从概率密度函数角度观察

pdf

- $x \sim N(\mu, \Sigma) = \frac{1}{(2\pi)^{p/2}|\Sigma|^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$

$x_i \sim \mathbb{R}^p$.

$x = \begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix} \quad \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \cdots & \sigma_{1p} \\ \\ \sigma_{11} \cdots & & \sigma_{pp} \end{pmatrix}_{p\times p}$. 正定的.

马氏距离

$(x-\mu)^T\Sigma^{-1}(x-\mu)$ ：马氏距离.（x与μ之间）

$1\times p \quad p\times p \quad p\times 1 \longrightarrow 1\times 1$.

若 $\Sigma = I$ 则 马氏距离为欧式距离.

$\Sigma = U\Lambda U^T \qquad U = (u_1, \cdots, u_p) \quad \Lambda = diag(\lambda_1, \cdots \lambda_p)$

$= (u_1 \cdots u_p)\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_p \end{bmatrix}\begin{pmatrix} u_1^T \\ \vdots \\ u_p^T \end{pmatrix}$

$= \sum_{i=1}^{p} u_i \lambda_i u_i^T$.

$\Sigma^{-1} = (U\Lambda U^T)^{-1} = U\Lambda^{-1}U^T$ $\quad \Lambda^{-1} \to diag(\lambda_1^{-1} \cdots \lambda_p^{-1})$

$= \sum_{i=1}^{p} u_i \lambda_i^{-1} u_i^T$.

$(x-\mu)^T\Sigma^{-1}(x-\mu) = (x-\mu)^T \sum_{i=1}^{p} u_i\lambda_i^{-1}u_i^T(x-\mu)$.

$= \sum \frac{1}{\lambda_i}(x-\mu)^T u_i u_i^T (x-\mu)$.

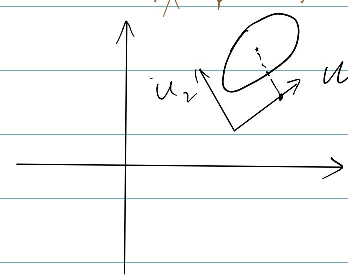
x在u_i上的投影.

$y_i = (x-\mu)^T u_i \qquad = \sum \frac{1}{\lambda_i} y_i y_i^T$.

$1\times p \quad p\times 1 \qquad = \sum \frac{1}{\lambda_i} y_i^2$.

1×1.

$x \to \mu$ 平移与投影



$\Delta = \frac{y_1^2}{\lambda_1} + \frac{y_2^2}{\lambda_2} = 1$ 椭圆曲线.