已知联合概率求边缘概率及条件概率

$$x \sim N(\mu, \Sigma) = \frac{1}{(2\pi)^{p/2}|\Sigma|^{\frac{1}{2}}} \cdot \exp\left[-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right]$$

- 设 $x = \begin{pmatrix} x_a \\ x_b \end{pmatrix} \begin{matrix} \to m \text{ 已知} \\ \to n \end{matrix}$ $\quad \mu = \begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix} \begin{matrix} p \\ q \end{matrix}$. $\Sigma = \begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{pmatrix}$.

then. ① X_1 和 $X_{21} = X_2 - \Sigma_{21}\Sigma_{11}^{-1}X_1$ 是统计意义独立.

② $X_1 \sim N_p(\mu_1, \Sigma_{11})$ $\quad X_{21} \sim N_q(\mu_{21}, \Sigma_{21})$.

$\mu_{21} = \mu_2 - \Sigma_{21}\Sigma_{11}^{-1}\mu_1$ $\quad \mu_{22.1} = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$

证:

$$x_a = \begin{pmatrix} I_m & O_n \end{pmatrix} \begin{pmatrix} x_a \\ x_b \end{pmatrix}$$

A 易求 $\quad X \leftarrow$ 已知量

$E[x_a] = AE(X) = \mu_a$.

$$Var[x_a] = (I_m, O_n)\begin{pmatrix} \Sigma_{aa} & \Sigma_{ba} \\ \Sigma_{ab} & \Sigma_{bb} \end{pmatrix}\begin{pmatrix} I_m \\ O_n \end{pmatrix} = \Sigma_{aa}$$

$x_a \sim N(\mu_a, \Sigma_{aa})$.

$P(x_a)$

$$x_{b \cdot a} = \begin{pmatrix} -\Sigma_{ba}\Sigma_{aa}^{-1}, & I \end{pmatrix} \begin{pmatrix} x_a \\ x_b \end{pmatrix} \leftarrow X$$

$$E(x_{b\cdot a}) = (-\Sigma_{ba}\Sigma_{aa}^{-1}, I)\begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix} = \mu_b - \Sigma_{ba}\Sigma_{aa}^{-1}\mu_a = \mu_{b\cdot a}$$

$$var[x_{b\cdot a}] = (-\Sigma_{ba}\Sigma_{aa}^{-1}\ I_n)\begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{pmatrix}\begin{pmatrix} -\Sigma_{aa}^{-1}\Sigma_{ba}' \\ I \end{pmatrix} = \Sigma_{bb\cdot a}.$$