$\mathcal{F}_K = \left\{ \sum_{i=1}^{m} \alpha_i \Phi(\underline{x}_i) \right\}$ vector space

Definition (Inner Product Space)

An inner product space over $\mathcal{R}$ is a vector space V over $\mathcal{R}$ together with an inner product, i.e., with a map

$$< \cdot, \cdot >: V \times V \to \mathcal{R}$$

that satisfies the following three axioms for all vectors $\mathbf{x}, \mathbf{y}, \mathbf{z} \in V$ and all scalars $a \in \mathcal{R}$:

1. Symmetry: $< \mathbf{x}, \mathbf{y} > = < \mathbf{y}, \mathbf{x} >$
2. Linearity in the first argument:

$$< a\mathbf{x}, \mathbf{y} > = a < \mathbf{x}, \mathbf{y} >,$$
$$< \mathbf{x} + \mathbf{y}, \mathbf{z} > = < \mathbf{x}, \mathbf{z} > + < \mathbf{y}, \mathbf{z} >$$

3. Positive-definiteness:

$$< \mathbf{x}, \mathbf{x} > > 0 \text{ if } \mathbf{x} \neq \mathbf{0},$$
$$< \mathbf{x}, \mathbf{x} > = 0 \Leftrightarrow \mathbf{x} = \mathbf{0},$$