Ejercicio genérico de demanda

$Max\ \ U = u(x)$
$s.a.\ \ x \in B$

Forma operativa

$$\left.\begin{array}{l} Max\ \ U = u(x) \text{ Estrictamente cuasi-cóncava} \\ s.a.\ \ px \le m \text{ Convexo, cerrado y acotado} \end{array}\right\} \text{Puedo aplicar K-T}$$

$$L = u\left(x\right) + \lambda\left(m - px\right)$$

C.P.O.

$$\frac{\partial L}{\partial x_i} \le 0 \ \wedge\ \frac{\partial L}{\partial x_i} x_i = 0 \ \ \forall i \qquad \frac{\partial u}{\partial x_i} - \lambda p_i \le 0 \ \wedge\ \left(\frac{\partial u}{\partial x_i} - \lambda p_i\right) x_i = 0$$

$$\frac{\partial L}{\partial \lambda} \ge 0 \ \wedge\ \frac{\partial L}{\partial \lambda_j} \lambda_j = 0 \ \ \forall j \qquad m - px \ge 0 \ \wedge\ \left(m - px\right)\lambda = 0$$

$$x_i, \lambda_j \ge 0 \ \ \forall i \forall j$$

Sí $x_i > 0$

$$\frac{\partial u}{\partial x_i} - \lambda p_i = 0 \Rightarrow \frac{\partial u}{\partial x_i} = \lambda p_i$$

Y por insaciabilidad $\frac{\partial u}{\partial x_i} > 0 : \ \lambda > 0$

Si $\lambda > 0 \ m - px = 0 \Rightarrow m = px$
Esto quiere decir que en el óptimo se agota el presupuesto.

Especificamos $u\left(x\right) = \prod_i x_i$ que cumple insaciabilidad local $\frac{\partial u}{\partial x_i} = \frac{\partial \prod x_i}{\partial x_i} > 0$ y cuasi-concavidad estricta.

$$L = \prod_i x_i + \lambda\left(m - \sum_i p_i x_i\right)$$

C.P.O.

$$\frac{\partial L}{\partial x_i} = \frac{\prod x_j}{x_i} - \lambda p_i \le 0 \ \ \forall i \ \ \wedge\ \left(\frac{\prod x_j}{x_i} - \lambda p_i\right) x_i = 0 \, (1)$$

$$\frac{\partial L}{\partial \lambda} = m - \sum_i p_i x_i \ge 0 \ \wedge\ \left(m - px\right)\lambda = 0$$

$x_i \geq 0 \quad \lambda \geq 0 \quad \forall i$

Si $x_i > 0$ entonces $\dfrac{\prod x_j}{x_i} = \lambda p_i$ (2)

Dado $p_i > 0 \wedge$ Insaciabilidad local $\lambda > 0 \quad m = \sum p_i x_i$ (3)

Solución sin hallar combinación óptima

1. Despejar x_i de (2): $x_i = \dfrac{\prod x_j}{\lambda p_i}$ (4)

2. Sustituir x_i en la restricción (3): $m = \sum p_i \dfrac{\prod x_j}{\lambda p_i} \Rightarrow m = \sum_i \dfrac{\prod x_j}{\lambda}$

3. Despejo λ y $\prod x_j$ que son variables no incluidas en la función de demanda de $x_i(p,m)$

$$m = \sum \frac{\prod x_j}{\lambda} \Rightarrow m = n\frac{\prod x_j}{\lambda} \Rightarrow \frac{m}{n} = \frac{\prod x_j}{\lambda}$$

4. sustituyo en la CPO de la variable de decisión (4) $x_i = \dfrac{\prod x_j}{\lambda p_i} \Rightarrow x_i = \dfrac{1}{p_i}\left(\dfrac{\prod x_j}{\lambda}\right)$

$x_i = \dfrac{m}{np_i}$ Curva de demanda Marshaliana

Hallo $V(p,m) \equiv u\left(x(p,m)\right)$

$$V(p,m) = \prod_{i=1}^{n}\left(\frac{m}{np_i}\right) = \frac{m^n}{n^n \prod p_i}$$

Dual

$$\begin{aligned} &Min \quad m = px \\ &s.a. \quad \prod x_i \geq \bar{u} \end{aligned}$$

$$L = \sum_i p_i x_i + \lambda\left(\bar{u} - \prod x_i\right)$$

C.P.O.

$$\frac{\partial L}{\partial x_i} = p_i - \frac{\lambda \prod x_j}{x_i} \geq 0 \ \wedge \ \left(p_i - \frac{\lambda \prod x_j}{x_i} \right) x_i = 0 \ \ \forall i \ \ (1)$$

$$\frac{\partial L}{\partial \lambda} = \bar{u} - \prod x_i \leq 0 \ \wedge \ \left(\bar{u} - \prod x_i \right) \lambda = 0 \ \ (2)$$

$$x_i, \lambda \geq 0 \ \ \forall i$$

Si $x_i > 0$ entonces $p_i = \dfrac{\lambda \prod x_j}{x_i}$ (3)

Dado que $p_i > 0$ entonces $\lambda > 0 \Rightarrow u = \prod x_i$ (4)

Solución sin hallar combinación óptima

1. Despejar la variable de decisión de la derivada de la función objetivo (1) (Ídem punto 1 anterior) $x_i = \dfrac{\lambda \prod x_j}{p_i}$ (5)
2. Sustituir x_i en la restricción (2)

   $$u = \prod_{i=1}^{n} \frac{\lambda \prod x_j}{p_i}$$

   $$u = \left(\lambda \prod x_j \right)^n \prod \frac{1}{p_i}$$

   Porque la $\prod$ es en i y la 2da $\prod$ es en j $\therefore$ es constante para $\prod_i$

   $$u = \frac{\left(\lambda \prod x_j \right)^n}{\prod p_i}$$

3. Despejo $\lambda \ \wedge \ \prod x$

   $$u = \frac{\left(\lambda \prod x_j \right)^n}{\prod p_i} \Rightarrow \left(\lambda \prod x_j \right)^n = u \prod p_i \Rightarrow$$

   $$\Rightarrow \lambda \prod x_j = \left(u \prod p_i \right)^{\frac{1}{n}}$$

   Sustituyo en (5): $x_i = \dfrac{\left(u \prod p_i \right)^{\frac{1}{n}}}{p_i} \Rightarrow h_i = \dfrac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i}$ Demanda Hicksiana

   Hallo $e(p,u) = ph(p,u)$

   $$e(p,u) = \sum p_i \frac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i} = n \left(u \prod p_j \right)^{\frac{1}{n}}$$

Comprobamos equivalencias

$$V\left(p,e\left(p,u\right)\right)\equiv u$$

$$V\left(p,m\right)=\frac{m^n}{n^n\prod p_i}$$

$$V\left(p,e\left(p,u\right)\right)=\frac{\left[n\left(u\prod p_j\right)^{\frac{1}{n}}\right]^n}{n^n\prod p_i}$$

$$V\left(p,e\left(p,u\right)\right)=u$$

$$e\left(p,V\left(p,m\right)\right)\equiv m$$

$$e\left(p,u\right)=n\left(u\prod p_i\right)^{\frac{1}{n}}$$

$$e\left(p,V\left(p,m\right)\right)=n\left(\frac{m^n}{n^n\prod p_i}\prod p_i\right)^{\frac{1}{n}}=m$$

$$h\left(p,u\right)\equiv x_i\left(p,e\left(p,u\right)\right)$$

$$\frac{\left(u\prod p_j\right)^{\frac{1}{n}}}{p_i}=\frac{e\left(p,u\right)}{np_i}$$

$$\frac{\left(u\prod p_j\right)^{\frac{1}{n}}}{p_i}=\frac{n\left(u\prod p_i\right)^{\frac{1}{n}}}{np_i}$$

$$x_i\left(p,m\right)\equiv h_i\left(p,V\left(p,m\right)\right)$$

$$\frac{m}{np_i}=\frac{\left[V\left(p,m\right)\prod p_j\right]^{\frac{1}{n}}}{p_i}$$

$$\frac{m}{np_i}=\frac{\left[\frac{m^n}{n^n\prod p_i}\prod p_j\right]^{\frac{1}{n}}}{p_i}=\frac{m}{np_i}$$

Slutsky

$$\frac{\partial x_i}{\partial p_i} = \frac{\partial h_i}{\partial p_i} - \frac{\partial x_i}{\partial m} x_i$$

$$\frac{\partial x_i}{\partial p_i} = \frac{\partial \left(\frac{m}{np_i} \right)}{\partial p_i} = -\frac{m}{np_i^2}$$

$$\frac{\partial h_i}{\partial p_i} = \frac{\partial \left[\frac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i} \right]}{\partial p_i} = \left[\frac{1}{n} \left(u \prod p_j \right)^{\frac{1}{n}-1} \frac{u \prod p_j}{p_i} p_i - \left(u \prod p_j \right)^{\frac{1}{n}} \right] \frac{1}{p_i^2}$$

$$\frac{\partial h_i}{\partial p_i} = \frac{\frac{1}{n} \left(u \prod p_j \right)^{\frac{1}{n}-1}}{p_i} \frac{\left(u \prod p_j \right)}{p_i} - \frac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i^2}$$

$$\frac{\partial h_i}{\partial p_i} = \frac{\frac{1}{n} \left(u \prod p_j \right)^{\frac{1}{n}-1+1}}{p_i^2} - \frac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i^2}$$

$$\frac{\partial h_i}{\partial p_i} = \frac{\frac{1}{n} \left(u \prod p_j \right)^{\frac{1}{n}}}{p_i^2} - \frac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i^2}$$

$$\frac{\partial h_i}{\partial p_i} = \left(\frac{1-n}{n} \right) \frac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i^2}$$

$$\frac{\partial x_i}{\partial m} = \frac{\partial \left(\frac{m}{np_i} \right)}{\partial m} = \frac{1}{np_i}$$

Armo Slutsky

$$-\frac{m}{np_i^2} = \left(\frac{1-n}{n} \right) \frac{\left(u \prod p_j \right)^{\frac{1}{n}}}{p_i^2} - \frac{1}{np_i} \frac{m}{np_i} =$$

$$-\frac{m}{np_i^2} = \left(\frac{1-n}{n} \right) \frac{\left(\frac{m^n}{n^n \prod p_j} \prod p_j \right)^{\frac{1}{n}}}{p_i^2} - \frac{m}{n^2 p_i^2}$$

$$-\frac{m}{np_i^2}=\left(\frac{1-n}{n}\right)\frac{m}{np_i^2}-\frac{m}{n^2p_i^2}$$

$$-\frac{m}{np_i^2}=\left(\frac{1-n}{n}-\frac{1}{n}\right)\frac{m}{np_i^2}$$

$$-\frac{m}{np_i^2}=-\frac{m}{np_i^2}$$

Identidad de Roy

$$-\frac{\frac{\partial V(p,m)}{\partial p_i}}{\frac{\partial V(p,m)}{\partial m}}=x_i$$

$$\frac{\partial V(p,m)}{\partial p_i}=\frac{\partial\frac{m^n}{n^n\prod p_j}}{\partial p_i}=-\frac{m^n}{n^n}\frac{1}{\left(\prod p_j\right)p_i}$$

$$\frac{\partial V(p,m)}{\partial m}=\frac{\partial\frac{m^n}{n^n\prod p_j}}{\partial m}=n\frac{m^{n-1}}{n^n\prod p_j}$$

$$-\frac{-\frac{m^n}{n^n}\frac{1}{\left(\prod p_j\right)p_i}}{n\frac{m^{n-1}}{n^n\prod p_j}}=\frac{m^n}{n^n}\frac{1}{p_i\prod p_j}\frac{n^{n-1}}{m^{n-1}}\prod p_j=\frac{m}{np_i}$$

Relación UMP y EMP