Detecting Heterogeneity in the MPC:
A Machine Learning Approach

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1 Introduction

How do households respond to income shocks, and how do their responses differ given their personal characteristics and economic circumstances? These questions are not only at the center of a wide academic debate in economics but also of major importance for policymakers. While the former revolves around verifying or neglecting the main mechanisms of the Permanent Income Hypothesis (PIH), the latter are interested in improving government transfers to more efficiently use public funds. These two sides have sparked many investigations using a wide array of approaches to quantify households' responses to income shocks - the Marginal Propensity to Consume (MPC). At the center of macroeconomics since Keynes introduced it at the heart of his General Economic Theory, it quantifies how much households will spend on consumption of each dollar they receive from an income shock. Research has long focused on testing whether the MPC out of income shocks is zero in general and, thus, in line with the PIH, but the focus has shifted over the last decade. Most studies support the notion of an average zero MPC, but more recent evidence suggests that for specific groups, the response is significantly different.

Empirical research related to the MPC and its heterogeneity has used several settings to identify income shocks. One of the most prominent is to use natural experiments in which households receive exogenous income shocks. Following Parker et al. (2013) and Misra and Surico (2014), we exploit the 2008 tax rebate in the USA to estimate households' MPC using data collected by the Consumer Expenditure Survey (CEX). Similar to these two prior studies, we are able to use the rich information on consumption the CEX provides to not only identify heterogeneities in the overall MPC but also to analyze which categories of consumption goods households spend their rebate on. However, our econometric approach sets us apart as it is more sophisticated and more precise in detecting heterogeneities compared to any contribution we are aware of.

We use the Double Machine Learning Framework (DML) developed by Chernozhukov et al. (2017) to estimate individual-level point estimates of the MPC as well as standard errors for each household. This enables us to run hypothesis tests on whether a household's MPC is statistically significantly different from zero. The DML allows us to estimate the conditional average treatment effect (CATE) of the tax rebate on changes in consumption. Meanwhile, thanks to the semi-parametric nature of the DML framework, we can use reliable Machine Learning models to control for any confounding factors without having to define their relationship with the outcome. Moreover, one of the two estimators we employ retrieves the CATE without assuming a specified relationship between variables we condition on (X) and the CATE itself. I.e., we do not assume that the CATE is linear in variables we condition on but leave this relationship unspecified.

Our results underline the heterogeneity of the Marginal Propensity to Consume out of

the tax stimulus documented in Parker et al. and Misra and Surico. We find a large mass of households shows no significant reaction upon receiving the stimulus payment, whereas a smaller fraction of households shows strong and significant reactions above an estimated MPC of 0.5. Our analysis suggests that liquidity is indeed the main driver of MPC heterogeneities and that low liquidity households are the ones reacting most sharply. Contrary to prior work, our estimated CATE does not rely on specifying subsets of the data across which we assume heterogeneity to exist. We employ modern methods to quantify the effects of single variables on the estimated MPCs to understand the role of these characteristics. Our non-linear estimators suggest the heterogeneities presented in other work are imprecise. Next to our contribution to the MPC literature by providing an empirically more robust analysis, we also see our contribution in introducing modern and flexible estimation approaches to the macroeconomic literature. Frameworks such as the DML offer a gateway to new methods and identifications in the macroeconomic literature. We stress the importance of further research into the theoretical and applied nature of these procedures and their usage in more settings.

It is important to highlight that the Economic Stimulus Act in 2008 was signed into law by President Bush in February 2008. The tax rebate payments, which were part of this policy, started in April of the same year and are therefore an anticipated income shock. This becomes relevant when investigating the role of various factors, especially liquidity. Also, the tax rebate was disbursed to USA taxpayers during a time of national and global economic downturn. Many households receiving the stimulus might have been in economic turmoil when receiving the payment and actually spend it to cover regular expenses that they otherwise would not have been able to cover (e.g., rent, utilities, or other necessities of daily life). However, Parker et al. (2013) emphasize that some rebates were reported to be received outside of the disbursal window, which suggests that the income shocks might not have been anticipated and only noticed after their arrival.

The fine-grained consumption data of the CEX allows us to identify what kind of goods households consumed and what they spent their stimulus money on. As Kaplan and Violante (2014) note, the tax stimulus is anticipated and is subject to these special circumstances. Therefore, one might also speak of our estimated coefficients as a 'Propensity to consume the rebate' or 'rebate coefficient,' which is not necessarily equivalent to households' overall MPC. While a government stimulus program might not be perfectly appropriate to verify theoretical models concerned with the MPC, providing evidence on their effect on individuals is of major importance for future policymaking. When economic relief is urgent, non-targeted stimuli can present a viable option; targeted transfers can play a major role in many policy settings. Thus, understanding what households spend government transfers on and what households actually use them for consumption is an important part of efficient policymaking. Additionally, quantifying the effectiveness

of untargeted transfers is necessary to assess whether they are actually helping to boost the economy. Aggregate estimates of the MPC suggest that this is not the case, but taking a closer look and adjusting for household characteristics reveals heterogeneities and effectiveness of these transfers.

The rest of the paper is structured as follows: Section 2 summarizes the theoretical and empirical literature on MPC heterogeneity. Section 3 discusses the data source and challenges connected with it. The empirical methodology we use is described in Section 4, while Section 5 presents the identification and estimation results of the MPC. We further investigate sources of heterogeneity in responses in Section 6. Section 7 concludes.

2   Liquid Channels of heterogeneity and the current state of the literature

Jappelli and Pistaferri (2010) provide a simple life-cycle model to illustrate various channels in the theoretical literature that have been identified as potential drivers of MPC heterogeneity. We will briefly summarize the channels most relevant to our analysis before we turn to the more recent empirical literature on MPC heterogeneity.

The first channel is the precautionary savings motive. Some households will show no response to income shocks because they tend to save them as a precautionary measure. This is a mechanism we could expect to play a role in our setting because we investigate a tax stimulus paid out during a strong recession. Uncertainty over future outcomes might increase - or be perceived as higher - by households, and thus their savings motives increase. At the same time, we might argue that low-income households especially need the stimulus to pay their day-to-day bills. This ambiguity might be one driver of the heterogeneity we observe.

However, these precautionary savings by the households themselves are necessary because they do not have access to insurance markets (Carroll, 2001; Blundell et al., 2008; Kaplan and Violante, 2009). These incomplete insurance markets are another driver of MPC heterogeneity. Given heterogeneous risks across the spectrum of households, access to insurance markets varies for each household. Hence, households without access rely on precautionary savings to insure themselves against future income shocks. In general, the literature takes a look at any form of the credit market to which households' access might be constrained. This brings us to the most global driver identified in the literature, which is the liquidity channel.

It is the most important and most prominent in the MPC literature and has been subject to the most and earliest attention (Carroll 1997; Deaton, 1991; and Zeldes, 1989). Its role is linked to the nature of the income shock and borrowing constraints. For example,

Kaplan and Violante (2009) show that in a calibrated life-cycle model, households that have no access to credit markets and can therefore not borrow react substantially more to transitory income shocks than do households without such constraints. In general, if a positive income shock is anticipated, households that are already close to or at their borrowing constraint cannot borrow new funds to smooth consumption in anticipation of a higher future income. Thus, once the shock is realized, we will observe an increase in consumption. On the other hand, saving is always possible for any household, and hence we will not see a reaction once the shock is realized in case of a negative anticipated shock. Thus, more liquid households react less to a positive anticipated shock in comparison with liquidity constrained ones. In contrast, in case of an unanticipated shock, we expect the opposite. Think of an agent that is temporarily out of work and has no liquid wealth at their disposal. In case of a negative shock, the agent is forced to adjust their consumption behavior downward. Meanwhile, a positive shock will always be saved and stretched over future periods, no matter the level of households' liquidity. E.g., Bunn et al. (2018) document this asymmetry depending on the sign of the shock.

Another potential driver of heterogeneity is households' age. Attanasio and Weber (2010) document the life-cycle dynamics and summarize the 'retirement-consumption puzzle'; however, this is concerned with the amount spent overall, not changes in consumption. Hence, while older households might operate on a lower level of consumption expenditures, changes in amounts are not necessarily subject to this. Moreover, we will see that estimates concerning age capture the effects of other confounders, most prominently liquidity, as they are correlated.

Lastly, a source of heterogeneity might be individual-level differences in preferences, which are subject to investigations in behavioral studies. Especially households' patience, represented by their discount factor in theoretical models, is of importance when considering the reaction to income shocks (need citation of the papers doing this; look for it again in notes). The applied empirical literature investigating the Marginal Propensity to Consume can be categorized into two strands. The first uses data on households' expenditures and observed income shocks; the second relies on surveys that ask respondents directly for their MPC out of hypothetical or experienced income shocks. Parker and Souleles (2019) coin them the 'revealed preferences' and the latter the 'reported preferences' approach, respectively. Our contribution firmly sits within the revealed preferences part of the literature. Similarly, many studies have investigated the MPC using expenditure data. One common approach is to exploit surveys of lottery winners. The odds of winning the lottery are so low that a win can be interpreted as an unanticipated income shock. Studies mostly use state lotteries which have a wide range of small amounts that

can be won.[1] For example, Fagereng et al. (2020) use Norwegian administrative panel data to estimate the MPC using winners from a national lottery as the treatment group. They find that households winning the lottery spend almost half of their win within one year and 90% after five years. Moreover, the authors report that liquidity and age are the only variables correlated with the MPC after controlling for confounders. However, correlations are a bad instrument to assess drivers of the MPC as we cannot assess which of the variables is the driving force. E.g., age could be correlated with liquidity and only be correlated with the MPC through this liquidity channel. In a similar vein, Golosov et al. (2020) construct a dataset of lottery winners of state lotteries in the USA. While their main goal is to estimate households' labor market response, they also construct a measure to recover the MPC using their estimates. They find that on average, the MPC out of lottery wins is 60ct of each dollar won. Further supporting the liquidity channel, they find that the highest quartile of the liquidity distribution spends only 49ct while the lowest quartile spends almost 80ct of each dollar won in the lottery. However, these two lottery-based approaches suffer from the drawback that they do not measure consumption directly. Instead, they have to either construct consumption out of households' balance sheet data (Fagereng et al., 2018) or model consumption as a function of their observed variables (Golosov et al., 2019).

Fuster et al. (2020) investigate the MPC out of unanticipated gains. They also find a vast heterogeneity in responses. On the other hand, they show that news about future income gains leads to no reaction on the side of the household. Most interesting and outstanding from other literature, however, is their approach to identify extensive and intensive margins of the MPC. As the size of windfalls gains increases, households report that they would increase their spending (reported preferences approach), but, conditional on responding, they find that households' reactions decline in the size of gains. Overall, the extensive margin effect is stronger in that more households are spending a significant amount of payments. They calibrate a precautionary savings model that is capable of replicating all their findings.

Recent examples of the reported preferences approach are Bunn et al. (2018) and Christelis et al. (2019). The former uses data from the Bank of England to estimate the MPC of British households to income shocks. In the BoE survey, participants are asked about past income shocks they experienced and how they reacted. Their results further support the liquidity channel as the important driver behind MPC heterogeneity and show that indeed households have asymmetric reactions to positive and negative income shocks. Their estimated average MPC is around 0.46 to 0.86 for negative income shocks and 0.07 to 0.17 for positive shocks. In a theoretical exercise, they show that a model

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[1]Using lottery winners who win hundreds of thousands or even millions of dollars would be fruitless since the size of the shock is unreasonably large and probably changes the complete underlying choice-set of households. Additionally, sample sizes would be very small.

with occasionally binding borrowing constraints can replicate their results.

Christelis et al. (2019) use Dutch data of reported preferences to also investigate the asymmetry of the MPC and the role of the size of the shock. They find an average MPC of 15% to 25%. In line with the above described liquidity channel, households react more to negative shocks. Moreover, their results reveal strong heterogeneity in responses. 40% of households in their sample react the same to positive and negative shocks, while another 40% respond asymmetrically. The latter suggests a strong role of liquidity in decision-making making how to use income shocks. The last 20% of households reveal asymmetric behavior in the opposite direction, which the authors connect to other behavioral models and the lack of financial sophistication. This is in line with Parker (2017), who also finds a strong relationship between MPC and sophistication.

The already mentioned Parker and Souleles (2019) evaluate the two different approaches of the applied literature. They summarize that households that self-report a larger propensity to spend income shocks indeed have larger estimates using the revealed preferences approach. Interestingly they show that self-reported MPCs do not vary conditional on liquidity, the channel which is supported the most by existing studies.

Parker (2017) uses data from the Nielsen Consumer Panel to also investigate the 2008 tax stimulus. The Nielsen Consumer Panel provides a large set of questions about how households used their rebate and on their personal preferences. He finds that the MPC out of the rebate is 1.5% of the stimulus received within one week of receipt and 3.5% within the first four weeks. He shows that spending responses are strongly related to personal characteristics. For example, households' response is heavily related to self-reported characteristics such as sophistication, planning, and impatience. Moreover, he also finds evidence supporting the liquidity channel. Also, whether households report being savers or spenders is strongly correlated with their MPC. Based on his findings, Parker suggests that the relationship between liquidity and MPC is not situational but rather must a long period of low liquidity persist before it affects the households response. I.e., households that only have low liquidity situationally do not respond differently than other households.

Lastly, we want to discuss the two studies immediately related to this paper: Parker et al. (2013) and Misra and Surico (2014). The former collaborated with the BLS to add questions about the 2008 tax rebate to the CEX in 2008 and 2009. They estimate the average response of households using OLS and 2SLS, where they instrument the rebate amount with a dummy signaling rebate receipt. Their motivation behind the latter will be subject to discussion when we present our identification strategy in section 5.1. Estimating various specifications, Parker et al. find MPCs out of the tax rebate that range between 12% and 30% when considering changes in non-durable consumption. Taking into account all expenditure categories reported in the CEX, they even find responses

between 50% to 90%. These higher estimates can be traced back to a small set of households making large purchases in the new vehicle category - a phenomenon that is also documented in Misra and Surico and our own results.

To investigate heterogeneity, Parker et al. follow one baseline approach in the literature. They define thresholds along the distribution of liquidity and create dummy variables that signal to which group observation i belongs. In their case, they set the cutoff points along the liquidity distribution to cut it into terciles and that each group has the same amount of households receiving the rebate in a given quarter. Parker et al. report that indeed lower liquidity households show a stronger response to receiving the tax rebate across all their specifications. However, as Misra and Surico point out as well, they only rely on the economic significance but cannot reject the null that the point estimates across groups are significantly different from each other.

Additionally, similar to another approach present in the literature - splitting the sample into subsamples and estimating the MPC in each sample on its own - this approach can miss out on patterns of heterogeneity. More precisely, we will only find out whether heterogeneity exists between the three terciles, but any patterns within these terciles are ruled out by construction. In that manner, our estimation approach is superior as we are able to find heterogeneous patterns without pre-defining any sub-groups.

Meanwhile, Misra and Surico (henceforth MS) use Quantile Regression on the same data to estimate the conditional distribution of the marginal propensity to consume. They find a distribution across all consumption categories that supports the notion by Kaplan and Violante (2014) that a large amount of households shows no significant response, while a substantial share has an MPC of around 0.5, and some households react even stronger. In fact, the lower and the upper end of the consumption change distribution are reacting significantly differently from zero, where the reaction is increasing along with the distribution. These findings are consistent across all consumption categories they investigate. They then check how specific variables - such as liquidity or home-ownership rates - are distributed across the distribution of consumption change. They show that in areas where households show significant reactions to tax rebate receipt - at the lower and upper end of the distribution - the median income is higher than in the center part of the distribution. They conclude that high-income households have either strong positive reactions or zero reactions, while low-income households show a consistently positive reaction of 10% to 40%. This explains previously contradicting findings by Sahm, Shapiro, and Slemrod (2010) and others, who provide evidence on high-income households having the largest MPC, and Johnson, Parker, and Souleles (2006) and Parker et al. (2013), who find that low income is associated with higher MPCs. However, these findings are only based on correlations that do not necessarily imply a causal relationship between these factors. However, Misra and Surico's results must be taken with a grain of salt as they ignore a

fundamental assumption of quantile regression, which is necessary to interpret their estimates as the MPC and draw the conclusions they present. The assumption in question is the *rank-invariance*, or *rank-preservation*, assumption. Discussing the inner workings of quantile regression is out of the scope of this work here, but we briefly lay out how MS interpretation is flawed as long as we do not make this restrictive assumption. The coefficient in a quantile regression when estimating the τth quantile of the outcome signals how much a one-unit change affects this quantile of the outcome's distribution - in our case, consumption change. However, individuals in this quantile before and after treatment must not be the same. Actually, we would exactly expect the opposite if some individuals react strongly to receiving the rebate and others do not. If, for example, individuals previously did not change their consumption much but now react strongly, they are part of a different quantile than before, and the coefficient only reveals how much the τth quantile changes - e.g., because households reacting strongly move out of it. Therefore, their coefficients do not reveal the MPC as we do not look at individuals but only at the distribution. Assuming rank-invariance implies that the rank in the distribution before and after treatment stays the same, and we thus compare the same individuals within the quantiles. As we have laid out already, this is not reasonable to assume in our setting and is actually quite counterintuitive. Keeping this issue in mind, Misra and Surico's results can still be helpful when we are interested to understand the distributional effects of the tax rebate - e.g., whether it moves closer together or spreads further apart.

3 Data

We use data collected by the Consumer Expenditure Survey (CEX) that is administered by the Bureau of Labor Statistics. Its main purpose is to provide information on the consumption preferences of US households to adjust the goods basket that is used to calculate various inflation measures (BLS, 2021). However, in an effort to understand the effects of the 2008 tax stimulus, Parker et al. (2013) added questions about these payments to the questionnaire between June 2008 and March 2009. Due to its original purpose, the CEX provides a finely-grained set of information on the type of goods households consume. This enables us to analyze what kind of goods households with a non-zero MPC spend their rebate on. In the following, we briefly outline the stimulus program and describe the CEX data.

3.1 The 2008 Tax Stimulus Program

Due to the global financial crisis and the subsequent recession, the United States government passed the Economic Stimulus Act (ESA) in February 2008. With projected costs

of more than 150 billion USD, it was the largest relief program passed in the history of the USA up to this point. Next to the stimulus payments, which made up roughly two-thirds of the program, the ESA also enacted other steps meant to provide economic relief, such as enabling government-owned entities (Fannie Mae and Freddie Mac) to buy up more mortgages. However, we only focus on the effects of the stimulus payments.

The rebate was paid out to any household that filed for income taxes. Households that fell beneath the minimum amount of income required to have to file for federal income taxes had to file for taxes anyway and were eligible for the minimum amount of rebate as long as they had a minimum annual income of 3,000 USD. Eligible households received their net tax liability as their rebate; however, the payments were bounded by a minimum of 300 and a maximum of 600 USD. For couples filing jointly, the limits were 600 and 1,200 USD, respectively. Parents of children under the age of 17 received an additional 300 USD per child. Additionally, the rebate was capped for high-income households. The rebate was reduced by 5% of the amount that the reported income exceeded 75,000 USD (150,000 USD for couples), which led the program to target mostly low to medium-income households.

3.2   Consumer Expenditure Survey

The CEX is a representative survey of households in the USA interviewing households about their consumption patterns on a quarterly basis. Once a household is selected to participate, they are interviewed a total of five times. The first interview is a baseline interview during which some general household characteristics, employment-related variables, and their stock of non-durable goods are documented.[2] The next four interviews are administered every three months, and households are asked to document their expenditures over the period since the last interview. The final interview collects data on global financial variables such as amounts saved in savings or checking accounts, which we use as our measures for liquidity. After this interview, the household is rotated out of the CEX and replaced with a new one. Hence, each month of data documented in the CEX contains a different set of households as new ones are added, and others are rotated out of the survey. Figure 1 is taken from the CEX website and illustrates this procedure. Note that a household is defined as a Consumer Unit (CU), which can represent either the number of blood or legally related persons living in one household (e.g., foster children), a single individual - even if living with other people as long as the individual is financially independent - or unrelated people who are pooling their income. All information about a Consumer Units' members is collected regarding their relationship to the reference person. This person is defined as the one named when asked who rents or owns the home. For

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[2]The baseline interview has only been conducted until 2015. Since then, the first interview has covered these questions.

Figure 1: CEX quarterly rotation procedure

Interview year and month		Interview set			
		1	2	3	4
2015	APR	a			
	MAY	b			
	JUN	c			
	JUL	d	a		
	AUG	e	b		
	SEPT	f	c		
	OCT		d	a	
	NOV		e	b	
	DEC		f	c	
2016	JAN			d	a
	FEB			e	b
	MAR			f	c
	APR				d
	MAY				e
	JUN				f
	JUL				
	AUG				
	SEPT				

Columns show number of interview and a letter signals a specific household. Source: `https://www.bls.gov/opub/hom/cex/data.htm`

personal traits such as age, we follow the convention by Parker et al. (2013) and take the average of the characteristic of all CU members.

It is important to highlight the limitations set by the usage of CEX data. As mentioned, the main objective of the CEX is to assess what goods the average household consumes to create the goods basket for inflation measurements. This focus results in a lack of interest in dense documentation of household characteristics and income-related variables. For example, the lack of asking for liquidity-related measures in each quarter prevents us from controlling for changes in liquidity, but we can only control for households' overall self-reported levels of liquidity. Also, the variables collected are only crude measures for liquidity.

While this is a disadvantage in comparison with other data sources, the CEX's richness in information on consumption behavior is unmatched. Keeping in mind the risk of measurement error through the self-reported consumption measurement, the CEX enables us to analyze not only the MPC for overall consumption but to dissect it and see which goods drive responses and heterogeneity seen in higher level estimates.

4 Methodology

To estimate the causal effect of tax rebate receipt on changes in consumption, we use the Double Machine Learning (DML) framework developed by Chernozhukov et al. (2017). This new kind of estimation approach allows to efficiently estimate semi-parametric models of treatment effects using Machine Learning methods. The semi-parametric approach

we follow has the major advantage that it does not restrict the effect of confounders on the outcome to a specific functional form. Moreover, specific DML estimators enable us to estimate heterogeneity given observables without defining in which form the observable affects the treatment effect. Past contributions that were looking into heterogeneity had to rely on choosing the correct interactions with observables. Sophisticated DML estimators can detect these interactions without knowing them beforehand. Meanwhile, its implementation procedure deals with common biases arising in more naive estimation procedures that employ Machine Learning methods. This opens the door to combining powerful machine learning algorithms with causal inference. Many ML estimators, such as Random Forests or Neural Nets, have proven as valuable assets in detecting complex patterns in data.

From a more theoretical perspective, the DML estimator yields very efficient properties when it comes to its asymptotic behavior. Under certain assumptions, Chernozhukov et al. (2017) are able to prove root-n consistency of the estimator, a rate of convergence not achieved in other Machine Learning based estimation approaches. However, we will not further elaborate on these details and refer the reader to Chernozhukov et al. (2017) for a more technical discussion. Instead, we focus on the general idea behind the DML framework and the different estimation methods we use in our analysis.

4.1 Setup

We start with considering a Partially Linear Model of treatment and outcome

$$Y_{it} = \theta(X_{it})D_{it} + g(X_{it}, W_{it}) + \epsilon_{it} \tag{1}$$

$$D_{it} = h(X_{it}, W_{it}) + u_{it}, \tag{2}$$

where Y_{it} is the outcome, D_{it} is the treatment and X_{it} and W_{it} are observable variables. We distinct between simple confounders W_{it} which affect the outcome and also potentially the treatment and X_{it}, which additionally are considered to impact the treatment effect of D_{it} on Y_{it}. The choice of these variables is left to the researcher. We also assume that $E[\epsilon_{it}|X_{it}, W_{it}] = 0$ and $E[u_{it}|X_{it}, W_{it}] = 0$.

We are interested in $\theta(X)$, the conditional average treatment effect (CATE). In Rubin's potential outcomes framework (**citation missing**) it is defined as

$$\theta(X) = E[Y_1 - Y_0|X = x]$$

where Y_d is the outcome when treatment is $D = d$. In our setting, treatment is not binary but continuous, hence $\theta(X)$ represents the marginal CATE

$$\theta(X) = E\left[\left.\frac{\delta Y(d)}{\delta d}\right| X = x\right].$$

The marginal CATE measures how much a marginal increase in the continuous treatment changes the outcome for individuals that have a set of characteristics $X = x$. Note that in our setting, we assume that the CATE is linear in treatment, i.e., the treatment effect is independent of the size of the treatment. The task is now to find an appropriate estimator $\theta(X_{it})$.

4.2 Regularization bias and how to get rid of it - alternative title: A quest to avoid biases

As Chernozhukov et al. (2017) point out, we could come up with some seemingly straightforward approach to estimate the PLM using machine learning methods. For example, approximating the function $g(X, W)$ with a high polynomial and using a Lasso regression for regularization or using a combination of random forests for predicting $g(X, W)$ and then an OLS regression to find $\theta(X)$. However, any machine learning-based approach that follows this notion will suffer from a bias due to regularization. To avoid overfitting and the resulting large variance of the estimator, machine learning methods deliberately induce a bias into their predictions. This bias does not vanish asymptotically, leading to inconsistent results.[3] However, we can deal with this regularization bias using orthogonalization. For this, we define

$$E[Y_{it}|X_{it}, W_{it}] \equiv f(X_{it}, W_{it}) \tag{3}$$

$$E[D_{it}|X_{it}, W_{it}] \equiv h(X_{it}, W_{it}) \tag{4}$$

where (4) follows from (2). It is straightforward to estimate these conditional means using any ML method of choice. Using these and the PLM defined above, we can find

$$Y_{it} - f(X_{it}, W_{it}) = \theta(X_{it})(D_{it} - h(X_{it}, W_{it})) + \epsilon_{it}. \tag{5}$$

Subtracting the conditional means from Y and D is known as orthogonalization and removes the impact of X and W on them, respectively. The residuals only contain variation that does not stem from any of the confounders. In Section 5.1 we discuss what this means in our setting in more detail. Indeed, the estimate of $\theta(X)$ retrieved from estimating the orthogonalized PLM in (5) is no longer suffering from the regularization bias. Excitingly,

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[3]See Appendix X.X (or only the paper?).

the authors are able to prove that even in case that the first stage estimators of $\hat{f}$ and $\hat{h}$ are converging at slower rates than root-n to the true parameter value, in the final estimator the regularization bias converges and the estimation error converges to zero at a potential rate of root-n.

In practice, the first stage of the estimation process consists of choosing an appropriate Machine Learning method, predicting the conditional expectation functions f and h and calculating residuals

$$\tilde{Y}_{it} = Y_{it} - \hat{f}(X_{it}, W_{it})$$
$$\tilde{D}_{it} = D_{it} - \hat{h}(X_{it}, W_{it}).$$

A welcome property of the DML estimation is its agnostic to the first stage estimator. Thus, it allows choosing the appropriate prediction method for the given setting.

4.3 Cross- against Overfitting

While orthogonalization takes care of the regularization bias plaguing more naive ML-based estimators, it implicitly induces a new bias. Machine Learning estimators are prone to overfitting models. Instead of picking up signals in features to predict the outcome, they start interpreting noise in the training data we feed them. To avoid this behavior, one can tune hyperparameters of the algorithm of choice to minimize this issue. Still, it is not unlikely that noise in the data is interpreted as a signal.

While orthogonalization takes care of the regularization bias plaguing more naive ML-based estimators, it implicitly induces a new bias. Machine Learning methods are often prone to overfit, i.e., they start interpreting noise in the data as signals from observables. However, this same individual level noise is contained in the structural error terms of the PLM, ϵ_{it} and u_{it}. Thus, our predictions of f and h are not independent of these. Using the orthogonalized outcomes and treatments to estimate $\theta(X)$ then leads to terms such as $u_{it}(\hat{f}(X_{it}, W_{it}) - f(X_{it}, W_{it})$ to show up in the estimation error $\hat{\theta}(X) - \theta(X)$. The dependence of the structural errors and the prediction errors - both driven by the individual level noise - are then not vanishing asymptotically. Similar to the regularization bias, this lets the asymptotic variance of the estimator explode and prohibits any convergence. However, it is rather easy to resolve this issue using sample splitting - a procedure called "crossfitting."

Instead of using all observations to find the estimates of f and h and then estimate $\theta(X)$ using the whole sample, consider the case in which we split the sample into two. The first sample is used to retrieve the first stage predictions. Those are used to predict the conditional means of the second sample, which are then subsequently used for orthogonalization and the second stage estimation. Since noise is independent across individuals, the

noise affecting the first stage prediction error and the structural errors coming into play in the second stage estimation are independent as well. It is then easy to show that terms leading to problems when using the whole sample are vanishing asymptotically now. In case we are interested in the unconditional average treatment effect (ATE), this procedure is repeated with the role of the samples reversed and the resulting estimators averaged. However, in the CATE case, we are interested in individual-level point estimates. Therefore, while the role of both samples is switched, we do not average any results but keep the individual-level estimates of all observations. The cross-fitting procedure for splitting up the sample into any K folds is described in Algorithm 1, which summarizes the whole DML estimation procedure.[4]

4.4   Retrieving the CATE

After retrieving the residualized outcome and treatment, the second stage estimates the conditional average treatment effect as defined in (6). We assume that it takes the following form

$$\theta(X) = \phi(X) \times \Theta, \tag{6}$$

where Θ is the baseline treatment effect of each individual and $\phi(X)$ is a mapping of our controls X. The form of the latter depends on the estimator chosen for the second stage. In Chernozhukov et al. (2017) estimators are proposed which have a linear second stage, either using a standard OLS estimator or Lasso to regress $\tilde{Y}_{it}$ on $\tilde{D}_{it}$. In these cases, the second stage boils down to a linear regression in which the residualized outcome is regressed on interactions $\tilde{D}_{it}$ and each element of X_{it}. This implies that the treatment effect we estimate is linear in the covariates X. It is also possible to include polynomials of or interactions between different elements of X_{it}. However, we choose a simple linear mapping of X for our linear DML approach presented in Section 5. To identify nonlinearities in the CATE, we use a nonparametric approach that allows us to uncover these without defining them beforehand. Namely, we use a Generalized Random Forest estimator introduced by Athey et al. (2018). It has been developed to take advantage of the powerful random forest predictor for causal inference. Similar to DML, the GRF is an estimation framework. The GRF replaces the original objective function of the random forest algorithm (Breiman, 2001) with a moment condition containing some loss function that can be defined by the researcher. When using it for moment conditions such as (7) to identify conditional average treatment effects, the GRF is also known as a Causal Forest, which is presented in earlier work by Athey and Wager (2016). The Generalized

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[4]Note that Chernozhukov et al. (2017) argue that K=4 or K=5 performs reasonably well, even for smaller samples.

Random Forest framework allows for causal inference as Athey et al. (2018) develop the theory that allows retrieving standard errors of the estimated coefficients. Appendix A elaborates in more detail how the Causal Forest algorithm works and how it identifies the treatment effect. In our case, the moment condition is defined as

$$E\left[\left(\tilde{Y} - \theta(X) \times \tilde{D}_{it} - \beta(x)\right) \times (\tilde{D}_{it}; 1)\right] = 0 \tag{7}$$

where we choose the CATE $\theta(X)$ and constants $\beta(x)$ to solve it. The causal forest non-parametrically estimates $\theta(X)$ and therefore puts no assumption on the form of the mapping $\phi(X)$. The term $(\tilde{D}_{it}; 1)$ represents a matrix consisting of the vector of orthogonalized treatments and ones to capture the constant effects.

As part of our analysis we will compare the results to check whether the relationship is indeed linear or whether we discover non-linear heterogeneities that the linear DML approach does not account for and have not been considered in the literature yet. However, note that when using a nonparametric second stage the convergence rate of the estimator declines. This implies that the Causal Forest based approach is more demanding when it comes to the number of observations.

This has to look better and be more 'algorithmic'.

Algorithm 1 Double Machine Learning Estimator

1: Split up sample into K folds.
2: To estimate $\widehat{h}$ and $\widehat{f}$ for for the kth fold use observations $j \notin k$.
3: To get residuals for observations in k, calculate $\widehat{h}(X_i)$ and $\widehat{f}(X_i, W_i)$ for $i \in k$ and use to retrieve residuals.
4: Once residuals of each fold retrieved, estimate $\theta(X_i)$.
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5 Estimation and Results

We investigate the heterogeneity of the Marginal Propensity to Consume by estimating the following partially linear model

$$\Delta C_{it+1} = \theta(X_{it})R_{it+1} + g(X_{it}, W_{it}) + \epsilon_{it} \tag{8}$$

$$R_{it} = h(X_{it}, W_{it}) + u_{it} \tag{9}$$

where our outcome of interest is the change in consumption between two quarters, ΔC_{it} and our treatment is the rebate amount household i receives. The choice of confounders X_{it} and W_{it} depends on the specification we estimate, as does which variables we consider to be part of X_{it} and thus have an effect on the treatment effect. Which variables are included in each specification is listed in Table X.X. We follow Parker et al. by including

monthly dummies to account for seasonality and to capture any unobserved effects that might appear in periods in which households learn about the upcoming rebate. By canceling these effects stemming from the anticipation of the treatment, our estimate represents the effect of actually receiving the rebate.

In total, we distinguish between three different levels in our estimations: we investigate different outcomes ΔC_{it} by using the rich information on expenditure categories included in the CEX. With the term 'specifications,' we distinguish between the different sets of confounders X and W we use. Lastly, we estimate each outcome-specification pair twice: once using the linear and once using the causal forest second stage. Since our estimation procedure predicts MPCs and we retrieve standard errors, we can run hypothesis tests on whether the estimated response to the tax rebate is statistically significant for each household.

However, one drawback in our specifications, including liquidity, salary, and income, is the already mentioned lack of detailed documentation of household characteristics in the CEX. Our sample size shrinks because they are not consistently documented for each interviewed household. This sample reduction can induce a sample selection bias because it is possible that households that answer questions on their liquidity are systematically different from households that provide information on these measures. Although the DML framework achieves fast convergence rates even in cases in which the first stage predictions do not converge as rapidly, we have to keep this drawback in mind.

5.1 Identifying the Income Shock

Since we use the same data and event to estimate the MPC, our identification is based on the approach by Parker et al. (2013). The main factor is the design of the stimulus rollout, which we can exploit to identify the income shock. The tax stimulus was paid out to households over several weeks as administrative and technological restrictions made it impossible to pay out all rebates at once. Instead, the date of rebate receipt depended on the last two digits of tax filers' social security number. These digits are randomly distributed, and therefore the timing of the treatment is random, rendering it exogenous from any household characteristics. Therefore, we observe rebate receipts at different points in time, which allows us to use all other households that received their rebate in a different quarter as the control group.

In the following, we depart from Parker et al.'s identification strategy given their findings as well as our inclusion of more control variables. They argue that using the actual amount of tax rebate received can lead to an omitted variable bias. This concern arises because of how households' stimulus payments are determined. Remember that the tax rebate directly depends on the number of children, which certainly affects the absolute

level of households' expenditures, as each dependent child adds 300 USD to the stimulus received. However, this is not a problem because we - as Parker et al. - control for the number of children in each specification. The stimulus excluding the child bonuses equals the household's net income tax liability (NTL; in the following, also referred to as the net tax liability) as long as it is within the exogenously defined boundaries we discussed in Section 3.1. Parker et al. argue that the NTL might also drive changes in consumption, rendering the treatment endogenous. Their solution is to instrument the amount received with a dummy variable that only signals whether the stimulus was received or not in the given quarter. While their results and the authors themselves suggest that this is not much of a concern, we decisively disagree with their identification approach. Parker et al. do not control for any variable related to households income or salary. These variables are without a doubt directly connected to our treatment because the NTL - i.e., how much a household owes in income taxes - is a function of the household's income. Excluding these variables leads to omitted variable biases causing inconsistent estimates. However, other than through the channel of income, we deem it highly unlikely that the net tax liability itself is driving changes in consumption. It might be possible that in other years the NTL plays a role for households' income as it can be perceived as an anticipated income - or liquidity - shock.[5]However, in 2008, the NTL affected households via their tax rebate, i.e., it does not affect the consumption change through other channels than what is captured by the tax rebate. Therefore, we argue in favor of using the actual rebate amount since it has two advantages: for one, we have an additional source of variation, and second, it allows us to estimate the continuous treatment effect and interpret it as the actual MPC.

5.2   Main Results

still missing some details on comparison between causal forest and linear model: Even in spec 1 negative MPCs found, there are less significant results and significant MPCs spread out more; in spec2 the cf picks up a ; and missing food expenditures

We analyze our results in several steps and begin by looking at the empirical distribution of the estimated MPCs. Figure X.X shows the distribution of MPCs for the four main expenditure categories considered by Parker et al.: Food (FD), Strictly Non-Durables (SND) as defined by Lusardi (1996), Non-Durables (ND), and Total (TOT) expenditure. These categories are increasing in their level of aggregation, e.g., SND includes expenditures on food. A detailed list of all sub-components of each of these categories is listed in Appendix X.X. Here, we only want to point out that the difference between SND and ND consumption categories are so-called 'semi-durables,' such as health expenditures, which

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[5]Households usually should know that they will have to pay this/receive this because of past experience and because the NTL also depends on how much income tax was already paid during the previous year.

are not included in the SND category.

A single plot of the empirical distribution in Figure X.X is retrieved as follows. We slice the range between the minimum and maximum of the point estimates into 20 equidistant bins and calculate the share of estimated MPCs that fall into each bin. The x-axis signals the borders of the different bins, and the y-axis shows the respective frequency. The blue part of a bar signals the total frequency of this bin. To illustrate how many of these estimates are actually rejecting the null of a zero MPC, we calculate the share of point estimates that reject the null at the 10% level within each bin. This is depicted by the red part of the frequency bars. I.e., a completely red bar implies that all observations within this bin are statistically significant, whereas a bar that is only red up to half of its height signals that only half of the point estimates within this bin are statistically significant. The vertical dashed line marks the average CATE - the average treatment effect across all households - as a benchmark. The plot description notes whether this ATE is significant or not.

Overall, we find strong support for heterogeneity in the Marginal Propensity to Consume. Plots in Figure X.X show a large variation in households' MPC. This underlines the importance of accounting for heterogeneous responses to income shocks. The heterogeneity is similar to what Kaplan and Violante's theoretical model suggests and Misra and Surico's empirical findings. Our results show a large mass of households having a Marginal Propensity to Consume that is distributed around 0 and for many households, we cannot reject the null hypothesis of a zero MPC. On the other hand, there is a smaller share of households that show strong, significant responses. Table X.X depicts the shares of significant MPCs we estimate for each specification and model when we look at changes in non-durable consumption.[6]

Meanwhile, the ATE is very close to zero in all expenditure categories (except TOT). We see that it falls into bins that have the highest frequency - which makes sense by construction - but across all estimations the respective bin never contains more than 15% of all point estimates. This highlights the weak representativeness of the ATE and its inability to reliably assess the success of programs such as the 2008 tax stimulus. Accounting for heterogeneity reveals positive effects of the stimulus program on household spending that exceed the average responses a lot. In the case of ND and SND goods, our MPCs are very similarly distributed in the linear model with significant reactions ranging from roughly 0.12 to around 0.6, where the maximum MPCs found are slightly lower for the SND category. However, once we control for liquidity, the linear model shows no more significant responses.

The similarity of our results to Misra and Surico's results depends on our specification and

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[6]Significance shares for the other four main outcomes discussed in this section are reported in Appendix A.A.

choice of estimator. Using the linear DML, we find similar results as MS in Specification 1 across all expenditure categories. The most significant MPCs for ND, SND, and FD goods are between 0.5 and 1; however, our point estimates for changes in total expenditure are largely exceeding the estimates presented by Misra and Surico. In general, the TOT responses are very strong and, in part exceeding 1. This is probably due to the underlying composition of the total expenditure variable and our estimation approach. It is quite likely that some outliers within a specific spending category part of TOT are driving the learning behavior of our estimators **here mention outliers in vehicle purchases**. This is underlined by the fact that the causal forest estimator finds way larger responses than the linear-based estimator. Nonparametric machine learning estimators often struggle to deal with not seen before outlier combinations of outcome and confounders.

Also, it is important to note that in both estimators, the spread of the CATE is drastically reduced once we control for liquidity, salary, and income - variables we expect to be closely related to the MPC. Turning to the significance of the response, we see that adding more controls also reduces the number of significant MPCs found, suggesting that prior specifications pick up signals of the confounding factors not included and interpret them as signals of the rebate.

This notion becomes more visible when we turn to non-durable and strictly non-durable goods. Excluding large, durable categories such as new vehicles immediately reduces our estimated MPCs to ranges between 0.12 and 0.6. Moreover, looking at the two categories, we see that in specification 3, which includes liquidity, salary, and income, the linear model fails to reject the null for all households. Interestingly though, the causal forest model still finds a small fraction of large significant MPCs. This difference suggests that there are nonlinearities in the dependence of the MPC on the variables such as liquidity - and potentially with respect to their interactions - that are ignored by the linear model but detected by the causal forest. Accounting for these nonlinearities reduces the noise in the point estimates and reveals significant MPCs where the linear analysis fails to pick up any significant MPCs. In general, the causal forest results show an even larger heterogeneity of MPC than the baseline linear model. We see that the spread of estimated MPCs is substantially larger for estimates returned by the causal forest compared to the linear model. This hints at the fact that the linear model not including any interactions and nonlinearities in the CATE reduces the precision of the estimates.

Lastly, we want to address the negative MPCs we find throughout almost all estimations. The MPC is bounded by zero at its lower bound by the definition of the concept. However, in our setting, negative estimates are still possible, as Kaplan and Violante point out. This relates back to their argument that when using the tax stimulus, we estimate a 'rebate coefficient' and not necessarily the MPC. However, the rebate coefficient can very well be negative, as they show in estimations using their calibrated two-asset model.

In their model, they explain the heterogeneous response to the 2001 tax stimulus by the government by distinciting between hosueholds that are welathy but only hold illiquid assets (wealthy hand-to-mouth) and households that have no liquidity and hold no illiquid assets (poor hand-to-mouth). **This is a sentence for the literature review; instead here only relate to this** In their two-asset model, the households holding illiquid assets have to pay transaction costs to increase their holdings of the illiquid asset. Kaplan and Violante show that when these transaction costs are relatively low compared to the size of the income shock, households will choose to pay the costs and make a deposit once they receive the payment resulting in a negative effect on contemporaneous consumption.

6 Understanding the roots of heterogeneity

In the previous section, we discussed the conditional average treatment effect of each individual given their specific set of characteristics. Prior contributions have looked at the correlations between the significance of the estimated MPC and households characteristics to get a glimpse into which factors play a role in the MPC. However, this approach does not reliably tell us which variables really drive the response. The correlation might very well be spurious or driven by other factors that are correlated with the characteristic we are looking at. Therefore, it is more fruitful to look at measures that can help us identify what role a variable plays in our predicted MPCs. In the case of specifications using the linear DML estimator, we know that this relationship is linear by construction since the CATE is defined as a linear combination of the single effects of interactions between treatment and the respective variable (see equation (6)). However, the causal forest-based approach will help us reveal whether there are any non-linear patterns underlying the effect of characteristics on the MPC without assuming any functional form of these patterns.

For this, we turn to the Machine Learning literature, which has developed a number of tools to analyze the relationship between prediction and feature. Feature is a different term for control variables used in the Machine Learning literature. In our setting, these are the variables we condition on to find the CATE, i.e., variables contained in X. Since variables in W are assumed to not impact the CATE, they are not contained in the second stage and therefore play no role in predicting individuals' MPC. Machine Learning estimators such as random forests are black boxes as they only provide predictions but stay quiet on which variables are important to arrive at this prediction. The literature has proposed multiple approaches that help quantify the role of a single feature, some of which we look at in the following.

6.1 Marginal and Partial Dependence Plots

this section is quite long for something I do not show, right?

Two popular approaches are marginal plots (M-Plots) and Partial Dependence Plots (PDPs; Friedman, 2001). Both use the same general idea to quantify the impact of some feature x_S on our predictions: we replace the value of x_S of each observation with some value v_1. Then we fit our trained prediction model to this "counterfactual" dataset and take the average over all these predictions. For example, we predict for each individual what their MPC looks like if they had a certain age and average the predicted MPC. Then we continue with $x_S = v_2$ and so on, where the values v_j are chosen from a grid along the distribution of x_S. The difference between M-Plots and PDPs is the distribution of all other features $X_C = X \setminus x_S$ we average over. In the case of Marginal Plots, contrary to what the name might suggest, we use the conditional distribution of X_C given x_S, $p_{X_C|x_S}$, to obtain the impact of x_S on our prediction. On the other hand, PDPs use the marginal distribution of X_C, p_{X_C}. Partial Dependence Plots are more common in the Machine Learning literature as M-Plots suffer from a severe weakness when features in X_C are correlated with x_S. However, PDPs also fail to reliably uncover the effect of x_S in such a setting.

To illustrate the issues arising in M-Plots and PDPs when features are correlated, let us consider a simple example. Let's say we have some predictive model m that only depends on two predictors x_1 and x_2, which are positively correlated. To now calculate the M-Plot of x_1 at v_1 we use the conditional distribution $p_{x_2|x_1}$. In practice, we plug in $x_1 = v_1$ for each observation that is within a specified neighborhood of $x_1 = v_1$ (e.g. observations in the same quantile). Then we predict and average to obtain the M-Plot value at $x_1 = v_1$. Repeating this procedure for other values v_j then results in the M-Plot of x_1. However, because the two variables are correlated, we do not know which variable drives the observed effect - if x_1 is increased, the values of x_2 we use for our predictions also increase because we only use x_2 of observations that are close to having $x_1 = v_j$. This problem is known as 'conflation.'

On the other hand, Partial Dependence Plots do not suffer from this problem because they use the marginal distribution of x_2. We use all observations of x_2 instead of only looking at a neighborhood in which $x_1 = v_1$ and, therefore, average out the effect of x_2 on our predictions. Since we use the same set of x_2 values at each point v_j, we know that changes in our predictions must stem from x_1. Still, the PDPs are not a good tool when features are correlated, and this is connected to the machine learning estimators we apply them to. These are nonparametric estimators that are usually very weak in predicting outcomes based on observations they have never seen before. This extrapolation, however, becomes necessary when we create the "counterfactual" dataset by setting $x_1 = v_j$. By doing so, we effectively create observations that are extremely unlikely or even impossible

to observe in the real world because of the correlation between the features. For example, in our data, age and salary are strongly correlated, which is quite intuitive because once retired, households do not receive a salary anymore. When creating PDPs, we ignore this fact and create households that have a high salary and are very old. The weakness in extrapolation leads the model to create imprecise predictions, which then severly bias the Partial Dependence Plots. (Apley and Zhu, 2020)

Therefore, while PDPs do not suffer from theoretical drawbacks like M-Plots, in practice, they are unable to uncover the effects of x_1 on our predictions in a stable manner because of the underlying predictive estimator. If the true model is indeed linear and we use a linear prediction method with the correct specifications of any interaction terms etc., then this extrapolation issue is unlikely to occur. Moreover, by construction, a linear predictor will result in linear Partial Dependence Plots.

6.2   Accumulated Local Effects

To circumvent issues arising in M-Plots and PDPs from correlated features, Apley and Zhu (2019) propose Accumulated Local Effects (ALE). The extrapolation issue PDPs suffer from is bypassed by using the conditional distribution $p_{X_C|x_S}$ as we do in M-Plots. The 'conflation' effect that results from this is tackled by not using average predictions at $x_S = v_j$ but rather the average marginal change in predictions at this point. In other words, we use the partial derivative of our predictor in question m with respect to x_S at the point v_j. Although many machine learning methods such as tree-based learners have no concept of a gradient, Apley and Zhu are able to derive proofs for non-differentiable functions m (see Section X.X in Apley and Zhu), and further does this not play a role when it comes to the ALE estimation. The ALE is then defined as

$$\hat{m}_{S,ALE}(x_S) = \int_{z_{0,S}}^{x_S} E_{X_C|X_S=x_S}[\hat{m}^S(X_S, X_C)|X_S = z_S]dz_S - constant, . \quad (10)$$

Looking at this equation step-by-step reveals how the ALE recovers the effect of x_S on our predictions even when features are correlated. As already mentioned, the ALE avoids 'conflation' by using the partial derivative of m, where we have $m^S = \frac{\partial m}{\partial x_S}|_{x_S=v_j}$ as the partial derivative of m evaluated at the point we want to find the ALE for. Since we only look at an infinitesimally small change, this change in x_S will not affect the features that are correlated with it in X_C unless the correlation is extremely high. In our analysis, we would want to avoid this case anyways to avoid problems in the estimation itself (e.g., multicollinearity). Once the changes in prediction are obtained for each observation, we average them over the conditional distribution, i.e., only using observations that are within a neighborhood of $x_S = v_j$ and actually exist. Now we have the average local effect of x_S on our prediction. To better visualize the global role of this feature, Apley and Zhu

argue that this can be achieved by accumulating all local effects up to $x_S = v_j$. Thus, we simply integrate overall local effects up to $x_S = v_j$, where $z_{0,S}$ is the lower bound of the distribution of x_S.[7]

To estimate the ALE we use the following estimator, which illustrates the procedure in more intuitive terms:

$$\hat{\tilde{m}}_{j,ALE}(x_S = v_j) = \sum_{k=1}^{k_j(x_S=v_j)} \frac{1}{n_j(k)} \sum_{i:x_{i,j} \in N_j(k)} [m(z_{k,j}, x_{i,\backslash j}) - m(z_{k-1,j}, x_{i,\backslash j})] \qquad (11)$$

The intuition behind the estimator is straightforward. First, we bin our data into n_b bins based on quantiles of the distribution of x_S. To mimic the marginal change represented by the partial derivative m^S in 10 we make two predictions for each individual. For an observation i that falls in bin k, we predict its outcome with $x_S = z_k$ and $x_S = z_{k-1}$, where z_k represents the upper bound value of bin k. We then average overall individuals that fall within this bin k denoted in equation (11) by the neighborhood $N_j(k)$. Finally, we accumulate all predicted preferences over all bins up to the bin at which point x falls, denoted by k(x). Only looking at individuals within the neighborhood $N(k)$ accounts for the conditional distribution used in 10.

As the last step, Apley and Zhu propose to center the effect around the average of all ALEs such that the mean effect is zero. Thus, the ALE has to be interpreted relative to the average prediction, and it shows whether for a given $x_S = v$ the effect of x_S is above or below the average prediction. I.e., whether x_S affects our predictions at $x_S = v$ more than it does on average. In practice, the *constant* in (10) is replaced by

$$\frac{1}{n} \sum_{i=1}^{n} \hat{m}_{j,ALE}(x_{i,j}).$$

Summarizing, the ALE shows the effect of a Note that we yet cannot say something meaningful about the statistical significance of these results. Most fields are only interested in the predictive power of machine learning methods and in understanding how these predictions are achieved, but there is no notion of statistical significance in these settings. Therefore, a specific approach to quantify the uncertainty of these measures has not yet been developed. However, a deeper look into this topic is out of the scope of this work. To briefly dive into the topic of statistical significance, we use a bootstrapping-based approach. We simulate the ALEs for $n_{bootstrap}$ samples. These create an empirical distribution on which basis we can calculate a pseudo Confidence interval using the reverse percentile approach (Davison and Hinkley, 1997, p. 194 eq 5.6). We report these

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[7]For more on this, see Section 5.2 of Apley and Zhu (2019) where they demonstrate how accumulation helps to improve the interpretability of ALE plots.

as the red lines surrounding the ALE in Figures X.X. We see that these bands are very wide in certain parts - especially in areas where there is a small number of observations. We strongly encourage a deeper investigation of the statistical properties of ALEs and a potential way of quantifying their uncertainty in a more rigorous way. While the ALEs show us the relationship between a specific variable and our predictions, having a sound theoretical foundation on which basis we could assess the statistical significance of these relationships is desirable.t.

6.3 Results

We now turn to the analysis of the Accumulated Local Effects of a selection of features used in our estimated specifications. As pointed out in Section 6.1, by the construction of the estimator and how the ALE is calculated, it will always depict a linear relationship when looking at the linear DML setting. However, we can still infer in what direction the relationship is going - e.g., whether predictions are above or below average for young people. More importantly, it is useful as a benchmark to compare the ALE of our estimates using the causal forest as the second stage estimator.

Two main channels of MPC heterogeneity discussed in the literature are the role of age and liquidity. We start by investigating the role of age and plot the ALE for households' age with respect when using changes in non-durable consumption as the outcome in Figure X.X. Two things become evident right away: First, the linear model finds that young people have a substantially lower MPC, while older households experience stronger reactions. Second, the relationship more or less breaks down once we control for liquidity, salary, and income. This is observed across all three main consumption categories. While the relationship actually turns around for strictly non-durables and total consumption, the non-durables still seem to associate a positive relationship between age and MPC. However, we see that the deviations from the average predictions decline drastically and are almost zero. Taking a look at the difference between SND and ND categories, we see that in the case of health expenditures, the MPC has a positive relationship with higher age that actually strengthens once we include liquidity, income, and salary (see Figure X.X in Appendix A.A). The effect seems to be strong enough to keep a linearly increasing relationship between MPC and age in the ND consumption category, while this does not appear in the SND category. In the case of total expenditure, the effect does not seem to be strong enough compared to the overall direction of the relationship between MPC predictions and age.

Still, we have established in Section 5.3 that the causal forest estimator reveals more significant MPCs in specification 3, where we control for liquidity, which is likely to occur because of nonlinearities not picked up in the linear CATE model. Instead of a clear linear

relationship, the ALE plots for AGE when using the causal forest estimator reveal a quite more varying relationship. Similar to the linear model, the overall structure of the relationship in specifications 1 and 2 is similar across all three main consumption categories. However, the magnitude of the ALEs varies a lot. In the case of total expenditures, we see large effects, while they decline more and more once we reduce the number of goods categories considered. Additionally, it is clearly visible that the ALEs for the causal forest model are, in part, varying widely. Mostly, the 95 and 5 percentiles of our bootstrapped ALEs are so wide that we cannot boldly state that the effects are positive or negative as our pseudo-CIs include zero along most parts of the age distribution. While this is not a valid statement on any hypothesis testing, it hints at a rather unstable relationship between age and the MPC. Once we turn to specification 3, the CIs become much more narrow but still include zero. Moreover, the widely varying ALEs are more closely fluctuating around zero. The only range where we find CI bounds above zero is in the case of TOT expenditures. This effect vanishes once we look at the less aggregated measures. Thus, we take a closer look into the sub-categories that are only included in TOT. We have to stress that these are not causal relationships we establish between any of these ALEs, but we only try to infer directions from which the effects we find in aggregated measures might stem.

Summarizing the ALEs of age, it is reasonable to say that the linear model fails to account for the correct relationship between age and MPC, while our findings support that households at the upper end of the age distribution experience higher MPCs out of the rebate shock than on average. As we have laid out in Section 6.1, we cannot make any substantial claims on whether these reactions are significantly different from zero but rather only look at the role age plays for our MPC predictions. In our case, this means that a higher age implies that a household's MPC will be larger than the average MPC of other households.

The main channel identified in the literature so far is liquidity. Our discussion of the underlying theory of binding borrowing constraints and lacking access to liquid assets provides the intuition for the following analysis. We consider the change in non-durable consumption here but note that this pattern is evident across all main consumption categories. ALE plots for these can be found in Appendix A.A. Figure X.X provides the ALE plots for specification 3, which includes liquidity, income, and salary for both estimation procedures.

Using the linear estimator, we find that the predictions rise in liquidity; however, the deviations calculated are very small - remember though we cannot test whether they are significant. The causal forest estimations are more informative. Here we see a strong spike in low levels of liquidity that falls as rapidly as it rises once liquidity is sufficiently large (around 5000). This underlines the important role of liquidity documented in the

literature and by our results presented in where we have seen that controlling for and conditioning on liquidity has a large impact on the MPCs and their significance. The ALE now provides further hints on what the role of liquidity might look like. We see that for very low levels of liquidity, the reaction of households is even below average, while once it is slightly above zero, we see that there is a steep jump in the ALE, which more or less declines immediately again at increasing levels of liquidity. This potentially signals that households with no liquid assets at all will actually not react more strongly than the average household. Since we find that the average household does not significantly react, we infer from this that households with no liquidity are not reacting to the income shock. In our data, income and salary are positively correlated, i.e., households with low liquidity also have low income. Thus, it is reasonable to assume that for these households, the rebate checks make up a larger share of their total income within this quarter. Given the times of economic hardship for many of these households during the time of the survey, it is possible that these households did not spend a significant amount all within one quarter but stretched out their expenditures over several months, using the rebate checks as fall back savings. However, this is only one interpretation of our results, and our data is too limited to infer robust causal relationships and the existence of such channels.

The reaction for low liquidity households is in line with our expectations from the liquidity channel we discussed. As soon as households have enough liquid assets, it seems that they are capable of borrowing to smooth out the income shock before it arrives and thus shows no significant reaction in consumption.

Last, we would like to discuss the effect of income and salary on the predicted MPC. As we can see from the ALEs in Figure X.X (they display the causal forest estimates for spec 3 of chNDexp for salary and income), we observe that salary and income have a strongly negative impact on the predicted MPCs.

6.4   Takeaways from a policy-perspective

From a policy perspective, it might not be of interest on which exact sub-category of consumption people spend their rebate on - at least when being interested in providing a stimulus to the economy overall. Still, our analysis reveals useful information for making stimuli more targeted to be more effective or to get a general sense of what people spend additional income on, given their characteristics. Although natural experiments such as the 2008 tax stimulus and connected analysis have mostly little external validity outside of their context, the heterogeneity analysis provides a hint on spending patterns and reactions of households given their personal characteristics and financial circumstances. This can at least be a starting point when designing more targeted transfer programs.

7 Conclusion

(from intro 2 draft but doesn't fit that much anymore) Our contribution is twofold: for one, we estimate the conditional MPC out of the tax stimulus in the most precise and rigorous manner thus far. Second, we use an estimator that exploits the power of machine learning methods for causal inference and contribute to the wider understanding and promoition of this method among applied researchers. Machine Learning predictors are powerful tools when it comes to handling large data and/or complex relationships between variables without any specification of those.

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