举例.(t统计量) • 令 $X_1 \cdots X_n$ 为 iid, $E[X_1]=0$, $E[X_1^2]<\infty$.

t-统计量 $\frac{\sqrt{n}\,\bar{X}_n}{S_n}$, where $S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X}_n)^2$, 是样本的标准差.

证明:

$$S_n^2 = \frac{n}{n-1}\Big(\underbrace{\sum_{i=1}^{n} X_i^2}_{\downarrow\, E[X_1^2]} - \underbrace{\bar{X}_n^2}_{\downarrow\, \mu^2}\Big) \xrightarrow{P} E[X_1^2]-\mu^2 = E[X_1^2]-(E[X])^2$$

← 弱中心极限定理.

$$S_n \xrightarrow{P} \sqrt{var(X_1)} = std(X_1)$$

$$\sqrt{n}\,\bar{X}_n \rightsquigarrow N(0, var(X_1))$$

则 $\frac{\sqrt{n}\,\bar{X}_n}{S_n} \sim N(0,I)$.

the Delta 方法 • 假设 $X_n=(X_{n1},\cdots,X_{nk})$ 是随机向量的序列, $\sqrt{n}(X_n-\mu)\rightsquigarrow N(0,\Sigma)$

令: $g\ \ \mathbb{R}^k\to\mathbb{R}$ 并且 $\nabla g = \begin{pmatrix}\frac{\partial g}{\partial x}\\ \frac{\partial g}{\partial x_k}\end{pmatrix}$

Let ∇_μ denote $\nabla g(y)\ x=\mu$ ∇_μ 非 0.

则 $\sqrt{n}(g(X_n)-g(\mu)) \rightsquigarrow N(0,\ \nabla_\mu^T\Sigma\nabla_\mu)$.

证明:

$$g(X_n)=g(\mu)-\nabla_\mu^T(X_n-\mu)+\cdots$$ 泰勒展开

$$\sqrt{n}(g(X_n)-g(\mu)) \approx \nabla_\mu^T\sqrt{n}(X_n-\mu).$$