
Algorithm 1 delta-stepping CPU SSSP

Input: $G(V, E)$, source vertex s , length function $l : E \rightarrow \mathbb{R}$;

Output: $dist(v)$, ($v \in V$), the weight of the shortest path from s to v ;

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1: function relax( $v$ , weight request  $x$ )                                ▷ assignment of  $v$  to appropriate bucket;
2:   if  $x < dist(v)$  then
3:      $B[\lfloor dist(v)/\Delta \rfloor] \leftarrow B[\lfloor dist(v)/\Delta \rfloor] \setminus \{v\}$ ;      ▷ remove the vertex from the old bucket;
4:      $B[\lfloor x/\Delta \rfloor] \leftarrow B[\lfloor x/\Delta \rfloor] \cup \{v\}$ ;              ▷ put it to appropriate bucket;
5:      $dist(v) \leftarrow x$ ;                                              ▷ update the dist;
6:   end if
7: end function
8:
9: function initial( $s, G(V, E)$ )
10:  for each  $v \in V$  do
11:     $dist(v) \leftarrow +\infty$ ;                                          ▷ initialize  $dist$  to positive infinity;
12:     $heavy(v) \leftarrow \{(v, w) \in E : l(v, w) > \Delta\}$ ;           ▷ set the heavy edge set;
13:     $light(v) \leftarrow \{(v, w) \in E : l(v, w) \leq \Delta\}$ ;         ▷ set the light edge set;
14:  end for
15:  relax( $s, 0$ );                                                        ▷ set the source distance to 0, and put it to the appropriate bucket;
16: end function
17:
18: initial( $s, G(V, E)$ );
19:
20:  $i \leftarrow 0$ ;
21: while  $B$  is not empty do
22:    $S \leftarrow \emptyset$ ;
23:   while  $B[i] \neq \emptyset$  do
24:      $Req \leftarrow \{(w, dist(v) + l(v, w)) : v \in B[i] \cap (v, w) \in light(v)\}$ ;  ▷ the vertices is in this bucket;
25:      $S \leftarrow S \cup B[i]$ ;                                          ▷ the vertices have been in this buckets;
26:      $B[i] \leftarrow \emptyset$ ;                                          ▷ set this bucket is empty;
27:     for each  $(v, x) \in Req$  do
28:       relax( $v, x$ );
29:     end for
30:   end while
31:
32:    $Req \leftarrow \{(w, dist(v) + l(v, w)) : v \in S \cap (v, w) \in heavy(v)\}$ ;
33:   for each  $(v, x) \in Req$  do
34:     relax( $v, x$ );
35:   end for
36:    $i \leftarrow i + 1$ ;
37: end while
38:
39: return result
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