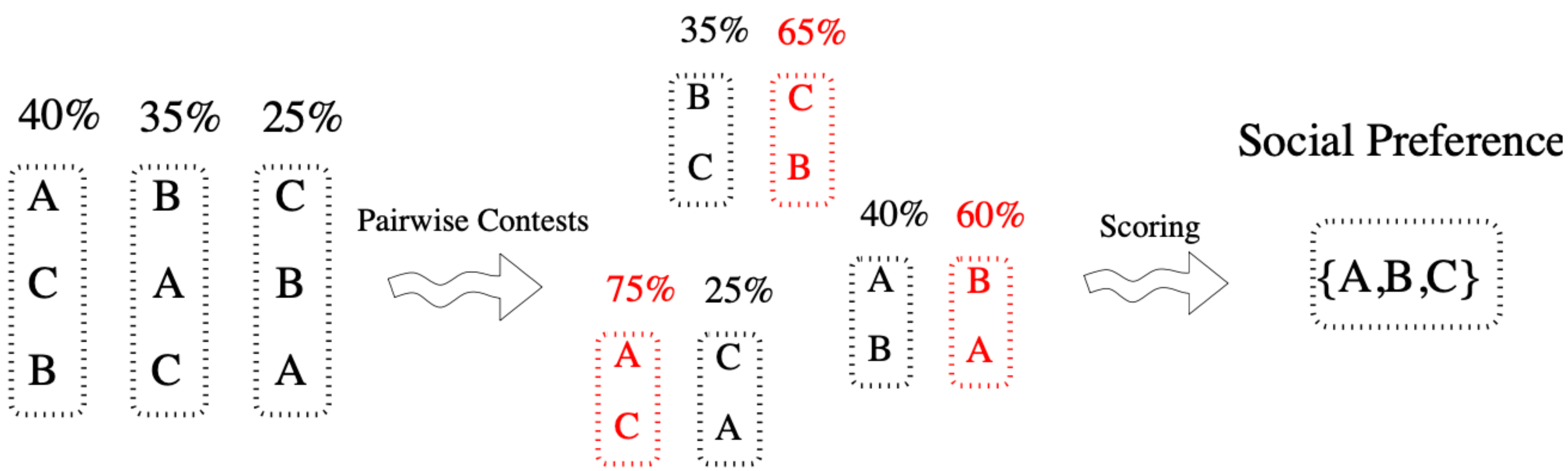

FIGURE 13.1. In pairwise contests A defeats C and C defeats B, yet B defeats A.

Plurality Voting

Choosing the favorite.

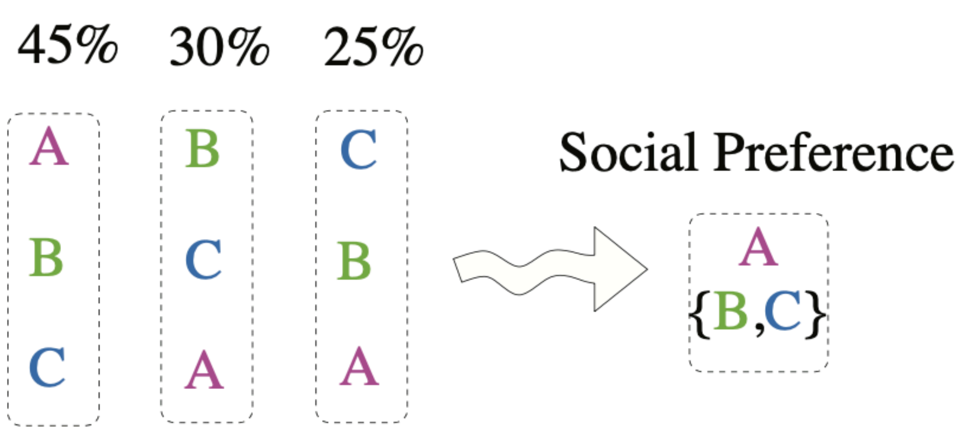

FIGURE 13.2. Option A is preferred by 45% of the population, option B by 30%, and option C by 25%, and thus A wins a plurality vote. However, A is the least favorite for 55% of the population.

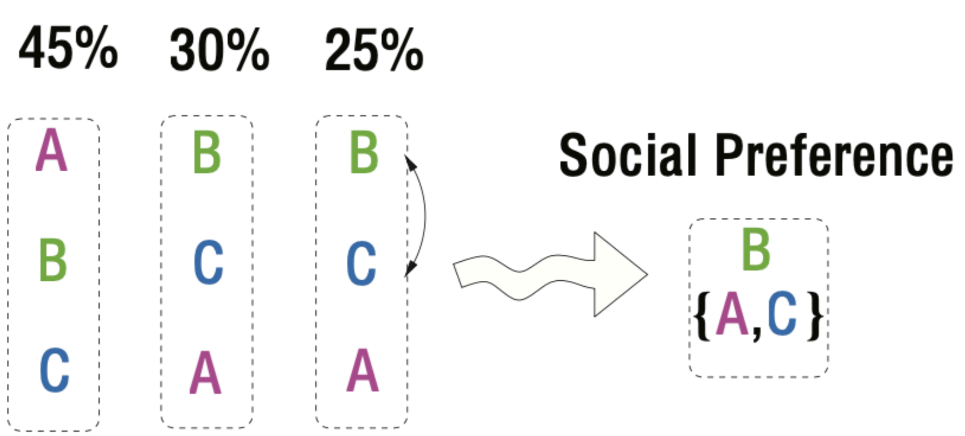

FIGURE 13.3. When 25% strategically switch their votes from C to B, the relative ranking of A and B in the outcome changes.

Runoff Elections

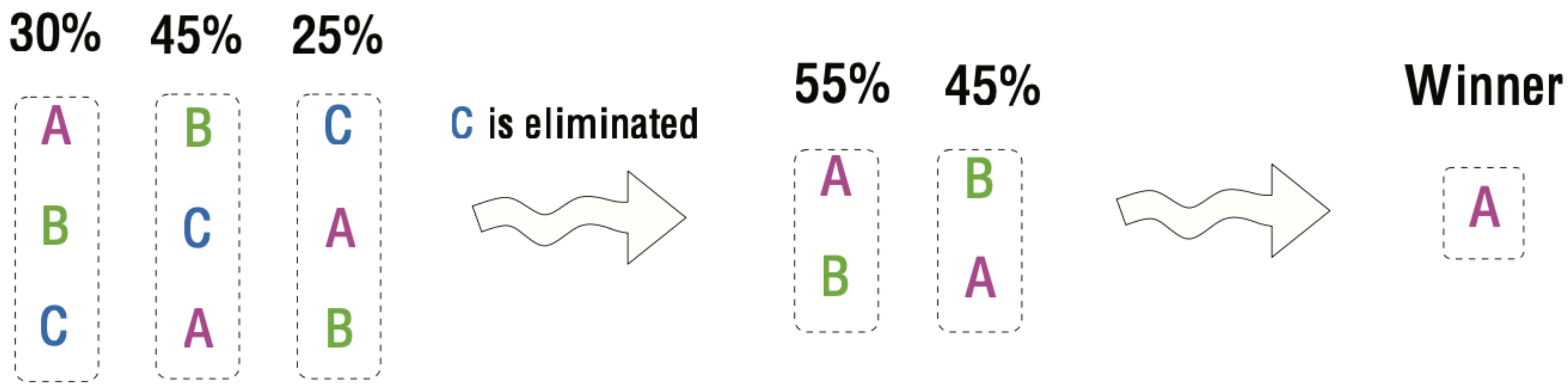

FIGURE 13.4. In the first round C is eliminated. When votes are redistributed, B gets the majority.

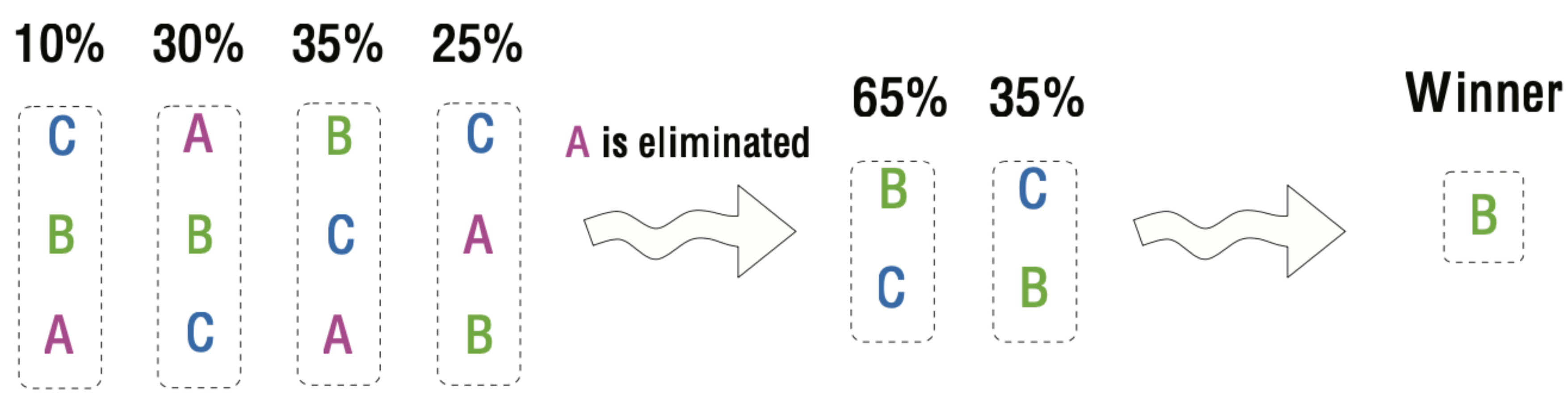

FIGURE 13.5. Some of the voters from the second group in Figure 13.4 misrepresent their true preferences, ensuring that A is eliminated. As a result, B wins the election.

Manipulations

- We would like to use voting rules that are strategy-proof, and always incentivize the agents to truthfully report their true preferences over the alternatives

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Theorem [Gibbard,1973 & Satterthwaite, 1975]
For at least three alternatives, any strategy-proof and onto the set of alternatives social choice function must be a dictatorship

Dealing with manipulations

- In general, the impossibility result of Gibbard-Satterthwaite indicates that there is now way to avoid manipulative behaviour, unless the voting rule is a dictatorship

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  - Computing a manipulation is easy for positional scoring rules and Copeland, but NP-complete for Ranked Pairs

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- Another way to "avoid" this is to focus on special cases, where the preferences of the agents are more structured

Facility location on the line

- A set of agents positioned on a line
- One **facility** to be built somewhere

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- The goal is to decide where to build the facility so that no agent manipulates, and without using a dictatorship

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- If the blue agent reports a position smaller than the median position, nothing will change
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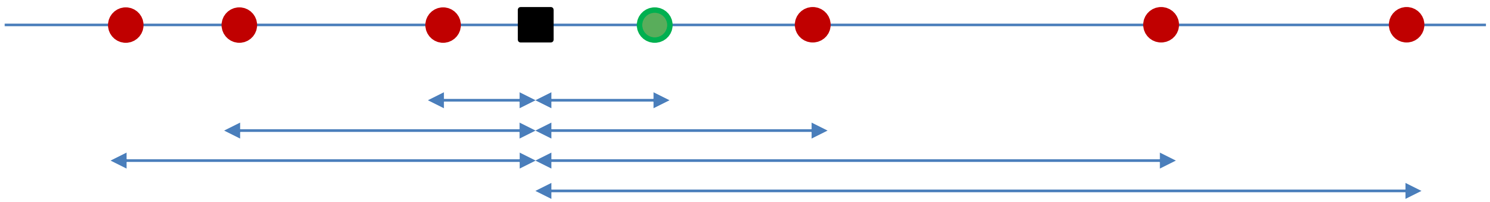

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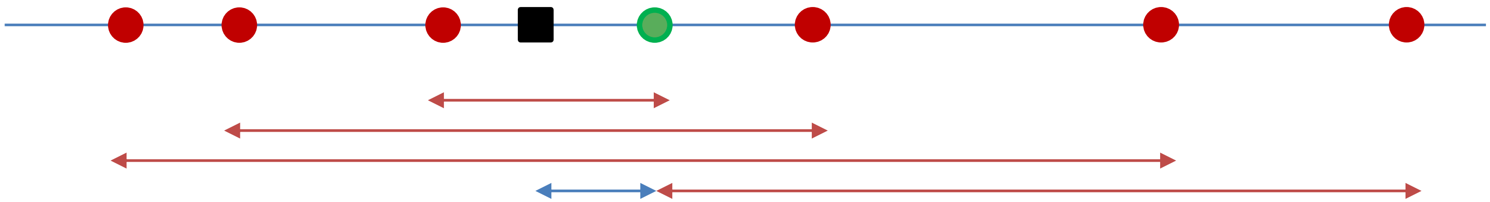

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