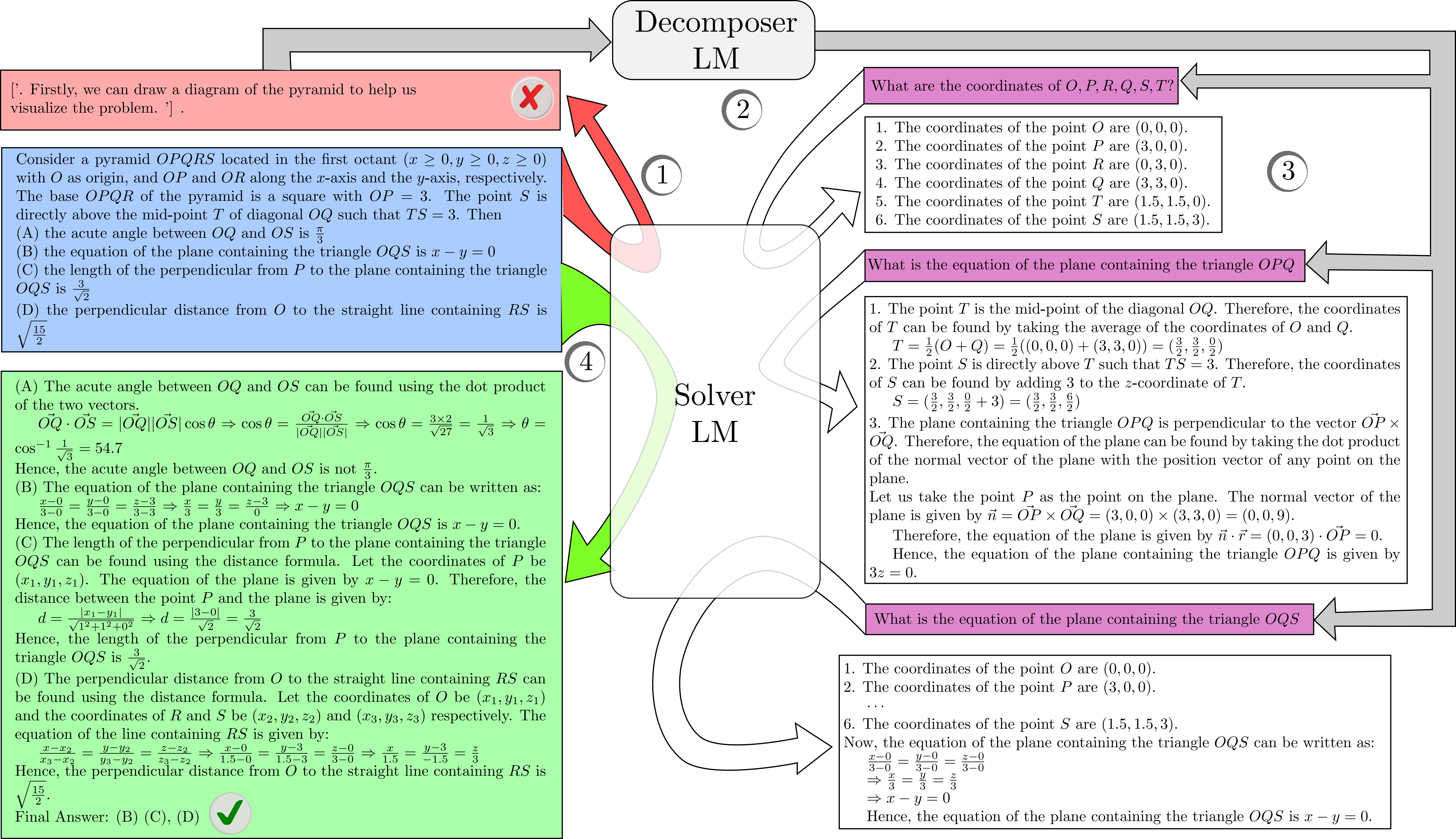
Decomposer LM
Solver LM
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['. Firstly, we can draw a diagram of the pyramid to help us visualize the problem. '] .
Consider a pyramid $OPQRS$ located in the first octant ($x \geq 0, y \geq 0, z \geq 0$) with $O$ as origin, and $OP$ and $OR$ along the $x$-axis and the $y$-axis, respectively. The base $OPQR$ of the pyramid is a square with $OP = 3$. The point $S$ is directly above the mid-point $T$ of diagonal $OQ$ such that $TS = 3$. Then
(A) the acute angle between $OQ$ and $OS$ is $\frac{\pi}{3}$
(B) the equation of the plane containing the triangle $OQS$ is $x - y = 0$
(C) the length of the perpendicular from $P$ to the plane containing the triangle $OQS$ is $\frac{3}{\sqrt{2}}$
(D) the perpendicular distance from $O$ to the straight line containing $RS$ is $\sqrt{\frac{15}{2}}$
(A) The acute angle between $OQ$ and $OS$ can be found using the dot product of the two vectors.
$\vec{OQ} \cdot \vec{OS} = |\vec{OQ}||\vec{OS}| \cos\theta \Rightarrow \cos\theta = \frac{\vec{OQ}\cdot\vec{OS}}{|\vec{OQ}||\vec{OS}|} \Rightarrow \cos\theta = \frac{3\times 2}{\sqrt{27}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\frac{1}{\sqrt{3}} = 54.7$
Hence, the acute angle between $OQ$ and $OS$ is not $\frac{\pi}{3}$.
(B) The equation of the plane containing the triangle $OQS$ can be written as:
$\frac{x-0}{3-0} = \frac{y-0}{3-0} = \frac{z-3}{3-3} \Rightarrow \frac{x}{3} = \frac{y}{3} = \frac{z-3}{0} \Rightarrow x - y = 0$
Hence, the equation of the plane containing the triangle $OQS$ is $x - y = 0$.
(C) The length of the perpendicular from $P$ to the plane containing the triangle $OQS$ can be found using the distance formula. Let the coordinates of $P$ be $(x_1, y_1, z_1)$. The equation of the plane is given by $x - y = 0$. Therefore, the distance between the point $P$ and the plane is given by:
$d = \frac{|x_1 - y_1|}{\sqrt{1^2+1^2+0^2}} \Rightarrow d = \frac{|3-0|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$
Hence, the length of the perpendicular from $P$ to the plane containing the triangle $OQS$ is $\frac{3}{\sqrt{2}}$.
(D) The perpendicular distance from $O$ to the straight line containing $RS$ can be found using the distance formula. Let the coordinates of $O$ be $(x_1, y_1, z_1)$ and the coordinates of $R$ and $S$ be $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ respectively. The equation of the line containing $RS$ is given by:
$\frac{x-x_2}{x_3-x_2} = \frac{y-y_2}{y_3-y_2} = \frac{z-z_2}{z_3-z_2} \Rightarrow \frac{x-0}{1.5-0} = \frac{y-3}{1.5-3} = \frac{z-0}{3-0} \Rightarrow \frac{x}{1.5} = \frac{y-3}{-1.5} = \frac{z}{3}$
Hence, the perpendicular distance from $O$ to the straight line containing $RS$ is $\sqrt{\frac{15}{2}}$.
Final Answer: (B) (C), (D)
What are the coordinates of $O, P, R, Q, S, T$?
1. The coordinates of the point $O$ are $(0, 0, 0)$.
2. The coordinates of the point $P$ are $(3, 0, 0)$.
3. The coordinates of the point $R$ are $(0, 3, 0)$.
4. The coordinates of the point $Q$ are $(3, 3, 0)$.
5. The coordinates of the point $T$ are $(1.5, 1.5, 0)$.
6. The coordinates of the point $S$ are $(1.5, 1.5, 3)$.
What is the equation of the plane containing the triangle $OPQ$
1. The point $T$ is the mid-point of the diagonal $OQ$. Therefore, the coordinates of $T$ can be found by taking the average of the coordinates of $O$ and $Q$.
$T = \frac{1}{2}(O + Q) = \frac{1}{2}((0,0,0) + (3,3,0)) = (\frac{3}{2}, \frac{3}{2}, \frac{0}{2})$
2. The point $S$ is directly above $T$ such that $TS = 3$. Therefore, the coordinates of $S$ can be found by adding 3 to the $z$-coordinate of $T$.
$S = (\frac{3}{2}, \frac{3}{2}, \frac{0}{2} + 3) = (\frac{3}{2}, \frac{3}{2}, \frac{6}{2})$
3. The plane containing the triangle $OPQ$ is perpendicular to the vector $\vec{OP} \times \vec{OQ}$. Therefore, the equation of the plane can be found by taking the dot product of the normal vector of the plane with the position vector of any point on the plane.
Let us take the point $P$ as the point on the plane. The normal vector of the plane is given by $\vec{n} = \vec{OP} \times \vec{OQ} = (3,0,0) \times (3,3,0) = (0,0,9)$.
Therefore, the equation of the plane is given by $\vec{n} \cdot \vec{r} = (0,0,3) \cdot \vec{OP} = 0$.
Hence, the equation of the plane containing the triangle $OPQ$ is given by $3z = 0$.
What is the equation of the plane containing the triangle $OQS$
1. The coordinates of the point $O$ are $(0, 0, 0)$.
2. The coordinates of the point $P$ are $(3, 0, 0)$.
...
6. The coordinates of the point $S$ are $(1.5, 1.5, 3)$.
Now, the equation of the plane containing the triangle $OQS$ can be written as:
$\frac{x-0}{3-0} = \frac{y-0}{3-0} = \frac{z-0}{3-0}$
$\Rightarrow \frac{x}{3} = \frac{y}{3} = \frac{z}{3}$
$\Rightarrow x - y = 0$
Hence, the equation of the plane containing the triangle $OQS$ is $x - y = 0$.