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Machine Learning

Session 14 - T

Support Vector Machines – Part 1

Degree in Applied Data Science

2024/2025

Support Vector Machines (SVMs) - Basics

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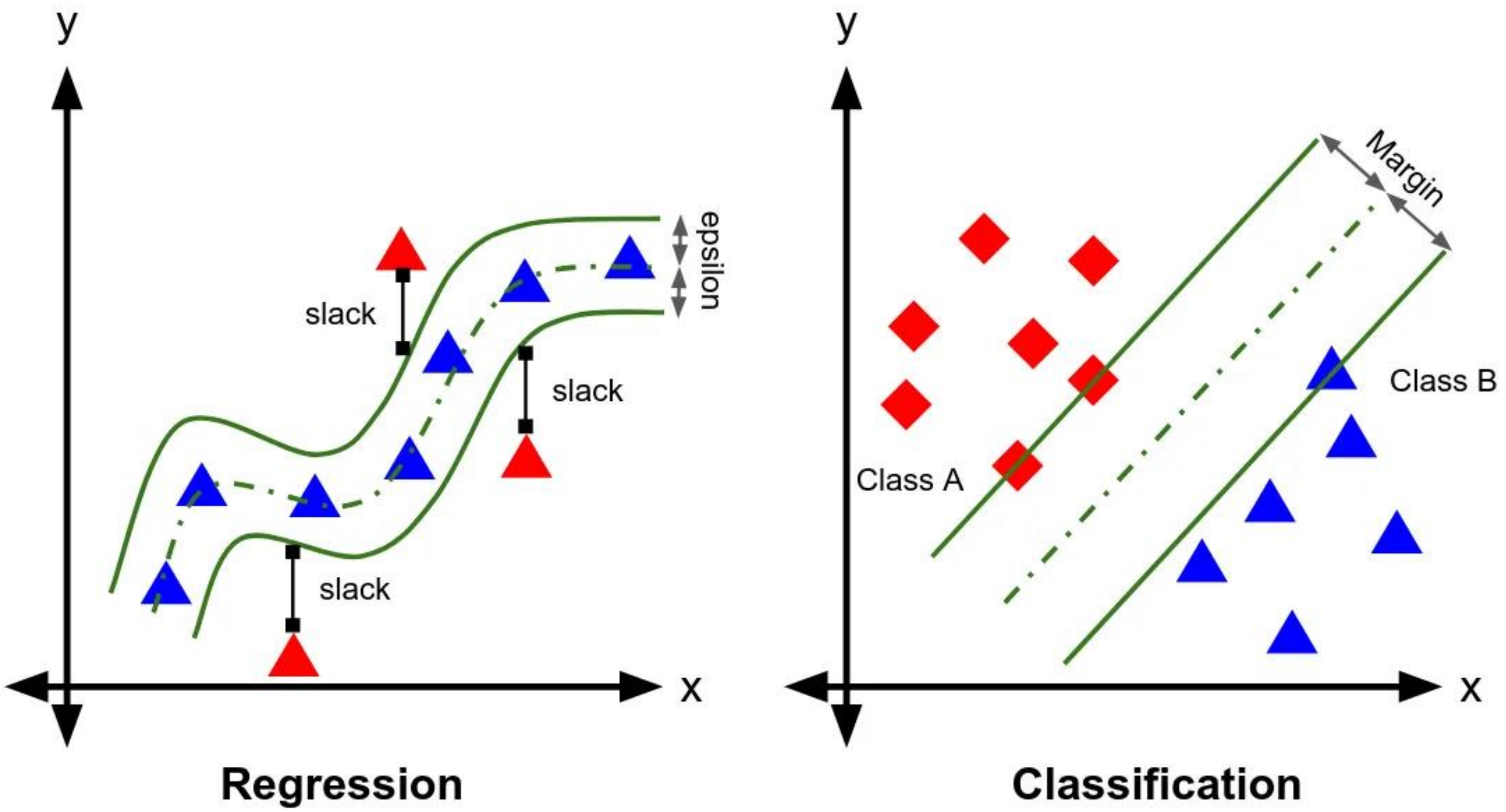
- Supervised machine learning algorithm suitable for both classification and regression.

- **Objective:** Find the optimal hyperplane or decision boundary in the feature space.

- **How?** By maximizing the margin or distance between data points of different classes or regression targets.

SVMs - Basics


y
slack
slack
slack
epsilon
x
Regression
y
Margin
Class A
Class B
x
Classification

SVMs - Hyperplanes

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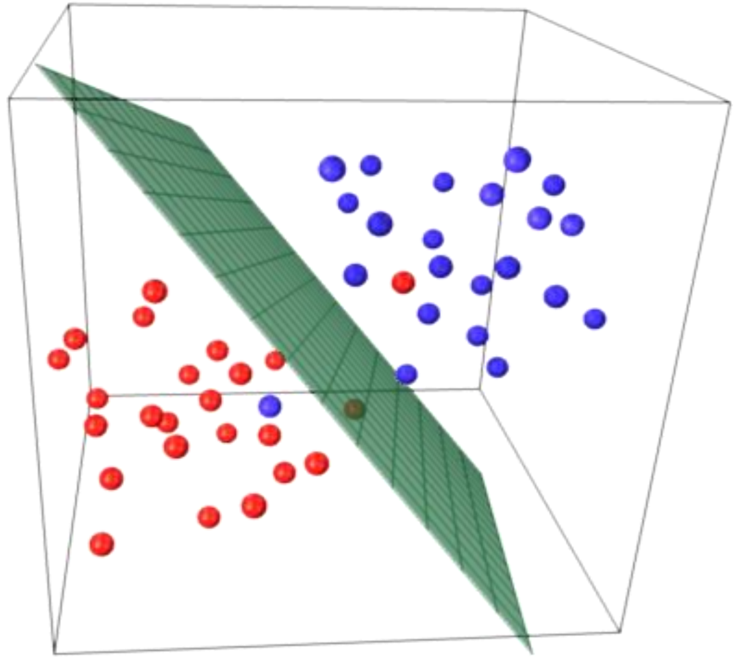
- A **hyperplane** is a subspace with **one dimension less** than the number of variables in the dataset (n-1).

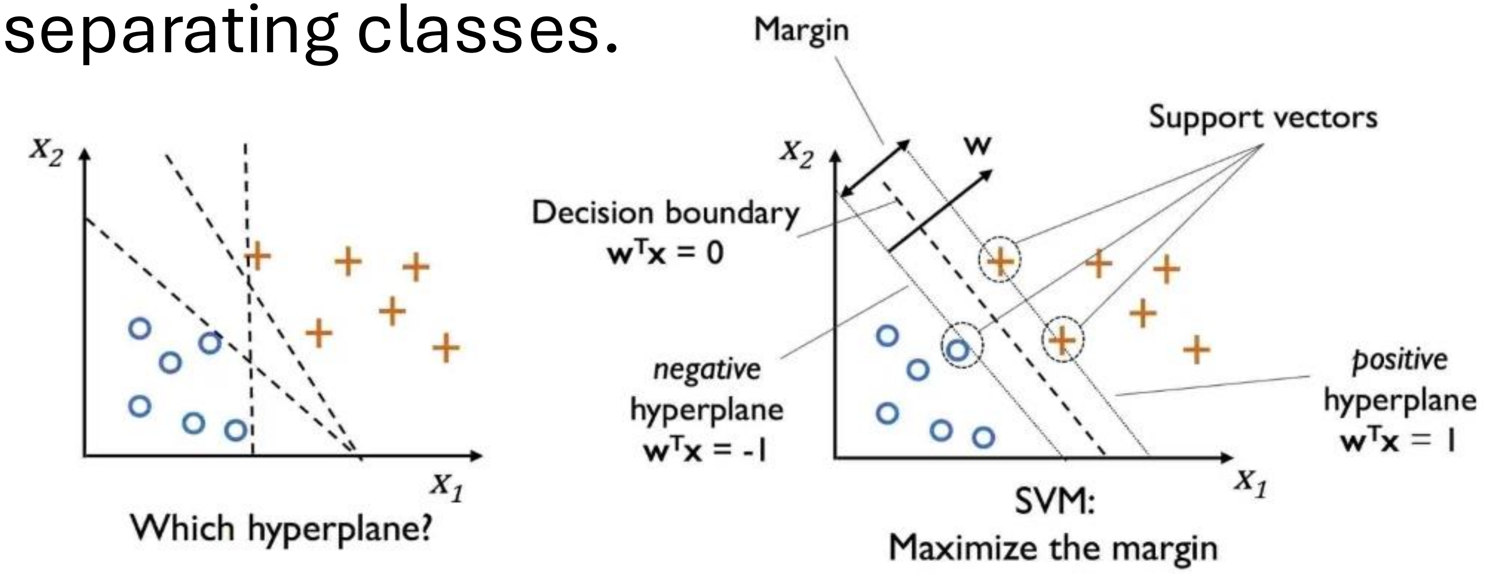
- For a 3-dimensional dataset, a 2-dimensional plane can be used to separate the data into two distinct groups.

- Multiple hyperplanes can separate the data. The goal is to find the **hyperplane with the maximum margin**.



Maximum Margin Classifier

- A **Margin Classifier** is a type of classifier that provides a distance to the discriminant.
  - The hyperplane serves as the boundary separating classes in a linear classifier.
- A **Maximum Margin Classifier** seeks the discriminant with the maximum margin, maximizing the distance to the nearest points (**support vectors**) for separating classes.

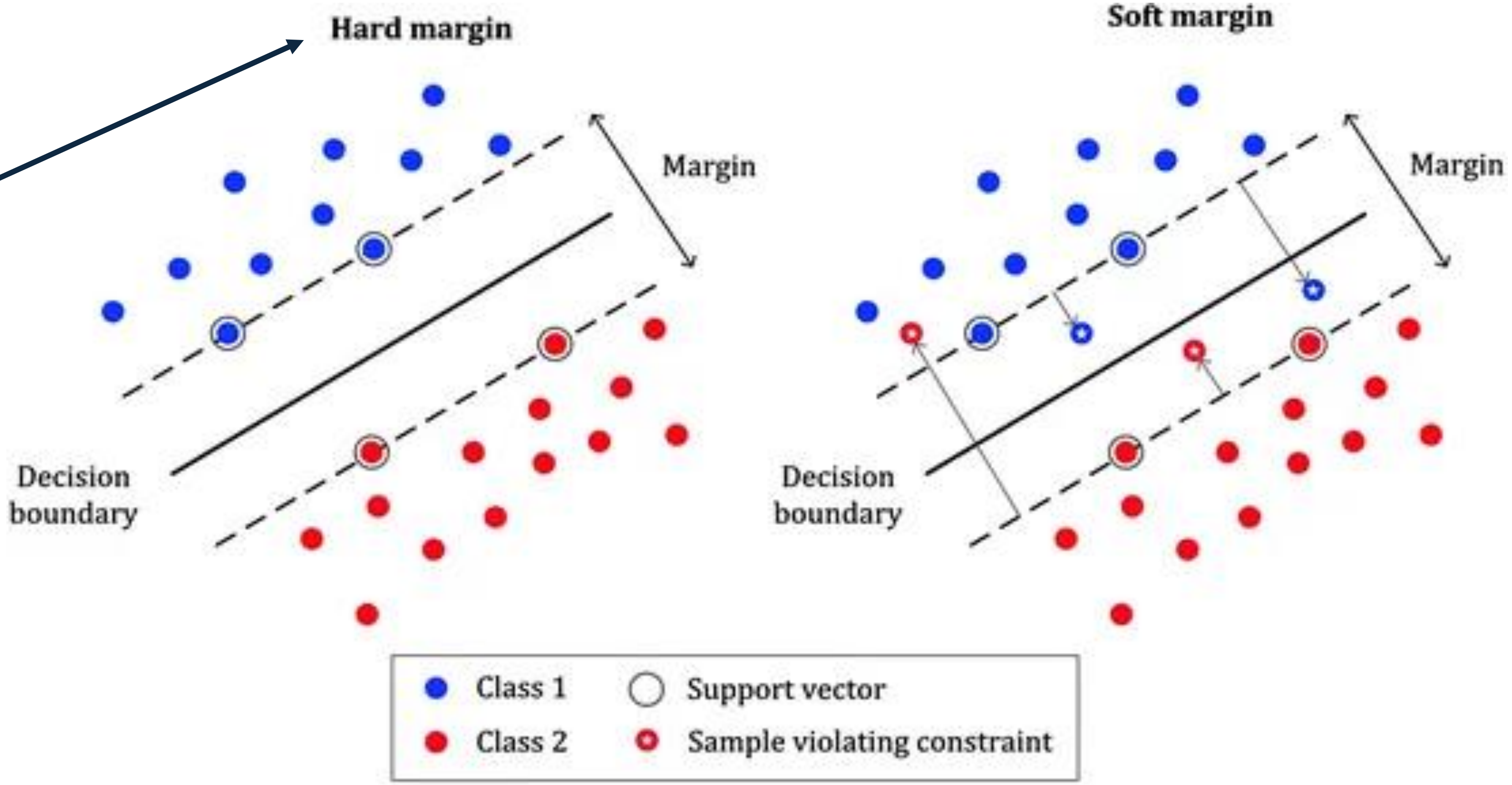


Maximum Margin Classifier

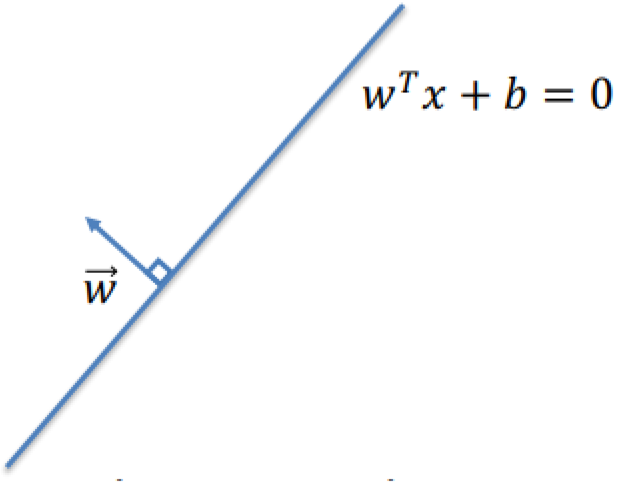
- Hard vs Soft Margin

- Very sensitive to outliers.
- All training instances need to be correctly classified.
- Only for linearly separable data.

- Uses cross validation to find the best support vectors.



Some Geometry



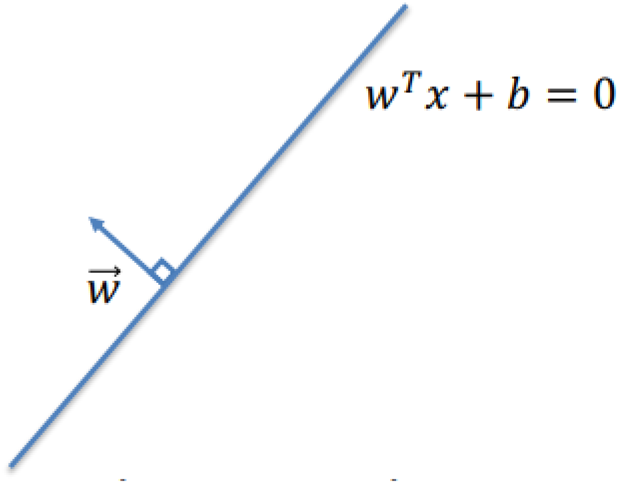
- In n dimensions, a hyperplane is a solution to the equation:

$$w^T x + b = 0$$

with $w \in \mathbb{R}^n, b \in \mathbb{R}$

- The vector $\vec{w}$ is called the **normal vector** of the hyperplane.

Some Geometry



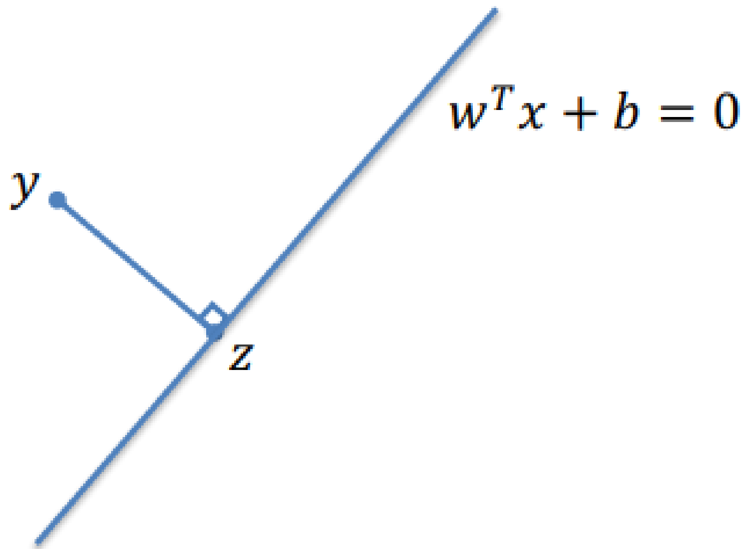
- In n dimensions, a hyperplane is a solution to the equation:

$$w^T x + b = 0$$

- Note that this equation is **scale invariante** for any scalar c

$$c \cdot (w^T x + b) = 0$$

Some Geometry



- The distance between a point y and a hyperplane $w^T x + b = 0$

 is the length of the segment **perpendicular** to the line to the point y

Note that:

Length of vector $x(x_1, x_2, x_3)$ *is calculated as :*

$\|x\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$

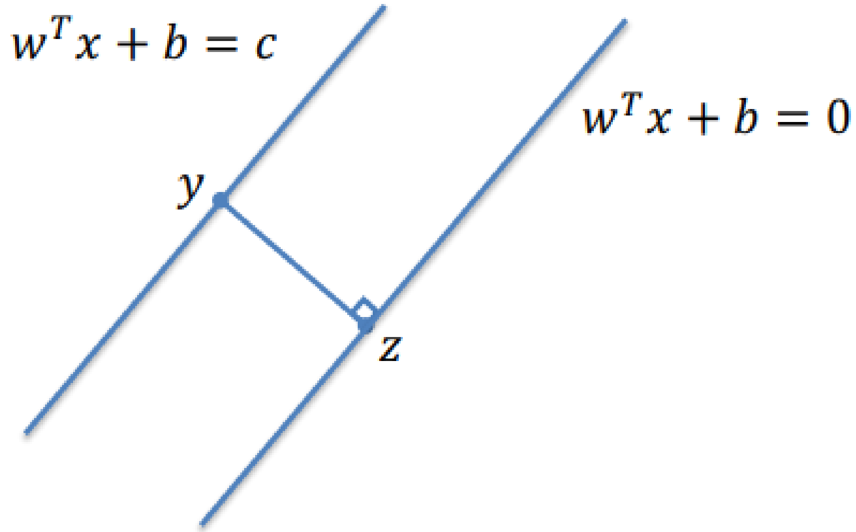
Direction of vector $x(x_1, x_2, x_3)$ *is calculated as:*

$\{ \frac{x_1}{\|x\|}, \frac{x_2}{\|x\|}, \frac{x_3}{\|x\|} \}$

- The vector from y to z is given by:

$$y - z = \|y - z\| \frac{w}{\|w\|}$$

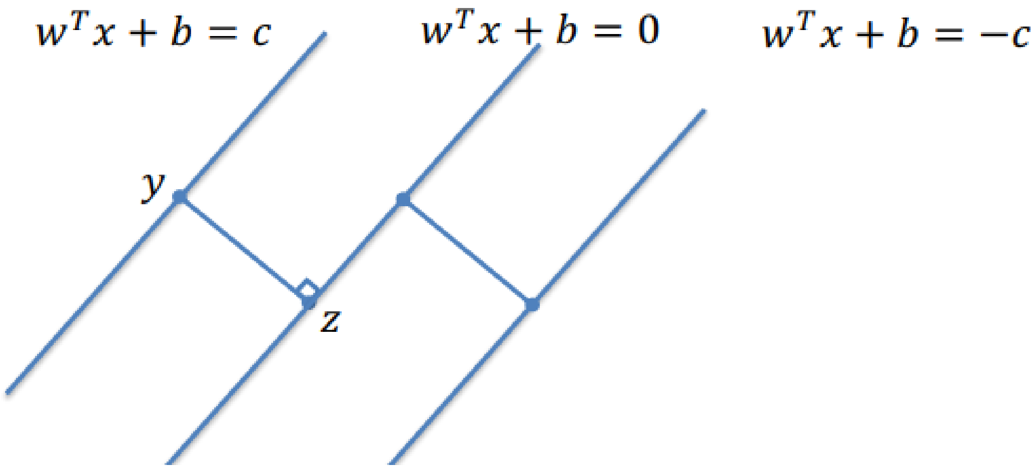
Scale Invariance



- By scale invariance, we can assume that c = 1

- The maximum margin is always obtained by choosing $w^T x + b = 0$ so that it is **equidistant** from the closest data point from each class.

Scale Invariance

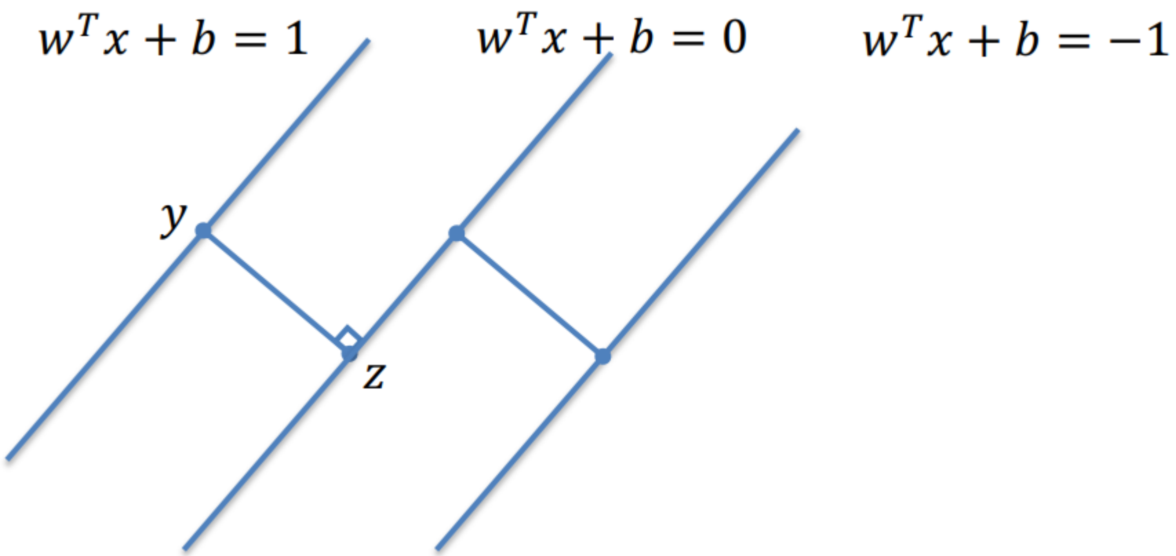


- We want to **maximize the margin** subject to the constraints that

$$y^{(i)}\left(w^T x^{(i)} + b\right) \geq 1$$

- But how do we compute the size of the margin?

Some Geometry



- Putting it all together:

$$y - z = \|y - z\| \frac{w}{\|w\|}$$

and

$$w^T y + b = 1$$
$$w^T z + b = 0$$

$$w^T (y - z) = 1$$

and

$$w^T (y - z) = \|y - z\| \, \|w\|$$

which gives:

$$\|y - z\| = 1/\|w\|$$

SVMs

- From the previous analysis we get the following optimization problem:

$$\max_{w,b} \frac{1}{||w||} \qquad \text{OR} \qquad \min_{w,b} ||w||^2$$

such that:

$$y^{(i)}\left(w^T x^{(i)} + b\right) \geq 1, \text{for all } i$$

SVMs

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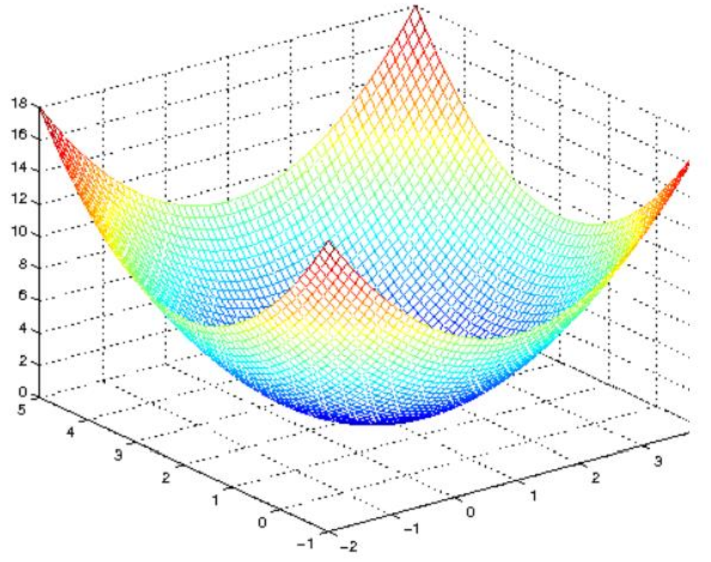
$$\min_{w,b} ||w||^2$$

such that:

$$y^{(i)}\left(w^T x^{(i)} + b\right) \geq 1, \text{for all } i$$

- This is a standard **quadratic programming problem**!
  - Convex optimization problems



SVMs

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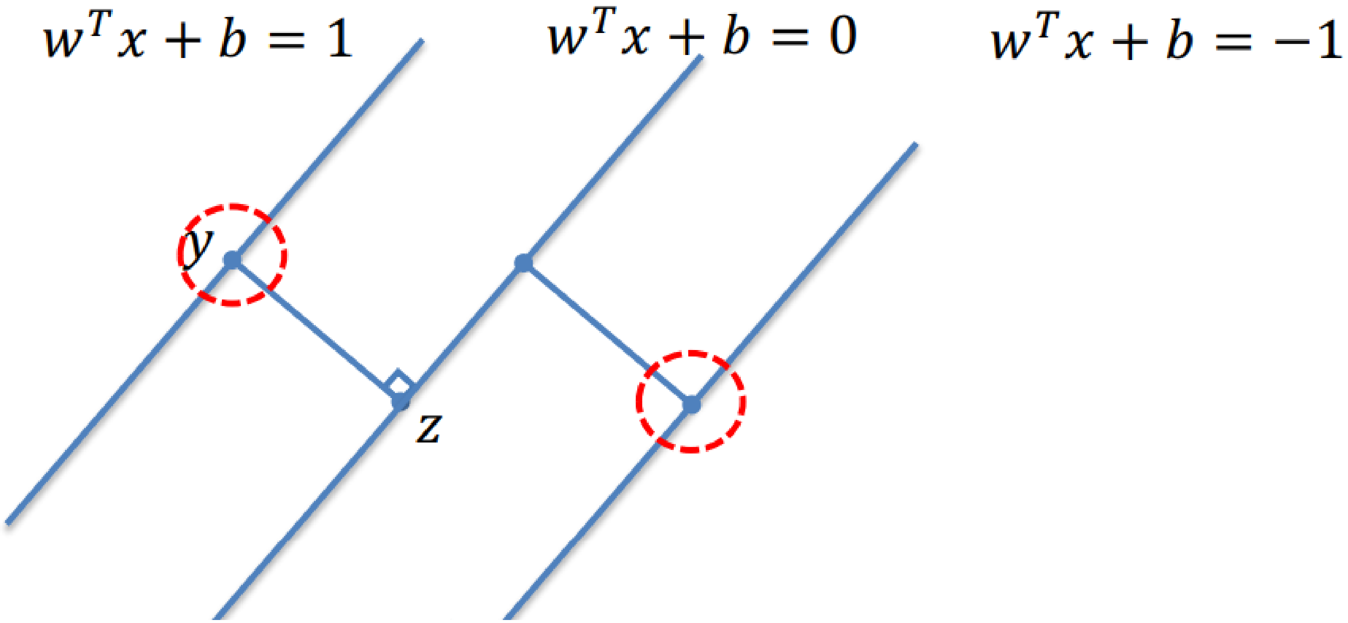


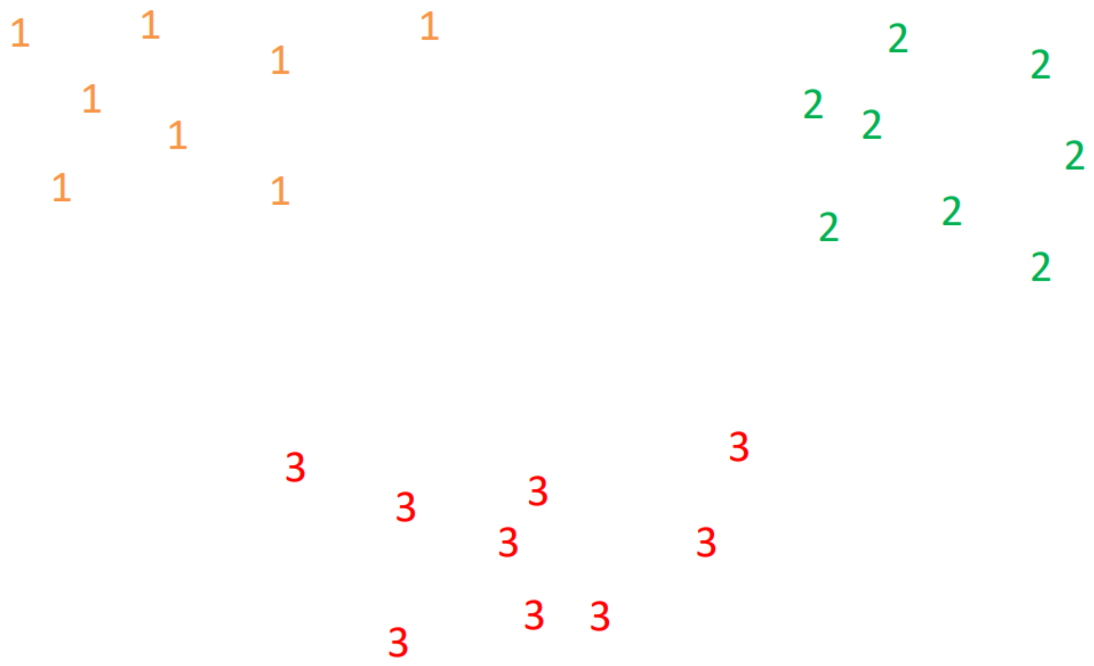
- Where does the name come from?
  - The set of data points such that: $y^{(i)}(w^T x^{(i)} + b) = 1$
    are called **support vectors**.
  - The SVM classifier is completely determined by the support vectors (the **other data points dont influence the algorithm**).

SVMs

- **Assumptions made so far:**
  - The data is linearly separable!
  - We are dealing with binary classification tasks!

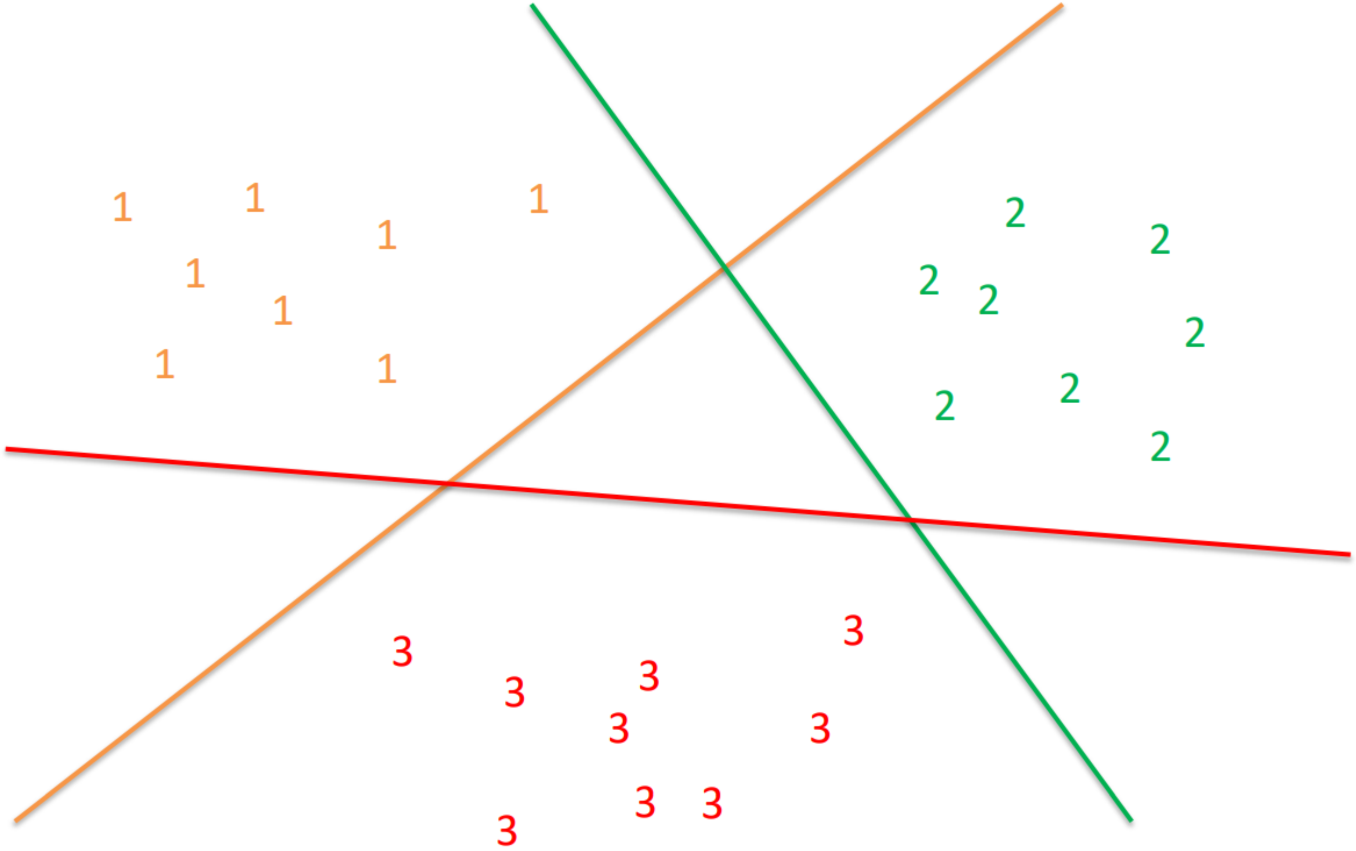
SVMs – Multiclass Classification

- What if we want to do more than just binary classification
  - for instance if $y \in \{1,2,3\}$)



One-Versus-All SVMs

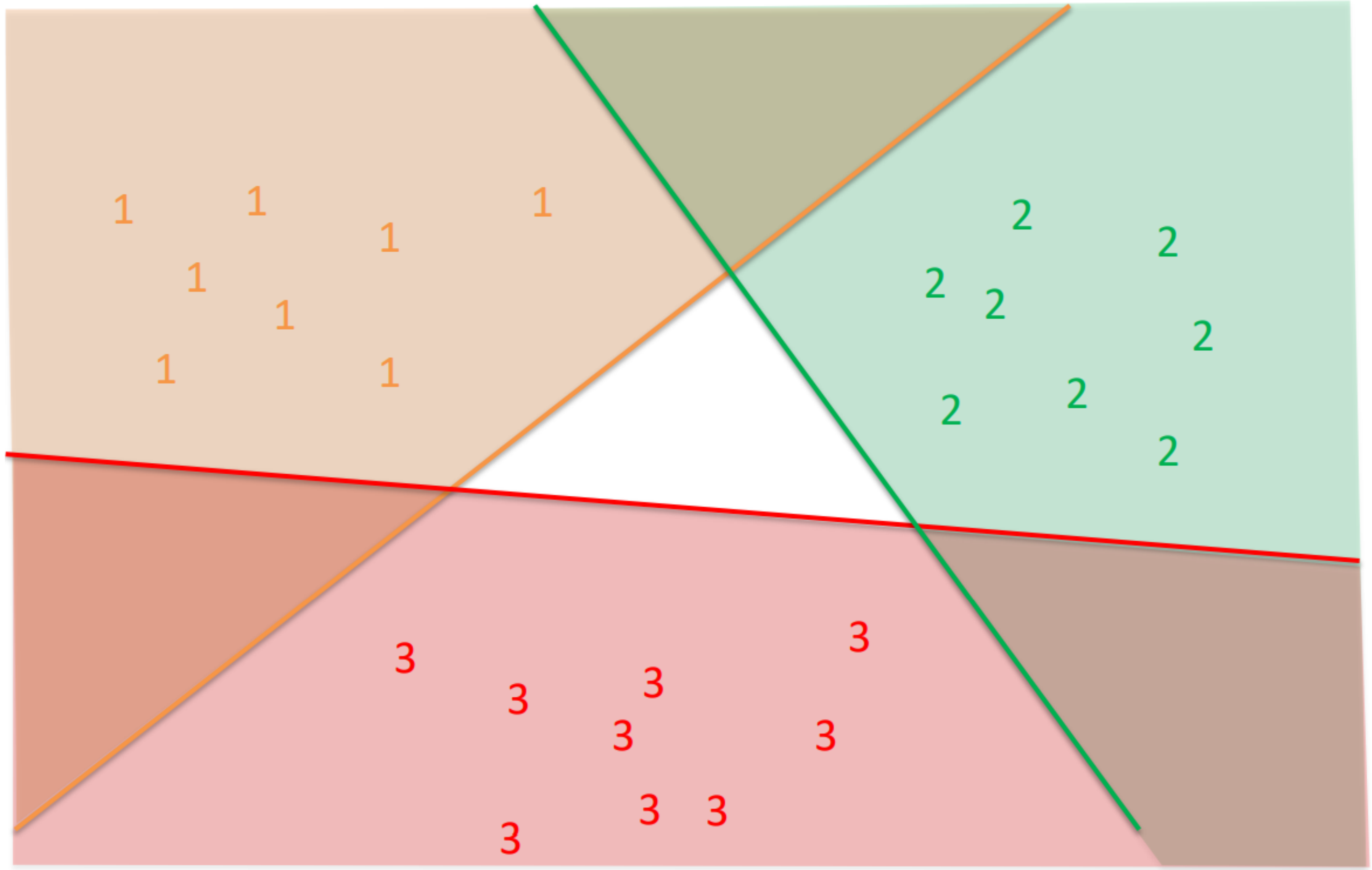
- What if we want to do more than just binary classification
  - for instance if $y \in \{1,2,3\}$)



One-Versus-All SVMs

- What if we want to do more than just binary classification
  - for instance if $y \in \{1,2,3\}$)

Regions correctly classified by exactly one classifier.



One-Versus-All SVMs

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- **Compute a classifier for each label versus all other labels;**

- Let $f^k(x) = w^{(k)^T} x + b^{(k)}$ be the classifier for the kth label

- For a new datapoint x, classify it as:

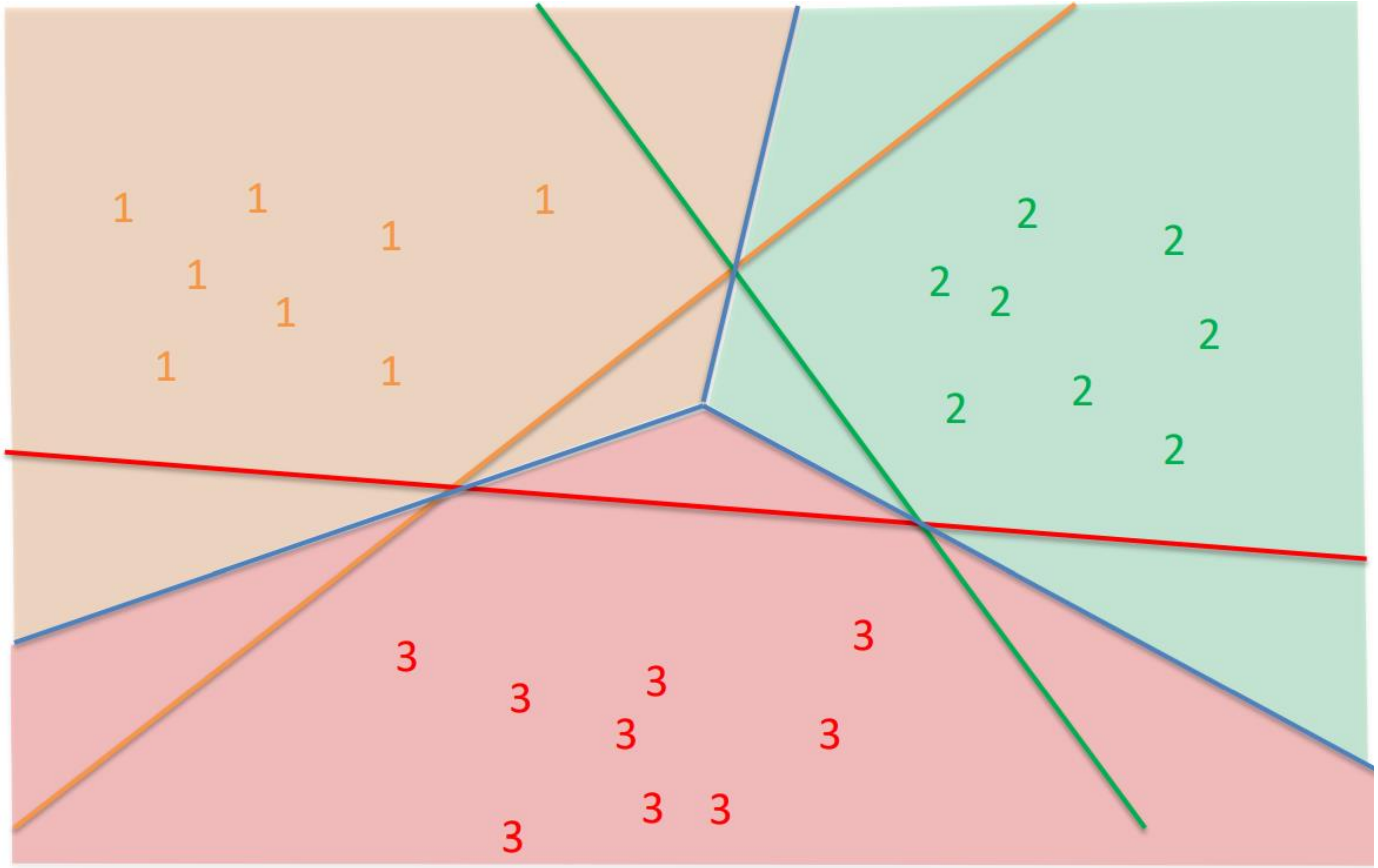
$$k' \in \mathrm{argmax}_k f^k(x)$$

- Drawbacks:
  - If there are L possible labels, requires **learning L classsifiers** over the entire dataset.

One-Versus-All SVMs

Regions in which points are classified by the highest value of:

$$w^T x + b$$



One-Versus-One SVMs

- Alternatively, it is possible to learn a classifier for **all possible pairs of labels**;

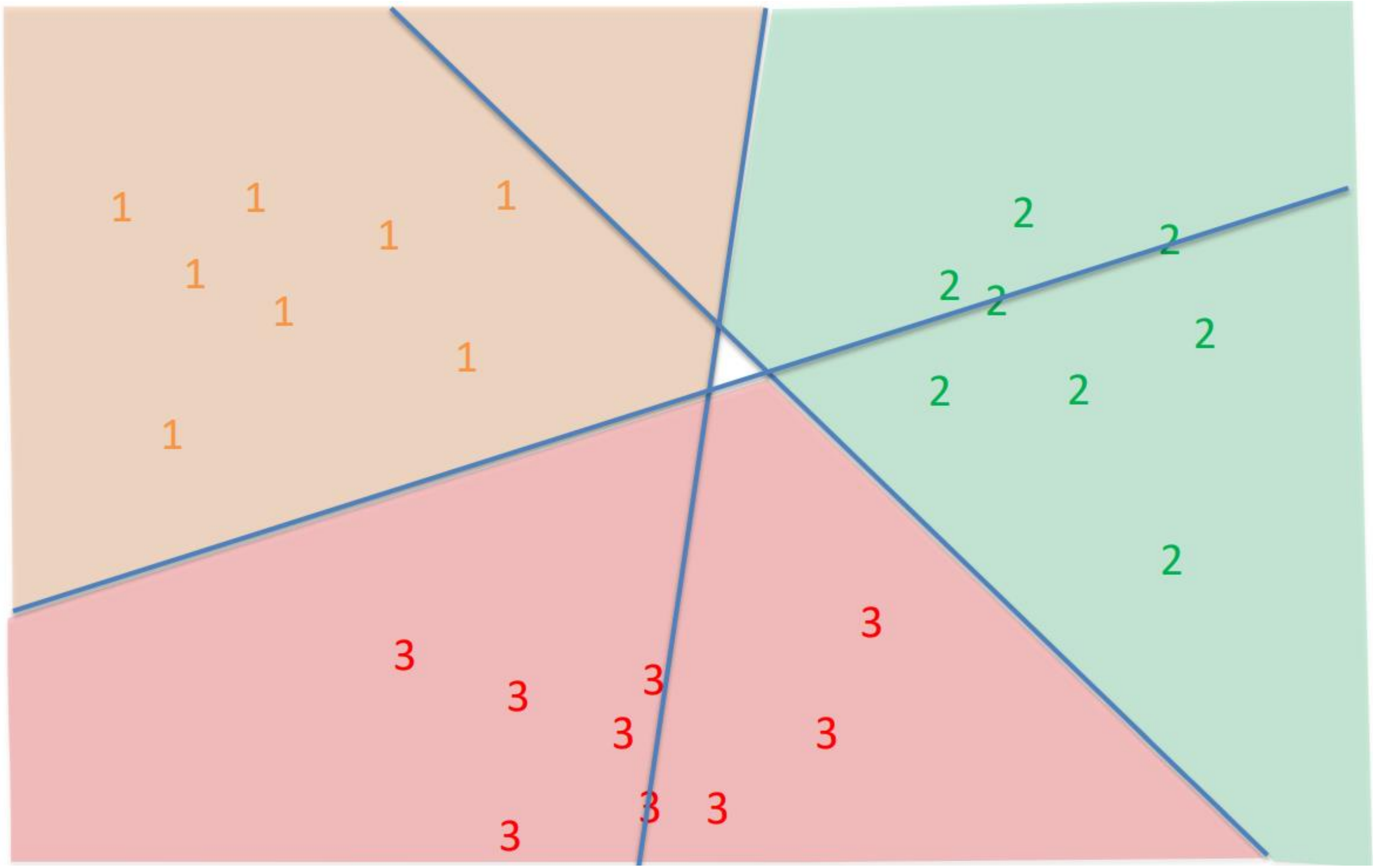
- Given a new data point, it will be classified by **majority vote** (by finding the most common lable among all possible classifiers);

- If there are L labels, requires learning $\frac{L(L-1)}{2}$ different classifiers using different fractions of the data.

- **Drawbacks:**
  - Can overfit some pairs of labels;
  - Computationaly expensive.

One-Versus-One SVMs

Regions determined by majority vote over all classifiers



Resources

- https://www.youtube.com/watch?v=efR1C6CvhmE