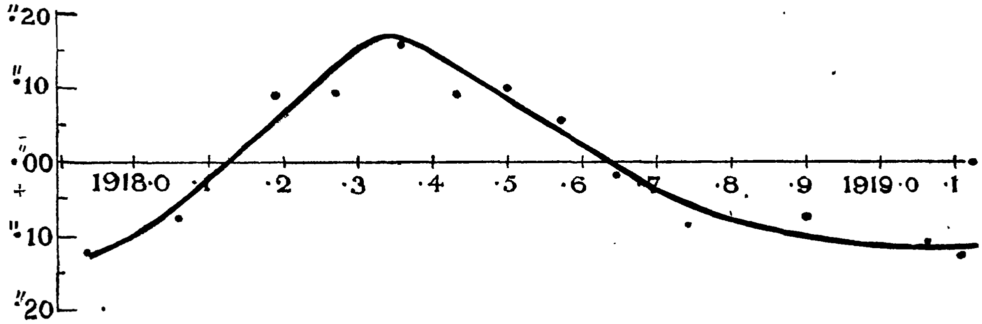
A table of corrections applicable to observed N.P.D.'s for variation of latitude on selected dates is also included. These



were read from a smooth curve drawn through the observed values plotted to scale on a diagram.

Corrections applicable to observed N.P.D.'s. (Longitude of Greenwich.)

1918 Jan. 1	− ·10	1918 May 1	+ ·17	1918 Sept. 1	− ·02
11	$-\cdot 08_5$	11	+ ·17	11	− ·03
21	$-\cdot 06_5$	21	+ ·16	21	− ·05
Feb. 1	− ·04	June 1	+ ·14	Oct. 1	− ·06
11	− ·02	11	$+\cdot 12_5$	11	− ·07
21	+ ·01	21	$+\cdot 10_5$	21	− ·08
Mar. 1	$+\cdot 02_5$	July 1	$+\cdot 08_5$	Nov. 1	$-\cdot 08_5$
11	+ ·05	11	+ ·07	11	− ·09
21	+ ·08	21	$+\cdot 05_5$	21	− ·10
Apr. 1	$+\cdot 10_5$	Aug. 1	+ ·04	Dec. 1	− ·10
11	$+\cdot 12_5$	11	+ ·02	11	$-\cdot 10_5$
21	+ ·15	21	·00	21	− ·11
				31	− ·11

Decentred Lens-Systems. By A. E. Conrady.

In general discussions of the properties of lens-systems as well as in the computation of the data of new designs it is always assumed that the surfaces are accurately centred, that is, that all the centres of curvature lie on an exact straight line known as the optical axis of the system.

In the making of the real lenses this centring depends entirely on the care and skill of the workers, and can only be attained within limits which, assuming good workmanship, certainly are narrow, but which must nevertheless be regarded as of finite magnitude.

It follows that no actual lens-system possesses an optical axis in the rigorous sense of the theoretical discussions and calculations; in reality, the centres of curvature are located within small distances of the intended axis, and are scattered around this in various azimuths according to chance.

The results of this departure from the assumed conditions do not seem to have been systematically investigated hitherto. Opticians know from practical experience that in an otherwise well-corrected system any decided centring error causes uniform coma over the entire field, accompanied in gross cases by one-sided colour-fringes, and they look out for this defect; it is therefore rarely seen by users of lenses. The fact, however, that the discovery of this defect in the preliminary workshop-tests of carefully made systems is not a very rare or unusual event proves that the residual centring errors are not negligible quantities.

Nothing appears to be known as to possible additional effects which might slightly distort the image without sensibly affecting its definition, which would therefore escape detection by the maker and user, and might thus introduce systematic errors into those extremely refined measurements of large fields which play so important a part in modern photographic methods of research.

The new method of dealing with the aberrations of oblique pencils, which I gave in my paper for the meeting of 1918 November, lends itself to a simple solution of this problem, and proves that such distortional effects *do* result from errors of centring.

In fig. 1 let OF represent the optical axis of an ideally centred optical system, AP one of the refracting surfaces with centre of curvature at C, OO_1 the object as presented to it by the surfaces in front of it, *i.e.* OO_1 is the image of the real object produced by the intervening refracting surfaces. Then my method of discussing the aberrations begins with an ideal image, free from aberration, which would be formed by thin pencils along the radial lines OC and O_1C, and would yield an image at F and F^1 according to the simple laws of pencils at very small angles of incidence. Now suppose that the refracting surface is tilted so that its centre now falls at C′. We can again trace radial pencils through the new position of the centre which will yield an image $F'F_1'$ tilted at an angle of the same order of magnitude as that by which the refracting surface is tilted, and which will be distorted according to the ordinary laws of central projection because we are now taking an *oblique* view of the surface in which the object OO_1 lies. This well-known distortion is that seen in snapshots of tall buildings taken on a plate inclined from the vertical, and is there characterised by convergence of vertical lines and by a falsification of the relative distances of horizontal lines, but in such a way that all straight lines in the objects are still rendered as straight lines in the image.

For the very small tilts which we are now considering the mathematical law of this distortion is a very simple one. The position angles of extra-axial points with reference to the centre of the field are not affected, but their distance d from it is altered by $b \cdot d^2 \cdot \cos \alpha$ if α is the position-angle reckoned from the direction defined by CC′ and b a small constant. It is easily shown that any number of small tilts in different azimuths of successive refractive surfaces lead to a final resultant distortion of precisely the same analytical form. This part of the effect of centring errors therefore becomes inextricably mixed with the same type of effect resulting from imperfect "squaring on" of the lens-system and of the plate-holder (or eyepiece), and is of no separate interest.

The real image is, however, produced by pencils of considerable aperture, which in the case of the more remote parts of a large field may not even include the thin radial pencil. These pencils of finite aperture are then subject to aberrations, and we must next examine the effect of centring errors on these aberrations.

Proceeding as in the November paper, we take an end view of the refracting surface along the radial line F_1A_1 in fig. 1 and obtain fig. 2, in which the large circle represents the lens-surface, the small circle the diameter of the oblique pencil passing through it, and A_1 the point where the radial line F_1C pierces it. Then the ray passing through any point R on the circumference of the pencil will suffer a lateral spherical aberration with reference to F_1 equal to $k(A_1R)^3$ if k is the aberration-constant for that surface, and the resulting displacement will be in the direction of A_1R. So far we have assumed that the surface is perfectly centred. Now consider the decentred surface. The line O_1F_1' (fig. 1) will pierce the surface in some point A_1', which we will define by the distance $A_1A_1' = r$ and by the position-angle $PA_1A_1' = O$. The pencil of rays, on the other hand, will cut the tilted surface on a line which differs only by negligible small quantities of the third order from the original position. The ray through R will now suffer lateral spherical aberration $= k(A_1'R)^3$, and this will lie in the direction of $A_1'R$.

Drawing a parallel to A_1P through A_1' and adopting the nomenclature of the former paper, viz. $A_1Q = Y$, $QR = y$, angle $RQR' = E$, we can take from the diagram

$$(A_1'R)^2 = (Y + y\cos E - r\cos O)^2 + (y\sin E - r\sin E)^2.$$

As before, we shall have the components of the aberrational displacement

$$\text{Horizontal component} = k(A_1'R)^3 \frac{Y + y\cos E - r\cos O}{A_1'R}$$

$$\text{Vertical component} = k(A_1'R)^3 \frac{y\sin E - r\sin O}{A_1'R}$$

and introducing the value of $(A_1'R)^2$, collecting terms and making slight reductions, we obtain the components of the lateral aberration:

$$\begin{aligned}\text{Horiz. comp.} &= k(y^3\cos E + Yy^2(2 + \cos 2E) + 3Y^2y\cos E + Y^3)\\ &\quad - k\,.\,r(y^2(2\cos O + \cos(2E - O)) + 2Yy(3\cos E\cos O + \sin E\sin O)\\ &\quad + 3Y^2\cos O) + \text{terms in } kr^2 \text{ and in } kr^3\end{aligned}$$

$$\begin{aligned}\text{Vert. comp.} &= k(y^3\sin E + Yy^2\sin 2E + Y^2y\sin E)\\ &\quad - k\,.\,r(y^2(2\sin O + \sin(2E - O)) + 2Yy(\sin E\cos O + \cos E\sin O)\\ &\quad + Y^2\sin O) + \text{terms in } kr^2 \text{ and in } kr^3.\end{aligned}$$

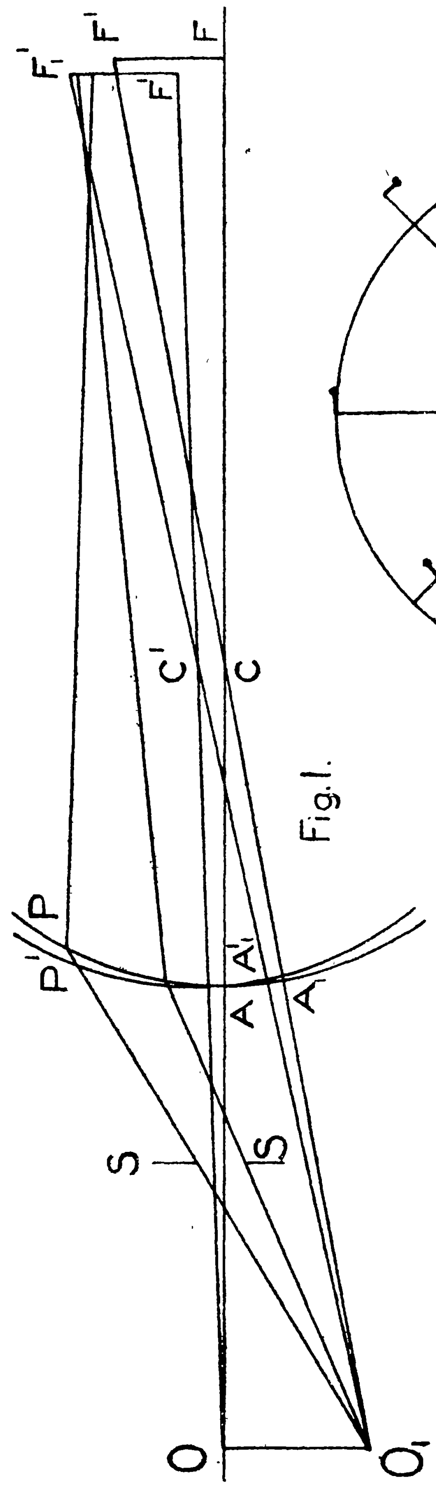


Fig. 1.

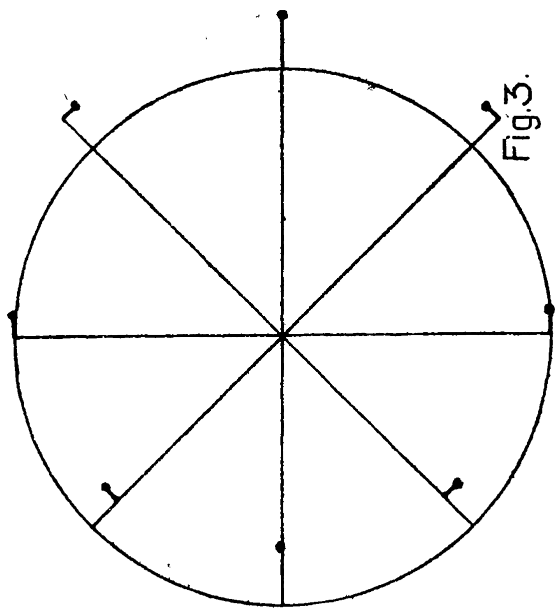


Fig. 3.

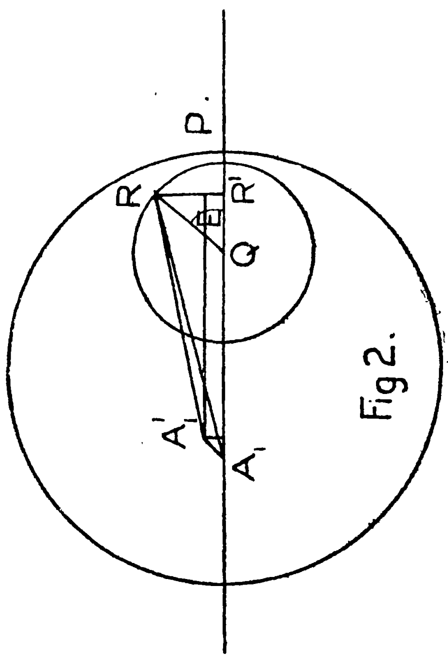


Fig 2.

The first line in each component represents the Seidel-aberrations of the perfectly centred surface, which thus reappear. They were discussed in the former paper, and do not interest us in the present investigation. The important terms resulting from the centring error appear in the second lines, and will be discussed. There are additional terms involving the second and third power of the centring error; optically these are quite interesting, but for our present purpose they may be safely ignored as utterly evanescent.

In most cases the centring defect will be chiefly due to one surface, for as its magnitude is proportional to the aberration-constant, a sensible effect will usually only be produced when it happens that a surface for which the constant has a high value is untrue. In ordinary object-glasses the contact-surfaces answer to this description.

The investigation may, however, easily be extended to any number of slightly decentred surfaces. As in the former paper, the displacements arising at any surface may be transferred to the following or to the final image by applying the magnification. We can also express the y and Y of every surface in terms of the constant semi-aperture S of the stop and of the angle V of the field of view. We can thus collect the algebraical sum of all the terms in the final image-surface. By applying simple transformations familiar to astronomers these sums again assume the analytical form of those found for a single surface, but with more or less *independent* coefficients for the terms in S^2, in SV, and in V^2. For a system of any number of decentred surfaces we thus obtain the final result, that the aberrational displacement of any ray due to the centring errors is of the form

$$\text{Horiz. comp.} = p_1 S^2(2\cos\phi + \cos(2E - \phi)) \\ + p_2 SV(3\cos\psi\cos E + \sin\psi\sin E) + 3p_3 V^2\cos\chi$$

$$\text{Vert. comp.} = p_1 S^2(2\sin\phi + \sin(2E - \phi)) \\ + p_2 SV(\cos\psi\sin E + \sin\psi\cos E) + p_3 V^2\sin\chi,$$

in which p_1, p_2, and p_3 are constants determining the magnitude of the resultant centring defects, and ϕ, ψ, and χ angles indicating their orientation in the field in the same sense in which the angle O indicated the direction of the decentration in our investigation of the effect at a single surface.

We will now discuss the three terms separately:—

1. The terms in S^2. As they stand, these define the displacement of any ray from its ideal position at the final focus of the thin radial pencils in cartesian co-ordinates, the "horizontal" component being taken along a radial line of the field and the "vertical" component in the tangential direction. If we refer the co-ordinates to a new axis forming the angle ϕ with the original one, they become

$$x = p_1 S^2(2 + \cos(2E - \phi))$$
$$y = p_1 S^2\sin(2E - \phi),$$

and on comparison with the Seidel-aberrations are immediately recognised as an orthodox coma-figure in the direction defined by the angle ϕ. For any given semi-aperture S this is constant all over the field (for it is independent of V), and evidently represents the uniform coma which was mentioned in the opening paragraphs as the only known effect of centring errors. As it is of the same amount and direction throughout the field of view, it does not affect the relative distances or position angles of points in it, and therefore is harmless as far as systematic errors in measurements are concerned. Like the Seidel coma, it grows with the square of the aperture.

2. The terms in VS. These assume their simplest analytical form by turning the co-ordinates through an angle equal to $\frac{1}{2}\psi$. The co-ordinates then become

$$x = p_2 \mathrm{SV} \frac{\sin \frac{3}{2}\psi}{\sin \frac{1}{2}\psi} \cos (\mathrm{E} - \tfrac{1}{2}\psi)$$

$$y = p_2 \mathrm{SV} \frac{\cos \frac{3}{2}\psi}{\cos \frac{1}{2}\psi} \sin (\mathrm{E} - \tfrac{1}{2}\psi).$$

The analogy with the Seidel-astigmatism is unmistakable, and closer discussion (which is really very interesting) shows that the terms represent astigmatic ellipses varying in length, eccentricity, and orientation in different parts of the field of view. But as this is a perfectly symmetrical deformation of the image points it is quite harmless, for it will of course be borne in mind that in any respectable instrument all these terms are bound to be very small.

3. There remain the terms V^2, namely,

$$x = 3p_3 \mathrm{V}^2 \cos \chi$$

$$y = p_3 \mathrm{V}^2 \sin \chi.$$

These are the only ones of astronomical interest, for they represent an extremely unpleasant dislocation of the image. As they contain neither S nor E they affect all the rays equally, just like the Seidel-distortion. Taken by themselves, the terms therefore represent simply a displacement of the sharp image.

The angle χ marks a fixed direction in the field. If we consider a point at any given distance from the centre of the field in the direction of χ, we shall obtain a radial displacement of the image $= 3p_3\mathrm{V}^2$, and in the opposite direction a displacement $= -3p_3\mathrm{V}^2$. For points at right angles to the direction of χ we shall have no radial displacement, but a tangential one $= p_3\mathrm{V}^2$ always in the direction of χ. In any other direction both components have finite values, with the result that we obtain displacements in both the radial and tangential direction as roughly indicated in fig. 3. Straight lines in any position or direction other than that of the *diameter* of the field in the direction of χ are rendered as curved lines in the image.

If sensible, this effect of centring errors will therefore be extremely objectionable. It must be borne in mind that the absence of the uniform coma-effect does *not* necessarily imply that this distortion is also absent. I have tried to estimate its reasonably possible magnitude, and feel confident that it could not reach one second of arc in the marginal parts of the ordinary astrographic plate. But in such instruments as the one used for the Franklin-Adams charts it might easily go into whole seconds.

Fortunately it will be a comparatively simple matter to test any given photographic instrument for the presence of this error. Let a rich field of stars near the meridian be photographed with instrument say West of the pier. Let the instrument be reversed to East of the pier and the plate turned through 180° in its own plane. If the same field is exposed again, it will be possible to secure the second impression everywhere very close to the first. There will thus be only a small parallel displacement plus a small difference of position angle between the first and second images of each star, and it should not be difficult to allow for these and to ascertain the direction and magnitude of the centring error. As an alternative two different plates could be exposed in the two positions of the instrument and a glass-positive from one of the negatives superposed upon the other negative, on Evershed's principle. In the absence of the error, perfect superposition would be attainable; in its presence, the reduction of the discrepancies to the possible minimum would everywhere display twice the aberrational centring error.

This seems such an obvious test that very possibly it has already been carried out with a view to the discovery of unsymmetrical displacements in the field, but in any case the knowledge of the law of distribution of the displacements caused by residual errors of centring should prove helpful in deducing the best possible corrective terms from a test of this description.

Imperial College of Science and Technology:
1919 *Jan.* 9.

Postscript.—My attention has recently been called to a paper by Mr. S. D. Chalmers, M.A. (*Proc. Phys. Soc.*, **30**, 100, 1918), in which the lateral aberration was used for the purpose of deducing the third-order aberrations of lens-systems in a manner very similar to that employed in my paper of 1918 November (*M.N.*, **79**, 60) on the same subject. So far as this method of attacking the problem is concerned, Mr. Chalmers can therefore undoubtedly claim priority.

1919 *March* 15.