分布公式

- 高斯分布

$X \sim N(\mu, \sigma^2)$.

均值

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right).$$

$\int f_X(x)\,dx = 1$

- $\int_{-\infty}^{+\infty} \exp\{x^2 \cdot \frac{1}{-2\sigma^2}\}dx = \sqrt{2\pi}\sigma$

证:

$$\int_{-\infty}^{+\infty} \exp\{-\frac{1}{2}\frac{x^2}{\sigma^2}\}dx \int_{-\infty}^{+\infty} \exp\{-\frac{1}{2}\frac{y^2}{\sigma^2}\}dy = 2\pi\sigma^2$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\{-\frac{1}{2}\frac{(x^2+y^2)}{\sigma^2}\}dx\,dy = 2\pi\sigma^2$$

设为极坐标:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad 则\ J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$|J| = r\cos^2\theta + r\sin^2\theta = r.$$

$$\int_0^{2\pi}\int_0^{+\infty} r \exp\{-\frac{1}{2}\cdot\frac{r^2}{\sigma^2}\}dr\,d\theta.$$

$$= 2\pi \int_0^{+\infty} r \exp\{-\frac{1}{2}\cdot\frac{r^2}{\sigma^2}\}dr$$

$$= 2\pi \cdot \sigma^2$$

广义逆高斯

让先验与后验同分布.

- 广义逆高斯

$$f_X(x) = \frac{(\alpha/\beta)^{r/2}}{2K_r(\sqrt{\alpha\beta})} x^{r-1} \exp\left(-\frac{\alpha x + \beta x^{-1}}{2}\right).$$

① $K_r(u) = K_{-r}(u)$

② $K_{r+1}(u) = 2K_r(u)\cdot\frac{r}{u} + K_{r-1}(u)$.

③ $K_{\frac{1}{2}}(u) = K_{-\frac{1}{2}}(u) = \sqrt{\frac{u}{2\pi}}\exp(-u)$.

④ $u \to 0 \quad \begin{cases} K_r(u) \sim \frac{1}{2}\Gamma(r)\left(\frac{u}{2}\right)^{-r}, & r>0 \\ K_0(u) \sim \ln(u). \end{cases}$

⑤ $u \to \infty \quad K_r \sim \sqrt{\frac{2\pi}{u}}\exp(-u)$.

- $\beta = 0, \quad r > 0 \quad \alpha > 0$

$f_X(x) = \frac{\alpha^r}{2^r\Gamma(r)}\cdot x^{r-1}\exp\left(-\frac{\alpha x}{2}\right)$

$x \sim Ga\left(r, \frac{\alpha}{2}\right)$

- $\beta = 0 \quad \alpha > 0.$

$r = 1$ 指数分布.

- 逆 Gamma. $\alpha = 0. \quad r < 0 \quad \beta > 0$

$f(x) = \frac{\beta^\tau}{2^\tau\Gamma(\tau)} x^{-(\tau+1)}\exp\left(-\frac{\beta}{x}\right)$

$\tau = -r \quad IG\left(\tau, \frac{\beta}{2}\right)$.

- $r = \frac{1}{2}$ 逆高斯.

$f(x) = \left(\frac{\beta}{2\pi}\right)^{\frac{1}{2}} \exp(\sqrt{2\beta})x^{-\frac{3}{2}}\exp\left(-\frac{\alpha x + x\beta}{2}\right)$