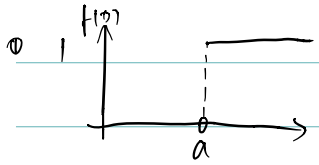
马尔科夫不等式

$P(X \geq a) \leq \frac{E[X]}{a}$，　X是随机变量≥ 0　a为常数>0.



$$f(y)=\begin{cases}0 & x<a\\ 1 & x\geq a\end{cases}$$

② $a\cdot f(y)=\begin{cases}0, & x<a \Rightarrow af(y)=0<y\\ a, & x\geq a \Rightarrow a\cdot f(y)=a\leq x\end{cases}$

$af(y)\leq x$

③ $E[af(y)]\leq E[x]$

$aE[f(y)]\leq E[x]$　　$\geq P(y)\cdot x=E[x]$

$$\boxed{P(y<a)f(y)+P(y\geq a)f(y)=P(y\geq a)}$$

$a\cdot P(y\geq a)\leq E[x]$

$$P(x\geq a)\leq \frac{E[x]}{a}$$

切比雪夫不等式

- 对 $\forall t \geq 0$, $Y \mathbb{1}\{Y \geq t\} \geq t \mathbb{1}\{Y \geq t\}$

$$E[Y \mathbb{1}\{Y \geq t\}] \geq E[t \mathbb{1}\{Y \geq t\}] = t E[\mathbb{1}\{Y \geq t\}]$$

($\mathbb{1}\{Y \geq t\}$: $Y \geq t$ 的概率)

马尔可夫不等式

($\mathbb{1}\{Y \geq t\}$: $Y \geq t$ 的概率)

$$Pr(\mathbb{1}\{Y \geq t\}) \leq \frac{E[Y \mathbb{1}\{Y \geq t\}]}{t} \leq \frac{E[Y]}{t}$$

(Y 的期望)

e.g.

- $Y = |Z - E[Z]|$

$$Pr(|Z - E[Z]| \geq t) \leq \frac{E[|Z - E[Z]|]}{t}$$

- ϕ 是非 递减的非负的函数,

$$Pr(Y \geq t) \leq Pr(\phi(Y) \geq \phi(t)) \leq \frac{E[\phi(Y)]}{\phi(t)}$$

切比雪夫

- $\phi(t) = t^2$. $Y = |Z - E[Z]|$

$$Pr(|Z - E[Z]| \geq t) \leq \frac{Var(Z)}{t^2}$$

切比雪夫不等式

$Z = X_1 + \cdots + X_n$ 独立变量

$$Var(Z) = \sum Var(X_i)$$

$$Pr\left(\frac{1}{n}\left|\sum (X_i - E[X_i])\right| \geq t\right) \leq \frac{s^2}{nt^2} \quad = \frac{1}{n}\sum Var(X_i)$$

- $\phi(t) = e^{\lambda t}$ $\lambda \geq 0$.

$$Pr(Z \geq t) \leq \frac{E[e^{\lambda Z}]}{e^{\lambda t}}$$