Date & time of lab: 04.04.2021 8:15-12:00
Author: Sławomir Batruch

EDISP Lab2

Fourier Transform, DFT, FFT

Task 1

Investigate a single square impulse of 1 ms length, sampled under different conditions

To get f at null I used following formulas:

$$\theta_k = \frac{2\pi k}{N}$$

$$\theta = 2\pi \frac{f}{f_s}$$

So f = (k*fs)/N

case	fs	T	N	N1	A_max	K_null	f_n at null	f at null
x1[n]	1 MHz	2 ms	2000	1000	1000	2	0,001	1000
x2[n]	10 kHz	2 ms	20	10	10	2	0,1	1000
x3[n]	10 kHz	4 ms	40	10	10	4	0,1	1000

Refer to the MATLAB code attached in the .zip file with the report to view realization of the task.

Task 2

Simulate and investigate various signals of 1024 samples and their FFTs

Signals:

(a) a 512 points square impulse (so you need to add 512 zeros to get 1024 samples)

(b) other (narrower) square impulses – fill them up to 1024

(c) sine wave with integer number of periods in window of 1024

(d) sine wave with non-integer number of periods in window of 1024

(e) $e^{jn\theta_c}$ - use `exp()` in Matlab; how many peaks do you see? why? Try different values of $0 < \theta_c \leq \pi$.

Case	No (samples/period)
a	1024
b	128
c	256
d	256
e	4

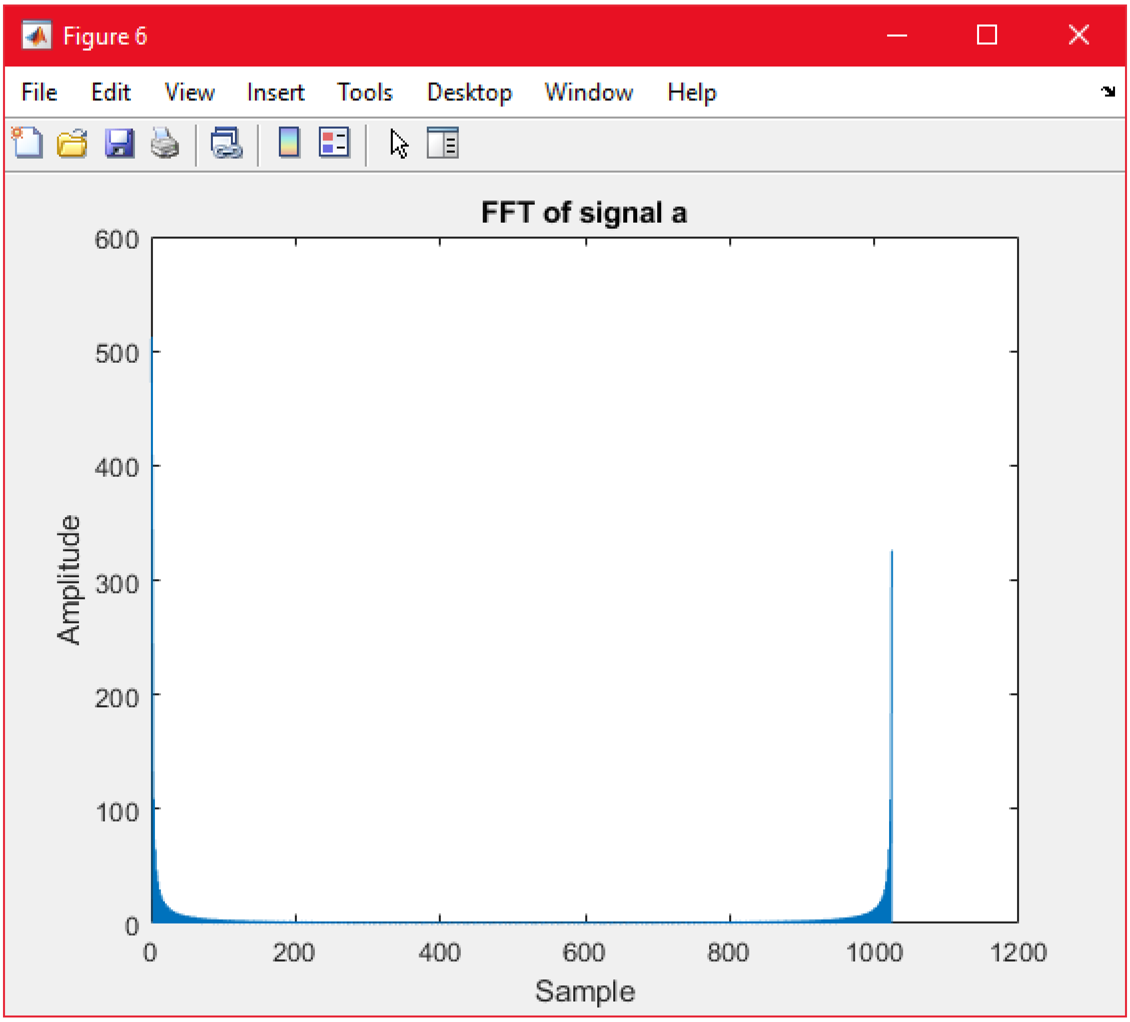
Note b: my own chosen, [ones(1, 64), zeros(1,64)] so 128 for one period
Note c: my own chosen time = 0.5 and f = 8 Hz, with fs = 2048 Hz yields 1024 samples, 4 periods, which results in 256 samples/period
Note e: result plot is rhombus which has 4 points

c signal fft peaks: x: 1021 y: 512; x: 5 y: 512
d signal fft peaks: x: 1006 y: 506; x: 5 y: 506
For signal d the peaks are wider than for signal c.

Zero points in a = 511; zero crossings in b = 959

For the MATLAB script code, refer to the .zip file

Plots of all FFTs:



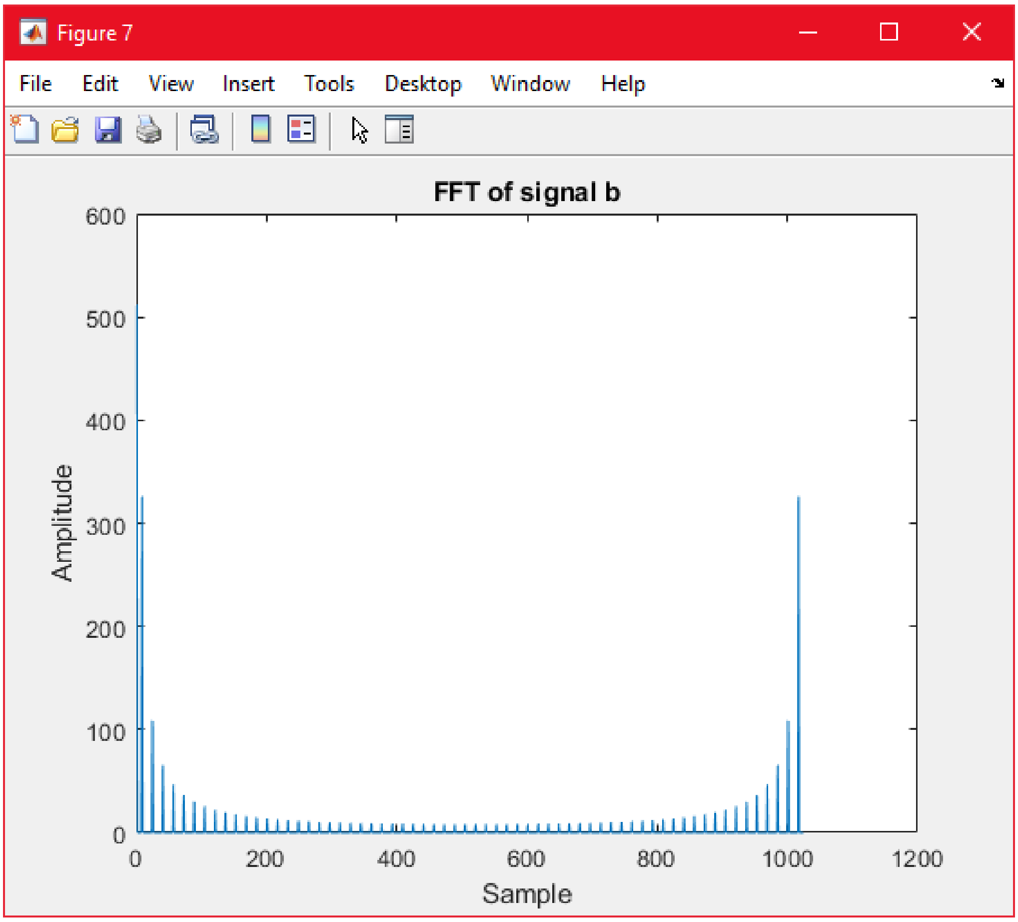


Figure 7
File Edit View Insert Tools Desktop Window Help
FFT of signal b
Amplitude
Sample
0 100 200 300 400 500 600
0 200 400 600 800 1000 1200

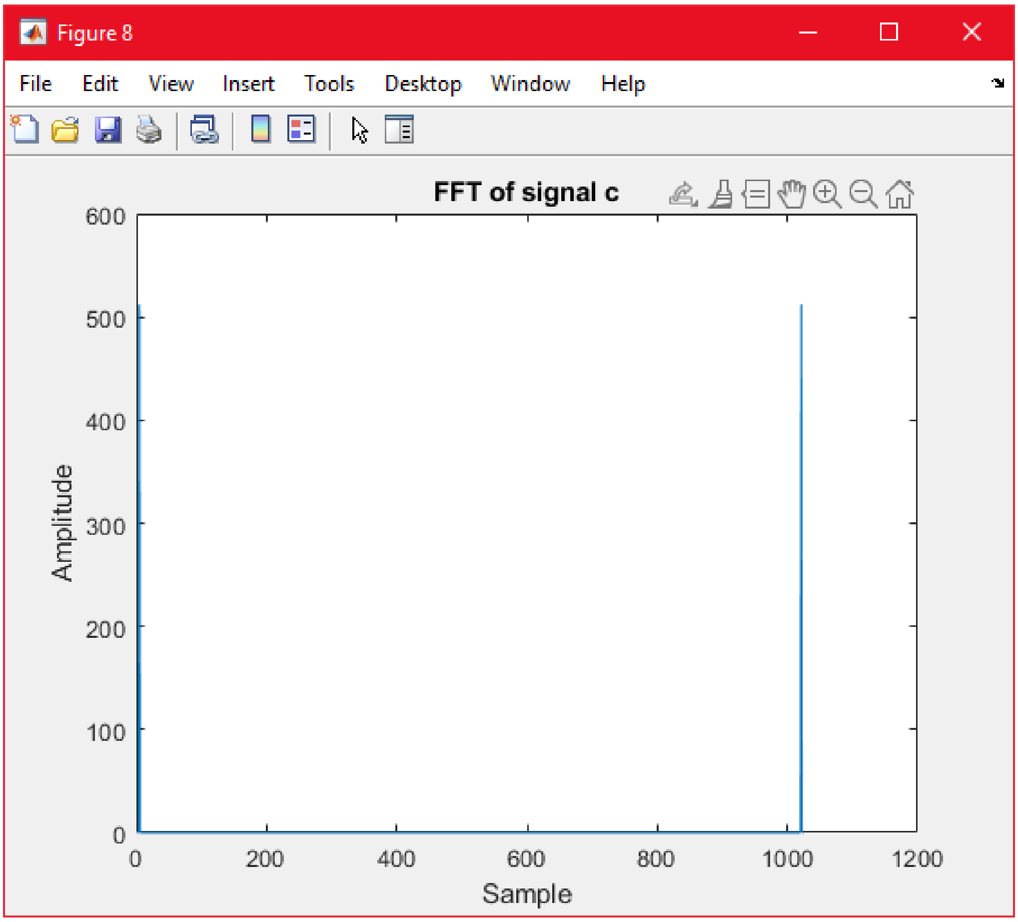


Figure 8
File Edit View Insert Tools Desktop Window Help
FFT of signal c
Amplitude
Sample
0 100 200 300 400 500 600
0 200 400 600 800 1000 1200

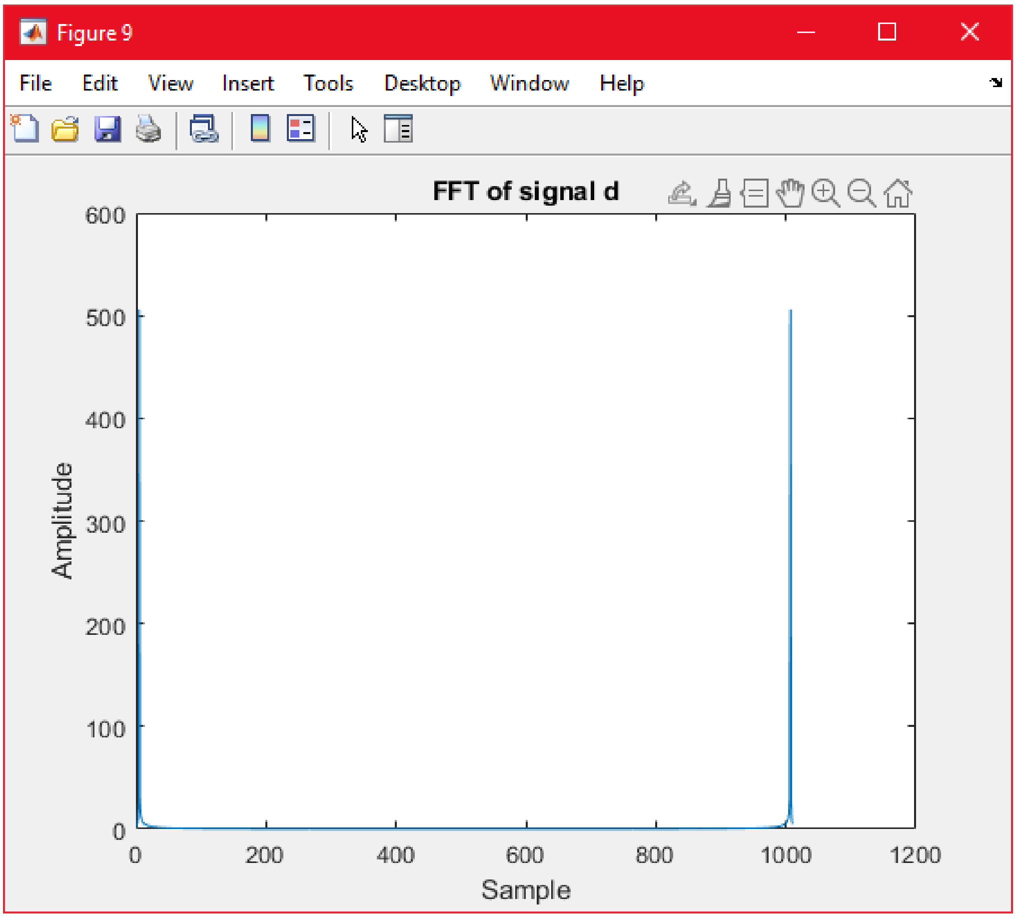


Figure 9
File Edit View Insert Tools Desktop Window Help
FFT of signal d
Amplitude
Sample
0
100
200
300
400
500
600
0
200
400
600
800
1000
1200

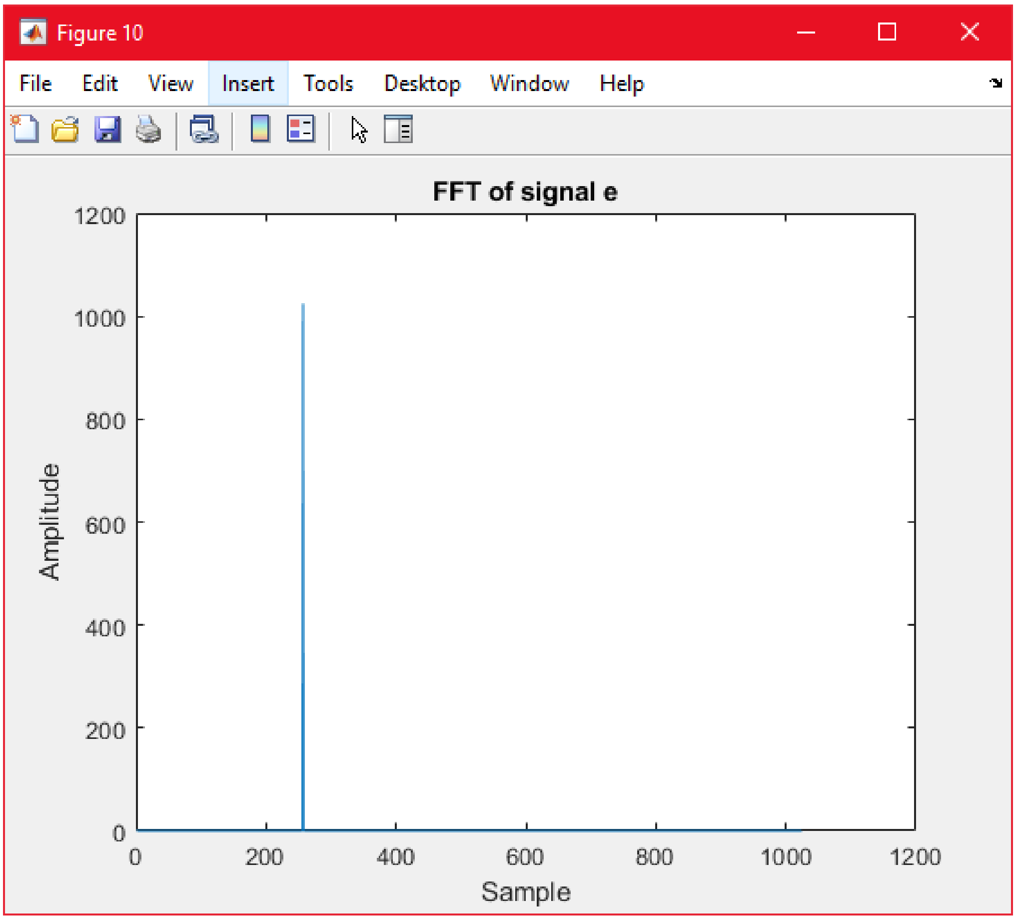


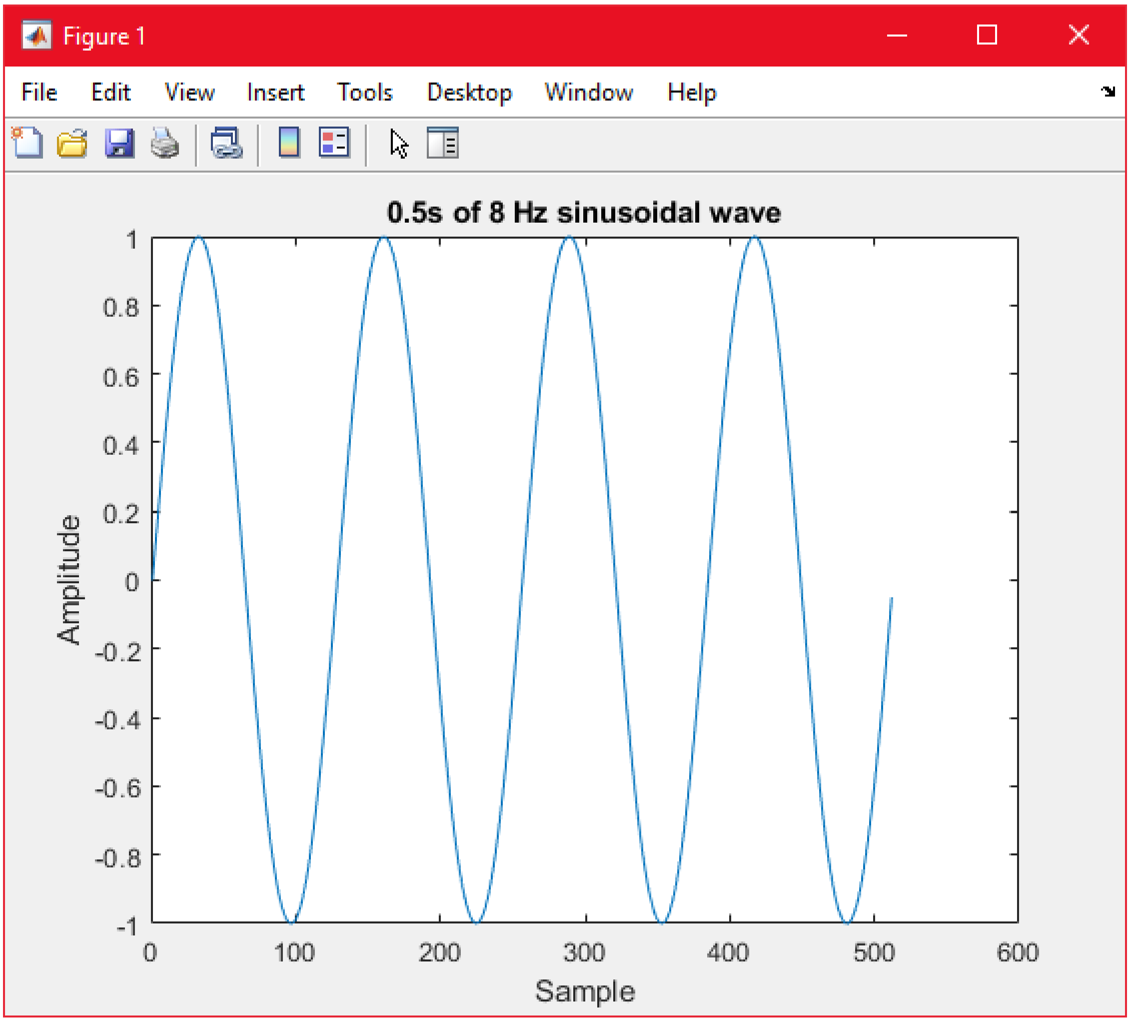
Figure 10
File Edit View Insert Tools Desktop Window Help
FFT of signal e
Amplitude
Sample
0
200
400
600
800
1000
1200
0
200
400
600
800
1000
1200

Task 3

Analyze sine wave spectrum and zero-padding impact on FFT and iFFT.

I decided to sample 0.5s of a sine wave with frequency of 8 Hz, with fs = 1024 Hz. I got 512 samples. FFT resolution is sampling rate fs divided by FFT size:

Type	Resolution [Hz]	Peak Width [Samples]
No padding	2	1
1024 padding	1	3
2048 padding	0,5	7



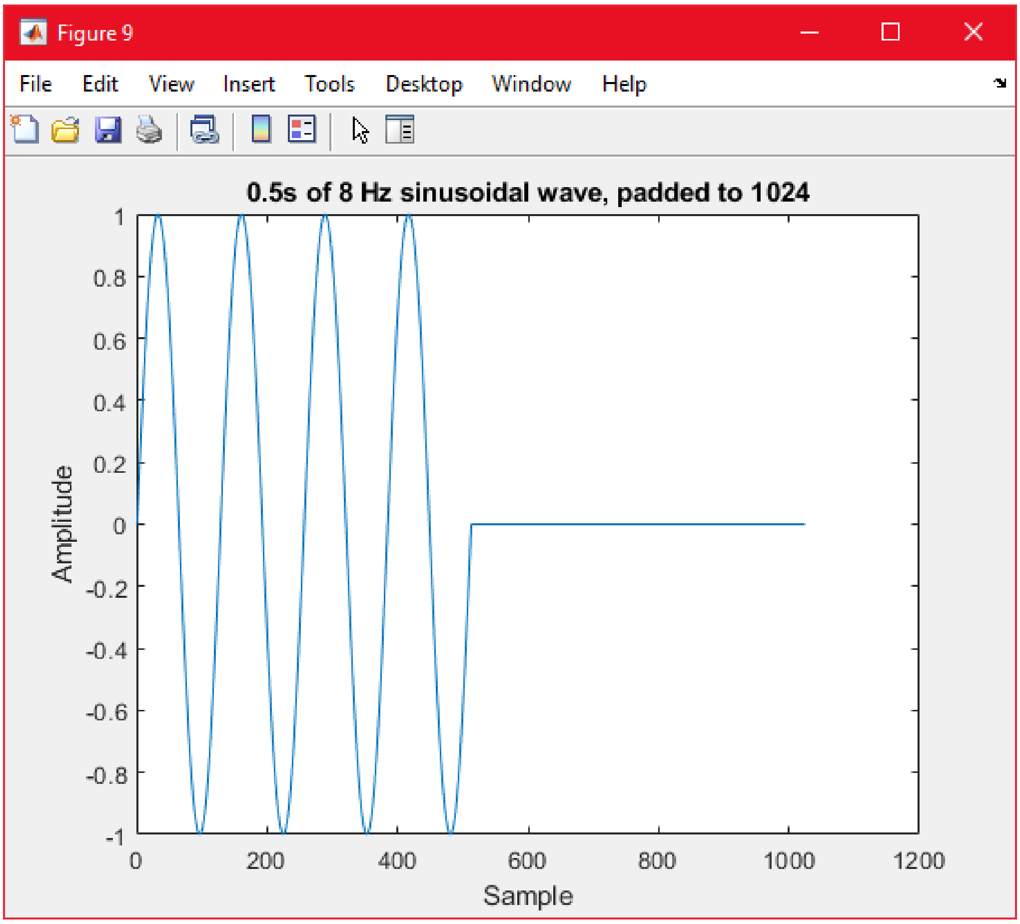


Figure 9
File Edit View Insert Tools Desktop Window Help
0.5s of 8 Hz sinusoidal wave, padded to 1024
Amplitude
Sample
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
200
400
600
800
1000
1200

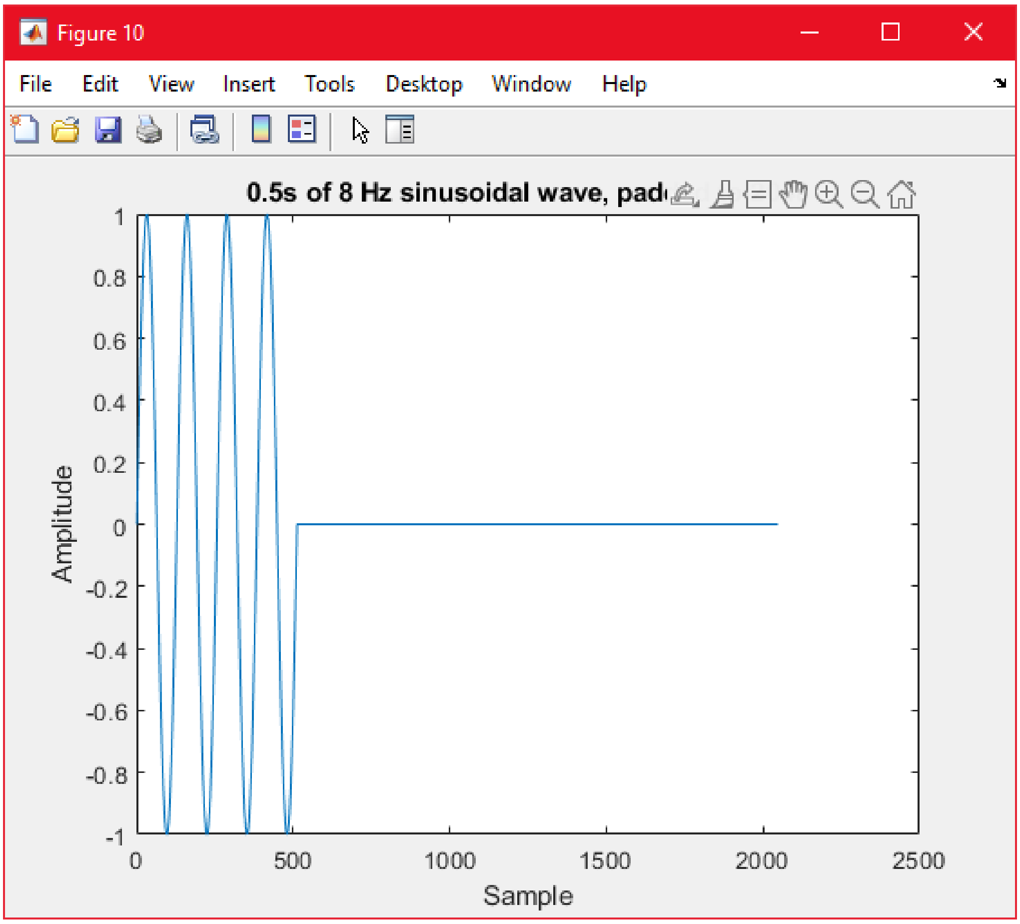


Figure 10
File Edit View Insert Tools Desktop Window Help
0.5s of 8 Hz sinusoidal wave, pad
Amplitude
Sample
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
500
1000
1500
2000
2500

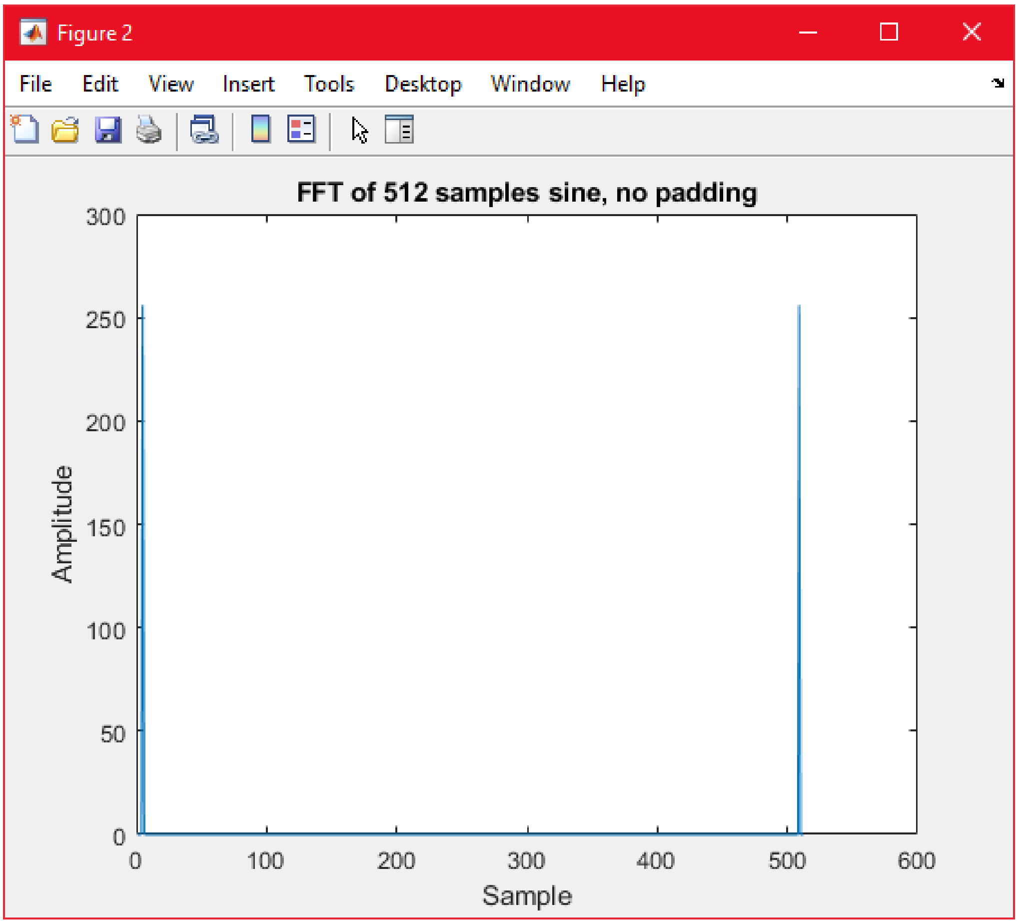


Figure 2
File Edit View Insert Tools Desktop Window Help
FFT of 512 samples sine, no padding
Amplitude
Sample
300
250
200
150
100
50
0
0 100 200 300 400 500 600

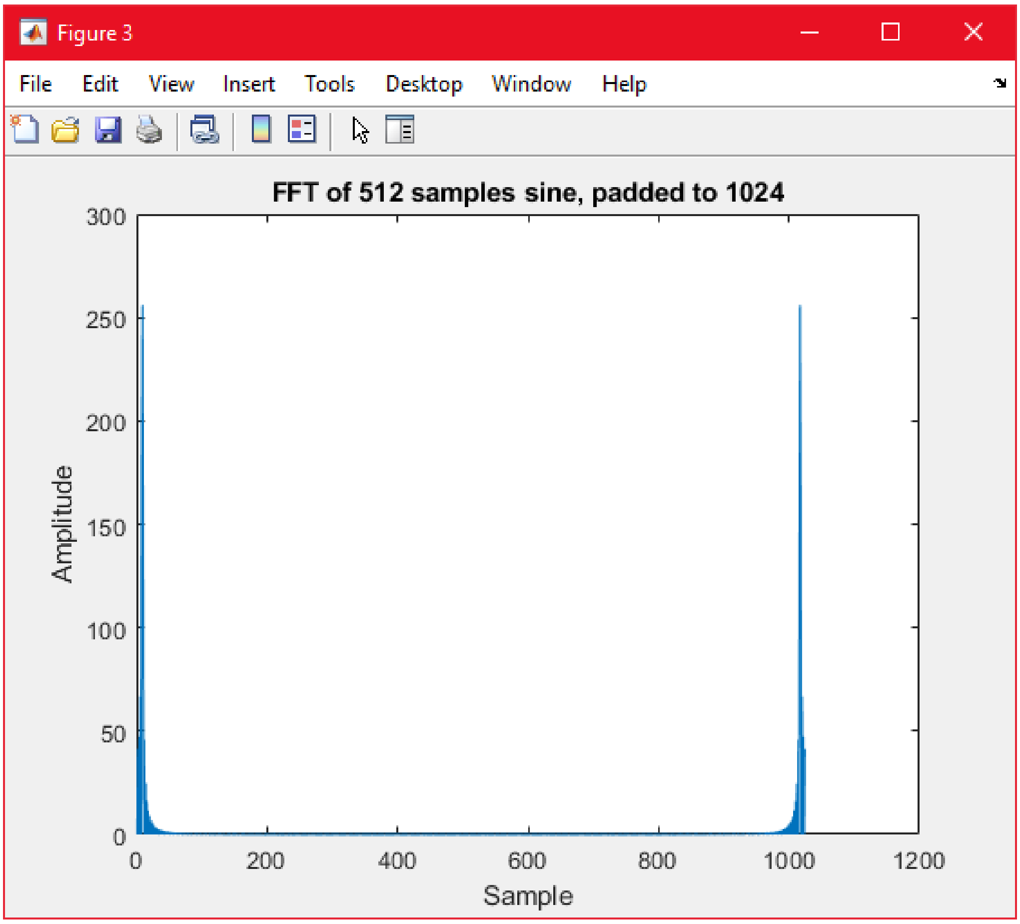


Figure 3
File Edit View Insert Tools Desktop Window Help
FFT of 512 samples sine, padded to 1024
Amplitude
Sample
300
250
200
150
100
50
0
0 200 400 600 800 1000 1200

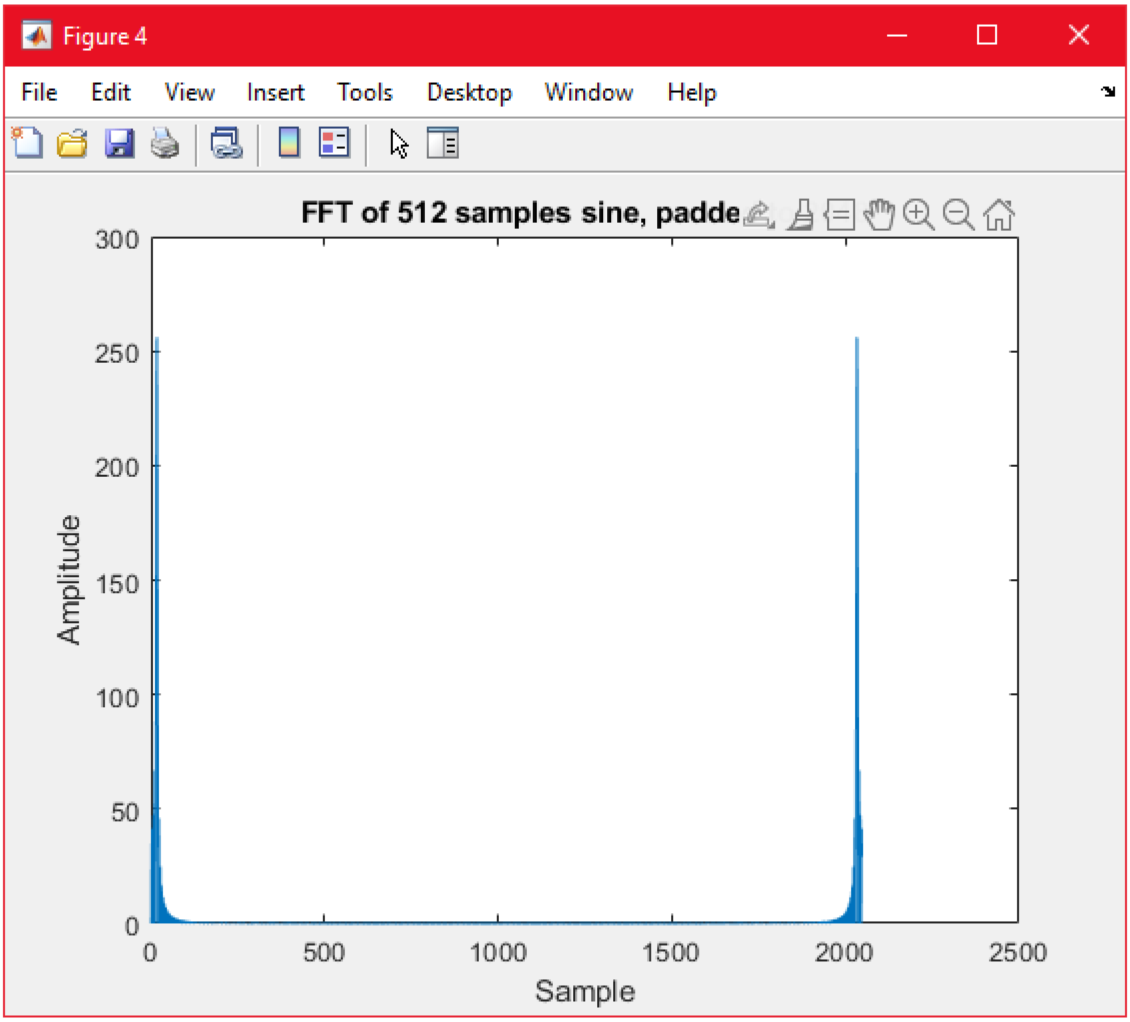


Figure 4
File Edit View Insert Tools Desktop Window Help
FFT of 512 samples sine, padde
Amplitude
Sample
0
50
100
150
200
250
300
0
500
1000
1500
2000
2500

IFFTs:

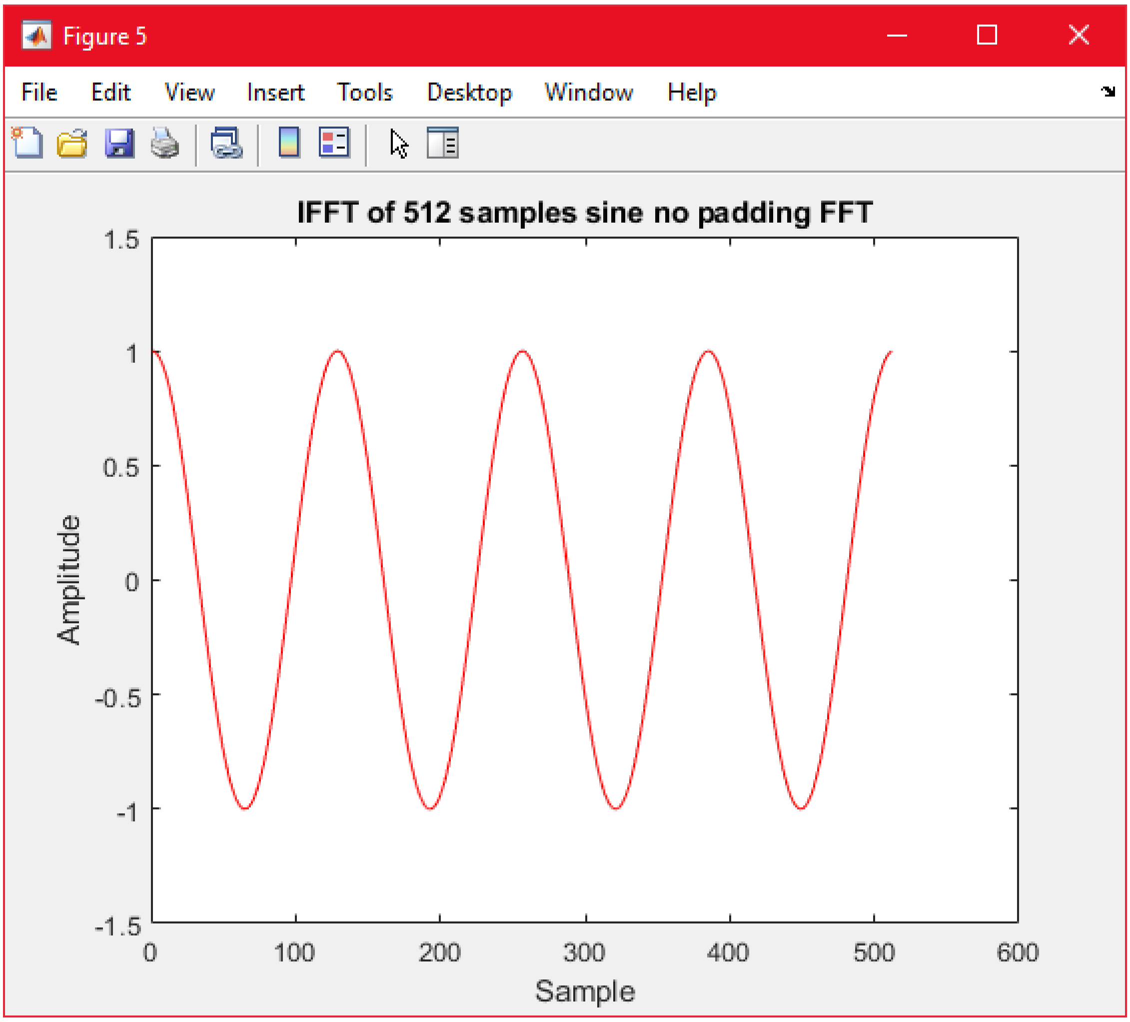


Figure 5
File Edit View Insert Tools Desktop Window Help
IFFT of 512 samples sine no padding FFT
Amplitude
Sample
1.5
1
0.5
0
-0.5
-1
-1.5
0
100
200
300
400
500
600

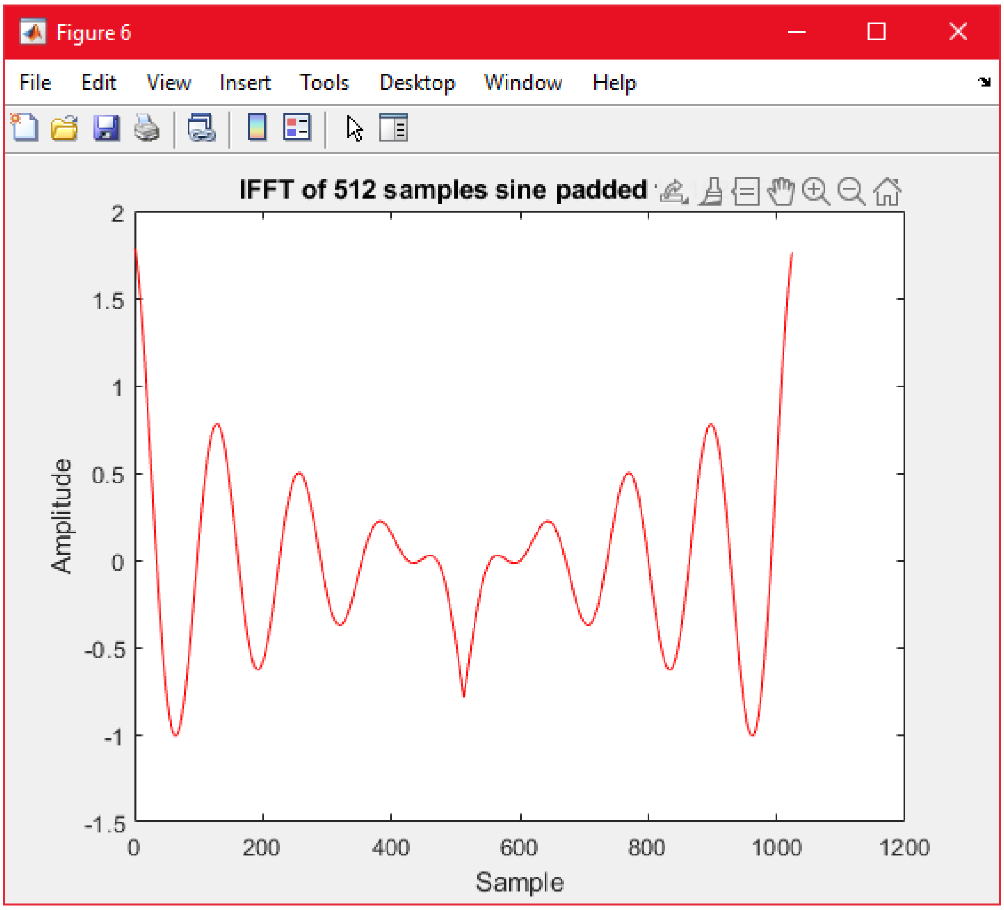


Figure 6
File Edit View Insert Tools Desktop Window Help
IFFT of 512 samples sine padded
Amplitude
Sample
2
1.5
1
0.5
0
-0.5
-1
-1.5
0 200 400 600 800 1000 1200

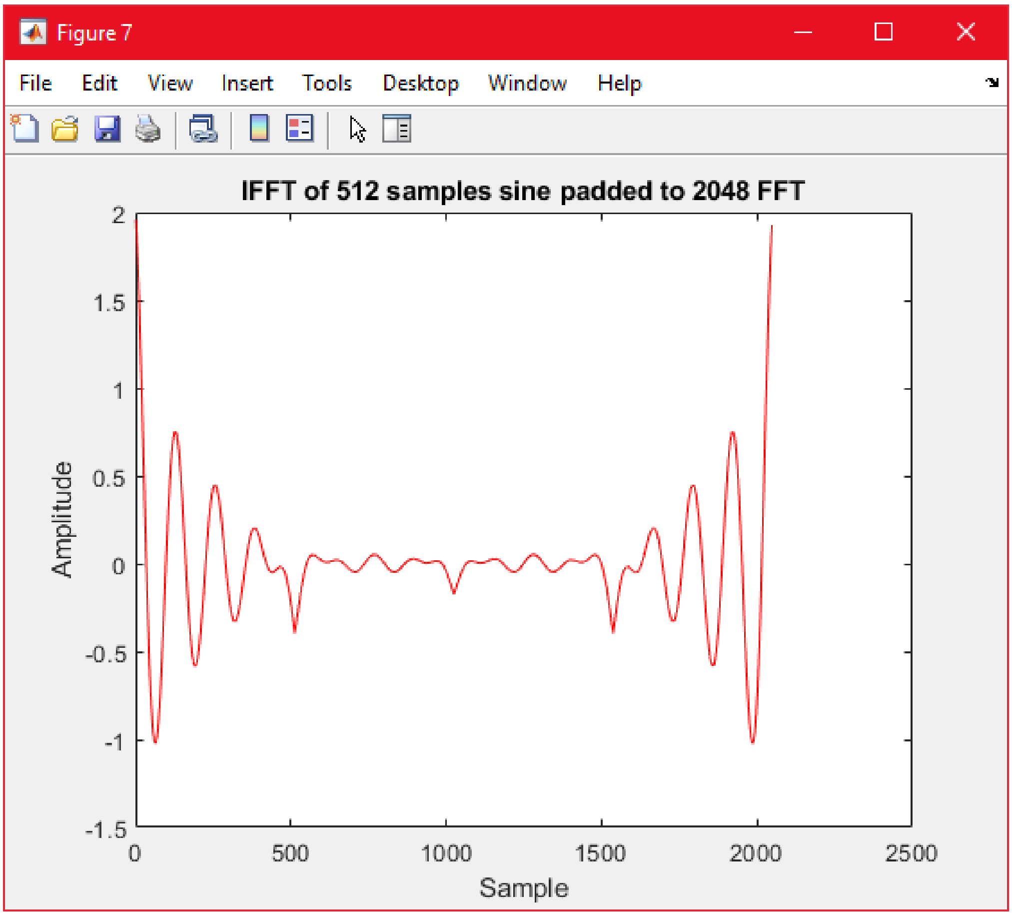


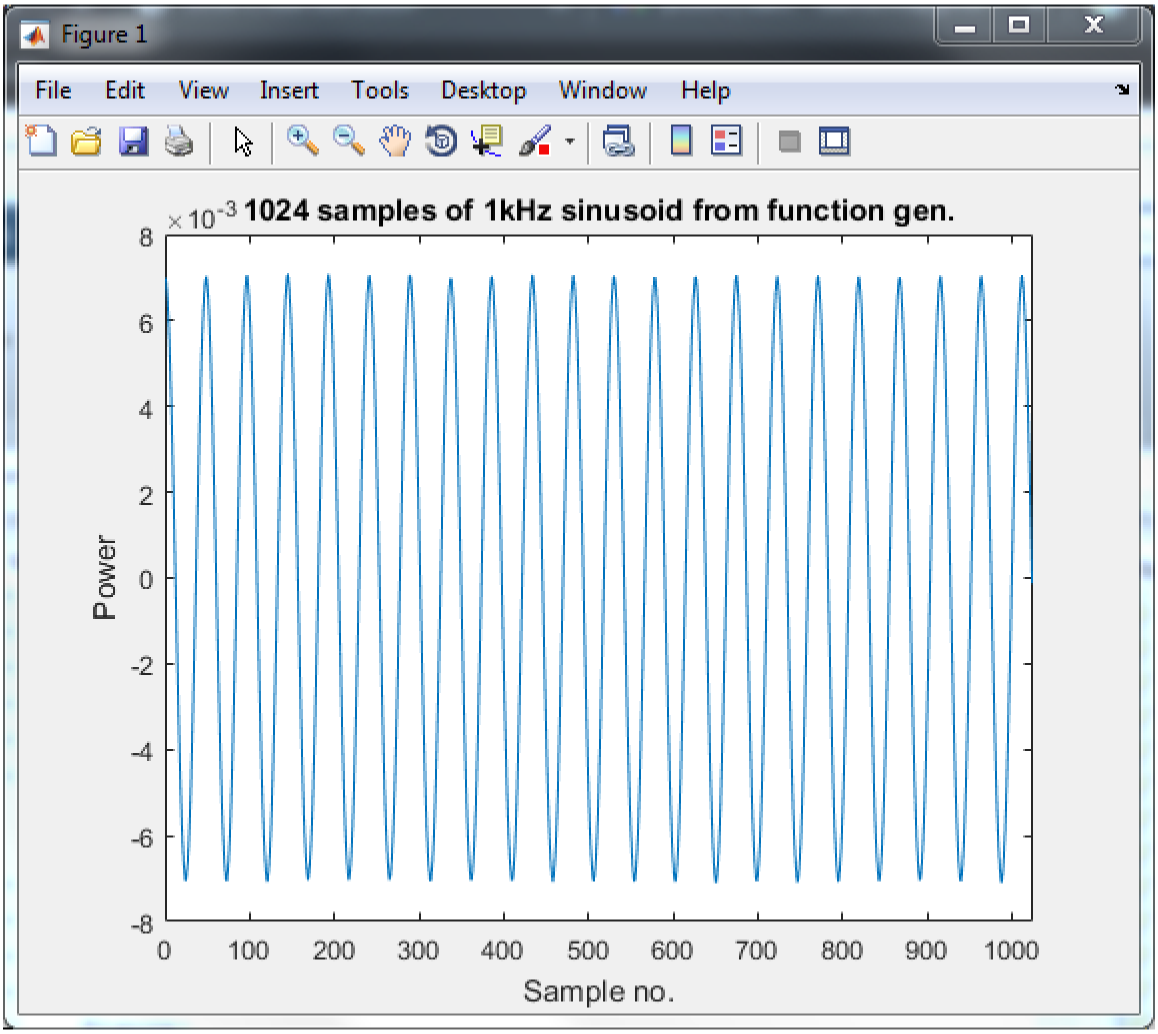
Figure 7
File Edit View Insert Tools Desktop Window Help
IFFT of 512 samples sine padded to 2048 FFT
Amplitude
Sample
2
1.5
1
0.5
0
-0.5
-1
-1.5
0 500 1000 1500 2000 2500

As it can be seen, only non-padded sine was properly reconstructed with the use of IFFT. Other IFFTs have created some signals that might be similar to a sinusoid shape at some points, but otherwise are incorrect and do not resemble the padded sinusoid signal.

Task 4 & 5

Analyze FFTs of a real-life signal from a function generator

Signal properties from generator (values are rounded):
amplitude 5 V, freq 1 kHz, sinusoidal signal
Raw signal:



FFTs:

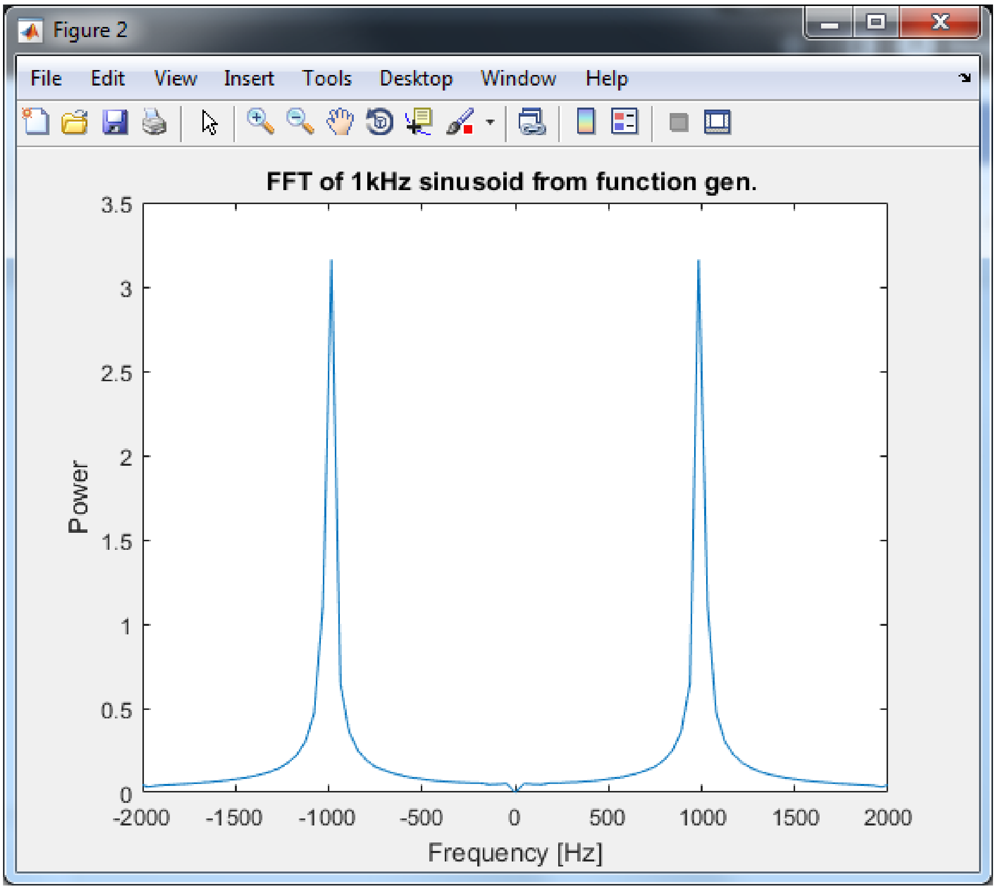


Figure 2
File Edit View Insert Tools Desktop Window Help
FFT of 1kHz sinusoid from function gen.
Power
Frequency [Hz]
3.5
3
2.5
2
1.5
1
0.5
0
-2000 -1500 -1000 -500 0 500 1000 1500 2000

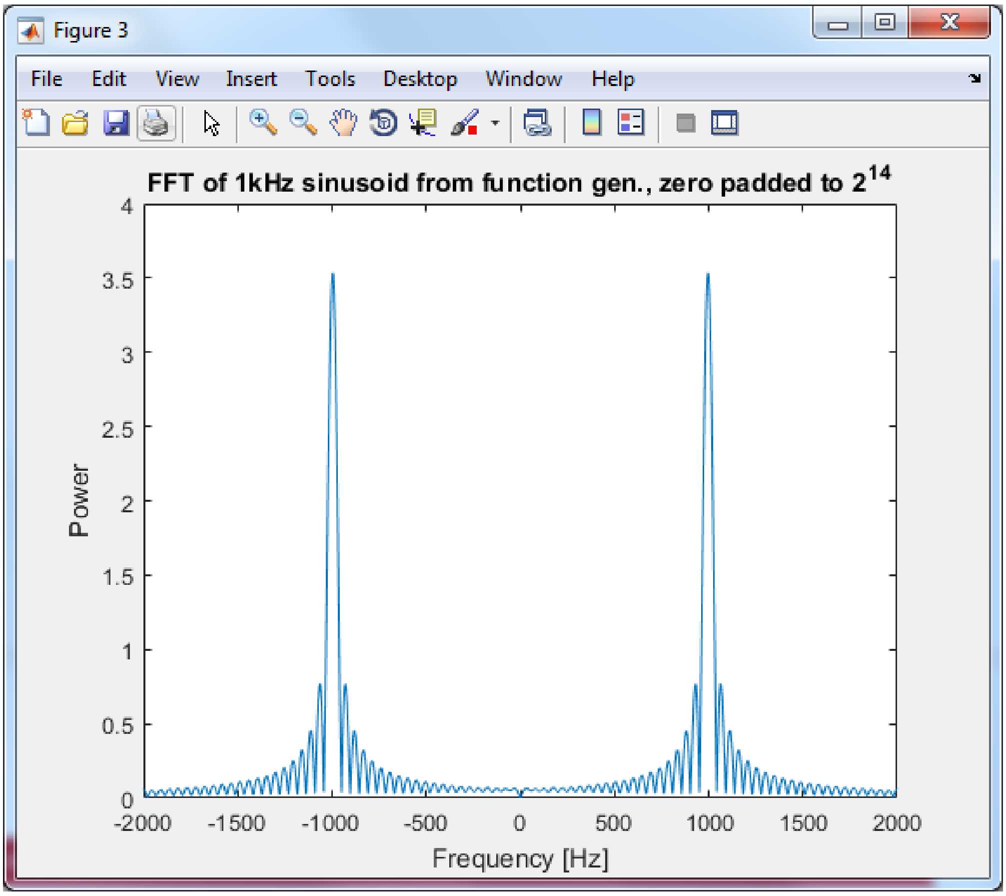
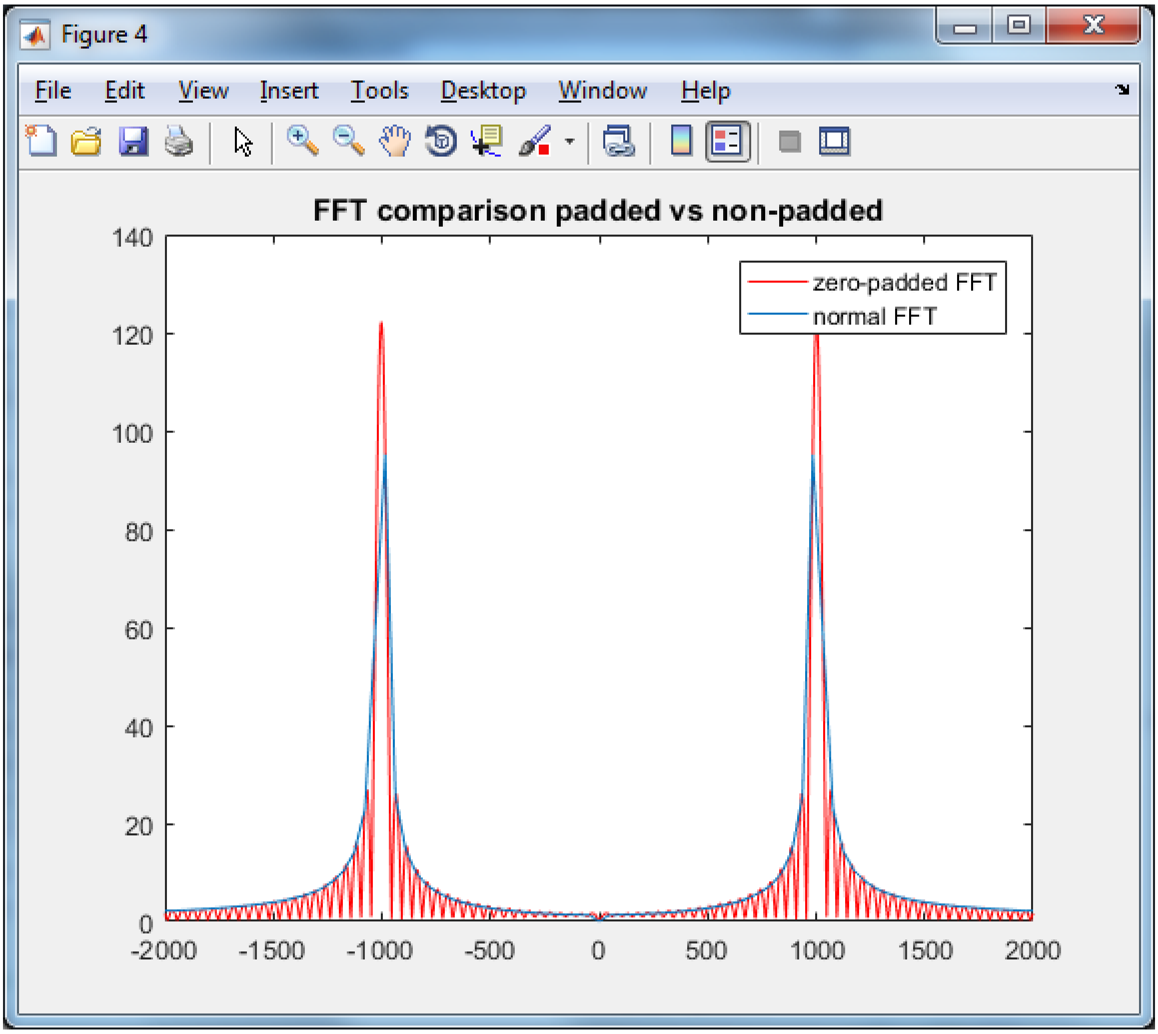


Figure 3
File Edit View Insert Tools Desktop Window Help
FFT of 1kHz sinusoid from function gen., zero padded to 2^14
Power
Frequency [Hz]
4
3.5
3
2.5
2
1.5
1
0.5
0
-2000 -1500 -1000 -500 0 500 1000 1500 2000

Zero-padded compared with non-padded FFT:

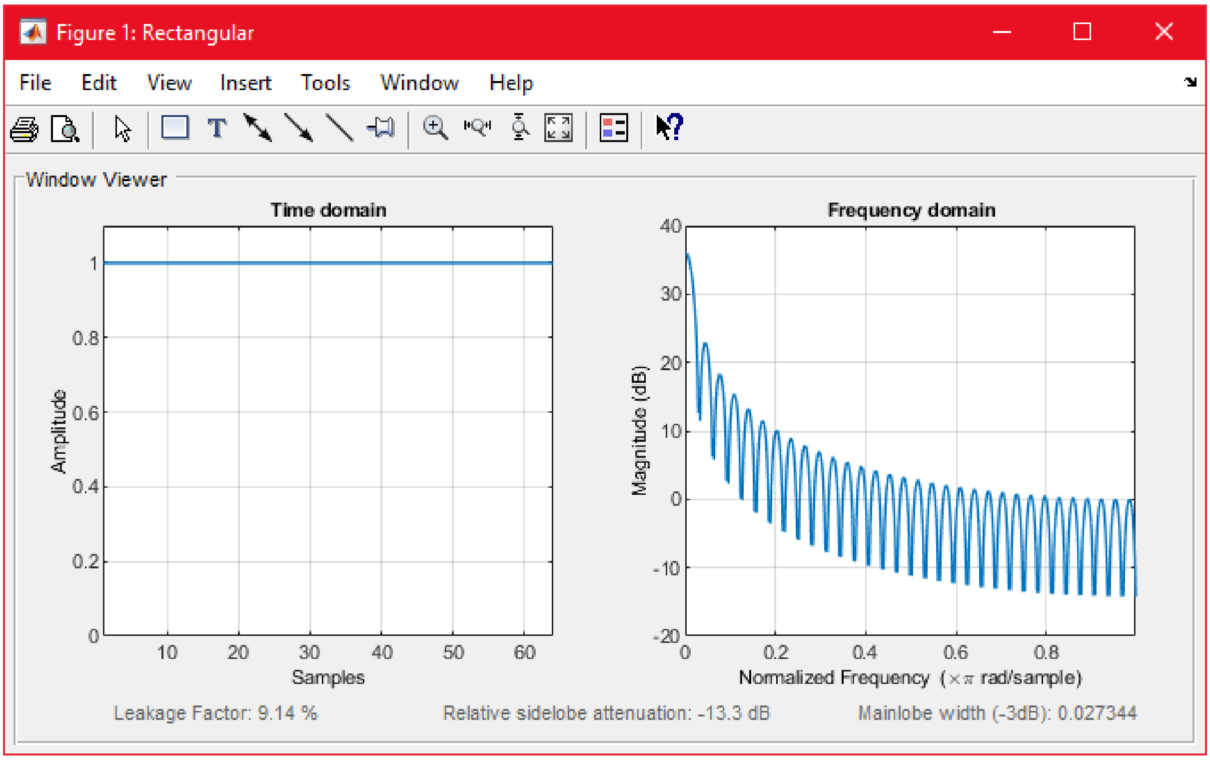


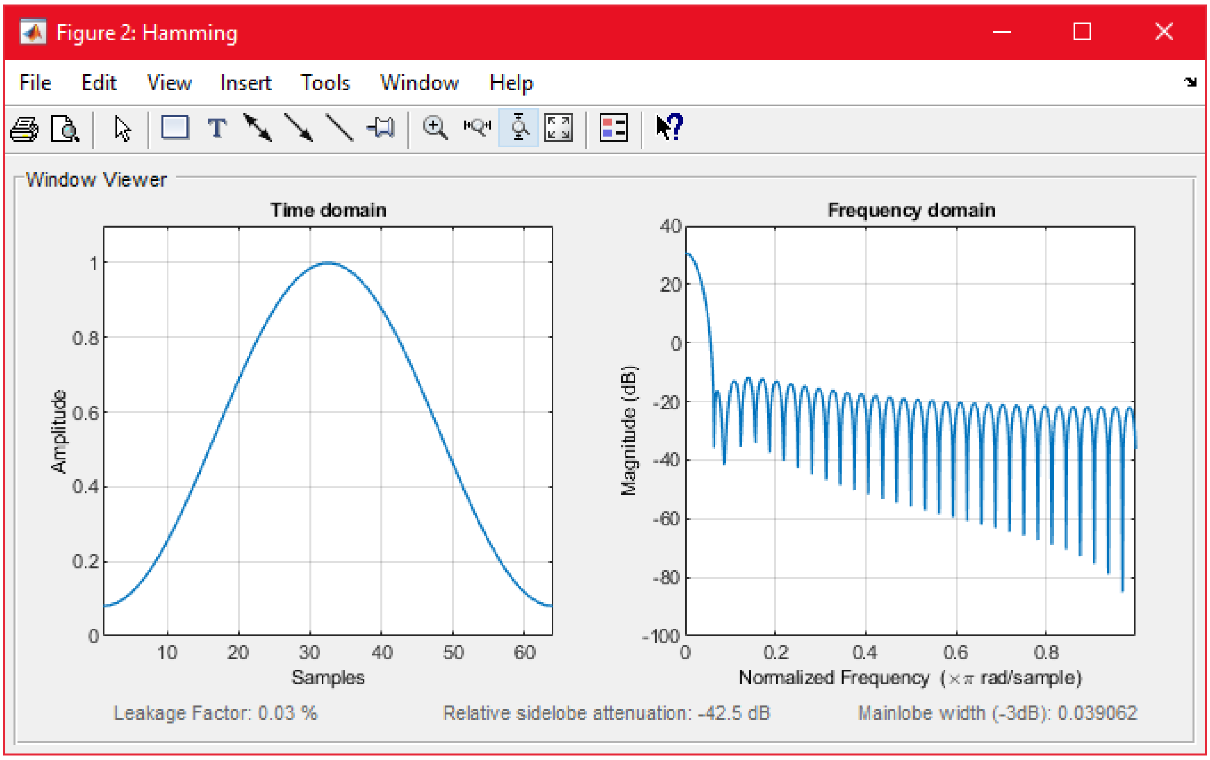
As it can be seen from the 3 figures above, the zero-padded FFT of the signal from the function gen. is more “jagged” at points different than +-1000 Hz. Peaks are visible on both FFTs. Zero-padded FFT has a bigger peak amplitude, so it is more sensitive.

Task 6

Analyze spectra of windows

Every window is 64-point, symmetric. Kaiser has beta = 4





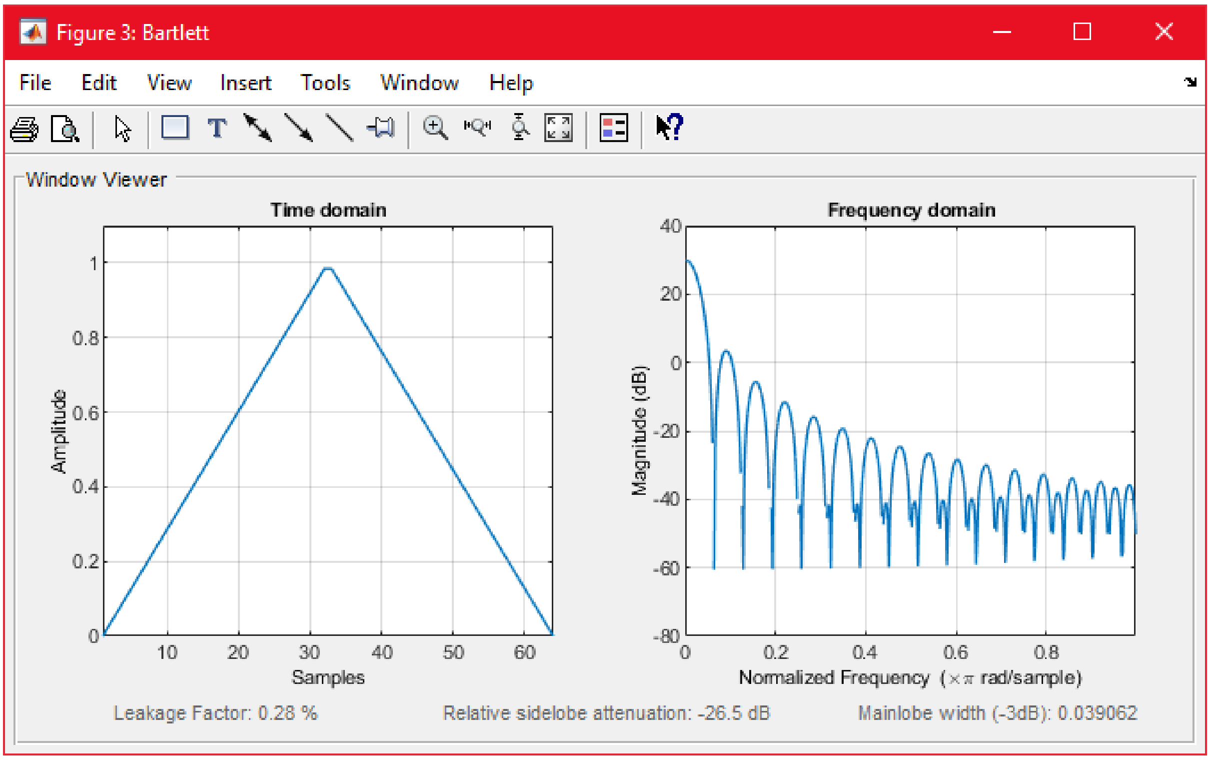


Figure 3: Bartlett
File Edit View Insert Tools Window Help
Window Viewer
Time domain
Amplitude
Samples
Frequency domain
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 0.28 %
Relative sidelobe attenuation: -26.5 dB
Mainlobe width (-3dB): 0.039062

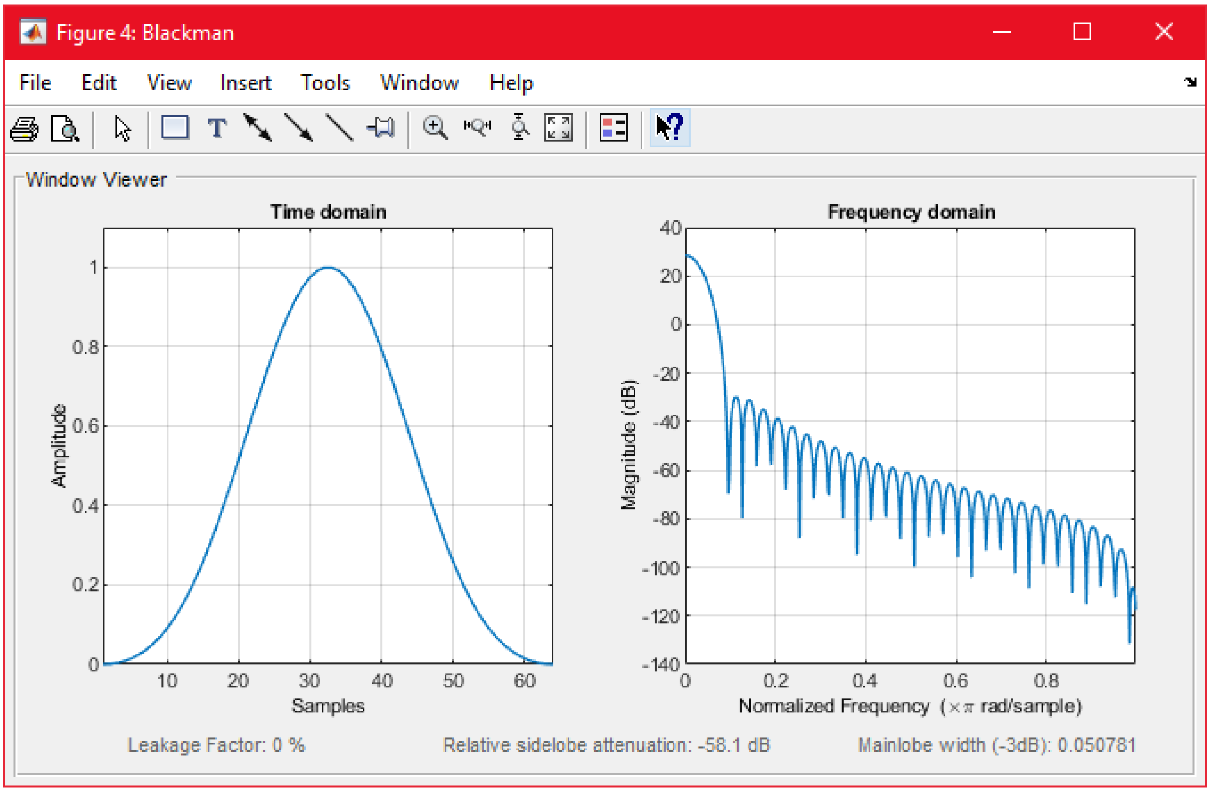


Figure 4: Blackman
File Edit View Insert Tools Window Help
Window Viewer
Time domain
Amplitude
Samples
Frequency domain
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 0 %
Relative sidelobe attenuation: -58.1 dB
Mainlobe width (-3dB): 0.050781

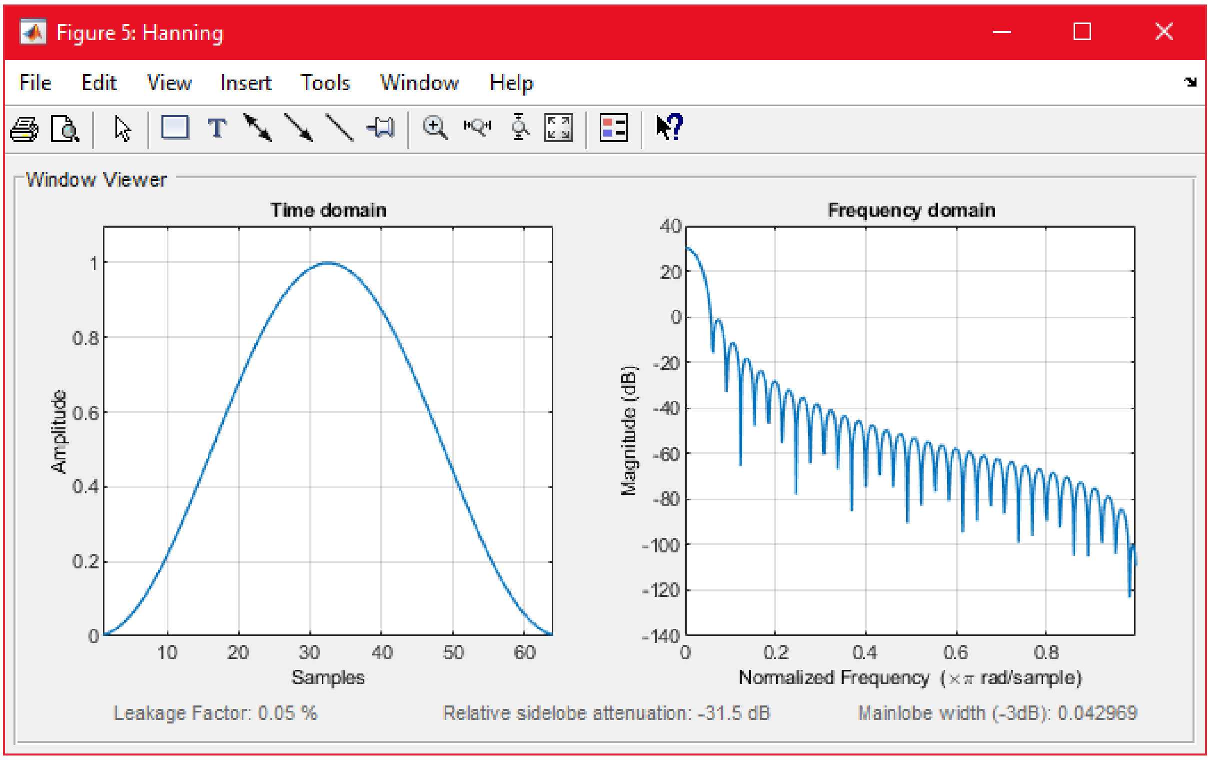


Figure 5: Hanning
File Edit View Insert Tools Window Help
Window Viewer
Time domain
Frequency domain
Amplitude
Samples
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 0.05 %
Relative sidelobe attenuation: -31.5 dB
Mainlobe width (-3dB): 0.042969

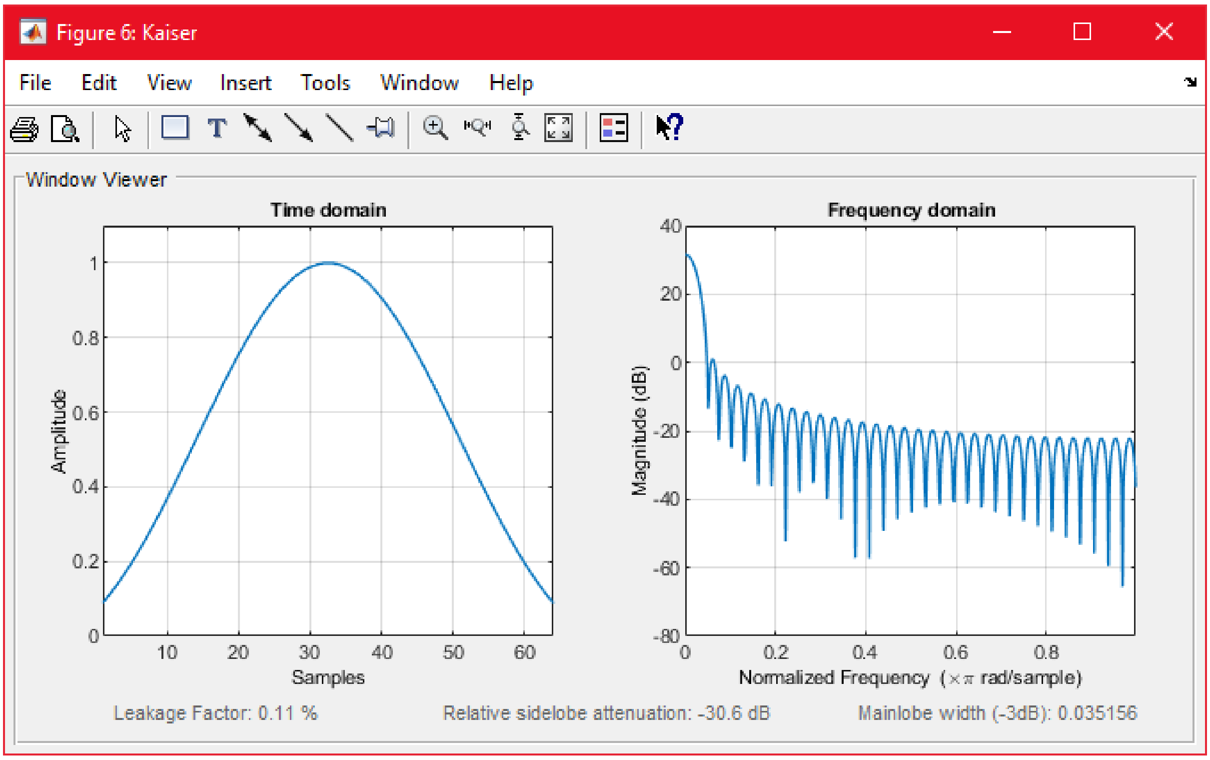
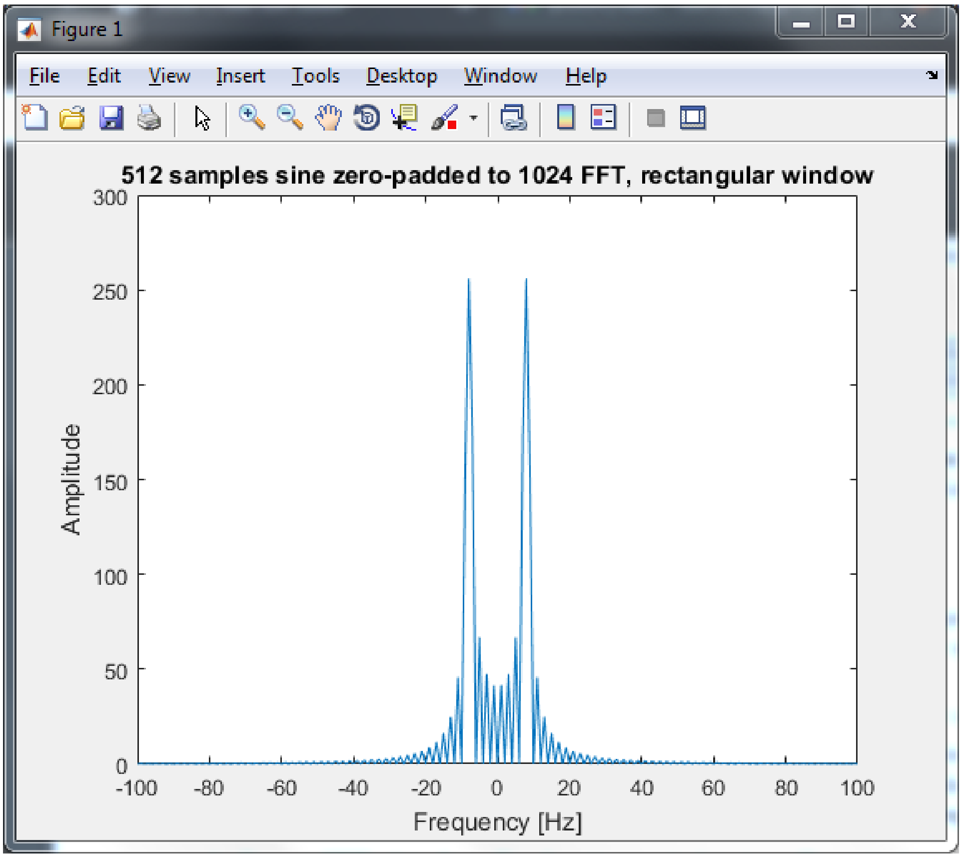


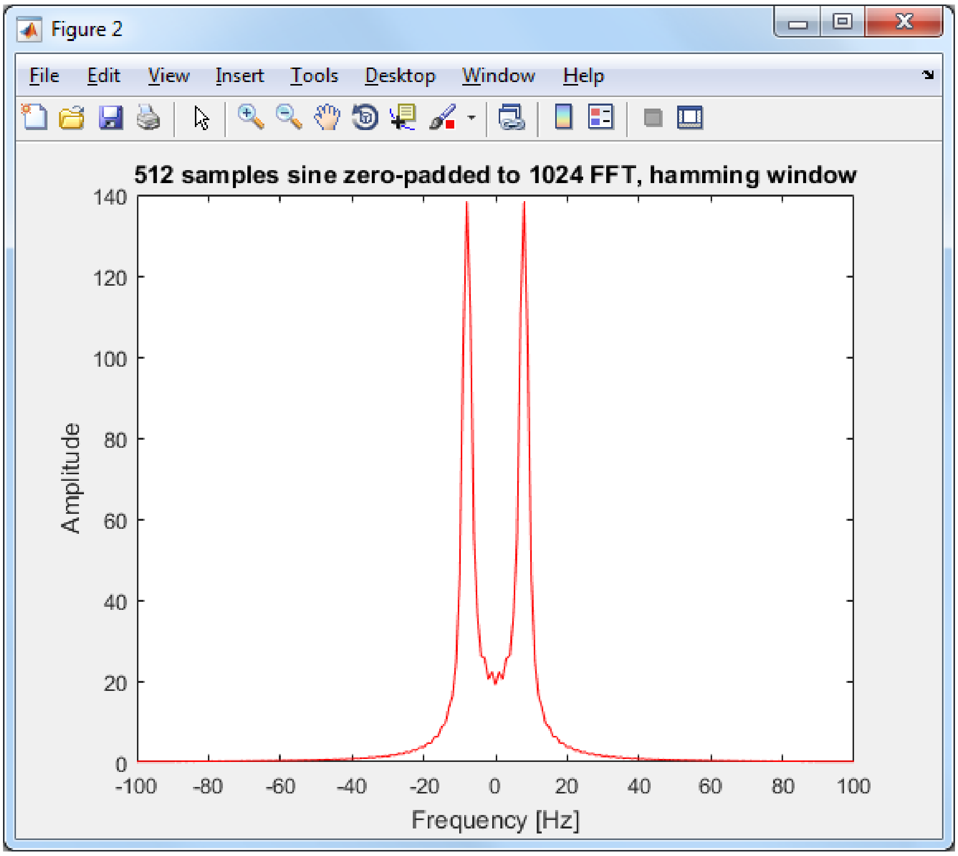
Figure 6: Kaiser
File Edit View Insert Tools Window Help
Window Viewer
Time domain
Frequency domain
Amplitude
Samples
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 0.11 %
Relative sidelobe attenuation: -30.6 dB
Mainlobe width (-3dB): 0.035156

Window	Mainlobe width at -3 dB [normalized freq]	First sidelobe attentuation [dB]	Highest sidelobe [dB]	Sidelobe changes with f
Rectangular	0,027	-13,3	22,87	Magnitude decreases as f gets higher, width is constant
Hamming	0,039	-42,5	-11,83	Magnitude slightly decreases. Magnitude drops get higher with f. Width const.
Bartlett	0,039	-26,5	3,43	Magnitude decreases. Some windows start at lower magnitudes, some at higher. As f gets higher, sidelobes of smaller magnitude start to form inbetween existing sidelobes
Blackman	0,051	-58,1	-29,69	Magnitude decreases, reaching very low level close to end of normalised freq. axis. Some windows start a lower magnitude, some at higher.
Hanning	0,043	-31,5	-1,24	Similar as to Blackman, but starts at higher magnitude and drops more rapidly.
Kaiser	0,035	-30,6	1,05	Peak magnitude drops slightly, reaching const. peak levels at around -20 dB. Sidelobes start at different magnitude levels - some higher, some lower. From normalised freq. 0.4 to 1.0, sidelobe starts and ends form a kind of parabole

Task 7

Analyze the effect of windowing





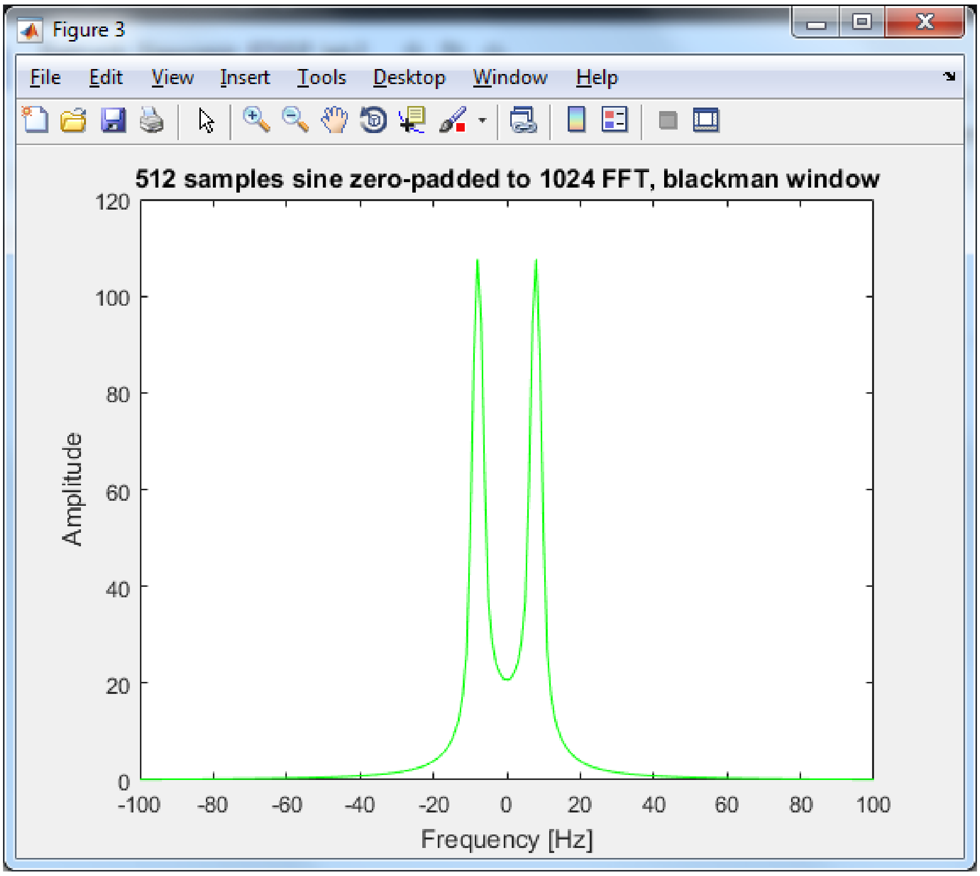


Figure 3
File Edit View Insert Tools Desktop Window Help
512 samples sine zero-padded to 1024 FFT, blackman window
Amplitude
Frequency [Hz]
0
20
40
60
80
100
120
-100
-80
-60
-40
-20
0
20
40
60
80
100

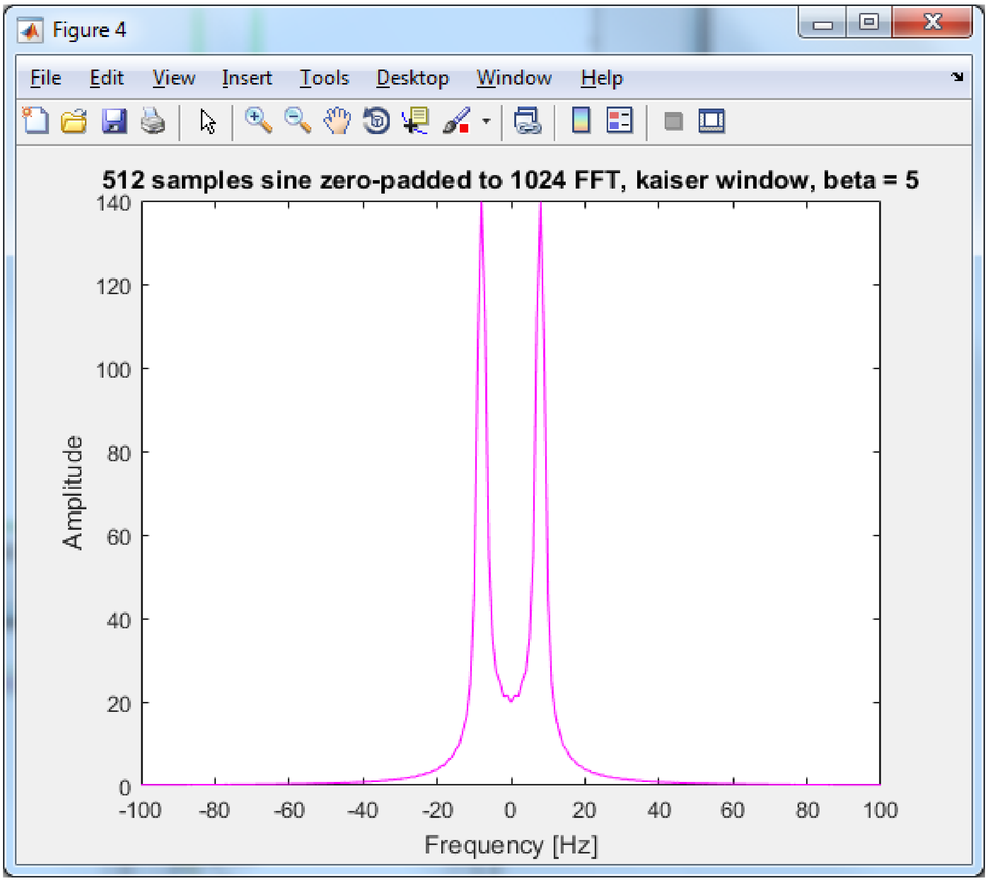
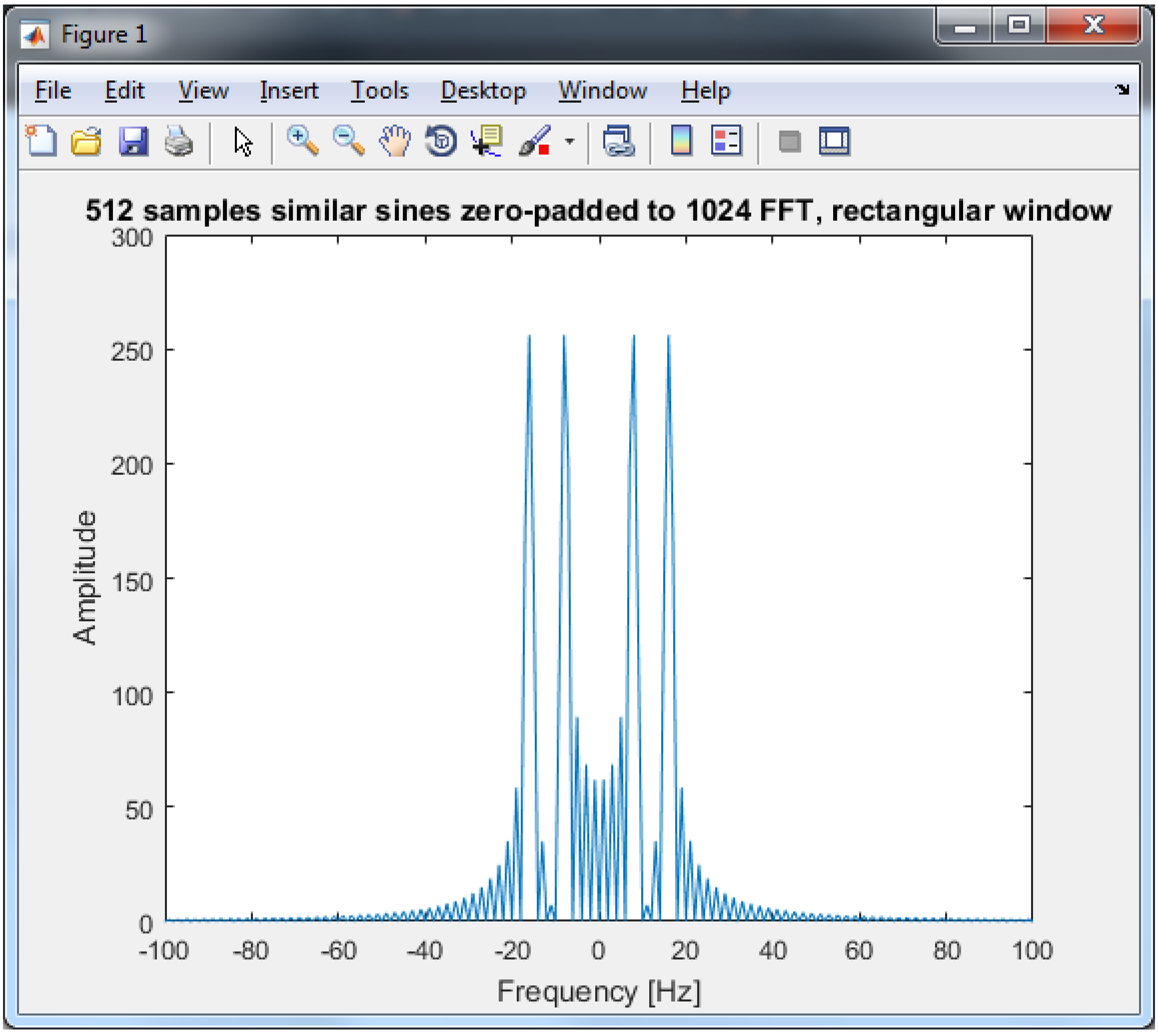


Figure 4
File Edit View Insert Tools Desktop Window Help
512 samples sine zero-padded to 1024 FFT, kaiser window, beta = 5
Amplitude
Frequency [Hz]
0
20
40
60
80
100
120
140
-100
-80
-60
-40
-20
0
20
40
60
80
100

From the set of 3 figures above, we can see that not using the window yields worse results than with windows. Kaiser window with beta = 5 seems to be the best in my case, as it has highest amplitudes of the spikes. Blackman looks a bit smoother than Kaiser, but Blackman has smaller peak amplitudes, which makes it worse.

FFTs of the “two sine” case.

Similar frequency sines, same amplitude:



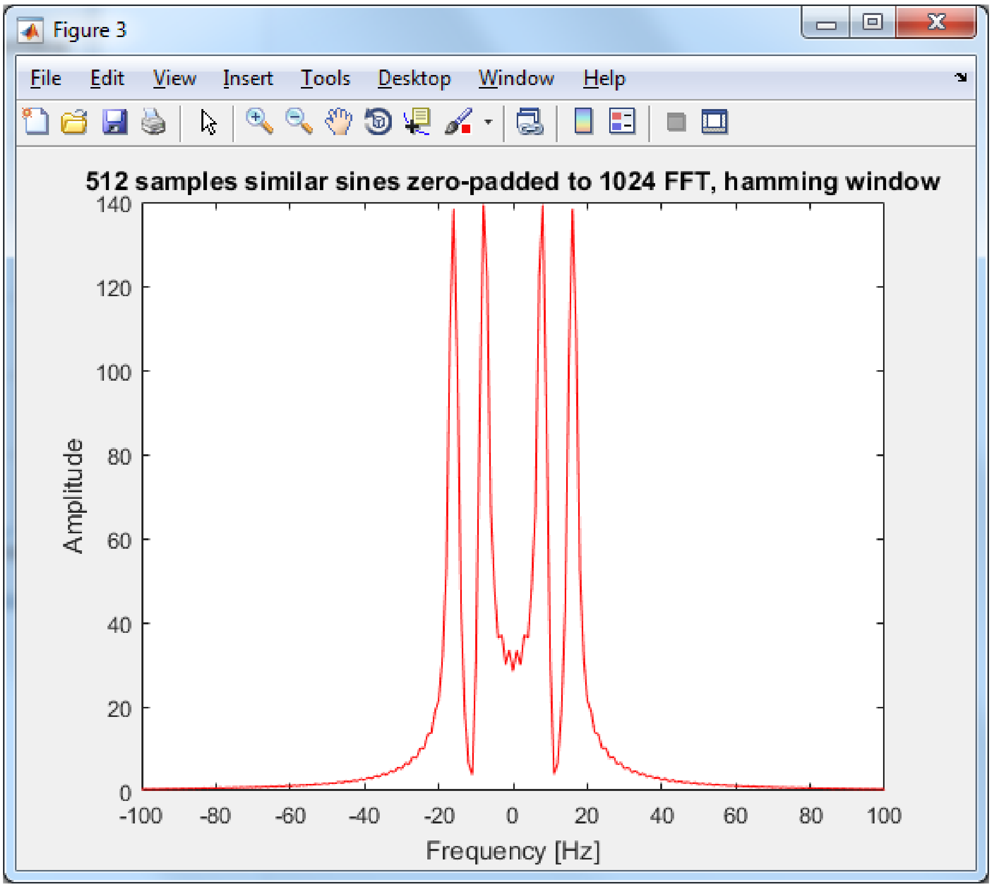


Figure 3
File Edit View Insert Tools Desktop Window Help
512 samples similar sines zero-padded to 1024 FFT, hamming window
Amplitude
Frequency [Hz]
0
20
40
60
80
100
120
140
-100
-80
-60
-40
-20
0
20
40
60
80
100

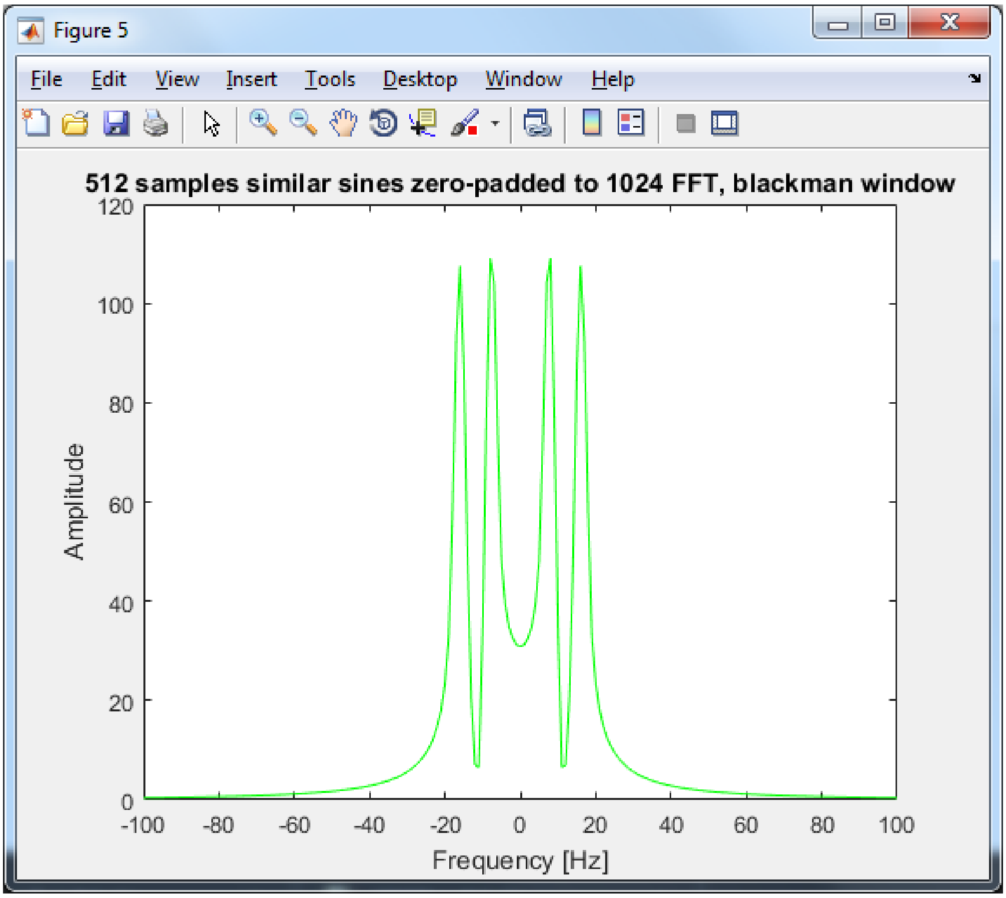


Figure 5
File Edit View Insert Tools Desktop Window Help
512 samples similar sines zero-padded to 1024 FFT, blackman window
Amplitude
Frequency [Hz]
0
20
40
60
80
100
120
-100
-80
-60
-40
-20
0
20
40
60
80
100

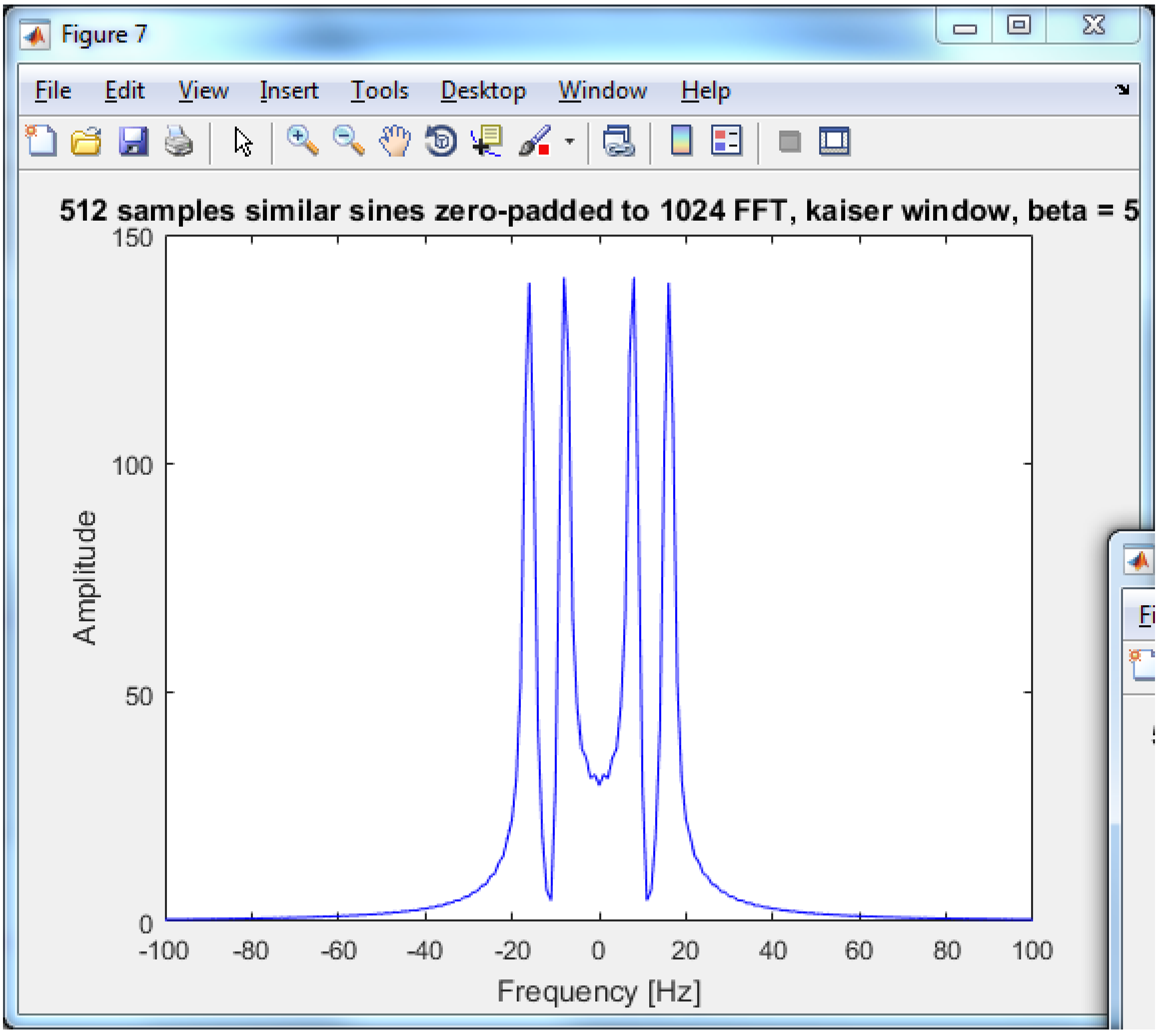
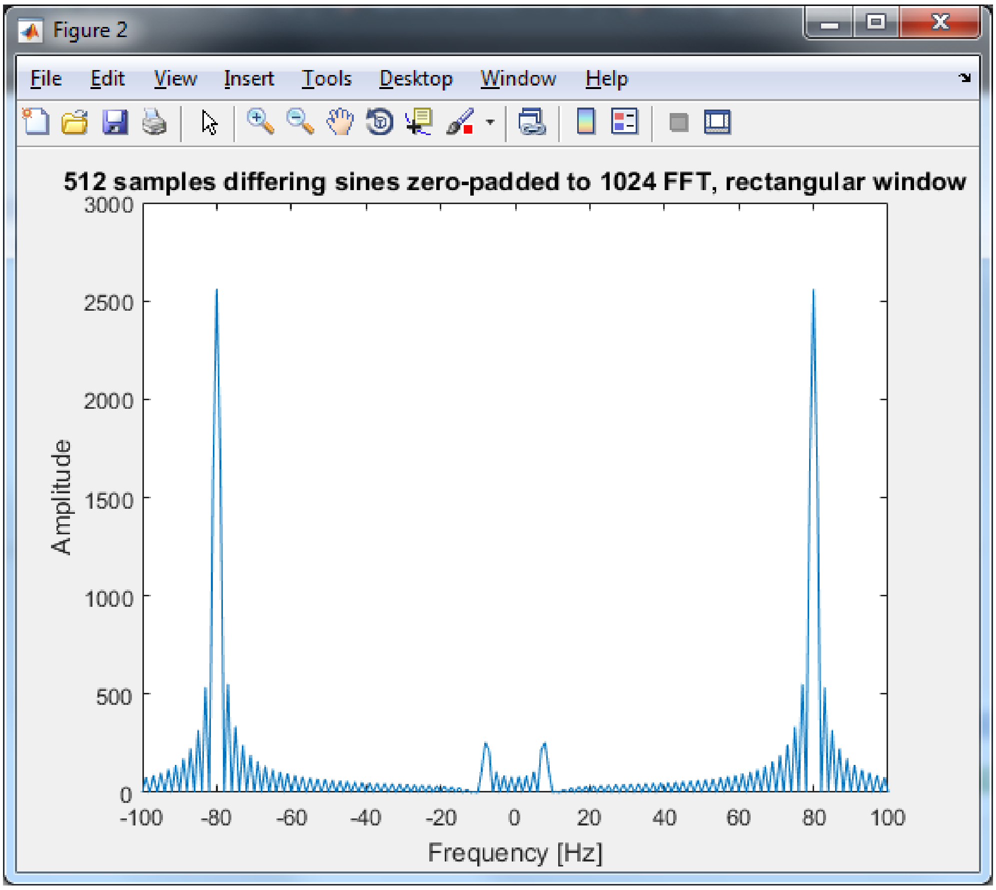
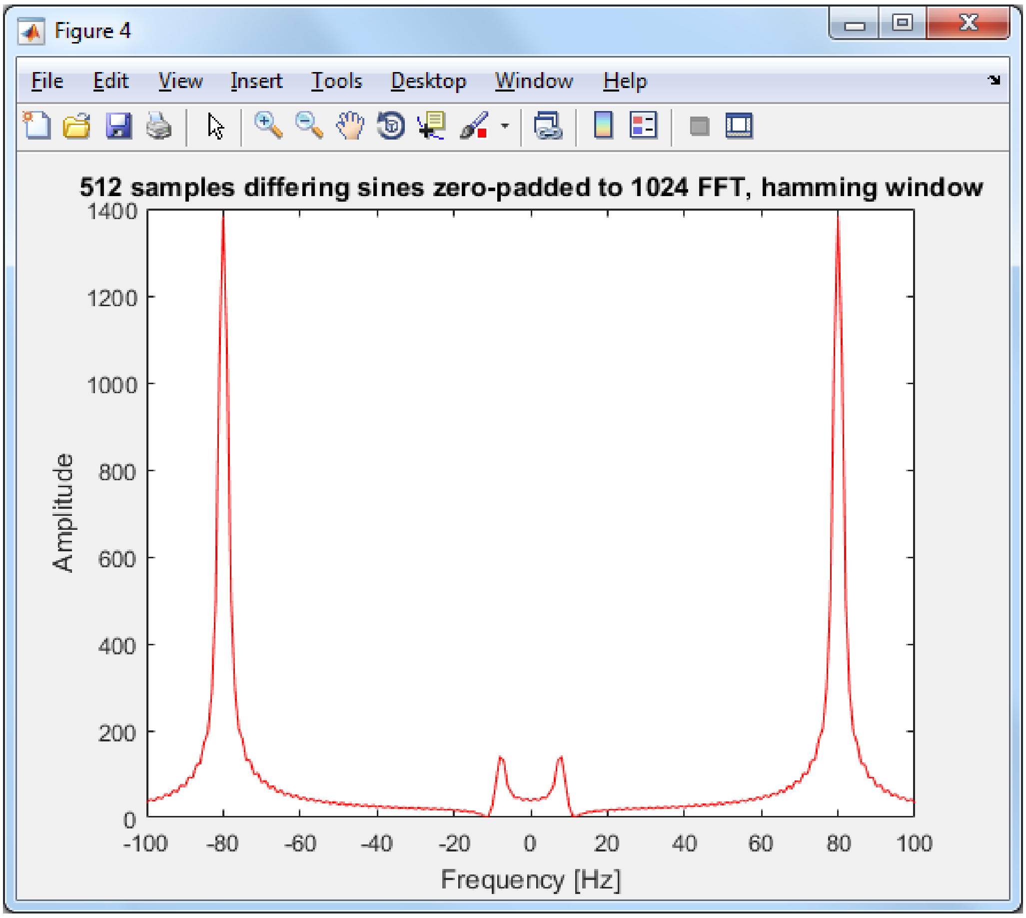


Figure 7
File Edit View Insert Tools Desktop Window Help
512 samples similar sines zero-padded to 1024 FFT, kaiser window, beta = 5
Amplitude
Frequency [Hz]
150
100
50
0
-100 -80 -60 -40 -20 0 20 40 60 80 100

Different frequency sines, different amplitude:





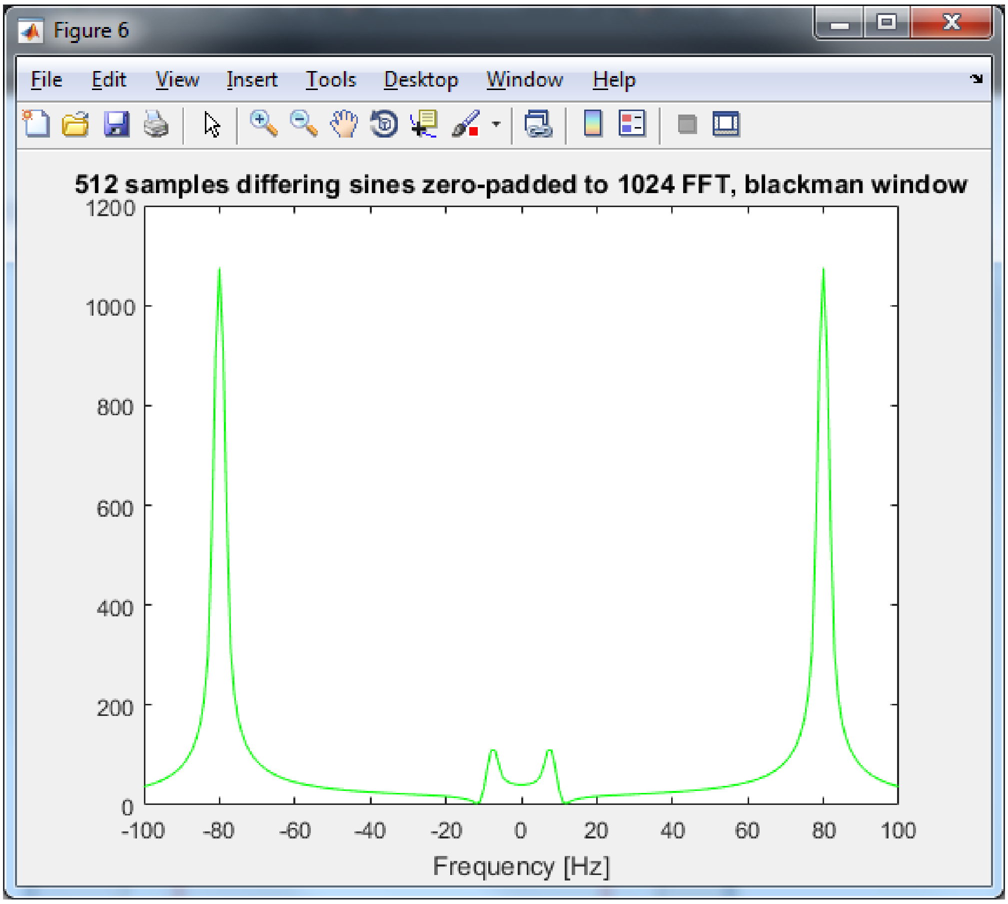


Figure 6
File Edit View Insert Tools Desktop Window Help
512 samples differing sines zero-padded to 1024 FFT, blackman window
Frequency [Hz]

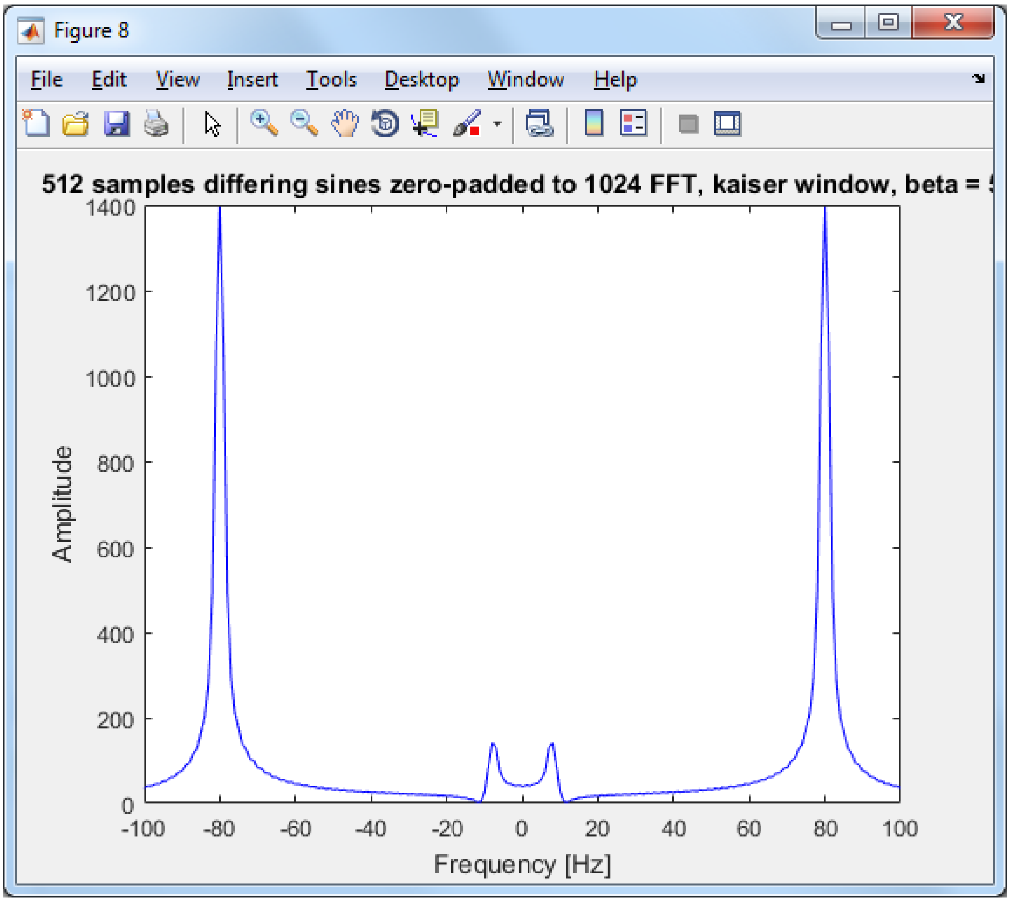


Figure 8
File Edit View Insert Tools Desktop Window Help
512 samples differing sines zero-padded to 1024 FFT, kaiser window, beta =
Amplitude
Frequency [Hz]

As it can be again seen, for the similar sines case - Kaiser window with beta = 5 seems to have the best amplitude and frequency sensitivity. Hamming window has slightly lower amplitude at peaks, but it also has jagged lines. Rectangular window has the biggest problem with jagged lines.

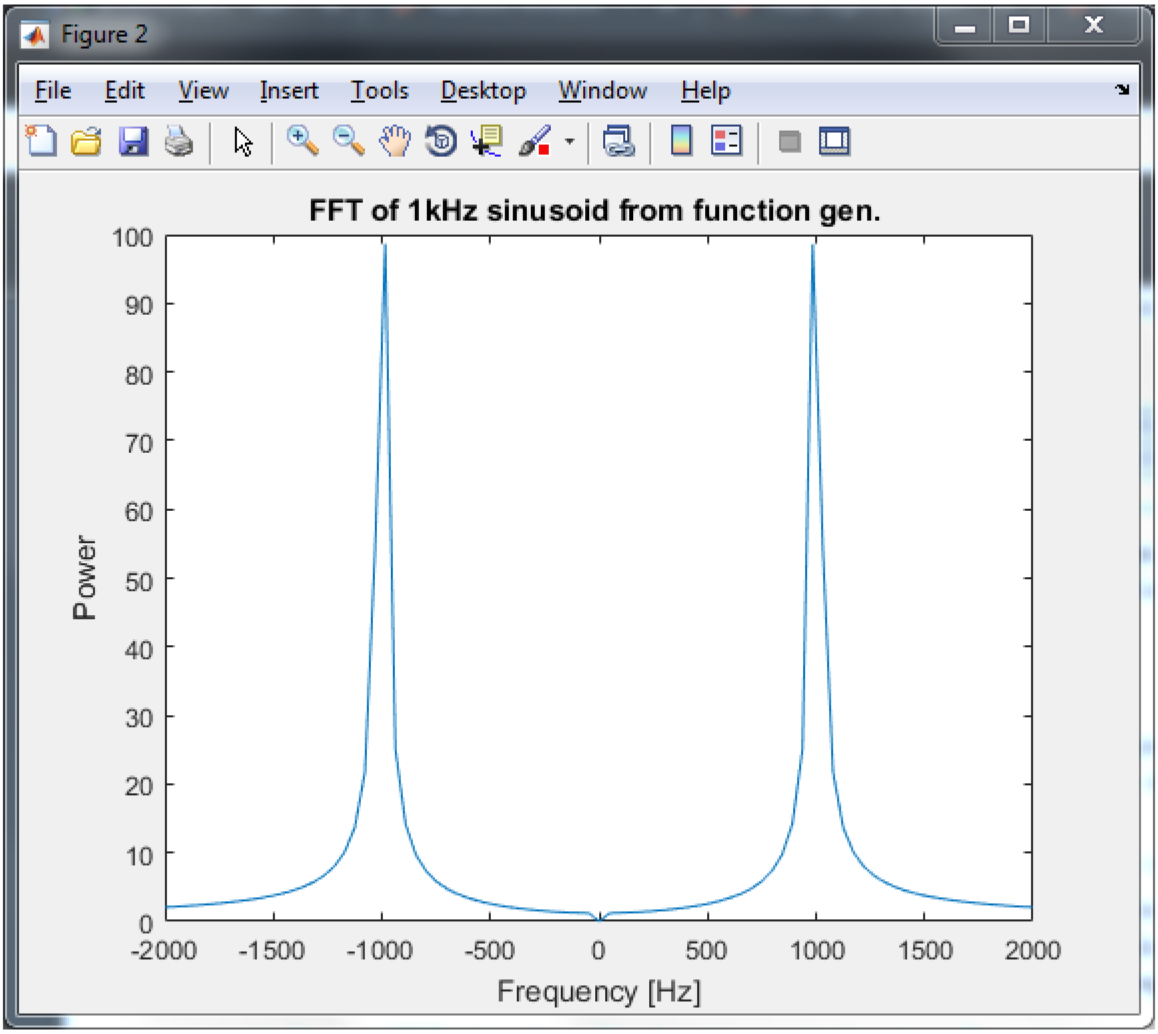
For the sum of different sines (different frequencies and amplitudes), it should be noted that the rectangular window has the perfect amplitude sensitivity. At 8 Hz, the amplitude is 256, and for 80 Hz it is 2560, which checks out with the signal that is generated from the matlab code. 80 Hz sine has amplitude of 10, 8 Hz has A = 1:

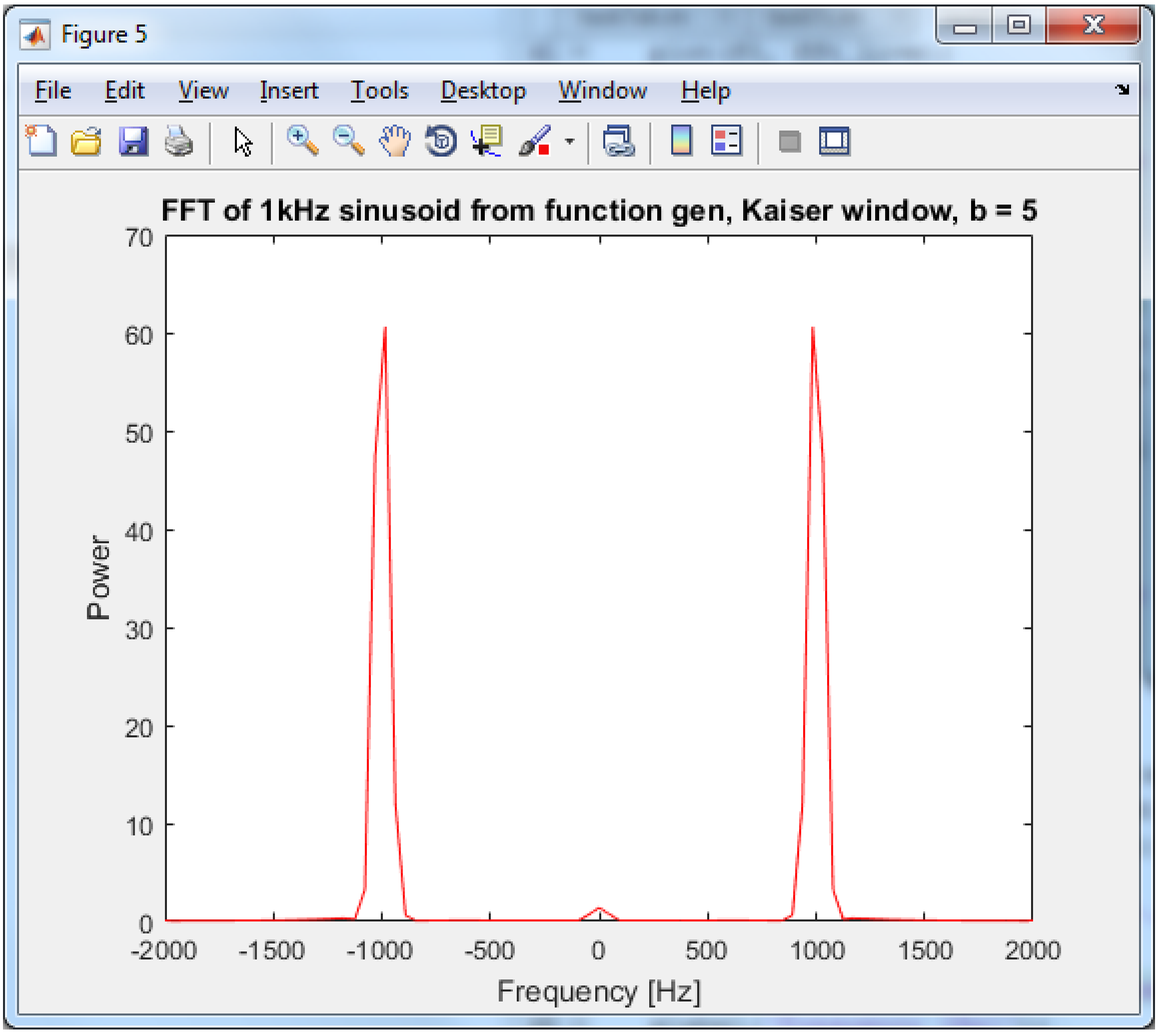
```
x2 = sin(2*pi*f*t) + 5*sin(2*pi*10*f*t);
```

Regarding other windows, the conclusions are similar to those from the “similar sines” case. Kaiser has a small amount of jagged line parts, and has the best amplitude sensitivity (apart from boxcar) compared to Hamming and Blackman. Blackman is smooth, but has lower amplitude sensitivity, and Hamming has similar amplitude sensitivity as Kaiser, but has a jagged line.

Task 8

Effect of windowing on a live signal





As it can be seen on two figures above, Kaiser window has better frequency sensitivity, but worse amplitude sensitivity compared to boxcar.

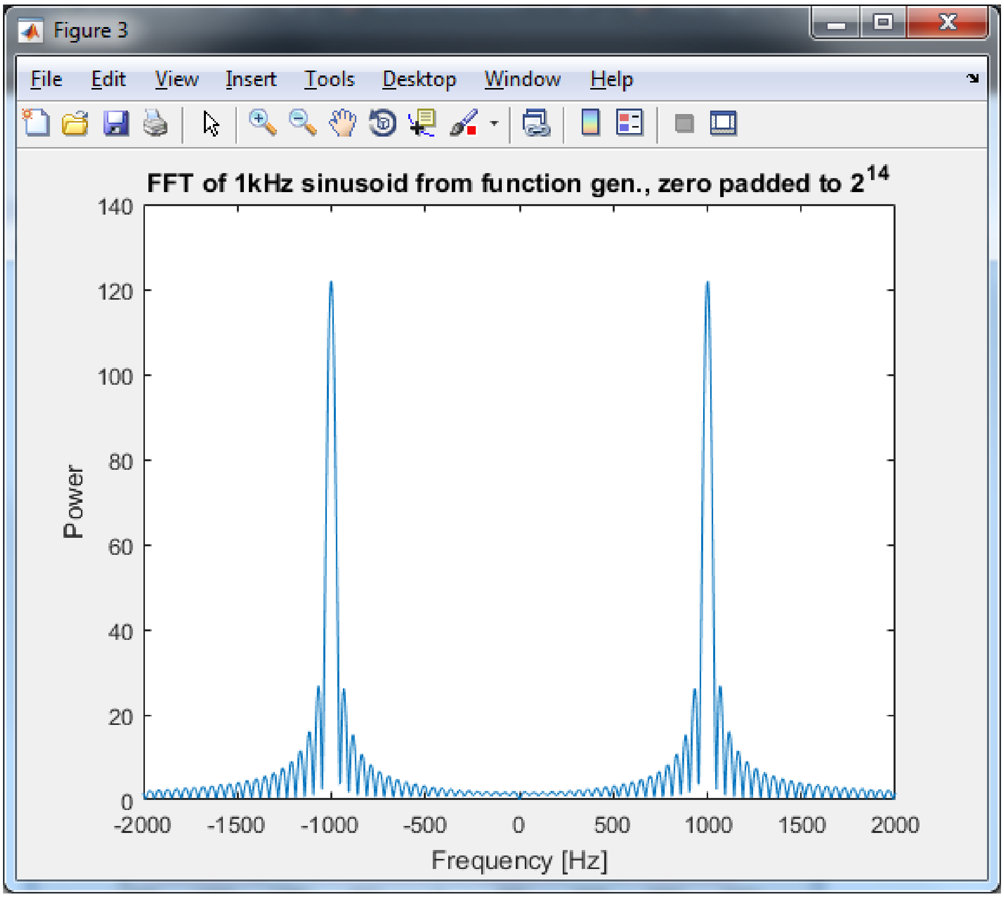


Figure 3
File Edit View Insert Tools Desktop Window Help
FFT of 1kHz sinusoid from function gen., zero padded to 2^14
Power
Frequency [Hz]
140
120
100
80
60
40
20
0
-2000 -1500 -1000 -500 0 500 1000 1500 2000

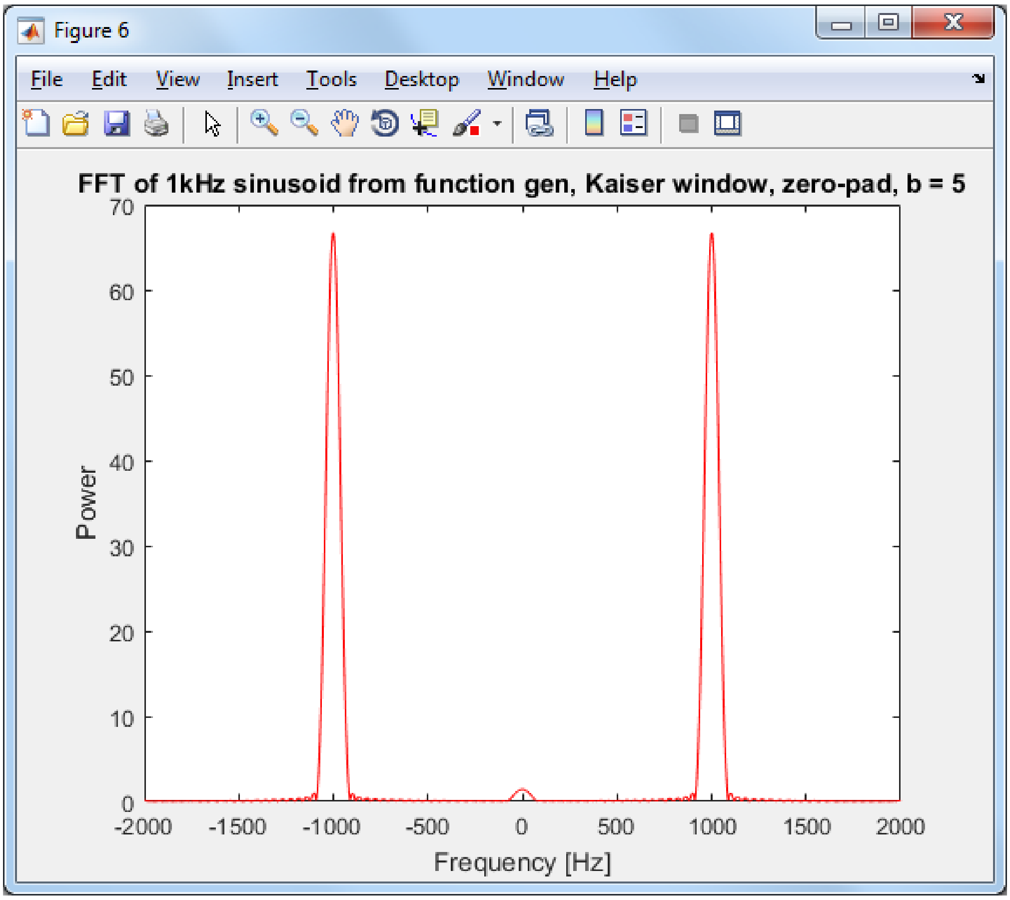


Figure 6
File Edit View Insert Tools Desktop Window Help
FFT of 1kHz sinusoid from function gen, Kaiser window, zero-pad, b = 5
Power
Frequency [Hz]
70
60
50
40
30
20
10
0
-2000 -1500 -1000 -500 0 500 1000 1500 2000

We see that the Kaiser window in the case of zero-padded signal has better amplitude resolution, compared to non-padded. Furthermore, for boxcar the line is significantly jagged compared to non-padded, but it has higher amplitude.

The conclusion that can be drawn is that windows are highly recommended for frequency analysis. Boxcar window has high amplitude sensitivity, but low frequency sensitivity compared to windows