矩阵元分布

- $X_1 \cdots X_n$ 独立 $N(\mu, \Sigma)$

$$p(X) = \prod_{i=1} p(x_i)$$

$1\times n \quad n\times 1 \quad n \times 1 \quad \to 1\times 1$

$$= \prod \frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x_i-\mu_i)^T \Sigma^{-1}(x_i-\mu_i)\right)$$

$$= \frac{1}{(2\pi)^{np/2}|\Sigma|^{n/2}} \exp\left(-\frac{1}{2} tr\left(\Sigma^{-1}\sum(x_i-\mu_i)(x_i-\mu_i)^T\right)\right)$$

$$= \frac{1}{(2\pi)^{np/2}|\Sigma|^{n/2}} \exp\left(-\frac{1}{2} tr\left(\Sigma^{-1}(X-\mu)^T I (X-\mu)\right)\right)$$

矩阵元的正态分布

- $\underset{n\times p}{X} \qquad \underset{n\times p}{M} \qquad \underset{p\times p}{A > 0}$ 正定 . $\underset{n\times n}{B > 0}$ 正定

$$\underbrace{vec(X^T)}_{np\times 1} \qquad \underbrace{vec(M^T)}_{np\times 1} \qquad B\otimes A$$

$$vec(X^T) \sim N(vec(M^T), B\otimes A)$$

则 X 为矩阵元分布.

$\exp(tr \to etr$

$$pdf = \frac{1}{(2\pi)^{np/2}|A|^{p/2}|B|^{p/2}} etr\left(-\frac{1}{2}A^{-1}(X-M)^T B^{-1}(X-M)\right)$$