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Study of the internal dynamics of an autonomous mobile robot

M. Eghtesad[a,*], D.S. Necsulescu[b]

[a] Department of Mechanical Engineering, Shiraz University, Shiraz, Iran
[b] Department of Mechanical Engineering, University of Ottawa, Ottawa, Ontario, Canada

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Abstract

The requirement of ideal rolling without sideways slipping for wheels imposes nonholonomic (non-integrable) constraints on the motion of the wheels and consequently on the motion of wheeled mobile robots. From the control point of view, the dynamics of nonholonomic systems can be divided in two parts: external and internal dynamics. The dimension of the external dynamics of nonholonomic systems depends on the number of inputs to the system and the dimension of the internal dynamics depends on the number of independent nonholonomic constraints. For different motion control problems of nonholonomic systems, a smooth (model based) state feedback control law only deals with the system external dynamics; therefore, the system internal dynamics must be examined separately and its stability has to be analyzed and proved.

In this paper, the internal dynamics of a three-wheel mobile robot with front wheel steering and driving is investigated. In particular, its internal dynamics stability is analyzed for two different situations, when the mobile robot is moving and when it is stationary.

Keywords: Internal dynamics; Autonomous mobile robots; Wheeled ground vehicles; Feedback linearization; Nonholonomic systems

1. Introduction

Control problems involving autonomous wheeled ground vehicles, i.e., mobile robots and AGVs, are challenging problems and they have attracted considerable attention in the control community. These vehicles have applications in industrial, household, military, security, space, office automation and scientific laboratory systems and require further improvements of their performance, safety, and reliability.

A conventional wheel has only two degrees of freedom, since its third degree of freedom in a planar pair model (the wheel and the ground) is restricted when the sideways component of wheel velocity assumed to be set to zero to eliminate sideways slip. Another assumption is that the wheel rolls without slip. These two assumptions are called ideal rolling conditions. Ideal rolling conditions impose nonholonomic constraints on the wheel and consequently on the vehicle motion [4,6]. The control problems for the nonholonomic systems are challenging problems because, in general, unlike holonomic constraints, nonholonomic constraints involve one or more velocities and are non-integrable. Given that the nonholonomic constraints are non-integrable, variables involved cannot be eliminated and will appear in the state space model of the system. Consequently, the rank of the system dynamics is lower than the number of state variables and, therefore, part of the system dynamics becomes unobservable. The presence of this unobservable (internal) dynamics is a result of nonholonomic constraints in the system and its consequences on system stability will be discussed in this paper.

There are several motion control problems of the vehicles, for example path following, trajectory tracking and point stabilization. A number of solutions for path following and trajectory tracking control of ground vehicles have been suggested, for example, Lyapunov direct methods [14,21], piecewise continuous controllers [27], fuzzy controllers [11, 31], the H_∞ method [20,29], sliding mode controllers [33], learning controllers [28], time-varying controllers [16,24], backstepping design [21,22], and hybrid controllers [3]. For point stabilization of the nonholonomic vehicles, three different approaches have been proposed. The first two are based on using non-smooth, namely, piecewise continuous [27], and time-varying [23,24], controllers. The third method refers to a

* Corresponding author. Fax: +98 711 628 7294.
E-mail address: eghtesad@shirazu.ac.ir (M. Eghtesad).

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particular control problem, the motion on a path which ends at the desired position and orientation [9,10]. The majority of the published research results have concentrated mostly on the use of kinematic models of the vehicles (where the velocities are the inputs), and less research has been done on solving the problem of integrating the kinematic nonholonomic constraints with the dynamics of the system [1]. Dynamic based models are needed for the verification of ideal rolling assumptions, for taking into consideration dynamic constraints, like motor-torque limits, and for including dynamic components like the suspension system into the model. Also, if the path of the vehicle contains some relatively sharp turns, a dynamic based control is expected to result in a more efficient motion control.

Nonlinear control theory can be applied to motion control problems based on a dynamic model of the vehicle. Frequently, feedback linearization, a systematic technique in nonlinear control theory that transforms the nonlinear system into a linear system, is used for stabilizing the original system around a point or about a trajectory. Application of this technique to nonholonomic vehicle trajectory tracking and stabilization to a manifold was suggested and used by d'Andrêa-Novel et al. [8], Yun and Yamamoto [34], Sarkar et al. [25], Caracciolo et al. [7], Necsulescu et al. [17] and Wang and Xu [30]. Application of the feedback linearization technique to nonholonomic vehicle point stabilization was investigated by Eghtesad and Necsulescu [9, 10]. The input–output feedback linearization is an efficient control approach when certain invertibility conditions are satisfied and the internal dynamics of the system is stable. There are only a few publications regarding internal dynamics of the nonholonomic vehicles and its stability. Yun and Yamamoto studied the internal dynamics using the dynamic model of a mobile robot with two independently driven wheels and four passive wheels when the mobile robot moves along a straight line backward or forward [34]. Wang and Xu investigated the stability of zero dynamics using kinematic models of a mobile robot with steerable wheels and a car-like robot, but the specific values of the internal dynamics dimension and its variable(s) were not determined [30]. Wang and Xu work referred to trajectory tracking and included simulations, while Yun and Yamamoto carried out experimental work as well.

For point stabilization, when the vehicle is moving, the internal dynamics analysis is the same as the trajectory tracking problem. When the vehicle is slowing down, the nonholonomic constraint becomes trivial and gives no solutions for internal dynamics governing state variable(s). In this case, to verify whether the vehicle has arrived at the desired orientation, more realistic assumptions (other than point contact between the wheel and the ground) are required. In this case, the friction force and moment between the wheel and the ground will act as damping terms and can (ultimately uniformly) stabilize the system.

A three-wheel vehicle with the front wheel steering and driving, and two rear wheels, shown in Fig. 1, is chosen as the example for this paper. The dynamic model of the mobile robot was derived and feedback linearization in Cartesian space was applied to this model [17]. The main points of this paper are finding the vehicle internal dynamics dimension

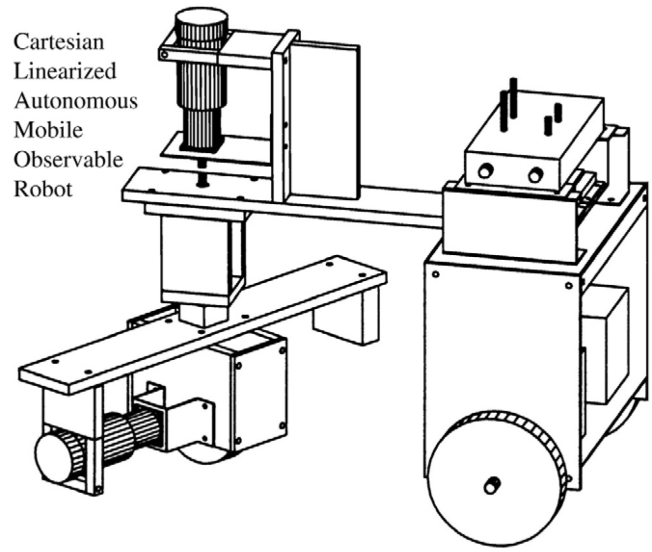

Fig. 1. A schematic diagram of the CLAMOR platform [15].

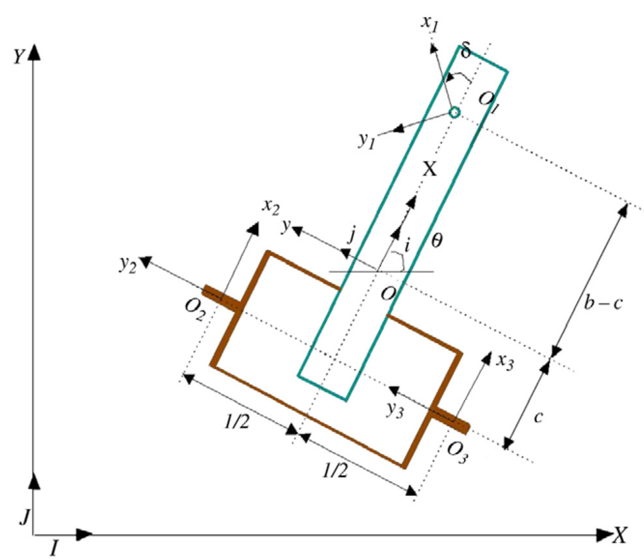

Fig. 2. Vehicle reference frames.

and its governing equation(s), and examining the internal dynamics stability when the vehicle is moving (for trajectory tracking) and when it is stationary (for point stabilization). To illustrate mathematical developments, vehicle simulation and experimental results are also presented.

2. Vehicle kinematics in Cartesian space

Wheeled vehicles correspond to multibody mechanisms and require multiple reference frames chosen in such a way as to facilitate the development of kinematic and dynamic models as well as control systems. For the case of planar motion of a three-wheel vehicle with a rigid structure, steering assembly and wheels, one inertial and four moving reference frames are chosen (Fig. 2). One moving frame, x–y–z, is attached to the vehicle main body (x-axis along the vehicle, y-axis parallel to the rear axle with the origin O at the centre of mass). Three others, x_i–y_i–z_i $(i = 1, 2, 3)$, with origins O_i, are attached to the three wheels (one at the front along with the steering assembly and two at the rear) with x_i-axes along the direction of motion of the wheels and z_i-axes parallel to z. A global inertial frame X–Y–Z is chosen with Z parallel to z. The angle θ, between the x- and X-axes is called the orientation angle.

The velocity of vehicle center of mass has components V_x and V_y in the vehicle moving frame:

$$V_x = r_1\omega_1 \cos\delta, \qquad V_y = \frac{c}{b} r_1\omega_1 \sin\delta \tag{1}$$

and the components $\dot{X}$, $\dot{Y}$ in the inertial frame:

$$\begin{aligned}\dot{X} &= V_x \cos\theta - V_y \sin\theta = r_1\omega_1 \\ &\quad\times \left(\cos\delta\cos\theta - \frac{c}{b}\sin\delta\sin\theta\right) = r_1\omega_1 A_1\end{aligned} \tag{2}$$

$$\begin{aligned}\dot{Y} &= V_x \sin\theta - V_y \cos\theta = r_1\omega_1 \\ &\quad\times \left(\cos\delta\sin\theta - \frac{c}{b}\sin\delta\cos\theta\right) = r_1\omega_1 A_1\end{aligned} \tag{3}$$

where b, c and δ are shown in Fig. 2 and r_1 and ω_1 are the radius and the angular velocity of the driving wheel, respectively. Also, since the velocity of the center of the rear axle along y_2 (or y_3) is assumed zero (no sideways slip for rear wheels), the following nonholonomic constraint can be written using rigid body kinematics:

$$V_y = c\dot{\theta} \Rightarrow \dot{\theta} = \frac{V_y}{c} = \frac{r_1}{b}\omega_1 \sin\delta. \tag{4}$$

Eq. (4) is nonholonomic since the angle δ represents the relative rotation of the front wheel about the z-axis with respect to the vehicle x–y moving frame, and therefore it cannot be integrated to obtain the orientation angle, θ. Eqs. (2)–(4) constitute the first part of the state space dynamic model of the vehicle.

3. Dynamic model of the vehicle in Cartesian space

A Newtonian dynamic model of the vehicle can be obtained using free body diagrams for the rigid structure of the vehicle, for each of the three wheels and for the steering assembly. For the rigid structure, three equations are obtained for the planar translations and for the rotation about axis Z. For each wheel, five equations of motion can be obtained for the planar translations (x_i, y_i) and for the rotation about the x_i-, y_i- and z_i-axes. For the steering assembly, five similar equations can be written in the x_1–y_1–z_1 frame.

Other than the moments at the wheel–ground contact areas, the forces and moments considered in the derivation of the above-mentioned equations are the most complete set of forces/moments for this system [9,10]. Some of these forces/moments could be negligible in some cases. Ground forces are functions of several factors including friction coefficients, weights and the slip angle [32]. Internal forces and moments are functions of the accelerations and the vehicle's characteristics [9]. By manipulating and eliminating the internal and ground forces and moments, this set of equations can be reduced to a set of six ordinary differential equations, including the three kinematic equations, (2)–(4) and the following equations [9]:

$$\dot{\omega}_1 = \frac{\tau_d - Num(\delta) r_1^2 \omega_1\omega_\delta - \frac{r_1}{b}\sin\delta\,\tau_s}{Den(\delta)} \tag{5}$$

$$\dot{\delta} = \omega_\delta \tag{6}$$

$$\begin{aligned}\dot{\omega}_\delta &= \tau_s\left[\frac{1}{J_{SA}+J_1} + \frac{r_1^2\sin^2\delta}{b^2 Den(\delta)}\right] - \frac{r_1\sin\delta\,\tau_d}{bDen(\delta)} \\ &\quad + \frac{r_1}{b}\omega_1\omega_\delta\left[\frac{Num(\delta)}{Den(\delta)} r_1^2\sin\delta - \cos\delta\right]\end{aligned} \tag{7}$$

where *Num* and *Den* are functions of vehicle dynamic parameters and the steering angle, δ [9]; J_{SA} and J_1 are the moments of inertia of the steering assembly and the front about axis z_1, and τ_d and τ_s are applied steering and driving torques, respectively. Eqs. (5) and (7) are Newton–Euler equations of motion for the front wheel steering and driving, and Eq. (6) is a definition required for the state space form of the system dynamics.

4. Cartesian space feedback linearization

Feedback linearization is a well-established approach to nonlinear control design. The central idea of the approach is to algebraically transform a nonlinear system dynamics into a (fully or partially) linear one, so that linear control techniques can be applied [12,26]. In input–output feedback linearization, the goal is to find a direct and simple relation between the system outputs and the control inputs [12,26]. To generate this direct relationship, the derivatives of the outputs should be taken until all the inputs (or their derivatives) appear independently in the decoupling matrix, $\mathbf{E}(\mathbf{x})$, which relates the output derivative and input vectors.

Given that the vehicle has only two inputs (steering torque τ_s and driving torque τ_d), only two out of the six state variables of the state vector $\underline{\mathbf{x}} = (X, Y, \theta, \omega_1, \delta, \omega_\delta)^{\mathrm{T}}$ can be controlled. For the case of choosing Cartesian coordinates X and Y as outputs, Eqs. (2) and (3) give the output derivatives. For the first derivatives, the decoupling matrix is $\mathbf{E}(\mathbf{x}) = \mathbf{0}$ and thus no relations between $\dot{X}$, $\dot{Y}$ and the inputs τ_s and τ_d are obtained [26]. When taking derivatives of Eqs. (2) and (3), both τ_s and τ_d appear, but one can easily show that det $\mathbf{E}(\mathbf{x}) = 0$. This is one of the consequences of the presence of nonholonomic constraints in the dynamic model of mechanical systems in which, because of Newton's second law, usually up to the second derivative of position vector should appear in the model based control algorithm. This leads to a dynamically extended input–output feedback linearization (that contains the third derivatives of the outputs and an input derivative) while for kinematically modeled mobile robots, static input–output feedback linearization requires only taking up to the second derivatives of the outputs.

Before taking the next derivatives, we introduce the intermediate input vector:

$$\underline{u} = [u_{i1}, u_{i2}]^{\mathrm{T}} = \left[\tau_d - \tau_s\frac{r_1}{b}\sin\delta, \tau_s\right]^{\mathrm{T}}. \tag{8}$$

Then [9],

$$E(\bar{x})\underline{u}_i = \begin{bmatrix} \dfrac{r_1 A_1}{Den(\delta)} & -r_1\left[A_3 + A_1\dfrac{Num(\delta)}{Den(\delta)}\right]\omega_1 \\ \dfrac{r_1 A_2}{Den(\delta)} & -r_1\left[A_4 + A_2\dfrac{Num(\delta)}{Den(\delta)}\right]\omega_1 \end{bmatrix}\begin{bmatrix}\dot{u}_{i1}\\ u_{i2}\end{bmatrix} \quad (9)$$

such that:

$$\det E(\bar{x}) = \frac{r_1^2(-A_1A_4 + A_2A_3)}{Den(\delta)}\omega_1 = \frac{1}{Den(\delta)}\frac{c}{b}r_1^2\omega_1 \quad (10)$$

where

$$A_3 = \sin\delta\cos\theta + \frac{c}{b}\cos\delta\sin\theta,$$
$$A_4 = \sin\delta\sin\theta - \frac{c}{b}\cos\delta\cos\theta. \quad (11)$$

This shows that $\det \mathbf{E}(\mathbf{x}) = 0$ only if $\omega_1 = 0$ or $c = 0$, i.e. only if the front wheel stops driving (when the nonholonomic constraint becomes trivial), or if the centre of mass is located on the rear wheels axle (Fig. 2).

5. Internal dynamics of the vehicle

Trajectory tracking (as well as path following) and point stabilization of the nonholonomic vehicles are two different motion control problems. For trajectory tracking, it is only required to show that the center of mass of the vehicle is on (or close to) the desired path. In this case the control problem consists of controlling two outputs (usually the Cartesian coordinates of the center of mass of the vehicle) using two inputs (usually, driving and steering torques, in the case of dynamic based control of the vehicle). For point stabilization, the control problem is more complicated since the vehicle's orientation and (its center of mass) position should be controlled by the same two inputs. For this case, Brockett's theorem applies and the system cannot be asymptotically stabilized by smooth state feedback laws [5].

When a nonholonomic system is in motion (trajectory tracking), its internal dynamics consists of one or more differential equation(s) and when the systems stops (point stabilization), this internal dynamics becomes trivial and gives no solutions. This is because, in general, the nonholonomic constraints involve one or more velocities and, when the system stops, these constraints become equal to zero. Therefore, for path following and trajectory tracking, the study of the internal dynamics of a nonholonomic vehicle reduces to study of its governing equation(s) and to finding the conditions when these generally nonlinear equations (or their linearized approximations) are stable.

(a) Finding the internal dynamics dimension

To find the dimension of the internal dynamics, Singh's inversion algorithm may be used [13]. For the Cartesian space dynamic model of the mobile robot considered in this paper, $n = 6$ and $\eta = \sum p_k$, where n is the number of the states and $p_{k+1} = p_k - (\rho_k - \rho_{k-1})$ with $p_0 = 2$ (starting with two degrees of freedom of the system), and ρ_k is the number of independent inputs present in the system's equations starting from outputs X, Y, and then their first derivatives $\dot{X}$, $\dot{Y}$, second derivatives $\ddot{X}$, $\ddot{Y}$, etc. At four different steps of the algorithm, $\rho_0 = 0$, $\rho_1 = 0$ (first derivatives), $\rho_2 = 1$ (second derivatives) and $\rho_3 = 2$ (third derivatives). Thus $\eta = 2+2+1+0 = 5$ and the system's internal dynamics is one dimensional ($n - \eta = 1$).

(b) Internal dynamics governing equation(s); zero dynamics

Knowing the dimension of the internal dynamics, its structure can be obtained using the zero-dynamics algorithm [12,26]. If we choose Cartesian coordinates X and Y as outputs, the following steps will result: (1) $X = 0$ and $Y = 0$; (2) $\omega_1^{(k)} = 0$ $(k = 1, \ldots)$.

Thus, this algorithm does not give any direct information about the other states, θ, δ and ω_δ. Eq. (4) and $\omega_1 = 0$ give: $\dot{\theta} = 0$. Eq. (7), for $\omega_1 = 0$, $\tau_s = 0$, $\tau_d = 0$ (for zero dynamics, the inputs are set to be zero) gives $\dot{\omega}_\delta = 0$, i.e., $\omega_\delta =$ const. and $\delta = \omega_\delta t + \delta_0$. This indicates that δ should be the internal dynamics variable. The intuition confirms this finding. If the internal dynamics referred to θ, the vehicle could not make any moves (X, Y and ω_1 and their derivatives would be zero). The internal dynamics can refer to δ because δ can keep changing since the front wheel, assumed to have point contact with the ground, can rotate about a vertical axis at constant speed $\omega_\delta =$ const., even when the vehicle center of mass is stationary ($\omega_1 = 0$) and the vehicle has no rotations. This leads to the conclusion that the dynamics of the steering angle δ is the vehicle's internal dynamics.

(c) Internal stability for $\omega_1 \neq 0$

An alternative approach to zero-dynamics analysis for path following and trajectory tracking of the nonholonomic vehicles is analyzing the stability of internal (steering) dynamics while the system is moving. Yun and Yamamoto suggested studying internal dynamics of the nonholonomic vehicle when it is moving on a straight line in order to analyze its local stability [34]. If the vehicle's motion under some conditions is locally (asymptotically) stable around its equilibrium points, $\delta = k\pi$, then it is acceptable to assume that the vehicle moves along a straight line. If there are other conditions under which the system's motion is unstable, there may be a chance of eventual recovery. This recovery has to satisfy the conditions for (asymptotic) stability around $\delta = k\pi$. To find those conditions, it can be assumed that, without loss of generality, the motion is along the X-axis,

$$\dot{X} = f(t), \quad \dot{Y} = 0. \quad (12)$$

For $\omega_1 \neq 0$, Eqs. (2), (3) and (12) give

$$\tan\delta = \frac{bV_y}{cV_x} = \frac{b(-\dot{X}\sin\theta + \dot{Y}\cos\theta)}{c(\dot{X}\cos\theta + \dot{Y}\sin\theta)} = \frac{b[-f(t)\sin\theta]}{c[f(t)\cos\theta]}$$
$$= \frac{-b\tan\theta}{c} \Rightarrow \delta = \tan^{-1}\left(\frac{-b\tan\theta}{c}\right). \quad (13)$$

Taking the time derivative of δ and using Eq. (4) will result in

$$\dot{\delta} = -(r_1\omega_1 \sin\delta) \Big/ \left(c\cos^2\theta + \frac{b^2}{c}\sin^2\theta\right). \tag{14}$$

The solution of Eq. (14) is

$$\delta = 2\tan^{-1}\left[\exp\left(-\int \frac{r_1\omega_1}{(c\cos^2\theta + \frac{b^2}{c}\sin^2\theta)}\mathrm{d}t\right)\right] + \text{const.} \tag{15}$$

For $\delta(t_0) = 0$ and const. $= 0$, it can be easily shown that a sufficient condition for having δ approaching zero for $t \to \infty$ is that the integrant has a lower positive bound $a > 0$; then, because of the minus sign, $\delta < 2\tan^{-1}(\mathrm{e}^{-at})$. In this case, given that $\lim_{t\to\infty}(\mathrm{e}^{-at}) = 0$, the result is $\lim_{t\to\infty}\delta = 0$.

Knowing that r_1 and c are positive, the assumption on the integrant means that, as long as the vehicle keeps moving forward ($\omega_1 > 0$), its internal dynamics is stable. This indicates that, in the event of a small perturbation of δ away from the zero value, eventually δ comes back to $\delta = 0$, i.e., to a straight line motion.

(d) Internal dynamics for $\omega_1 = 0$; wheel–ground contact area

The steering dynamics governing equation can be obtained by summing Eq. (5) multiplied by $(r_1\sin\delta)/b$ and Eq. (7):

$$\tau_s = (J_{SA} + J_1)\left[\frac{r_1}{b}(\dot{\omega}_1\sin\delta + \omega_1\dot{\delta}\cos\delta) + \ddot{\delta}\right]. \tag{16}$$

When a vehicle is stationary, $\omega_1 = 0$, and there are no input torques to the system, Eq. (16) reduces to $\ddot{\delta} = 0$. This is a result of the simplifying assumption that the contact between wheel and ground takes place at a point or a line (with a length equal to the width of the wheel). The reality is that, because of the weight of the wheel and the fact that the wheel is not completely rigid, there is a contact surface which is known as the Hertzian surface [18]. As a result, contact friction leads not only to a resultant force applied to the center of the area but also to a non-vanishing moment about the normal axis through the center of that area [18]. This moment, M_A, is a function of the size of the contact area A, wheel material, type of wheel–ground contact, weight of the vehicle, etc. Since M_A opposes the steering motion, it should be added to Eq. (16) using a sign function, such that

$$\tau_s = (J_{SA} + J_1)\left[\frac{r_1}{b}(\dot{\omega}_1\sin\delta + \omega_1\dot{\delta}\cos\delta) + \ddot{\delta}\right] + M_A\,\mathrm{sgn}\,\dot{\delta}. \tag{17}$$

Eq. (17) is a non-homogeneous nonlinear differential equation. Since M_A is nonzero as long as the resultant friction force at the contact area is nonzero, Eq. (17) shows that the friction term transforms a marginally stable system corresponding to Eq. (16) into a uniformly bounded stable system. In fact, the dry friction term leads to energy dissipation during motion and eventually to sticking in the vicinity of the desired orientation.

6. Simulation and experimental results

(a) The vehicle

A three-wheel vehicle, with the front wheel steering and driving (Fig. 1), is used for simulation and experiments. It has five mechanical parts: a rigid structure to which the other parts are attached, two rear wheels, a front wheel and the steering assembly, which is attached to the front wheel, to the main body and to two steering and driving motors. The electrical and electronic components are: two DC servo-motors, two 12 V batteries, an interface card and two amplifiers, a pair of signal buffers and two encoders for measuring the steering angle and the driving angular velocity. The main dimensions of the vehicle are: radius of the front wheel, $r_1 = 0.0625$ m, wheelbase, $b = 0.381$ m, track width, $l = 0.36$ m, and distance from the rear wheel axle to the vehicle center of mass, $c = 0.187$ m (Fig. 1).

The vehicle main dynamic parameters (i.e., masses and moments of inertia) are: mass of the rigid structure (including batteries), $m_0 = 18.0$ kg, mass of the steering assembly, $m_{\mathrm{SA}} = 6.1$ kg, moment of inertia of the steering assembly about axis z_1, $J_{\mathrm{SA}} = 0.354$ kg m^2, moment of inertia of the rigid structure about axis z_1, $J_0 = 2.01$ kg m^2, and moment of inertia of the front wheel about axis z_1, $J_1 = 0.000844$ kg m^2 [15].

In order to validate the results of the analytical investigation of the stability of the internal dynamics of the three-wheel mobile robot, several simulation and experimental tests have been performed.

(b) Trajectory tracking and internal stability for $\omega_1 \neq 0$

Since trajectory tracking is a simpler task, only the results of two simulations when the vehicle does not start on the desired trajectory, with and without saturation limits on input torques (10 N m), are presented. The results of Figs. 3 and 4 are for a vehicle path that starts at (0.0, 0.5) parallel to the X-axis and a desired path that starts at (0.5, 0.0) with 45° orientation. The heading velocity of a virtual vehicle moving on the desired path is assumed to be 0.25 m/s and the rate of change of its orientation is 0.1 rad/s. The results show an excellent trajectory tracking by the vehicle. The behavior of the vehicle is asymptotically stable, with regard to both external and internal dynamics.

Another simulation was carried out for the case when the vehicle is moving on a straight line ($\omega_1 > 0$), to verify that any relatively small perturbation in the steering angle will eventually disappear. In this simulation the vehicle starts from the origin with $\delta = 0.1$ rad, $\omega_1 = 0.1$ rad/s, $\theta = 0.0$ rad and moves at a constant heading velocity equal to 0.1 m/s (Fig. 5). The results from Fig. 6 show that the steering angle will reach zero relatively fast. Thus, this simulation illustrates the validity of our analytical investigation of the internal dynamics stability when $\omega_1 \neq 0$.

(c) Point stabilization and internal stability for $\omega_1 = 0$

For the case of $\omega_1 = 0$, analytical results reported in this paper are verified by simulations and experiments. In order to

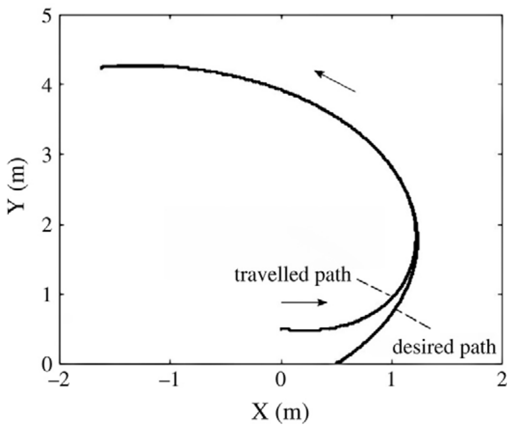

Fig. 3. A simple trajectory tracking without saturation.

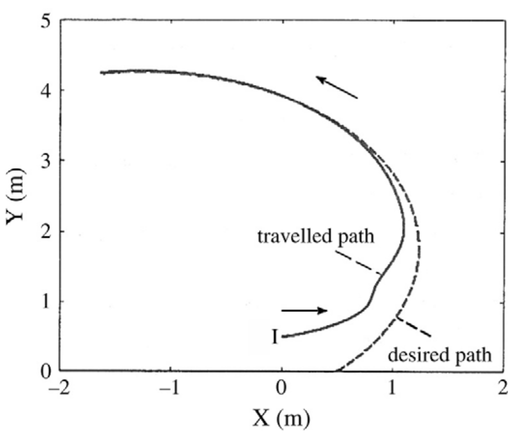

Fig. 4. A simple trajectory tracking with saturation.

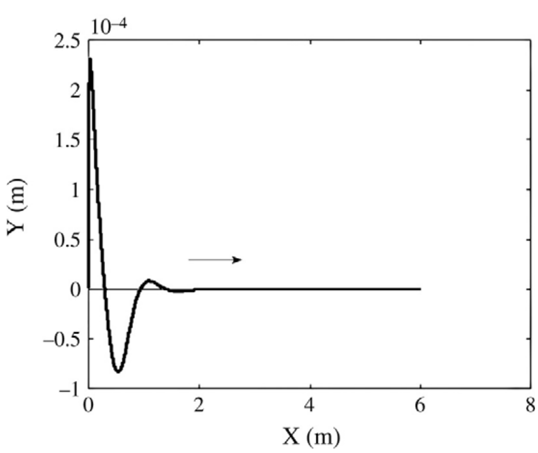

Fig. 5. Moving forward ($\omega_1 > 0$).

perform the experimental tests, several steps besides parameter identification, have been taken: friction compensation using the Bo–Pavelescu friction model [2,19], filtering of the measured data using a low-pass IIR filter, and real-time controller implementation on a TMS320C30 digital signal processor [9, 10,15]. In the simulation and experimental test, reported in this paper, the vehicle starts at (0.0, 0.0) parallel to the X-axis and is supposed to stop at (2.0, 1.0) with a 60° orientation. Figs. 7–12

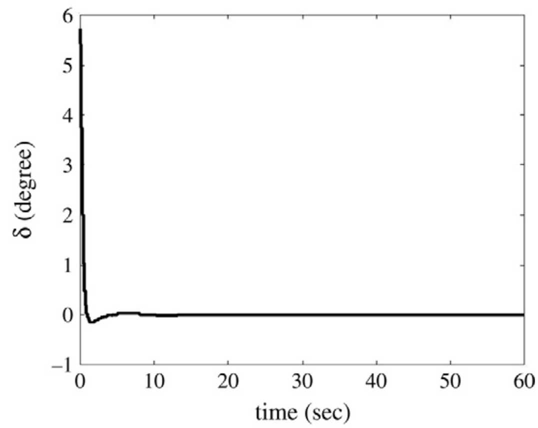

Fig. 6. Steering angle versus time (moving forward).

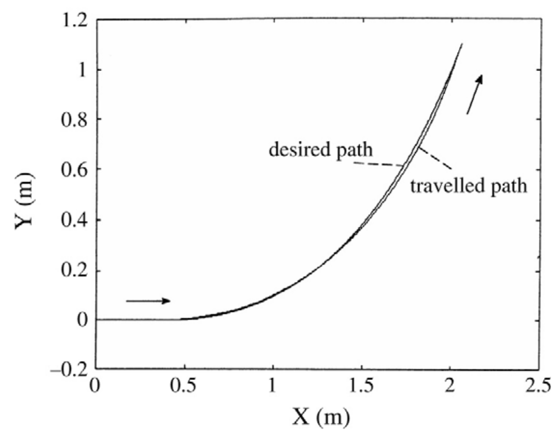

Fig. 7. Experimental results for the position of the CG versus time.

show the experimental and simulation results for Cartesian coordinates of the position of the vehicle center of mass, errors in X- and Y-directions, and the orientation angle for this test, respectively. Experimental and simulation results from Figs. 7 and 8 as well as from Figs. 11 and 12 resemble each other closely. X-direction and Y-direction errors from Figs. 9 and 10 are small compared with the path traveled, shown in Fig. 7. These results confirm that the proposed controller can achieve, as expected, point stabilization of the mobile robot, since the errors of position and orientation are in acceptable ranges from a practical point of view. Other experimental results have been reported in [9,10].

7. Conclusions

One of the major issues involved in applying smooth model based state feedback control laws like input–output feedback linearization to a nonholonomic system is that the dynamics of the system will be divided into two parts, external and internal dynamics. Smooth model based state feedback control laws in the case of feedback linearization can only deal with the external dynamics. In this paper, the internal dynamics of a three-wheel vehicle was examined. The vehicle considered in this paper has a one-dimensional internal dynamics with the steering angle as its governing state variable. The examination

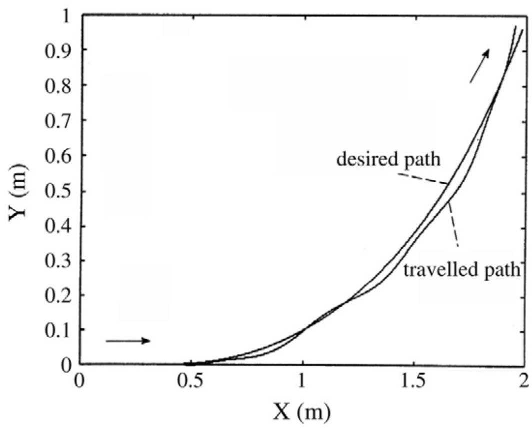

Fig. 8. Simulation results for the position of the CG versus time.

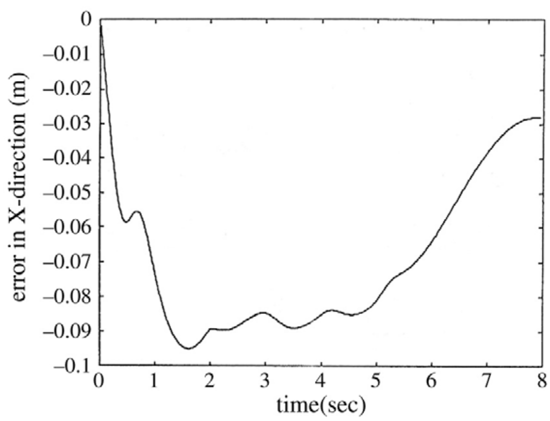

Fig. 9. Experimental results for the error in the X-direction versus time.

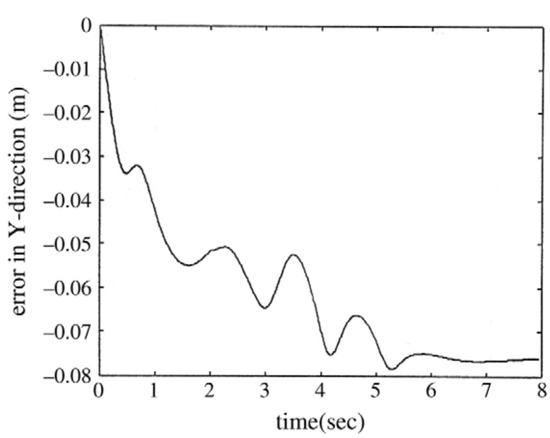

Fig. 10. Experimental results for the error in the Y-direction versus time.

of the steering dynamics indicates that, when the steering torque is zero, this dynamics is asymptotically stable as long as the vehicle keeps moving forward. This result relates to the asymptotic stabilization of trajectory tracking procedure.

For point stabilization, if the path close to the end point is almost a straight line, internal stability analysis leads to the result that, as long as the front wheel angular velocity is positive, the overall system is stable and that, when this velocity

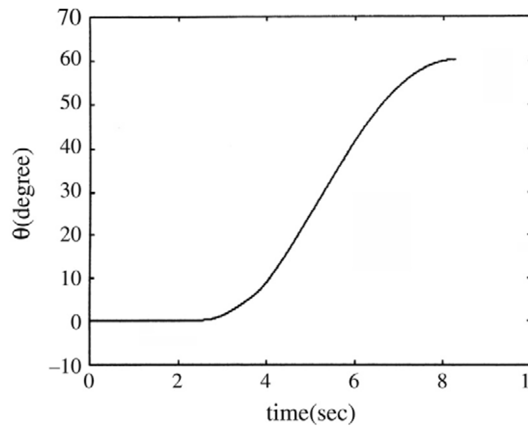

Fig. 11. Experimental results for the orientation angle, θ, versus time.

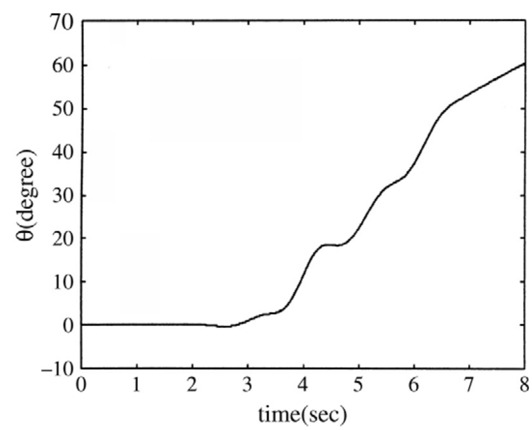

Fig. 12. Simulation results for the orientation angle, θ, versus time.

approaches zero (in the vicinity of the vehicle stopping point), the vehicle motion is bounded and ultimately uniformly stable.

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Mohammad Eghtesad, Ph.D.
B.Sc., University of Tehran (1987),
M.Sc., University of Tehran (1989),
Ph.D., University of Ottawa (1996).
Dr. Eghtesad joined the Department of Mechanical Engineering at the University of Shiraz in 1997. He has taught and done research in the areas of robotics, mechatronics and control. His research includes both theoretical and experimental studies.

Dan S. Necsulescu, Ph.D., P.Eng.
Eng. Dipl., Politecnica University of Bucharest (1966),
Licentiate in Philosophy, University of Bucharest (1973),
Ph.D., Politecnica University of Bucharest (1974).
Following his university study, Dr. Necsulescu worked as a design and research engineer until he left Romania in 1980. Since 1980 he has been professor in the Department of Mechanical Engineering at the University of Ottawa, where he has carried out collaborative research with various companies and governmental organizations including CRC, Thomson CSF Systems Canada and the Canadian Space Agency and Defense Research and Development Canada.