Group Assignment #2
Math 290, Dr. Walnut

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1 Proof 1

Let p and q be natural numbers such that $\gcd(p, q) = 1$. Prove that for all $a \in \mathbb{N}$, if $p|a$ and $q|a$ then $pq|a$.

Proof: Let p and q be natural numbers such that $\gcd(p, q) = 1$. Let $p|a$, and $q|a$. We want to show that $pq|a$. Then, there exist integers n and m such that $pn = a$ and $qm = a$. Thus, $pn = qm$. Since $n \in \mathbb{Z}$, then $p|qm$. Since $\gcd(p, q) = 1$, then $p|m$. That means there exists an integer x such that $px = m$. By multiplying q to both sides, we get $pqx = qm = a$. Thus, $pqx = a$. Since $x \in \mathbb{Z}$, then $pq|a$. We are done.

2 Proof 2

Prove that for all natural numbers a and b, $\gcd(a, b) = 1$ if and only if $a\mathbb{Z} \cap b\mathbb{Z} = ab\mathbb{Z}$.

Proof: Let a and b be natural numbers.

($\Rightarrow$) Let $\gcd(a, b) = 1$. We want to show that $a\mathbb{Z} \cap b\mathbb{Z} = ab\mathbb{Z}$. This means we want to show that $a\mathbb{Z} \cap b\mathbb{Z} \subseteq ab\mathbb{Z}$ and $ab\mathbb{Z} \subseteq a\mathbb{Z} \cap b\mathbb{Z}$.Since $k\mathbb{Z}$ notates a set containing all integer multiples of natural number k, then if $x \in k\mathbb{Z}$, $k|x$. This means we want to show that if $a|x$ and $b|x$ then $ab|x$ and if $ab|x$ then $a|x$ and $b|x$.

Let $a|x$ and $b|x$. By the first theorem we proved, since $\gcd(a, b) = 1$, $ab|x$.

Let $ab|x$. That means there exists an integer y such that $aby = x$. Since b and y are integers, by is an integer. Thus, $a|x$. Since a and y are integers, ay is an integer. Thus, $b|x$.

($\Leftarrow$) (By contrapositive) Let $\gcd(a, b) \neq 1$. This means there exists a natural number d such that $\gcd(a, b) = d \neq 1$. This means there exists integers n and m such that $dn = a$ and $dm = b$. Also, assume that $ab\mathbb{Z} \subseteq a\mathbb{Z} \cap b\mathbb{Z}$. We want to show that $a\mathbb{Z} \cap b\mathbb{Z} \not\subseteq ab\mathbb{Z}$. This means we want to show that there exists an integer k such that $k \in a\mathbb{Z} \cap b\mathbb{Z}$ and $k \notin ab\mathbb{Z}$. This means that we want to show that there exists an integer k such that $a|k$, $b|k$, and $ab|k$. Let $k = dmn$. Then, $k = am$ and $k = bn$. Since m and n are integers, $a|k$ and $b|k$.

Since $a = dn$ and $b = dm$, $ab = d^2mn$. Since $d > 1$, $d^2mn > dmn$. Thus, $ab > k$. Since ab and k are natural numbers, ab cannot divide k. This means there exists a k such that $k \in a\mathbb{Z} \cap b\mathbb{Z}$ and $k \notin ab\mathbb{Z}$. We are done.