the weak law of 大数定理 • $X_1, X_2, \cdots, X_n$ 是为独立同分布, $\mu = E[X_1]$ $\delta^2 = Var[X_1]$.

$\bar{X}_n = \frac{1}{n}\sum X_i$, 则 $\bar{X}_n \xrightarrow{P} \mu$.

证明:

$$P(|\bar{X}-\mu| > \varepsilon) \leq \frac{Var(\bar{X})}{\varepsilon^2} = \frac{\delta^2}{n\varepsilon^2} \to 0.$$

$\begin{cases} E[\bar{X}] = \frac{1}{n} \cdot n\mu = \mu \\ Var[\bar{X}] = \frac{n\delta^2}{n^2} = \frac{\delta^2}{n} \end{cases}$

中心极限定理 • $X_1 \cdots X_n$ 是 iid, $\mu, \bar{\Sigma}$. $\hat{X} = \frac{1}{n}\sum X_n$, 则.

$$Z_n = \Sigma^{\frac{1}{2}}(\bar{X}_n - \mu) \rightsquigarrow N(0, I).$$