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# Identification of Source Terms in Nonlinear Convection Diffusion Phenomena by Sinc Collocation-Interpolation Methods

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**Abstract**—The solution of an inverse problem related to the identification of a source term for convection diffusion equations in one space dimension is dealt with in this paper. The solution technique is based on collocation-interpolation methods which use sinc functions and on a suitable application of domain decomposition methods.

**Keywords**—Nonlinear problems, Evolution equations, Sinc functions, Collocation, Interpolation, Pollutants hydrodynamics.

## 1. INTRODUCTION

This paper deals with the solution of a class of inverse problems related to the identification of a source term for convection diffusion equations in one space dimension. The problem is such that the source term is well localized in space; however, the intensity of the source is not known in time. The mathematical problem is an initial-boundary value problem with well-defined initial and boundary conditions. In addition, the dependent variable is assumed to be known, obtained by experimental measurements, in a fixed point of the space variable. This additional information is exploited to recover the needed information on the source term.

This problem can be classified, as documented in [1, Chapter 4], as an inverse (ill-posed) problem. Dealing with such a problem involves instability problems that have to be carefully controlled. Some ideas to deal with this problem in the case of parabolic purely diffusive equations are given in the above cited book on the basis of the analysis proposed in [2]. The solution method proposed in [2] is based on decomposition of domains techniques and on solution in each domain by collocation-interpolation methods based on Lagrange type interpolation. This method is certainly efficient and stable for parabolic diffusive equations and for source terms that have a smooth monotone behaviour in time. However, further developments are necessary in the case of convective equations and of source terms characterized by fast oscillations. The technical problem is here solved by use of interpolations by sinc functions. The analysis of [2] will still be a useful reference point.

As known, see [3–5], the approximation of functions on the real line can be obtained by the so-called sinc functions. In order to deal with such a problem, consider the collocation on the real line

$$I_x = \{\dots, x_{-i} = -ih, \dots, x_0 = 0, \dots, x_i = ih, \dots\}, \tag{1.1}$$

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and the corresponding sinc functions

$$S_i = (x; h) = \frac{\sin z_i}{z_i}, \qquad z_i = \frac{\pi}{h}(x - ih), \qquad S_i(x_j) = \delta_{ij}, \tag{1.2}$$

where $\delta_{ij}$ denotes the Kronecker delta function. Then a function

$$f = f(x_i) : [0,1] \to \Re \tag{1.3}$$

can be approximated and interpolated, see [3,4], as follows:

$$f \cong f^n = \sum_{i=-n}^{i=n} f_i S_i(x; h), \tag{1.4}$$

where $f_i = f(x_i)$. If $f$ is defined over the whole real line, then the series

$$C(f, h) = \sum_{-\infty}^{\infty} f_i S_i(x; h), \tag{1.5}$$

which is convergent under suitable integrability and decay to zero properties of $f$, can be used for the approximation; see [5, Chapter 1].

The functional space of the functions for which convergence of the series (1.5) is assured will be denoted by $B(h)$ and is known as the *Paley-Wiener space.* The set

$$\left\{ \frac{1}{\sqrt{i}} S_i(x; h) \right\}_{-\infty}^{\infty} \tag{1.6}$$

defines a complete orthonormal set in $B(h)$.

As known [1], the solution of nonlinear initial-boundary value problems for partial differential equations can be dealt with by suitable collocation and interpolation methods based upon collocation in space and approximation of the dependent variable and of the space derivatives by interpolation by fundamental polynomials. Generally, Lagrange polynomials and fundamental splines are used. Then technical calculations recover the evolution of the dependent variable in the collocation points in terms of a suitable set of ordinary differential equations. The system of ordinary differential equations can subsequently be solved by known solution techniques, see [1, Chapter 2].

Although efficient alternatives to this method can be proposed, collocation-interpolation methods can very naturally deal with mathematical problems governed by nonlinear partial differential equations. The use of sinc functions to deal with nonlinear initial-boundary value problems was first proposed in [6], where several technical aspects are dealt with and convergence criteria are discussed. Further developments are proposed in [7] with special attention to integro-differential equations.

Beyond its interest for mathematics, the specific physical problem under investigation was chosen for the relevance it has in engineering sciences. In recent years, interest in the study of the dynamics of pollutants has continually increased (see, for example, review [8]); and the phenomenon of convection diffusion investigated here is one of the main areas, describing as it does the longitudinal evolution of an effluent in both natural and artificial water courses.

With the aim of approaching the real behaviour of the pollutant, a nonlinear model is considered in place of the usual linear one: as will be seen below, nonlinearity plays a significant role which cannot be neglected in order to reach certainty of description of the phenomena.

The content of this paper is developed four sections. The first one is the Introduction. Section 2 provides the description of the physical system modelled by differential equations and of the related mathematical statement of the problem. Section 3 describes the proposed solution schemes and the technical solution of the problem. In Section 4, some simulations are reported with a final discussion.

## 2. THE PHYSICAL SYSTEM AND STATEMENT OF THE PROBLEM

As stated in the Introduction, the physical phenomenon studied in the present work concerns the dispersion due to shear effect of passive substances in turbulent flows. This phenomenon, in the case in which a puntiform source is also present, is described by the following convection-dispersion equation, with source term, determined in its linear form by Taylor (see, for example, [9,10])

$$\frac{\partial C'}{\partial t'} + U\frac{\partial C'}{\partial x'} = \frac{\partial}{\partial x'}\left(K(C')\frac{\partial C'}{\partial x'}x'\right) + s'(t')\delta(x' - x'_S), \tag{2.1}$$

where $C'$ is the mean concentration of the substance dispersed in the general cross section of the flow, $U$ is the mean flow velocity, $s'(t')$ is the intensity of the source, $\delta$ is the Dirac function (with $x'_S$ source coordinate), and the factor $K(C')$ is the dispersion coefficient. For the latter, a dependence on the concentration $C'$ is assumed, obtained through a Taylor series expansion cut off at the second term, that is,

$$K(C') = K_M\left(1 - \varepsilon\left(\frac{C' - C'_M}{C'_M}\right)\right), \tag{2.2}$$

with $\varepsilon$ (small) nonlinear parameter, $C'_M$ a suitable reference concentration (here chosen as the maximum concentration present in the problem treated), and $K_M$ the corresponding dispersion coefficient.

Equation (2.1) may be conveniently adimensionalized in the form

$$\frac{\partial C}{\partial t} + c_1\frac{\partial C}{\partial x} = c_2(1 - \varepsilon(C-1))\frac{\partial^2 C}{\partial x^2} - c_3\left(\frac{\partial C}{\partial x}\right)^2 + s(t)\delta(x - x_s), \tag{2.3}$$

where

- $C = C'/C_M$ is the dimensionless concentration of effluent $C'$ referred to the maximum value $C_M$,
- $t = t'/T$ is the dimensionless time variable obtained referring the time variable $t'$ to the duration $T$ of the simulation,
- $x = x'/L$ is the dimensionless space variable obtained referring the space variable $x'$ to the length $L$ of the channel,
- $s(t) = s'(t')T/C_M$ is the dimensionless intensity of the source.

Furthermore, the constants which characterize the model are defined as follows:

$$c_1 = \frac{TU}{L}, \qquad c_2 = \frac{TK_M}{L^2}, \qquad c_3 = -\frac{\varepsilon TK_M}{L^2} = -\varepsilon c_2. \tag{2.4}$$

In this way we obtain the definition of the domains of definition of independent and dependent variables as follows:

$$C = C(t,x) : [0,1] \times [0,1] \to [0,1]. \tag{2.5}$$

The inverse problem is investigated assuming that a source set on the known abscissa $x_S$ introduces passive substances into the current with an intensity described by an unknown time law $s(t)$. Having available an auxiliary measurement of the concentration $C_m(t)$ in a point set downstream of the source and the time behaviour of the concentration at the domain boundaries, we wish to reconstruct the source term $s(t)$.

The problem investigated is of great interest both in hydraulic engineering and in the broader realm of environmental engineering. Often, in fact, the need occurs to discover hidden pollutant sources or to reconstruct the evolution of accidental discharges of pollutants starting from information recorded in established measuring stations along the water courses.

The mathematical problem, which is an inverse problem (see [11, Chapter 1]), can be formalized as follows.

PROBLEM. Find the solution to the initial-boundary value problem for equation (2.3) linked to the initial conditions

$$C(0,x) = C_0(x), \quad \forall x \in [0,1], \tag{2.6}$$

to the boundary (Dirichlet) conditions

$$C(t,0) = \alpha(t), \qquad C(t,1) = \beta(t), \quad \forall t \in [0,1], \tag{2.7}$$

and to additional information

$$C(t,x_m) = C_m(t), \qquad x_s < x_m < 1, \quad \forall t \in [0,1], \tag{2.8}$$

while the source term $s = s(t)$, localized in $x_S$, is not known and has to be determined by the solution of the problem.

The solution technique will be developed bearing in mind that the related instabilities have to be controlled. The solution will be easily generalized to the case of Neumann boundary conditions to one of the two boundaries.

This problem for the parabolic equation without convection term was proposed in [2] (technically reported in [1]) based on Lagrange-type interpolation. This type of interpolation certainly works for smooth solutions with monotone behaviour. It can hardly capture steep solutions induced by the convection term or wavy solutions induced by oscillating behaviours of the initial or boundary conditions or induced by a source term oscillating in time.

The sinc type interpolation proposed in this paper is developed in order to overcome the above outlined difficulties. The whole matter is discussed in the last section of this paper.

## 3. SOLUTION METHOD

The solution method for the problem described in the preceding section is based on a collocation interpolation method linked to a decomposition of domain techniques. In particular, the domain $D = [0,1]$ of the space variable is decomposed as follows:

$$D = [0,1] = D_1 \cup D_2 \cup D_3 = [0,x_s] \cup [x_s,x_m] \cup [x_m,1]. \tag{3.1}$$

Then, in each domain different algorithms will be developed towards a parallel solution, related to the equation without the source term.

SOLUTION IN $D_3$. The problem in $D_3$ is a Dirichlet problem with two points boundary conditions in $x_m$ and 1, respectively. The solution can be obtained using a collocation interpolation method. Consider then the collocation

$$i = 0,\dots,n+1: \; I_x = \{x_0 = x_m,\dots,x_i = ih,\dots,x_{n+1} = 1\}, \qquad h = \frac{1-x_m}{n+1}. \tag{3.2}$$

The function $C = C(t,x)$ defined over $[0,1] \times [x_m,1]$ can be approximated by means of sinc functions

$$C(t,x) \cong C^n(t,x_i) = \sum_{i=0}^{n+1} S_i(x;h)C_i(t), \tag{3.3}$$

where $C_i(t) = C(t,x_i)$ denote the values of $C$ in the nodal points of a discretization of the variables $x$, and the sinc functions $S_i(x)$ have already been defined in the first section.

The partial derivatives can be approximated by

$$\frac{\partial C^n}{\partial x}(t,x_i) = \sum_{p=0}^{n+1} a_{pi}C_p(t), \qquad \frac{\partial^2 C^n}{\partial x^2}(t,x_i) = \sum_{p=0}^{n+1} b_{pi}C_p(t), \tag{3.4}$$

where

$$a_{pi} = \frac{dS_p}{dx}(x_i), \qquad b_{pi} = \frac{d^2S_p}{dx^2}(x_i). \tag{3.5}$$

Technical calculations yield

$$\begin{aligned} a_{pi} &= \left(\frac{\pi}{h}\right)\frac{z_{pi}\cos z_{pi} - \sin z_{pi}}{z_{pi}^2} = \frac{(-1)^{i-p}}{h(i-p)}, & a_{ii} &= 0, \\ b_{pi} &= \left(\frac{\pi}{h}\right)^2\frac{(2 - z_{pi}^2)\cos z_{pi} - 2z_{pi}\sin z_{pi}}{z_{pi}^2} = \frac{2(-1)^{i-p+1}}{h^2(i-p)^2}, & b_{ii} &= -\frac{1}{3}\left(\frac{\pi}{h}\right)^2. \end{aligned} \tag{3.6}$$

Substituting the expressions of the partial derivatives (3.4) into equation (2.3), without the source term, yields a system of ordinary differential equations which defines the evolution of the values $C_i$ of the variable $C$ in the nodal points. Eliminating, for simplicity of notations, the superscripts $n$ in the variable $C$ yields

$$\frac{dC_i}{dt} = -c_1\sum_{p=0}^{n+1} a_{pi}C_p(t) + c_2(1 - \varepsilon(C_i - 1))\sum_{p=0}^{n+1} b_{pi}C_p(t) - c_3\left(\sum_{p=0}^{n+1} a_{pi}C_p(t)\right)^2, \tag{3.7}$$

for $i = 1, \dots, n$. Moreover, the boundary conditions are enforced in $x = x_m$ and in $x = 1$

$$C_0(t) = C_m(t), \qquad C_{n+1}(t) = \beta(t). \tag{3.8}$$

The system can be solved by means of standard techniques for ordinary differential equations, see [1, Chapter 2]. The equation (3.7) provides the time-evolution of the variable $C$ in the nodal points. The time-space evolution is given by

$$C(t, x) \cong S_0(x; h)C_m(t) + \sum_{i=1}^{n} S_i(x; h)C_i(t) + S_{n+1}(x; h)\beta(t). \tag{3.9}$$

Moreover, the $x$-derivative of $C$ is obtained, at $x = x_m$, as

$$\frac{\partial C}{\partial x}(t, x_m) \cong \sum_{i=0}^{n+1} \frac{dS_i}{dx}(x_m)C_i(t) = \sum_{i=0}^{n+1} \frac{(-1)^{i+1}}{hi}C_i(t). \tag{3.10}$$

SOLUTION IN $D_2$. The problem in $D_2$ is now solved by a collocation interpolation in time, so that the differential system is reduced to a set of differential equations with ordinary derivatives with respect to the space variable. Integration is then performed over the $x$-variable.

The solution of the problem in $D_3$ is used to recover boundary conditions in $x = x_s$, which become initial conditions for the integration with respect to the $x$-variable.

Bearing this in mind, let us rewrite the evolution equation (2.3) without the source term as follows:

$$\frac{\partial C}{\partial x} = v, \qquad \frac{\partial v}{\partial x} = \frac{1}{c_2(1 - \varepsilon(C - 1))}\frac{\partial C}{\partial t} + \frac{c_1 v + c_3 v^2}{c_2(1 - \varepsilon(C_1))}. \tag{3.11}$$

Moreover, consider the space discretization (3.1) and the time discretization

$$r = 0, \dots, q+1 : \{t_0 = 0, \dots, t_r = rk^*, \dots, t_{q+1} = 1\}, \qquad k^* = \frac{1}{q+1}. \tag{3.12}$$

The following interpolation can be used:

$$C(t, x) \cong C^q(t_r, x) = \sum_{r=0}^{q+1} S_r(t, k^*)C_r(x), \tag{3.13a}$$

$$v(t, x) \cong v^q(t_r, x) = \sum_{r=0}^{q+1} S_r(t, k^*)v_r(x), \tag{3.13b}$$

where

$$C_r(x) = C(t_r, x), \qquad v_r(x) = v(t_r, x). \tag{3.14}$$

Accordingly, the time partial derivative can be approximated by

$$\frac{\partial C^q}{\partial q}(t_r, x) = \sum_{k=0}^{q+1} d_{kr} C_k(x), \tag{3.15}$$

where

$$d_{kr} = \left(\frac{\pi}{k^*}\right) \frac{z_{kr}\cos z_{kr} - \sin z_{kr}}{z_{kr}^2} = \frac{(-1)^{r-k}}{k^*(r-k)}, \qquad d_{kk} = 0. \tag{3.16}$$

Following the same mode described for the solution in $D_3$ yields the system of ordinary differential equations which defines the evolution of the values $C_r$ and $v_r$ of the variables $C$ and $v$ in the nodal points. Eliminating the superscript $q$, the system of equations is written as

$$\frac{dC_r}{dx} = v_r, \qquad \frac{dv_r}{dx} = \frac{1}{c_2(1-\varepsilon(C_r-1))} \sum_{k=1}^{q+1} d_{kr} C_k - \frac{c_1 v_r - c_3 v_r^2}{c_2(1-\varepsilon(C_r-1))}, \tag{3.17}$$

where $r = 1, \dots, q+1$, with initial conditions referred to by the variables $C$ and $v$ in $x = x_m$ defined by $C_r(x_m) = C_m(t_r)$ and $v_r(x_m) = v_m(t_r)$. Furthermore, the boundary conditions are enforced in $t = 0$:

$$C_0(x) = C(0, x), \qquad \frac{dC_r}{dx}(t, x_m) = \sum_{i=0}^{n+1} \frac{dS_i}{dx}(x_m) C_i(t). \tag{3.18}$$

Again the system can be solved by means of suitable techniques for ordinary differential equations. The time-space evolution is now given by

$$C(t, x) \cong \sum_{r=1}^{q+1} S_r(t; k^*) C_r(x), \tag{3.19a}$$

$$v(t, x) \cong \sum_{r=1}^{q+1} S_r(t; k^*) v_r(x). \tag{3.19b}$$

In particular, the value of $C$ and its $x$-derivative is obtained, at $x = x_S^+$, as

$$C(t, x) \cong \sum_{r=1}^{q+1} S_r(t; k^*) C_r(x_S^+), \tag{3.20a}$$

$$\frac{\partial C}{\partial x}(t, x_S^+) = v(t, x_S^+) \cong \sum_{r=1}^{q+1} S_r(t; k^*) v_r(x_S^+). \tag{3.20b}$$

This change of variable of integration stabilized the differential system which would be instable for collocation in space and integration in time. This change of variable produced an exchange of role between boundary and initial conditions.

SOLUTION IN $D_1$. The solution in domain $D_1$ is of the same type as that obtained for domain $D_3$. It is again a Dirichlet problem: the condition at $x = 0$ is a data of the problem, whilst the solution of the problem in $D_2$, in particular the result (3.20a), supplies the boundary conditions at $x_s$. This is in fact assumed to be

$$C(t, x_s^-) = C(t, x_s^+). \tag{3.21}$$

Then consider the collocation

$$j = 0, \dots, l+1 : I_x = \{x_0 = 0, \dots, x_j = jh^*, \dots, x_{l+1} = x_s\}, \qquad h^* = \frac{x_s}{l+1}. \tag{3.22}$$

The function $C = C(t,x)$ defined over $[0,1] \times [0,x_s]$ can again be approximated by means of sinc functions

$$C(t,x) \cong C^l(t,x_i) = \sum_{j=0}^{l+1} S_j(x;h)C_j(t), \tag{3.23}$$

where $C_j(t) = C(t,x_j)$ denote the values of $C$ in the nodal points of a discretization of the variables $x$. From here the solution proceeds in the same way as that for domain $D_3$, again producing a system of ordinary differential equations

$$\frac{dC_j}{dt} = -c_1 \sum_{p=0}^{l+1} a_{pj}C_p(t) + c_2(1-\varepsilon(C_j-1)) \sum_{p=0}^{l+1} b_{pj}C_p(t) - c_3 \left(\sum_{p=0}^{l+1} a_{pj}C_p(t)\right)^2, \tag{3.24}$$

with the boundary conditions in $x = 0$ and $x = x_s$

$$C_0(t) = \alpha(t), \qquad C_{l+1}(t) = C(t,x_s^-). \tag{3.25}$$

Thus we obtain the field $C(t,x)$ and, in particular, the value of $x$-derivative

$$\frac{\partial C}{\partial x}(t,x_s^-) \cong \sum_{j=0}^{l+1} \frac{dS_j}{dx}(x_s)C_j(t) = \sum_{j=0}^{l+1} \frac{(-1)^{l+1-j}}{h^*(l+1-j)} C_j(t). \tag{3.26}$$

Finally, it is possible to solve the inverse problem and to obtain the unknown flux of the source according to the (dimensionless) relationship

$$s(t) = c_2[1-\varepsilon(C-1)] \left[\frac{\partial C}{\partial x}(t,x_s^-) - \frac{\partial C}{\partial x}(t,x_s^+)\right], \tag{3.27}$$

in which the two derivatives are those obtained in (3.20) and (3.26).

## 4. SIMULATIONS AND DISCUSSION

With the aim of realizing simulations with real physical significance, the nonlinear parameter $\varepsilon$ present in (2.3) is evaluated as follows, with reference to the noted experiments conducted by Godfrey and Frederick [12] in a river. Two concentration surveys were available for this river one taken at 1731 m from the effluent introduction point and a second at 4300 m. Based on the hydraulic information provided by the authors from the experiments, we believe that the first measurement well satisfies the validity hypotheses of the equation (2.1) and, furthermore, that the characteristics of the water course limit the effects of the bends and the dead zones. A domain is then considered with origins which correspond to the first measurement and sufficiently extended to be able to set the downstream boundary condition at zero. Considering the first survey as upstream boundary condition and resolving the corresponding direct problem, the parameter $\varepsilon$ is adjusted in such a way as to obtain the best agreement between numerical simulation and behaviour measured at the survey station, producing an optimum value equal to $\varepsilon = 0.25$. This value enables considerable agreement decisively better than what was obtainable with the linear model.

In passing, it is worth noting that diverse numerical simulations corresponding to different hydraulic conditions underline the importance of the nonlinear term in the phenomenon under study. As anticipated in the introduction, a clear difference is observed between the linear and nonlinear models, accentuated with the reduction of velocity $U$, and the linear approximation always proves unfavourable to certainty.

The value of $\varepsilon$ thus fixed, two different examples of the inverse problem described above are treated. Both cases concern the following values:

$$L = 8000\,\text{m}, \qquad T = 11000\,\text{s}, \qquad U = 0.25\,\text{m/s}, \qquad K_M = 21\,\text{m}^2/\text{s}, \tag{4.1}$$

which correspond to

$$c_1 = 0.34375, \qquad c_2 = 0.00369, \qquad c_3 = 0.000922. \tag{4.2}$$

In both applications $x_S = 0.45$ and $x_m = 0.50$. The results shown were obtained considering 51 spatial nodes in domains $D_1$ and $D_3$ and 9 spatial nodes in domain $D_2$ (with 51 time nodes). In the second domain, the problem being ill-posed, this number of nodes represented the best compromise between simulation resolution and error control.

The relative systems of ordinary differential equations have been solved by means of a predictor-corrector method of the Adams-Basforth type with the Adams-Moulton modification, see [1, Chapter 2], using time-step $\Delta t = 0.001$ in the domains $D_1$ and $D_3$ and space-step $\Delta x = 0.005$ in domain $D_2$.

## Application 1

In this first example it is assumed that, with regard to conditions (2.7) and (2.8)

$$\alpha(t) = 0, \qquad \beta(t) = 0, \qquad \forall t \in [0,1], \tag{4.3}$$

and $C_m(t)$ has the Gaussian behaviour, typical for such problems, shown in Figure 1.

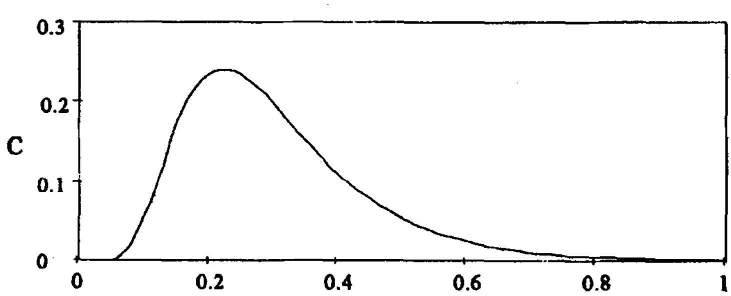
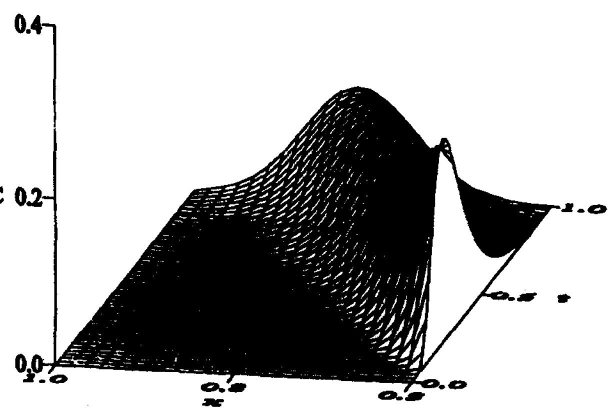


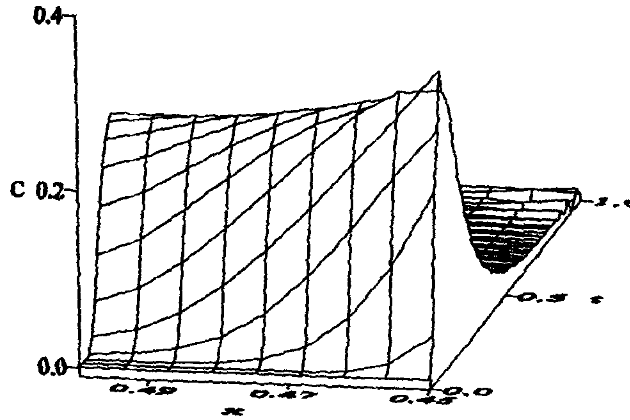

Figure 1. Evolution of concentration at measuring point $x_m$.

Figures 2a–2d and 3 show the results of the simulation: the former show the space-time behaviour of the concentration $C(t,x)$, separately in the three dimensions $D_1, D_2$, and $D_3$, and altogether in domain $D$; Figure 4 shows the result sought, that is, the unknown flow $s(t)$.

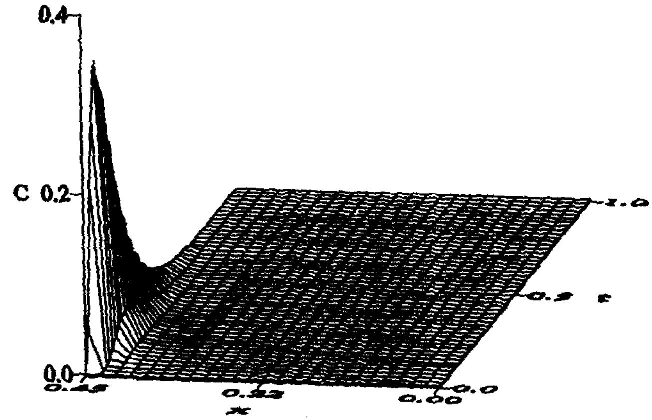



(a) Spatio-temporal behaviour of pollutant in $D_1$.

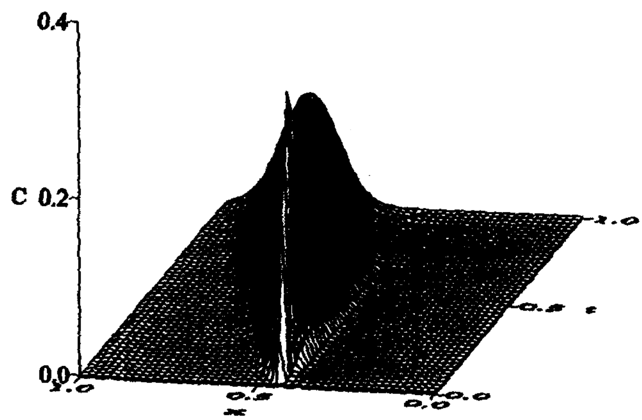
Figure 2.




(b) Spatio-temporal behaviour of pollutant in $D_2$.




(c) Spatio-temporal behaviour of pollutant in $D_3$.




(d) Spatio-temporal behaviour of pollutant in $D$.

Figure 2. (cont.)

The behaviours are regular, coherent between themselves, and well appropriate the physics of the problem. Note in particular how in domain $D_1$ the algorithm is very stable and simulates very well, without significant spurious disturbances, the weak rise upstream of the pollutant introduced into the source.

## Application 2

In contrast to the previous case, now we also assume an inflow, in the upward boundary of the domain, of a pollutant substance in accordance with the time law shown in Figure 4.

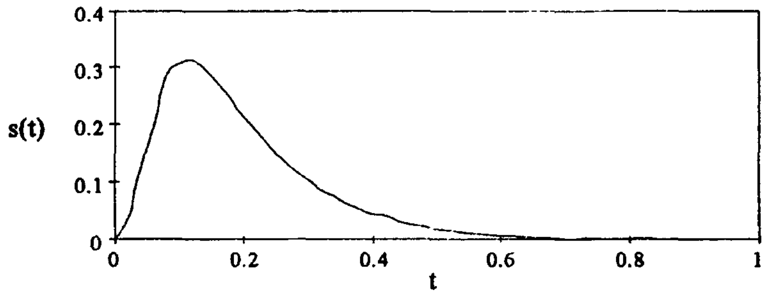



Figure 3. Time evolution of source intensity.

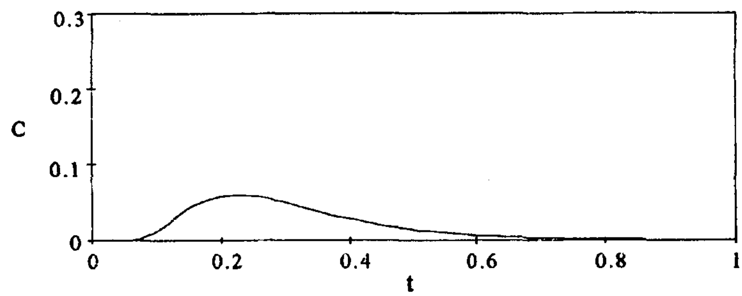
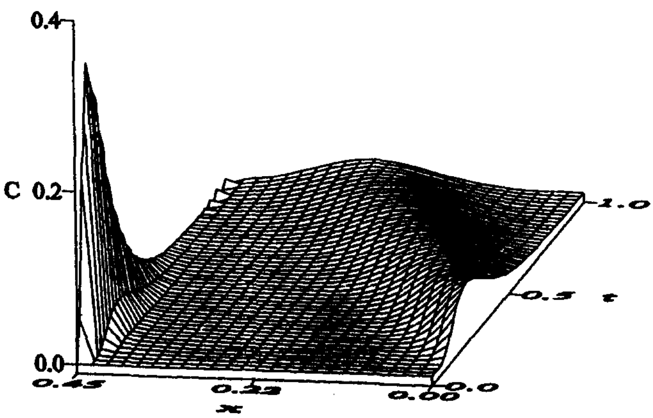
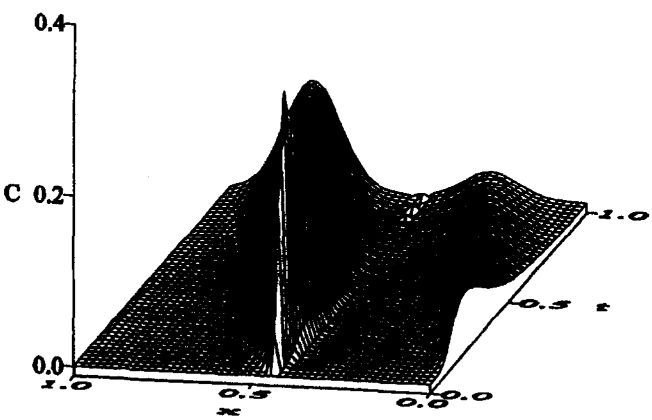



Figure 4. Evolution of concentration $\alpha(t)$ at $x = 0$.




(a) Spatio-temporal behaviour of pollutant in $D_1$.




(b) Spatio-temporal behaviour of pollutant in $D$.

Figure 5.

The corresponding results are shown in Figures 5a and 5b (the source intensity is the same as the previous). Also here the behaviours $C(t, x)$ in the three domains are very well described.

This paper deals with the solution and analysis of a nonlinear inverse problem in fluid mechanics related to the identification of a source term in a nonlinear diffusive convective system. The analysis was developed by the sinc collocation interpolation method organized by decomposition of domains methods.

As a conclusion to this paper, a list of remarks is proposed.

- It is important to deal with nonlinearities in the diffusion term. As a matter of fact, previsions by the linear model are defective with respect to the nonlinear term. Hence, it is necessary to include nonlinearity to have a reliable description of the phenomena.
- The use of sinc interpolation is certainly not the only one which can be adopted. As known and documented in [1], alternative interpolarity (say Lagrange polynomials, splines) can be used. Sinc functions are useful when the dependent variable is oscillating in time and/or in space. Oscillations may be induced by oscillations in the source term or by initial conditions. This statement is confirmed by computations additional to the ones which have been proposed here.
- The analysis of the problem is limited to the identification of the time evolution of the source term of a well localized source. The identification of the source requires additional measurements and techniques, which are not technical developments of the methods proposed in this paper.
- The one-dimensional description of the phenomenon is certainly an idealization that has to be regarded as a simplification of physical reality. This paper has developed one-dimensional analysis with special attention to nonlinearities in view of two-dimensional analysis.

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