Detecting Heterogeneity in the MPC: A Machine Learning Approach

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1 Introduction

How do households respond to income shocks and how do their responses differ given their personal characteristics and economic circumstances? These questions are not only at the centre of a wide academic debate in economics but also of major importance for policy makers. While the former revolves around verifying or neglecting the main mechanisms of the Permanent Income Hypothesis (PIH), the latter are interested in improving government transfers to more efficiently use public funds. These two sides have sparked many investigations using a wide array of approaches to quantify hosueholds' responses to income shocks - the Marginal Propensity to Consume (MPC). At the centre of macroeconomics since Keynes introduced it at the heart of his General Economic Theory, it quantifies how much households will spend on consumption of each dollar they receive from an income shock. While research has long focused on testing whether the MPC out of income shocks is zero in general and thus in line with the PIH, the literature has shifted its focus over the last decade. Most studies support the notion of an average zero MPC, but more recent evidence suggests that for specific groups the response is significantly different.

Empirical research related to the MPC and its heterogeneity has used several settings to identify income shocks. One of the most prominent is to use natural experiments in which households receive exogenous income shocks. Following Parker et al. (2013) and Misra and Surico (2014), we exploit the 2008 tax rebate in the USA to estimate households' MPC using data collected by the Consumer Expenditure Survey (CEX). Similar to these two prior studies, we are able to use the rich information on consumption the CEX provides to not only identify heterogeneities in the overall MPC but also to analyse which categories of consumption goods households spend their rebate on. However, our econometric approach sets us apart as it is more sophisticated and more precise in detecting heterogeneities compared to any contribution we are aware of.

We use the Double Machine Learning Framework (DML) developed by Chernozhukov et al. (2017) to estimate individual level point estimates of the MPC as well as standard errors for each household. This enables us to run hypothesis tests on whether a household's MPC is statistically significantly different from zero. The DML allows us to estimate the conditional average treatment effect (CATE) of the tax rebate on changes in consumption. Meanwhile, thanks to the semi-parametric nature of the DML framework we can use reliable Machine Learning models to control for any confounding factors without having to define their relationship with the outcome. Moreover, one of the two estimators we employ retrieves the CATE without assuming a specified relationship between variables we condition on (X) and the CATE itself. I.e. we do not assume that the CATE is linear in variables we condition on but let this relationship unspecified.

Our results underline the heterogeneity of the Marginal Propensity to Consume out of the tax stimulus documented in Parker et al. and Misra and Surico. We find the a large mass of households shows no significant reaction upon receiving the stimulus payment, whereas a smaller fraction of households shows strong and significant reactions above an estimated MPC of 0.5. Our analysis suggests that liquidity is indeed the main driver of MPC heterogeneities and that low liquid households are the ones reacting most sharply. Contrary to prior work, our estimated CATE does not rely on specifiying subsets of the data across which we assume heterogeneity to exist. We employ modern methods to quantify the effects of single variables on the estimated MPCs to understand the role of these characteristics. Our non-linear estimators suggest the heterogeneities presented in other work are imprecise. Next to our contribution to the MPC literature by providing a empirically more robust analysis, we also see our contribution in introducing modern and flexible estimation approaches to the macroeconomic literature. Frameworks such as the DML offer a gateway to new methods and identifications in the macroeconomic literature. We stress the importance of further research into the theoretical and applied nature of these procedures and their usage in more settings.

It is important to highlight that the Economic Stimulus Act in 2008 was signed into law by President Bush in February 2008. The tax rebate payments, which were part of this policy, started in April of the same year and are therefore an anticipated income shock. This becomes relevant when investigating the role of various factors, especially liquidity. Also, the tax rebate was disbursed to US taxpayers during a time of national and global economic downturn. Many households receiving the stimulus might have been in economic turmoil when receiving the payment and actually spend it to cover regular expenses that they otherwise would not have been able to cover (e.g. rent, utilities or other necessities of daily life). However, Parker et al. (2013) emphasise that some rebates were reported to be received outside of the disbursal window, which suggests that the income shocks might not have been anticipated and only noticed after their arrival.

The fine-grained consumption data of the CEX allows us to identify what kind of goods households consumed and what they spent their stimulus money on. As Kaplan and Violante (2014) note, the tax stimulus is anticiapted and is subject to these special circumstances. Therefore, one might also speak of our estimated coefficients as a 'Propensity to consume the rebate' or 'rebate coeffcient', which is not necessarily equivalent to households' overall MPC. While a government stimulus program might not be perfectly appropriate to verify theoretical models concerned with the MPC, providing evidence on their effect on individuals is of major importance for future policy making. When economic relief is urgent, non-targeted stimuli can present a viable option, targeted transfers can play a major role in many policy settings. Thus, understanding what households spend government transfers on and what households actually use them for consumption, is an

important part of efficient policy-making. Additionally, quantifying the effectiveness of untargeted transfers is necessary to assess whether they are actually helpful to boost the economy. Aggregate estimates of the MPC suggest that this is not the case, but taking a closer look and adjusting for household characteristics reveals heterogeneities and effectiveness of these transfers.

The rest of the paper is structured as follows: Section 2 summarizes the theoretical and empirical literature on MPC heterogeneity. Section 3 discusses the data source and challenges connected with it. The empirical methodology we use is described in Section 4, while Section 5 presents the identification and estimation results of the MPC. We further investigate sources of heterogeneity in responses in Section 6. Section 7 concludes.

2 Literature Review

use the channels as starting point of literature review, use 3 defined in Jappelli and Pistaferri, 2010, and then relate to the two ways of empirical studies, Parker and Souleles (2019) characterize; Parker et al. (why heterogeneity investigation bad method) and Misra and Surico (why QR based on critical assumption that is very unreasonable and not discussed by them) at the end of this section The two main channels identified to drive MPC heterogeneity are life-cycle dynamics and liquidity. The former is driven by a consumer's age and the associated fluctuation in income. As data consistently shows (Attanasio and Weber, 2010), consumption expenditures follow a hump shape over the life-cycle rather than being roughly constant as we would expect under the PIH. This anomaly is often referred to as the retirement-consumption puzzle and connected to an increased amount of free time which allows households to reduce the cost of their consumption. Therefore, we would expect a reduction in the measured MPC in age.

In the case of liquidity, its role is linked to the nature of the income shock and borrowing constraints. If a positive income shock is anticipated, households that are already close to or at their borrowing constraint cannot borrow new funds to smooth consumption in anticipation of a higher future income. Thus, once the shock realises, we will observe an increase in consumption. On the other hand, saving is always possible for any household and hence we will not see a reaction once the shock realises in case of a negative anticipated shock. Thus, more liquid households react less to a positive anticipated shock in comparison with liquidity constrained ones. In contrast, in case of an unanticipated shock, we expect the opposite. Think of an agent that is temporarily out of work and has no liquid wealth at their disposal. In case of a negative shock, the agent is forced to adjust their consumption behaviour downward. Meanwhile, a positive shock will always be saved and stretched over future periods, no matter the level of households' liquidity.

E.g. Bunn et al. (2018) document this asymmetry depending on the sign of the shock.
The literature investigating the size of the MPC and potential heterogeneity can broadly be categorized into three different strands. The first one uses quasi-experimental settings to exploit variation in income to estimate households' MPC. The second uses surveys that explicitly question participants about their MPC - be it out of actual or hyopthetical income shocks. Lastly, a vast literature focuses on building sophisticated macroeconomic models that are calibrated to match real world data and subsequently estimate the MPC agents experience in these models. Our work falls into the first of these categories.
In this section we briefly summarize the findings in all three and additionally discuss two studies - Parker et al. (2013) and Misra and Surico (2014) - in more detail as they investigate MPC heterogeneity using the same data as we do.
Quasi-experimental settings appear all the time in the real world, e.g. in case of a specific policy being implemented or another exogenous shock happening. Researchers interested in MPC heterogeneity focus on shocks that alter the income of a household. For example, Fagereng et al. (forthcoming) use panel data from Norwegian administrative data on winners of a state lottery in which most citizens participate. Receiving a payment from the lottery can be seen as an unanticipated income shock because the chances of winning are so low. They find that households winning the lottery spend almost half of their win within one year and 90% after 5 years. Moreover, liquidity and age are the only variables correlated with the MPC, providing evidence for the existence of the liquidity and age channels. In similar vein, Golosov et al. (2021) construct a dataset of lottery winners in the USA to estimate their MPC and labor market response. They make use of tax forms provided by the lottery winners and general income tax statements. Their main goal is to estimate the labor market responses to windfall gains in unearned income but their strategy allows them to identify the MPC as well. Using a Difference-in-Difference estimator, their estimated MPC is around 60ct out of each dollar earned on average, while labor earnings are reduced by 50ct. To investigate heterogeneity in these responses, the authors split their sample based on the quartile along the liquidity distribution. Further supporting the liqudity channel, they find that households in the highest quartile spend only 49ct while the lowest quartile spends almost 80ct of each dollar won in the lottery. However, these two lottery-based approaches suffer from the drawback that they do not measure consumption directly. Instead they have to either construct consumption out of households balance sheet data (Fagereng et al. (forthcoming)) or model consumption as a function of their observed variables (Golosov et al. (2021)).
Gelman et al. (2018) use the government shutdown in the U.S. as a transitory liquidity shock. Hence, contrary to other literature they only estimate how liquidity changes the consumption behavior and not the MPC directly. Still, their setup allows them to disentangle the pure effect a liquidity shock has on consumers spending as government

workers receive a payback of their wage once the government shutdown is over. Hence, there are no changes in expected income. Meanwhile, studies using income shocks cannot quantify what effect stems from the liquidity channel and what stems from changes in expected income. Their findings highlight that low liquid households react more to a negative liquidity shock as they have no assets to fall back on. Low liquid government workers started postponing their credit card payments, while simultaneously increasing the amount spend using them. **probably only add very short inside of this as not so much related to raw MPC and little/bad heterogeneity investigation**

The second strand of literature uses survey data from field surveys that question households about potential or actually relaized income shocks and how their reaction looks like. Bunn et al. (2018) use (**use twice here**) data collected by the Bank of England to assess the asymmetry that we expect in households' reaction depending on the sign of the income shock. As mentioned before, the liquidity channel suggests that an unanticipated shock calls for a stronger reaction if its negative. Indeed, the authors are able to provide ample evidence for such a reaction with their estimated MPC out of a negative shock being being between 5 to 12 times as high as the reaction to a positive shock. The balance sheet data their source provides also enables them to show that borrowers show a more pronounced asymmetry and reaction, which is also in line with the theoretical mechanisms of the liquidity channel. This also holds for households that face some kind of liquidity constraint. Additionally, Bunn et al. are capable of replicating their estimated MPCs in a model with households at the borrowing constraint. These findings are further underlined by Christelis et al. (2019) who use Dutch data for a similar study. Summarizing these studies strongly suggest the existence of the mechanisms related to households at their borrowing constraint and precautionatry saving motives (**the latter must be elaborated on in intro**).

at end of section & combine with Parker et al.: One major issue in the existing literature is the way how heterogeneity is measured. Most studies rely on either splitting their sample into smaller sub samples and estimating the MPC within each sample or use dummy variables that are defined by the authors based on some continuous variable. These approaches suffer from the severe issue that any heterogeneity that does not fall into this pre-defined pattern is not captured and will muddy the results of these investigations. In the worst case these procedures miss to pick up existing heterogeneity or missing patterns within these pre-defined subgroups. On the contrary, our Double Machine Learning (DML) approach allows us to estimate the conditional MPC out of the tax rebate of each individual household. Prior studies have to rely on looking at the correlation between their estimates and characteristics such as liquidity, but our setting enables us to calculate more sophisticated measures that capture the influence of each variable on the MPC.

2.1 2008 Tax Stimulus Studies

In using the 2008 tax rebate as the income shock and data collected by the CEX, we are by default closely related to Parker et al. (2013) and Misra and Surico (2014). The former collaborated with the Bureau of Labor Statistics (BLS) to add specific questions to the CEX and provide the first analysis using this data. Misra and Surico (2014) use the same dataset to assess the MPC out of the the tax stimulus applying Quantile Regression. This supposedly allows them to recover the whole conditional distribution of the MPC. However, there are reasons to doubt this claim, which we discuss further down.

Parker et al. (2013) estimate their rebate coefficient using OLS and Two Stage Least Squares (2SLS) estimators. We lay out their identification strategy in 5.1 and also address their motivation to instrument the amount of tax stimulus. Both estimators only allow for a limited investigation of heterogeneity. Parker et al. propopse interaction terms between tax stimulus received and proxy measures for liquidity. They create dummy variables that signal whether household i falls into the lowest, middle or the highest tercile along the liquidity distribution. However, the cutoffs for the terciles are not chosen based on the distribution of the proxy variable but such that each categroy has roughly the same number of tax rebate recipients within a given quarter. Next to simply splitting the sample based on such categorizations and estimating the MPC within each sample, this interaction based approach is quite common in the literature. However, it suffers from the major drawback that the cutoffs to identifiy specific subgroups are exogenously set by the researcher. This harbors the danger that heterogeneity patterns within these subgroups or across smaller subgroups are impossible to find. Consider the case in which heterogeneity is strongest within the lowest tercile of liquidity. Given the medium size of the liquidity shock, not a large amount of liquid assets is necessary to borrow beforehand and smooth consumption over time. Under the LCPIH, we expect only households with very small amounts of liquid asseets or no access to these at all to not smooth consumption. Therefore, setting the cutoff of liquidity too high for the lowest group can potentially lead to missing out the strongest heterogeneities driven by liquidity.

Finally, we want to briefly address Misra and Surico (2014) and their use of Quantile Regression (QR). QR was developed by Kohnker (FIND CITATION) to estimate the conditional quantile of a distribution and the regression coefficient of variables at this quantile. To do so, QR minimizes the least absolute deviation (LAD) of some $X\beta$ from the outcome instead of the least squares deviation as in OLS or other linear regression methods. The LAD is minimized by choosing coefficients β and to find the coefficients for the τ^{th} quantile of the conditional distribution of the outcome the LAD is weighted with

τ and $1-\tau$, respectively. A more detailed explanation is provided by Misra and Surico (2014) or in Kohnker's XXXX paper in which he introduced the quantile regression.
However, the QR approach by Misra and Surico (2014) suffers from a severe misinterpretation of what the QR coefficients represent as well as what underlying assumptions are made for QR to work out.

Issues with QR

- rank-invariance assumption
- the coefficient in a quantile regression shows how much variable x shifts the τ^{th} percentile of the conditional distribution of y
- problem is not that changing x by one unit moves individuals into a different quantile and therefore this doesn't show change for individual
- changing x does not move individuals away from the conditional quantile
- QR point estimate tells us by how much a one unit change in X changes the value of the τ^{th} quantile of the conditional distribution of Y
- it does not show how much INDIVIDUALS at the τ^{th} quantile react to a one unit change in X
- this is only the case when the rank-preservation/invariance condition holds
- parphrasing Angrist and Pischke in their hallmark book 'Mostly Harmless Econometrics': if the point estimate for a low decile is positive that doesn't mean that individuals with low change in consumption previously experience a strong increase in consumption. Instead it shows us that those in the lowest quantile of the distribution with treatment have a larger change in consumption than those in the lowest quantile of the distribution that have not yet received a rebate. Thus, it does not really identify the marginal propensity to consume because unless we assume rank-invariance, the coefficient doesn't tell us how much individuals (e.g. on average) changed their consumption. The previously described coefficient is not the MPC.

3 Data

We use data collected by the Consumer Expenditure Survey (CEX) that is administered by the Bureau of Labor Statistics. Its main purpose is to provide information on the consumption preferences of US households to adjust the goods basket that is used to calculate various inflation measures (BLS, 2021). However, in an effort to understand the effects of the 2008 tax stimulus, Parker et al. (2013) added questions about these payments to the questionnaire between June 2008 and March 2009. Due to its original purpose, the CEX provides a finely grained set of information on the type of goods households consume. This enables us to analyse on what kind of goods households with a non-zero MPC spend their rebate on. In the following, we briefly outline the stimulus program and describe the CEX data.

3.1 The 2008 Tax Stimulus Program

Due to the global financial crisis and the subsequent recession, the United States government passed the Economic Stimulus Act (ESA) in February 2008. With projected costs of more than 150 billion USD it was the largest relief program passed in the history of the USA up to this point. Next to the stimulus payments, which made up roughly two thirds of the program, the ESA also enacted other steps meant to provide economic relief such as enabling government owned entities (Fannie Mae and Freddie Mac) to buy up more mortgages. However, we only focus on the effects of the stimulus payments.
The rebate was paid out to any household that filed for income taxes. Households that fell beneath the minimum amount of income required to have to file for federal income taxes had to file for taxes anyway and were eligible for the minimum amount of rebate as long as they had a minimum annual income of 3,000 USD **lots of 'minimum' here**. Eligible households received their net tax liability as their rebate, however, the payment were bounded by a minimum of 300 and a maximum of 600 USD. For couples filing jointly the limits were 600 and 1,200 USD, respectively. Parents of children under the age of 17 received additional 300 USD per child. Additionally, the rebate was capped for high income households. The rebate was reduced by 5% of the amount that the reported income exceeded 75,000 USD (150,000 USD for couples), which led the program to target mostly low to medium income households.

3.2 Consumer Expenditure Survey

The CEX is a representative survey of households in the USA interviewing households about their consumption patterns on a quarterly basis. Once a household is selected to participate, they are interviewed a total of five times. The first interview is a baseline

Figure 1: CEX quarterly rotation procedure

Interview year and month		Interview set			
		1	2	3	4
2015	APR	a			
	MAY	b			
	JUN	c			
	JUL	d	a		
	AUG	e	b		
	SEPT	f	c		
	OCT		d	a	
	NOV		e	b	
	DEC		f	c	
2016	JAN			d	a
	FEB			e	b
	MAR			f	c
	APR				d
	MAY				e
	JUN				f
	JUL				
	AUG				
	SEPT				

Columns show number of interview and a letter signals a specific household. Source: `https://www.bls.gov/opub/hom/cex/data.htm`

interview during which some general household characteristics, employment related variables and their stock of nondurable goods are documented.[1] The next four interviews are administered every three months and households are asked to document their expenditures over the period since the last interview. The final interview collects data on global financial variables such as amounts saved in savings or checkings accounts, which we use as our measures for liquidity. After this interview, the household is rotated out of the CEX and replaced with a new one. Hence, each month of data documented in the CEX contains a different set of households as new ones are added and others are rotated out of the survey. Figure 1 is taken from the CEX website and illustrates this procedure. Note that a household is defined as a Consumer Unit (CU), which can represent either a number of blood or legally related persons (e.g. foster children), a single individual - even if living with other people as long as the individual is financially independent - or unrelated people who are pooling their income. All information about a Consumer Units members are collected regarding their relationship to the reference person. This person is defined as the one named when asked who rents or owns the home. For personal traits such as age we follow the convention by Parker et al. (2013) and take the average of the characteristic of all CU members.

It is important to highlight the limitations set by the usage of CEX data. As mentioned, the main objective of the CEX is to assess what goods the average household consumes to create the goods basket for inflation measurements. This focus results in a lack of interest

[1] The baseline interview has only been conducted until 2015. Since then the first interview covers these questions.

in a dense documentation of household characteristics and income related variables. For example, the lack of asking for liquidity related measures in each quarter prevents us from controlling for changes in liquidity but we can only control for households overall self-reported levels of liquidity. Also, the variables collected are only crude measures for liquidity.

While this is a disadvantage in comparison with other data sources, the CEX's richness in information on consumption behavior is unmatched. Keeping in mind the risk of measurement error through the self-reported consumption measurement, the CEX enables us to analyse not only the MPC for overall consumption but to dissect it and see which goods drive responses and heterogeneity seen in higher level estimates.

4 Methodology

To estimate the causal effect of tax rebate receipt on changes in consumption, we use the Double Machine Learning (DML) framework developed by Chernozhukov et al. (2017). This new kind of estimation approach allows to efficiently estimate semi-parametric models of treatment effects using Machine Learning methods. The semi-parametric approach we follow has the major advantage that it does not restrict the effect of confounders on the outcome to a specific functional form. Moreover, specific DML estimators enable us to estimate heterogeneity given observables without defining in which form the observable affects the treatment effect. Past contributions that were looking into heterogeneity had to rely on choosing the correct interactions with observables. Sophisticated DML estimators can detect these interactions without knowing them beforehand.Meanwhile, its implementation procedure deals with common biases arising in more naive estimation procedures that employ Machine Learning methods. This opens the door to combine powerful machine learning algorithms with causal inference. Many ML estimators, such as Random Forests oder Neural Nets, have proven as valuable assets in detecting complex patterns in data.

From a more theoretical perspective the DML estimator yields very efficient properties when it comes to its asymptotic behaviour. Under certain assumptions, Chernozhukov et al. (2017) are able to prove root-n consistency of the estimator, a rate of convergence not achieved in other Machine Learning based estimation approaches. However, we will not further elaborate on these details and refer the reader to Chernozhukov et al. (2017) for a more technical discussion. Instead we focus on the general idea behind the DML framework and the different estimation methods we use in our analysis.

4.1 Setup

We start with considering a Partially Linear Model of treatment and outcome

$$Y_{it} = \theta(X_{it})D_{it} + g(X_{it}, W_{it}) + \epsilon_{it} \tag{1}$$

$$D_{it} = h(X_{it}, W_{it}) + u_{it}, \tag{2}$$

where Y_{it} is the outcome, D_{it} is the treatment and X_{it} and W_{it} are observable variables. We distinct between simple confounders W_{it} which affect the outcome and also potentially the treatment and X_{it}, which additionally are considered to impact the treatment effect of D_{it} on Y_{it}. The choice of these variables is left to the researcher. We also assume that $E[\epsilon_{it}|X_{it}, W_{it}] = 0$ and $E[u_{it}|X_{it}, W_{it}] = 0$.
We are interested in $\theta(X)$, the conditional average treatment effect (CATE). In Rubin's potential outcomes framework (**citation missing**) it is defined as

$$\theta(X) = E[Y_1 - Y_0|X = x]$$

where Y_d is the outcome when treatment is $D = d$. In our setting, treatment is not binary but continuous, hence $\theta(X)$ represents the marginal CATE

$$\theta(X) = E\left[\frac{\delta Y(d)}{\delta d}\middle| X = x\right].$$

The marginal CATE measures how much a marginal increase in the continuous treatment changes the outcome for individuals that have a set of characteristics $X = x$. Note that in our setting we assume that the CATE is linear in treatment, i.e. the treatment effect is independent of the size of treatment. The task is now to find an appropriate estimator $\theta(X_{it})$.

4.2 Regularization bias and how to get rid of it - alternative title: A quest to avoid biases

As Chernozhukov et al. (2017) point out, we could come up with some seemingly straightforward approach to estimate the PLM using machine learning methods. For example, approximating the function $g(X, W)$ with a high polynomial and using a Lasso regression for regularization or use a combination of random forests for predicting $g(X, W)$ and then an OLS regression to find $\theta(X)$. However, any machine learning based approach that follows this notion will suffer from a bias due to regularization. To avoid overfitting and the resulting large variance of the estimator, machine learning methods deliberately induce a bias into their predictions. This bias does not vanish asymptotically, leading to inconsis-

tent results.[2] However, we can deal with this regularization bias using orthogonalization. For this, we define

$$E[Y_{it}|X_{it}, W_{it}] \equiv f(X_{it}, W_{it}) \tag{3}$$
$$E[D_{it}|X_{it}, W_{it}] \equiv h(X_{it}, W_{it}) \tag{4}$$

where (4) follows from (2). It is straightforward to estimate these conditional means using any ML method of choice. Using these and the PLM defined above, we can find

$$Y_{it} - f(X_{it}, W_{it}) = \theta(X_{it})(D_{it} - h(X_{it}, W_{it})) + \epsilon_{it}. \tag{5}$$

Subtracting the conditional means from Y and D is known as orthogonalization and removes the impact of X and W on them, respectively. The residuals only contain variation that does not stem from any of the confounders. In Section 5.1 we discuss what this means in our setting in more detail. Indeed, the estimate of $\theta(X)$ retrieved from estimating the orthogonalized PLM in (5) is no longer suffering from the regularization bias. Excitingly, the authors are able to prove that even in case that the first stage estimators of $\hat{f}$ and $\hat{h}$ are converging at slower rates than root-n to the true parameter value, in the final estimator the regularization bias converges and the estimation error converges to zero at a potential rate of root-n.
In practice, the first stage of the estimation process consists of choosing an appropriate Machine Learning method, predicting the conditional expectation functions f and h and calculating residuals

$$\tilde{Y}_{it} = Y_{it} - \hat{f}(X_{it}, W_{it})$$
$$\tilde{D}_{it} = D_{it} - \hat{h}(X_{it}, W_{it}).$$

A welcome property of the DML estimation is its agnostic to the first stage estimator. Thus, it allows choosing the appropriate prediction method for the given setting.

4.3 Cross- against Overfitting

While orthogonalization takes care of the regularization bias plaguing more naive ML based estimators, it impliclty induces a new bias. Machine Learning estimators are prone to overfitting models. Instead of picking up signals in features to predict the outcome, they start interpreting noise in the training data we feed them. To avoid this behaviour, one can tune hyperparameters of the algorithm of choice to minimize this issue. Still, it is not unlikely that noise in the data is interpreted as a signal.

[2] See Appendix X.X (or only the paper?).

While orthogonalization takes care of the regularization bias plaguing more naive ML based estimators, it implicitly induces a new bias. Machine Learning methods are often prone to overfit, i.e. they start interpreting noise in the data as signals from observables. However, this same individual level noise is contained in the structural error terms of the PLM, ϵ_{it} and u_{it}. Thus, our predictions of f and h are not independent of these. Using the orthogonalized outcomes and treatments to estimate $\theta(X)$ then leads to terms such as $u_{it}(\hat{f}(X_{it}, W_{it}) - f(X_{it}, W_{it})$ to show up in the estimation error $\hat{\theta}(X) - \theta(X)$. The dependence of the structural errors and the prediction errors - both driven by the individual level noise - are then not vanishing asymptotically. Similarly to the regularization bias this lets the asymptotic variance of the estimator explode and prohibits any convergence. However, it is rather easy to resolve this issue using sample splitting - a procedure called "crossfitting."

Instead of using all observations to find the estimates of f and h and then estimate $\theta(X)$ using the whole sample, consider the case in which we split the sample into two. The first sample is used to retrieve the first stage predictions. Those are used to predict the conditional means of the second sample, which are then subsequently used for orthogonalization and the second stage estimation. Since noise is independent across individuals, the noise affecting the first stage prediction error and the structural errors coming into play in the second stage estimation, are independent as well. It is then easy to show that terms leading to problems when using the whole sample are vanishing asymptotically now. In case we are interested in the unconditional average treatment effect (ATE), this procedure is repeated with the role of the samples reversed and the resulting estimators are averaged. However, in the CATE case we are interested in individual-level point estimates. Therefore, while the role of both samples are switched, we do not average any results but keep the individual level estimates of all observations. The cross-fitting procedure for splitting up the sample into any K folds is described in Algorithm 1, which summarizes the whole DML estimation procedure.[3]

4.4 Retrieving the CATE

After retrieving the residualized outcome and treatment, the second stage estimates the conditional average treatment effect as defined in (6). We assume that it takes the following form

$$\theta(X) = \phi(X) \times \Theta, \tag{6}$$

[3]Note that Chernozhukov et al. (2017) argue that K=4 or K=5 performs reasonably well, even for smaller samples.

where Θ is the baseline treatment effect of each individual and $\phi(X)$ is a mapping of our controls X. The form of the latter depends on the estimator chosen for the second stage. In Chernozhukov et al. (2017) estimators are proposed which have a linear second stage, either using a standard OLS estimator or Lasso to regress $\tilde{Y}_{it}$ on $\tilde{D}_{it}$. In these cases, the second stage boils down to a linear regression in which the residualized outcome is regressed on interactions $\tilde{D}_{it}$ and each element of X_{it}. This implies that the treatment effect we estimate is linear in the covariates X. It is also possible to include polynomials of or interactions between different elements of X_{it}. However, we choose a simple linear mapping of X for our linear DML approach presented in Section 5. To identify nonlinearities in the CATE, we use a nonparametric approach that allows us to uncover these without defining them beforehand. Namely, we use a Generalized Random Forest estimator introduced by Athey et al. (2018). It has been developed to take advantage of the powerful random forest predictor for causal inference. Similar to DML, the GRF is an estimation framework. The GRF replaces the original objective function of the random forest algorithm (Breiman, 2001) with a moment condition containing some loss function that can be defined by the researcher. When using it for moment conditions such as (7) to identify conditional average treatment effects, the GRF is also known as a Causal Forest, which is presented in earlier work by Athey and Wager (2016). The Generalized Random Forest framework allows for causal inference as Athey et al. (2018) develop the theory that allows retrieving standard errors of the estimated coefficients. Appendix A elaborates in more detail how the Causal Forest algorithm works and how it identifies the treatment effect. In our case the moment condition is defined as

$$E\left[\left(\tilde{Y} - \theta(X) \times \tilde{D}_{it} - \beta(x)\right) \times (\tilde{D}_{it}; 1)\right] = 0 \tag{7}$$

where we choose the CATE $\theta(X)$ and constants $\beta(x)$ to solve it. The causal forest non-parametrically estimates $\theta(X)$ and therefore puts no assumption on the form of the mapping $\phi(X)$. The term $(\tilde{D}_{it}; 1)$ represents a matrix consisting of the vector of orthogonalized treatments and ones to capture the constant effects.

As part of our analysis we will compare the results to check whether the relationship is indeed linear or whether we discover non-linear heterogeneities that the linear DML approach does not account for and have not been considered in the literature yet. However, note that when using a nonparametric second stage the convergence rate of the estimator declines. This implies that the Causal Forest based approach is more demanding when it comes to the number of observations.

This has to look better and be more 'algorithmic'.

Algorithm 1 Double Machine Learning Estimator

1: Split up sample into K folds.
2: To estimate $\widehat{h}$ and $\widehat{f}$ for for the k^{th} fold use observations $j \notin k$.
3: To get residuals for observations in k, calculate $\widehat{h}(X_i)$ and $\widehat{f}(X_i, W_i)$ for $i \in k$ and use to retrieve residuals.
4: Once residuals of each fold retrieved, estimate $\theta(X_i)$.

5 Estimation and Results

We investigate the heterogeneity of the Marginal Propensity to Consume by estimating the following partially linear model

$$\Delta C_{it+1} = \theta(X_{it}) R_{it+1} + g(X_{it}, W_{it}) + \epsilon_{it} \tag{8}$$

$$R_{it} = h(X_{it}, W_{it}) + u_{it} \tag{9}$$

where our outcome of interest is the change in consumption between two quarters, ΔC_{it} and our treatment is the rebate amount household i receives. The choice of confounders X_{it} and W_{it} depends on the specification we estimate as does which variables we consider to be part of X_{it} and thus have an effect on the treatment effect. Which variables are included in each specification is listed in Table X.X. We follow Parker et al. by including monthly dummies to account for seasonality and to capture any unobserved effects that might appear in periods in which households learn about the upcoming rebate. By cancelling these effects stemming from the anticipation of the treatment, our estimate represents the effect of actually receiving the rebate.

In total, we distinct between three different levels in our estimations: we investigate different outcomes ΔC_{it} by using the rich information on expenditure categories included in the CEX. With the term 'specifications' we distinct between the different sets of confounders X and W we use. Lastly, we estimate each outcome-specification pair twice: once using the linear and once using the causal forest second stage. Since our estimation procedure predicts MPCs and we retrieve standard errors, we can run hypothesis tests on whether the estimated response to the tax rebate is statistically significant for each household.

However, one drawback in our specifications including liquidity, salary and income is the already mentioned lack of detailed documentation of household characteristics in the CEX. Our sample size shrinks because they are not consistently documented for each interviewed households. This sample reduction can induce a sample selection bias because it is possible that households that answer questions on their liquidity are systematically different from households that provide informations on these measures. Although the DML framework achieves fast convergence rates even in cases in which the first stage predictions do not converge as rapidly, we have to keep this drawback in mind.

5.1 Identifying the Income Shock

Since we use the same data and event to estimate the MPC our identification is based on the approach by Parker et al. (2013). The main factor is the design of the stimulus rollout, which we can exploit to identify the income shock. The tax stimulus was paid out to households over several weeks as administrative and technological restrictions made it impossible to pay out all rebates at once. Instead the date of rebate receipt depended on the last two digits of tax filers' social security number. These digits are randomly distributed and therefore the timing of the treatment is random rendering it exogenous from any household characteristics. Therefore, we observe rebate receipts at different points in time, which allows us to use all other households that received their rebate in a different quarter as the control group.

In the following, we depart from Parker et al.'s identification strategy given their findings as well as our inclusion of more control variables. They argue that using the actual amount of tax rebate received can lead to an omitted variable bias. This concern arises because of how households' stimulus payments are determined. Remember that the tax rebate directly depended on the number of children, which certainly affects the absolute level of households' expenditures, as each dependent child add 300 USD to the stimulus received. However, this is not a problem because we - as Parker et al - control for the number of children in each specification. The stimulus excluding the child bonuses equals the household's net income tax liability (NTL; in the following also referred to as the net tax liability) as long as it is within the exogenously defined boundaries we discussed in Section 3.1. Parker et al. argue that the NTL might also drive changes in consumption, rendering the treatment endogenous. Their solution is to instrument the amount received with a dummy variable that only signals whether the stimulus was received or not in the given quarter. While their results and the authors themselves suggest that this is not much of a concern **rephrase this sentence**, we decisively disagree with their identification approach. Parker et al. do not control for any variable related to households income or salary. These variables are without a doubt directly connected to our treatment because the NTL - i.e. how much a household owes in income taxes - is a function of the households income. Exlcuding these variables leads to an omitted variable biases causing inconsistent estimates. However, other than through the channel of income, we deem it highly unlikely that the net tax liability itself is driving changes in consumption. It might be possible that in other years the NTL plays a role for households income as it can be perceived as an anticipated income - or liquidity - shock.[4]However, in 2008 the NTL affected households via their tax rebate, i.e. it does not affect the consumption change through other channels

[4]Households usually should know that they will have to pay this/receive this because of past experience and because the NTL is also depending on how much income tax was already paid during the previous year.

than what is captured by the tax rebate. Therefore, we argue in favor of using the actual rebate amount since it has two advantages: for one, we have an additional source of variation and second it allows us to estimate the continuous treatment effect and interpret it as the actual MPC.

5.2 Main Results

Have to add details on ranges of estimates and how the distributions change in more detail We analyse our results in several steps and begin by looking at the empirical distribution of the estimated MPCs. Figure X.X shows the distribution of MPCs for the four main expenditure categories considered by Parker et al.: Food (FD), Strictly Non-Durables (SND) as defined by Lusardi (1996), Non-Durables (ND) and Total (TOT) expenditure. These categories are increasing in their level of aggregation, e.g. SND includes expenditures on food. A detailed list of all sub-components of each of these categories is listed in Appendix X.X. Here we only want to point out that the difference between SND and ND consumption categories are so-called 'semi-durables', such as health expenditures, which are not included in the SND category.

A single plot of the empirical distribution in Figure X.X is retrieved as follows. We slice the range between the minimum and maximum of the point estimates into 20 equidistant bins and calculate the share of estimated MPCs that fall into each bin. The x-axis signals the borders of the different bins and the y-axis shows the respective frequency. The blue bar signals what the total frequency of this bin is. To illustrate how many of these estimates are actually rejecting the null of a zero MPC, we calculate the share of point estimates that reject the null at the 10% level within each bin. This is depicted by the red overlay over the frequency bars. I.e. a completely red bar implies that all observations within this bin are statistically significant whereas a bar that is only red up to half of its height signals that only half of the point estimates within this bar are statistically significant. The vertical dashed line marks the average CATE - the average treatment effect across all households - as a benchmark. The plot description notes whether this ATE is significant or not.

First, we have a look at global trends across all specifications and expenditure categories before we start taking a closer look at each category and estimation procedure.

We find strong support for heterogeneity in the Marginal Propensity to Consume. Plots in Figure X.X show a large variation in households' MPC. This underlines the importance of accounting for heterogenous responses to income shocks. The heterogeneity is similar to what Kaplan and Violante's theoretical model suggests and Misra and Surico's empirical findings. Namely, our results show a large mass of households having a Marginal Propensity to Consume that is closely distributed around 0 and for many households we

cannot reject the null hypothesis of a zero MPC. On the other hand, there is a smaller share of households that show strong, significant responses. Contrary to Misra and Surico, these shares are smaller and the size of the significant MPCs is also higher. Table X.X depicts the shares of significant MPCs we estimate for each specification and model when we look at changes in non-durable consumption. As illustrated in Figure X.X., Table X.X shows that once we control for liquidity in Specification 3, the number of significant MPCs decline. This is independent of which estimator we use for the second stage as well as from the outcome we look at.

Most importantly though, we find that the ATE is always very close to zero but the individual point estimates show a completely different pattern. We see that the ATE falls into bins that have the highest frequency - which makes sense by construction - but across all estimations the respective bin never contains more than 15% of all point estimates. This highlights the weak representativeness of the ATE and its inability to reliably assess the success of programs such as the 2008 tax stimulus.

Also we see that introducing more controls to our estimation reduces the spread of the point estimates no matter at which outcome and estimator we look at. The change is the most pronounced once we add liquid assets, income and salary as confounders to the estimation.

Curiously at first, the responses we find for total expenditures are unreasonably large for a bulk of individuals. This is probably due to the underlying composition of the total expenditure variable and our estimation approach. It is quite likely that some outliers within a specific spending category part of TOT are driving the learning behavior of our estimators **here mention outliers in vehicle purchases**. This is underlined by the fact that the causal forest estimator finds way larger responses than the linear based estimator. Nonparametric machine learning estimators are often performing weakly when they encounter unusual combinations of confounders and outcomes. Also, it is important to note that in both estimators the spread of the CATE is drastically reduced once we control for liquidity, salary and income - variables we expect to be closely related to the MPC. Turning to the significance of the response, we see that adding more controls also reduces the amount of significant MPCs found; suggesting that prior specifications pick up signals of the confounding factors not included and interpret them as signals of the rebate. Concluding, it seems that our estimation procedure is quite sensitive to extreme outliers in the underlying consumption categories as for total expenditure we find extreme responses as laid out above.

This notion becomes more clearly when we turn to the non-durable and strictly non-durable goods. Excluding large durable categories such as *new vehicles* immediately reduces our estimated MPCs and a range between ... and ... with most significant MPCs ranging around .... Moreover, looking at the ND category we see that in specification

3, which includes liquidity, the linear model fails to reject the null for all households. Interestingly though, the causal forest model still finds a small fraction of large significant MPCs. This difference suggests that there are non-linearities in the dependence of the MPC on the variables such as liquidity - and potentially with respect to their interactions - that are ignored by the linear model but detected by the causal forest. Accounting for these non-linearities reduces the noise in the point estimates and reveals significant MPCs where the linear analysis fails to pick up any significant MPCs. Comparing this to the Strictly Non-Durable category there is little difference in the distribution and significance of the recovered MPCs.

Turning to the comparison of the two estimators, we see that the spread of estimated MPCs is substantially larger for estimates returned by the causal forest compared to the linear model. This hints to the fact that the linear model not including any interactions and non-linearities in the CATE, reduces the precision of the estimates. However, we have to keep in mind that the causal forest can result in more spurious estimates when handling outliers as extrapolating from unseen combinations will lead to imprecise predictions by the causal forest.

Throughout almost all estimations, we find a substantial share of households that show a negative MPC - a concept that is bounded by zero as its lower bound. This relates back to their argument that when using the tax stimulus we estimate a 'rebate coefficient' and not necessarily the MPC. However, the rebate coefficient can very well be negative as they show in estimations using their calibrated two-asset model. In their model, they explain the heterogeneous response to the 2001 tax stimulus by the government by distinciting between hosueholds that are welathy but only hold illiquid assets (wealthy hand-to-mouth) and households that have no liquidity and hold no illiquid assets (poor hand-to-mouth). **This is a sentence for the literature review; instead here only relate to this** In their two-asset model the households holding illiquid assets have to pay transaction costs to increase their holdings of the illiquid asset. Kaplan and Violante show that when these transaction costs are relatively low compared to the size of the income shock, households will choose to pay the costs and make a deposit once they receive the payment resulting in a negative effect on consumption.

6 Understanding the roots of heterogeneity

In the previous section we discussed the conditional average treatment effect of each individual given their specific set of characteristics. Similarly to prior contributions, we also looked at correlations between the significance of the estimated MPC and households characteristics to get a glimpse into which factors play a role in the MPC. However, this approach does not reliably tell us which variables really drive the response. The

correlation might very well be spurious or driven by other factors that are correlated with the characteristic we are looking at. Therefore, it is more fruitful to look at measures that can help us identify what role a variable plays in our predicted MPCs. In case of specifications using the linear DML estimator, we know that this relationship is linear by construction since the CATE is defined as a linear combination of the single effects of interactions between treatment and the respective variable (see equation (6)). However, the causal forest based approach will help us reveal whether there are any non-linear patterns underlying in the effect of characteristics on the MPC without assuming any functional form of these patterns.

For this, we turn to the Machine Learning literature, which has developed a number of tools to analyse the relationship between prediction and feature. Feature is a different term for control variables. In our setting these are the variables we condition on to find the CATE, i.e. variables contained in X. Since variables in W are assumed to not impact the CATE they are not contained in the second stage and therefore play no role in predicting individuals' MPC. Machine Learning estimators such as random forests are blackboxes as they only provide predictions but stay quiet on which variables are important to arrive at this prediction. The literature has proposed multiple approaches that help quantify the role of a single feature, some of which we look at in the following.

6.1 Marginal and Partial Dependence Plots

this section is quite long for something I do not show Two popular approaches are marginal plots (M-Plots) and Partial Dependence Plots (PDPs; Friedman, 2001). Both use the same general idea to quantify the impact of some feature x_S on our predictions: we replace the value of x_S of each observation with some value v_1. Then we fit our trained prediction model to this "counterfactual" dataset and take the average over all these predictions. For example, we predict for each individual what their MPC looks like if they had a certain age and average the predicted MPC. Then we continue with $x_S = v_2$ and so on, where the values v_j are chosen from a grid along the distribution of x_S. The difference between M-Plots and PDPs is the distribution of all other features $X_C = X \setminus x_S$ we average over. In case of Marginal Plots, contrary to what the name might suggest, we use the conditional distribution of X_C given x_S, $p_{X_C|x_S}$, to obtain the impact of x_S on our prediction

$$\hat{f}_M(x_S = v_j) = \int p_{X_C|x_S=v_j} m(x_S = v_j, X_C) dX_C, \tag{10}$$

where m is our predictor and $\hat{m}_M(v_j)$ is the effect at $x_S = v_j$. On the other hand, PDPs use the marginal distribution of X_C, p_{X_C},

$$\hat{f}_{PDP}(x_S) = \int p_{X_C} f(x_S, X_C) dX_C \tag{11}$$

where $\hat{m}_{PDP}(v_j)$ is the Partial Dependence of our predictor on x_S at v_j. Using the marginal distribution of X_C effectively "marginalizes out" the effect of any other variables than x_S at some point v and therefore reveals what impact x_S has on our prediction at this point. Partial Dependence Plots are more common in the Machine Learning literature as M-Plots suffer from a severe weakness when features in X_C are correlated with x_S. However, PDPs also fail to reliably uncover the effect of x_S in such a setting.

To illustrate the issues arising in M-Plots and PDPs when features are correlated, let us consider a simply example. Lets say we have some predictive model m that only depends on two predictors x_1 and x_2, which are positively correlated. To now calculate the M-Plot of x_1 at v_1 we use the conditional distribution $p_{x_2|x_1}$. In practice, we plug in $x_1 = v_1$ for each observation that is within a specified neighborhood of $x_1 = v_1$ (e.g. observations in the same quantile). Then we predict and average to obtain the M-Plot value at $x_1 = v_1$. Repeating this procedure for other values v_j then results in the M-Plot of x_1. However, because the two variables are correlated, we do not know which variable drives the observed effect - if x_1 is increased, the values of x_2 we use for our predictions also increase because we only use x_2 of observations that are close to having $x_1 = v_j$. This problem is known as 'conflation'.

On the other hand, Partial Dependence Plots do not suffer from this problem because they use the marginal distribution of x_2. We use all observations of x_2 instead of only looking at a neighborhood in which $x_1 = v_1$ and, therefore, do average out the effect of x_2 on our predictions. Since we use the same set of x_2 values at each point v_j, we know that changes in our predictions must stem from x_1. Still, the PDPs are not a good tool when features are correlated and this is connected to the machine learning estimators we apply them to. These are nonparametric estimators that are usually very weak in predicting outcomes based on observations they have never seen before. This extrapolation however becomes necessary when we create the "counterfactual" dataset by setting $x_1 = v_j$. By doing so, we effectviely create observations that are extremely unlikely or even impossible to observed in the real world because of the correlation the features. For example, in our data age and salary are strongly correlated, which is quite intuitive because once retired, households do not receive a salary anymore. When creating PDPs we ignore this fact and create households that have a high salary and are very old. The weakness in extrapolation leads the model to create weak predictions, which then severly bias the Partial Dependence Plots. (Apley and Zhu, 2020)

Therefore, while PDPs do not suffer from theoretical drawbacks like M-Plots, in practice they are unable to uncover the effects of x_1 on our predictions in a stable manner because of the underlying predictive estimator. If the true model is indeed linear and we use a linear prediction method with the correct specifications of any interaction terms etc., then this extrapolation issue is unlikely to occur. Moreover, by construction, a linear predictor will results in linear Partial Dependence Plots. **Remember that in our linear DML approach, we assume that the CATE we estimate is linear in features X and the second stage - the fitted model we actually investigate here - is simply a linear regression, which results in a linear PDP by construction as our predicted MPC is simply the sum of all coefficients for individual i given their characteristics. → I am not so sure about this part yet**
Indeed, results of the partial dependence plots are rather spurious. They are reported in more detail in Appendix X.X, where we look at the PDPs for non-durable consumption. The effects have a high variation and point estimates out of line of the existing literature. **this is not a good reasoning of why I don't show them because they are too close to the ALEs - I guess**

6.2 Accumulated Local Effects

To circumvent issues arising in M-Plots and PDPs from correlated features, Apley and Zhu (XXX) propose Accumulated Local Effects (ALE). The extrapolation issue PDPs suffer from is bypassed by using the conditional distribution $p_{X_C|x_S}$ as we do in M-Plots. As with M-Plots we use the conditional distribution to bypass the extrapolation issues that PDPs suffer from. The 'conflation' effect that results from this is tackled by not using average predictions at $x_S = v_j$ but rather the average marginal change in predictions at this point. We apply this by using the partial derivative of our predictor m with respect to x_S at the point v_j. Although many machine learning methods such as tree based learners have no concept of a gradient, Apley and Zhu are able to derive proofs for non-differentiable functions m (see Section X.X) and further does this not play a role when it comes to the ALE estimation. The ALE is then defined as

$$\hat{f}_{S,ALE}(x_S) = \int_{z_{0,S}}^{x_S} E_{X_C|X_S=x_S}[\hat{f}^S(X_S, X_C)|X_S = z_S]dz_S - constant, . \tag{12}$$

Looking at this equation step-by-step reveals how the ALE recovers the effect of x_S on our predictions even when features are correlated. As already mentioned, the ALE avoids 'conflation' by using the partial derivative of m, where we have $m^S = \frac{\partial m}{\partial x_S}|_{x_S=v_j}$ as the partial derivative of m evaluated at the point we want to find the ALE for. Since we only look at an infinitesimally small change, this change in x_S will not affect the features that are correlated with it in X_C unless the correlation is extremely high. In our analysis we

would want to avoid this case anyways to avoid problems in the estimation itself (e.g. multicollinearity). Once the changes in prediction are obtained for each observation, we average them over the conditional distribution, i.e. only using observations that are within a neighborhood of $x_S = v_j$ and actually exist. Now we have the average local effect, but we are interested in how x_S affects our predictions and not how it affects changes in predictions. Thus, we simply integrate over all local effects up to $x_S = v_j$, where $z_{0,S}$ is the lower bound of the distribution of x_S.
To estimate the ALE we use the following estimator, which illustrates the procedure in more intuitive terms:

$$\hat{\tilde{f}}_{j,ALE}(x) = \sum_{k=1}^{k_j(x)} \frac{1}{n_j(k)} \sum_{i:x_{i,j} \in N_j(k)} [f(z_{k,j}, x_{i,\setminus j}) - f(z_{k-1,j}, x_{i,\setminus j})] \tag{13}$$

The intuition behind the estimator is straightforward. First, we bin our data into n_b bins based on quantiles of the distribution of x_S. To mimic the marginal change represented by the partial derivative m^S in 12 we make two predictions for each individual. For an observation i that falls in bin k, we predict its outcome with $x_S = z_k$ and $x_S = z_{k-1}$, where z_k represents the upper bound value of quantile k. We then averages over all individuals that fall within this bin k and finally accumulate all predicted differences from the lowest bin up to bin k. Only looking at individuals within a neighborhood $N(k)$ - effectively observations in the same quantile - accounts for the conditional distribution used in 12.
As a last step, Apley and Zhu propose to center the effect around the average of all ALEs such that the mean effect is zero. Thus, the ALE has to be interpreted relative to the average prediction and it shows whether for a given $x_S = v$ the effect of x_S is above or below the average prediction. I.e. whether x_S affects our predictions at $x_S = v$ more than it does on average. In practice, the *constant* in (12) is replaced by

$$averagetermhere$$

.
Note that we yet cannot say something meaningful about the statistical significance of these results. Most fields are only interested in the predictive power of machine learning methods and to understand how these predictions are achieved, but there is no notion of statistical significance in these settings. Therefore, a specific approach to quantify uncertainty of these measures has not yet been developed. A deeper look into this topic is, however, out of the scope of this work. To briefly dive into the topic of statistic significance, we use a bootstrapping based approach. We simulate the ALEs for $n_{bootstrap}$ samples. These create an empirical distribution (need at least e.g. 500-1000) on which basis we calculate pseudo-standard errors. Figure X.X reports the Confidence Intervals

using the reverse percentile approach (see Appendix X.X) and the mean point estimates in each bin. We see that these bands are very wide in certain parts - especially in areas where there is a small number of observations. We strongly encourage a deeper investigation of the statistical properties of ALEs and a potential way of quantifying their uncertainty in a more rigorous way. While the ALEs show us the relationship between a specific variable and our predictions, understanding whether this relationship is statistically significantly different from playing no role would be a major improvement.

6.3 Results

We now turn to the analysis of the Accumulated Local Effects of a selection of features used in our estimated specifications. As pointed out in Section 6.1, by construction of the estimator and how the ALE is calculated, it will always depict a linear relationship when looking at the linear DML setting. However, we can still infer in what direction the relationship is going - e.g. whether predictions are above or below average for young people. More importantly, it is useful as a benchmark to compare the ALE of our estimates using the causal forest as the second stage estimator.

Two main channels of MPC heterogeneity discussed in the literature are the role of age and liquidity. We start by investigating the role of age and plot the ALE for households' age with respect when using changes in non-durable consumption as the outcome in Figure X.X. Two things become evident right away: First, the linear model finds that young people have a substantially lower MPC, while older households experience stronger reactions. Second, the relationship more or less breaks down once we control for liquidity, salary and income. This is observed across all three main consumption categories. While the relationship actually turns around for strictly non-durables and total consumption, the non-durables still seem to associate a positive relationship between age and MPC. However, we see that the deviations from the average predictions decline drastically and are almost zero. Taking a look at the difference between SND and ND categories, we see that in the case of health expenditures the MPC has a positive relationship with higher age that actually strengthens once we include liquidty, income and salary (see Figure X.X in Appendix A.A). The effect seems to be strong enough to keep a linearly increasing relationship between MPC and age in the ND consumption category, while this does not appear in the SND category. In case of total expenditure, the effect does not seem to be strong enough compared to the overall direction of the relationship between MPC predictions and age.

Still, we have established in Section 5.3 that the causal forest estimator reveals more significant MPCs in specification 3, where we control for liquidity, which is likely to occur because of non-linearities not picked up in the linear CATE model. Instead of a clear

linear relationship, the ALE plots for AGE when using the causal forest estimator reveal a quite more varying relationship. Similar to the linear model, the overall structure of the relationship in specification 1 and 2 is similar across all three main consumption categories. However, the magnitude of the ALEs varies a lot. In case of total expenditures, we see large effects, while they decline more and more once we reduce the number of goods categories considered. Additionally, it is clearly visible, that the ALEs for the causal forest model are in part varying widely. Mostly, the 95 and 5 percentiles of our bootstrapped ALEs are so wide that we cannot boldly state that the effects are positive or negative as our pseudo-CIs include the zero along most parts of the age distribution. While this is not a valid statement on any hypothesis testing, it hints to a rather unstable relationship between age and the MPC. Once we turn to specification 3, the CIs become much more narrow but still include zero. Moreover, the widely varying ALEs are more closely fluctuating around zero. The only range where we find CI bounds above zero is in case of TOT expenditures. This effect vanishes once we look at the less aggregated measures. Thus, we take a closer look into the sub-categories that are only included in TOT. We have to stress that these are not causal relationships we establish between any of these ALEs but we only try to infer directions from which the effects we find in aggregated measures might stem.

Summarizing the ALEs of age it is reasonable to say that the linear model fails to account for the correct relationship between age and MPC, while our findings support that households at the upper end of the age distribution experience higher MPCs out of the rebate shock than on average. As we have laid out in Section 6.1 we cannot make any substantial claims on whether these reactions are significantly different from zero but rather only look at the role age plays for our MPC predictions. In our case this means that a higher age implies that a household's MPC will be larger than the average MPC of other households.

The main channel identified in the literature so far is liquidity. Our discussion of the underlying theory of binding borrowing constraints and lacking access to liquid assets provides the intuition for the following analysis. We consider the change in non-durable consumption here, but note that this pattern is evident across all main consumption categories. ALE plots for these can be found in Appendix A.A. Figure X.X provides the ALE plots for specification 3, which includes liquidity, income and salary, for both estimation procedures.

Using the linear estimator, we find that the predictions rise in liquidity, however, the deviations calculated are very small - remember though we cannot test whether they are significant. The causal forest estimations are more informative. Here we see a strong spike in low levels of liquidity that falls of as rapidly as it rises once liquidity is sufficiently large (around 5000). This underlines the important role of liquidity documented in the

literature and by our results presented in Section 5.2 where we have seen that controlling for and conditioning on liquidity has a large impact on the MPCs and their significance. The ALE now provides further hints on what the role of liquidity might look like. We see that for very low levels of liquidity, the reaction of households is even below average, while once it is slightly above zero, we see that there is a steep jump in the ALE, which more or less declines immediately again at increasing levels of liquidity. This potentially signals that households with no liquid assets at all will actually not react more strongly than the average household. Since we find that the average household does not significantly react we infer from this that households with no liquidity are not reacting to the income shock. In our data income and salary are positively correlated, i.e. households with low liquidity also have low income. Thus, it is reasonable to assume that for these households the rebate checks make up a larger share of their total income within this quarter. Given the times of economic hardship for many of these households during the time of the survey, it is possible that these households did not spend a significant amount all within one quarter but stretched out their expenditures over several months, using the rebate checks as fall back savings. However, this is only one interpretation of our results and our data is too limited to infer robust causal relationships and the existence of such channels.
The reaction for low liquidity households is in line with our expectations from the liquidity channel we discussed. As soon as households have enough liquid assets it seems that they are capable of borrowing to smooth out the income shock before it arrives and thus show no significant reaction in consumption.
Last, we would like to discuss the effect of income and salary on the predicted MPC. As we can see from the ALEs in Figure X.X (they display the causal forest estimates for spec 3 of chNDexp for salary and income), we observe that salary and income have a strongly negative impact on the predicted MPCs.

6.4 Takeaways from a policy-perspective

From a policy perspective it might not be of interest on which exact sub-category of consumption people spend their rebate on - at least when being interested in providing a stimulus to the economy overall. Still, our analysis reveals useful information for making stimuli more targeted to be more effective or to get a general sense of what people spend additional income on given their characteristics. Although natural experiments such as the 2008 tax stimulus and connected analysis have mostly little external validity outside of their context, the heterogeneity analysis provides a hint on spending patterns and reactions of households given their personal characteristics and financial circumstances. This can at least be a starting point when designing more targeted transfer programs.

7 Conclusion

(from intro 2 draft but doesn't fit that much anymore) Our contribution is twofold: for one, we estimate the conditional MPC out of the tax stimulus in the most precise and rigorous manner thus far. Second, we use an estimator that exploits the power of machine learning methods for causal inference and contribute to the wider understanding and promoition of this method among applied researchers. Machine Learning predictors are powerful tools when it comes to handling large data and/or complex relationships between variables without any specification of those.

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