Microeconomía II
Profesor Julio Ruiz

Notas de clase:
"Teoría de la Firma"

Caravaggio, Leonardo Andrés

Teoría de la empresa - Problema primal y dual

1) Planteo numérico
2) Resolver primal
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5) Resolver primal
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   8.1) Relación CM y CMg + Gráfico

1) Problema:

$$f(x_1,x_2)=x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}$$
$$p_{x_1}=p_{x_2}=10$$
$$CT=2000$$

2) Problema primal:

$$Max\ \ y=x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}$$
$$s.a.\ \ CT=2000=10x_1+10x_2$$

Armo el Lagrangiano

$$L=x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}-\lambda\left(10x_1+10x_2-2000\right)$$

Condiciones de primer orden

$$\frac{\partial L}{\partial x_1}=\frac{1}{2}x_2^{\frac{1}{2}}\frac{1}{x_1^{\frac{1}{2}}}-10\lambda=0$$
$$\frac{\partial L}{\partial x_2}=\frac{1}{2}x_1^{\frac{1}{2}}\frac{1}{x_2^{\frac{1}{2}}}-10\lambda=0$$
$$\frac{\partial L}{\partial \lambda}=10x_1+10x_2-2000=0$$

$$\frac{1}{2}x_2^{\frac{1}{2}}\frac{1}{x_1^{\frac{1}{2}}}=10\lambda$$
$$\lambda=\frac{\sqrt{x_2}}{20\sqrt{x_1}}$$
$$\frac{1}{2}x_1^{\frac{1}{2}}\frac{1}{x_2^{\frac{1}{2}}}=10\lambda$$
$$\lambda=\frac{\sqrt{x_1}}{20\sqrt{x_2}}$$
$$\frac{\sqrt{x_2}}{20\sqrt{x_1}}=\frac{\sqrt{x_1}}{20\sqrt{x_2}}$$
$$x_1=x_2$$

$$10x_1+10x_1-2000=0$$
$$20x_1=2000$$
$$x_1=x_2=100$$

$$\left(x_1^*,x_2^*\right)=\left(100,100\right)$$

Cantidad máxima alcanzable:

$$y=x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}=100^{\frac{1}{2}}100^{\frac{1}{2}}=100$$

3) Problema dual

$$Min\ \ CT=10x_1+10x_2$$
$$s.a.\ \ y=x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}=100$$

Por Lagrange:

$$L = CT - \lambda(y)$$
$$L = 10x_1 + 10x_2 - \lambda\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} - 100\right)$$

C.P.O.

$$\frac{\partial L}{\partial x_1} = 10 - \frac{1}{2}\lambda x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} = 0$$
$$\frac{\partial L}{\partial x_2} = 10 - \frac{1}{2}\lambda x_2^{-\frac{1}{2}} x_1^{\frac{1}{2}} = 0$$
$$\frac{\partial L}{\partial \lambda} = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} - 100 = 0$$

Despejo λ de $\frac{\partial L}{\partial x_1}$

$$10 = \frac{1}{2}\lambda x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$
$$\frac{20}{\lambda} = x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$
$$20 x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} = \lambda$$

Despejo λ de $\frac{\partial L}{\partial x_2}$

$$10 = \frac{1}{2}\lambda x_2^{-\frac{1}{2}} x_1^{\frac{1}{2}}$$
$$\frac{20}{\lambda} = x_2^{-\frac{1}{2}} x_1^{\frac{1}{2}}$$
$$20 x_2^{\frac{1}{2}} x_1^{-\frac{1}{2}} = \lambda$$

Igualo los λ

$$20 x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} = 20 x_2^{\frac{1}{2}} x_1^{-\frac{1}{2}}$$
$$x_2 = x_1$$

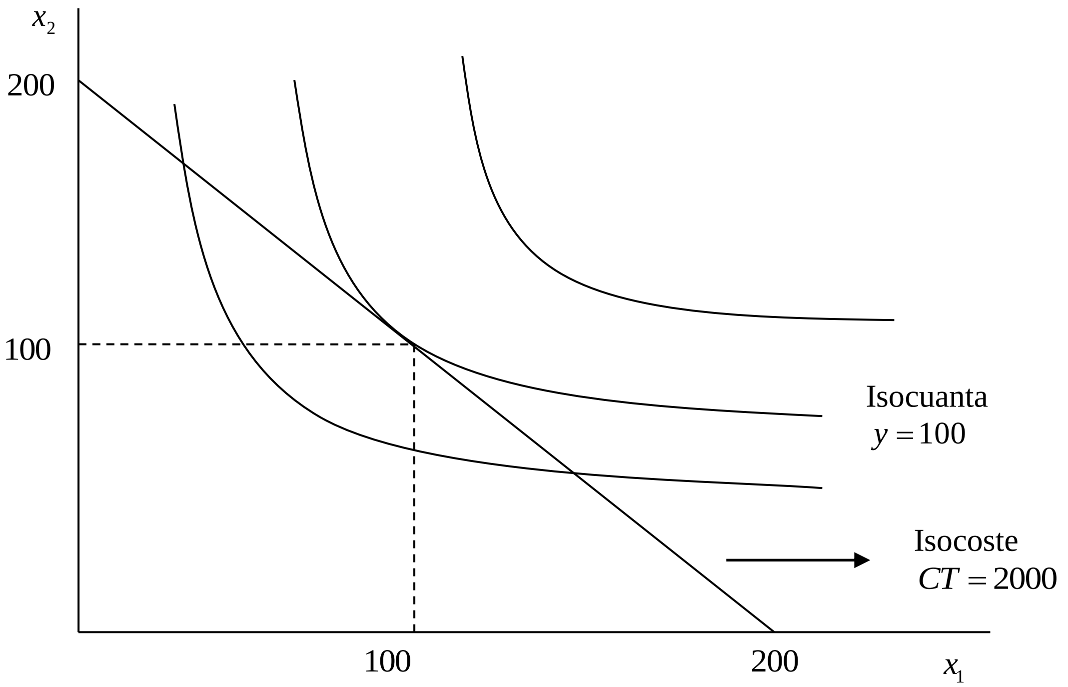
Sustituyo en la tercera:

$$x_1^{\frac{1}{2}} x_1^{\frac{1}{2}} = 100$$
$$x_1 = x_2 = 100$$

$$\left(x_1^*, x_2^*\right) = (100,100)$$

Calculo el costo total:
$$CT = 10x_1 + 10x_2 = 2000$$

3.1) Gráfico



4) Planteo con CT y r

$$Max\ \ y = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$
$$s.a.\ \ CT = r_1 x_1 + r_2 x_2$$

5) Resolver Primal

Armo el Lagrangiano

$$L = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} - \lambda\left(r_1 x_1 + r_2 x_2 - CT\right)$$

Condiciones de primer orden

$$\frac{\partial L}{\partial x_1} = \frac{1}{2} x_2^{\frac{1}{2}} \frac{1}{x_1^{\frac{1}{2}}} - r_1\lambda = 0$$
$$\frac{\partial L}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} \frac{1}{x_2^{\frac{1}{2}}} - r_2\lambda = 0$$
$$\frac{\partial L}{\partial \lambda} = r_1 x_1 + r_2 x_2 - CT = 0$$

Despejo los λ de $\frac{\partial L}{\partial x_1}$ y $\frac{\partial L}{\partial x_2}$

$$\frac{1}{2} x_2^{\frac{1}{2}} \frac{1}{x_1^{\frac{1}{2}}} = r_1\lambda$$
$$\lambda = \frac{\sqrt{x_2}}{2r_1\sqrt{x_1}}$$

$$\frac{1}{2} x_1^{\frac{1}{2}} \frac{1}{x_2^{\frac{1}{2}}} = r_2\lambda$$
$$\lambda = \frac{\sqrt{x_1}}{2r_2\sqrt{x_2}}$$

Igualo los λ

$$\frac{\sqrt{x_2}}{2r_1\sqrt{x_1}} = \frac{\sqrt{x_1}}{2r_2\sqrt{x_2}}$$
$$\frac{r_2}{r_1} = \frac{x_1}{x_2}$$
$$x_1 = x_2\frac{r_2}{r_1}$$

Tasa marginal de sustitución técnica (TMST):

$$\frac{x_1}{x_2} = \frac{r_2}{r_1}$$

Sustituyo en la CPO 3:

$$r_1 x_2 \frac{r_2}{r_1} + r_2 x_2 - CT = 0$$
$$2r_2 x_2 = CT$$
$$x_2 = \frac{CT}{2r_2}$$

$$x_1 = \frac{CT}{2r_2}\frac{r_2}{r_1}$$
$$x_1 = \frac{CT}{2r_1}$$
$$\left(x_1^*, x_2^*\right) = \left(\frac{CT}{2r_1}, \frac{CT}{2r_2}\right)$$

Cantidad máxima alcanzable:

$$y = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = \left(\frac{CT}{2r_1}\right)^{\frac{1}{2}}\left(\frac{CT}{2r_2}\right)^{\frac{1}{2}} =$$
$$= \frac{\sqrt{CT}}{\sqrt{2r_1}}\frac{\sqrt{CT}}{\sqrt{2r_2}} = \frac{CT}{2\sqrt{r_1}\sqrt{r_2}}$$

5.1) Verificar el numérico

$$y = \frac{CT}{2\sqrt{r_1}\sqrt{r_2}} = \frac{2000}{2\sqrt{10}\sqrt{10}} = 100$$

6) Armar DCF

$$x_1 = x_1\left(r, y\right)$$
$$x_2 = x_2\left(r, y\right)$$

$$x_1 = x_2 \frac{r_2}{r_1}$$

$$y = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$\sqrt{x_2} = \frac{y}{\sqrt{x_1}}$$

$$x_2 = \left(\frac{y}{\sqrt{x_1}} \right)^2 = \frac{y^2}{x_1}$$

$$x_1 = \frac{y^2}{x_1} \frac{r_2}{r_1}$$

$$\left(x_1\right)^2 = y^2 \frac{r_2}{r_1}$$

$$x_1 = y\sqrt{\frac{r_2}{r_1}}$$

La demanda del bien 1 es directamente proporcional al producto, e indirectamente proporcional a su precio.

Análogamente:

$$x_2 = y\sqrt{\frac{r_1}{r_2}}$$

7) Armar la función de costos

$$C = c\left(r, y\right)$$

$$C = r_1 x_1 + r_2 x_2$$

$$C = r_1 y\sqrt{\frac{r_2}{r_1}} + r_2 y\sqrt{\frac{r_1}{r_2}} =$$

$$= y\left(r_1\sqrt{\frac{r_2}{r_1}} + r_2\sqrt{\frac{r_1}{r_2}} \right) =$$

$$= 2y\sqrt{r_1 r_2}$$

7.1) Rendimientos constantes dan CMI constantes

$$CM = \frac{C}{y} = \frac{2y\sqrt{r_1 r_2}}{y} = 2\sqrt{r_1 r_2}$$

8) Planteo con α_1 y α_2

$$Max \quad y = x_1^{\alpha_1} x_2^{\alpha_2}$$

$$s.a. \quad CT = r_1 x_1 + r_2 x_2$$

$$L = x_1^{\alpha_1} x_2^{\alpha_2} - \lambda\left(r_1 x_1 + r_2 x_2 - CT\right)$$

$$\frac{\partial L}{\partial x_1} = \frac{\alpha_1 x_2^{\alpha_2} x_1^{\alpha_1}}{x_1} - r_1\lambda = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{\alpha_2 x_1^{\alpha_1} x_2^{\alpha_2}}{x_2} - r_2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = r_1 x_1 + r_2 x_2 - CT = 0$$

$$\frac{\alpha_1 x_2^{\alpha_2} x_1^{\alpha_1}}{x_1} = r_1\lambda$$

$$\frac{\alpha_2 x_1^{\alpha_1} x_2^{\alpha_2}}{x_2} = r_2\lambda$$

$$\lambda = \frac{\alpha_1 x_2^{\alpha_2} x_1^{\alpha_1}}{x_1 r_1} = \frac{\alpha_2 x_1^{\alpha_1} x_2^{\alpha_2}}{x_2 r_2}$$

Tasa marginal de sustitución: $\frac{r_2}{r_1} = \frac{\alpha_2}{\alpha_1}\frac{x_1}{x_2}$

$$\frac{1}{x_1} = \frac{\alpha_2}{\alpha_1}\frac{r_1}{x_2 r_2}$$

$$x_1 = \frac{\alpha_1 x_2 r_2}{\alpha_2 r_1}$$

Armo la DCF

$$y = x_1^{\alpha_1} x_2^{\alpha_2}$$

$$x_2^{\alpha_2} = \frac{y}{x_1^{\alpha_1}}$$

$$x_2 = \left(\frac{y}{x_1^{\alpha_1}} \right)^{\frac{1}{\alpha_2}}$$

$$x_1 = \frac{\alpha_1 \left(\frac{y}{x_1^{\alpha_1}}\right)^{\frac{1}{\alpha_2}} r_2}{\alpha_2 r_1}$$

$$x_1 = \frac{\alpha_1 y^{\frac{1}{\alpha_2}} x_1^{-\frac{\alpha_1}{\alpha_2}} r_2}{\alpha_2 r_1}$$

$$\frac{x_1}{x_1^{-\frac{\alpha_1}{\alpha_2}}} = \frac{\alpha_1 y^{\frac{1}{\alpha_2}} r_2}{\alpha_2 r_1}$$

$$x_1^{1+\frac{\alpha_1}{\alpha_2}} = \frac{\alpha_1 y^{\frac{1}{\alpha_2}} r_2}{\alpha_2 r_1}$$

$$x_1^{\frac{\alpha_2+\alpha_1}{\alpha_2}} = \frac{\alpha_1 y^{\frac{1}{\alpha_2}} r_2}{\alpha_2 r_1}$$

$$x_1 = \left(\frac{\alpha_1 y^{\frac{1}{\alpha_2}} r_2}{\alpha_2 r_1}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}}$$

$$x_2 = \left(\frac{\alpha_2 y^{\frac{1}{\alpha_1}} r_1}{\alpha_1 r_2}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}}$$

Armo la función de costos:

$$C = r_1 \left(\frac{\alpha_1 y^{\frac{1}{\alpha_2}} r_2}{\alpha_2 r_1}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}} + r_2 \left(\frac{\alpha_2 y^{\frac{1}{\alpha_1}} r_1}{\alpha_1 r_2}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}}$$

$$C = r_1 \left(\frac{\alpha_1}{\alpha_2} y^{\frac{1}{\alpha_2}}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}} \left(\frac{r_2}{r_1}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}} +$$

$$+ r_2 \left(\frac{\alpha_2}{\alpha_1} y^{\frac{1}{\alpha_1}}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}} \left(\frac{r_1}{r_2}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}}$$

$$C = r_2^{\frac{\alpha_2}{\alpha_2+\alpha_1}} r_1^{1-\frac{\alpha_2}{\alpha_2+\alpha_1}} \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}} y^{\frac{1}{\alpha_2+\alpha_1}} +$$

$$+ r_1^{\frac{\alpha_1}{\alpha_2+\alpha_1}} r_2^{1-\frac{\alpha_1}{\alpha_2+\alpha_1}} \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}} y^{\frac{1}{\alpha_1+\alpha_2}}$$

$$C = y^{\frac{1}{\alpha_2+\alpha_1}} \left(r_2^{\frac{\alpha_2}{\alpha_2+\alpha_1}} r_1^{1-\frac{\alpha_2}{\alpha_2+\alpha_1}} \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}} + r_1^{\frac{\alpha_1}{\alpha_2+\alpha_1}} r_2^{1-\frac{\alpha_1}{\alpha_2+\alpha_1}} \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}} \right)$$

$$1 - \frac{\alpha_2}{\alpha_2+\alpha_1} = \frac{\alpha_2+\alpha_1-\alpha_2}{\alpha_2+\alpha_1} = \frac{\alpha_1}{\alpha_2+\alpha_1}$$

$$C = y^{\frac{1}{\alpha_2+\alpha_1}} \left(r_2^{\frac{\alpha_2}{\alpha_2+\alpha_1}} r_1^{\frac{\alpha_1}{\alpha_2+\alpha_1}} \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}} + r_1^{\frac{\alpha_1}{\alpha_2+\alpha_1}} r_2^{\frac{\alpha_2}{\alpha_2+\alpha_1}} \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}} \right)$$

$$C = y^{\frac{1}{\alpha_2+\alpha_1}} r_2^{\frac{\alpha_2}{\alpha_2+\alpha_1}} r_1^{\frac{\alpha_1}{\alpha_2+\alpha_1}} \left(\left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha_2+\alpha_1}} + \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_2+\alpha_1}} \right)$$

$$C = y^{\frac{1}{\alpha_2+\alpha_1}} r_2^{\frac{\alpha_2}{\alpha_2+\alpha_1}} r_1^{\frac{\alpha_1}{\alpha_2+\alpha_1}} \left(\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2} + \left(\frac{\alpha_2}{\alpha_1}\right)^{\alpha_1} \right)^{\frac{1}{\alpha_2+\alpha_1}}$$

$$C = \left[y r_2^{\alpha_2} r_1^{\alpha_1} \left(\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2} + \left(\frac{\alpha_2}{\alpha_1}\right)^{\alpha_1} \right) \right]^{\frac{1}{\alpha_2+\alpha_1}}$$

Sea: $\phi = \left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2} + \left(\frac{\alpha_2}{\alpha_1}\right)^{\alpha_1}$

$$C = \left[y r_2^{\alpha_2} r_1^{\alpha_1} \phi \right]^{\frac{1}{\alpha_2+\alpha_1}}$$

8.1) Relación CM y CMg + Gráfico

Costos medios:

$$CM = \frac{C}{y} = \left[r_2^{\alpha_2} r_1^{\alpha_1} \phi \right]^{\frac{1}{\alpha_2+\alpha_1}} y^{\frac{1}{\alpha_2+\alpha_1}-1} =$$

$$= \left[r_2^{\alpha_2} r_1^{\alpha_1} \phi \right]^{\frac{1}{\alpha_2+\alpha_1}} y^{\frac{1-\alpha_1-\alpha_2}{\alpha_2+\alpha_1}}$$

Si hay rendimientos constantes a escala:

$\alpha_1 + \alpha_2 = 1$

$\frac{1-\alpha_1-\alpha_2}{\alpha_2+\alpha_1} = 0$

$y^0 = 1$

Hay costos medios constantes.

Si hay rendimientos decrecientes a escala:

$\alpha_1 + \alpha_2 < 1$

$\frac{1-\alpha_1-\alpha_2}{\alpha_2+\alpha_1} > 0$

Hay costos medios crecientes.

Si hay rendimientos crecientes a escala:

$\alpha_1 + \alpha_2 > 1$

$\frac{1-\alpha_1-\alpha_2}{\alpha_2+\alpha_1} < 0$

Hay costos medios decrecientes.

Costos marginales:

$$CMg = \frac{\partial C}{\partial y} = \left(r_2^{\alpha_2} r_1^{\alpha_1} \phi\right)^{\frac{1}{\alpha_2+\alpha_1}} \frac{1}{\alpha_2+\alpha_1} y^{\frac{1}{\alpha_2+\alpha_1}-1} =$$

$$= \left(r_2^{\alpha_2} r_1^{\alpha_1} \phi\right)^{\frac{1}{\alpha_2+\alpha_1}} \frac{1}{\alpha_2+\alpha_1} y^{\frac{1-\alpha_1-\alpha_2}{\alpha_2+\alpha_1}}$$

$$CMg = \frac{1}{\alpha_1+\alpha_2} CM$$

Con rendimientos constantes: $CMg = CM$

Con rendimientos crecientes: $CMg < CM$

Con rendimientos decrecientes: $CMg > CM$